



# Feature-based Alignment

## Chapter 6 R. Szelisky

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**Slide Credits:** Trevor Darrell, Berkeley (C280 CV Course), Steve Seitz, Kristen Grauman, Alyosha Efros, L. Lazebnik, Marc Pollefeys

Original Slides Prof. Trevor Darrel (08Alignment, 06LocalFeatures): please visit  
<http://www.eecs.berkeley.edu/~trevor/CS280Notes/>



# Today: Alignment



(a)



(b)



(c)



# Motivation: Mosaics

- Getting the whole picture
  - Consumer camera:  $50^\circ \times 35^\circ$

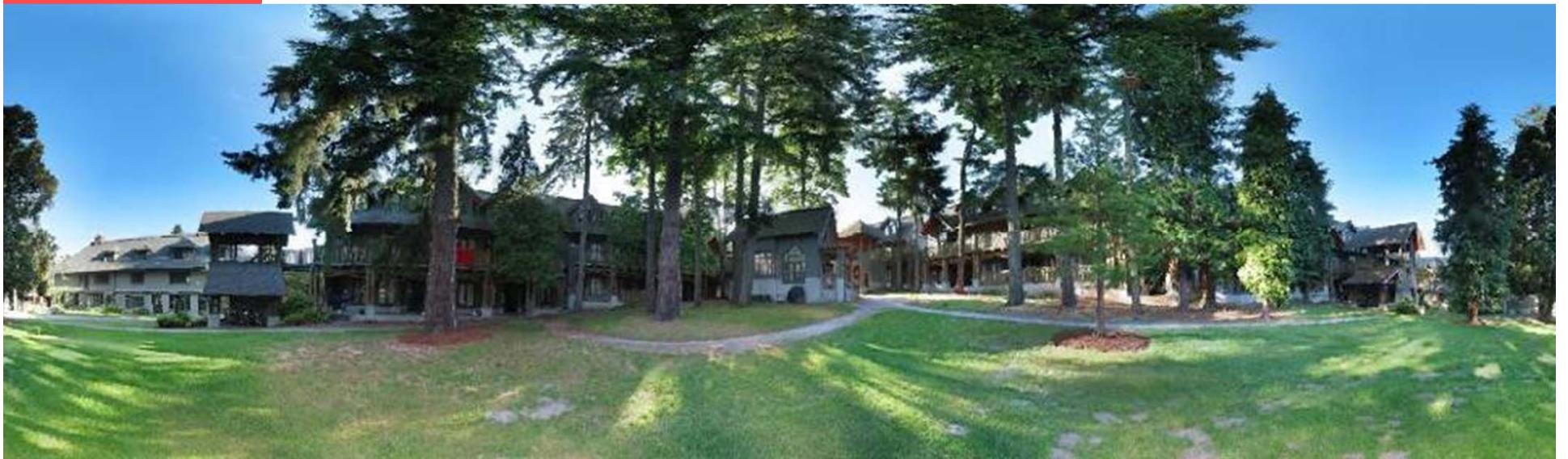


Slide from Brown & Lowe 2003



# Motivation: Mosaics

- Getting the whole picture
  - Consumer camera:  $50^\circ \times 35^\circ$
  - Human Vision:  $176^\circ \times 135^\circ$



Slide from Brown & Lowe 2003



# Motion models



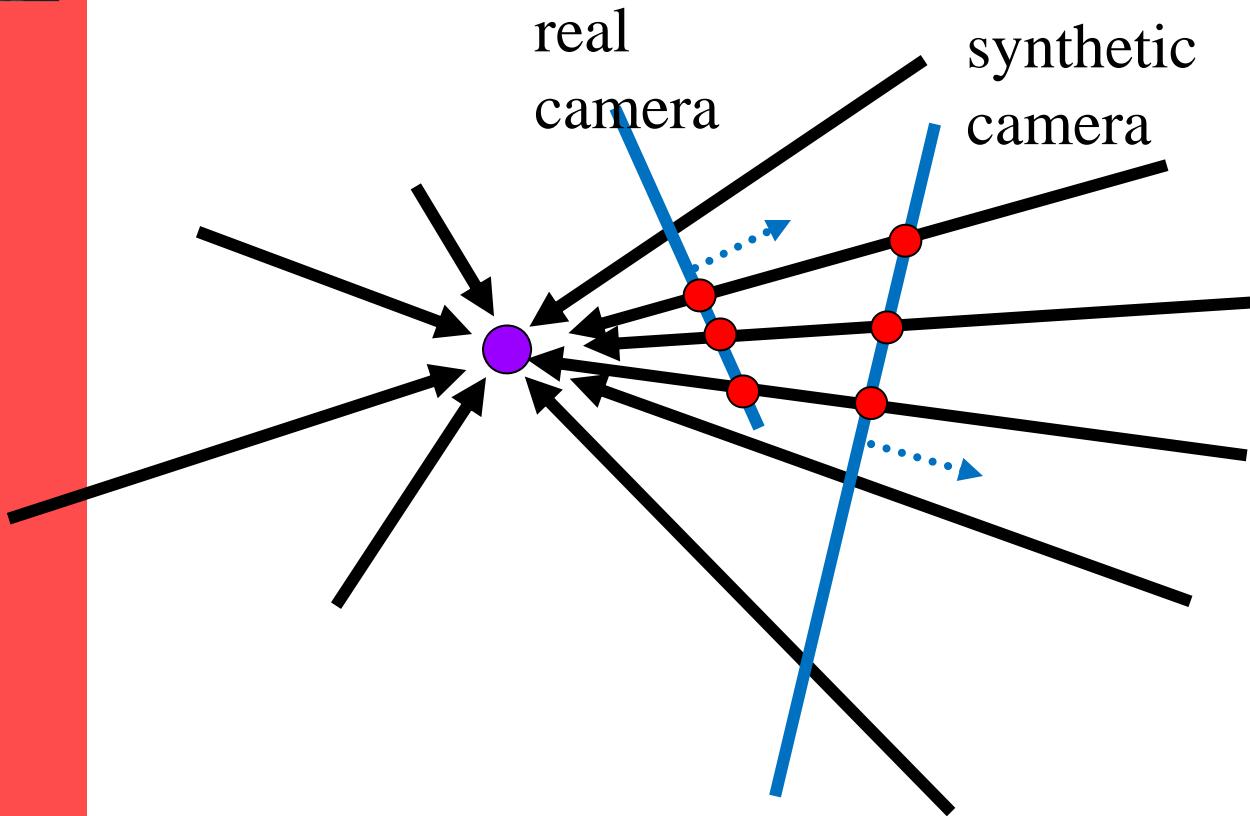
# Motion models

- What happens when we take two images with a camera and try to align them?
  - translation?
  - rotation?
  - scale?
  - affine?
  - perspective?





# Panoramas: generating synthetic views

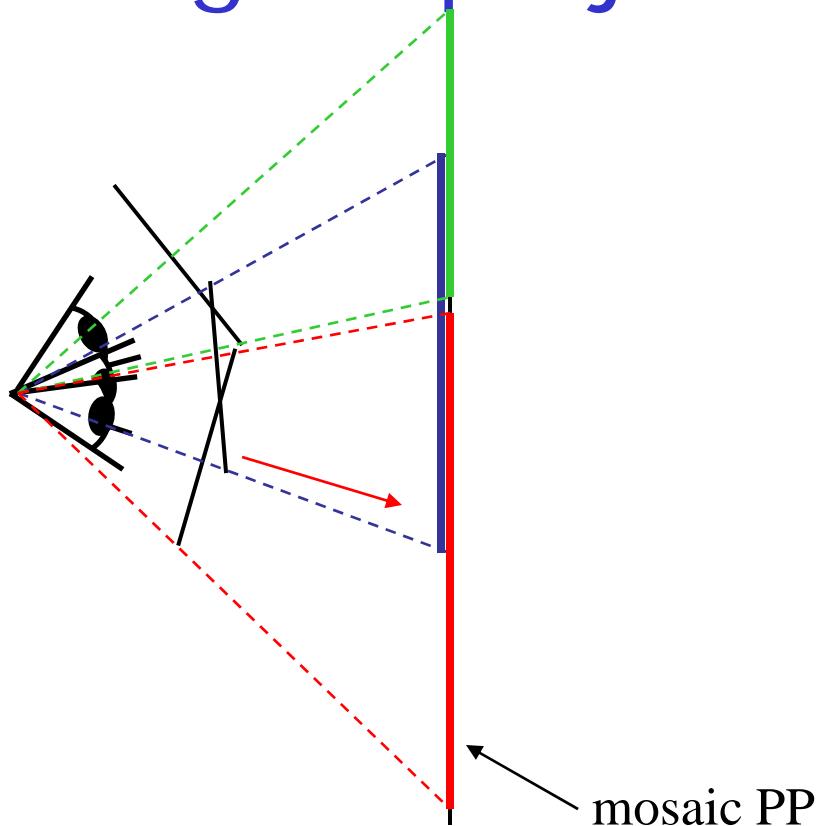


Can generate any synthetic camera view  
as long as it has **the same center of projection!**

Source: Alyosha Efros



# Image reprojection

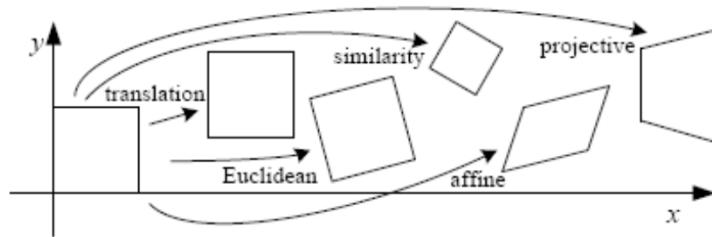


- The mosaic has a natural interpretation in 3D
  - The images are reprojected onto a common plane
  - The mosaic is formed on this plane
  - Mosaic is a *synthetic wide-angle camera*

Source: Steve Seitz



# Motion models

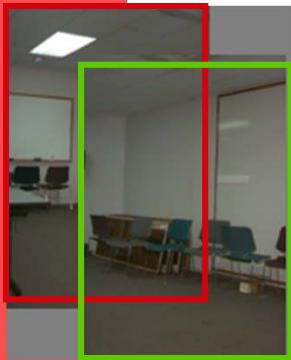


Translation

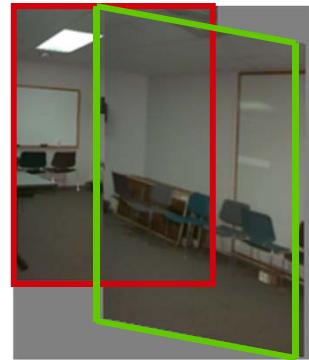
Affine

Perspective

3D rotation



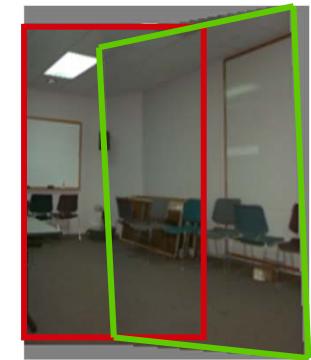
2 unknowns



6 unknowns



8 unknowns



3 unknowns

Szeliski



## 2D coordinate transformations

- translation:  $\mathbf{x}' = \mathbf{x} + \mathbf{t}$   $\mathbf{x} = (x, y)$
- rotation:  $\mathbf{x}' = \mathbf{R} \mathbf{x} + \mathbf{t}$
- similarity:  $\mathbf{x}' = s \mathbf{R} \mathbf{x} + \mathbf{t}$
- affine:  $\mathbf{x}' = \mathbf{A} \mathbf{x} + \mathbf{t}$
- perspective:  $\underline{\mathbf{x}}' \cong \mathbf{H} \underline{\mathbf{x}}$   $\underline{\mathbf{x}} = (x, y, 1)$   
( $\underline{\mathbf{x}}$  is a *homogeneous* coordinate)
- These all form a nested *group* (closed w/ inv.)



