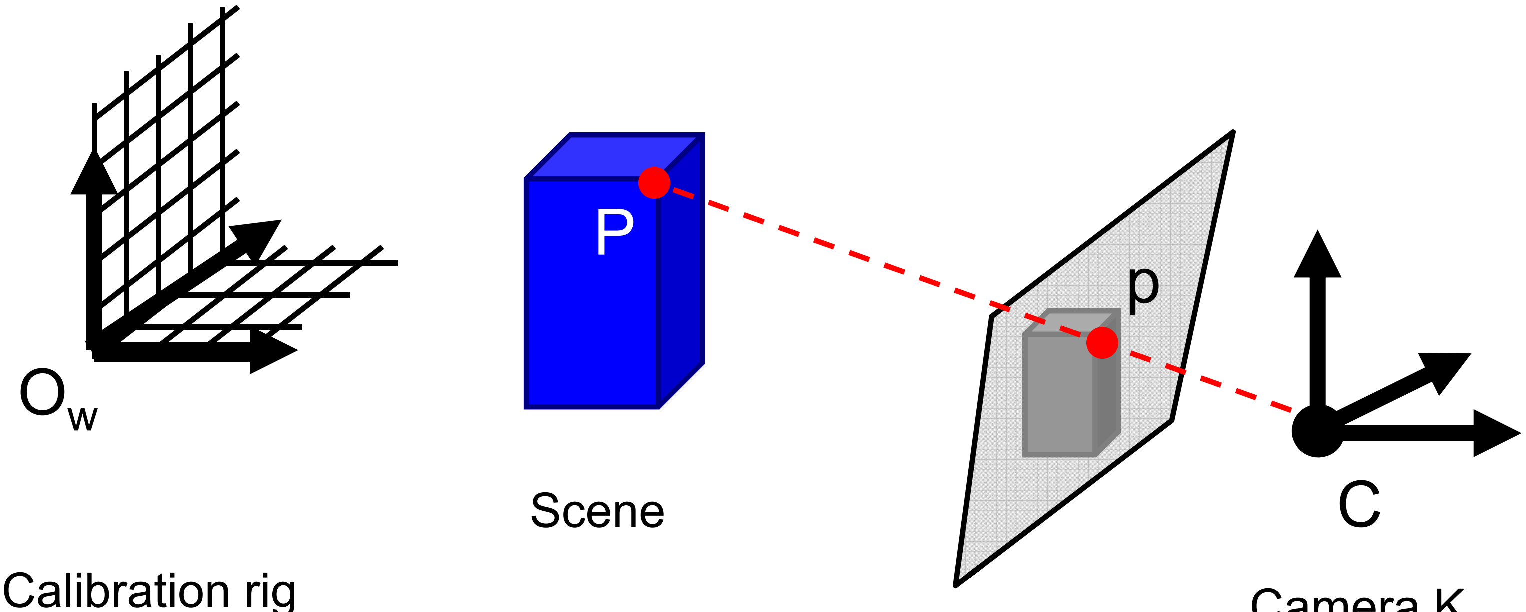
EECS 442 – Computer vision

Epipolar Geometry

- Why is stereo useful?
- Epipolar constraints
- Essential and fundamental matrix
- Estimating F
- Examples

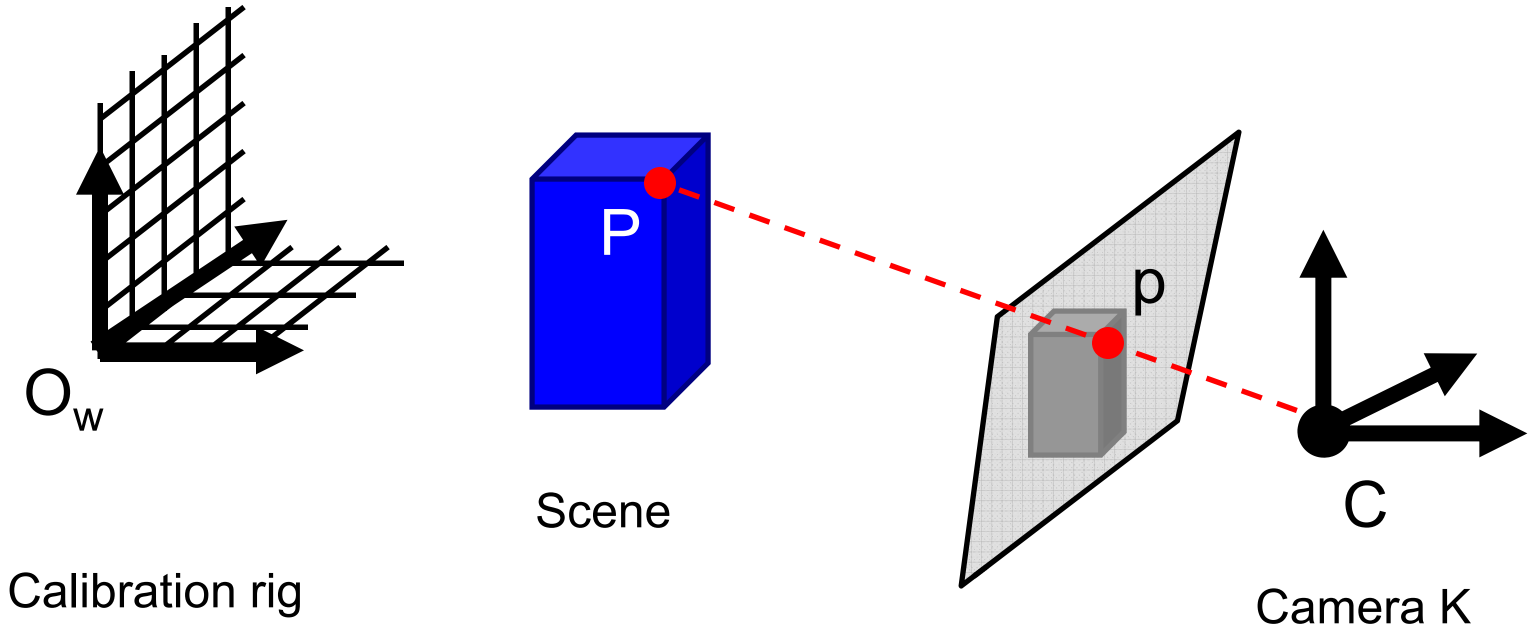
Reading: [AZ] Chapters: 4, 9, 11
[FP] Chapters: 10

Recovering structure from a single view



From calibration rig	→ location/pose of the rig, K
From points and lines at infinity + orthogonal lines and planes	→ structure of the scene, K
Knowledge about scene (point correspondences, geometry of lines & planes, etc…	

Recovering structure from a single view



Why is it so difficult?

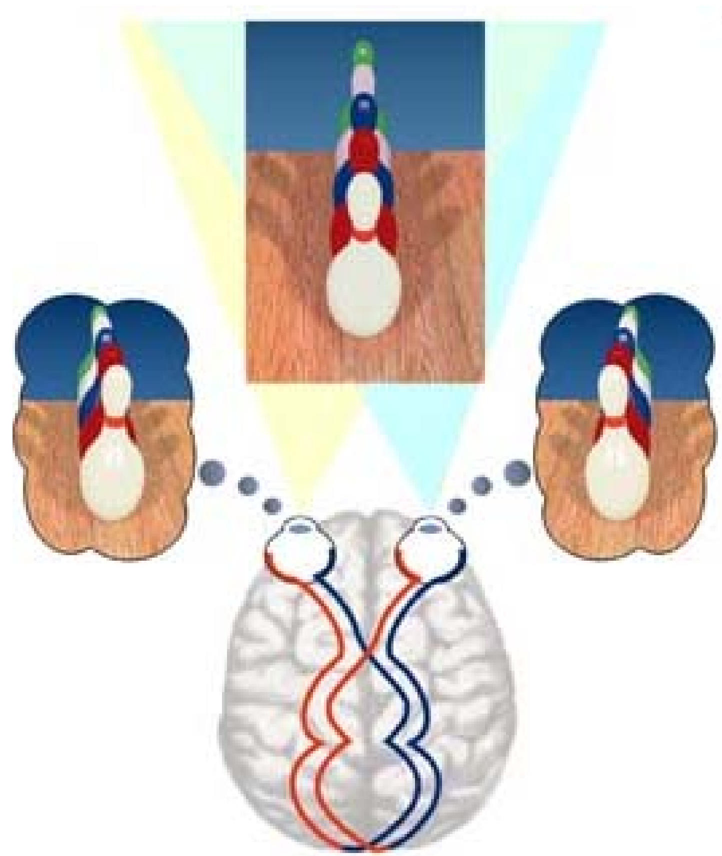
Intrinsic ambiguity of the mapping from 3D to image (2D)

Recovering structure from a single view

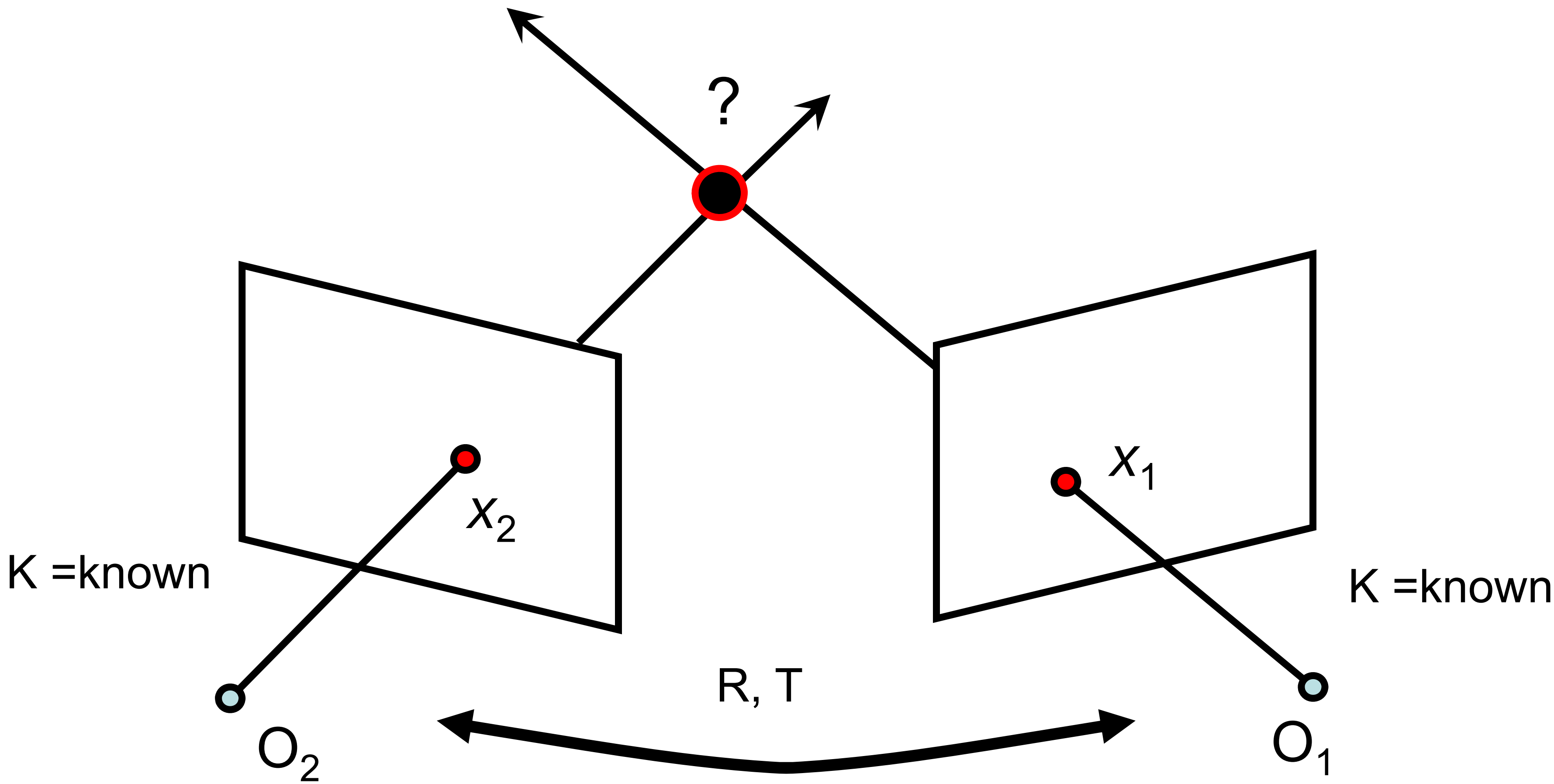
Intrinsic ambiguity of the mapping from 3D to image (2D)

Courtesy slide S. Lazebnik

Two eyes help!



Two eyes help!

