Documentation of modelling of permanent magnet synchronous motor considering iron core losses project

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Abstract

this paper presents a comprehensive modelling approach for Permanent Magnet Synchronous Motors the model is design to be used as a foundation for parameter optimization and accurate investigation of currents, power losses, efficiency and. the model accounts for iron core losses.

1 Introduction

A generalized model is introduced by Paul C. Krause ,Oleg Wasiynczuk and Scott D. Sudhoff in Analysis of Electric Machinery and Drives in chapter (5.5) they modeled a salient-pole synchronous generator with two damper windings in qd axes rotor reference frame, assuming a linear magnetic circuit ,and balanced conditions sinusoidally distributed flux,all rotor variables are referred to stator.it may be useful to get the model in terms of impedance(X_l, X_{md}) so we will use the model as a function of flux linkages per second ψ instead of flux linkage λ, this model can be used as a general case that we will use to obtain a model for the permanent-magnet synchronous motor as a special case.

2 Derivation of the PMSM model from general synchronous machine's model

To obtain the PMSM's mathematical equations we need to make some modifications on the model that is mentioned above.

- All variables and equations related to the field winding are eliminated since we use a permanent magnet rotor.
- The effect of the permanent magnet will be considered in the flux linkage per second relations by replacing $X_{md}i_f$ by $\omega_b\lambda_m$ (or ψ_m) where λ_m is the permanent magnet flux linkage and ψ_m
- Only one shorted winding in the rotor in each direction (d and q) will be enough to compensate for the hysteresis losses, eddy current losses and any parasite magnetization effect in the iron core and the air gap; all variables and equations related to the second damper winding is removed.
- In voltage equations all stator currents are positive since we make a model for the motor mode,
- Zero sequence equations is out of our scope since we are modelling the machine in balanced conditions.

By applying the discussed modification we get the following model

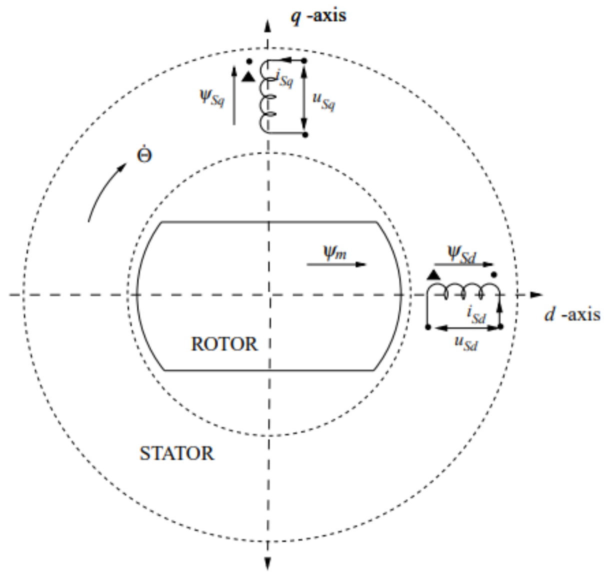

Figure 1: Circuit model of the synchronous machine with permanent magnet excitation

Voltage equations

$$\begin{aligned}
v_{qs}^r &= r_s i_{qs}^r + \frac{\omega_r}{\omega_b}\psi_{ds}^r + \frac{p}{\omega_b}\psi_{qs}^r \\
v_{ds}^r &= r_s i_{ds}^r - \frac{\omega_r}{\omega_b}\psi_{qs}^r + \frac{p}{\omega_b}\psi_{ds}^r \\
v_{kq}'^r &= r_{kq1}' i_{kq}'^r + \frac{p}{\omega_b}\psi_{kq}'^r \\
v_{kd}'^r &= r_{kd}' i_{kd}'^r + \frac{p}{\omega_b}\psi_{kd}'^r
\end{aligned} \tag{1}$$

Flux linkages per second

$$\begin{aligned}
\psi_{qs}^r &= X_{ls} i_{qs}^r + X_{mq}\left(i_{qs}^r + i_{kq}'^r\right) \\
\psi_{ds}^r &= X_{ls} i_{ds}^r + X_{md}\left(i_{ds}^r + i_{kd}'^r\right) + \omega_b \lambda_m \\
\psi_{kq}'^r &= X_{lkq}' i_{kq}'^r + X_{mq}\left(i_{qs}^r + i_{kq}'^r\right) \\
\psi_{kd}'^r &= X_{lkd}' i_{kd}'^r + X_{md}\left(i_{ds}^r + i_{kd}'^r\right) + \omega_b \lambda_m
\end{aligned} \tag{2}$$

Electromechanical torque equation

The electromechanical torque is defined a function of the stator variables

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left(\psi_{ds}^r i_{qs}^r - \psi_{qs}^r i_{ds}^r\right) \tag{3}$$