# Basic 2D Transformations

- Basic 2D transformations as  $3 \times 3$  matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear



# 2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

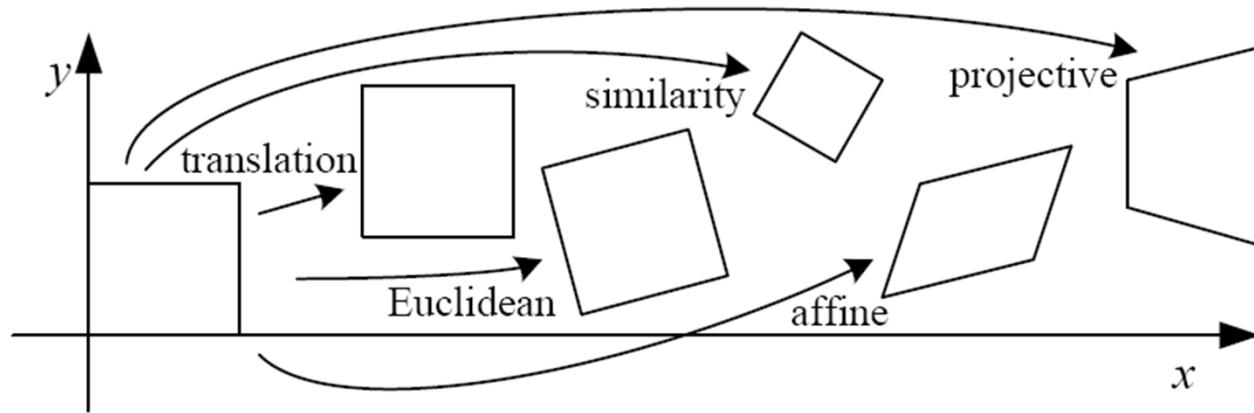
- Affine transformations are combinations of
  - ...
    - Linear transformations, and
    - Translations
- Parallel lines remain parallel



# Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

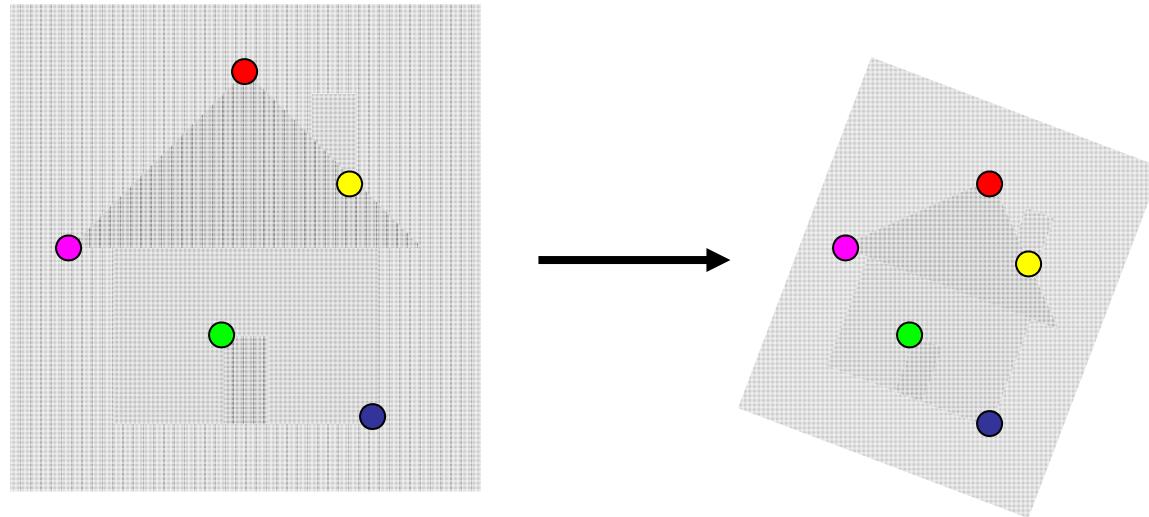
- Projective transformations:
  - Affine transformations, and
  - Projective warps
- Parallel lines do not necessarily remain parallel



Grauman



# Image alignment



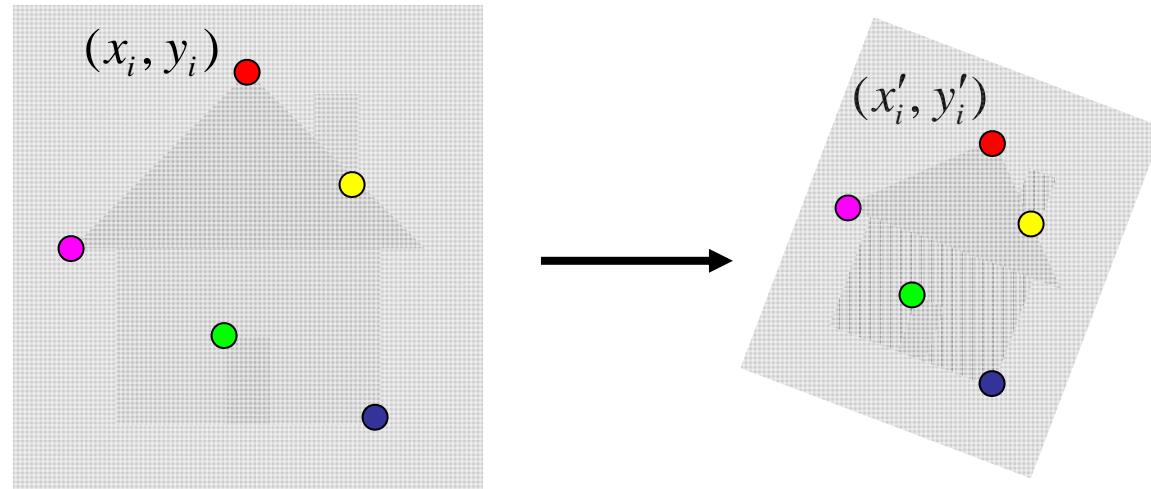
- Two broad approaches:
  - Direct (pixel-based) alignment
    - Search for alignment where most pixels agree
  - Feature-based alignment
    - Search for alignment where *extracted features* agree
    - Can be verified using pixel-based alignment

Source: L. Lazebnik



# Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?

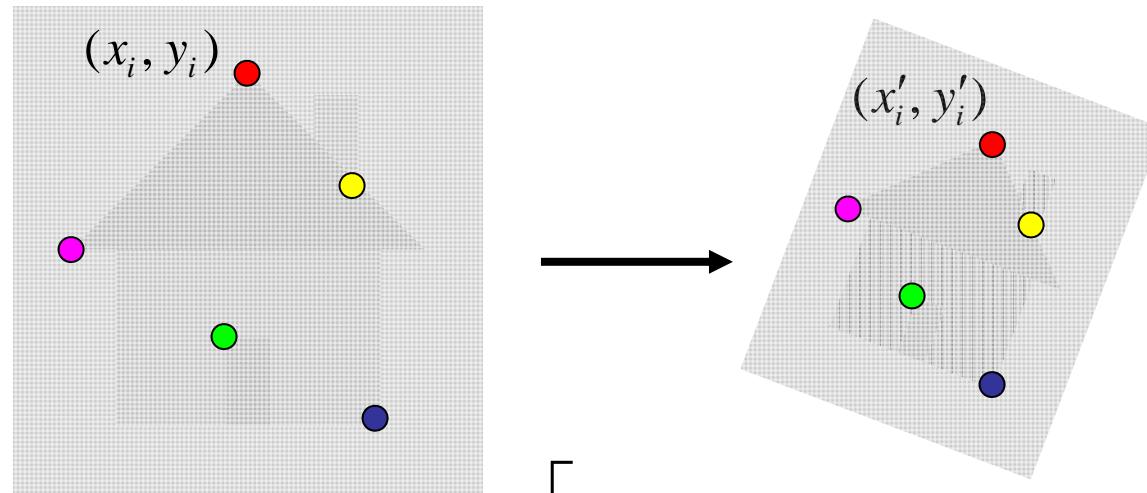


$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$



# Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \boxed{\quad}$$



# Fitting an affine transformation

$$\begin{bmatrix} & & \dots \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \dots \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix}$$

- How many matches (correspondence pairs) do we need to solve for the transformation parameters?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for  $(x_{new}, y_{new})$  ?

# Panoramas

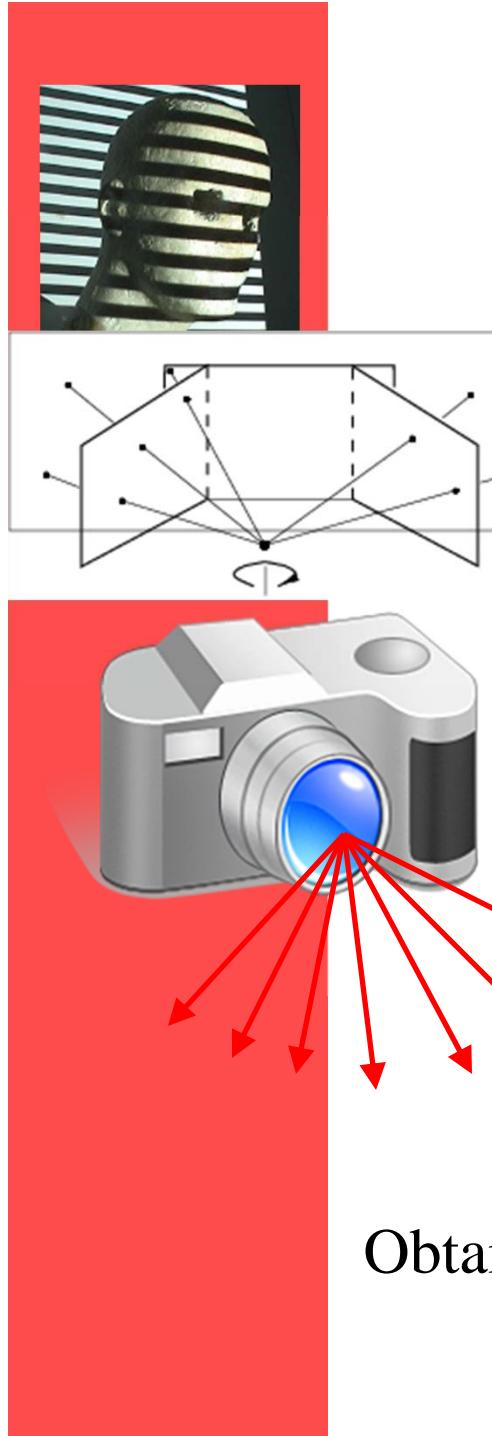


image from S. Seitz



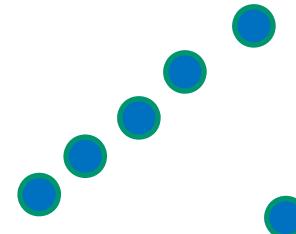
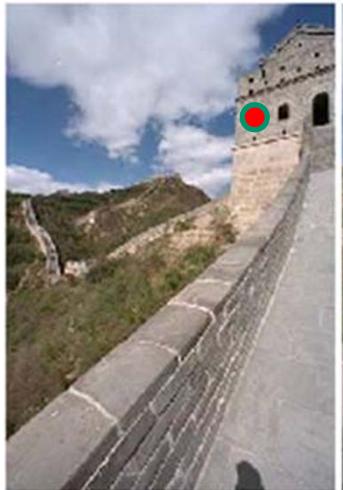
Obtain a wider angle view by combining multiple images.