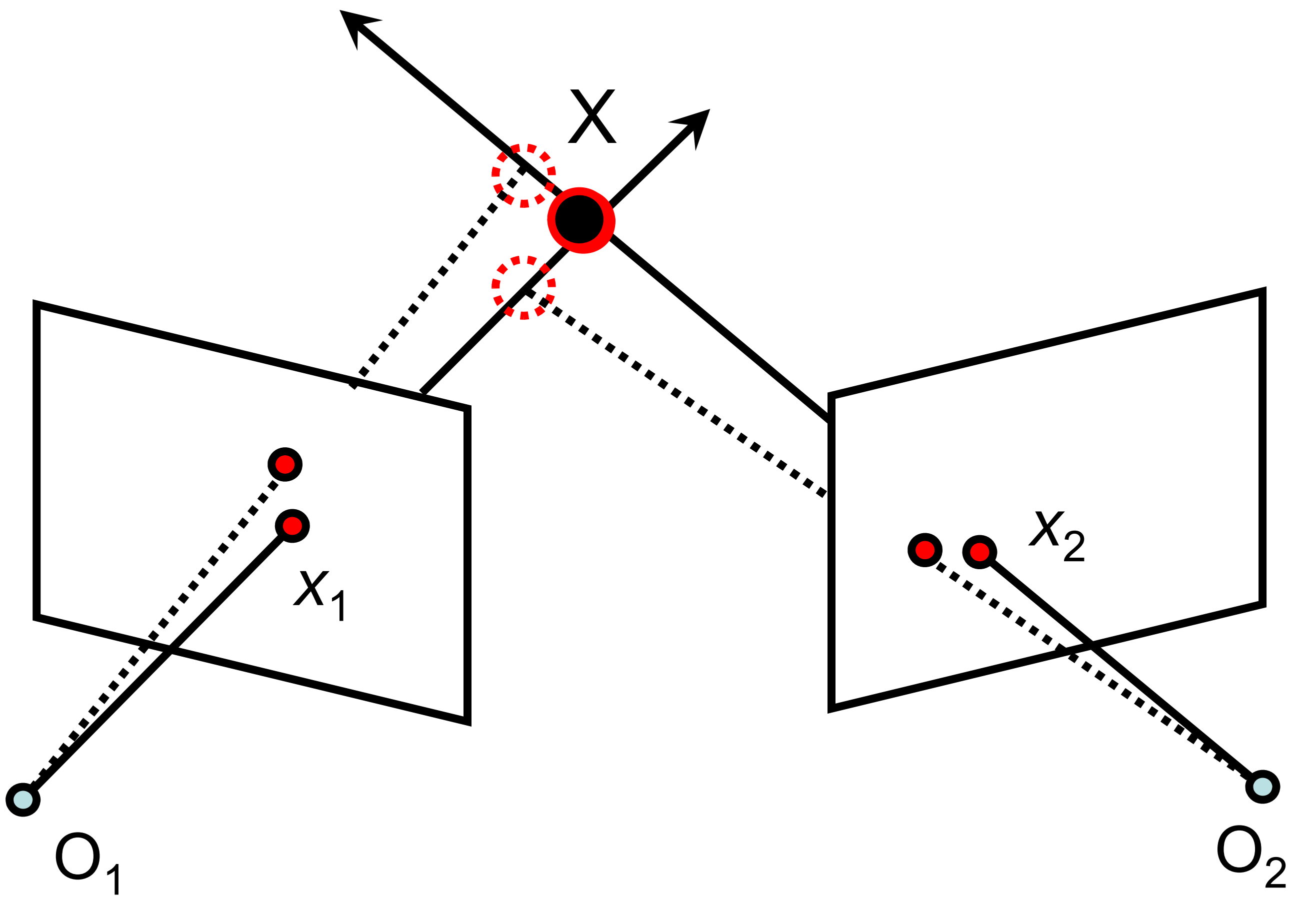


This is called triangulation

Triangulation

- Find X that minimizes

$$d^2(x_1, P_1 X) + d^2(x_2, P_2 X)$$



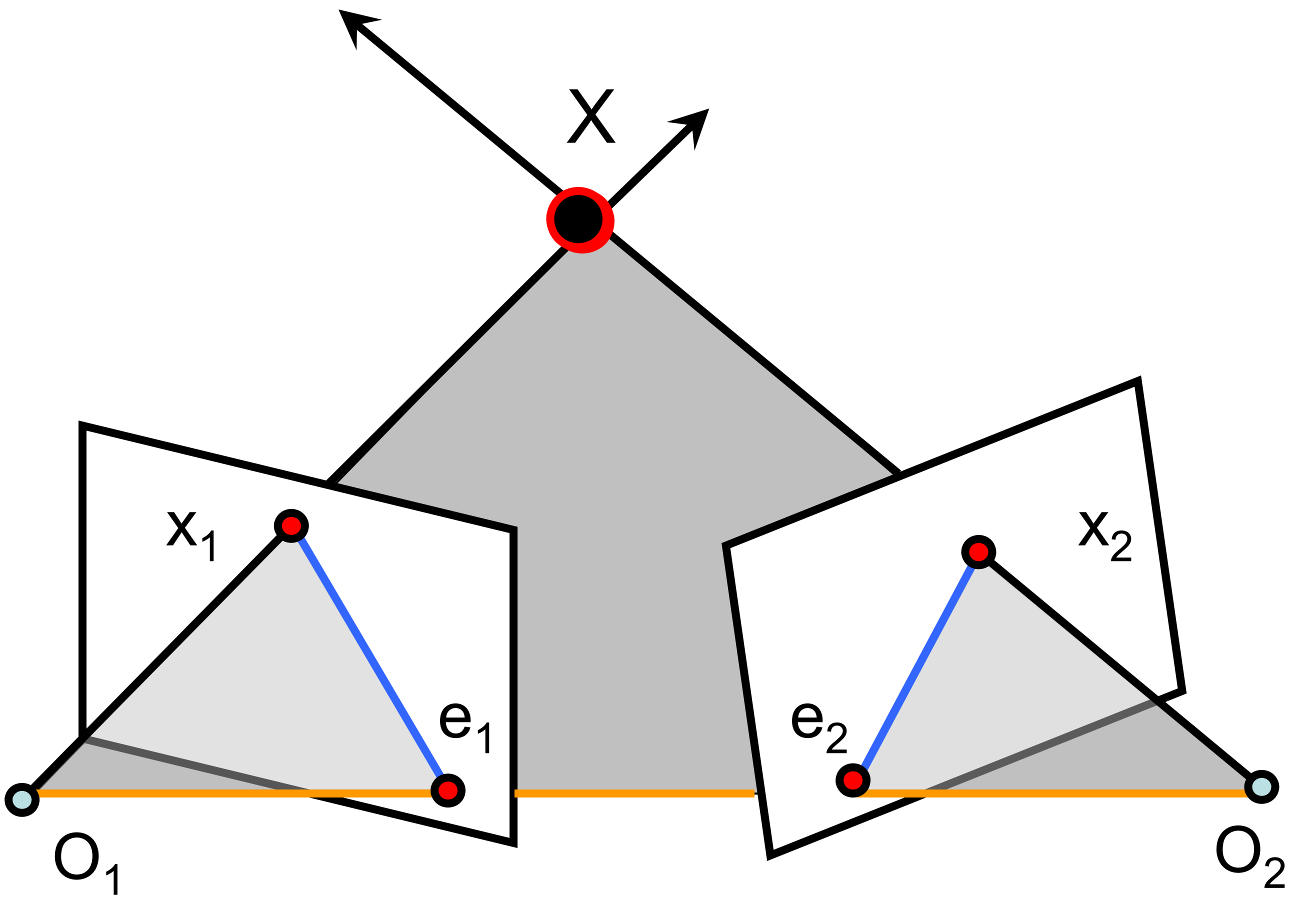
Stereo-view geometry

- **Scene geometry: Find coordinates of 3D point from its projection into 2 or multiple images.**

- **Correspondence:** Given a point in one image, how can I find the corresponding point x' in another one ?

- **Camera geometry:** Given corresponding points in two images, find camera matrices, position and pose.

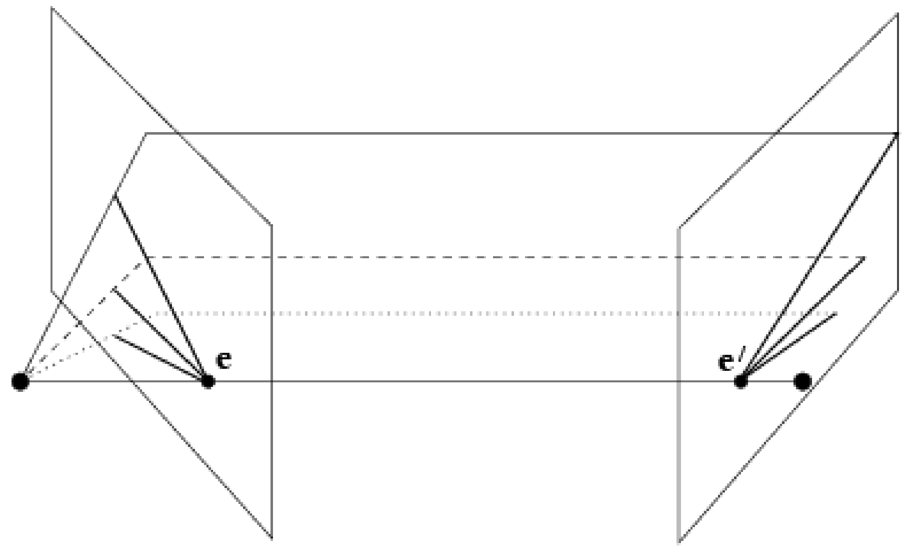
Epipolar geometry

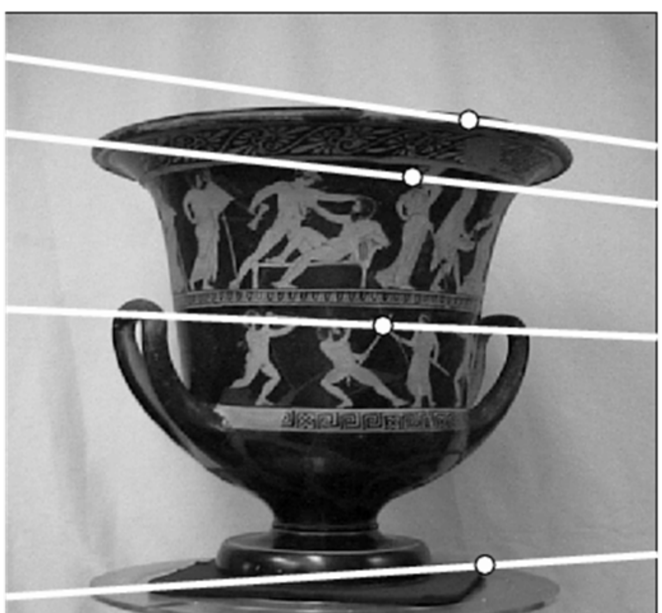


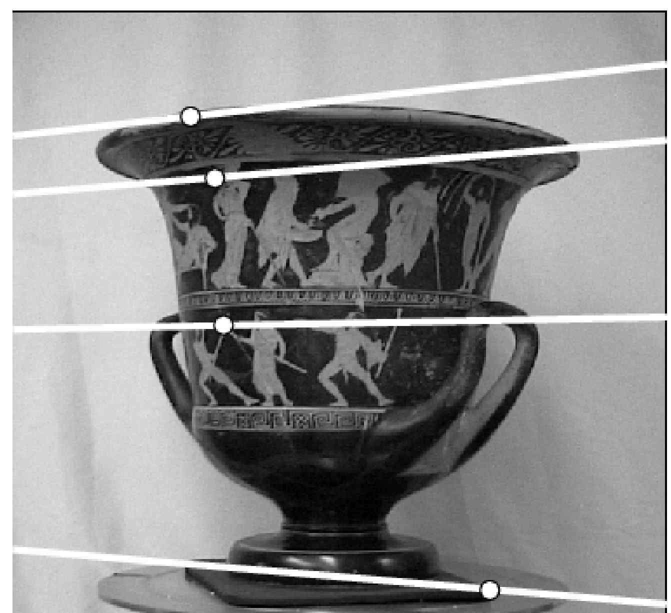
- Epipolar Plane
- Baseline
- Epipolar Lines
- Epipoles e_1, e_2

  = intersections of baseline with image planes
  = projections of the other camera center
  = vanishing points of camera motion direction

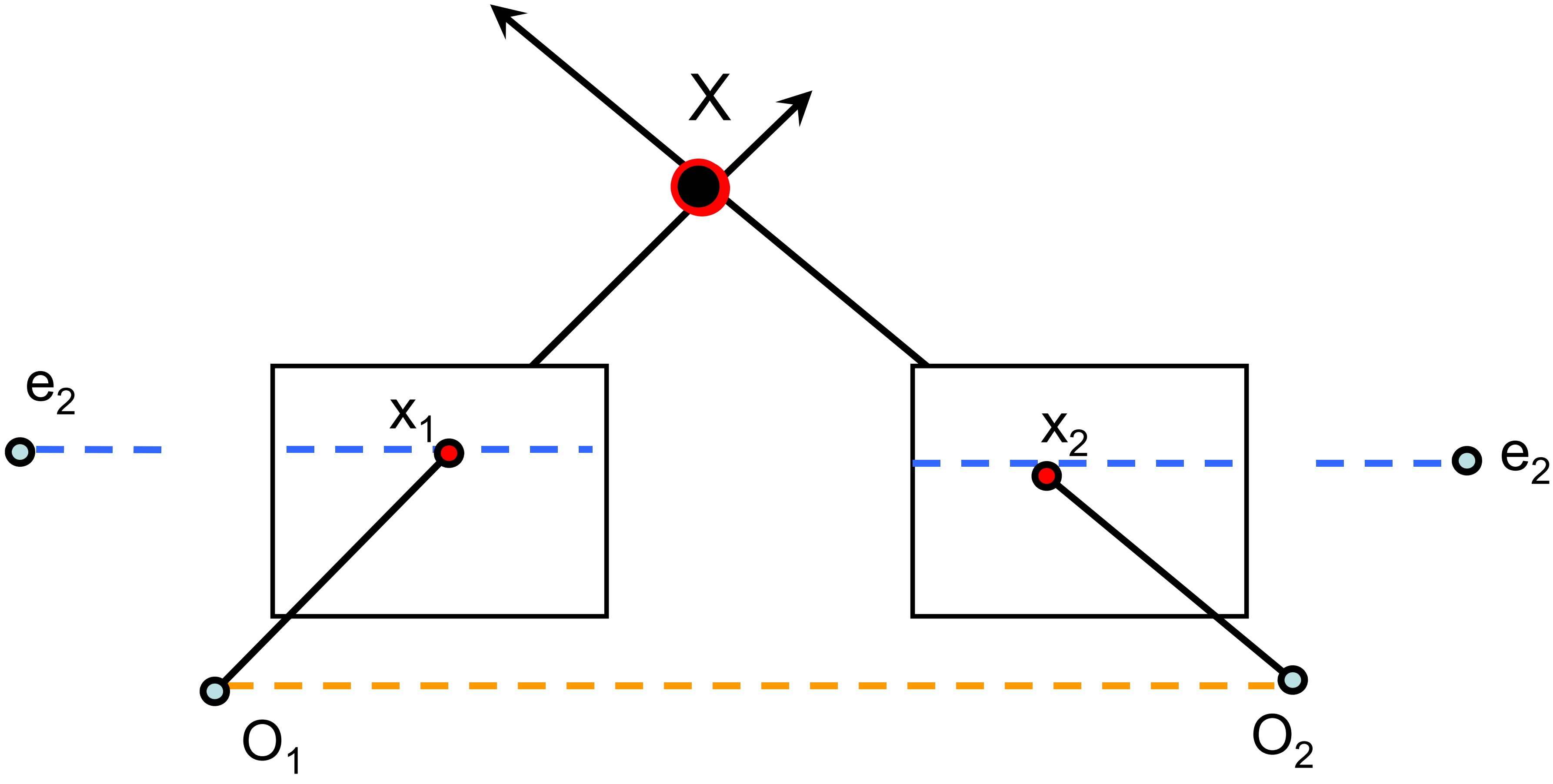
Example: Converging image planes





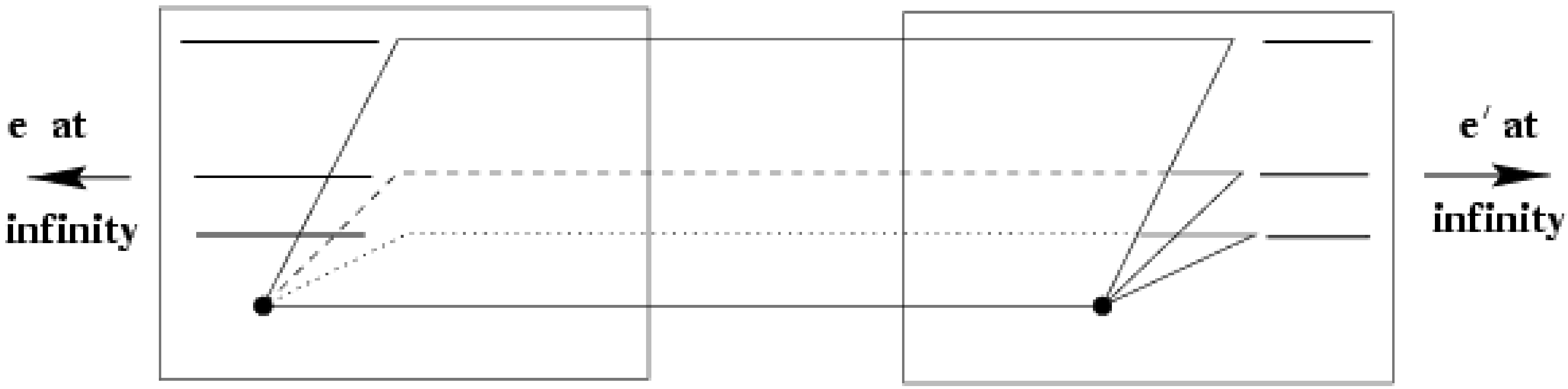


Example: Parallel image planes

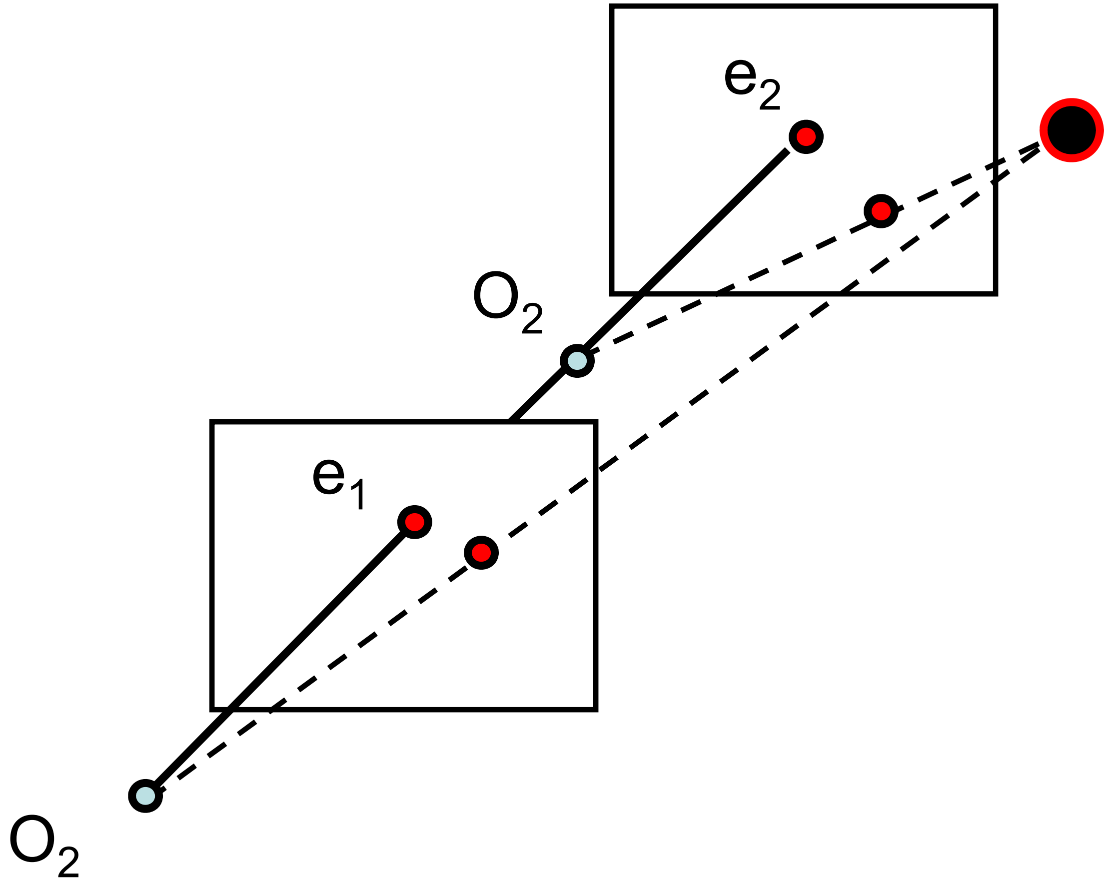


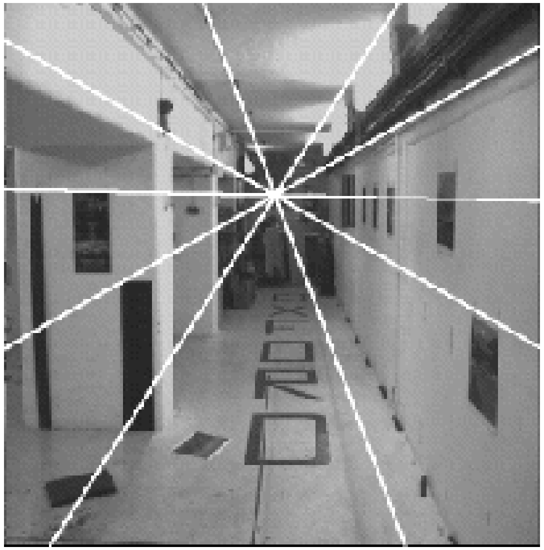
- Baseline intersects the image plane at infinity
- Epipoles are at infinity
- Epipolar lines are parallel to x axis

