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COMPUTERS & DATA SCIENCE

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Lecture: 10

Numerical Computations

lec (10): Numerical Computations

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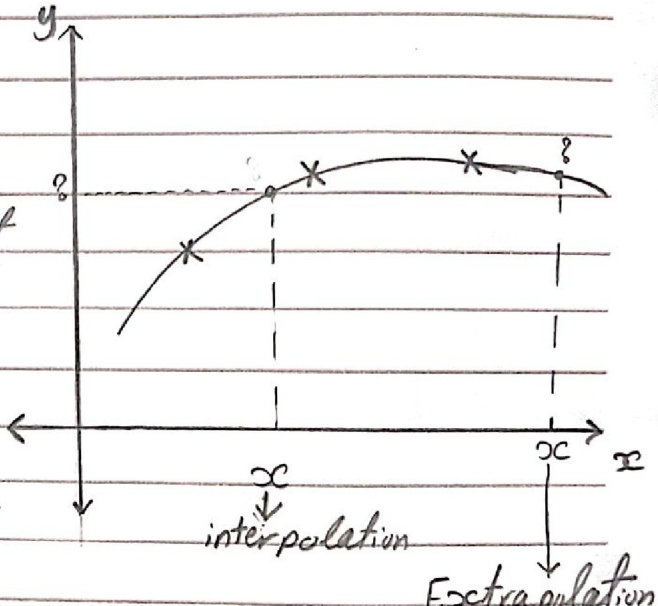
Interpolation

If we have data as following

x	x_1	x_2	x_3
y	y_1	y_2	y_3

we search for the Curve that passes by all the points actually.



illustrating the graph:-

1st:- I search for the Curve that passes through all points

2nd:- The importance of interpolation is:-

- we Can Calculate any point on the curve "Calculating (x, y)"

→ If this point lies between any two known points on the Curve
∴ This process is Called: Interpolation

→ If this point lies at the end of the Curve from left or right.
∴ This process is Called: Extrapolation

The difference between interpolation and Curve fitting:-

[1] Interpolation:-

1. The Curve (polynomial) posses through all points.
2. polynomial degree = [no. of data - 1]
3. Calculation are simple Compared to fitting.

[2] Curve fitting:-

1. Curve passes as near as possible to data
2. for polynomial fit the degree must be $\leq$ [data - 1]
3. Calculation are Complex Compared to interpolation.

* Divided difference

$$\# \therefore f(x_0, \dots, x_n) = \frac{f(x_0, \dots, x_n) - f(x_0, \dots x_{n-1})}{x_n - x_0}$$

$$\therefore \underbrace{F_n(x)}_{\text{poly. order}} = \underbrace{f(x_0)}_{(b_0)} + \underbrace{f(x_0, x_1)}_{\substack{(b_1) \\ 1^{st}\text{ finit diff.}}} (x - x_0) + \underbrace{f(x_0, x_1, x_2)}_{\substack{(b_2) \\ 2^{nd}\text{ finit diff.}}}$$

$$* (x - x_0)(x - x_1) + \text{----}$$

$$\# \therefore y = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \text{----}$$

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Newton's interpolation polynomials:-

Consider the following data:-

x	x_0	x_1	x_2	---	x_n
y	y_0	y_1	y_2	---	y_n

* We fit n^{th} order polynomial to $n+1$ data points

∴ The n^{th} order polynomial will be:

$$f_n(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + \cdots + b_n(x-x_0)\cdots(x-x_n)$$

"last general formula"

⇓

We want to calculate $b_0, b_1, \ldots, b_n$

steps:-

(1) Calculate first finite divided difference:-

$$b_1 = f(x_i, x_j) = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

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(2) Calculate second difference:

$$b_2 = f(x_i, x_j, x_k) = \frac{f(x_i, x_j) - f(x_j, x_k)}{x_i - x_k}$$

We continue in calculation of all divided differences till n^{th} difference

$$\therefore b_n = f(x_0, \dots x_n) = \frac{f(x_1, \dots x_n) - f(x_0, \dots x_{n-1})}{x_n - x_0}$$

Then substitute by $b_0, b_1, \dots, b_n$ in the general formula.

x_i	$y_i = f(x_i)$	first	second	third
x_0	$y_0 = f(x_0)$			
		$f(x_0,x_1) = \Delta y_0$		
x_1	$y_1 = f(x_1)$		$f(x_0,x_1,x_2) = \Delta^2 y_0$	
		$f(x_1,x_2) = \Delta y_1$		$f(x_0,x_1,x_2,x_3) = \Delta^3 y_0$
x_2	$y_2 = f(x_2)$		$f(x_1,x_2,x_3) = \Delta^2 y_1$	
		$f(x_2,x_3) = \Delta y_2$		
x_3	$y_3 = f(x_3)$			

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ex:

x	1	4	6	5
y	0	1.3863	1.7918	1.6094

$\hookrightarrow b_0$

Solution:

$F(x_0, x_1)$

$\Rightarrow \frac{1.3863-0}{4-1} = 0.4621 \rightarrow b_1$ (first Finit diff)

$\Rightarrow \frac{1.7918-1.3863}{6-4} = 0.2028$

$\Rightarrow \frac{1.6094-1.7918}{5-6} = 0.1824$

$F(x_0, x_1, x_2)$

$\rightsquigarrow \frac{0.2028-0.4621}{6-1} = -0.0519 \rightarrow b_2$

$\rightsquigarrow \frac{0.1824-0.2028}{5-4} = -0.0204$

$F(x_0, \dots, x_3)$

$* \frac{-0.0204+0.0519}{5-1} = 0.0079 \rightarrow b_3$

$$\boxed{\therefore y = 0 + 0.4621(x-1) - 0.0519(x-1)(x-4) + 0.0079(x-1)(x-4)(x-6)}$$

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• Calculate $f(x)$ when $x = 2$

∴ We will substitute by $x = 2$ in the previous equation

∴ $f(x) = y = 0.6288$ ⇒ numerical value

Note:-

$$R.P.E = \frac{|\text{exact value} - \text{numerical value}|}{\text{exact value}}$$

• exact value $= \ln(2) = 0.6931$

∴ $$R.P.E = \frac{|0.6931 - 0.6288|}{0.6931} * 100$$

$= 9.3\%$