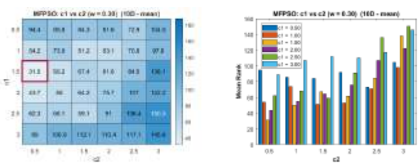
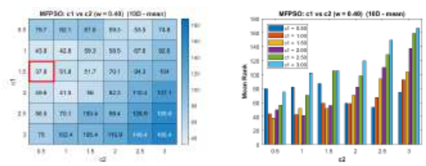
Supplementary Materials

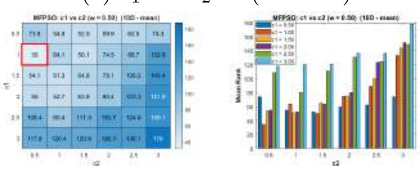
S1 Sensitivity Analysis Results



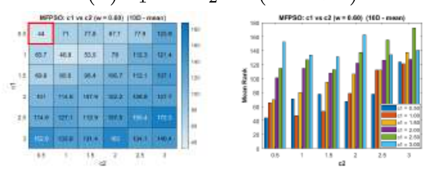
(a) c_1 vs. c_2 at $(w = 0.3)$.



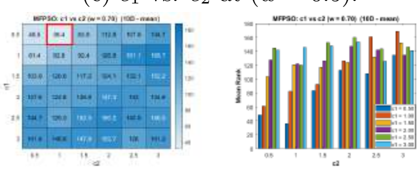
(b) c_1 vs. c_2 at $(w = 0.4)$.



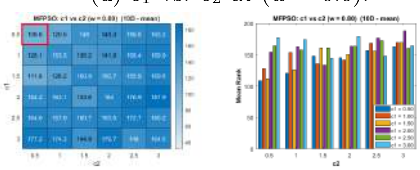
(c) c_1 vs. c_2 at $(w = 0.5)$.



(d) c_1 vs. c_2 at $(w = 0.6)$.

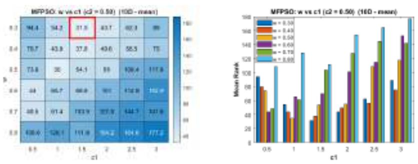


(e) c_1 vs. c_2 at $(w = 0.7)$.

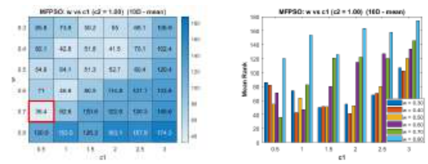


(f) c_1 vs. c_2 at $(w = 0.8)$.

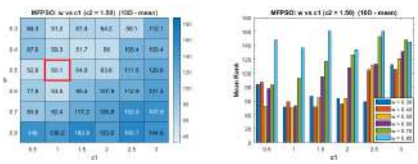
Figure S1: Heatmaps and bar charts for 10D functions (c_1 vs c_2 at fixed w).



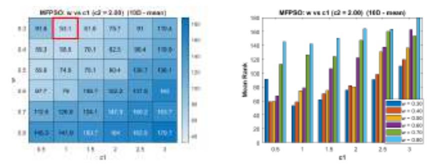
(a) c_1 vs. w at ($c_2 = 0.5$).



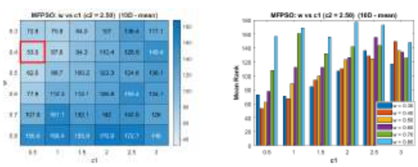
(b) c_1 vs. w at ($c_2 = 1.0$).



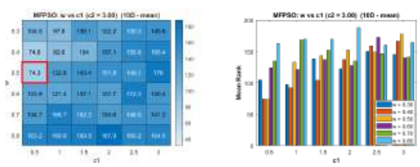
(c) c_1 vs. w at ($c_2 = 1.5$).



(d) c_1 vs. w at ($c_2 = 2.0$).

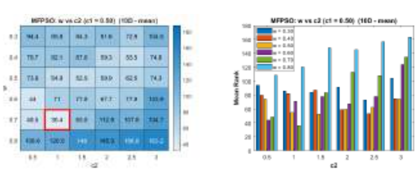


(e) c_1 vs. w at ($c_2 = 2.5$).

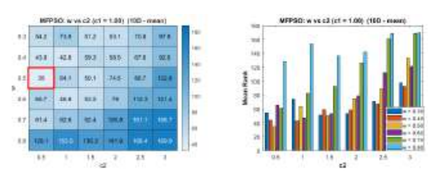


(f) c_1 vs. w at ($c_2 = 3.0$).

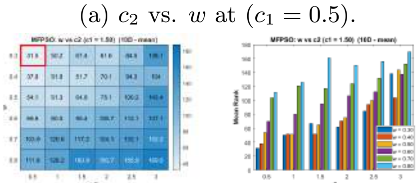
Figure S2: Heatmaps and bar charts for 10D functions(w vs c_1 at fixed c_2).



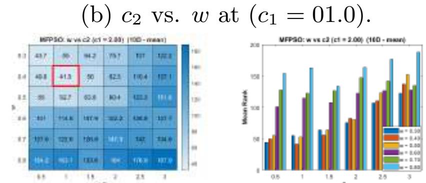
(a) c_2 vs. w at ($c_1 = 0.5$).



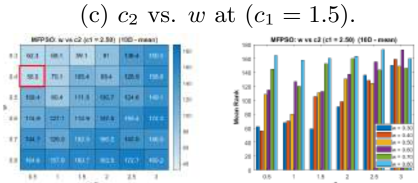
(b) c_2 vs. w at ($c_1 = 01.0$).



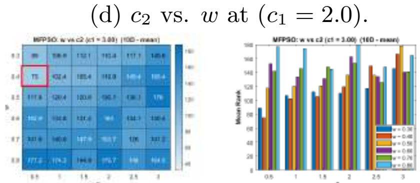
(c) c_2 vs. w at ($c_1 = 1.5$).



(d) c_2 vs. w at ($c_1 = 2.0$).

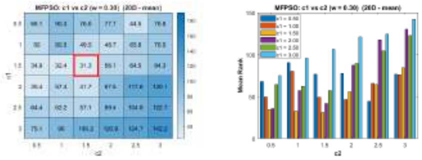


(e) c_2 vs. w at ($c_1 = 2.5$).

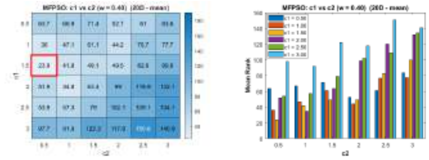


(f) c_2 vs. w at ($c_1 = 3.0$).

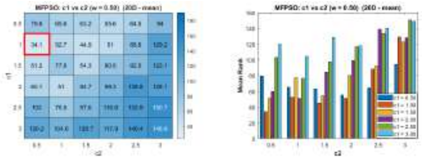
Figure S3: Heatmaps and bar charts for 10D functions (c_2 vs w at fixed c_1).



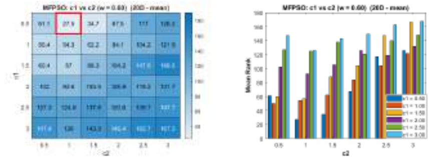
(a) c_1 vs. c_2 at $(w = 0.3)$.



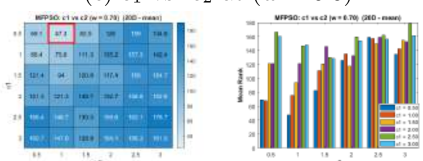
(b) c_1 vs. c_2 at $(w = 0.4)$.



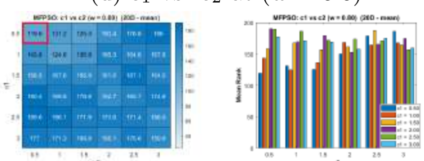
(c) c_1 vs. c_2 at $(w = 0.5)$.



(d) c_1 vs. c_2 at $(w = 0.6)$.

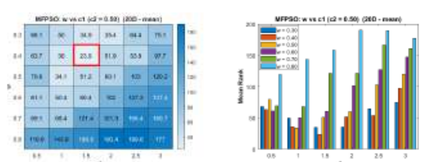


(e) c_1 vs. c_2 at $(w = 0.7)$.

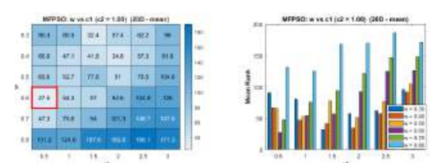


(f) c_1 vs. c_2 at $(w = 0.8)$.

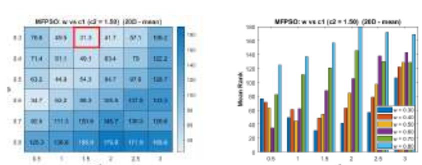
Figure S4: Heatmaps and bar charts for 20D functions (c_1 vs c_2 at fixed w).



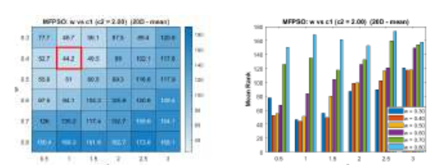
(a) c_1 vs. w at $(c_2 = 0.5)$.



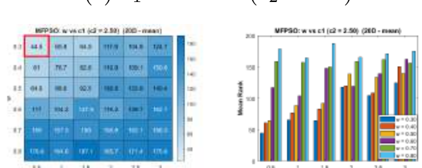
(b) c_1 vs. w at $(c_2 = 1.0)$.



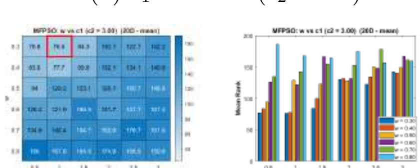
(c) c_1 vs. w at $(c_2 = 1.5)$.



(d) c_1 vs. w at $(c_2 = 2.0)$.

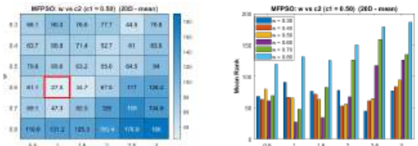


(e) c_1 vs. w at $(c_2 = 2.5)$.

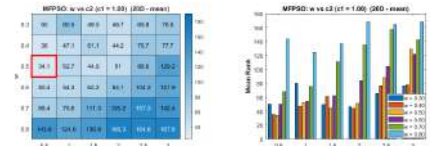


(f) c_1 vs. w at $(c_2 = 3.0)$.

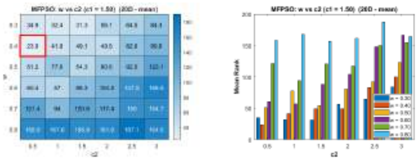
Figure S5: Heatmaps and bar charts for 20D functions (w vs c_1 at fixed c_2).



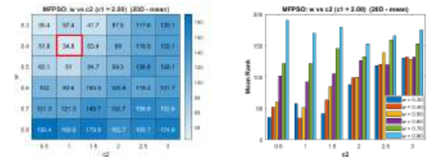
(a) c_2 vs. w at ($c_1 = 0.5$).