Grauman



# Outliers

- **Outliers** can hurt the quality of our parameter estimates, e.g.,
  - an erroneous pair of matching points from two images
  - an edge point that is noise, or doesn't belong to the line we are fitting.

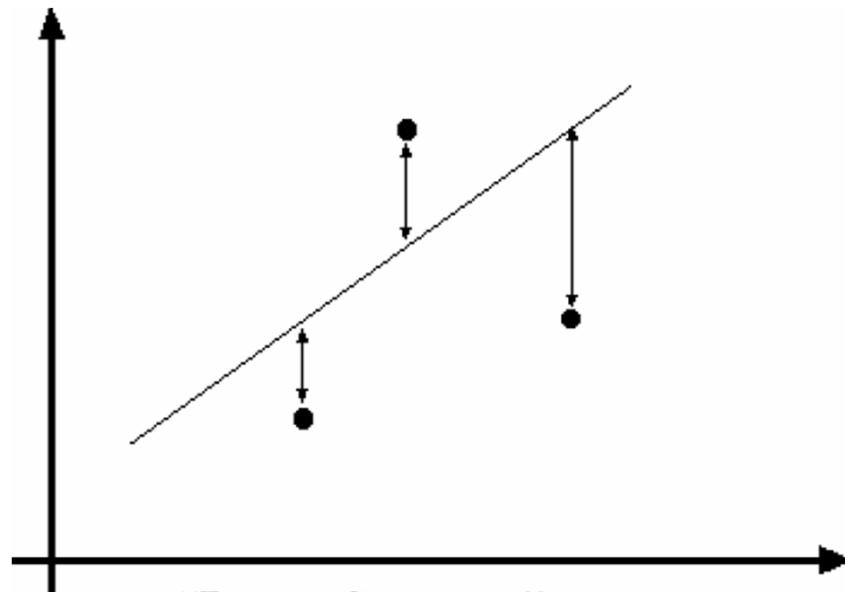


Grauman



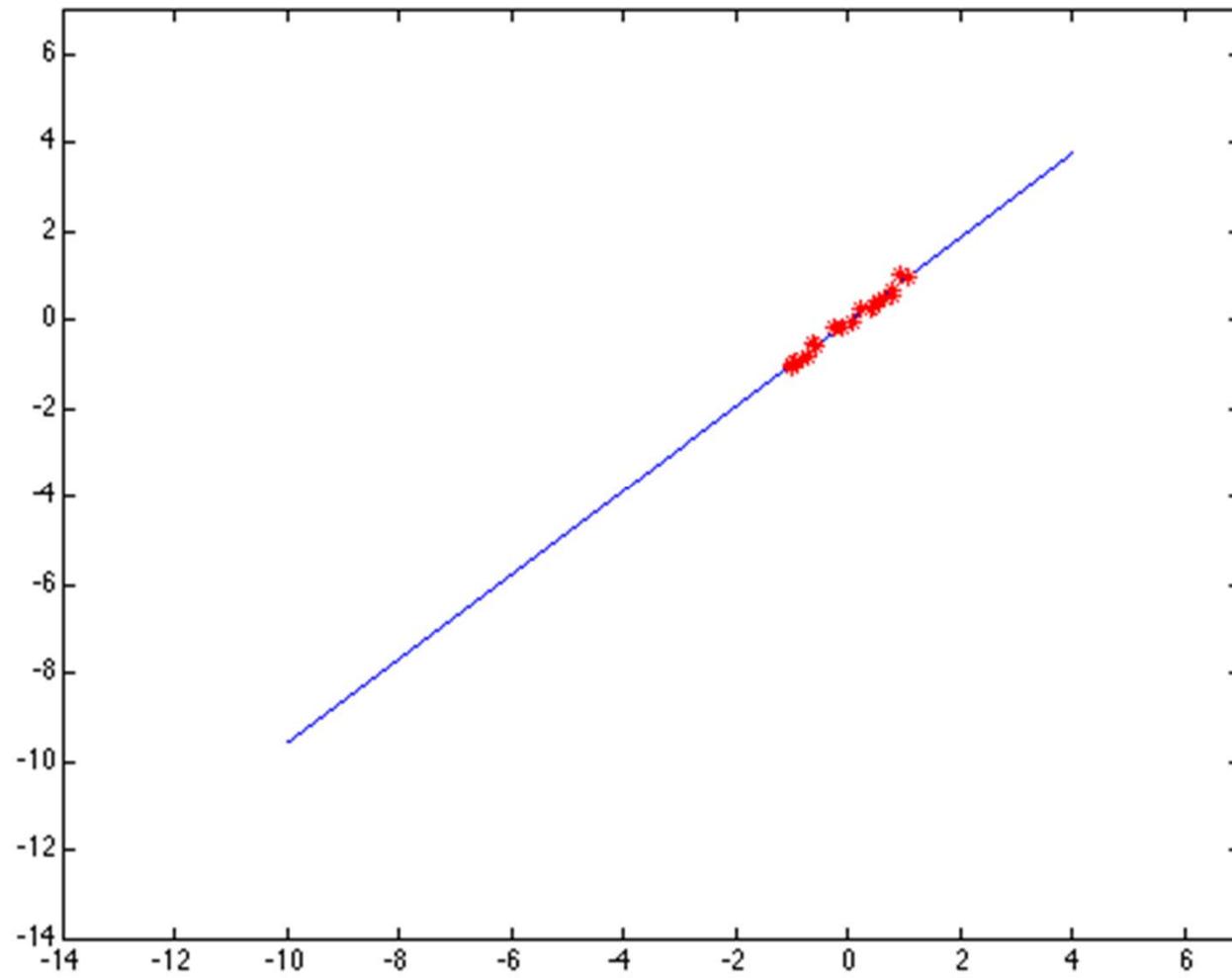
# Example: least squares line fitting

- Assuming all the points that belong to a particular line are known



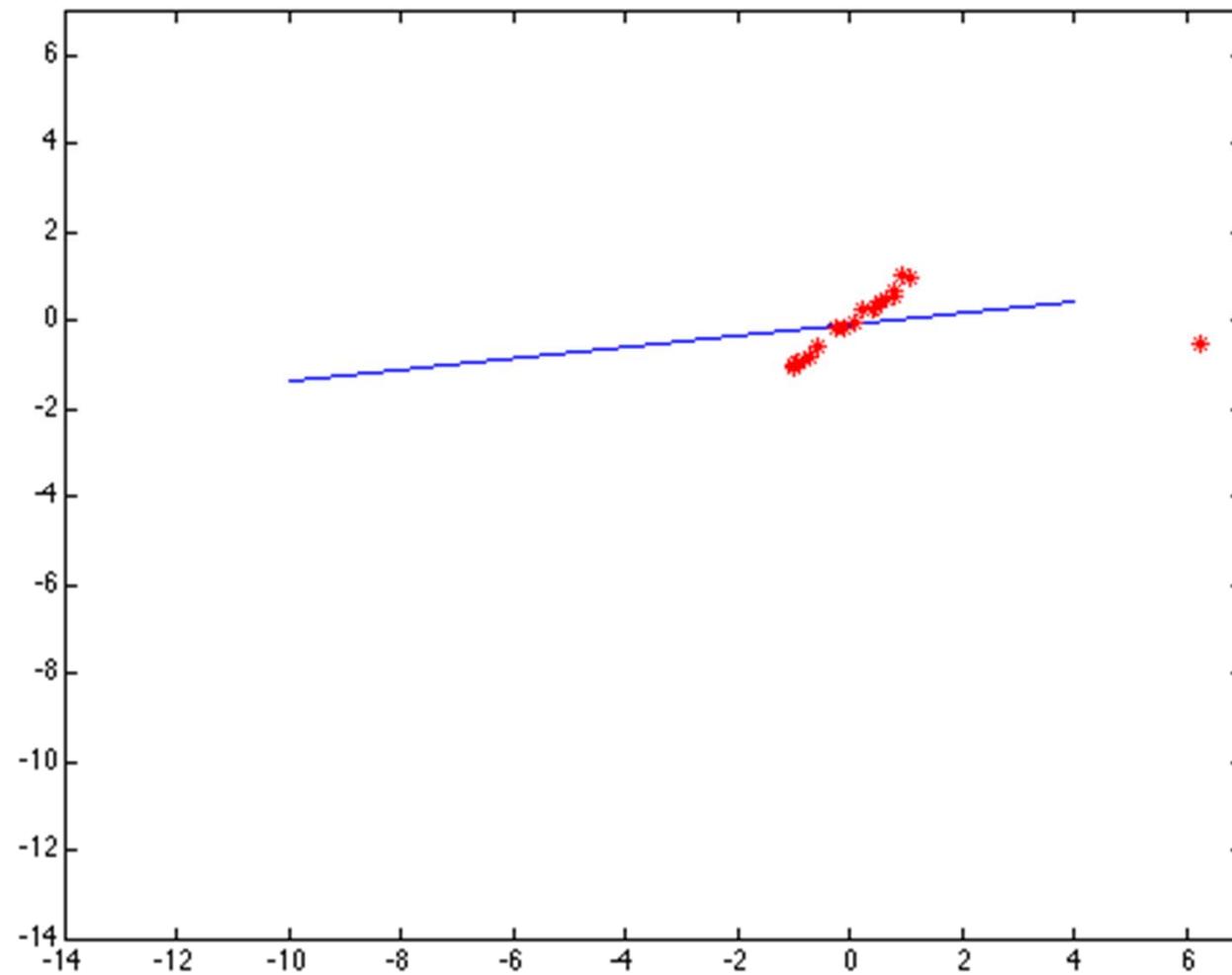


# Outliers affect least squares fit





# Outliers affect least squares fit





# RANSAC

- RANdom Sample Consensus
- Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use those only.
- Intuition: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.

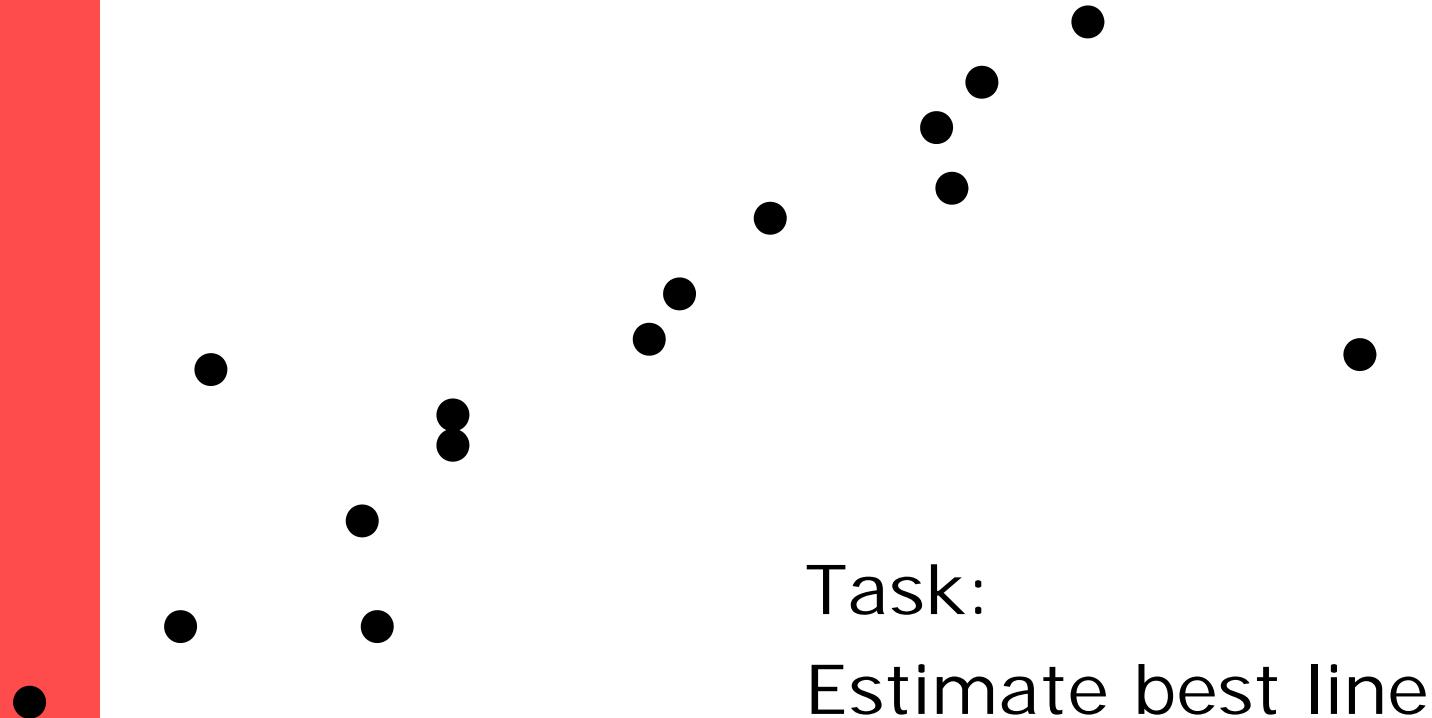


# RANSAC

- RANSAC loop:
  1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
  2. Compute transformation from seed group
  3. Find *inliers* to this transformation
  4. If the number of inliers is sufficiently large, recompute least-squares estimate of transformation on all of the inliers
- Keep the transformation with the largest number of inliers