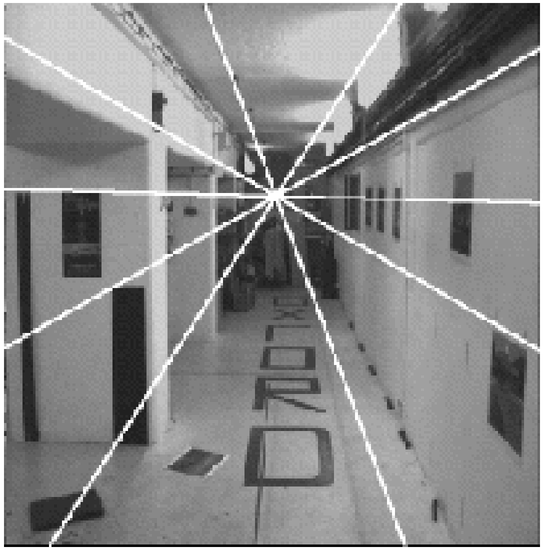
Example: Parallel image planes



Example: Forward translation

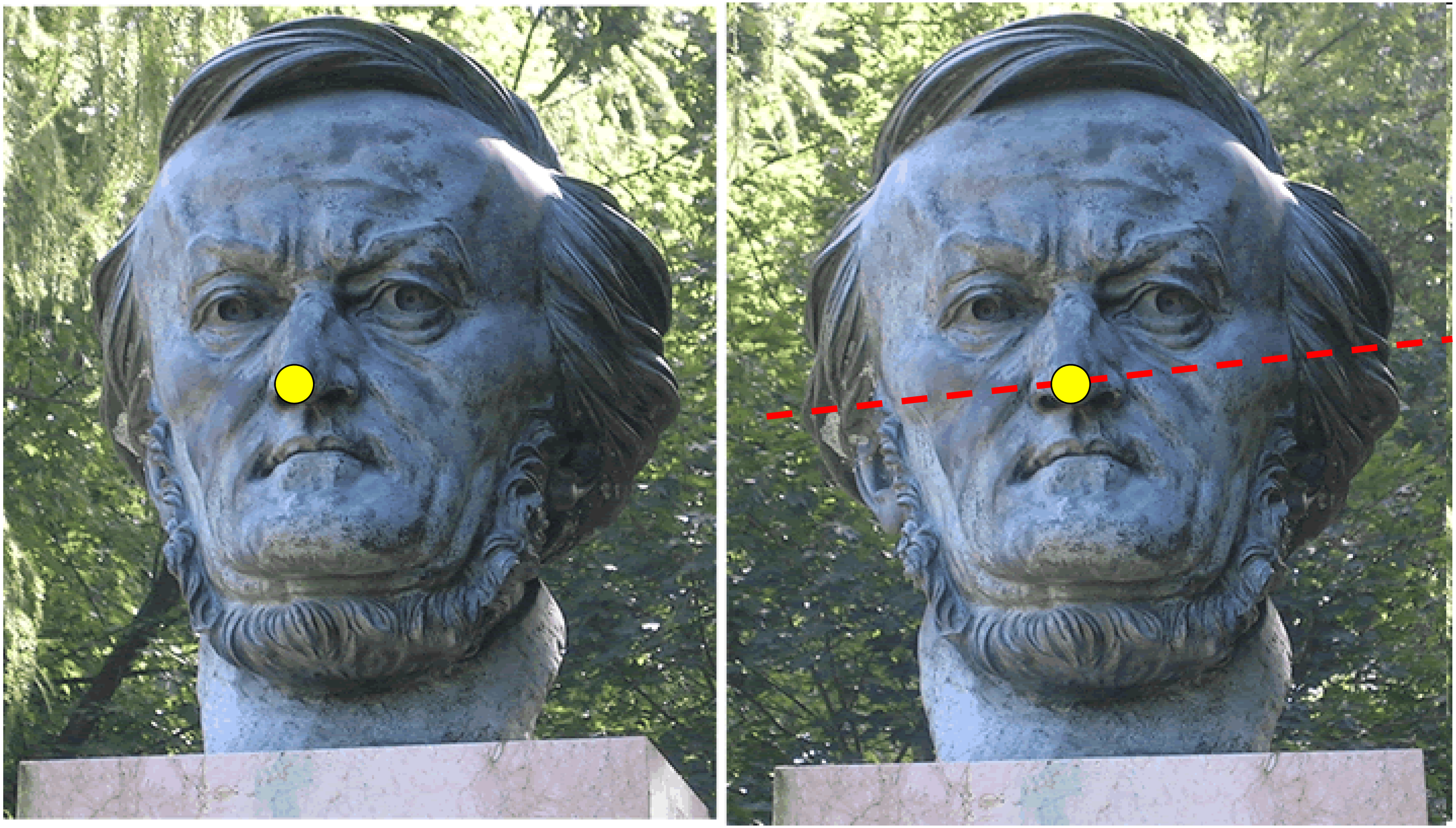






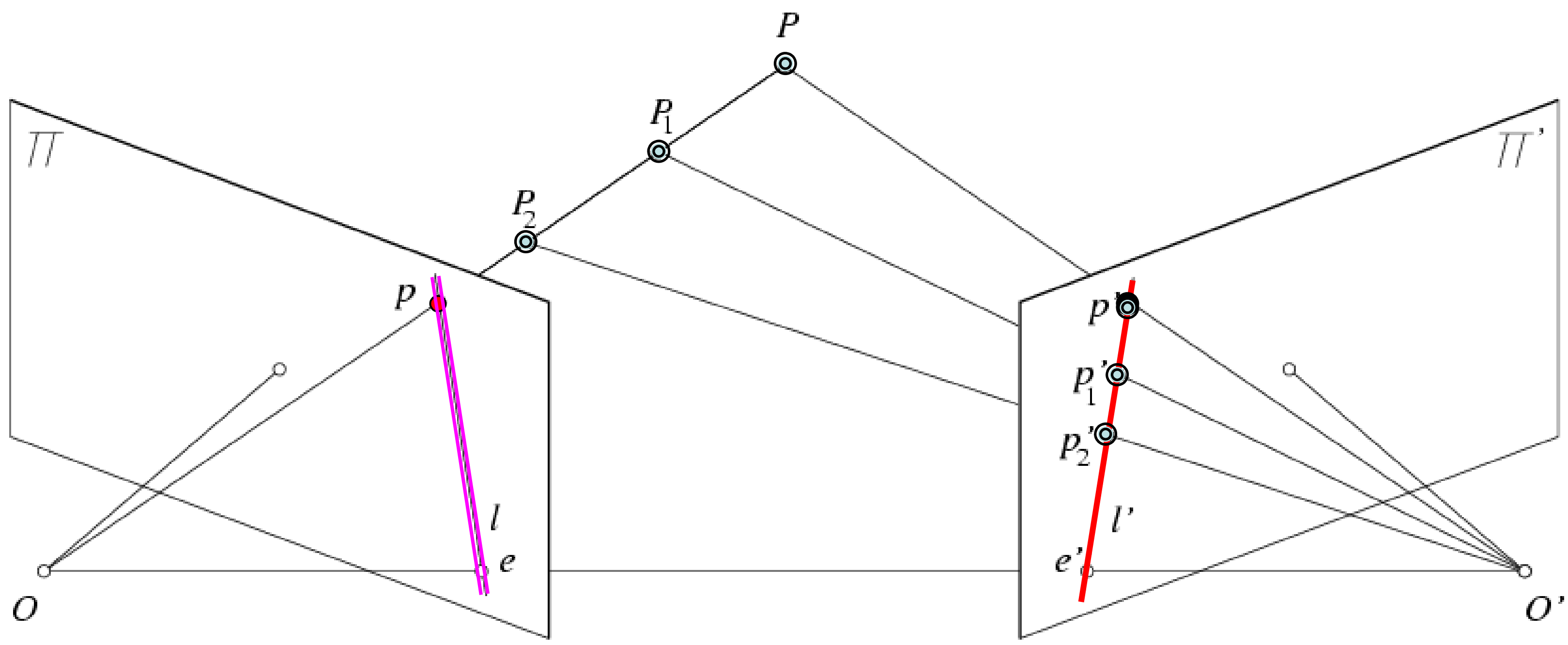
- The epipoles have same position in both images
- Epipole called FOE (focus of expansion)

Epipolar Constraint



- Two views of the same object
- Suppose I know the camera positions and camera matrices
- Given a point on left image, how can I find the corresponding point on right image?

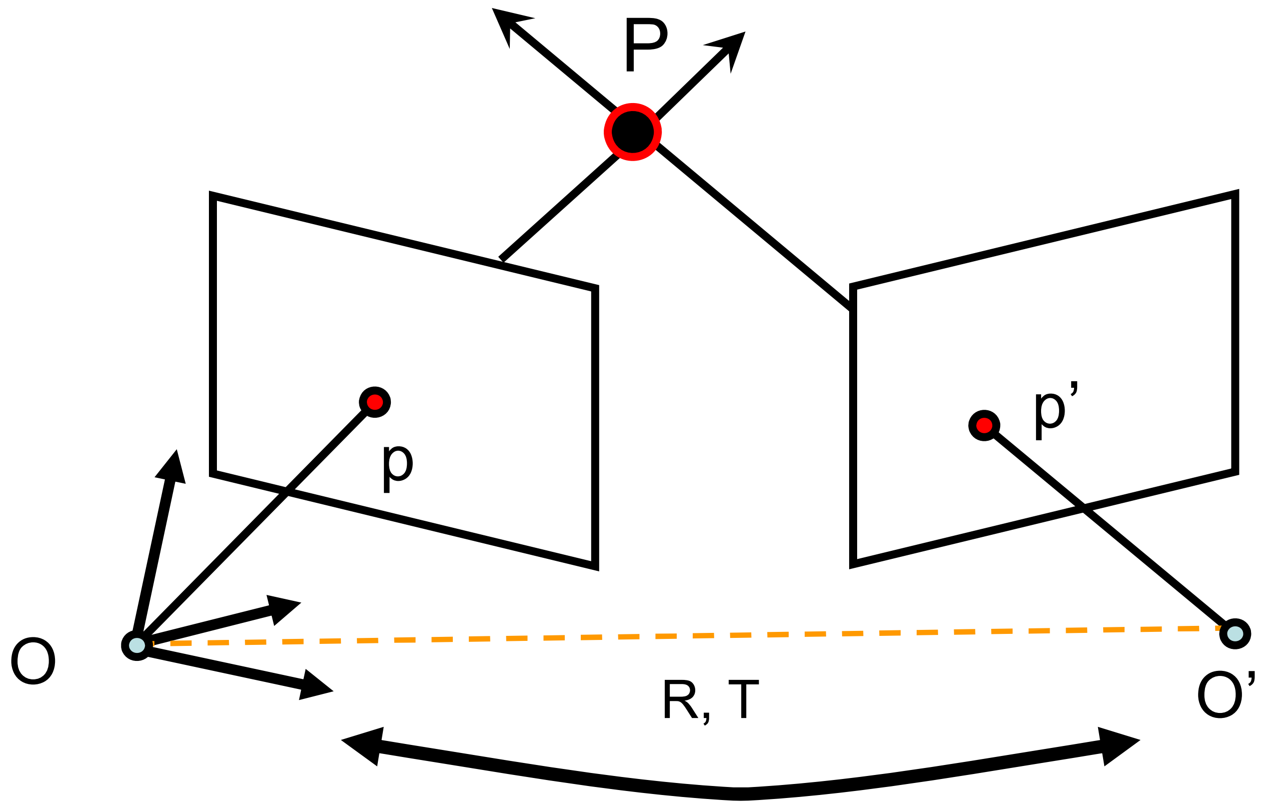
Epipolar Constraint



- Potential matches for *p* have to lie on the corresponding epipolar line *l'*.

- Potential matches for *p'* have to lie on the corresponding epipolar line *l*.

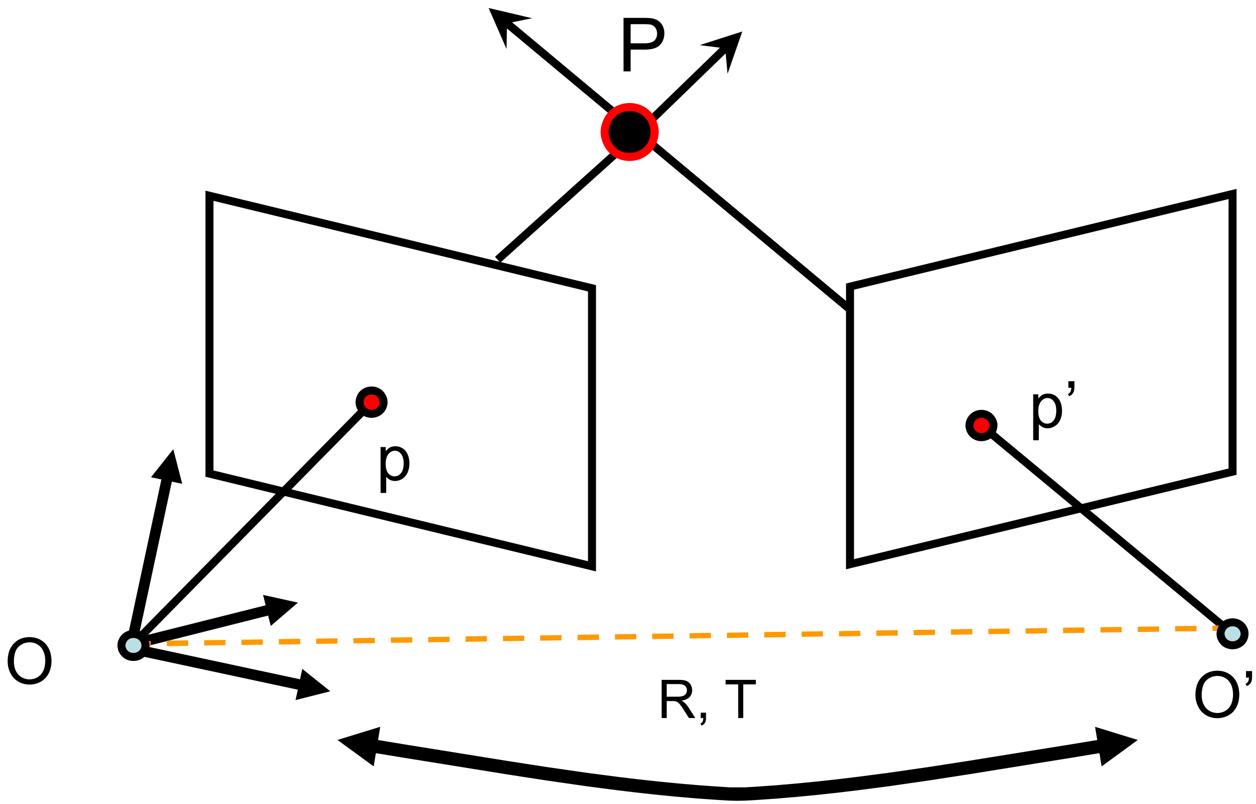
Epipolar Constraint



$$M = K\begin{bmatrix} I & 0 \end{bmatrix} \qquad\qquad M' = K\begin{bmatrix} R & T \end{bmatrix}$$

$$P \rightarrow M\,P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad\qquad P \rightarrow M'\,P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