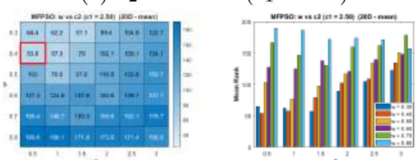
(b) c_2 vs. w at ($c_1 = 01.0$).



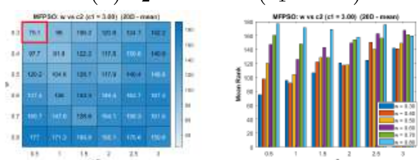
(c) c_2 vs. w at ($c_1 = 1.5$).



(d) c_2 vs. w at ($c_1 = 2.0$).



(e) c_2 vs. w at ($c_1 = 2.5$).



(f) c_2 vs. w at ($c_1 = 3.0$).

Figure S6: Heatmaps and bar charts for 20D functions (c_2 vs w at fixed c_1).

S2 Statistical Analysis Background

S2.1 Confidence interval (CI) analysis and Confidence curves

The Confidence interval (CI) analysis and the confidence curves are performed for each function, as introduced in [1] from a non-parametric and ranking perspective. A confidence interval is a range of values derived from the sample data likely to contain the true difference between the performances of two algorithms with a given confidence level (usually 95%).

The observations from the algorithms over one specific benchmark function represent the error value for each independent run. The independent runs represent unpaired observations for the two algorithms, where $n = 30$ is the number of runs. First, the differences for all the unpaired observations are calculated from Eq.(S1), where E_j^{ref} is the error of the reference algorithm (e.g. MFPSO) in the j-th run and E_i^{cur} is the error of the other algorithm in the i-th run. Then, these differences are sorted in ascending order.

$$\text{differences} = \{E_i^{cur} - E_j^{ref} \mid i = 1, \ldots, n;\ j = 1, \ldots, n\} \tag{S1}$$

The critical value K represents the number of observations you move away from the median difference to determine the lower and upper bounds of the confidence interval. In other words, K helps to identify the interval where the true difference in performance is most likely to fall, given the sampled data. K is calculated from Eq.(S2), where n is the number of samples (runs) and $z_{\alpha/2}$ is the critical value from the standard normal distribution, corresponding to the desired confidence level, typically of 95% ($\alpha = 0.05$). Finally, the CI is defined as the $k-th$ smallest difference to the $k-th$ largest difference in the n^2 sorted difference list, as in Eq.(S3).

$$K = \frac{n^2}{2} - z_{\alpha/2} \times \sqrt{\frac{n^2 \times (2n+1)}{12}} \tag{S2}$$

$$\text{CI} = [\text{differences}(\lceil K \rceil), \text{differences}(\lfloor n^2 - K \rfloor)] \tag{S3}$$

The confidence curve is a graphical representation that illustrates how the confidence interval (CI) changes across different confidence levels. The x-axis represents the range of potential differences in performance between the two algorithms, as captured by the CI at each significance level. The y-axis displays the corresponding p-value for each confidence level. The intersection of the confidence curve with the alpha level line (typically set at 0.05) identifies the confidence interval for the difference in performance between the two algorithms.

A zero-reference line, represented as a vertical dashed red line at zero on the x-axis, corresponds to the null hypothesis of no significant difference between the algorithms. The null hypothesis is rejected if the zero-reference line lies outside the confidence curve, indicating a significant difference between the two algorithms. Conversely, if the zero-reference line lies within the confidence curve, the null hypothesis cannot be rejected, suggesting that the performance of the two algorithms is similar [1].

S2.2 Convergence Trends using Page Test

Page's test is a non-parametric statistical test used to detect trends in ordered alternatives across several conditions or treatments. This test provides a statistical measure to compare two algorithms and determine if one converges faster than the other. In the Page test, the treatments in the convergence analysis refer to the different cut-points or iterations in the optimization process from the beginning to the algorithm termination, where $N = 19$ is the number of cut-points selected across the run. Each treatment represents the median error at a specific cut-point across the 30 runs. Samples correspond to the different test functions used to evaluate the algorithms, where $k = 10$ is the number of functions. Each sample is an individual function for which the algorithms' performance is measured across the various cut points [1].

First, the difference $D_{i,j}$ in the median error between two algorithms A and B is computed for each i-th function (sample) at each j-th cut-point (treatment), as in Eq.(S4), where $A_{i.j}$ and $B_{i,j}$ are the median error at the i-th function at the j-th cut-point. Second, these differences are ranked across all the cut-points for each function, where $R_{i,j}$ is the rank for the i-th function at the j-th cut-point. The largest difference gets the lowest rank (rank 1), and the smallest difference gets the highest rank (rank N) [2].

$$D_{i,j} = A_{i.j} - B_{i,j} \tag{S4}$$

The sum of ranks S_j is obtained by summing all the ranks $R_{i,j}$ for each j-th cut point across all the functions, as in Eq.(S5). Then, compute Page's Test Statistic L that represents the weighted sum for the sum ranks S_j across all the cut-points, as in Eq.(S6). The p-value is computed by the exact method from Table Q in [3] if $N \in [3, 8]$ and $k \in [2, 11]$. If the samples or the treatments have larger sizes, the p-value can be approximated using a normal distribution with a Z-score calculated using Eq.(S7). The previous steps are repeated for the two algorithms, A and B, but the difference $D_{i,j}$ is performed as $B_{i,j} - Ai, j$ instead of $A_{i.j} - B_{i,j}$. The interpretation of the results can be as follows [2]:

- If the p-value > 0.05, then the null hypothesis cannot be rejected, indicating no difference in convergence between the two algorithms.
- Case 1: In (A-B), If the p-value ≤ 0.05 and an increasing trend is detected, then B converges faster than A.
- Case 2: In (A-B), If the p-value ≤ 0.05 and a decreasing trend is detected, then A converges faster than B.
- Case 3: In (B-A), If the p-value ≤ 0.05 and an increasing trend is detected, then A converges faster than B.
- Case 4: In (B-A), If the p-value ≤ 0.05 and a decreasing trend is detected, then B converges faster than A.

$$S_j = \sum_{i=1}^{k} R_{i,j} \tag{S5}$$

$$L = \sum_{j=1}^{N} j \cdot S_j \tag{S6}$$

$$Z = \frac{12 \cdot (L - 0.5) - 3 \cdot k \cdot N \cdot (N+1)^2}{N \cdot (N+1) \cdot \sqrt{k \cdot (N-1)}} \tag{S7}$$

S2.3 Computational Time Complexity Analysis

Computation time for an algorithm is an essential measure of the algorithm's complexity. Complex algorithms consume more time and computational resources, representing crucial factors for real-time applications. Four-time metrics have been used to compare all the algorithms: T_0, T_1, T_2, and T_3. T_0 measures the speed of the machine by running a specific code snippet as a reference, which is presented in [4].

T_1 measures the average complexity of the problems using Eq.(S8), where t_f^i is the run time for calling the i-th function for 10000 times and $n = 10$ is the number of benchmark functions. T_2 measures the average algorithm run time on the benchmark functions using Eq.(S9), where t_a^i is the run time for the algorithm on the i-th function for a maximum function evaluations of 10000. T_3 is the relative complexity metric that reflects $T0$, T_1, and T_3, as in Eq.(S10)[5].

$$T_1 = \left(\sum_{i=1}^{n} t_f^i \right) / n \tag{S8}$$

$$T_2 = \left(\sum_{i=1}^{n} t_a^i \right) / n \tag{S9}$$

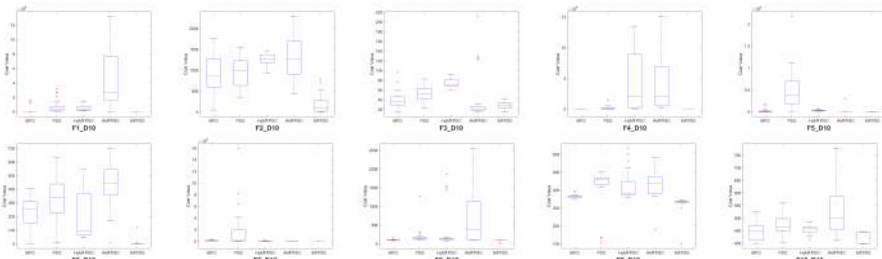
$$T_3 = \frac{(T_2 - T_1)}{T_0} \tag{S10}$$

S3 CEC2020/2021 Benchmark Results

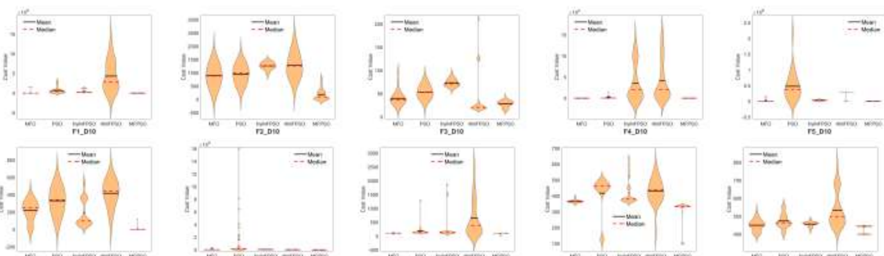
S3.1 Results Table, Box, and Violin Plots

Table S1: The best, worst, median, mean, and SD fitness results of all the algorithms in 30 runs for all the CEC2020/2021 benchmark functions for 10-Dim and 20-Dim.