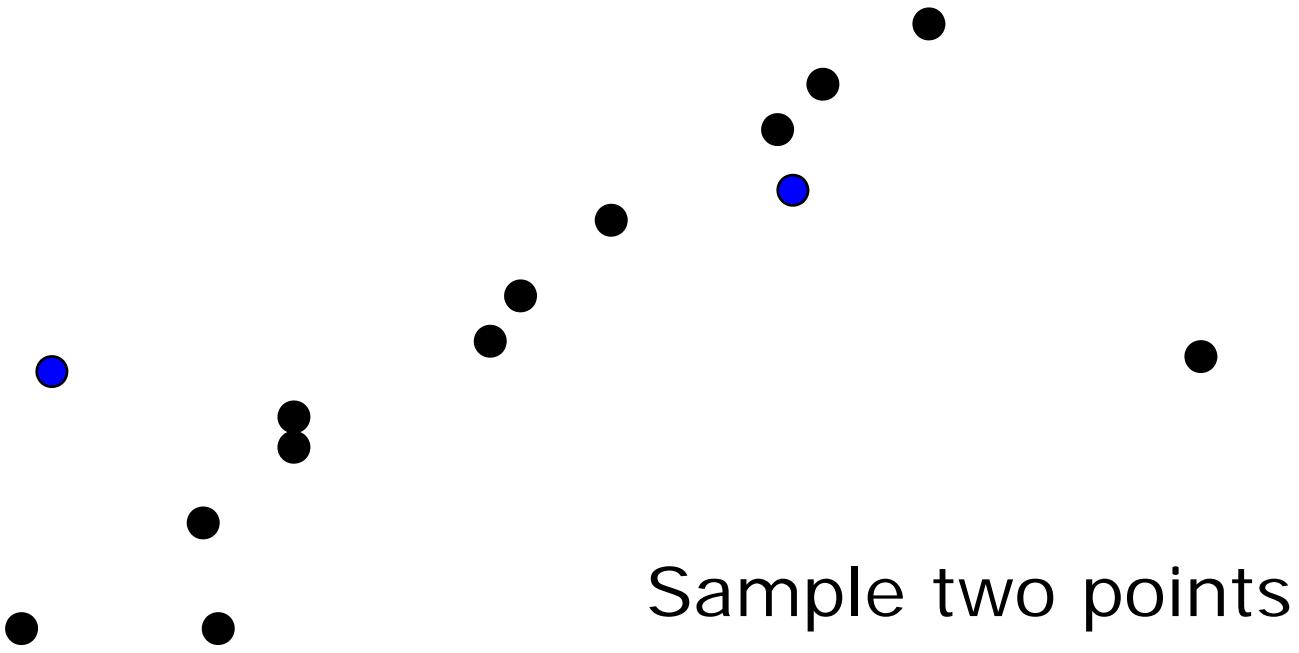
# RANSAC Line Fitting Example



Task:  
Estimate best line

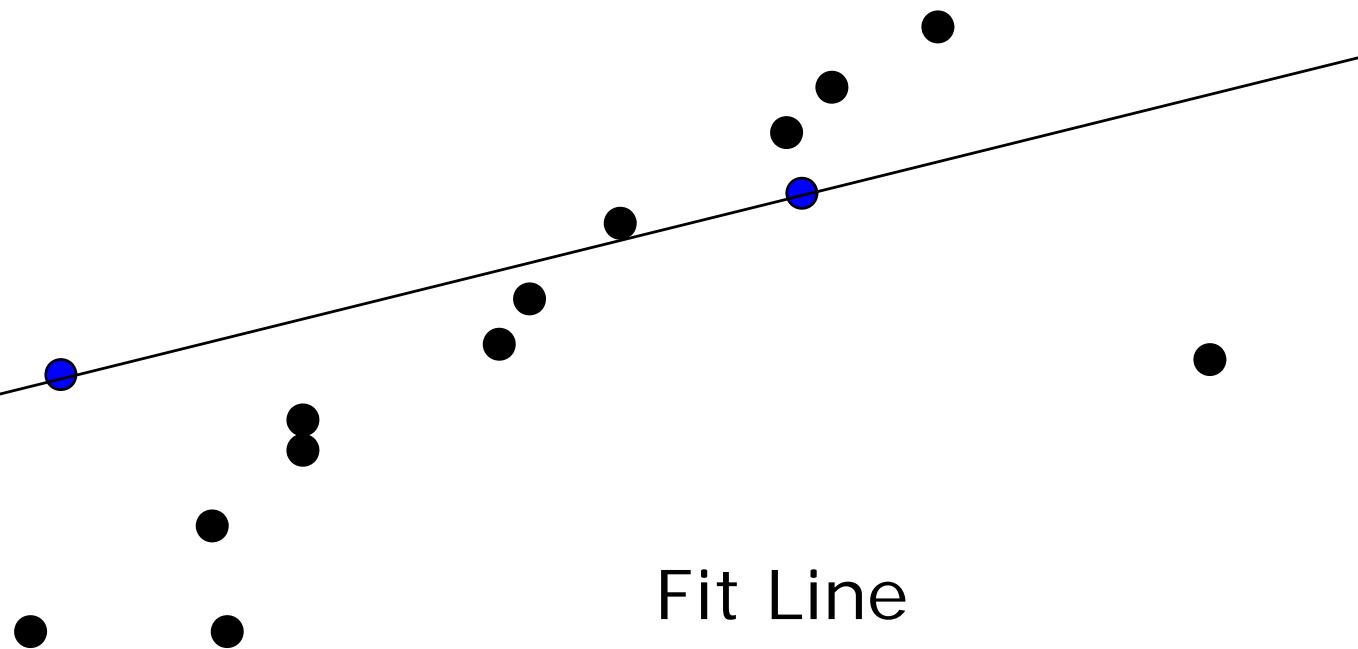


# RANSAC Line Fitting Example



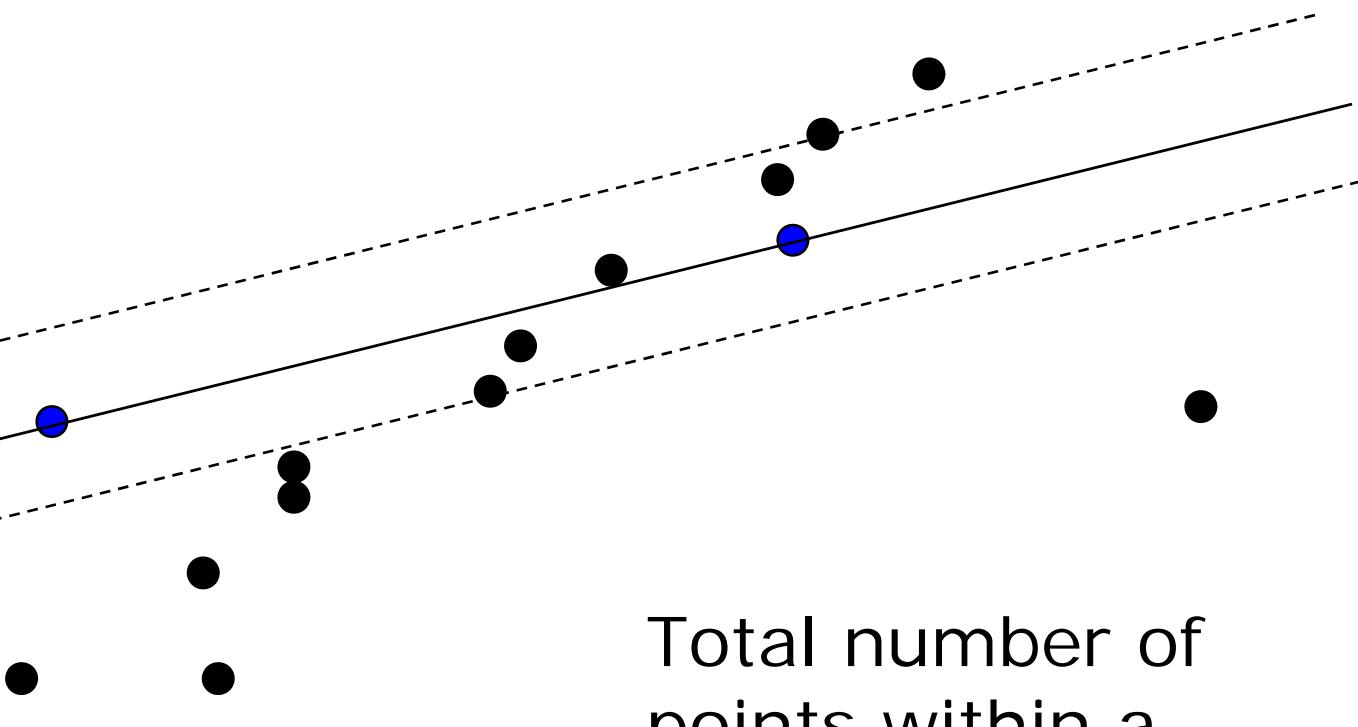


# RANSAC Line Fitting Example



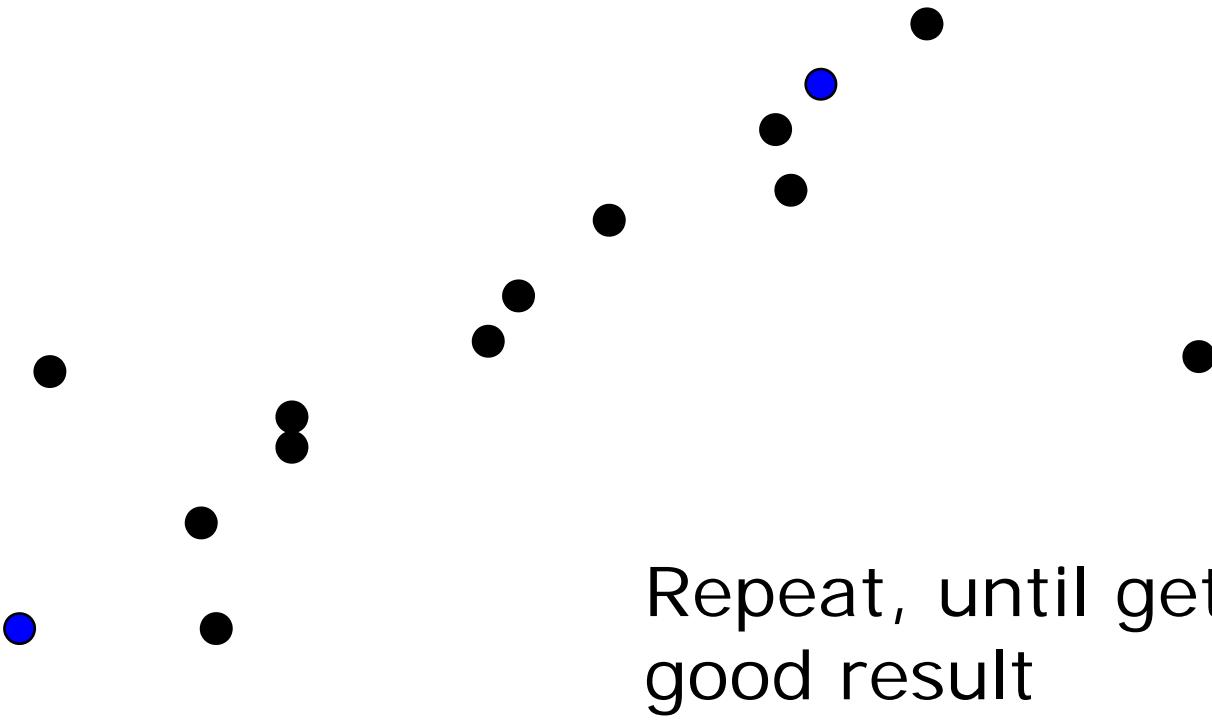


# RANSAC Line Fitting Example



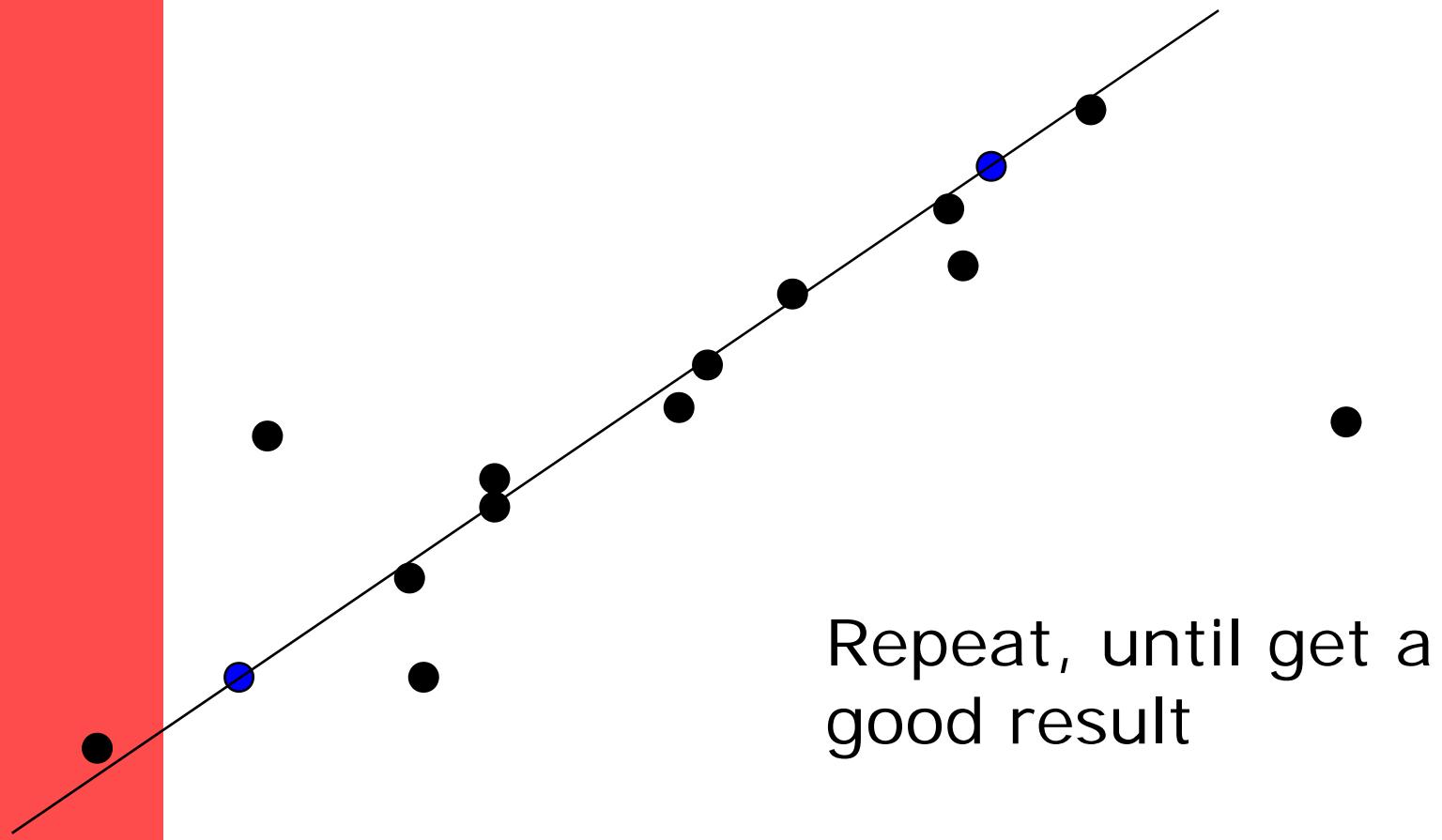


