STATISTICS

Chapter 12

Time Series, Forecasting, and Index Numbers

12 Time Series, Forecasting, and Index Numbers

- Using Statistics
- Trend Analysis
- Seasonality and Cyclical Behavior
- The Ratio-to-Moving-Average Method
- Exponential Smoothing Methods
- Index Numbers

12 LEARNING OBJECTIVES

After studying this chapter you should be able to:

- Differentiate between qualitative and quantitative methods of forecasting
- Carryout a trend analysis in time series data
- Identify seasonal and cyclical patterns in time series data
- Forecast using simple and weighted moving average methods
- Forecast using exponential smoothing method
- Forecast when the time series contains both trend and seasonality
- Assess the efficiency of forecasting methods using measures of error
- Make forecasts using templates
- Compute index numbers

12-1 Using Statistics

A ***time series*** is a set of measurements of a variable that are ordered through time. Time series analysis attempts to detect and understand regularity in the fluctuation of data over time.

Regular movement of time series data may result from a tendency to increase or decrease through time - ***trend***- or from a tendency to follow some cyclical pattern through time - ***seasonality or cyclical variation.***

Forecasting is the extrapolation of series values beyond the region of the estimation data. Regular variation of a time series can be forecast, while random variation cannot.

The Random Walk

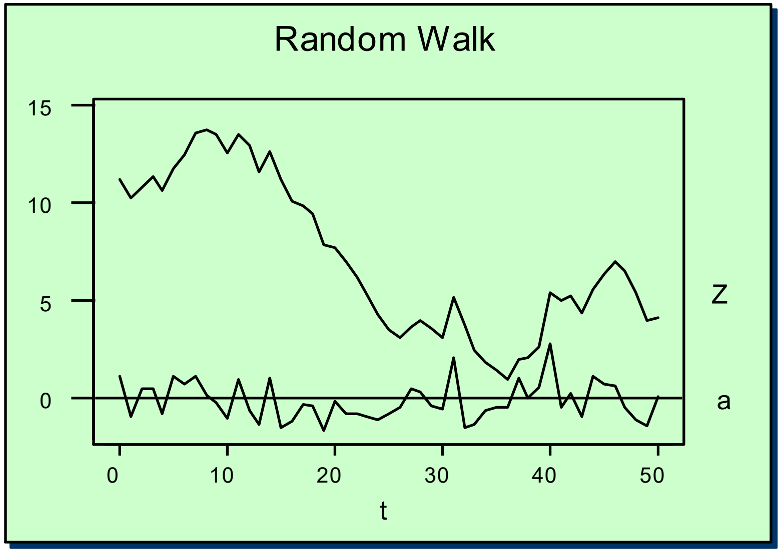
A **random walk:**

$$Z_t - Z_{t-1} = a_t$$

or equivalently:

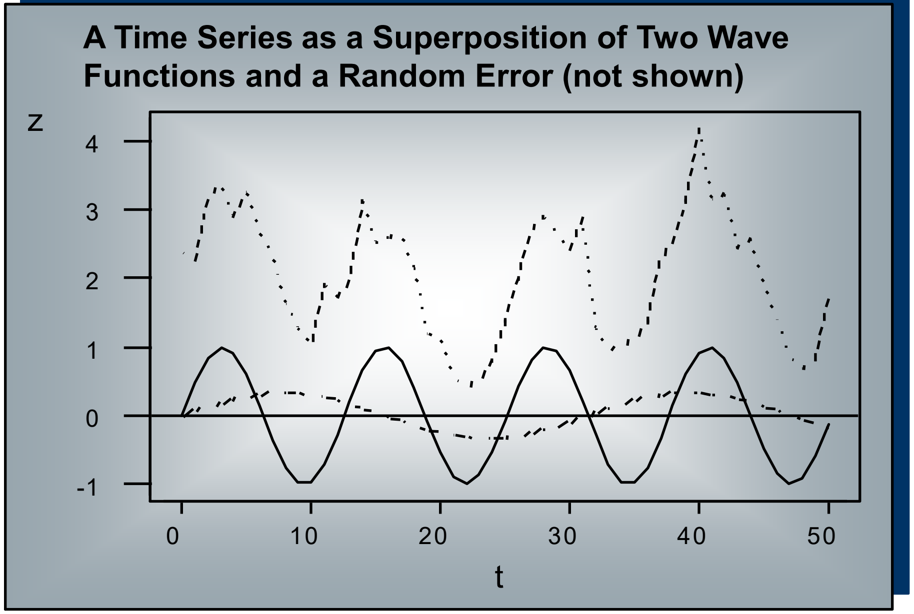
$$Z_t = Z_{t-1} + a_t$$

The difference between Z in time t and time $t\text{-}1$ is a random error.



There is no evident regularity in a random walk, since the difference in the series from period to period is a random error. A random walk is not forecastable.

A Time Series as a Superposition of Cyclical Functions



In contrast with a random walk, this series exhibits obvious ***cyclical fluctuation***. The underlying cyclical series - the regular elements of the overall fluctuation - may be analyzable and forecastable.

12-2 Trend Analysis: Example 12-1

The following output was obtained using the template.