Fun.	Alg.	Error (10-Dim)					Error (20-Dim)				
No.	Name	Best	Worst	Median	Mean	SD	Best	Worst	Median	Mean	SD
F1	MFO	5.23E+01	1.60E+09	9.42E+03	1.48E+08	4.24E+08	1.20E+03	9.08E+09	1.39E+09	2.47E+09	2.80E+09
	PSO	2.83E+07	3.12E+09	3.97E+08	6.84E+08	8.39E+08	3.87E+09	1.56E+10	6.48E+09	7.10E+09	2.64E+09
	HyMFPSO	1.21E+08	1.40E+09	2.61E+08	5.04E+08	4.56E+08	8.13E+08	4.25E+09	1.87E+09	2.02E+09	9.33E+08
	HMFPSO	3.99E+01	1.32E+10	2.74E+09	4.32E+09	3.76E+09	2.15E+09	5.21E+10	1.35E+10	1.56E+10	9.76E+09
	MFPSO	2.59E+01	7.51E+03	1.23E+03	1.71E+03	1.89E+03	2.86E+00	1.09E+04	1.28E+03	2.26E+03	2.72E+03
F2	MFO	4.52E+01	1.76E+03	8.75E+02	8.93E+02	4.32E+02	8.91E+02	3.92E+03	1.93E+03	2.00E+03	6.97E+02
	PSO	3.48E+02	1.56E+03	9.88E+02	9.50E+02	3.40E+02	1.22E+03	3.36E+03	2.28E+03	2.31E+03	5.59E+02
	HyMFPSO	9.28E+02	1.47E+03	1.26E+03	1.26E+03	1.39E+02	2.96E+03	4.88E+03	3.50E+03	3.58E+03	3.55E+02
	HMFPSO	4.25E+02	2.29E+03	1.26E+03	1.28E+03	4.69E+02	3.08E+03	5.34E+03	4.56E+03	4.39E+03	5.94E+02
	MFPSO	3.54E+00	7.88E+02	9.08E+01	1.83E+02	2.15E+02	1.28E+01	1.47E+03	7.18E+02	6.78E+02	3.78E+02
F3	MFO	1.54E+01	9.60E+01	3.66E+01	3.97E+01	1.67E+01	4.59E+01	2.97E+02	9.72E+01	1.13E+02	5.60E+01
	PSO	2.32E+01	8.26E+01	5.29E+01	5.32E+01	1.52E+01	9.48E+01	2.91E+02	1.65E+02	1.74E+02	4.78E+01
	HyMFPSO	6.07E+01	9.26E+01	7.11E+01	7.33E+01	8.17E+00	1.68E+02	2.58E+02	2.15E+02	2.18E+02	2.27E+01
	HMFPSO	1.54E+01	2.11E+02	2.05E+01	4.11E+01	4.87E+01	1.99E+01	5.85E+02	5.62E+01	1.30E+02	1.36E+02
	MFPSO	1.55E+01	4.09E+01	2.84E+01	2.75E+01	6.98E+00	3.01E+01	9.51E+01	4.38E+01	4.66E+01	1.37E+01
F4	MFO	6.49E-01	1.62E+01	3.09E+00	3.82E+00	3.56E+00	6.09E+00	5.61E+05	1.97E+03	2.40E+04	1.02E+05
	PSO	9.43E+00	1.43E+04	5.23E+02	1.62E+03	2.77E+03	1.04E+03	8.67E+04	7.53E+03	1.38E+04	1.75E+04
	HyMFPSO	8.67E+00	1.35E+05	2.06E+04	3.56E+04	4.44E+04	2.33E+01	1.16E+05	4.79E+01	8.56E+03	2.27E+04
	HMFPSO	1.13E+03	1.51E+05	2.06E+04	4.11E+04	4.86E+04	4.81E+04	5.72E+06	7.54E+05	9.23E+05	1.21E+06
	MFPSO	1.01E-02	2.00E+00	1.20E+00	1.15E+00	5.23E-01	8.85E-01	8.67E+00	2.85E+00	4.21E+00	2.45E+00
F5	MFO	5.37E+02	1.69E+05	7.19E+03	2.00E+04	3.89E+04	1.76E+03	2.69E+06	3.57E+05	5.50E+05	7.65E+05
	PSO	2.91E+03	2.18E+06	3.76E+05	4.79E+05	4.51E+05	1.78E+05	1.27E+07	2.20E+06	3.34E+06	3.10E+06
	HyMFPSO	4.44E+03	7.15E+04	3.00E+04	3.14E+04	1.49E+04	3.98E+05	2.88E+06	1.04E+06	1.24E+06	6.11E+05
	HMFPSO	7.01E+03	2.95E+05	9.18E+03	3.77E+04	8.73E+04	1.44E+04	3.31E+07	5.47E+05	5.03E+06	1.09E+07
	MFPSO	7.35E+01	6.28E+03	1.01E+03	1.38E+03	1.27E+03	8.32E+03	1.66E+05	5.00E+04	6.10E+04	4.56E+04
F6	MFO	2.29E+00	4.06E+02	2.51E+02	2.20E+02	1.18E+02	8.83E+01	1.09E+03	4.72E+02	5.13E+02	2.17E+02
	PSO	4.69E+00	6.34E+02	3.38E+02	3.30E+02	1.61E+02	3.94E+02	1.46E+03	8.58E+02	8.78E+02	2.94E+02
	HyMFPSO	4.30E+01	5.43E+02	9.62E+01	1.87E+02	1.59E+02	3.27E+02	9.17E+02	4.56E+02	5.06E+02	1.49E+02
	HMFPSO	6.62E+00	7.00E+02	4.41E+02	4.16E+02	1.59E+02	5.80E+02	1.99E+03	1.49E+03	1.49E+03	3.47E+02
	MFPSO	8.10E-02	1.21E+02	1.08E+00	5.35E+00	2.20E+01	9.43E-01	2.65E+01	2.42E+00	4.78E+00	6.33E+00
F7	MFO	1.39E+02	3.09E+04	2.13E+03	6.34E+03	8.41E+03	6.48E+03	3.52E+06	9.84E+04	5.44E+05	9.40E+05
	PSO	6.86E+02	1.60E+06	1.10E+04	1.63E+05	3.37E+05	1.16E+04	5.08E+06	7.12E+05	9.69E+05	1.03E+06
	HyMFPSO	1.59E+03	1.22E+04	4.60E+03	4.79E+03	2.42E+03	3.09E+04	7.90E+05	2.63E+05	2.92E+05	1.60E+05
	HMFPSO	4.18E+02	3.43E+03	1.88E+03	2.15E+03	9.48E+02	2.40E+04	2.54E+07	1.41E+06	1.91E+06	4.48E+06
	MFPSO	3.24E+00	2.97E+02	5.17E+01	7.21E+01	7.18E+01	1.33E+03	1.94E+04	9.34E+03	9.51E+03	4.96E+03
F8	MFO	5.66E+01	1.54E+02	1.02E+02	1.06E+02	1.59E+01	1.01E+02	4.31E+03	1.49E+03	1.84E+03	1.38E+03
	PSO	1.08E+02	1.27E+03	1.37E+02	1.87E+02	2.10E+02	2.69E+02	4.42E+03	1.43E+03	1.99E+03	1.29E+03
	HyMFPSO	6.64E+01	1.85E+03	1.25E+02	3.19E+02	5.12E+02	5.08E+02	5.15E+03	4.06E+03	3.96E+03	1.02E+03
	HMFPSO	1.04E+02	2.55E+03	3.84E+02	6.39E+02	6.72E+02	2.22E+02	6.59E+03	5.94E+03	5.27E+03	1.76E+03
	MFPSO	0.00E+00	1.05E+02	1.01E+02	9.22E+01	2.85E+01	1.00E+02	1.03E+02	1.01E+02	1.01E+02	9.23E-01
F9	MFO	3.48E+02	3.97E+02	3.64E+02	3.65E+02	9.97E+00	4.55E+02	5.34E+02	4.94E+02	4.92E+02	2.09E+01
	PSO	1.07E+02	5.03E+02	4.61E+02	4.17E+02	1.19E+02	7.07E+02	1.12E+03	9.24E+02	9.07E+02	1.03E+02
	HyMFPSO	3.61E+02	6.33E+02	3.79E+02	4.18E+02	7.58E+01	5.04E+02	9.77E+02	5.36E+02	5.85E+02	1.01E+02
	HMFPSO	1.77E+02	5.82E+02	4.37E+02	4.32E+02	7.80E+01	5.63E+02	1.17E+03	9.20E+02	9.06E+02	1.45E+02
	MFPSO	1.00E+02	3.46E+02	3.35E+02	3.12E+02	7.22E+01	4.11E+02	4.50E+02	4.25E+02	4.26E+02	9.29E+00
F10	MFO	3.99E+02	5.24E+02	4.51E+02	4.49E+02	3.26E+01	4.11E+02	1.02E+03	5.04E+02	5.49E+02	1.52E+02
	PSO	4.02E+02	5.60E+02	4.63E+02	4.73E+02	4.17E+01	5.74E+02	1.17E+03	7.91E+02	8.09E+02	1.65E+02
	HyMFPSO	4.15E+02	4.84E+02	4.58E+02	4.55E+02	1.47E+01	4.44E+02	6.95E+02	5.11E+02	5.25E+02	6.65E+01
	HMFPSO	4.10E+02	7.77E+02	4.98E+02	5.34E+02	9.78E+01	4.14E+02	4.14E+03	9.50E+02	1.28E+03	9.20E+02
	MFPSO	3.98E+02	4.46E+02	4.44E+02	4.23E+02	2.28E+01	4.10E+02	5.00E+02	4.61E+02	4.58E+02	3.22E+01

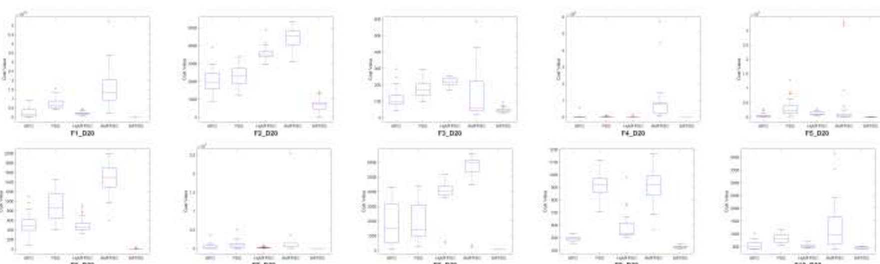


(a) Box plots.

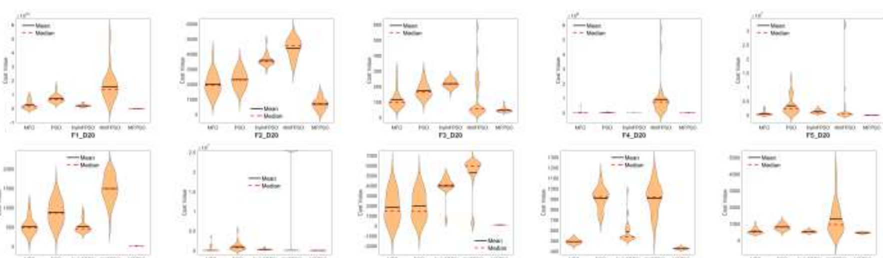


(b) Violin plots.

Figure S7: Box & violin plots in 10D CEC2020/2021 functions.



(a) Box plots.



(b) Violin plots.

Figure S8: Box & violin plots in 20D CEC2020/2021 functions.

S3.2 Confidence Curves

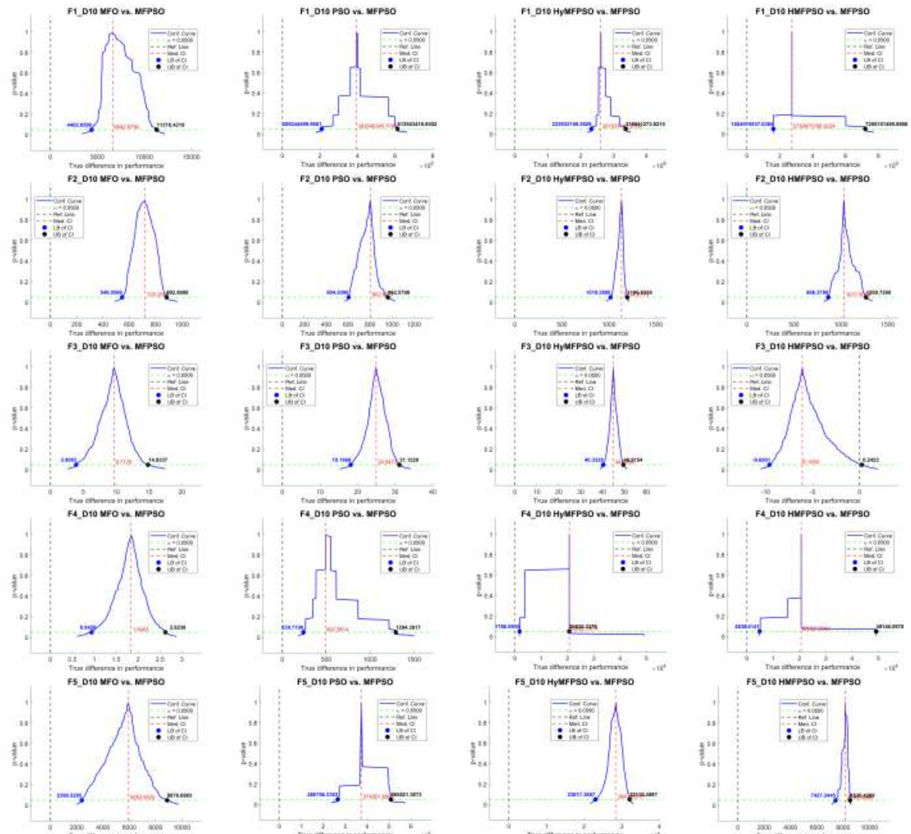


Figure S9: Confidence curves for all the 10-Dim CEC2020/2021 benchmark functions across the 30 runs for all possible significant levels, where MFPSO is compared with the MFO, PSO, HyMFPSO, and HMFPSO algorithms.