# RANSAC Line Fitting Example



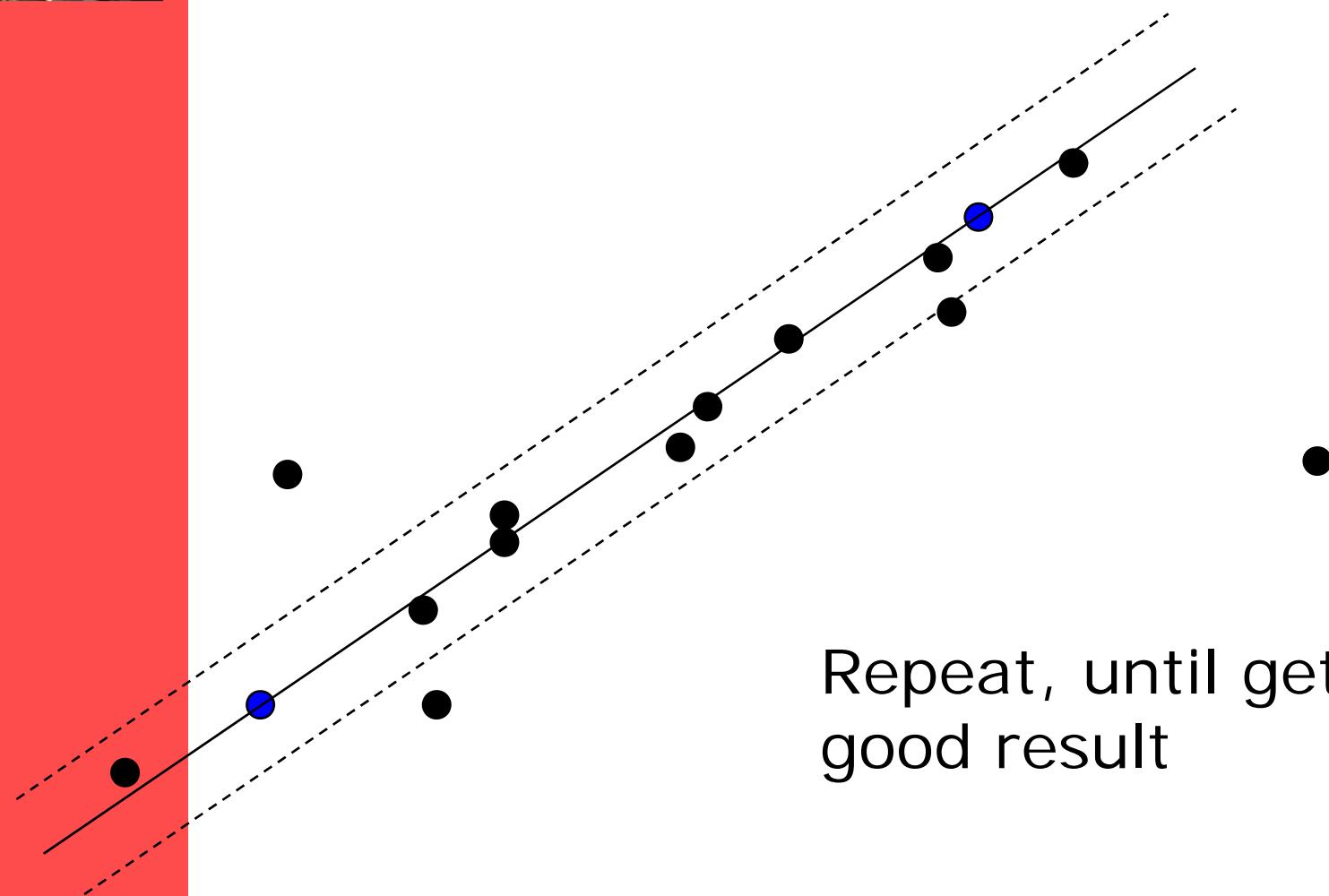


# RANSAC Line Fitting Example



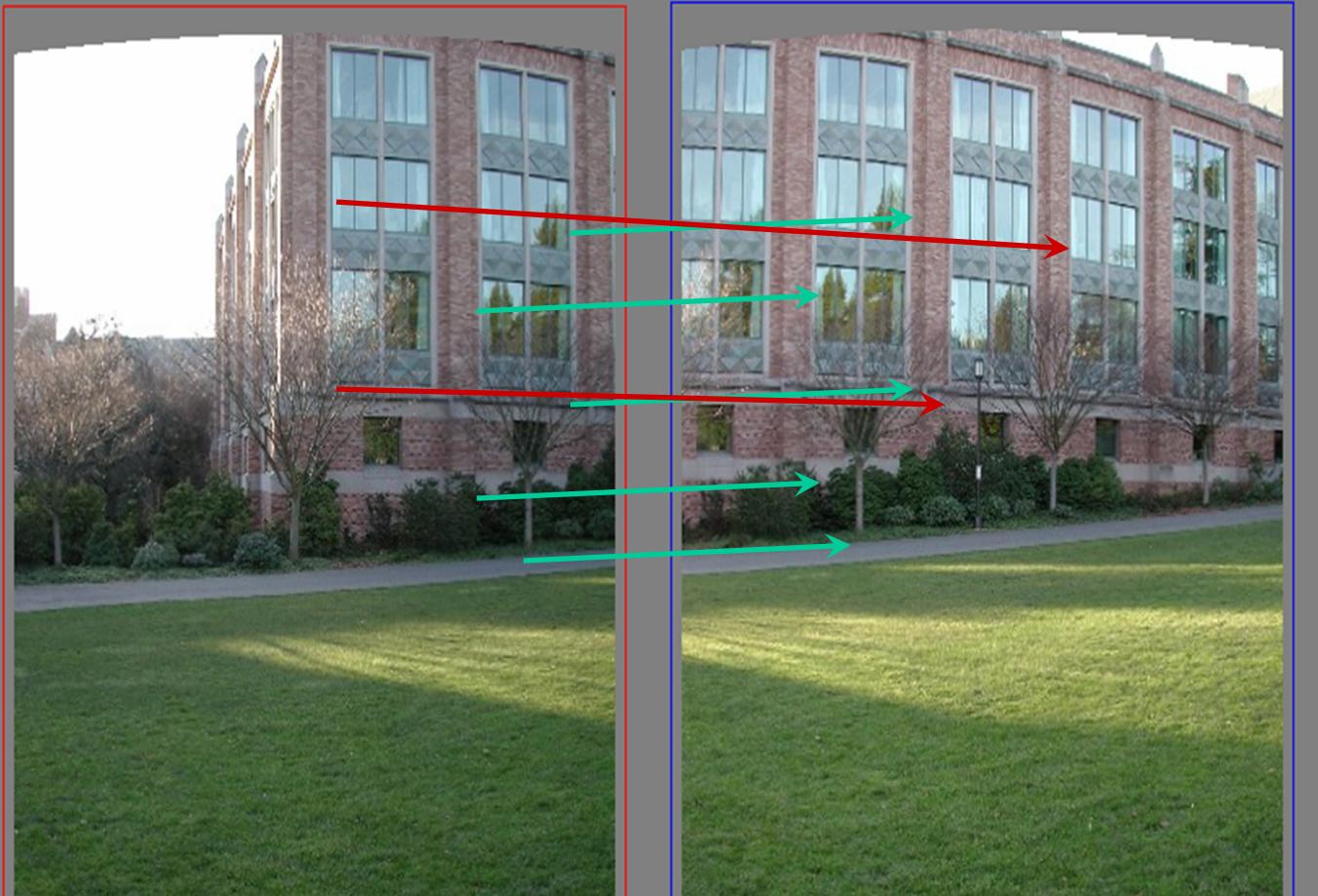


# RANSAC Line Fitting Example





# RANSAC example: Translation

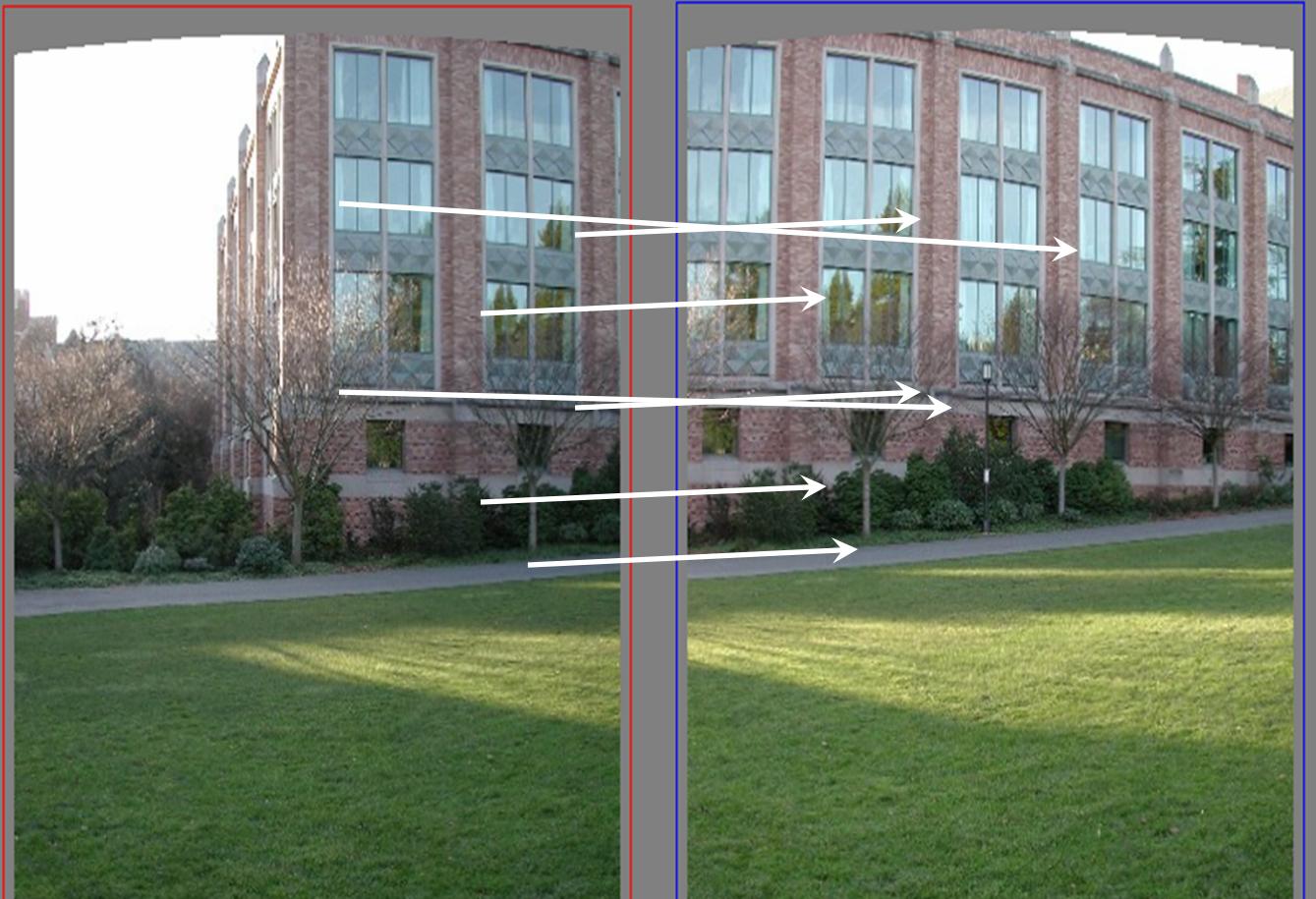


Putative matches

Source: Rick Szeliski