Forecasting with Trend — Loans

Data		
Period	*t*	Z_t
1994	1	833
1995	2	936
1996	3	1006
1997	4	1120
1998	5	1212
1999	6	1301
2000	7	1490
2001	8	1608

Forecast	
t	Z-hat
9	1679.61
10	1788.8
11	1897.99
12	2007.18
13	2116.37
14	2225.56
15	2334.75
16	2443.94
17	2553.13
18	2662.32
19	2771.51
20	2880.7

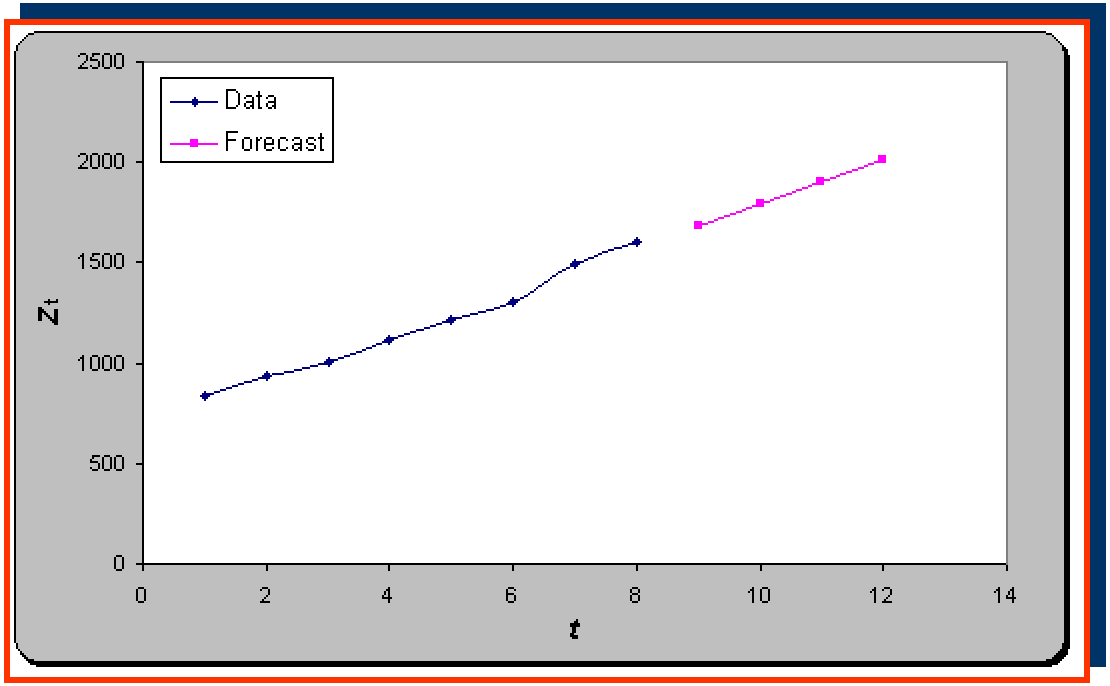
Forecast for selected *t*	
t	Z-hat
25	3426.65
11	1897.99
5	1242.85

Regression Statistics	
r^2	0.9853
MSE	1246.329
Slope	109.1905
Intercept	696.8929

$$Z_t = 696.89 + 109.19t$$

Note: The template contains forecasts for t = 9 to t = 20 which corresponds to years 2002 to 2013.

12-2 Trend Analysis: Example 12-1



Straight line trend.

Example 12-2

Forecasting with Trend

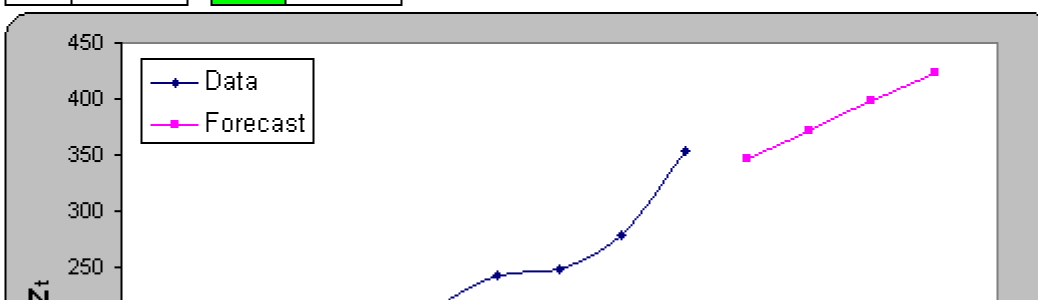
Price Index

Data		
Period	t	Z_t
1993	1	122.55
1994	2	140.64
1995	3	164.93
1996	4	167.24
1997	5	211.28
1998	6	242.17
1999	7	247.08
2000	8	277.72
2001	9	353.4

Forecast	
t	Z-hat
10	345.268
11	371.499
12	397.73
13	423.962
14	450.193
15	476.424
16	502.655
17	528.886
18	555.117
19	581.349
20	607.58
21	633.811