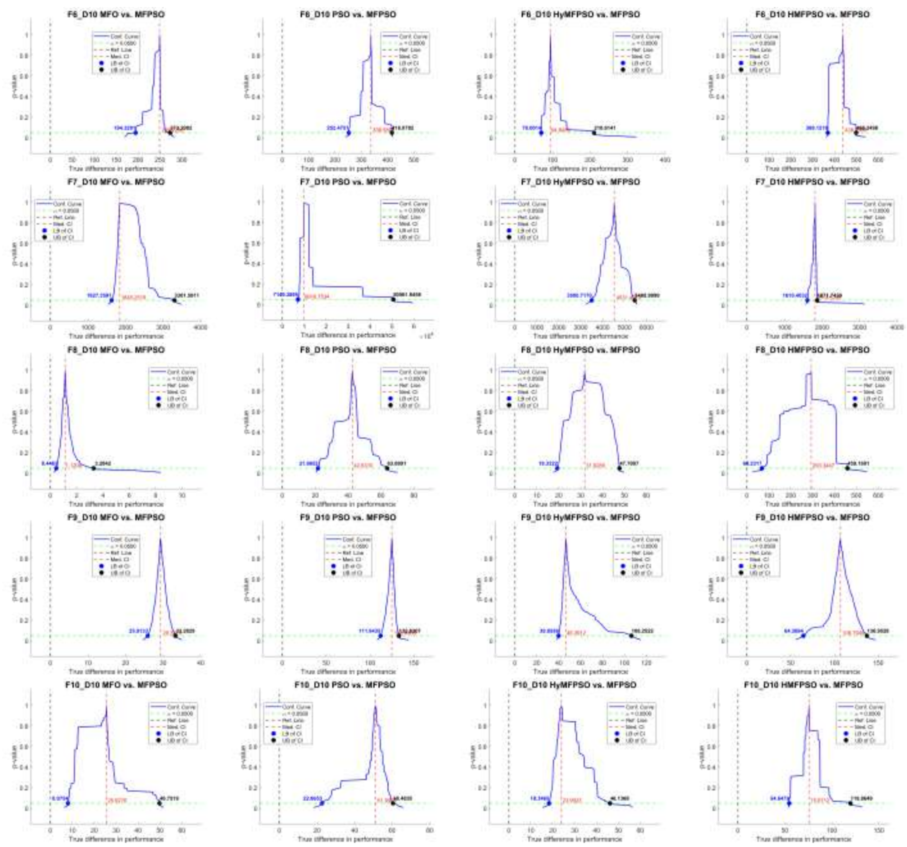


Fig. S9 (Cont.): Confidence curves for all the 10-Dim CEC2020/2021 benchmark functions across the 30 runs for all possible significant levels, where MFPSO is compared with the MFO, PSO, HyMFPSO, and HMFPSO algorithms.

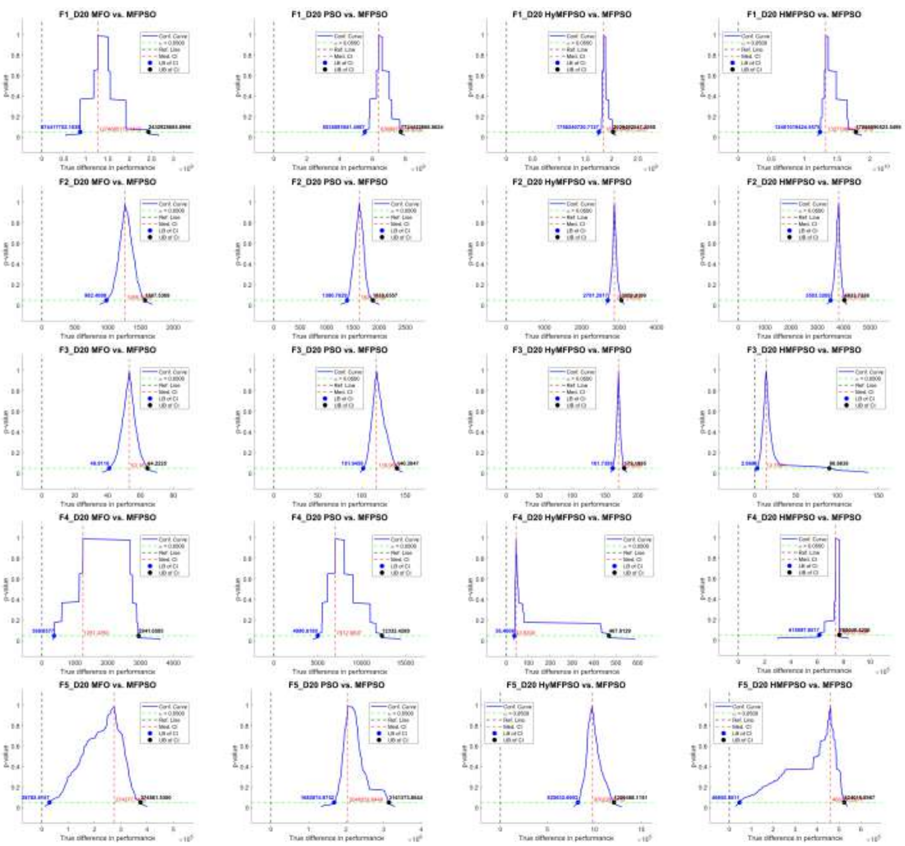


Figure S10: Confidence curves for all the 20-Dim CEC2020/2021 benchmark functions across the 30 runs for all possible significant levels, where MFPSO is compared with the MFO, PSO, HyMFPSO, and HMFPSO algorithms.

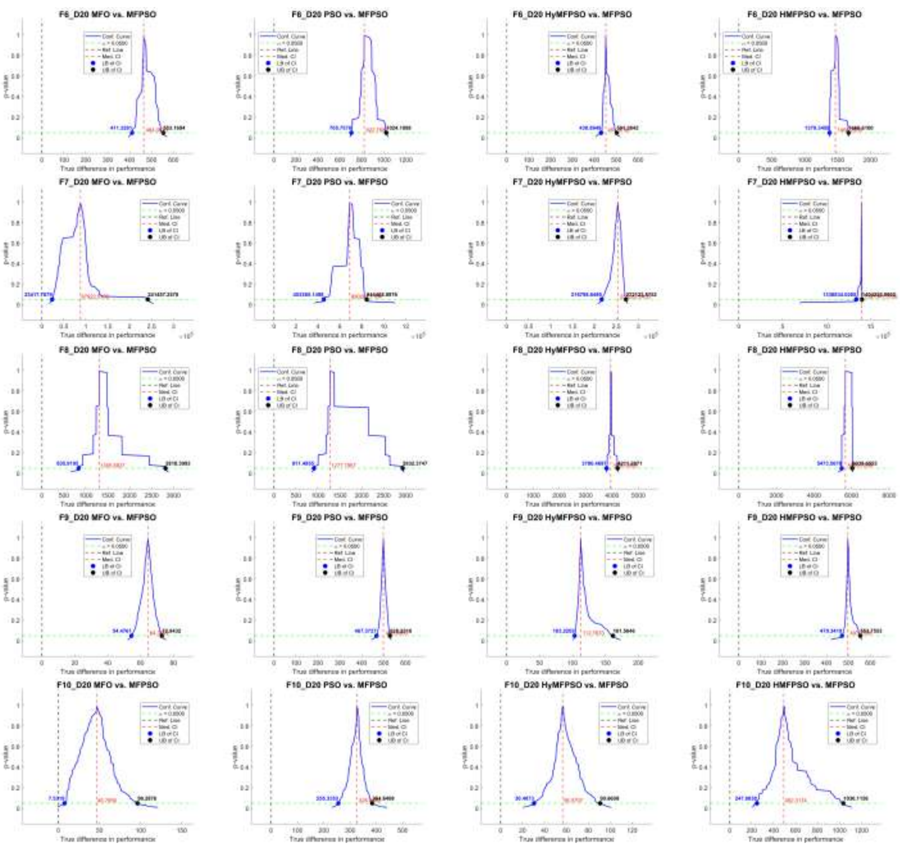
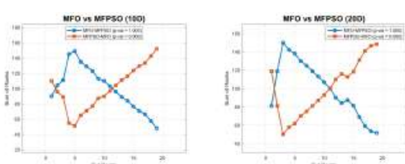


Figure S11: Fig. S10 (Cont.): Confidence curves for all the 20-Dim CEC2020/2021 benchmark functions across the 30 runs for all possible significant levels, where MFPSO is compared with the MFO, PSO, HyMFPSO, and HMFPSO algorithms.

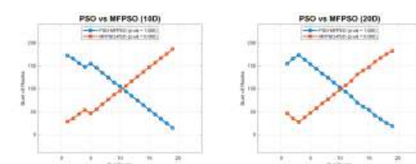
S3.3 Convergence Trends using Page Test

Table S2: The Page's test results for all the comparisons between the MFPSO algorithm and all the other algorithms. The test is performed to measure the convergence of the algorithms across all the 10 functions (samples) at different cut-points from C1 to C19. The comparisons are done twice at the difference in the median errors for the two algorithms A-B and B-A. The sum ranks for the median error differences at each cut-point across all the functions are labeled from C1 to C19.