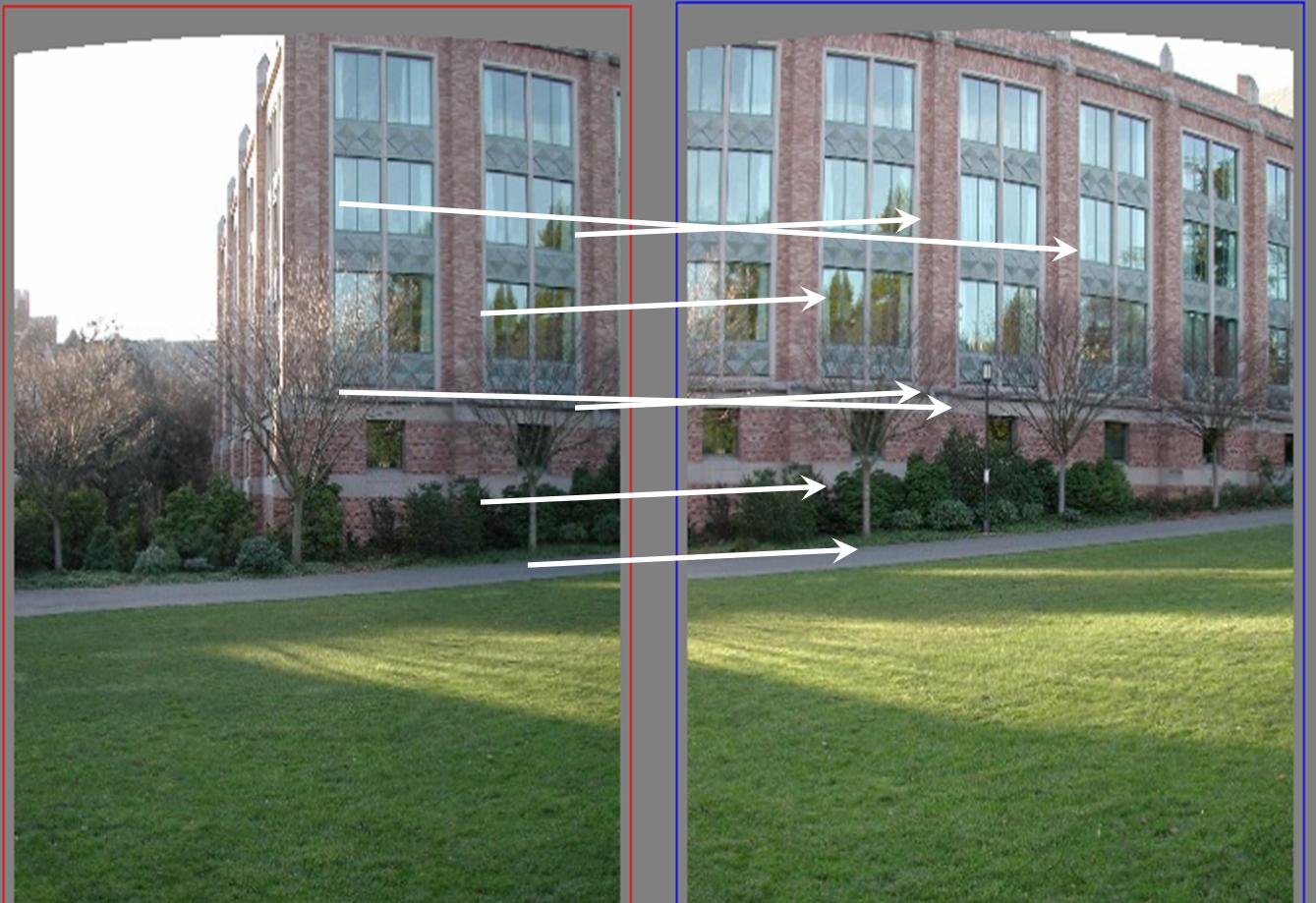
## RANSAC example: Translation



Select *one* match, count *inliers*



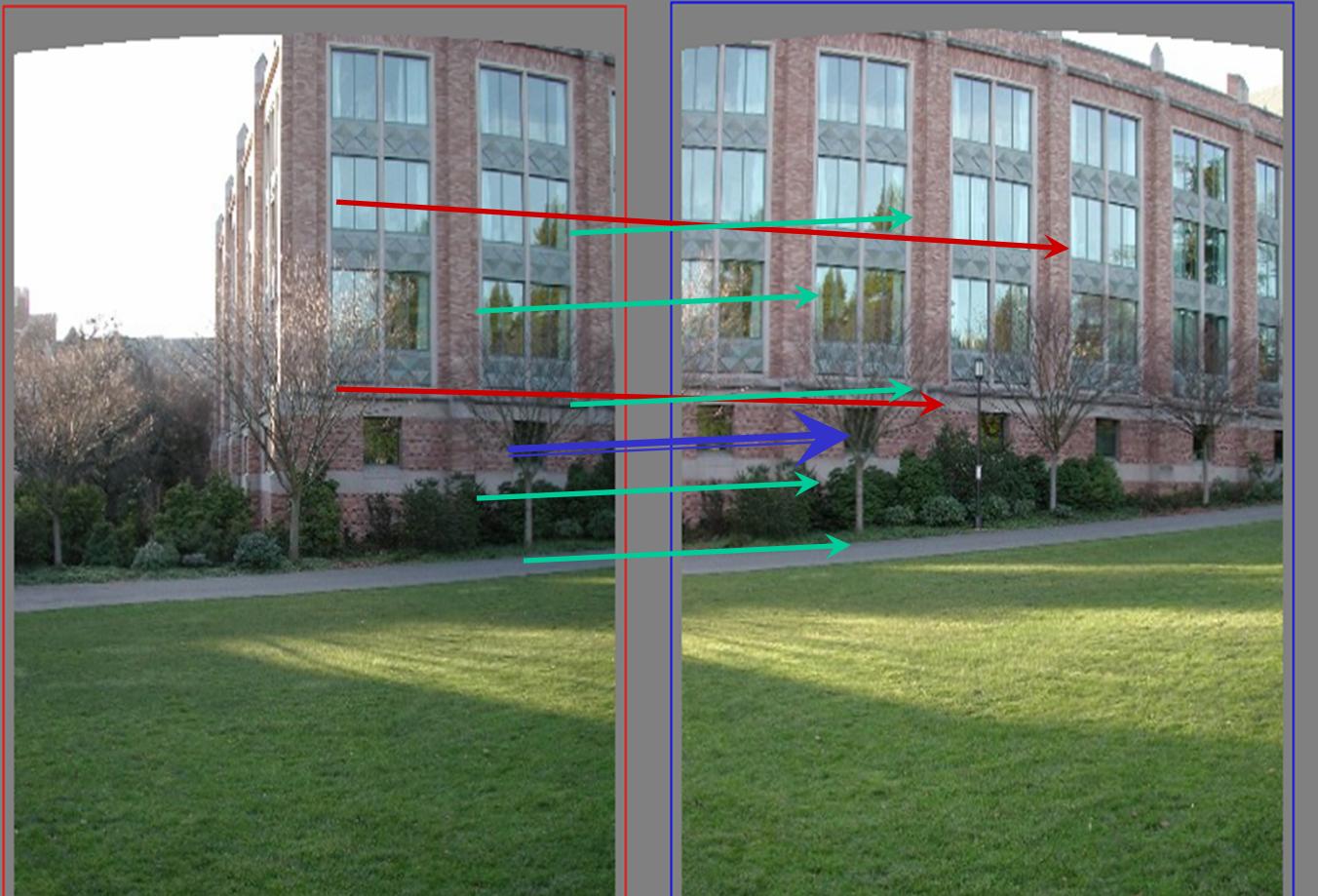
## RANSAC example: Translation



Select *one* match, count *inliers*



## RANSAC example: Translation



Find “average” translation vector

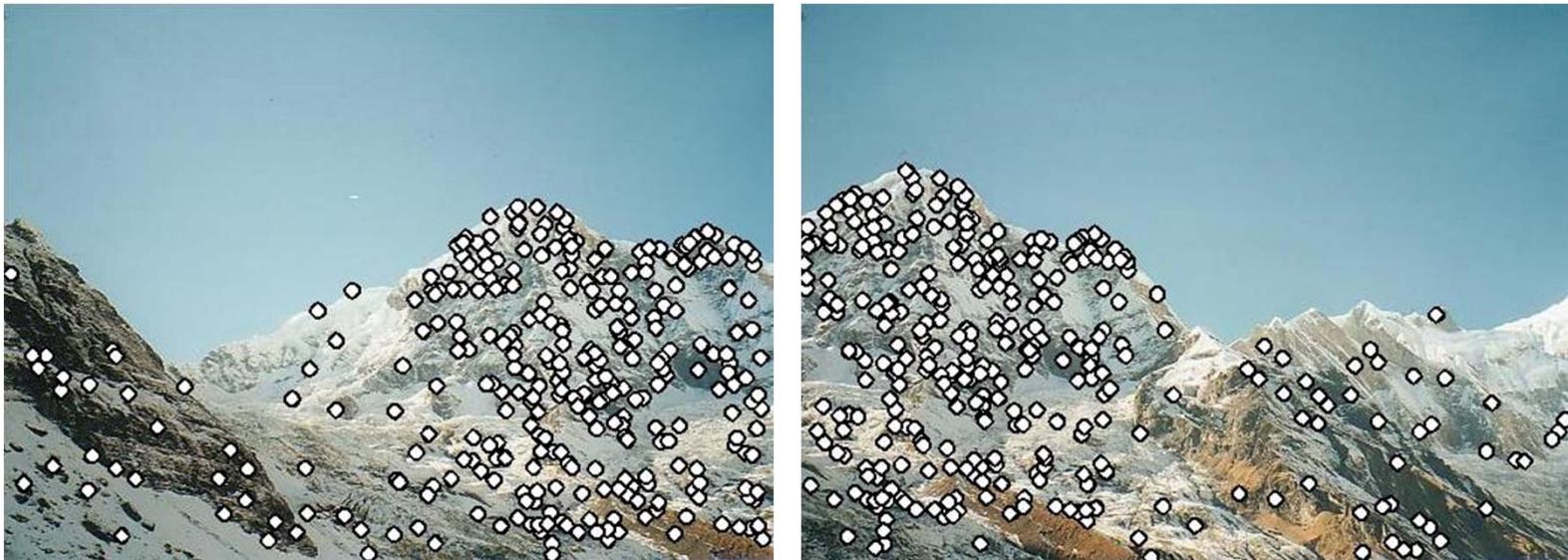
Towards large-scale mosaics...

