

Example:

It is felt that three drugs ( $X_1$ ,  $X_2$  and  $X_2$ ) may lead to changes in the level of a certain biochemical compound found in the brain. Thirty mice of the same strain were randomly divided into three groups and received the drugs. The amount of the compound (in micrograms per gram of brain tissue) is recorded before and after the treatments. The responses are in given in the following table. Test the hypothesis of no treatment effect at 5% level of significance.

Before				After		
$X_{1i1}$	$X_{1i2}$	$X_{1i3}$		$X_{2i1}$	$X_{2i2}$	$X_{2i3}$
1.21	0.61	0.70		1.26	0.50	0.81
0.92	0.43	0.71		1.07	0.39	0.69
0.80	0.35	0.71		1.33	0.24	0.70
0.85	0.48	0.68		1.39	0.37	0.72
0.98	0.42	0.71		1.38	0.42	0.71
1.15	0.52	0.72		0.98	0.49	0.70
1.10	0.50	0.75		1.41	0.41	0.70
1.02	0.53	0.70		1.30	0.47	0.67
1.18	0.45	0.7		1.22	0.29	0.68
1.09	0.40	0.69		1.00	0.30	0.70

$d_{i1}$	$d_{i2}$	$d_{i3}$
-0.050	0.110	-0.110
-0.150	0.040	0.020
-0.530	0.110	0.010
-0.540	0.110	-0.040
-0.400	0.000	0.000
0.170	0.030	0.020
-0.310	0.090	0.050
-0.280	0.060	0.030
-0.040	0.160	0.020
0.090	0.100	-0.010

$$\bar{d} = \begin{bmatrix} -0.204 \\ 0.081 \\ -0.001 \end{bmatrix}$$

$$S_{\underline{d}} = \begin{bmatrix} 0.06160 & -0.00011 & -0.00060 \\ -0.00011 & 0.00228 & -0.00052 \\ -0.00060 & -0.00052 & 0.00205 \end{bmatrix}, S_{\underline{d}}^{-1} = \begin{bmatrix} 16.28866 & 1.98818 & 5.27173 \\ 1.98818 & 465.77088 & 118.72867 \\ 5.27173 & 118.73867 & 519.46436 \end{bmatrix}$$

The hypothesis to be tested is

$$H_a : \underline{\mu_d} \neq 0$$

$$T_0^2 = n \underline{\bar{d}}^T [S]^{-1} \underline{\bar{d}} = 36.515$$

The critical value is

$$c^* = \frac{(n-1)p}{n-1} F_\alpha(p, n-p) = 16.779$$

**There is a significant treatment effect at 5% level of significance**

Hence, the 95% confidence intervals are:

$$\underline{\bar{d}}_j \pm \sqrt{c^*} \sqrt{\frac{s_{djj}}{n}}$$

$$\underline{\mu}_{d_1} = (-0.5255, 0.1175)$$

$$\underline{\mu}_{d_2} = (0.0191, 0.1429)$$

$$\underline{\mu}_{d_3} = (-0.0596, 0.0576)$$

The confidence interval for  $\underline{\mu}_{d_2}$  does not include zero. Thus,  $H_0 : \underline{\mu_d} = 0$  was rejected due to the second component ( $X_2$ ). In other words, it is the second drug ( $X_2$ ) that led to a significant change in the level of the biochemical compound found in the brain at 5% level of significance.