Epipolar Constraint



$$\underbrace{p^T \cdot \left[T \times (R\, p') \right]}_{\text{Perpendicular to epipolar plane}} = 0$$

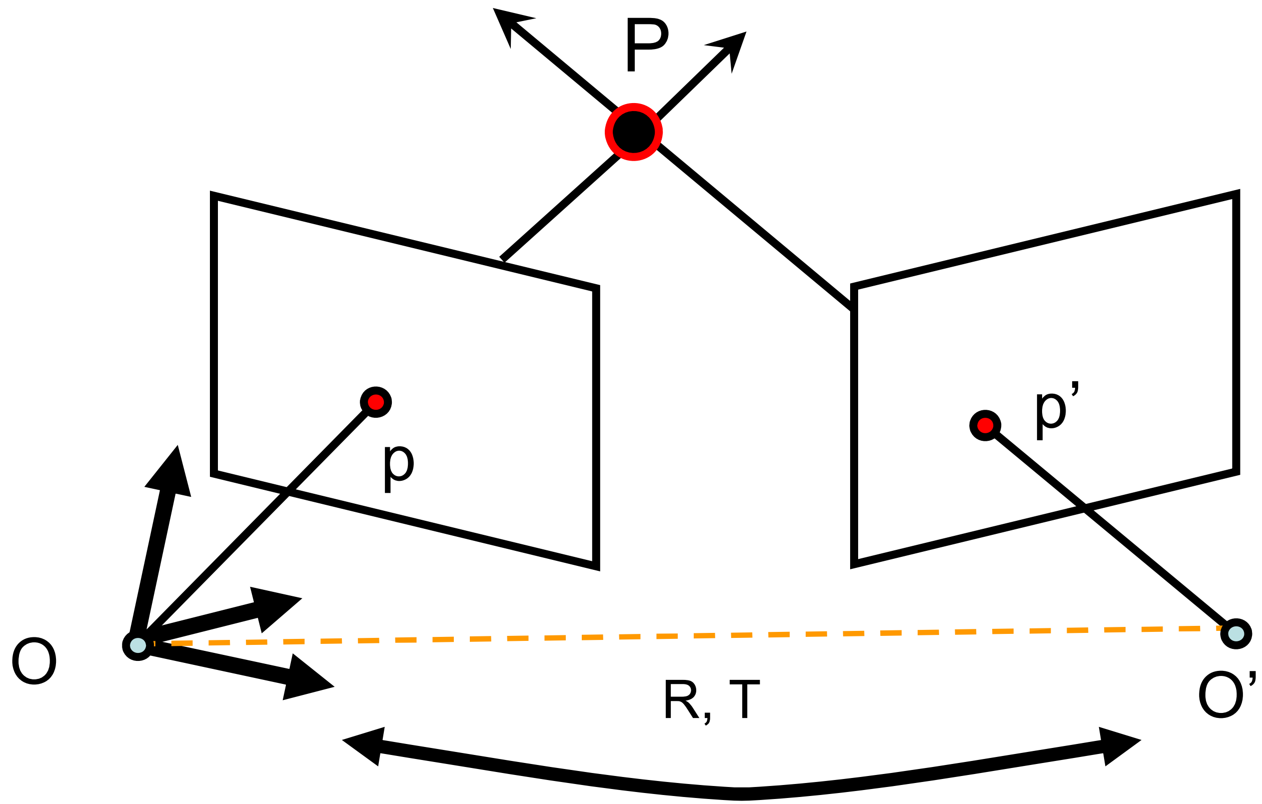
Perpendicular to epipolar plane

K_1 and K_2 are known (calibrated cameras)

Cross product as matrix multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Epipolar Constraint

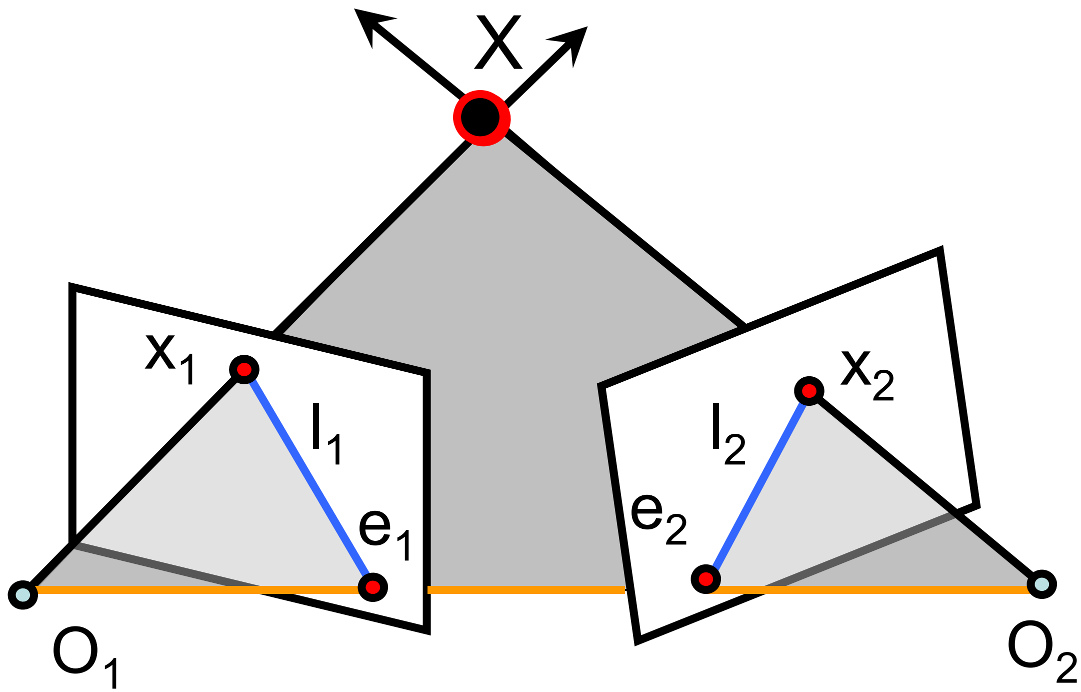


$$p^T \cdot [T \times (R\,p')] = 0 \rightarrow p^T \cdot \boxed{[T_\times] \cdot R}\, p' = 0$$

E = essential matrix

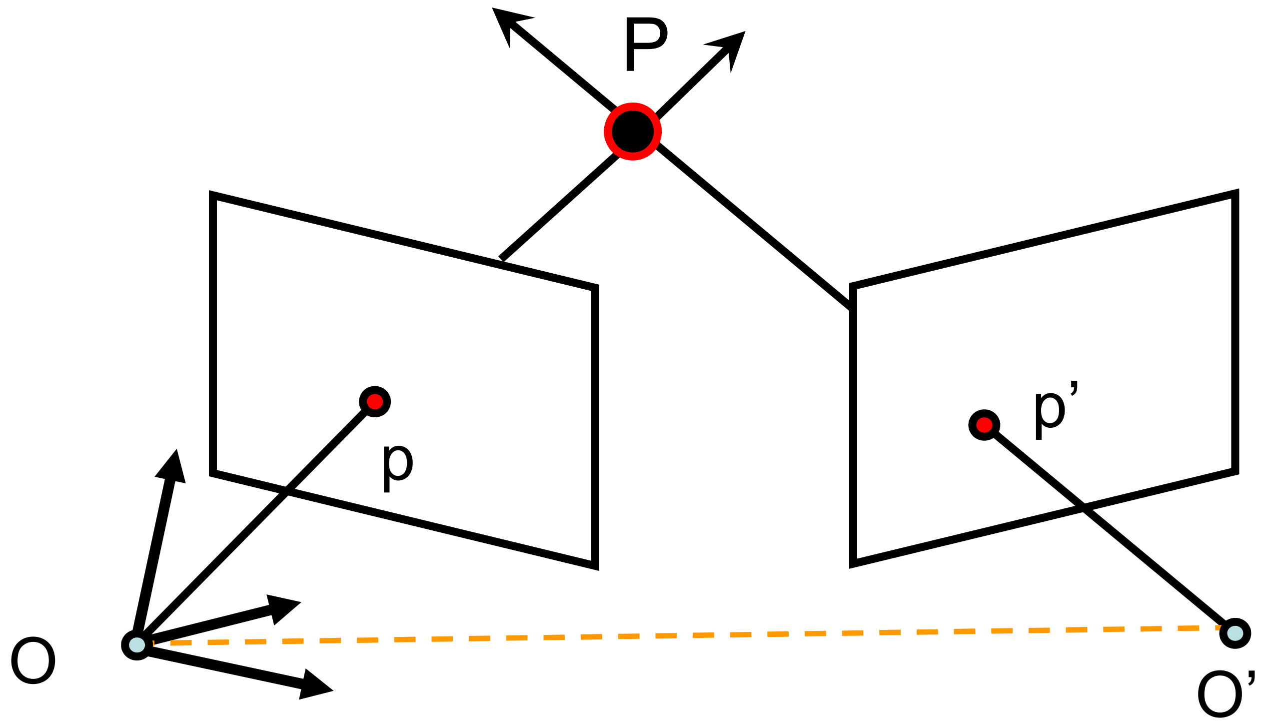
(Longuet-Higgins, 1981)

Epipolar Constraint



- $E\,x_2$ is the epipolar line associated with x_2 ($l_1 = E\,x_2$)
- $E^T x_1$ is the epipolar line associated with x_1 ($l_2 = E^T x_1$)
- E is singular (rank two)
- $E\,e_2 = 0$ and $E^T e_1 = 0$
- E is 3x3 matrix; 5 DOF

Epipolar Constraint

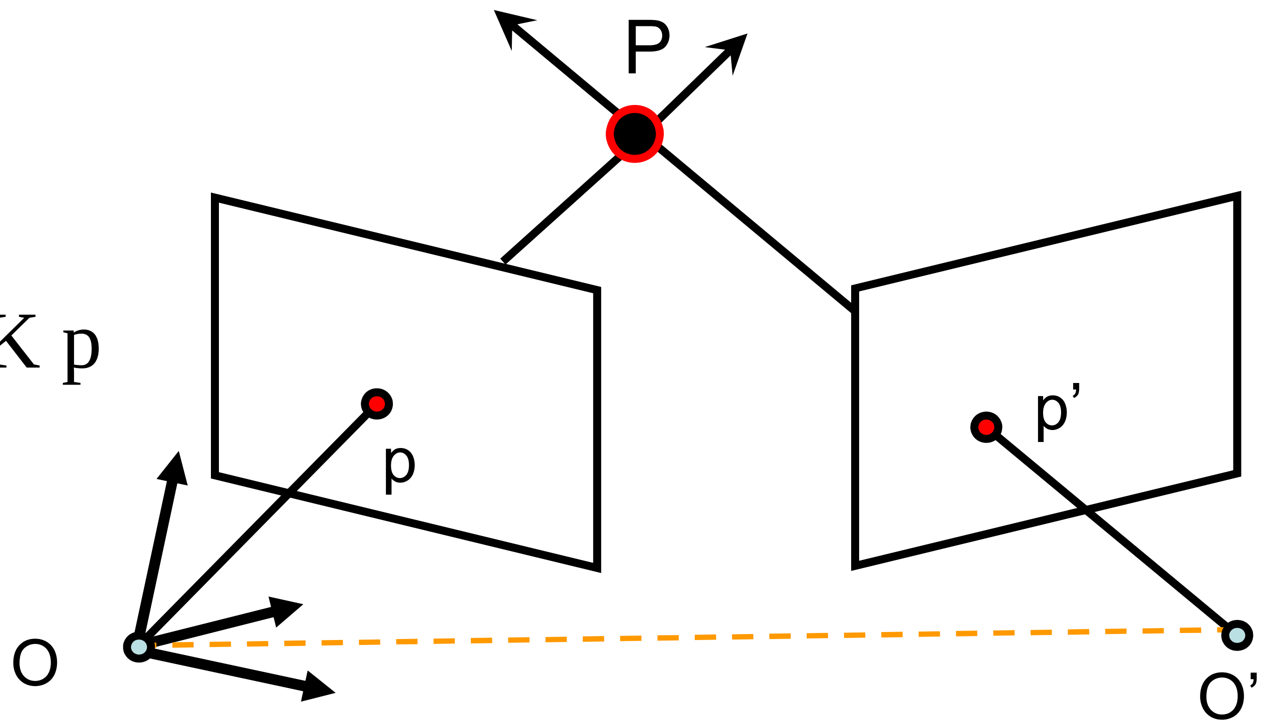


$$P \rightarrow M\,P \quad \rightarrow p = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \boxed{K}\begin{bmatrix} I & 0 \end{bmatrix}$$

unknown

Epipolar Constraint



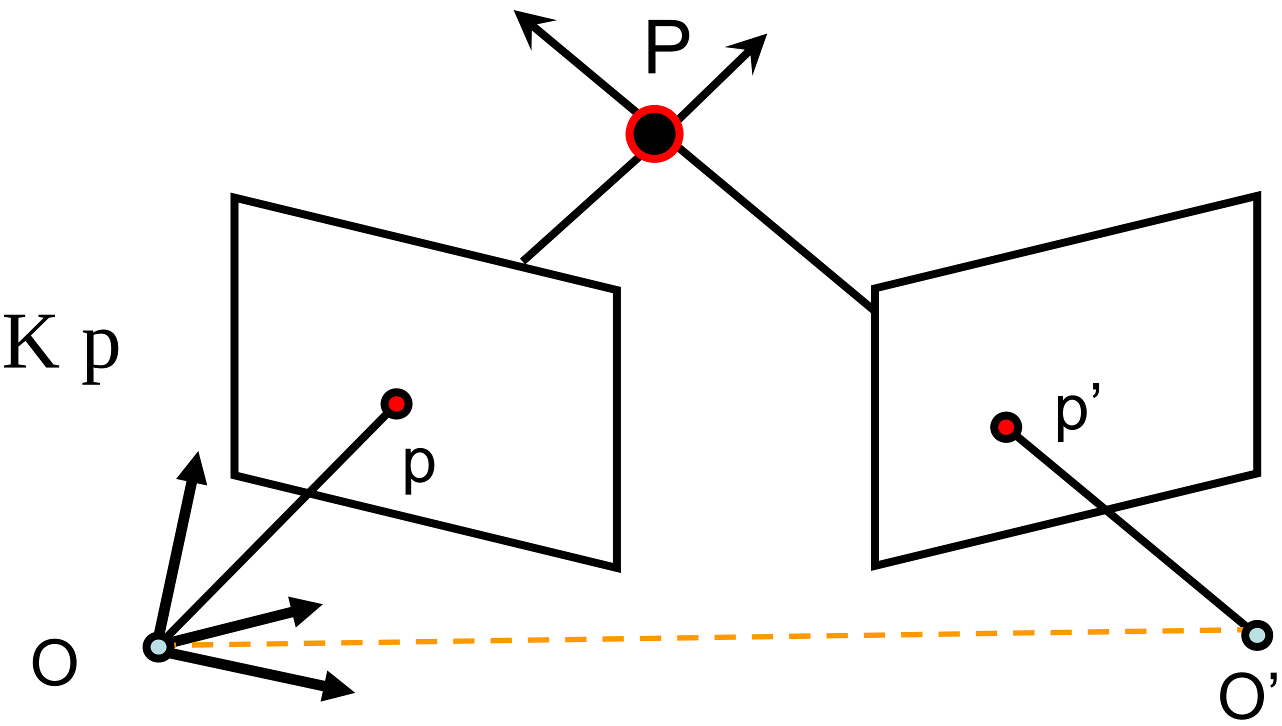
$$p \rightarrow K\,p$$

$$p' \rightarrow K'\,p'$$

$$p^T \cdot [T_\times] \cdot R\,p' = 0 \rightarrow (K^{-1}\,p)^T \cdot [T_\times] \cdot R\,K'^{-1}\,p' = 0$$

$$p^T\,\boxed{K^{-T} \cdot [T_\times] \cdot R\,K'^{-1}}\,p' = 0 \rightarrow p^T\,\boxed{F}\,p' = 0$$