# Feature-based alignment outline



Source: L. Lazebnik

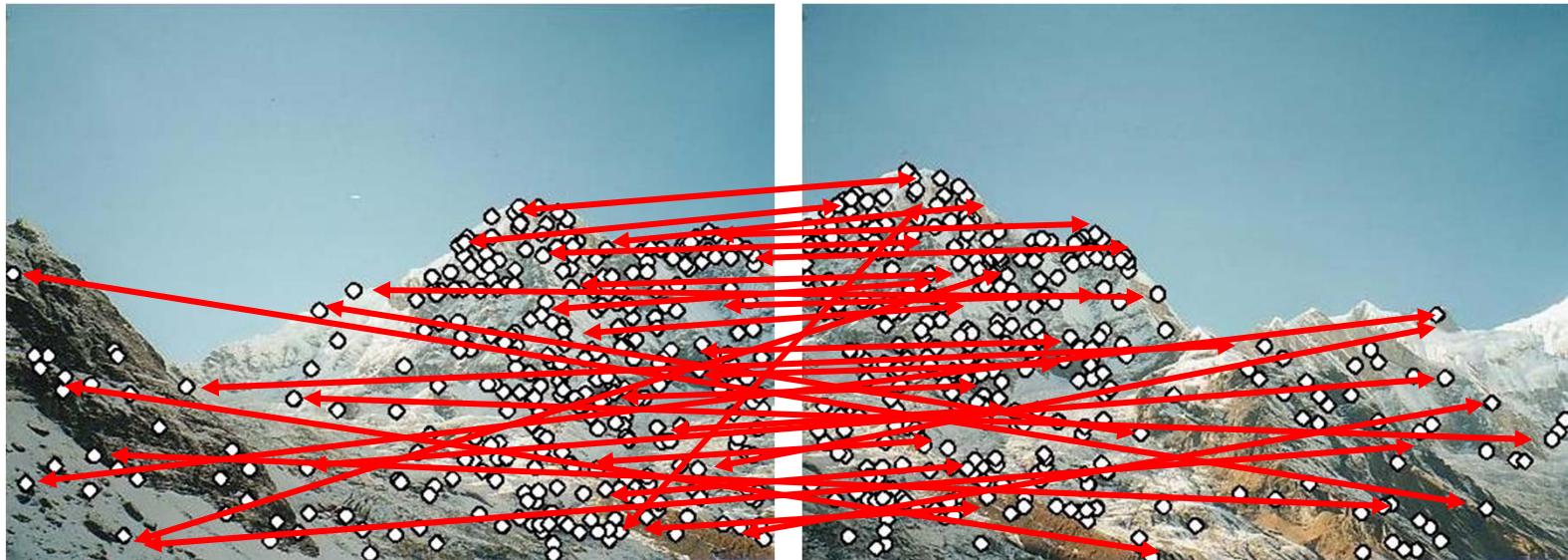
# Feature-based alignment outline



- Extract features

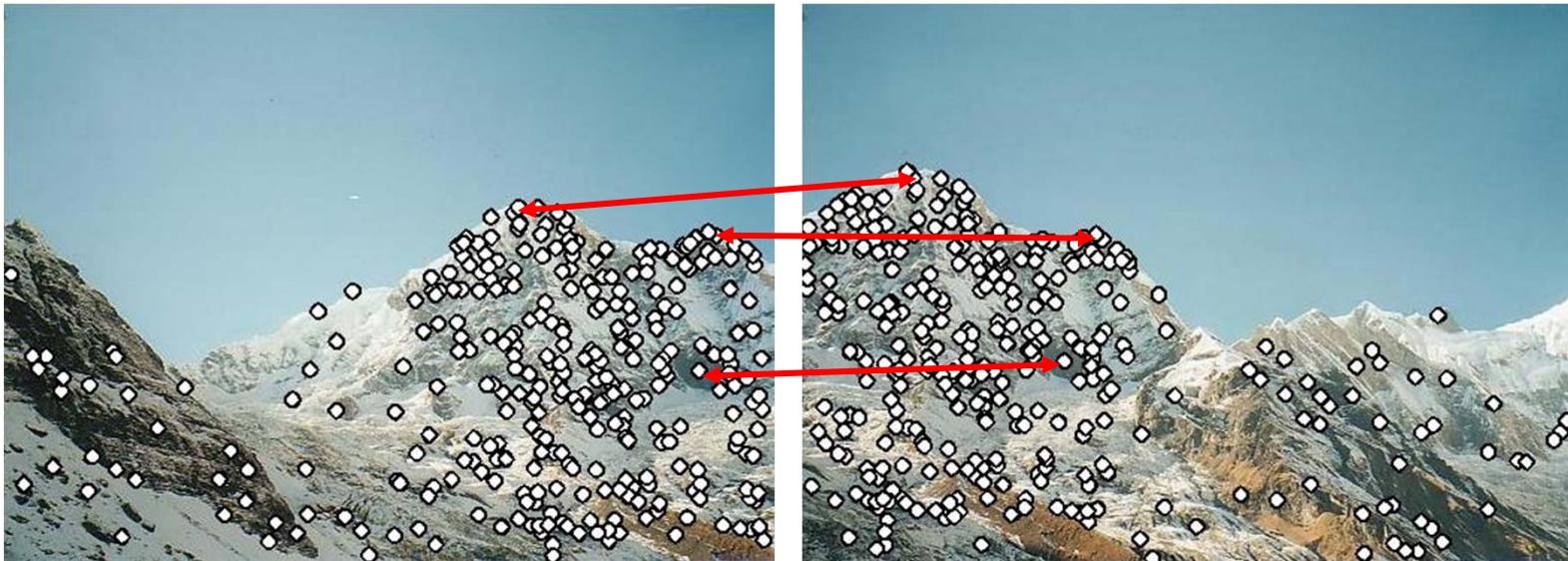
Source: L. Lazebnik

# Feature-based alignment outline



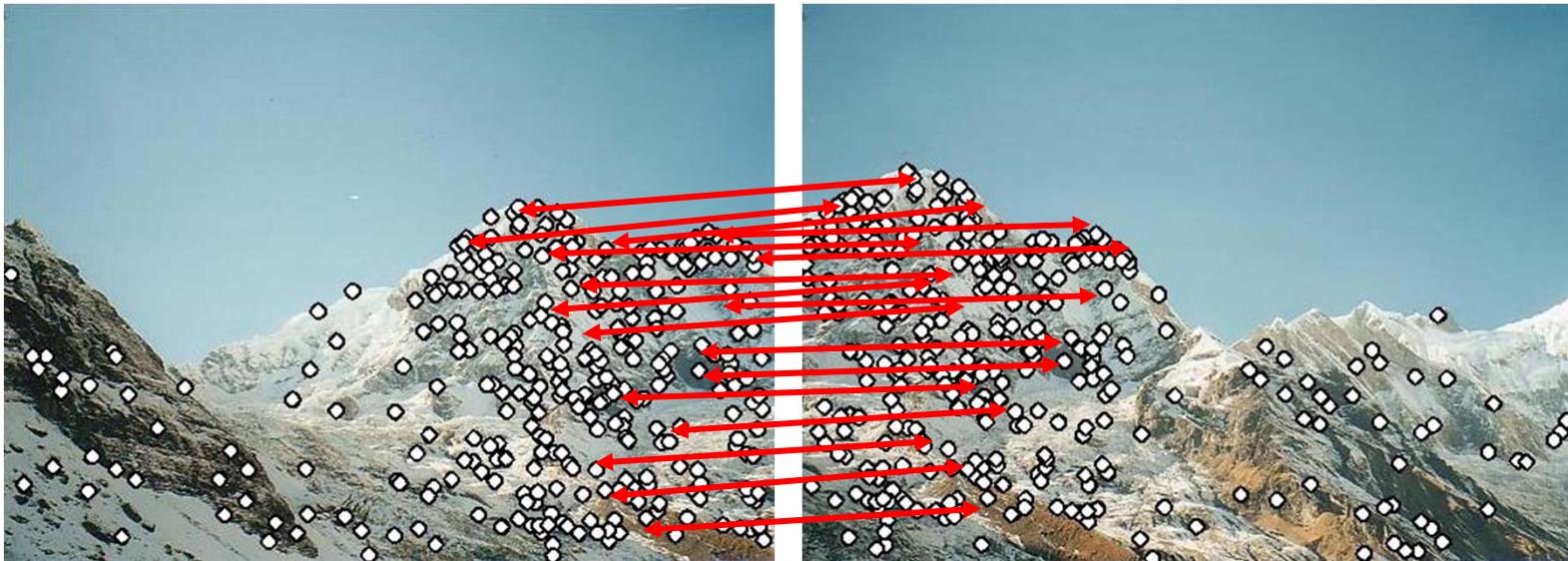
- Extract features
- Compute *putative matches*

