Chapter 4:

Kepler Laws

- gravitational force:

$$F_G = \frac{G M_S M_E}{r^2}$$

$$F_{G,x} = -\frac{G M_S M_E\, x}{r^3} \qquad F_{G,y} = -\frac{G M_S M_E\, y}{r^3}$$

$$G M_S = 4\pi^2\ AU^3/yr^2$$

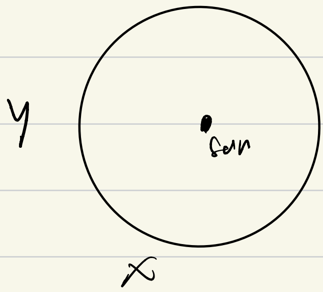
- Euler-Cromer:

$$v_{x,i} = v_{x,i-1} - \frac{4\pi^2 x_{i-1}}{r_{i-1}^3}\Delta t$$

$$x_i = x_{i-1} + v_{x,i}\Delta t$$

$$v_{y,i} = v_{y,i-1} - \frac{4\pi^2 y_{i-1}}{r_{i-1}^3}\Delta t$$

$$y_i = y_{i-1} + v_{y,i}\Delta t$$



The Inverse Square Laws...

$$v_{max} = \sqrt{GM_s}\sqrt{\frac{1+e}{a(1-e)}\left(1+\frac{M_p}{M_s}\right)}$$

$$v_{min} = \sqrt{GM_s}\sqrt{\frac{1-e}{a(1+e)}\left(1+\frac{M_p}{M_s}\right)}$$

- Kepler's 1st Law: sun at one focus of ellipse
- Kepler's 2nd Law: conservation of angular momentum
- Kepler's 3rd Law: $T^2 = a^3 \cdot 4\pi^2/(G(M_s+M_p))$

Precession of the Perihelion of Mercury

- Force w/ general relativity:

$$F_G = \frac{GM_s M_m}{r^2}\left(1+\frac{\alpha}{r^2}\right)$$

$$\alpha = 1.1\times10^{-8}\ AU^2$$

$$v_1 = \sqrt{\frac{GM_s(1-e)}{a(1+e)}} = 8.2\ AU/yr$$

$$r_1 = (1+e)a = 0.47\ AU$$

Three Body Problem: Jupiter & Earth

- $F_{E,J} = \frac{G M_J M_E}{r_{EJ}^2}$

$$F_{EJ,x} = -\frac{G M_J M_E (x_e - x_j)}{r_{EJ}^3}$$

$$F_{EJ,y} = -\frac{G M_J M_E (y_e - y_j)}{r_{EJ}^3}$$

- Euler Cromer:

$$r_{e,i} = \sqrt{x_e(i)^2 + y_e(i)^2}$$

$$r_{j,i} = \sqrt{x_j(i)^2 + y_j(i)^2}$$

$$r_{EJ} = \sqrt{(x_e(i) - x_j(i))^2 + (y_e(i) - y_j(i))^2}$$

$$v_{e,x}(i) = v_{e,x}(i-1) - \frac{4\pi^2 x_e(i-1)}{r_e(i-1)^3} \Delta t$$

$$- \frac{4\pi^2 (M_J/M_S)(x_e(i-1) - x_j(i-1))}{r_{ej}(i-1)^3} \Delta t$$

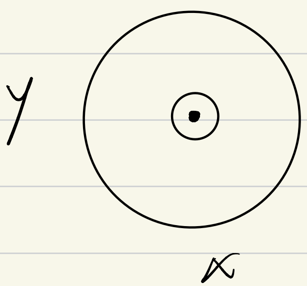
$$v_{j,x}(i) = v_{j,x}(i-1) - \frac{4\pi^2 x_j(i-1)}{r_j(i-1)^3} \Delta t$$

$$- \frac{4\pi^2 (M_E/M_S)(x_j(i-1) - x_e(i-1))}{r_{ej}(i-1)^3} \Delta t$$

- likewise for $v_{e,y}$ & $v_{J,y}$

$$x_e(i) = x_e(i-1) + v_{e,x}(i)\Delta t$$
$$y_e(i) = y_e(i-1) + v_{e,y}(i)\Delta t$$

- likewise for x_J & y_J



- different results for greater mass of Jupiter