Epipolar Constraint



$$\mathrm{p} \rightarrow \mathrm{K}\,\mathrm{p}$$

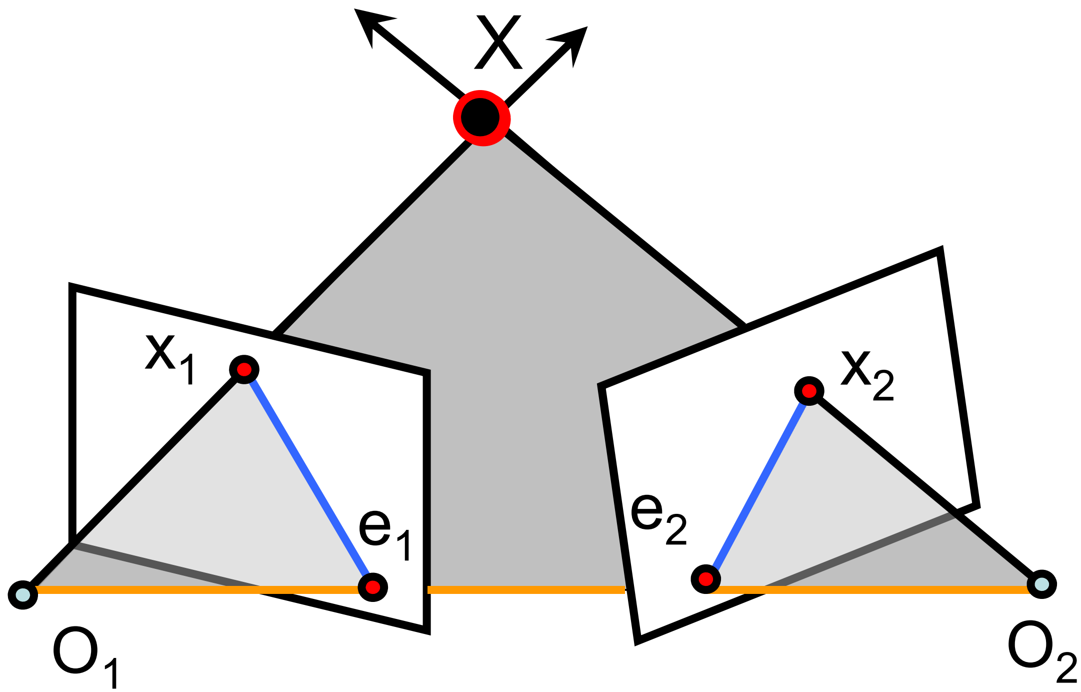
$$\mathrm{p}' \rightarrow \mathrm{K}'\,\mathrm{p}'$$

$$\boxed{\mathrm{p}^{\mathrm{T}}\,\mathrm{F}\,\mathrm{p}' = 0}$$

F = Fundamental Matrix

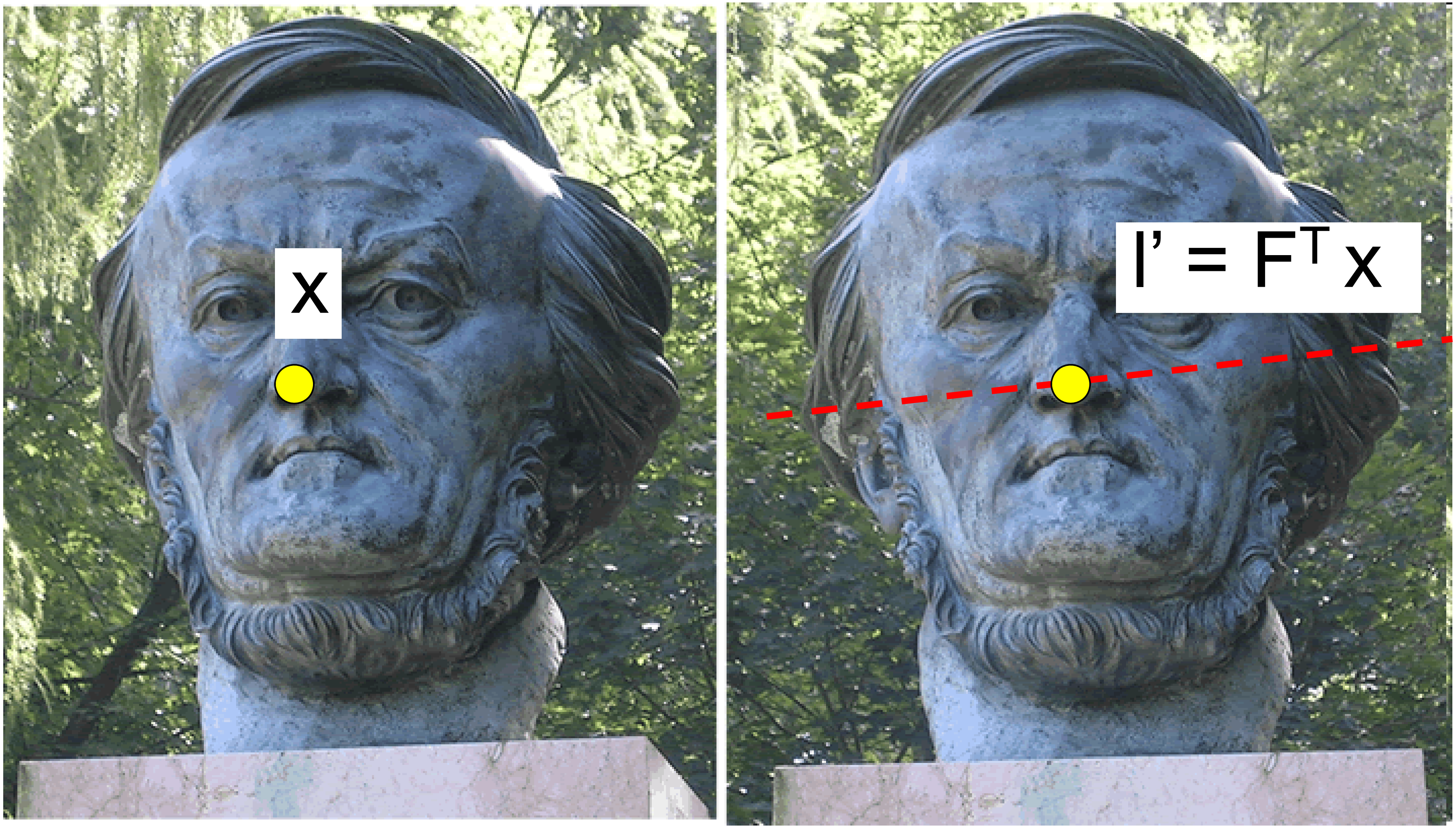
(Faugeras and Luong, 1992)

Epipolar Constraint



- F x_2 is the epipolar line associated with x_2 (l_1 = F x_2)
- $F^T x_1$ is the epipolar line associated with x_1 ($l_2 = F^T x_1$)
- F is singular (rank two)
- F $e_2 = 0$ and $F^T e_1 = 0$
- F is 3x3 matrix; 7 DOF

Why F is useful?



- Suppose F is known
- No additional information about the scene and camera is given
- Given a point on left image, how can I find the corresponding point on right image?

Why F is useful?

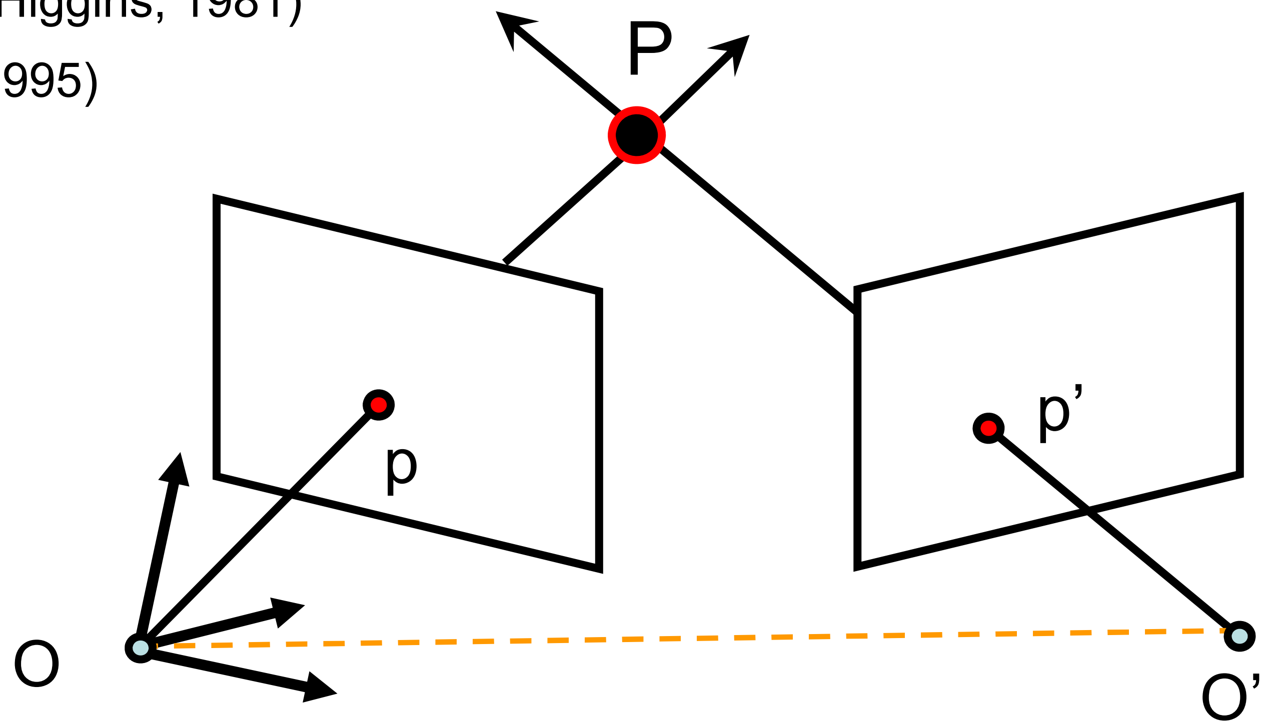
- F captures information about the epipolar geometry of 2 views + camera parameters
- MORE IMPORTANTLY: F gives constraints on how the scene changes under view point transformation (without reconstructing the scene!)
- Powerful tool in:
  - 3D reconstruction
  - Multi-view object/scene matching

Estimating F

The Eight-Point Algorithm

(Longuet-Higgins, 1981)

(Hartley, 1995)



$$P \rightarrow p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad P \rightarrow p' = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} \qquad p^{T}\, F\, p' = 0$$

Estimating F

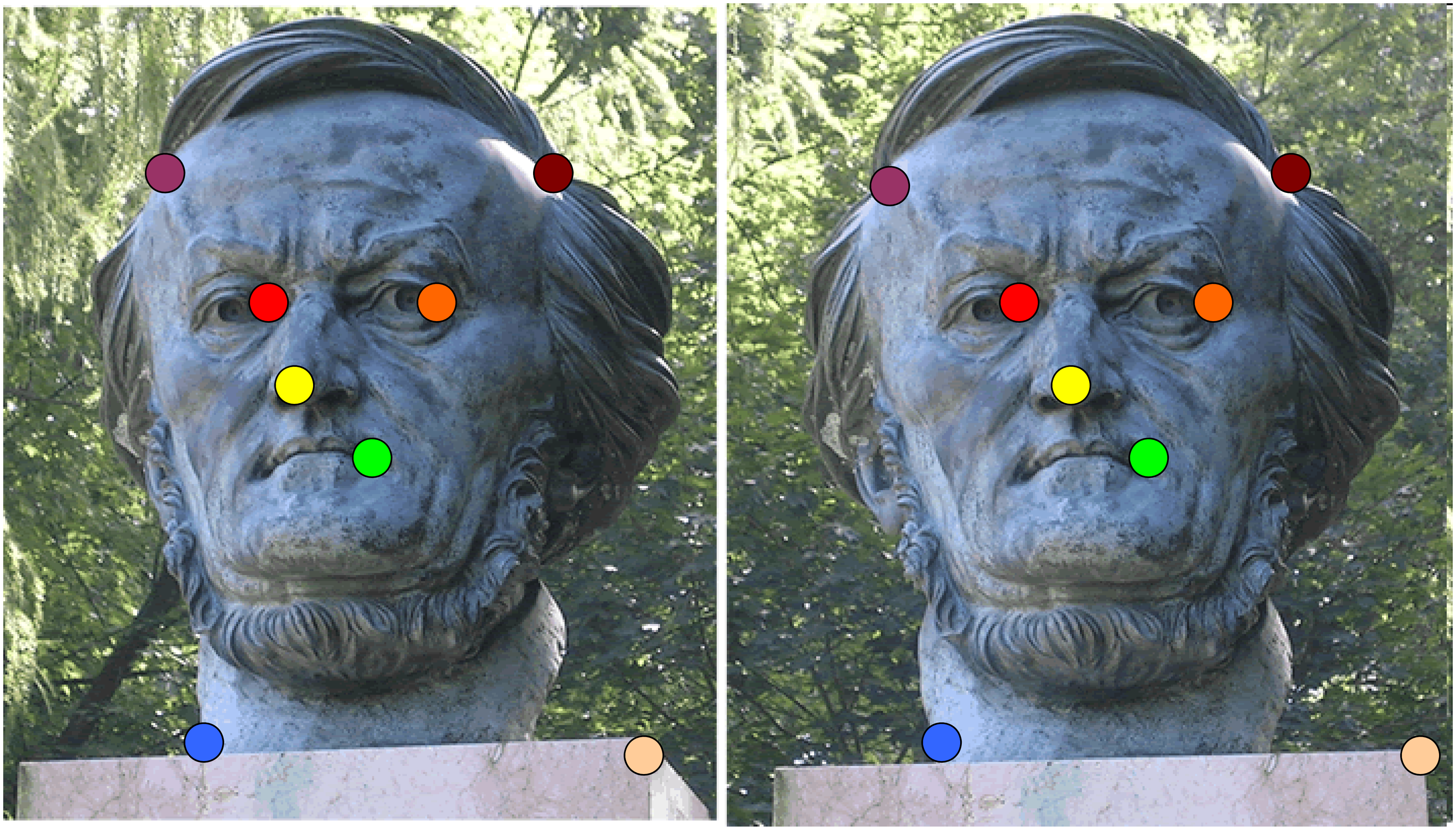
$$p^T F p' = 0 \quad \Rightarrow$$

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$\Rightarrow \quad (uu', uv', u, vu', vv', v, u', v', 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

Let's take 8 corresponding points

Estimating F



Estimating F

$$\mathbf{W}\ \begin{pmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' & 1 \\ u_3u_3' & u_3v_3' & u_3 & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' & 1 \\ u_4u_4' & u_4v_4' & u_4 & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' & 1 \\ u_5u_5' & u_5v_5' & u_5 & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' & 1 \\ u_6u_6' & u_6v_6' & u_6 & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' & 1 \\ u_7u_7' & u_7v_7' & u_7 & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' & 1 \\ u_8u_8' & u_8v_8' & u_8 & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8' & 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} \mathbf{f} = 0$$

- Homogeneous system $\mathbf{W}\,\mathbf{f} = 0$
- Rank 8 → A non-zero solution exists (unique)
- If N>8 → Lsq. solution by SVD! → $\hat{\mathrm{F}}$