# Feature-based alignment outline



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation  $T$  (small group of putative matches that are related by  $T$ )

# Feature-based alignment outline



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation  $T$  (small group of putative matches that are related by  $T$ )
  - *Verify* transformation (search for other matches consistent with  $T$ )

Source: L. Lazebnik

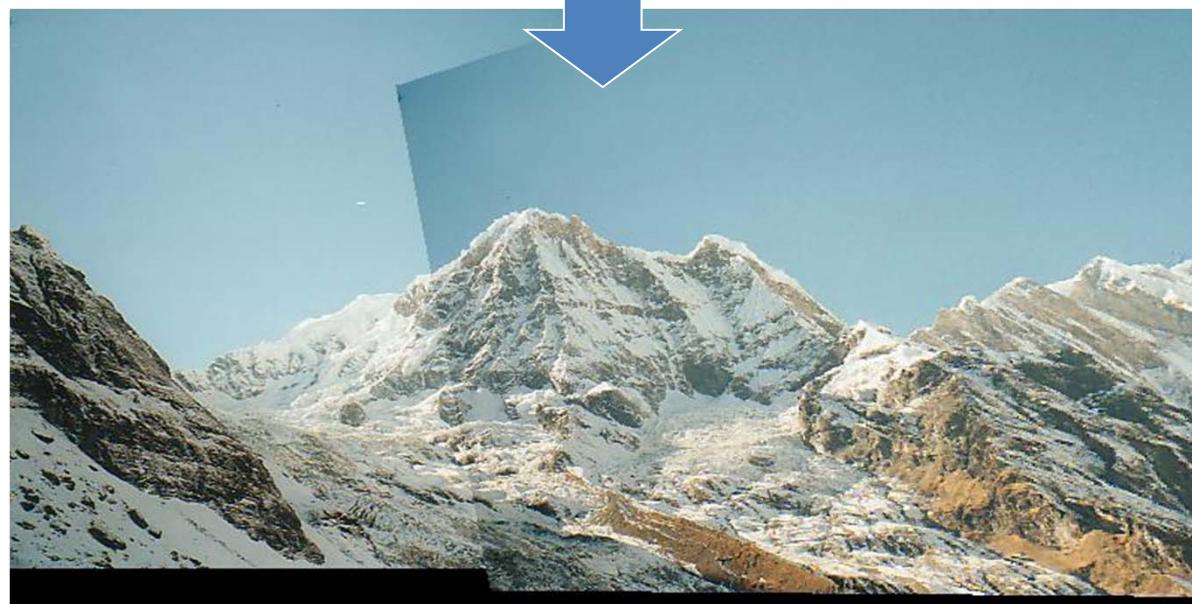
# Feature-based alignment outline



- Extract features
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Source: L. Lazebnik

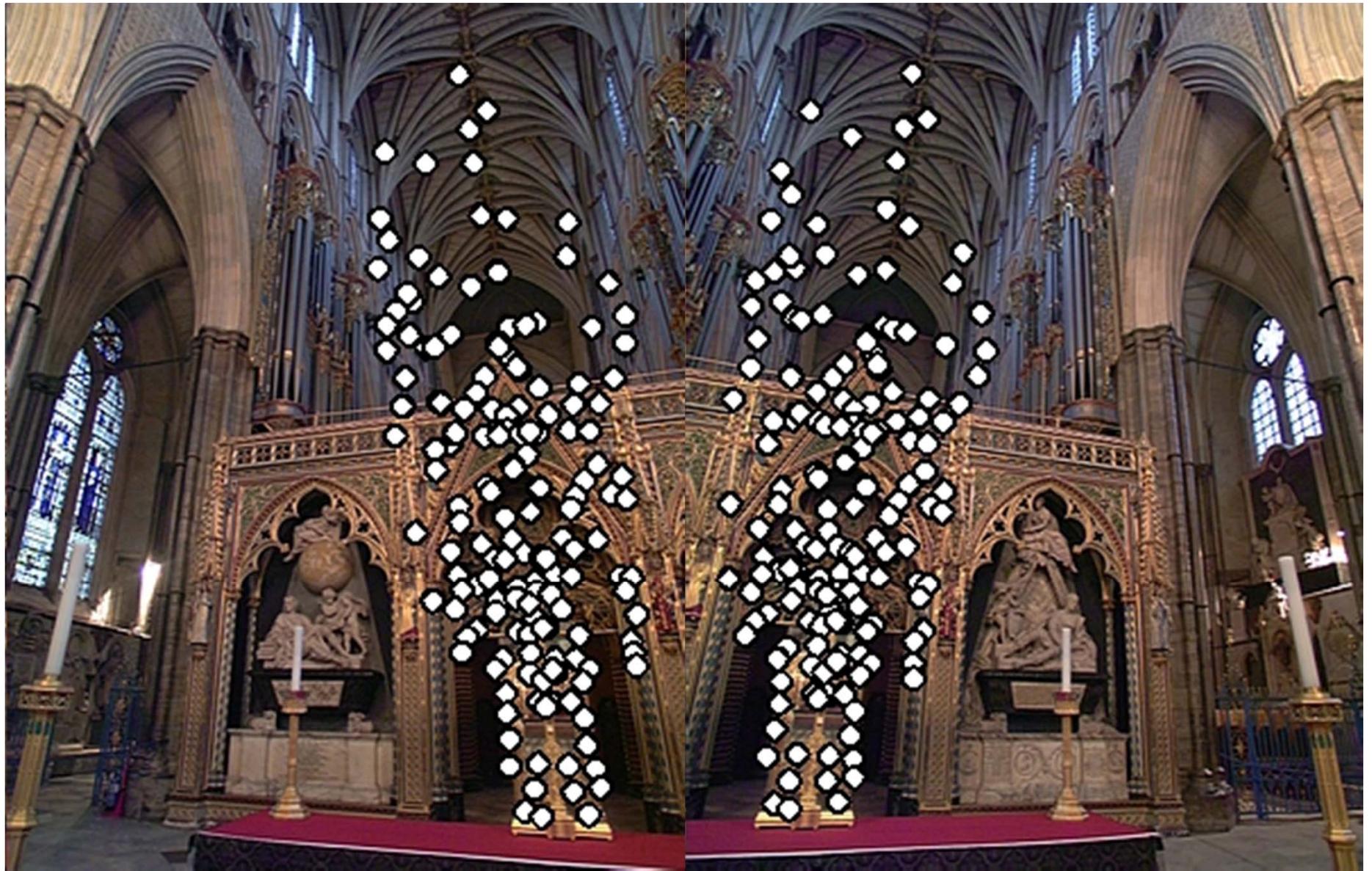
# RANSAC motion model



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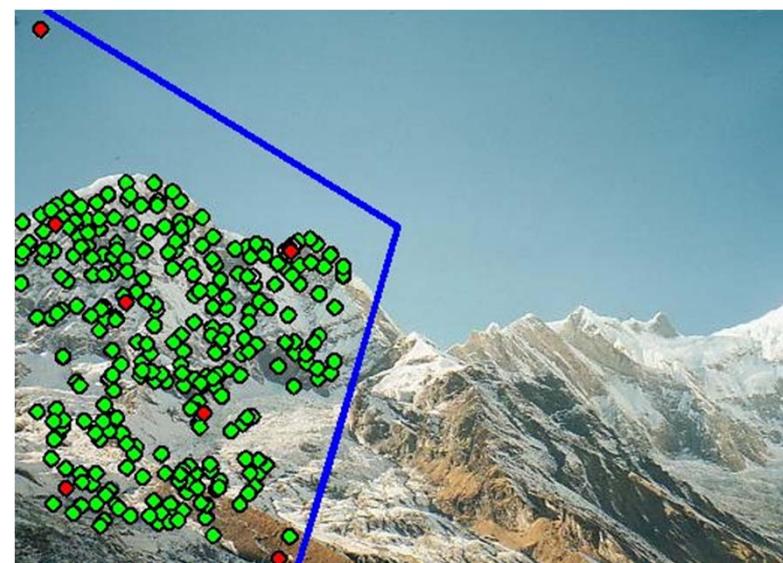
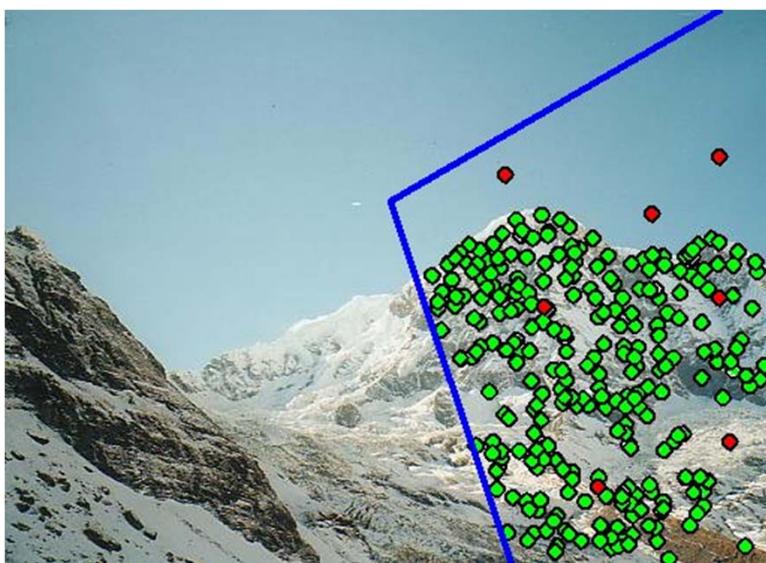
Towards large-scale mosaics...

# Probabilistic Feature Matching



Richard Szeliski

# Probabilistic model for verification



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## Other types of mosaics



- Can mosaic onto *any* surface if you know the geometry
  - See NASA's [Visible Earth project](#) for some stunning earth mosaics
    - <http://earthobservatory.nasa.gov/Newsroom/BlueMarble/>



# Final thought: What is a “panorama”?

- Tracking a subject
- Repeated (best) shots
- Multiple exposures
- “Infer” what photographer wants?



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## Next time: 6.2 Pose Estimation

- 6.2 Pose Estimation
- Chapter 7: Structure from Motion