Forecast for selected t	
t	Z-hat
25	738.736
11	371.499
5	214.112

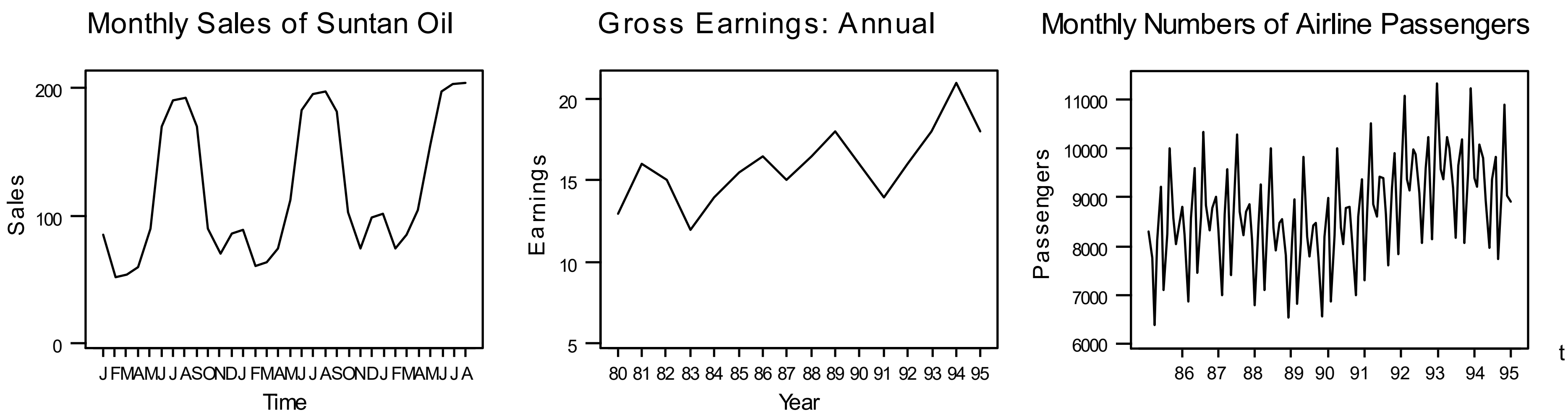
Regression Statistics	
r^2	0.9441
MSE	348.9182
Slope	26.23117
Intercept	82.95639



The forecast for t = 10 (year 2002) is 345.27

Observe that the forecast model is $Z_t = 82.96 + 26.23t$

12-3 Seasonality and Cyclical Behavior



When a cyclical pattern has a period of one year, it is usually called **seasonal variation**. A pattern with a period of other than one year is called **cyclical variation**.

Time Series Decomposition

- Types of Variation
  - ✓ Trend (T)
  - ✓ Seasonal (S)
  - ✓ Cyclical (C)
  - ✓ Random or Irregular (I)
- Additive Model
  - ➢ $Z_t = T_t + S_t + C_t + I_t$
- Multiplicative Model
  - ➢ $Z_t = (T_t)(S_t)(C_t)(I_t)$

Estimating an Additive Model with Seasonality

An additive regression model with seasonality:

$$Z_t = \beta_0 + \beta_1 t + \beta_2 Q_1 + \beta_3 Q_2 + \beta_4 Q_3 + a_t$$

where

- ✓ $Q_1 = 1$ if the observation is in the first quarter, and 0 otherwise
- ✓ $Q_2 = 1$ if the observation is in the second quarter, and 0 otherwise
- ✓ $Q_3 = 1$ if the observation is in the third quarter, and 0 otherwise

12-4 The Ratio-to-Moving- Average Method

A ***moving average*** of a time series is an average of a fixed number of observations that moves as we progress down the series.

Time, t:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Series, Z_t:	15	12	11	18	21	16	14	17	20	18	21	16	14	19
Five-period moving average:			15.4	15.6	16.0	17.2	17.6	17.0	18.0	18.4	17.8	17.6		

Time, t:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Series, Z_t:	15	12	11	18	21	16	14	17	20	18	21	16	14	19

$(15 + 12 + 11 + 18 + 21)/5 = 15.4$

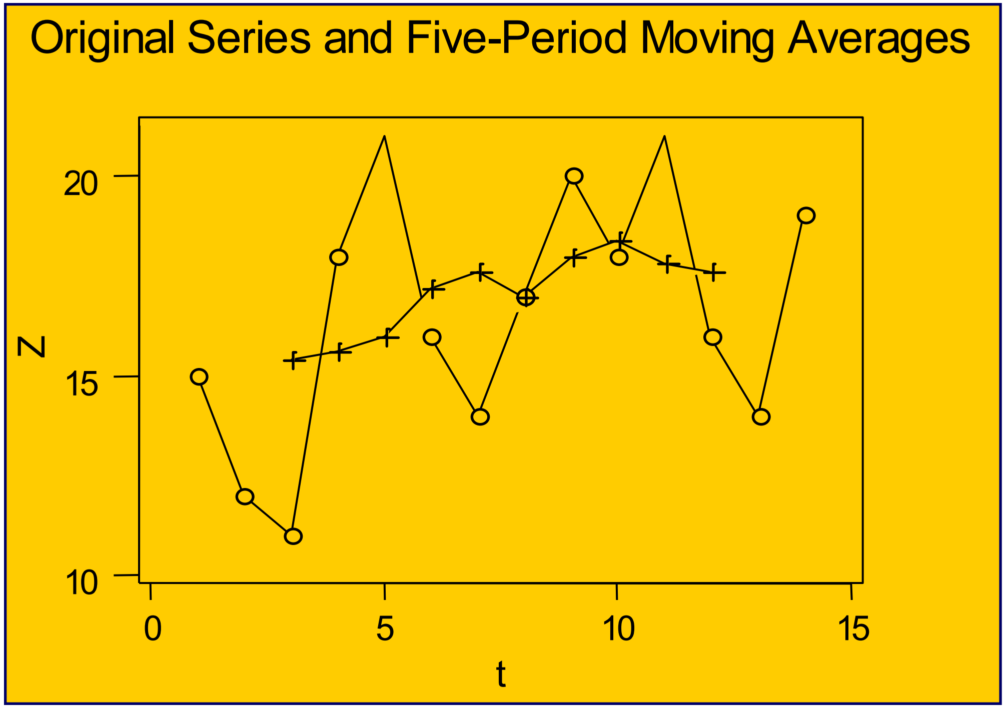
$(12 + 11 + 18 + 21 + 16)/5 = 15.6$

$(11 + 18 + 21 + 16 + 14)/5 = 16.0$

. . . . .