$\|\mathbf{f}\| = 1$

Rank-2 constraint

$$p^T \hat{F} p' = 0$$

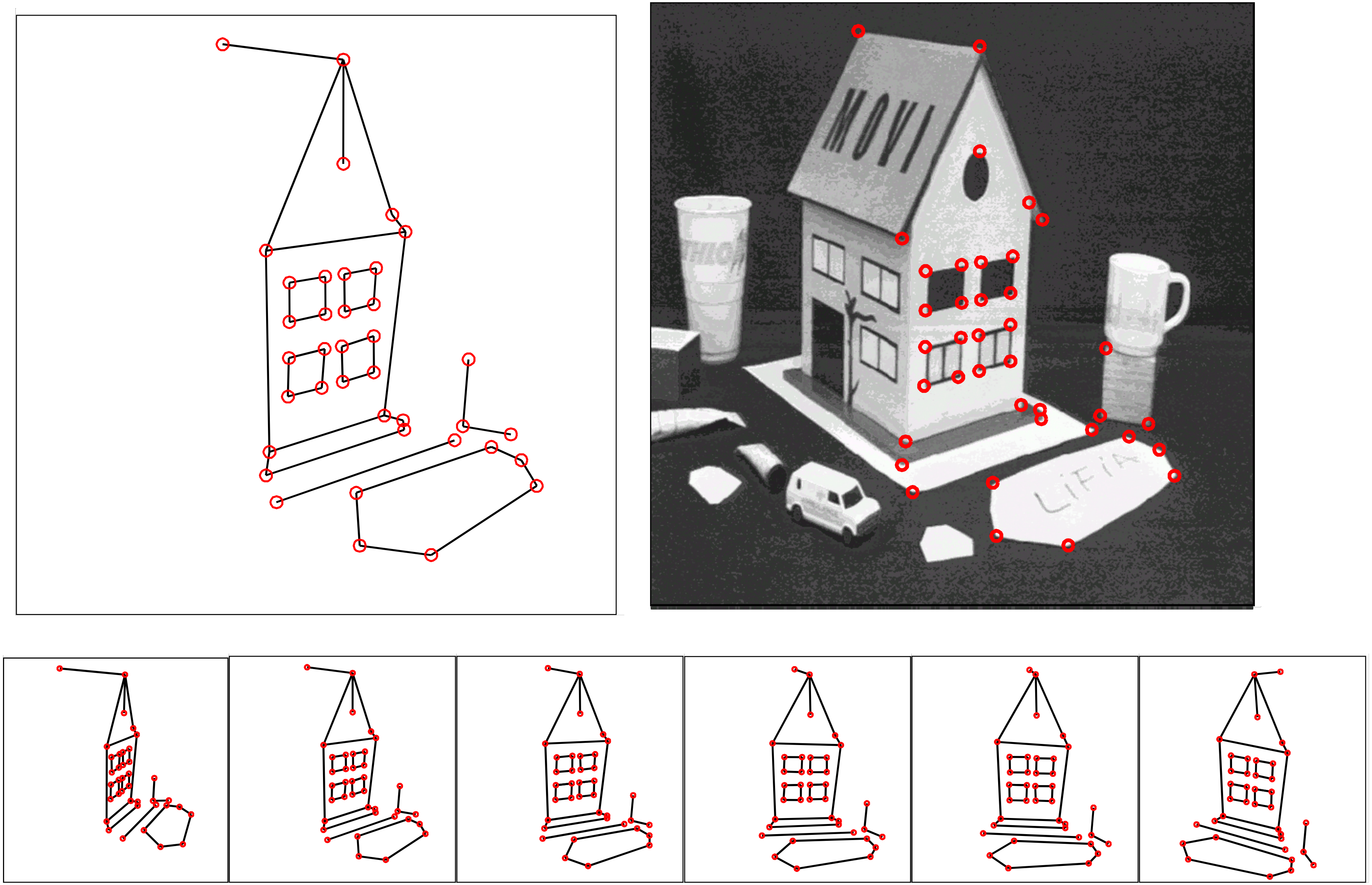
The estimated $\hat{F}$ may have full rank ($\det(\hat{F}) \neq 0$)
(F should have rank=2 instead)

Find F that minimizes $\left\| F - \hat{F} \right\| = 0$

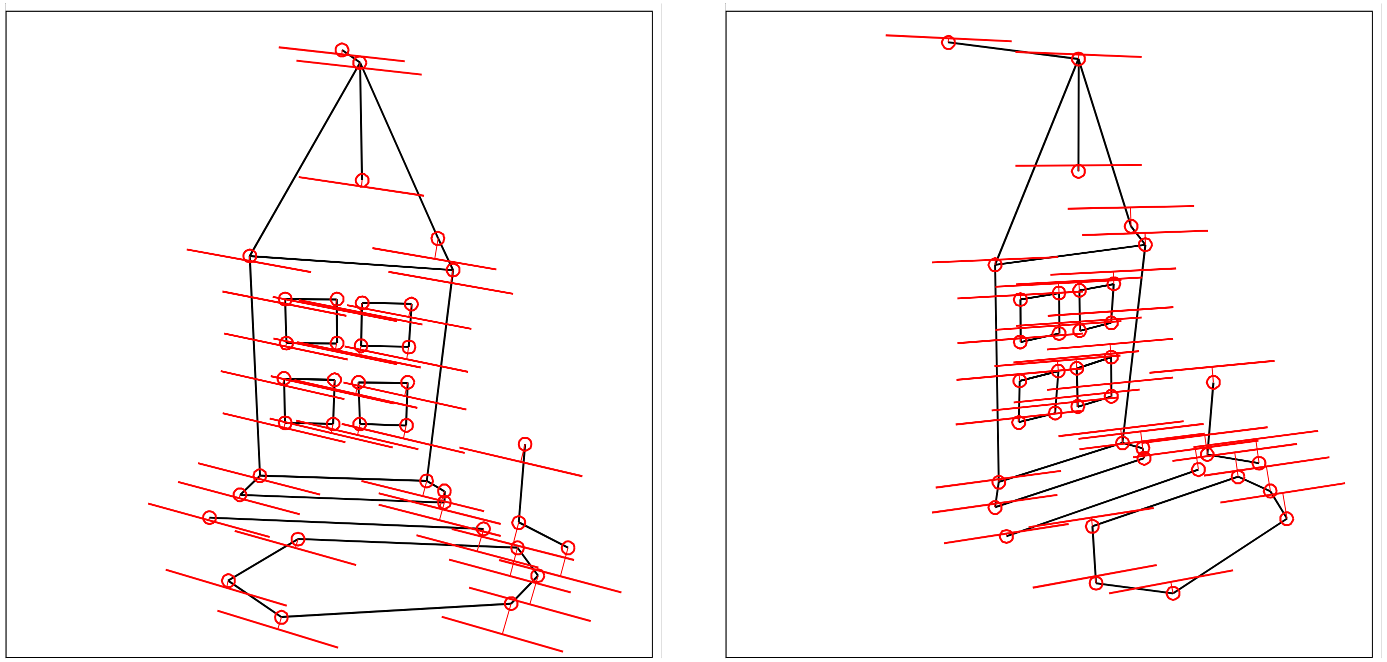
Frobenius norm

Subject to det(F)=0

SVD (again!) can be used to solve this problem

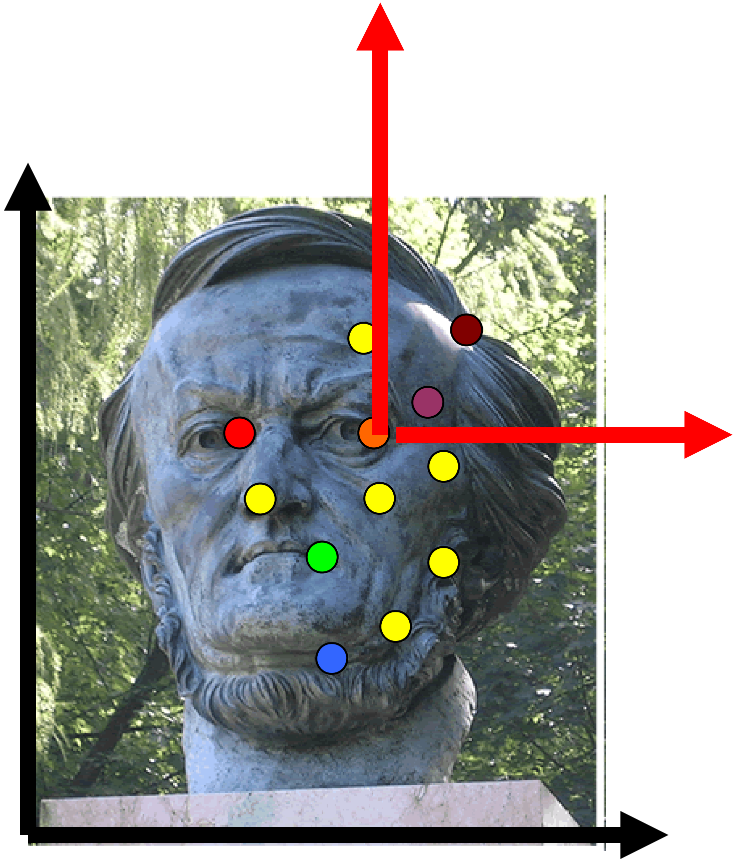


Data courtesy of R. Mohr and B. Boufama.



Mean errors:
10.0pixel
9.1pixel

Normalization



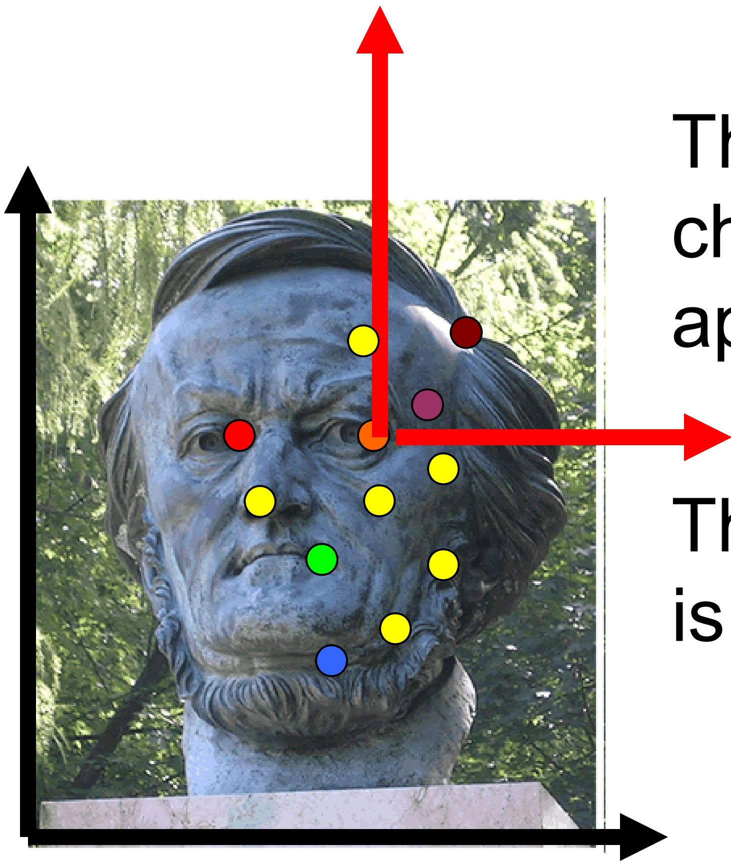
Is the accuracy in estimating F function of the ref. system in the image plane?

E.g. under similarity transformation (T = scale + translation):

$$q_i = T_i \, p_i \qquad q_i' = T_i' \, p_i'$$

Does the accuracy in estimating F change if a transformation T is applied?

Normalization



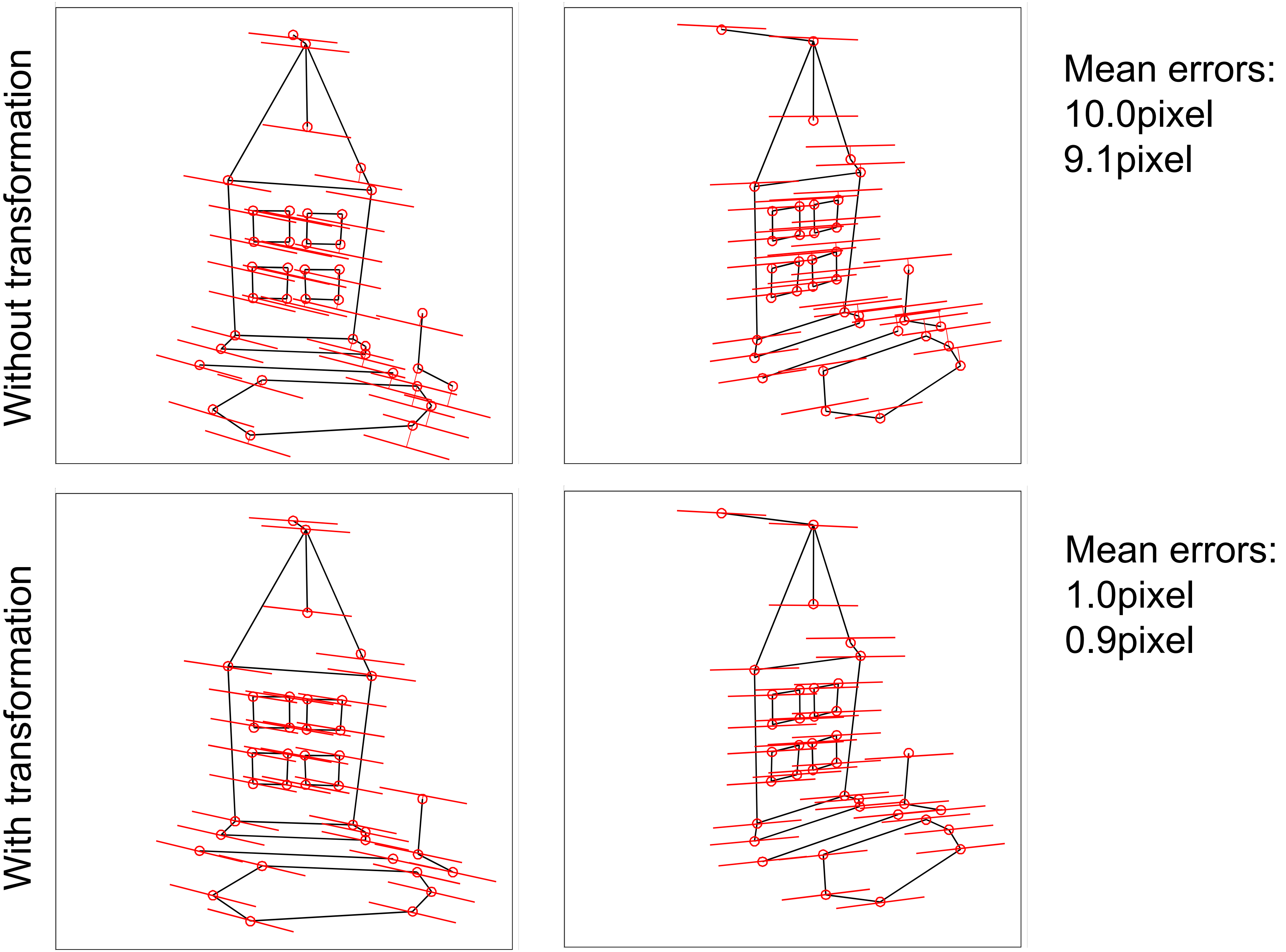
The accuracy in estimating F does change if a transformation T is applied

There exists a T for which accuracy is maximized

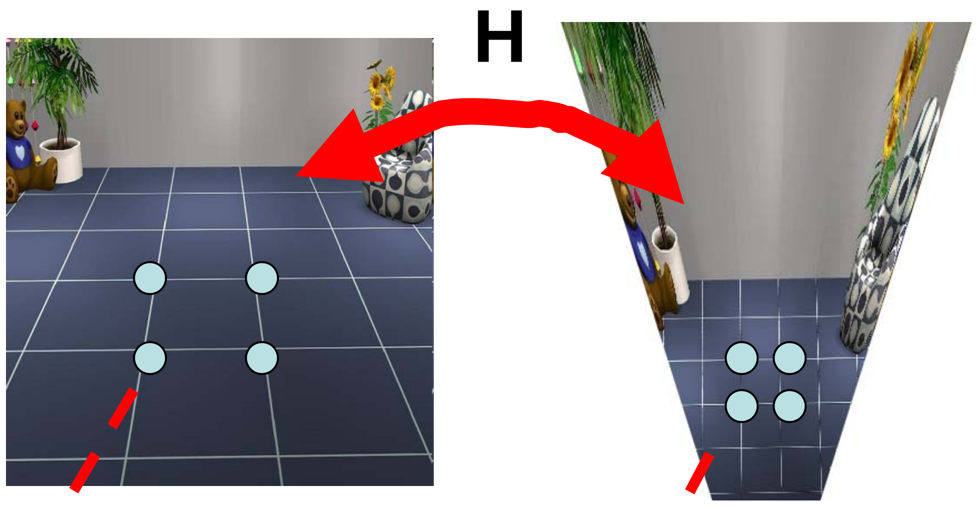
Why?

$$\mathbf{W}\,\mathbf{f} = 0, \quad \boxed{\left\|\mathbf{f}\right\| = 1} \longrightarrow \mathrm{F}$$

The constrain under which |W f| is minimized is
not invariant under similarity transformation


Without transformation
With transformation
Mean errors:
10.0pixel
9.1pixel
Mean errors:
1.0pixel
0.9pixel