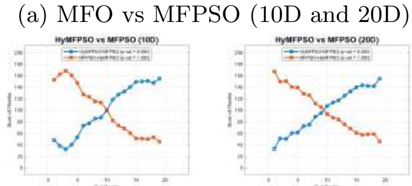
Dim	Comparison	L	p-val	C1	C3	C5	C7	C9	C11	C13	C15	C17	C19
10	MFO-MFPSO	16741	1.00E+00	90	111	149	129	113	103	89	76.5	66.5	48
	MFPSO-MFO	21259	5.30E-08	110	89	51	71	87	97	111	123.5	133.5	152
	PSO-MFPSO	13883.5	1.00E+00	172	155	154	134	113	94	74	53.5	34	14
	MFPSO-PSO	24116.5	0.00E+00	28	45	46	66	87	106	126	146.5	166	186
	HyMFPSO-MFPSO	23349.5	0.00E+00	48	32	53	77	87	118	132	149	150.5	155
	MFPSO-HyMFPSO	14650.5	1.00E+00	152	168	147	123	113	82	68	51	49.5	45
	HMFPSO-MFPSO	22485.5	1.11E-16	94	82	62	42	19.5	116	149	172.5	158.5	127
	MFPSO-HMFPSO	15514.5	1.00E+00	106	118	138	158	180.5	84	51	27.5	41.5	73
20	MFO-MFPSO	16489	1.00E+00	81	150	138	125	113	100	84	81.5	59	51.5
	MFPSO-MFO	21511	1.72E-09	119	50	62	75	87	100	116	118.5	141	148.5
	PSO-MFPSO	13831.5	1.00E+00	154	173	153	132.5	113	93	69	53.5	33.5	18
	MFPSO-PSO	24168.5	0.00E+00	46	27	47	67.5	87	107	131	146.5	166.5	182
	HyMFPSO-MFPSO	22820.5	0.00E+00	33	50	61	74.5	95	113	125	139.5	142	154.5
	MFPSO-HyMFPSO	15179.5	1.00E+00	167	150	139	125.5	105	87	75	60.5	58	45.5
	HMFPSO-MFPSO	21717	8.08E-11	108	88	66	44.5	25	131	142.5	156.5	142	124.5
	MFPSO-HMFPSO	16283	1.00E+00	92	112	134	155.5	175	69	57.5	43.5	58	75.5



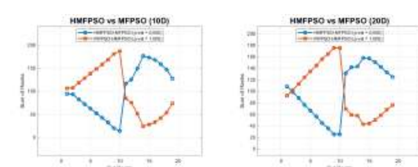
(a) MFO vs MFPSO (10D and 20D).



(b) PSO vs MFPSO (10D and 20D).



(c) HyMFPSO vs MFPSO (10D and 20D).



(d) HMFPSO vs MFPSO (10D and 20D).

Figure S12: Convergence trends between the algorithms using the Page's test.

S4 Engineering Optimization Benchmark Problem Definitions [6]

S4.1 F1: Speed Reducer

$$
\begin{aligned}
\text{Minimize:}\quad & f(X) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) \\
\text{Subject to:}\quad & g_1(X) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0, \\
& g_2(X) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0, \\
& g_3(X) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0, \\
& g_4(X) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leq 0, \\
& g_5(X) = \frac{\sqrt{745x_4/x_2x_3}^2 + 16.9 \times 10^6}{110x_6^3} - 1 \leq 0, \\
& g_6(X) = \frac{\sqrt{745x_5/x_2x_3}^2 + 157.5 \times 10^6}{85x_7^3} - 1 \leq 0, \\
& g_7(X) = \frac{x_2x_3}{40} - 1 \leq 0, \\
& g_8(X) = \frac{5x_2}{x_1} - 1 \leq 0, \\
& g_9(X) = \frac{x_1}{12x_2} - 1 \leq 0, \\
& g_{10}(X) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0, \\
& g_{11}(X) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0, \\
\text{Variable Range:}\quad & 2.6 \leq x_1 \leq 3.6, \\
& 0.7 \leq x_2 \leq 0.8, \\
& x_3 \in \{17, 18, 19, \ldots, 28\}, \\
& 7.3 \leq x_4, \\
& x_5 \leq 8.3, \\
& 2.9 \leq x_6 \leq 3.9, \\
& 5 \leq x_7 \leq 5.5.
\end{aligned}
\tag{S11}
$$

Where: x_1 : Face width (b),
x_2 : Module of teeth (m),
x_3 : Number of teeth in the pinion (z),
x_4 : Length of the first shaft between bearings (l_1),
x_5 : Length of the second shaft between bearings (l_2),
x_6 : Diameter of the first shaft (d_1),
x_7 : Diameter of the second shaft (d_2).

S4.2 F2: Design of Tension/Compression Spring

$$
\begin{aligned}
&\text{Minimize:} \quad f(X) = (x_3+2)x_2x_1^2,\\
&\text{subject to:}\\
&g_1(X) = 1 - \frac{x_3x_2^3}{71785x_1^4} \leq 0,\\
&g_2(X) = \frac{4x_2^2 - x_1x_2}{12566(x_3x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0,\\
&g_3(X) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0,\\
&g_4(X) = \frac{x_1+x_2}{1.5} - 1 \leq 0,\\
&\text{variable range:}\\
&0.05 \leq x_1 \leq 2,\\
&0.25 \leq x_2 \leq 1.3,\\
&2 \leq x_3 \leq 15,
\end{aligned}
\tag{S12}
$$

where:
x_1 is the mean coil diameter (D),
x_2 is the wire diameter (d),
x_3 is the number of active coils (N).

S4.3 F3: Design of Pressure Vessel

$$
\begin{aligned}
&\text{Minimize:}\quad f(X) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1x_3^3,\\
&\text{subject to:}\\
&g_1(X) = -x_1 + 0.0193x_3 \le 0,\\
&g_2(X) = -x_2 + 0.00954x_3 \le 0,\\
&g_3(X) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1,296,000 \le 0,\\
&g_4(X) = x_4 - 240 \le 0,\\
&\text{variable range:}\\
&x_1, x_2 \in \{1 \times 0.0625, 2 \times 0.0625, 3 \times 0.0625, \ldots, 1600 \times 0.0625\},\\
&10 \le x_3,\\
&x_4 \le 200,\\
&\text{where:}\\
&x_1 \text{ is the thickness of the shell } (T_s),\\
&x_2 \text{ is the thickness of the head } (T_h),\\
&x_3 \text{ is the inner radius } (R),\\
&x_4 \text{ is the length of the cylindrical section of the vessel } (L).
\end{aligned}
\tag{S13}
$$

S4.4 F4: Three-Bar Truss Design Problem

$$
\begin{aligned}
&\text{Minimize:}\quad f(X) = (2\sqrt{2}\,x_1 + x_2)\cdot l,\\
&\text{Subject to:}\\
&\quad g_1(X) = \frac{\sqrt{2}\,x_1 + x_2}{\sqrt{2}\,x_1^2 + 2x_1x_2}P - \sigma \le 0,\\
&\quad g_2(X) = \frac{x_2}{\sqrt{2}\,x_1^2 + 2x_1x_2}P - \sigma \le 0,\\
&\quad g_3(X) = \frac{1}{\sqrt{2}\,x_2 + x_1}P - \sigma \le 0,\\
&\text{Variable range:}\quad 0 \le x_1, \quad x_2 \le 1.\\
&\text{where:}\\
&\quad x_1 \text{ : Cross-sectional area of the first bar } (A_1),\\
&\quad x_2 \text{ : Cross-sectional area of the second bar } (A_2),\\
&\quad l \text{ : Length of the bars (100 cm)},\\
&\quad P \text{ : Load applied per unit area (2 kN/cm}^3\text{)},\\
&\quad \sigma \text{ : Allowable stress (2 kN/cm}^3\text{)}
\end{aligned}
\tag{S14}
$$

S4.5 F5: Design of Gear Train

$$
\begin{aligned}
&\text{Minimize:} \quad f(X) = \left(\frac{1}{6.931} - \frac{x_3 x_2}{x_1 x_4}\right)^2, \\
&\text{Subject to:} \quad \text{(No explicit constraints are provided for this problem.)} \\
&\text{Variable range:} \quad x_1, x_2, x_3, x_4 \in \{12, 13, 14, \ldots, 60\}, \\
&\text{Where:} \\
&x_1 : \text{Number of teeth of gear } n_A, \\
&x_2 : \text{Number of teeth of gear } n_B, \\
&x_3 : \text{Number of teeth of gear } n_C, \\
&x_4 : \text{Number of teeth of gear } n_D.
\end{aligned}
\tag{S15}
$$

S4.6 F6: Cantilever Beam Problem

$$
\begin{aligned}
&\text{minimize:} \quad f(X) = 0.0624(x_1 + x_2 + x_3 + x_4 + x_5), \\
&\text{subject to:} \quad g(X) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \le 0, \\
&\text{variable range:} \quad 0.01 \le x_i \le 100, \quad i = 1, \ldots, 5.
\end{aligned}
\tag{S16}
$$

S4.7 F7: Optimal Design of I-Shaped Beam

$$
\begin{aligned}
&\text{Minimize:} \\
&f(X) = \frac{5000}{x_3(x_2 - 2x_4)^3/12 + (x_1 x_4^3/6) + 2b x_4 (x_2 - x_4/2)^2}, \\
&\text{Subject to:} \\
&g_1(X) = 2x_1 x_3 + x_3(x_2 - 2x_4) \le 300, \\
&g_2(X) = \frac{18 x_2 \times 10^4}{x_3(x_2 - 2x_4)^3 + 2x_1 x_3 (4x_2 + 3x_2(x_2 - 2x_4))} + \frac{15 x_1 \times 10^3}{(x_2 - 2x_4)x_3^2 + 2x_3 x_1} \le 56, \\
&\text{Variable range:} \\
&10 \le x_1 \le 50, \quad 10 \le x_2 \le 80, \quad 0.9 \le x_3 \le 5, \quad 0.9 \le x_4 \le 5, \\
&\text{where:} \\
&x_1\text{: Width of the flange } (b), \\
&x_2\text{: Height of the section } (h), \\
&x_3\text{: Thickness of the web } (t_w), \\
&x_4\text{: Thickness of the flange } (t_f).
\end{aligned}
\tag{S17}
$$

S4.8 F8: Tubular Column Design

$$
\begin{aligned}
&\text{minimize:}\\
&f(X) = 9.8x_1x_2 + 2x_1,\\
&\text{subject to:}\\
&g_1(X) = \frac{P}{\pi x_1 x_2 \sigma_y} - 1 \leq 0,\\
&g_2(X) = \frac{8PL^2}{\pi^3 E x_1 x_2 (x_1^2 + x_2^2)} - 1 \leq 0,\\
&g_3(X) = \frac{2.0}{x_1} - 1 \leq 0,\\
&g_4(X) = \frac{x_1}{14} - 1 \leq 0,\\
&g_5(X) = \frac{0.2}{x_2} - 1 \leq 0,\\
&g_6(X) = \frac{x_2}{8} - 1 \leq 0,\\
&\text{variable range:}\\
&2 \leq x_1 \leq 14,\\
&0.2 \leq x_2 \leq 0.8,\\
&\text{where:}\\
&x_1 : \text{Mean diameter of the column } (d),\\
&x_2 : \text{Thickness of the tube } (t).
\end{aligned}
\tag{S18}
$$