$(18 + 21 + 16 + 14 + 19)/5 = 17.6$

Comparing Original Data and Smoothed Moving Average



- Moving Average:
  - "Smoother"
  - Shorter
  - Deseasonalized
    - Removes seasonal and irregular components
  - Leaves trend and cyclical components

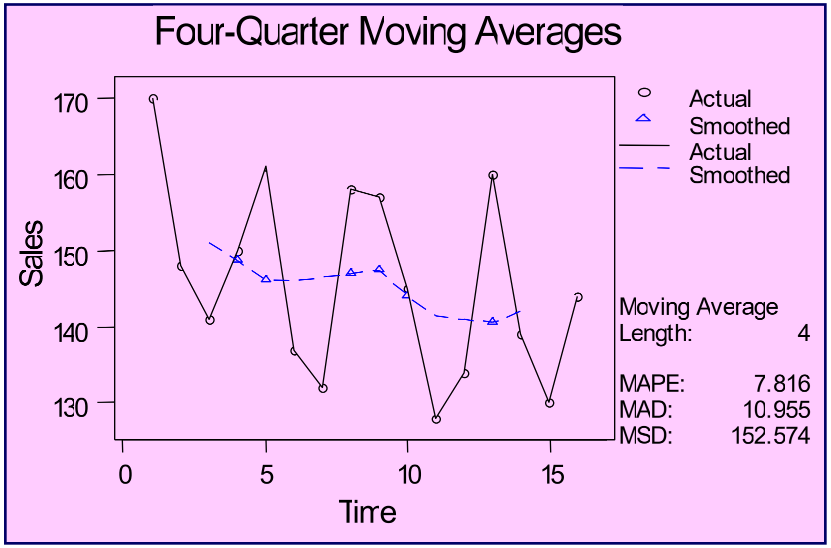
$$\frac{Z_t}{MA} = \frac{TSCI}{TC} = SI$$

Ratio-to-Moving Average

- Ratio-to-Moving Average for Quarterly Data
  - Compute a four-quarter moving-average series.
  - Center the moving averages by averaging every consecutive pair and placing the average between quarters.
  - Divide the original series by the corresponding moving average. Then multiply by 100.
  - Derive **quarterly indexes** by averaging all data points corresponding to each quarter. Multiply each by 400 and divide by sum.

Ratio-to-Moving Average: Example 12-3

Quarter	Sales	Simple Moving Average	Centered Moving Average	Ratio to Moving Average
1998W	170	*	*	*
1998S	148	*	*	*
1998S	141	*	151.125	93.3
1998F	150	152.25	148.625	100.9
1999W	161	150.00	146.125	110.2
1999S	137	147.25	146.000	93.8
1999S	132	145.00	146.500	90.1
1999F	158	147.00	147.000	107.5
2000W	157	146.00	147.500	106.4
2000S	145	148.00	144.000	100.7
2000S	128	147.00	141.375	90.5
2000F	134	141.00	141.000	95.0
2002W	160	141.75	140.500	113.9
2002S	139	140.25	142.000	97.9
2002S	130	140.75	*	*
2002F	144	143.25	*	*



Seasonal Indexes: Example 12-3

		Quarter		
Year	**Winter**	**Spring**	**Summer**	**Fall**
1998			93.3	100.9
1999	110.2	93.8	90.1	107.5
2000	106.4	100.7	90.5	95.0
2002	113.9	97.9		
Sum	330.5	292.4	273.9	303.4
Average	110.17	97.47	91.3	101.13