Same issue for the DLT algorithm

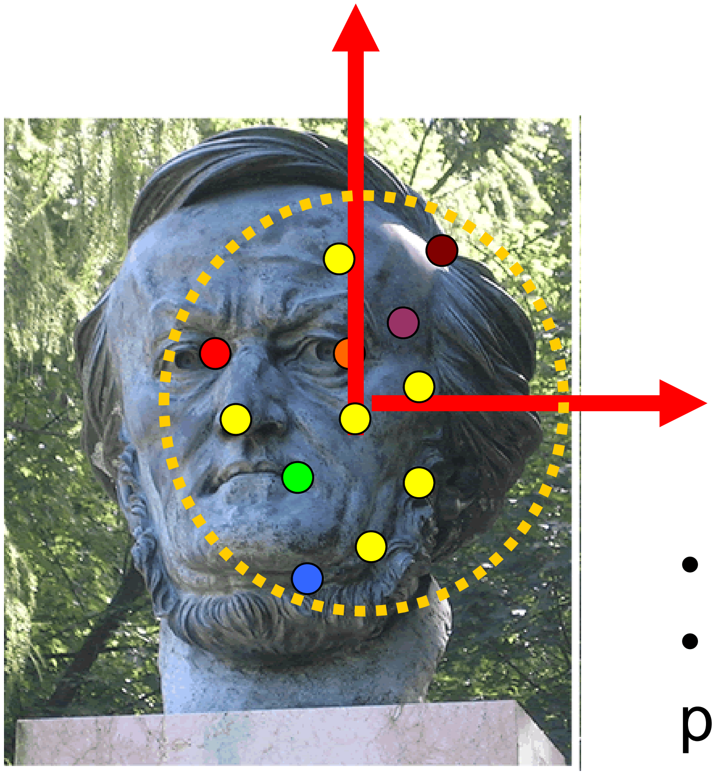


x_i x'_i

$$x'_i = H\, x_i$$

[Section 4.4 in AZ]

Normalization



Transform image coordinate system (T = translation+scaling) such that:

- Origin = centroid of image points
- Mean square distance of the data points from origin is 2 pixels

$$q_i = T_i\ p_i \qquad q_i' = T_i'\ p_i' \qquad \text{(normalization)}$$

The Normalized Eight-Point Algorithm

0. Compute T_i and T_i'

1. Normalize coordinates:

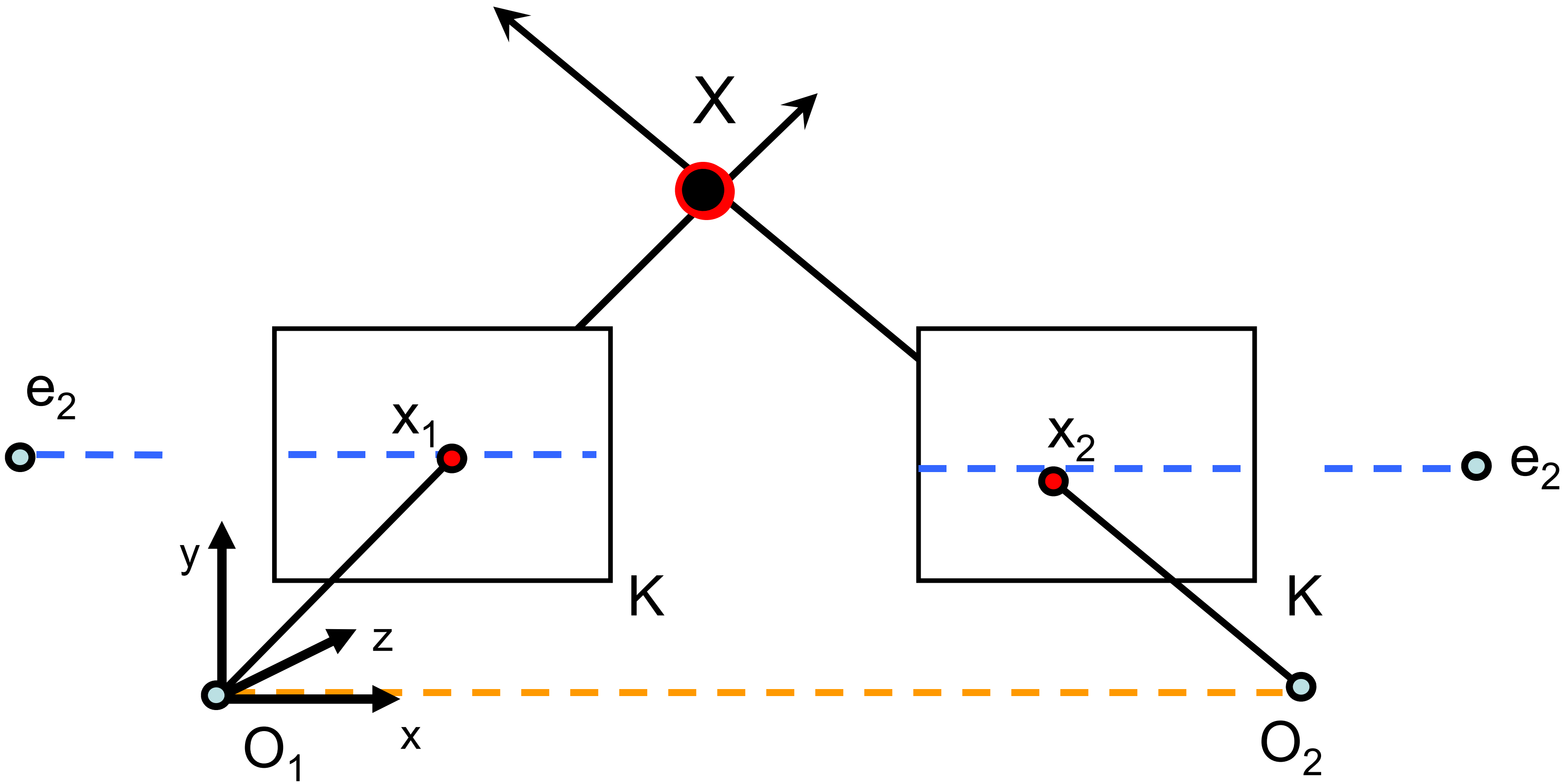
$$q_i = T_i \; p_i \qquad\qquad q_i' = T_i' \, p_i'$$

2. Use the eight-point algorithm to compute F'$_q$ from the points q_i and q'$_i$.

1. Enforce the rank-2 constraint. $\rightarrow F_q$ $\begin{cases} q^T \; F_q \; q' = 0 \\ \det(\, F_q \,) = 0 \end{cases}$

2. De-normalize F_q: $F = T'^T F_q T$

Example: Parallel image planes



K_1=K_2 = known

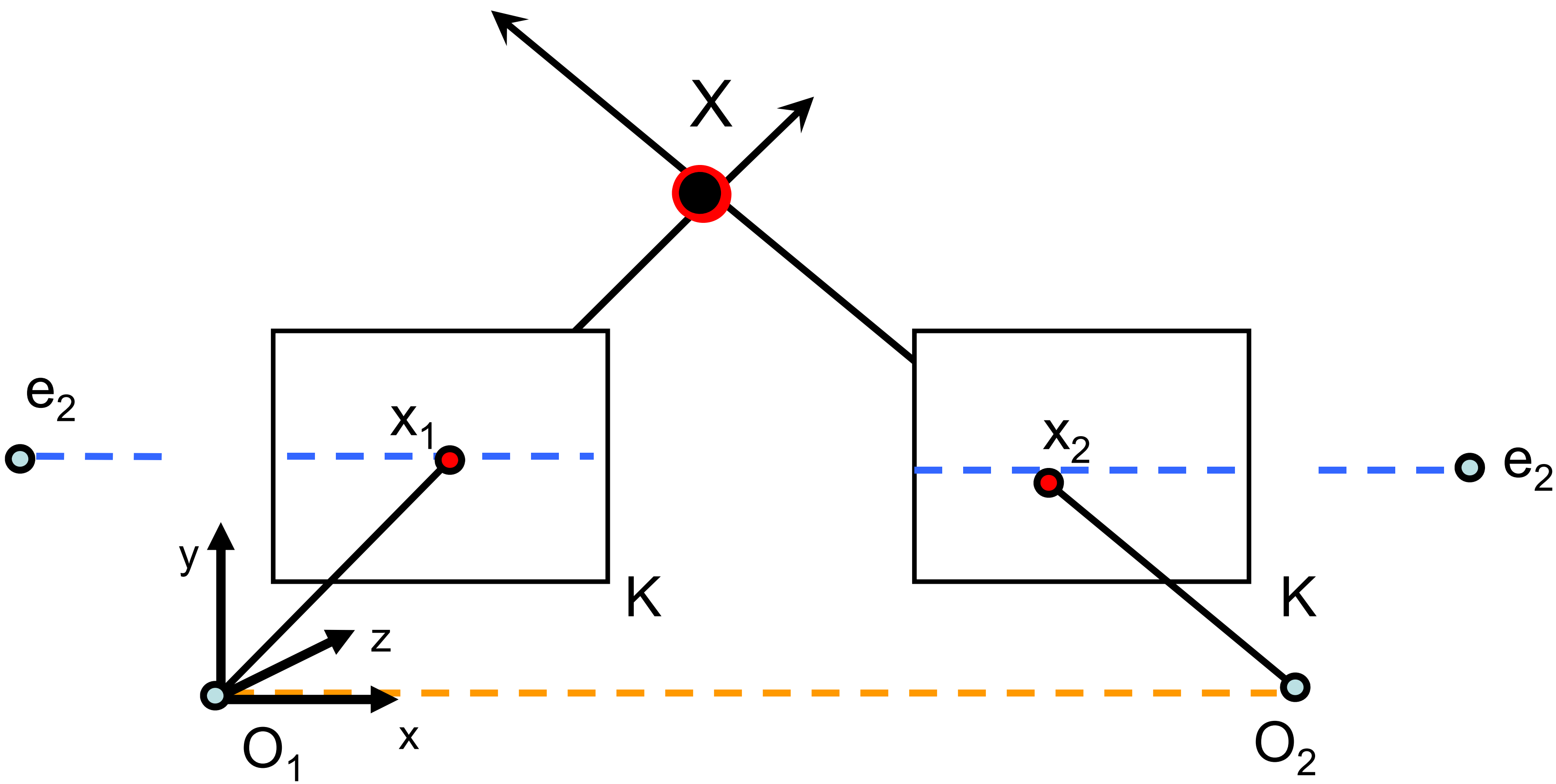
x parallel to O_1O_2

$\boxed{E=?}$

Hint :

$R = I \qquad t = (T, 0, 0)$

Example: Parallel image planes



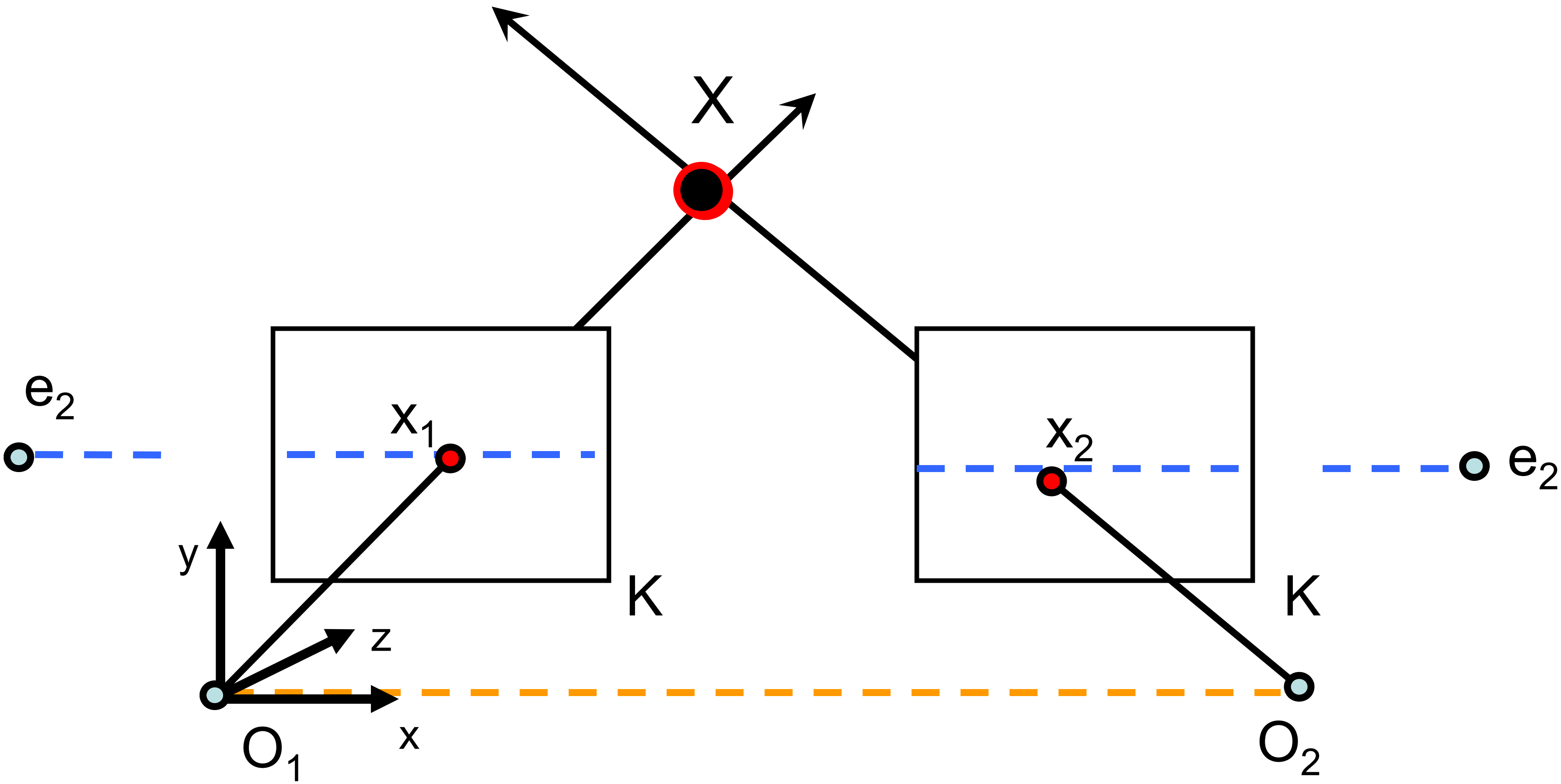
K_1=K_2 = known

x parallel to O_1O_2

$\boxed{E=?}$

$$E = [t_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

Example: Parallel image planes

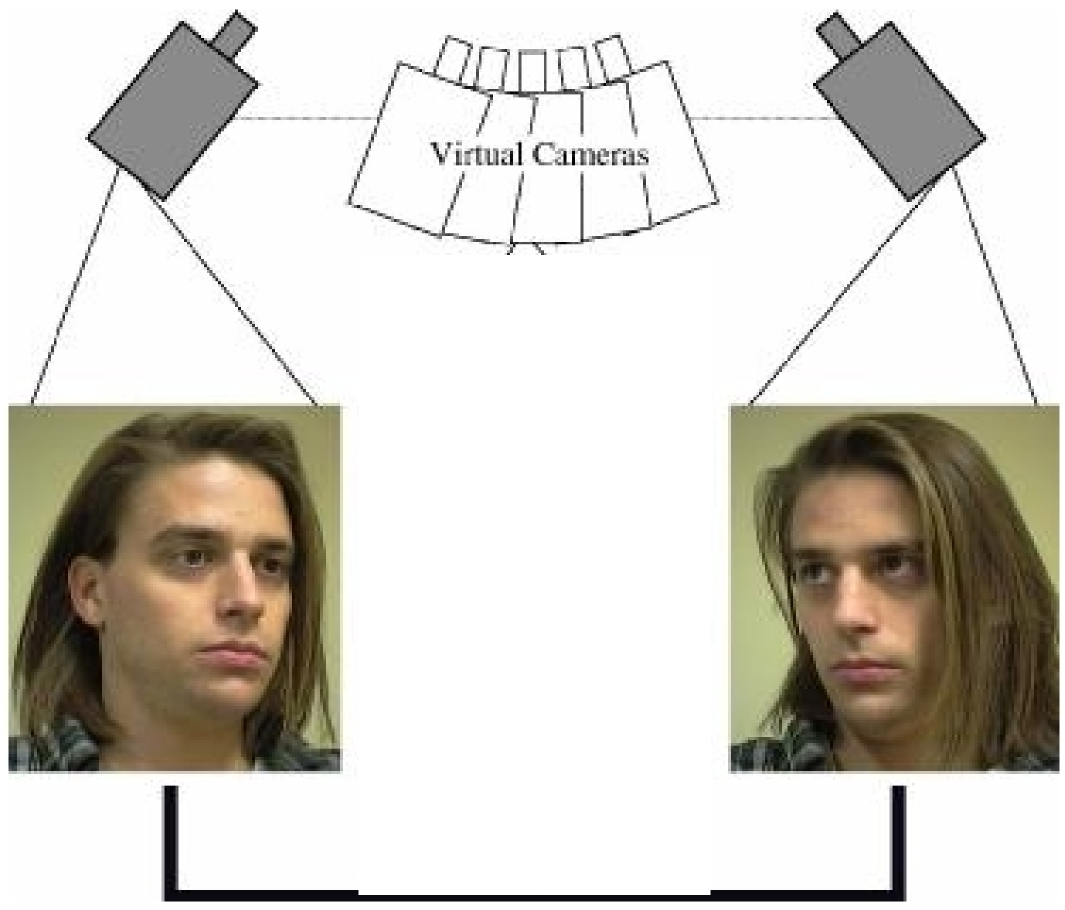


Rectification: making two images “parallel”