S4.9 F9: Piston Lever

$$
\begin{aligned}
\text{Minimize:}\quad & f(X) = \frac{1}{4}\pi x_3^2 (L_2 - L_1),\\
\text{Subject to:}\quad & g_1(X) = QL\cos\theta - R \times F \leq 0,\\
& g_2(X) = Q(L - x_4) - M_{\max} \leq 0,\\
& g_3(X) = 1.2(L_2 - L_1) - L_1 \leq 0,\\
& g_4(X) = \frac{x_3}{2} - x_2 \leq 0,\\
\text{Where:}\quad & R = \left| \frac{-x_4(x_4\sin\theta + x_1) + x_1(x_2 - x_4\cos\theta)}{\sqrt{(x_4 - x_2)^2 + x_1^2}} \right|,\\
& F = \frac{\pi P x_3^2}{4},\\
& L_1 = \sqrt{(x_4 - x_2)^2 + x_1^2},\\
& L_2 = \sqrt{(x_4\sin\theta + x_1)^2 + (x_2 - x_4\cos\theta)^2}.
\end{aligned}
\tag{S19}
$$

$$\text{Constants:}\quad \begin{aligned} &\theta = 45^\circ, \\ &Q = 10,000 \text{ lbs}, \\ &L = 240 \text{ in}, \\ &M_{\max} = 1.8 \times 10^6 \text{ lbs in}, \\ &P = 1500 \text{ psi}. \end{aligned}$$

$$\text{Variable range:}\quad \begin{aligned} 0.05 \le x_1, x_2, x_4 \le 500, \\ 0.05 \le x_3 \le 120. \end{aligned}$$

$$\text{Where:}\quad \begin{aligned} &x_1 : \text{Horizontal location of the piston component (H)}, \\ &x_2 : \text{Vertical location of the piston component (B)}, \\ &x_3 : \text{Diameter of the piston rod (D)}, \\ &x_4 : \text{Horizontal displacement of the piston end (X)}. \end{aligned}$$

S4.10 F10: Corrugated Bulkhead Design

$$\begin{aligned}
&\text{Minimize:}\quad f(X) = \frac{5.885x_4(x_1 + x_3)}{x_1 + \sqrt{x_3^2 - x_2^2}}, \\
&\text{subject to:} \\
&g_1(X) = -x_4x_2\left(0.4x_1 + \frac{x_3}{6}\right) + 8.94\left(x_1 + \sqrt{x_3^2 - x_2^2}\right) \le 0, \\
&g_2(X) = -x_4x_2\left(0.2x_1 + \frac{x_3}{12}\right) + 2.2\left(8.94\left(x_1 + \sqrt{x_3^2 - x_2^2}\right)\right)^{4/3} \le 0, \\
&g_3(X) = -x_4 + 0.0156x_1 + 0.15 \le 0, \\
&g_4(X) = -x_4 + 0.0156x_3 + 0.15 \le 0, \\
&g_5(X) = -x_4 + 1.05 \le 0, \\
&g_6(X) = -x_3 + x_2 \le 0, \\
&\text{variable range:}\quad 0 \le x_1, x_2, x_3 \le 100, \quad 0 \le x_4 \le 5, \\
&\text{where:} \\
&x_1 \text{ is the width}, \\
&x_2 \text{ is the depth}, \\
&x_3 \text{ is the length}, \\
&x_4 \text{ is the plate thickness}.
\end{aligned} \tag{S20}$$

S4.11 F11: Car Side Impact Design

minimize: $f(X) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7,$

subject to:

$$
\begin{aligned}
g_1(X) &= 1.16 - 0.3717x_2x_4 - 0.00931x_3x_{10} - 0.484x_5x_9 + 0.01343x_6x_{10} - 1 \leq 0,\\
g_2(X) &= 46.36 - 9.9x_2 - 12.9x_1x_2 + 0.1107x_3x_{10} - 32 \leq 0,\\
g_3(X) &= 33.86 + 2.95x_3 + 0.1792x_3 - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10}\\
&\quad - 9.98x_9x_6 + 22.0x_3x_9 - 32 \leq 0,\\
g_4(X) &= 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_3x_{10} + 6.63x_9x_8 - 7.77x_3x_9\\
&\quad + 0.32x_6x_9 - 32 \leq 0,\\
g_5(X) &= 0.261 - 0.0159x_1 - 0.188x_2 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.000875x_6x_9\\
&\quad + 0.00139x_8x_{11} + 0.0000157x_{10}x_{11} - 0.32 \leq 0,\\
g_6(X) &= 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7\\
&\quad + 0.0208x_3x_9 + 0.121x_3x_6 - 0.00364x_5x_6 + 0.0007715x_5x_9\\
&\quad - 0.000535x_4x_6x_9 + 0.00121x_3x_{11} + 0.00184x_6x_{11} - 0.022x_2 - 0.32 \leq 0,\\
g_7(X) &= 0.74 - 0.61x_2 - 0.163x_3x_8 + 0.00123x_3x_{10} - 0.166x_7 + 0.227x_2^2 - 0.32 \leq 0,\\
g_8(X) &= 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2 - 4 \leq 0,\\
g_9(X) &= 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_9 - 9.9 \leq 0,\\
g_{10}(X) &= 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} - 0.000786x_{11} - 15.7 \leq 0,
\end{aligned}
$$

variable range:

$0.5 \leq x_1, x_2, x_3, x_4, x_5, x_6, x_7 \leq 1.5,$

$x_8, x_9 \in \{0.192, 0.345\},$

$-30 \leq x_{10},$

$x_{11} \leq 30,$

where:

x_1 : Thickness of B-pillar inner,

x_2 : Thickness of B-pillar reinforcement,

x_3 : Thickness of floor side inner,

x_4 : Thickness of cross members,

x_5 : Thickness of door beam,

x_6 : Thickness of door beltline reinforcement,

x_7 : Thickness of roof rail,

x_8 : Material of B-pillar inner,

x_9 : Material of floor side inner,

x_{10} : Barrier height,

x_{11} : Hitting position.

(S21)

S4.12 F12: Design of Welded Beam

$$
\begin{aligned}
&\text{Minimize:} \quad f(X) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2),\\
&\text{Subject to:}\\
&g_1(X) = \tau(x) - \tau_{\max} \le 0,\\
&g_2(X) = \sigma(x) - \sigma_{\max} \le 0,\\
&g_3(X) = \delta(x) - \delta_{\max} \le 0,\\
&g_4(X) = x_1 - x_4 \le 0,\\
&g_5(X) = P - P_c(x) \le 0,\\
&g_6(X) = 0.125 - x_1 \le 0,\\
&g_7(X) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \le 0,\\
&\text{where:}\\
&\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{R} + (\tau'')^2}, \quad \tau' = \frac{P}{\sqrt{2}x_1x_2}, \quad \tau'' = \frac{MR}{J},\\
&M = P\left(L + \frac{x_2}{2}\right), \quad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},\\
&J = 2\left[x_1x_2\sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}\right],\\
&\sigma(x) = \frac{6PL}{x_4x_3^2}, \quad \delta(x) = \frac{6PL^3}{Ex_3x_4^3},\\
&P_c(x) = \frac{4.013E\sqrt{x_3^2x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),
\end{aligned}
\tag{S22}
$$

$$
\text{Constants:} \quad
\begin{array}{l}
P = 6000\,\text{lb}, \quad L = 14\,\text{in}, \quad \delta_{\max} = 0.25\,\text{in},\\
E = 30 \times 10^6\,\text{psi}, \quad G = 12 \times 10^6\,\text{psi},\\
\tau_{\max} = 13,600\,\text{psi}, \quad \sigma_{\max} = 30,000\,\text{psi}.
\end{array}
$$

$$
\text{Variable ranges:} \quad
\begin{array}{l}
0.1 \le x_1, \quad x_4 \le 2,\\
0.1 \le x_2, \quad x_3 \le 10.
\end{array}
$$

S4.13 F13: A Reinforced Concrete Beam Design

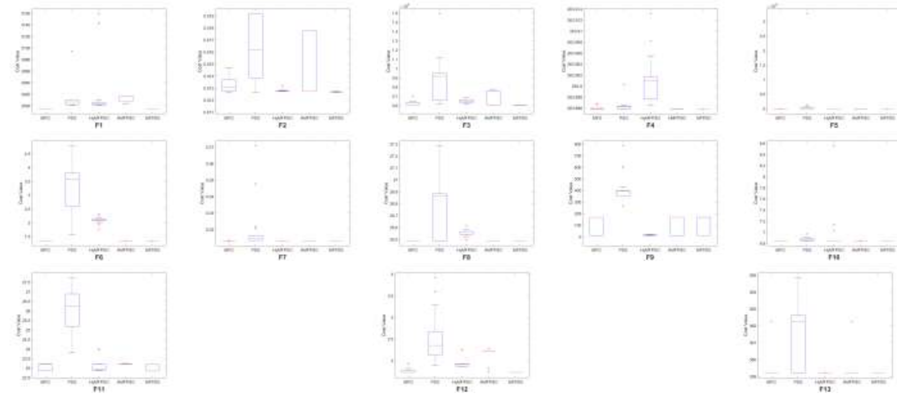
$$
\begin{aligned}
&\text{Minimize:} \quad f(X) = 2.9x_1 + 0.6x_2x_3,\\
&\text{Subject to:}\\
&g_1(X) = \frac{x_2}{x_3} - 4 \le 0,\\
&g_2(X) = 180 + 7.375\frac{x_1^2x_2}{x_3} - x_1x_2 \le 0,\\
&\text{Variable Range:}\\
&x_1 \in \{6, 6.16, 6.32, 6.6, 7, 7.11, 7.2, 7.8, 7.9, 8, 8.4\},\\
&x_2 \in \{28, 29, 30, \ldots, 40\},\\
&5 \le x_3 \le 10.\\
&\text{Where:}\\
&x_1 : \text{Area of the reinforcement } (A_s),\\
&x_2 : \text{Width of the beam } (b),\\
&x_3 : \text{Depth of the beam } (h).
\end{aligned}
\tag{S23}
$$

S5 Engineering Optimization Benchmark Results

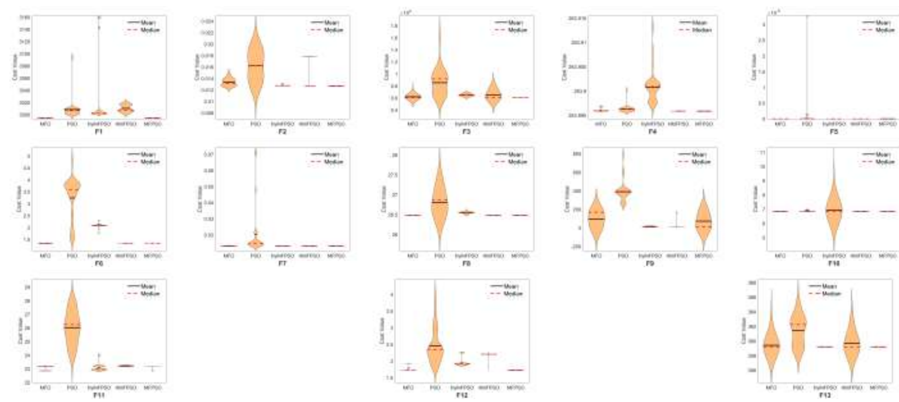
S5.1 Results Table and Box Plots

Table S3: The best, worst, median, mean, and SD fitness results of all the algorithms in 30 runs for all the engineering optimization benchmark problems.