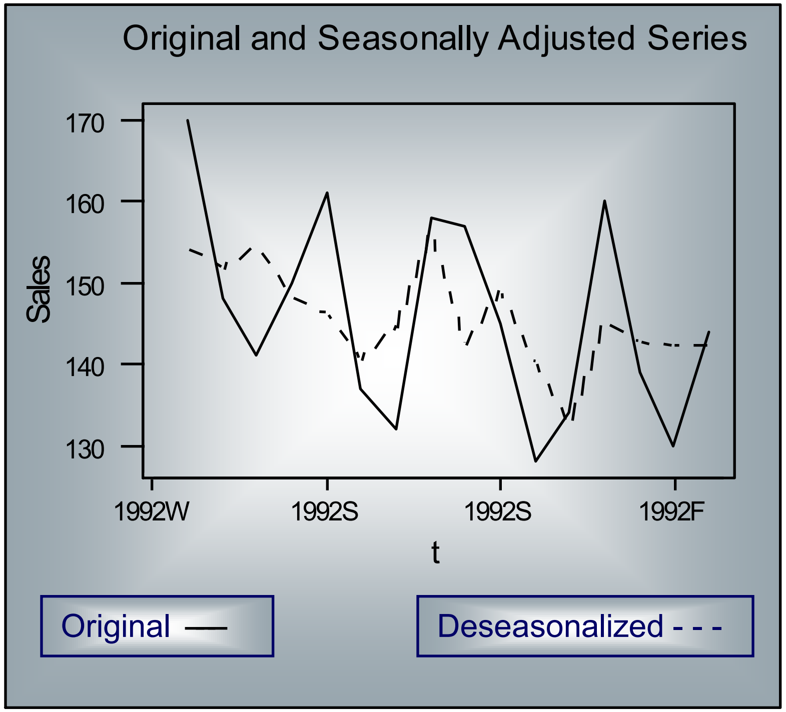
Sum of Averages = 400.07

Seasonal Index = (Average)(400)/400.07

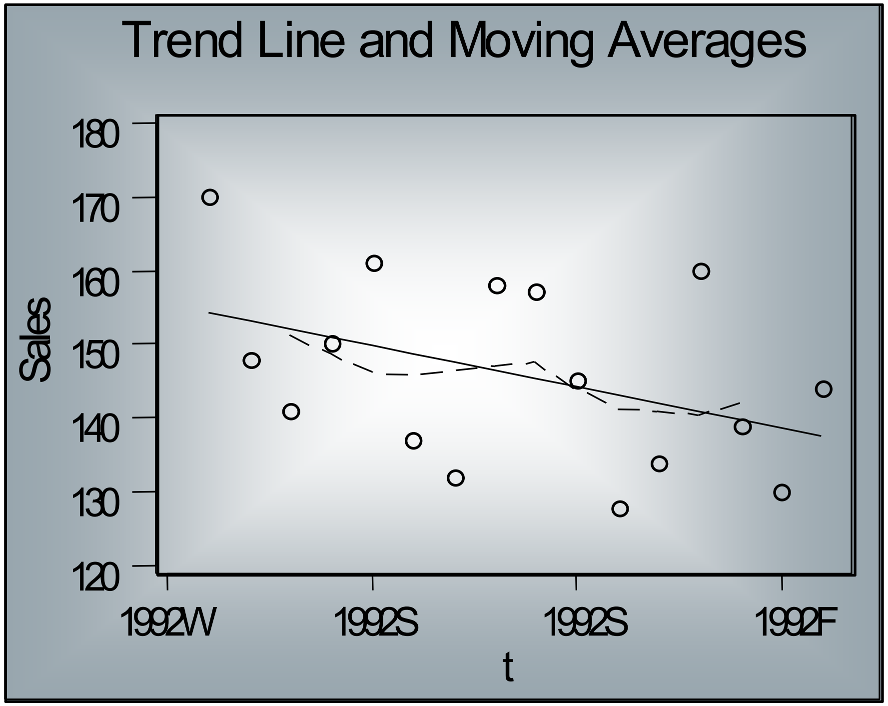
Seasonal Index	**110.15**	**97.45**	**91.28**	**101.11**

Deseasonalized Series: Example 12-3

Quarter	Sales	Seasonal Index (S)	Deseasonalized Series(Z/S)*100
1998W	170	110.15	154.34
1998S	148	97.45	151.87
1998S	141	91.28	154.47
1998F	150	101.11	148.35
1999W	161	110.15	146.16
1999S	137	97.45	140.58
1999S	132	91.28	144.51
1999F	158	101.11	156.27
2000W	157	110.15	142.53
2000S	145	97.45	148.79
2000S	128	91.28	140.23
2000F	134	101.11	132.53
2002W	160	110.15	145.26
2002S	139	97.45	142.64
2002S	130	91.28	142.42
2002F	144	101.11	142.42



The Cyclical Component: Example 12-3



The **cyclical component** is the remainder after the moving averages have been detrended. In this example, a comparison of the moving averages and the estimated regression line:

$$\hat{Z} = 155.275 - 1.1059t$$

illustrates that the cyclical component in this series is negligible.

Example 12-3 using the Template

The centered moving average, ratio to moving average, seasonal index, and deseasonalized values were determined using the Ratio-to-Moving-Average method.

Forecasting with Trend and Seasonality

Northern Natural Gas

Data

t	Year	Q	Y	Deseason
1	2001	1	170	154.351
2	2001	2	148	151.876
3	2001	3	141	154.451
4	2001	4	150	148.335
5	2002	1	161	146.179
6	2002	2	137	140.588
7	2002	3	132	144.593
8	2002	4	158	156.246
9	2003	1	157	142.548
10	2003	2	145	148.797
11	2003	3	128	140.211
12	2003	4	134	132.513
13	2004	1	160	145.271
14	2004	2	139	142.64
15	2004	3	130	142.402
16	2004	4	144	142.402

Forecasts

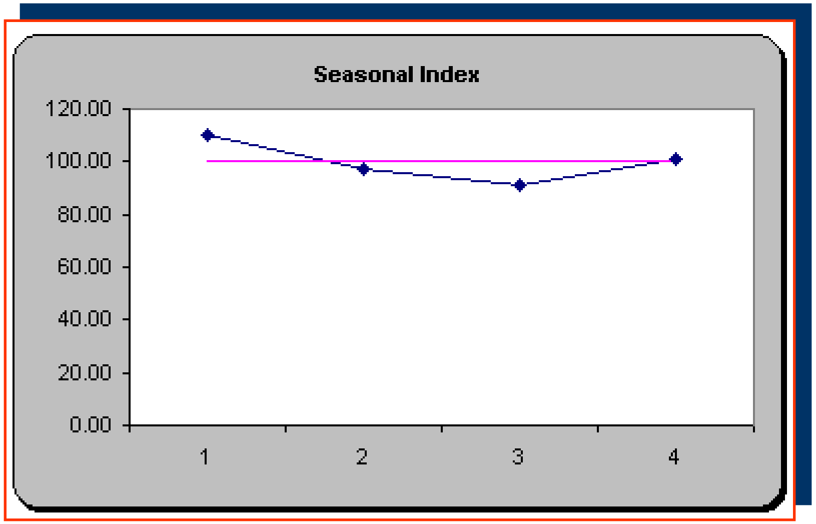
t	Year	Q	Y
17	2005	1	152.02
18	2005	2	133.69
19	2005	3	124.48
20	2005	4	137.04
21	2006	1	148.33
22	2006	2	130.43
23	2006	3	121.42
24	2006	4	133.65
25	2007	1	144.64
26	2007	2	127.16
27	2007	3	118.36
28	2007	4	130.26