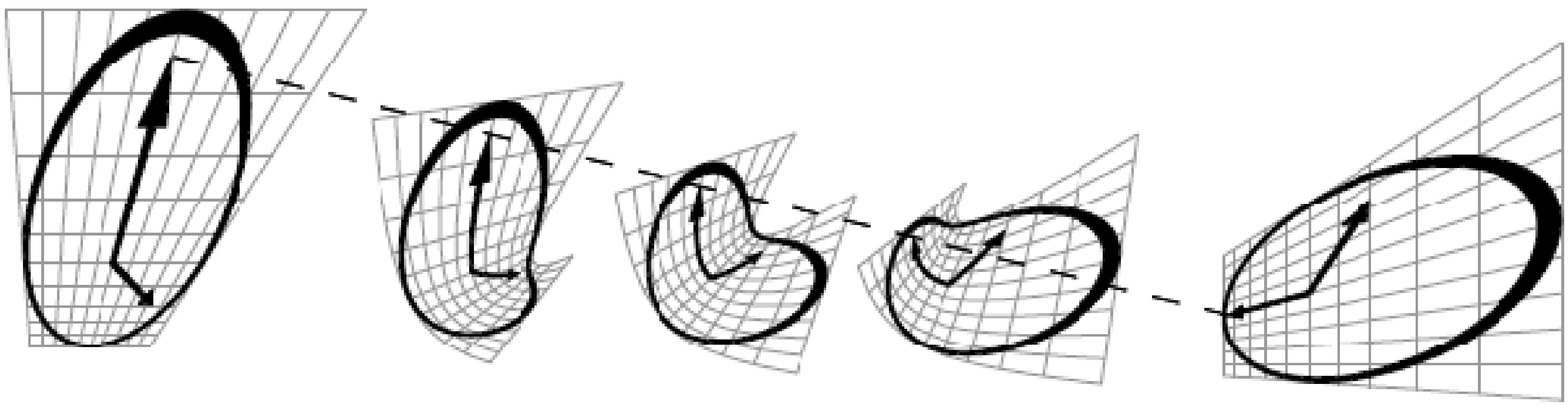
Why it is useful? Epipolar constraint $\rightarrow$ y = y’

Application: view morphing

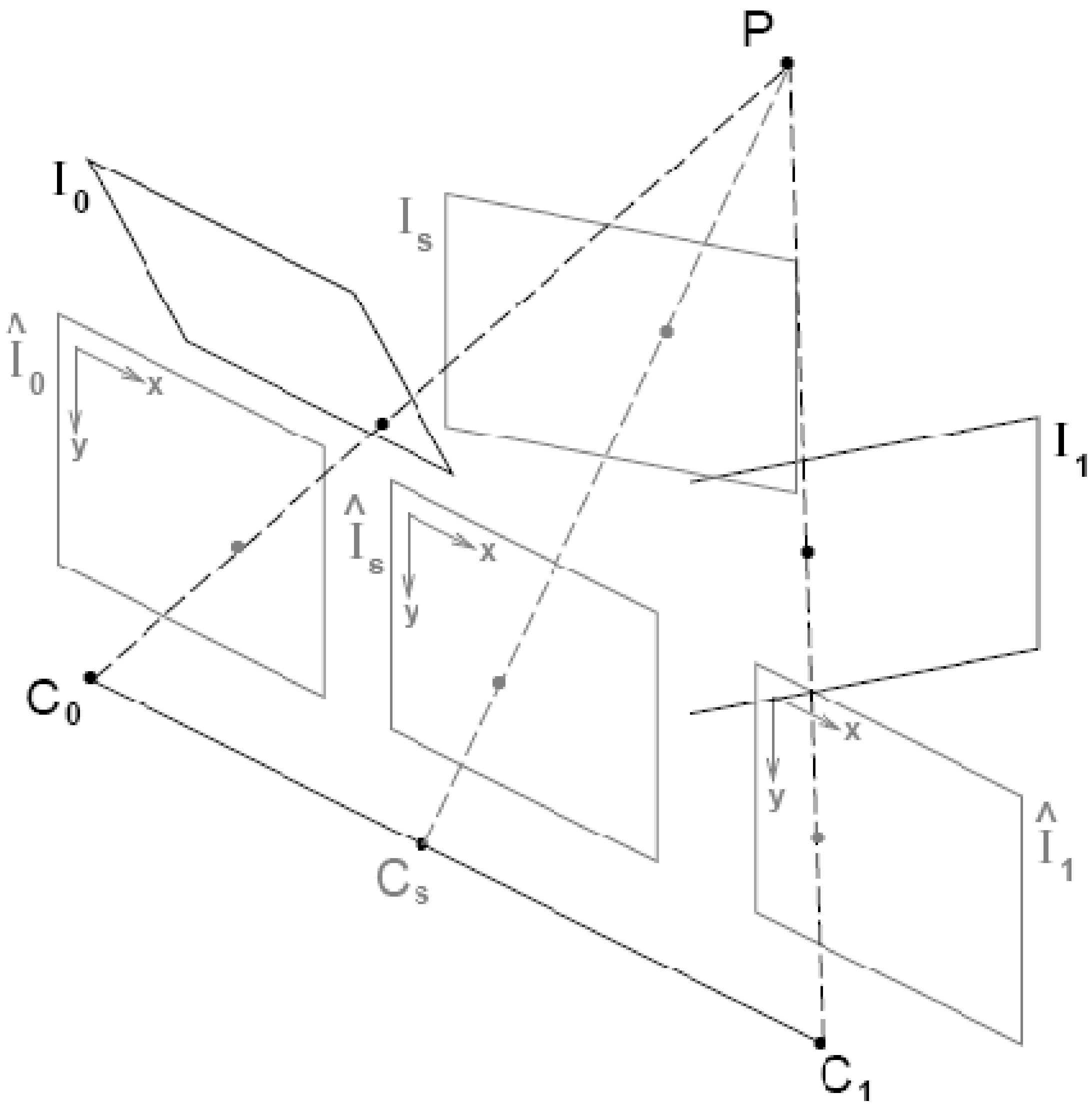
S. M. Seitz and C. R. Dyer, *Proc. SIGGRAPH 96*, 1996, 21-30



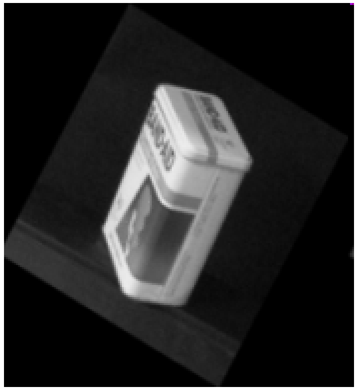
Morphing without using geometry

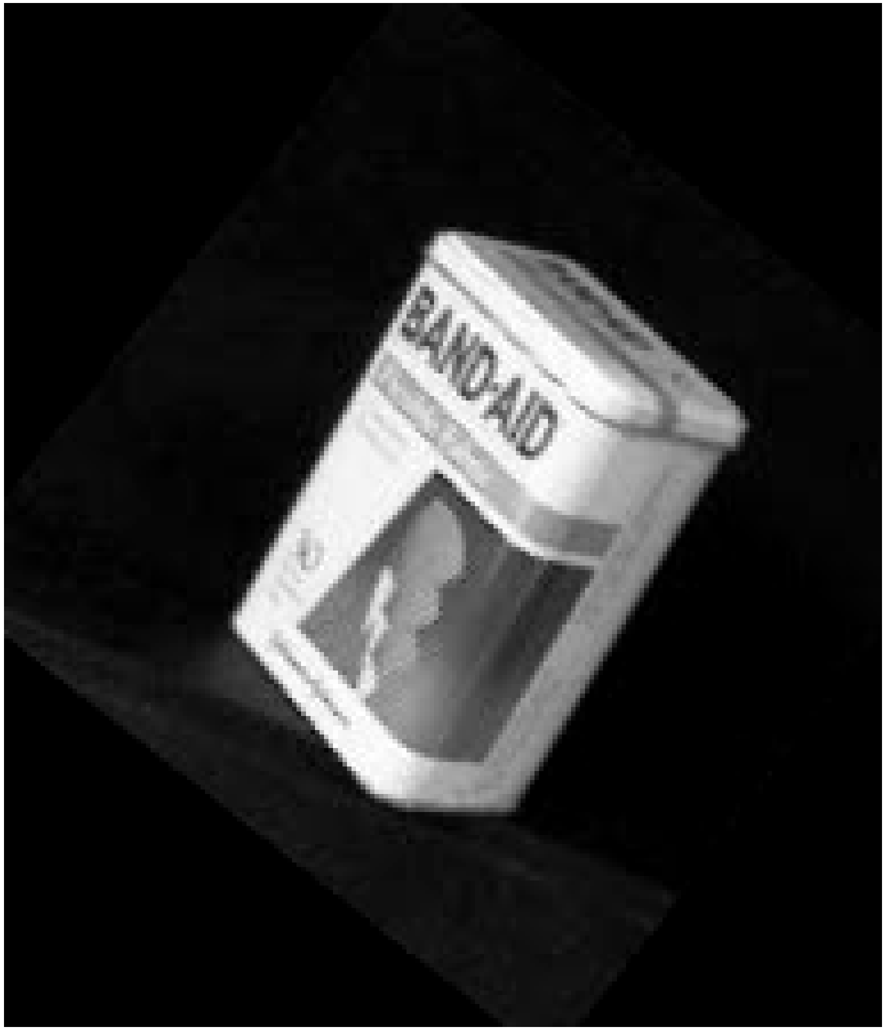


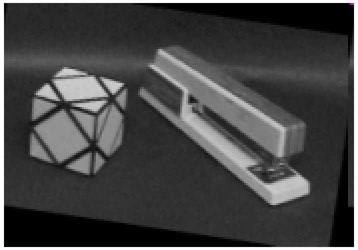
Rectification

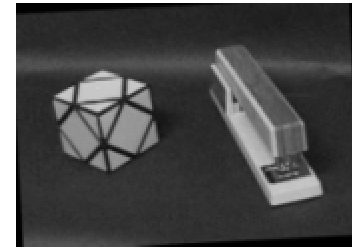


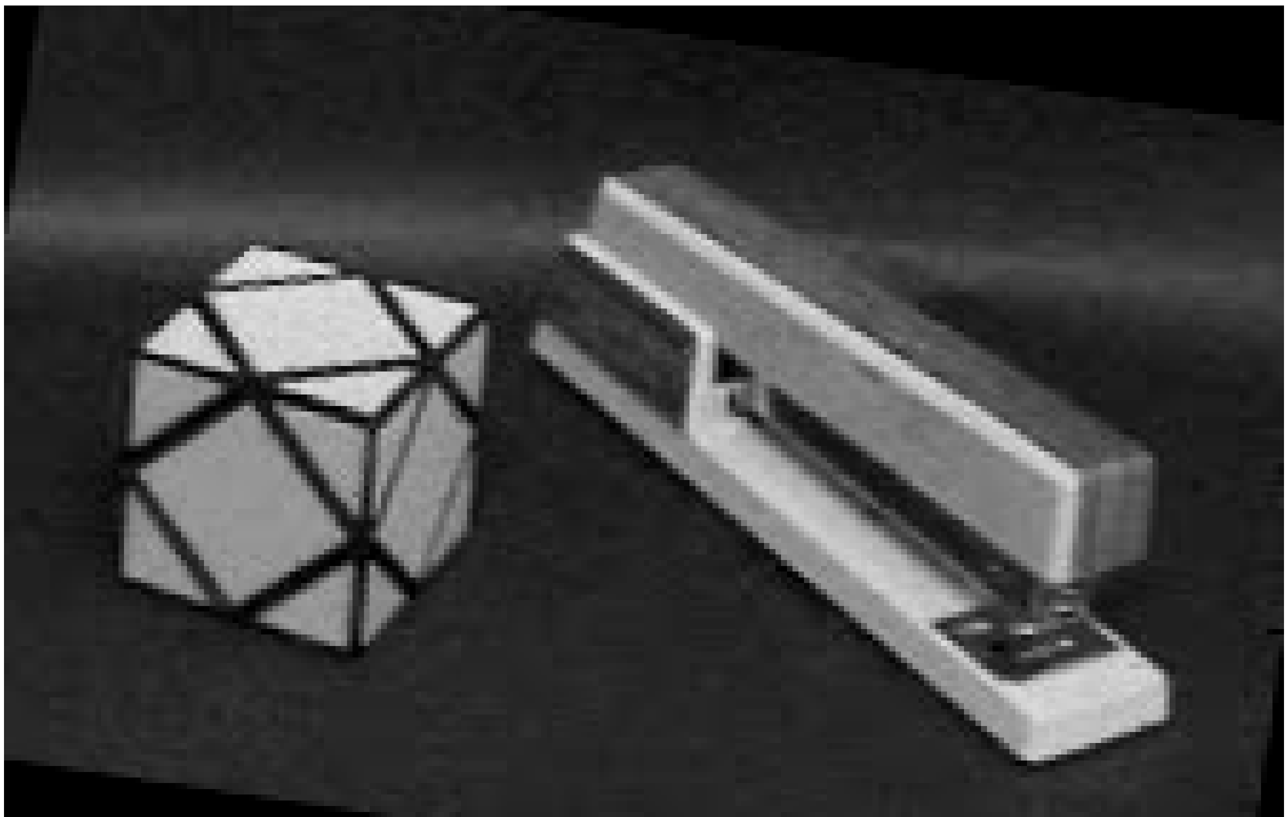

BAND-AID

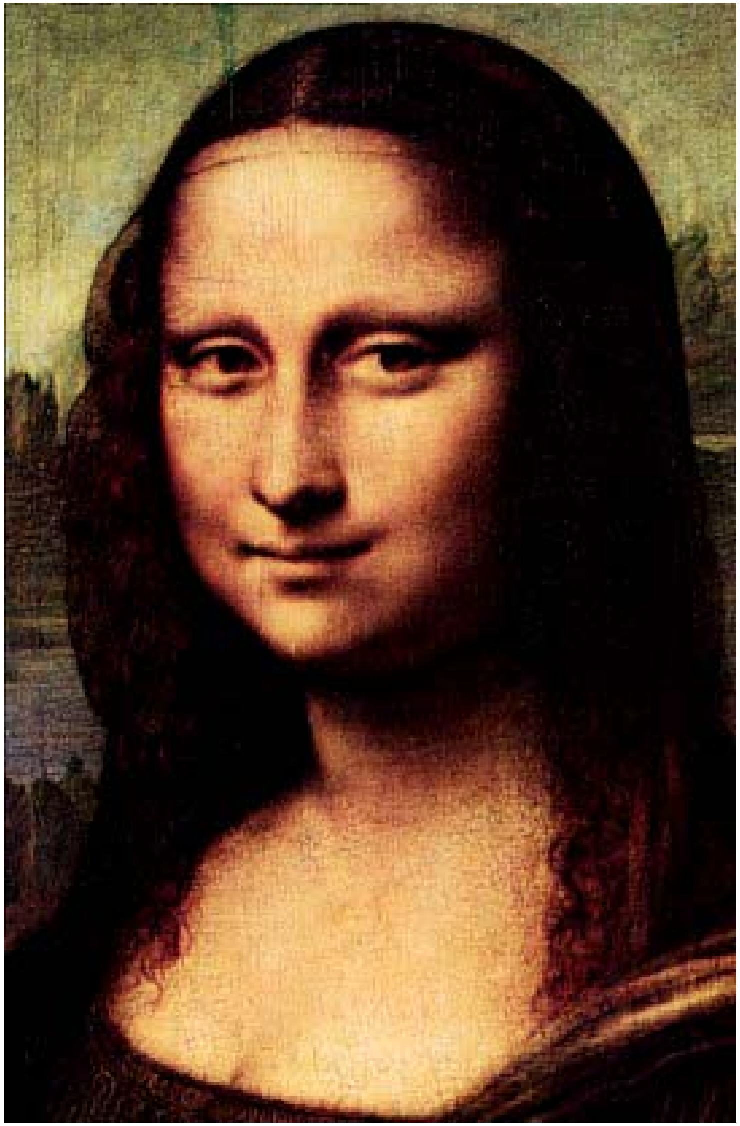



BAND-AID









From its reflection!

Next lecture:

Reconstruction using stereo systems