FNo.	Alg.	Best	Worst	Median	Mean	SD
F1	MFO	2.9944E+03	2.9944E+03	2.9944E+03	2.9944E+03	4.6252E-13
	PSO	3.0003E+03	3.0939E+03	3.0065E+03	3.0083E+03	1.6589E+01
	HyMFPSO	3.0006E+03	3.1594E+03	3.0023E+03	3.0280E+03	5.6839E+01
	HMFPSO	3.0038E+03	3.0167E+03	3.0074E+03	3.0103E+03	5.5335E+00
	MFPSO	2.9944E+03	2.9944E+03	2.9944E+03	2.9944E+03	4.6252E-13
F2	MFO	1.2674E-02	1.4694E-02	1.3114E-02	1.3322E-02	6.2680E-04
	PSO	1.2680E-02	1.9194E-02	1.6186E-02	1.6208E-02	2.5114E-03
	HyMFPSO	1.2734E-02	1.3193E-02	1.2754E-02	1.2790E-02	1.1078E-04
	HMFPSO	1.2719E-02	1.7773E-02	1.2719E-02	1.4741E-02	2.5183E-03
	MFPSO	1.2668E-02	1.2718E-02	1.2696E-02	1.2696E-02	1.4521E-05
F3	MFO	6.0696E+03	7.0478E+03	6.0696E+03	6.1817E+03	2.1487E+02
	PSO	6.2224E+03	1.5942E+04	9.1701E+03	8.5250E+03	2.1378E+03
	HyMFPSO	6.1236E+03	6.8782E+03	6.4615E+03	6.4746E+03	2.2373E+02
	HMFPSO	6.0597E+03	7.6663E+03	6.0696E+03	6.5307E+03	7.1836E+02
	MFPSO	6.0597E+03	6.0905E+03	6.0597E+03	6.0690E+03	1.4354E+01
F4	MFO	2.6390E+02	2.6390E+02	2.6390E+02	2.6390E+02	3.6912E-04
	PSO	2.6390E+02	2.6390E+02	2.6390E+02	2.6390E+02	7.9025E-04
	HyMFPSO	2.6390E+02	2.6391E+02	2.6390E+02	2.6390E+02	3.3989E-03
	HMFPSO	2.6390E+02	2.6390E+02	2.6390E+02	2.6390E+02	1.0761E-05
	MFPSO	2.6390E+02	2.6390E+02	2.6390E+02	2.6390E+02	3.9683E-08
F5	MFO	1.1661E-10	2.7265E-08	8.8876E-10	3.6552E-09	5.5822E-09
	PSO	1.5450E-10	3.2726E-05	1.1173E-08	2.3125E-06	8.2720E-06
	HyMFPSO	2.7009E-12	1.3616E-09	1.1661E-10	3.8470E-10	4.4776E-10
	HMFPSO	2.7009E-12	2.7265E-08	9.9216E-10	8.2969E-09	1.1813E-08
	MFPSO	2.7009E-12	9.7457E-10	2.7009E-12	1.7011E-10	3.6602E-10
F6	MFO	1.3401E+00	1.3407E+00	1.3403E+00	1.3403E+00	2.2551E-04
	PSO	1.5781E+00	4.7939E+00	3.5699E+00	3.2355E+00	8.7332E-01
	HyMFPSO	1.7726E+00	2.3088E+00	2.0724E+00	2.0728E+00	1.0679E-01
	HMFPSO	1.3401E+00	1.3420E+00	1.3403E+00	1.3406E+00	7.5462E-04
	MFPSO	1.3400E+00	1.3400E+00	1.3400E+00	1.3400E+00	1.3427E-06
F7	MFO	1.3074E-02	1.3302E-02	1.3074E-02	1.3085E-02	4.2012E-05
	PSO	1.3365E-02	7.0917E-02	1.4742E-02	2.0190E-02	1.5212E-02
	HyMFPSO	1.3074E-02	1.3075E-02	1.3074E-02	1.3074E-02	2.0764E-07
	HMFPSO	1.3074E-02	1.3074E-02	1.3074E-02	1.3074E-02	8.8219E-18
	MFPSO	1.3074E-02	1.3074E-02	1.3074E-02	1.3074E-02	8.8219E-18
F8	MFO	2.6486E+01	2.6486E+01	2.6486E+01	2.6486E+01	7.2269E-15
	PSO	2.6486E+01	2.7287E+01	2.6862E+01	2.6805E+01	3.1232E-01
	HyMFPSO	2.6494E+01	2.6612E+01	2.6544E+01	2.6546E+01	2.7160E-02
	HMFPSO	2.6486E+01	2.6486E+01	2.6486E+01	2.6486E+01	7.2269E-15
	MFPSO	2.6486E+01	2.6486E+01	2.6486E+01	2.6486E+01	7.2269E-15
F9	MFO	8.4127E+00	1.6747E+02	1.6747E+02	9.3245E+01	8.0710E+01
	PSO	2.6488E+02	7.8712E+02	3.9277E+02	3.8933E+02	1.0994E+02
	HyMFPSO	9.8160E+00	2.0653E+01	1.3998E+01	1.3754E+01	3.2448E+00
	HMFPSO	8.4127E+00	1.6749E+02	1.6747E+02	9.8548E+01	8.0168E+01
	MFPSO	8.4127E+00	1.6747E+02	8.4127E+00	7.2037E+01	7.9255E+01
F10	MFO	6.8430E+00	6.8430E+00	6.8430E+00	6.8430E+00	4.5168E-15
	PSO	6.8430E+00	6.9687E+00	6.8547E+00	6.8727E+00	3.9219E-02
	HyMFPSO	6.8430E+00	8.5601E+00	6.8430E+00	6.9353E+00	3.2046E-01
	HMFPSO	6.8430E+00	6.8451E+00	6.8430E+00	6.8431E+00	4.5385E-04
	MFPSO	6.8430E+00	6.8430E+00	6.8430E+00	6.8430E+00	4.5168E-15
F11	MFO	2.2845E+01	2.3196E+01	2.3185E+01	2.3053E+01	1.7257E-01
	PSO	2.3800E+01	2.7743E+01	2.6241E+01	2.6019E+01	1.2050E+00
	HyMFPSO	2.2882E+01	2.3986E+01	2.2955E+01	2.3090E+01	2.7736E-01
	HMFPSO	2.3196E+01	2.3245E+01	2.3196E+01	2.3209E+01	1.7181E-02
	MFPSO	2.2843E+01	2.3185E+01	2.2844E+01	2.2946E+01	1.5894E-01
F12	MFO	1.7249E+00	1.9286E+00	1.7250E+00	1.7666E+00	7.7139E-02
	PSO	1.8816E+00	3.9196E+00	2.3491E+00	2.4502E+00	5.2440E-01
	HyMFPSO	1.8695E+00	2.2511E+00	1.9166E+00	1.9499E+00	1.2216E-01
	HMFPSO	1.7286E+00	2.2821E+00	2.2040E+00	2.1650E+00	1.2992E-01
	MFPSO	1.7249E+00	1.7249E+00	1.7249E+00	1.7249E+00	6.7752E-16
F13	MFO	3.5921E+02	3.6225E+02	3.5921E+02	3.5941E+02	7.7178E-01
	PSO	3.5921E+02	3.6485E+02	3.6225E+02	3.6142E+02	1.9686E+00
	HyMFPSO	3.5921E+02	3.5922E+02	3.5921E+02	3.5921E+02	1.8351E-03
	HMFPSO	3.5921E+02	3.6225E+02	3.5921E+02	3.5972E+02	1.1531E+00
	MFPSO	3.5921E+02	3.5921E+02	3.5921E+02	3.5921E+02	5.7815E-14



(a) Box plots.



(b) Violin plots.

Figure S13: Box & violin plots for all the algorithms for the Engineering Optimization Problems.

S5.2 Confidence Interval and Confidence Curves

Table S4: The confidence interval (CI) of the mean results for all the engineering optimization benchmark problems. The MFPSO algorithm is set as a reference in the unpaired comparisons across the 30 independent runs.

Fn.	MFO			PSO			HyMFPSO			HMFPSO		
	LB	UB	pCI	LB	UB	pCI	LB	UB	pCI	LB	UB	pCI
F1	0.00E+00	0.00E+00	FALSE	7.50E+00	1.45E+01	TRUE	7.83E+00	1.13E+01	TRUE	1.30E+01	2.23E+01	TRUE
F2	5.33E-05	1.01E-03	TRUE	2.02E-03	5.19E-03	TRUE	5.34E-05	7.38E-05	TRUE	2.55E-05	5.06E-03	TRUE
F3	9.87E+00	3.07E+01	TRUE	6.85E+02	3.41E+03	TRUE	3.24E+02	4.02E+02	TRUE	9.87E+00	9.87E+00	TRUE
F4	8.81E-05	8.88E-05	TRUE	1.46E-05	4.52E-04	TRUE	3.83E-03	5.06E-03	TRUE	2.96E-06	1.58E-05	TRUE
F5	8.86E-10	1.36E-09	TRUE	1.26E-09	7.13E-08	TRUE	1.76E-11	1.14E-10	TRUE	9.69E-10	1.34E-09	TRUE
F6	2.12E-04	3.83E-04	TRUE	2.05E+00	2.46E+00	TRUE	7.32E-01	7.35E-01	TRUE	1.98E-04	4.61E-04	TRUE
F7	0.00E+00	1.92E-06	FALSE	1.08E-03	3.19E-03	TRUE	3.89E-08	1.78E-07	TRUE	0.00E+00	0.00E+00	FALSE
F8	0.00E+00	0.00E+00	FALSE	0.00E+00	3.76E-01	FALSE	5.70E-02	6.78E-02	TRUE	0.00E+00	0.00E+00	FALSE
F9	0.00E+00	3.69E-13	FALSE	2.56E+02	3.84E+02	TRUE	-1.51E+02	2.49E+00	FALSE	5.51E-05	5.00E-03	TRUE
F10	0.00E+00	0.00E+00	FALSE	4.15E-03	3.22E-02	TRUE	0.00E+00	0.00E+00	FALSE	0.00E+00	0.00E+00	FALSE
F11	1.43E-03	3.41E-01	TRUE	2.76E+00	3.40E+00	TRUE	4.80E-02	1.11E-01	TRUE	3.53E-01	3.53E-01	TRUE
F12	3.58E-07	4.95E-04	TRUE	5.55E-01	6.73E-01	TRUE	1.81E-01	2.04E-01	TRUE	4.79E-01	4.79E-01	TRUE
F13	0.00E+00	0.00E+00	FALSE	0.00E+00	3.43E+00	FALSE	0.00E+00	0.00E+00	FALSE	0.00E+00	0.00E+00	FALSE