Seasonal Indices

Q	Index
1	110.14
2	97.45
3	91.29
4	101.12
	400

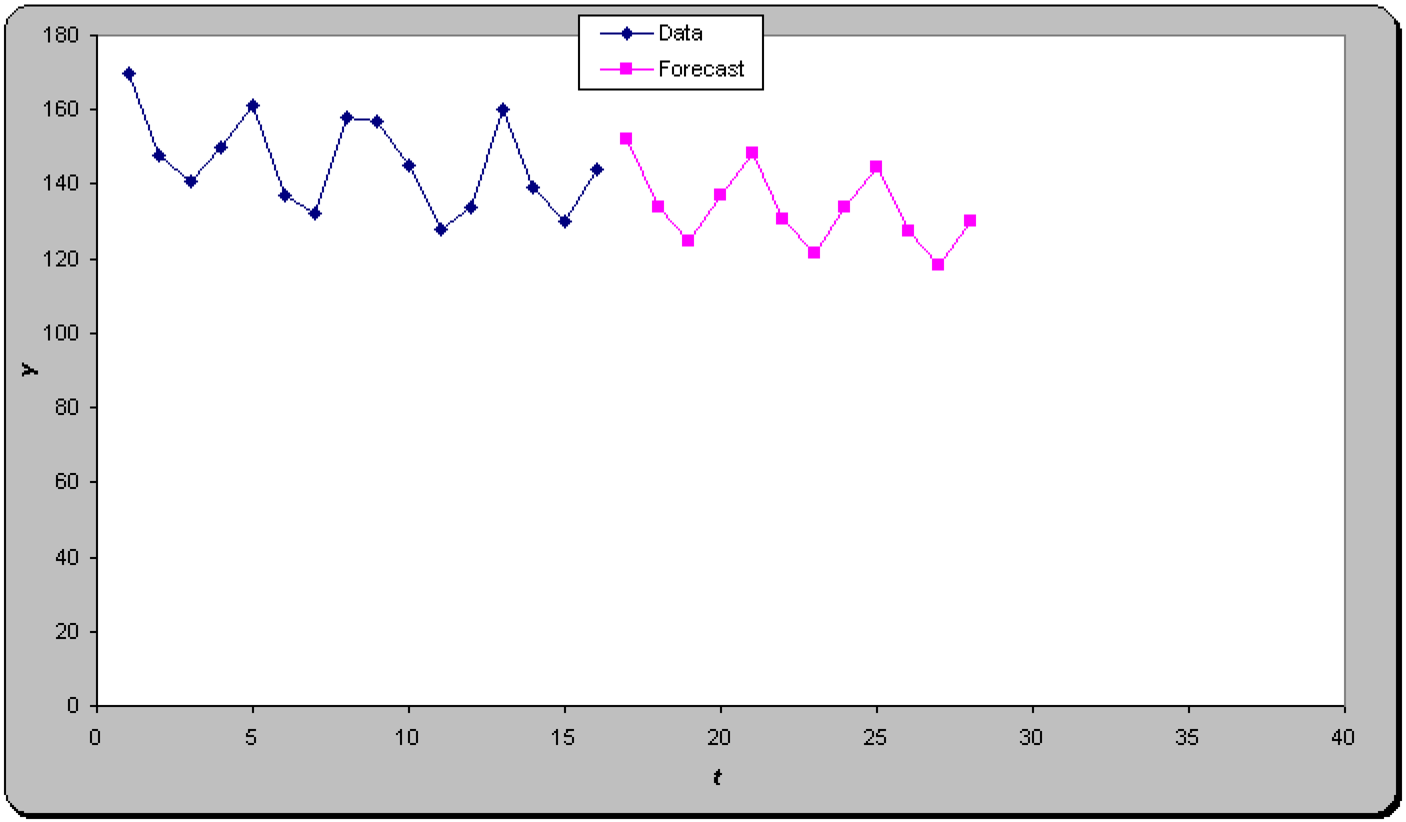
This is a partial output for the quarterly forecasts.

Example 12-3 using the Template



Graph of the quarterly Seasonal Index

Example 12-3 using the Template



Graph of the Data and the quarterly Forecasted values

Example 12-3 using the Template

The centered moving average, ratio to moving average, seasonal index, and deseasonalized values were determined using the Ratio-to-Moving-Average method.