Mechanical equation

$$T_e = T_L - J\frac{d\omega_m}{dt} - B\omega_m \tag{4}$$

where

$$\omega_m = \frac{P}{2}\ \omega_r$$

Torque angle equation

$$\frac{d}{dt}\delta = \omega_e - \omega_r \tag{5}$$

3 Equations for computer simulation

This model's equations has to be manipulated in away that makes it possible to make a computer program ,We will use the same sequence that is commonly used in induction machine modelling which is also the most common with synchronous machines by defining the flux linkages per second as functions of voltages using the equations (1) and substituting for the current from equations(2)

Flux linkage per second and current relations

$$\begin{aligned}
\frac{d}{dt}\psi_{qs}^r &= \omega_b\left[v_{qs}^r - \frac{\omega_r}{\omega_b}\psi_{ds}^r - \frac{r_s}{X_{ls}}\left(\psi_{mq}^r - \psi_{qs}^r\right)\right]\\
\frac{d}{dt}\psi_{ds}^r &= \omega_b\left[v_{ds}^r + \frac{\omega_r}{\omega_b}\psi_{qs}^r - \frac{r_s}{X_{ls}}\left(\psi_{md}^r - \psi_{ds}^r\right)\right]\\
\frac{d}{dt}\psi_{kq}'^r &= \omega_b\left[v_{kq}'^r - \frac{r_{kq}'}{X_{lkq}'}\left(\psi_{mq}^r - \psi_{kq}'^r\right)\right]\\
\frac{d}{dt}\psi_{kd}'^r &= \omega_b\left[v_{kd}'^r - \frac{r_{kd}'}{X_{lkd}'}\left(\psi_{md}^r - \psi_{kd}'^r\right)\right]
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
i_{qs}^r &= \frac{1}{X_{ls}}\left(\psi_{qs}^r - \psi_{mq}^r\right)\\
i_{ds}^r &= \frac{1}{X_{ls}}\left(\psi_{ds}^r - \psi_{md}^r\right)\\
i_{kq}'^r &= \frac{1}{X_{lkq}'}\left(\psi_{kq}'^r - \psi_{mq}^r\right)\\
i_{kd}'^r &= \frac{1}{X_{lkd}'}\left(\psi_{kd}'^r - \psi_{md}^r\right)
\end{aligned} \tag{7}$$

Since the d-axes currents are coupled and so the q-axes currents we define $\psi_{mq}^r$and ψ_{md}^r as functions of flux linkages per second to replace the variables $X_{mq}\left(i_{qs}^r + i_{kq}'^r\right)$ and $X_{md}\left(i_{ds}^r + i_{kd}'^r\right)$ in equations 2.

$$\begin{aligned}
\psi_{mq}^r &= X_{aq}\left(\frac{\psi_{qs}^r}{X_{ls}} + \frac{\psi_{kq}'^r}{X_{lkq}'}\right)\\
\psi_{md}^r &= X_{ad}\left(\frac{\psi_{ds}^r}{X_{ls}} + \frac{\psi_{kd}'^r}{X_{lkd}'}\right) + \omega_b\lambda_m
\end{aligned} \tag{8}$$

$$\begin{aligned}
X_{aq} &= \left(\frac{1}{X_{mq}} + \frac{1}{X_{ls}} + \frac{1}{X_{lkq}'}\right)^{-1}\\
X_{ad} &= \left(\frac{1}{X_{md}} + \frac{1}{X_{ls}} + \frac{1}{X_{lkd}'}\right)^{-1}
\end{aligned} \tag{9}$$

Mechanical angular velocity

$$\frac{d}{dt}\omega_m = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left(\psi_{ds}^r i_{qs}^r - \psi_{qs}^r i_{ds}^r\right) - T_l - B\omega_m \tag{10}$$

Torque angle equation

$$\frac{d}{dt}\delta = \omega_e - \omega_r \tag{11}$$

Source voltage transformation

$$V_{qdo} = K_s^r V_{abc} \tag{12}$$

where $\theta_r$is the rotor position $\theta_r = \int \frac{P}{2}\omega_m$

$$K_s^r = \frac{2}{3}\left[\begin{array}{ccc} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right] \tag{13}$$

Output currents transformation

$$i_{qdo} = K_s^{r-1} i_{abc} \tag{14}$$

where

$$\left[\begin{array}{c} i_{as} \\ i_{bs} \\ i_{cs} \end{array}\right] = \left[\begin{array}{ccc} \cos\theta_r & \sin\theta_r & 1 \\ \cos\left(\theta_r - 120^\circ\right) & \sin\left(\theta_r - 120^\circ\right) & 1 \\ \cos\left(\theta_r + 120^\circ\right) & \sin\left(\theta_r + 120^\circ\right) & 1 \end{array}\right]\left[\begin{array}{c} i_{qs} \\ i_{ds} \\ i_{os} \end{array}\right] \tag{15}$$