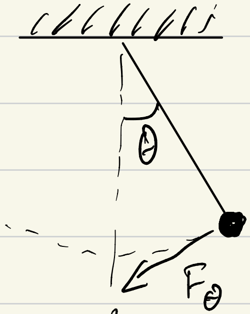
Chapter 8:

Simple Harmonic Motion

- simple pendulum:



$$F_\theta = -mg\sin\theta$$

$$F_\theta = m\frac{d^2\theta}{dt^2} = -mg\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -g\sin\theta \approx -\frac{g}{l}\theta$$

↑ small angle approx.

$$\theta = \theta_0 \sin(\Omega t + \phi)$$

$$\Omega = \sqrt{\frac{g}{l}}$$

- numerical solution:

$$\frac{d\omega}{dt} = -\frac{g}{l}\theta \quad , \quad \frac{d\theta}{dt} = \omega$$

$$\omega_{i+1} = \omega_i - g/l\,\theta_i\,\Delta t$$
$$\theta_{i+1} = \theta_i + \omega_i\,\Delta t$$

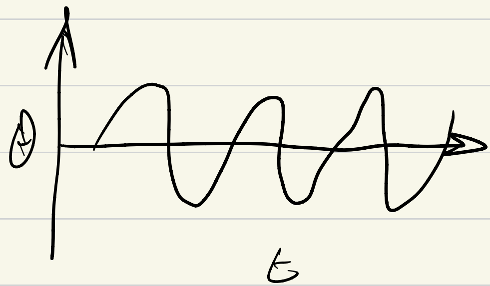
Euler Method

$\omega_{i+1} = \omega_i - g/l\,\theta_i \Delta t$

$\theta_{i+1} = \theta_i + \omega_{i+1} \Delta t$

Euler–Cromer Method

Euler



Euler–Cromer

Adding Dissipation, Non-linearity, & Driving Force

- damped pendulum:

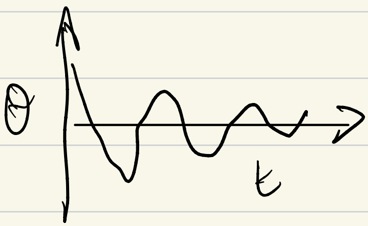
$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt}$$

$$\theta(t) = \theta_0 e^{-qt/2} \sin\left(\sqrt{\Omega^2 - q^2/4}\, t + \phi\right)$$

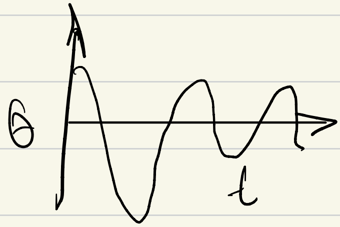
- driving force:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t)$$

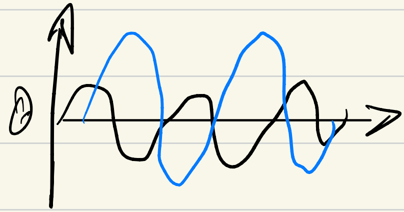
$$\theta(t) = \theta_0 \sin(\Omega_D t + \phi)$$



damped



driving force



non-linear

Chaos

- numerical solution:

$$\frac{d\omega}{dt} = -\frac{g}{l}\sin(\theta) - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t)$$

$$\frac{d\theta}{dt} = \omega$$

$$\omega_{i+1} = \omega_i - [g/l \sin\theta_i - q\omega_i + F_D \sin(\Omega_D t_i)]\Delta t$$

$$\theta_{i+1} = \theta_i + \omega_{i+1}\Delta t$$