Forecasting with Trend and Seasonality

Average Pay

Data

t	Year	Month		Y	Deseason
1	2001	4	Apr	1306	1315.33
2	2001	5	May	1282	1300.87
3	2001	6	Jun	1297	1308.15
4	2001	7	Jul	1315	1319.1
5	2001	8	Aug	1310	1307.39
6	2001	9	Sep	1323	1318.83
7	2001	10	Oct	1335	1305.09
8	2001	11	Nov	1322	1310.79
9	2001	12	Dec	1317	1315.81
10	2002	1	Jan	1325	1330.1
11	2002	2	Feb	1327	1323.51
12	2002	3	Mar	1328	1332.14
13	2002	4	Apr	1322	1331.44
14	2002	5	May	1320	1339.43
15	2002	6	Jun	1339	1350.51
16	2002	7	Jul	1343	1347.18
17	2002	8	Aug	1353	1350.31
18	2002	9	Sep	1358	1353.72
19	2002	10	Oct	1399	1367.66
20	2002	11	Nov	1379	1367.31
21	2002	12	Dec	1369	1367.77

Forecasts

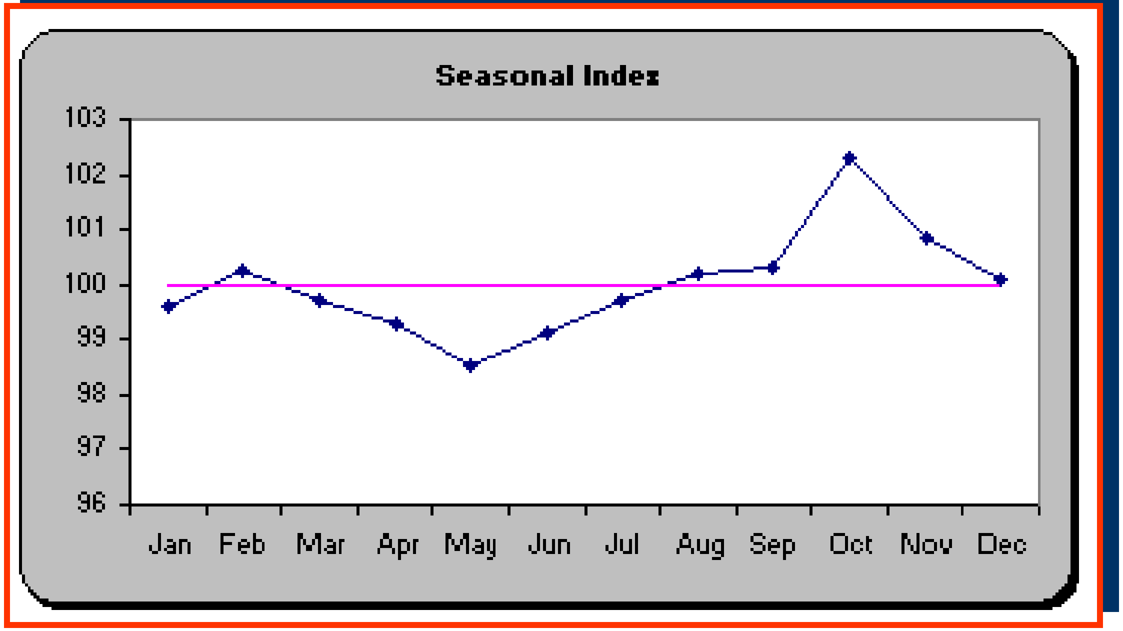
t	Year	Month		Y
34	2004	1	Jan	1397.7
35	2004	2	Feb	1410
36	2004	3	Mar	1405.1
37	2004	4	Apr	1402.7
38	2004	5	May	1395.4
39	2004	6	Jun	1407
40	2004	7	Jul	1417.9
41	2004	8	Aug	1428.3
42	2004	9	Sep	1433.2
43	2004	10	Oct	1464.7
44	2004	11	Nov	1447.4
45	2004	12	Dec	1439.6

Seasonal Indices

Mont	Index
Jan	99.62
Feb	100.26
Mar	99.69
Apr	99.29
May	98.55
Jun	99.15
Jul	99.69
Aug	100.20
Sep	100.32
Oct	102.29
Nov	100.86
Dec	100.09