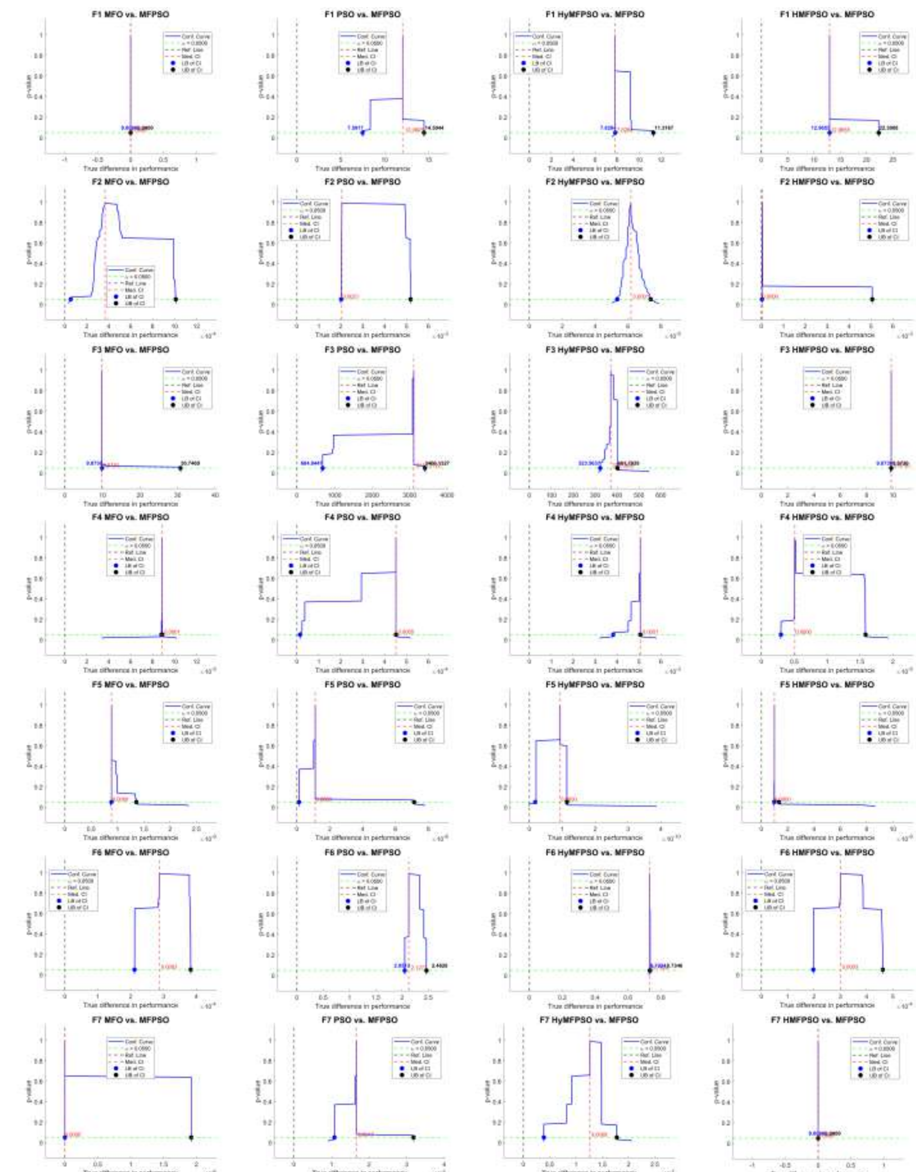


Figure S14: Confidence curves for all the engineering optimization benchmark problems across the 30 runs, where MFPSO is compared with the MFO, PSO, HyMFPSO, and HMFPSO algorithms.

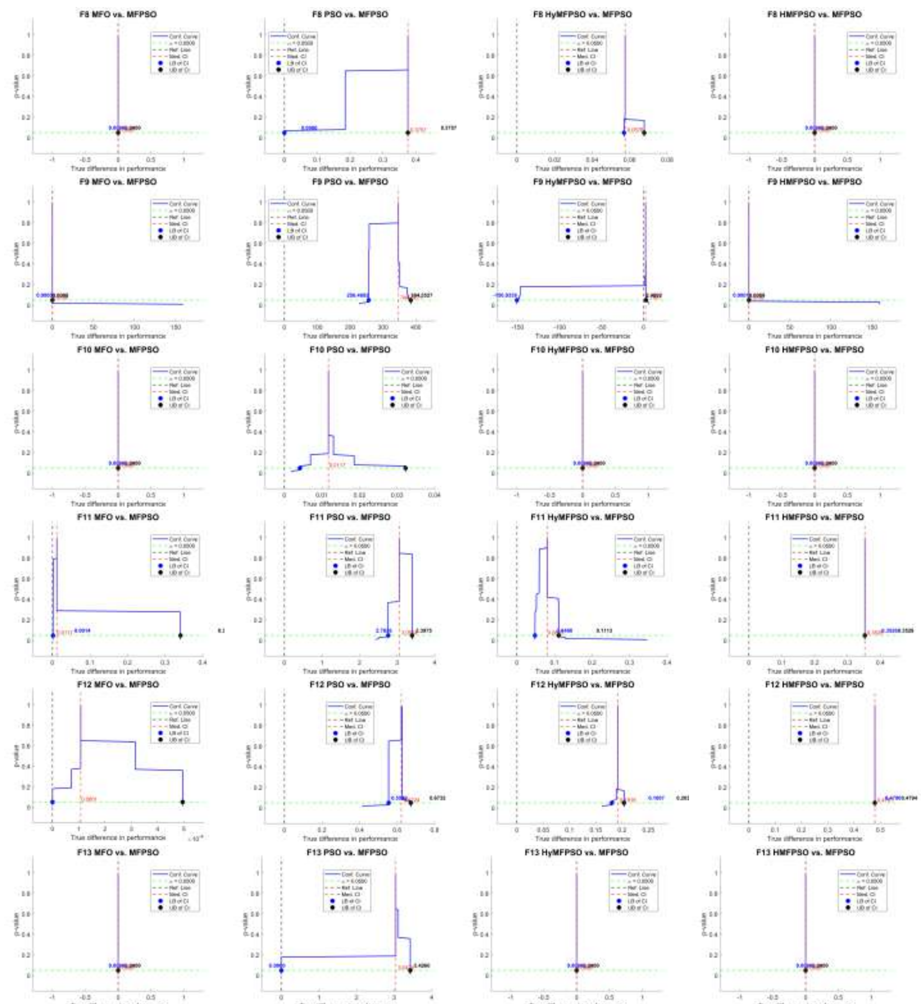


Fig. S14 (Cont.): Confidence curves for all the engineering optimization benchmark problems across the 30 runs, where MFPSO is compared with the MFO, PSO, HyMFPSO, and HMFPSO algorithms.

S5.3 Page Test and Convergence Trends

Table S5: The Page's test results for all the comparisons between the MFPSO algorithm and all the other algorithms on the Engineering Optimization problems.

Comparison	L	p-val	C1	C3	C5	C7	C9	C11	C13	C15	C17	C19
MFO-MFPSO	21512.5	1.00E+00	164	174	153.5	143	136.5	125	113.5	104	90	80
MFPSO-MFO	29615.5	0.00E+00	70	62	87.5	117	135.5	159	181.5	162	172	187
PSO-MFPSO	19880.5	1.00E+00	185	192	176.5	177.5	156.5	142	109.5	91	65.5	25
MFPSO-PSO	30900.5	0.00E+00	47	49	76.5	87.5	110.5	134	159.5	182.5	214	236
HyMFPSO-MFPSO	29598	0.00E+00	102	75	80	99	117.5	146	149.5	179	199	198
MFPSO-HyMFPSO	21235	1.00E+00	165	175	178	156	130.5	114	101.5	88.5	87.5	107
HMFPSO-MFPSO	28003	4.63E-12	138	116	101.5	85	63.5	132.5	164.5	193	188	175
MFPSO-HMFPSO	22809	1.00E+00	117	133	159.5	172	194.5	110.5	103.5	84	84	125

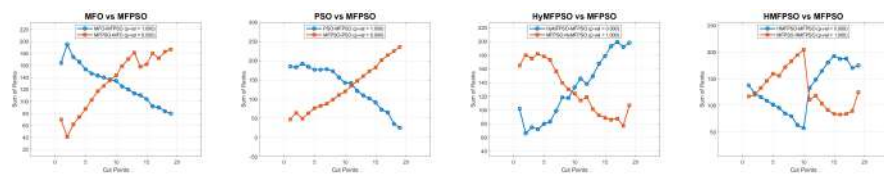
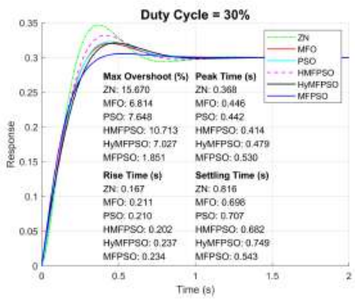
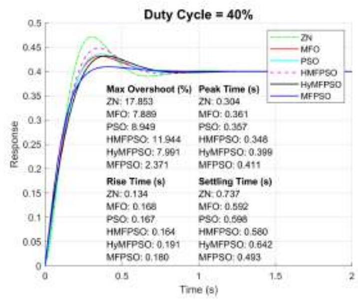


Figure S15: Convergence trends between the algorithms using the Page's test.

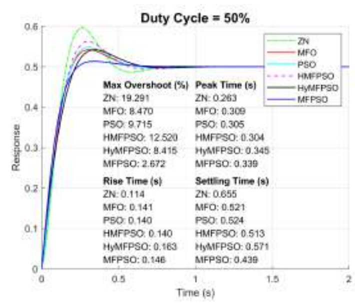
S6 Transient Response for PID Speed Control Testing



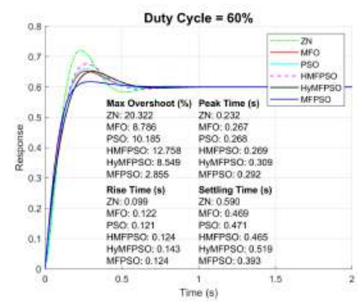
(a) 30% duty cycle.



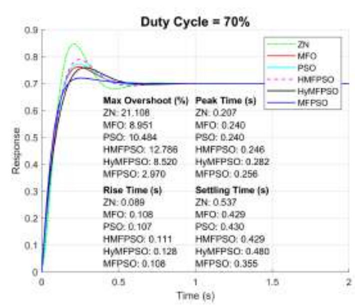
(b) 40% duty cycle.



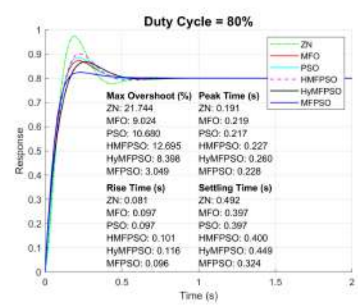
(c) 50% duty cycle.



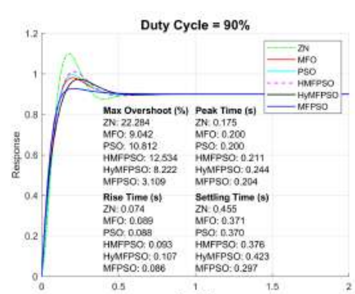
(d) 60% duty cycle.



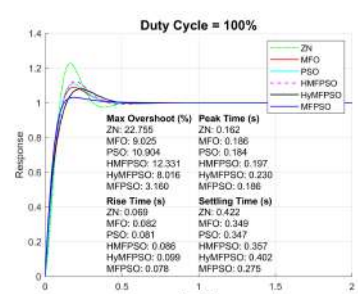
(e) 70% duty cycle.



(f) 80% duty cycle.



(g) 90% duty cycle.



(h) 100% duty cycle.

Figure S16: Transient response of the speed control using PID controller.

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