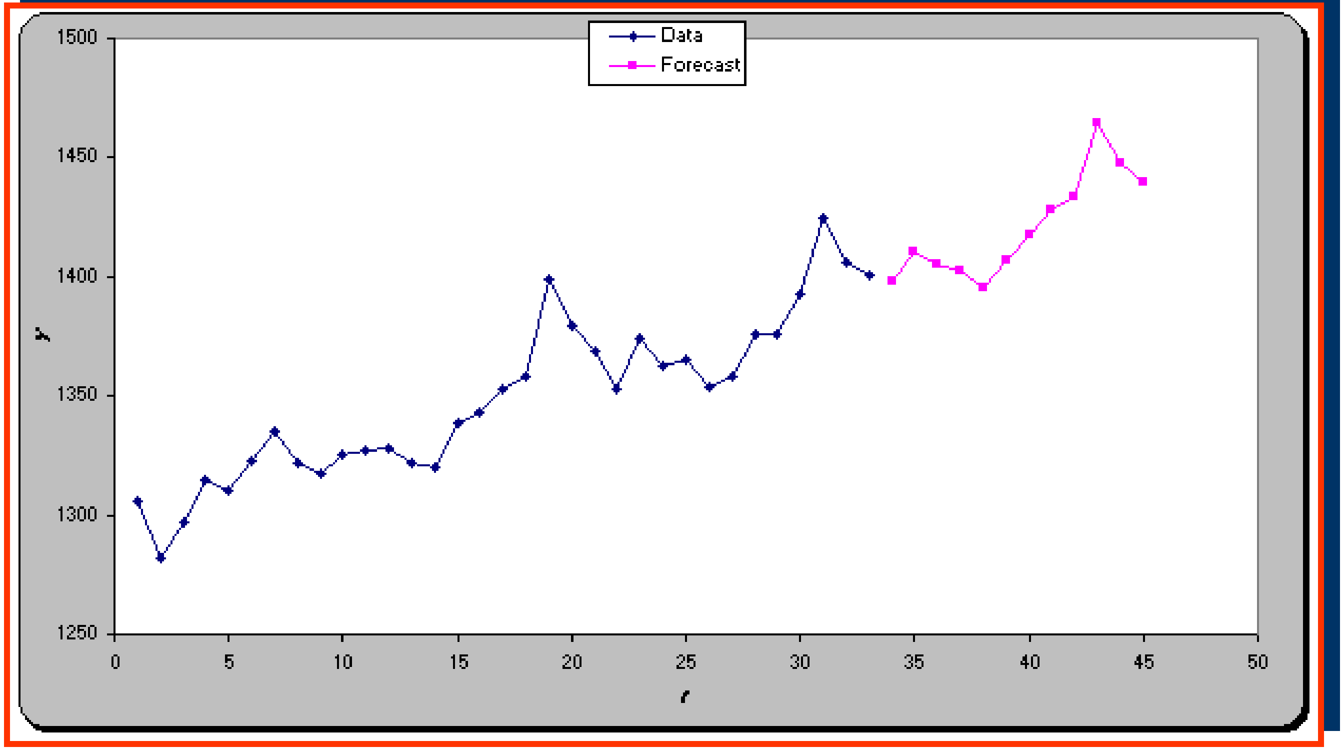
This displays just a partial output for the monthly forecasts.

Example 12-3 using the Template



Graph of the monthly Seasonal Index

Example 12-3 using the Template



Graph of the Data and the monthly Forecasted values

Forecasting a Multiplicative Series: Example 12-3

The forecast of a multiplicative series:

$$\hat{Z} = TSC$$

Forecast for Winter 2002 (t = 17):

Trend: $\hat{z} = 152.26 - (0.837)(17) = 138.03$

$S = 1.1015$

$C \approx 1$ (negligible)

$$\hat{Z} = TSC$$
$$= (1)(138.03)(1.1015) = 152.02$$

Multiplicative Series: Review

$$\begin{aligned} Z &= (Trend)(Seasonal(Cyclical)(Irregular) \\ &= TSCI \end{aligned}$$

$$\begin{aligned} MA &= (Trend)(Cyclical) \\ &= TC \end{aligned}$$

$$\frac{Z}{MA} = \frac{TSCI}{TC} = SI$$

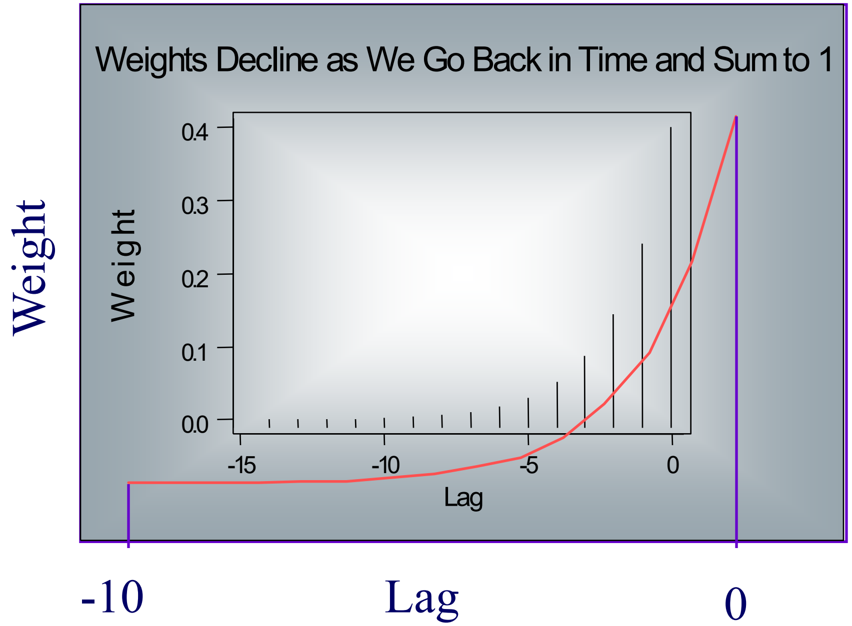
$$\text{S} = \text{Average of SI (Ratio - to - Moving Averages)}$$

$$\frac{Z}{S} = \frac{TSCI}{S} = CTI \text{ (Deseasonalized Data)}$$

12-5 Exponential Smoothing Methods

Smoothing is used to forecast a series by first removing sharp variation, as does the moving average.

Weights Decline as we go back in Time



Exponential smoothing is a forecasting method in which the forecast is based in a ***weighted average*** of current and past series values. The largest weight is given to the present observations, less weight to the immediately preceding observation, even less weight to the observation before that, and so on. ***The weights decline geometrically as we go back in time.***

The Exponential Smoothing Model

Given a weighting factor: $0 < w < 1$:

$$\hat{Z}_{t+1} = w(Z_t) + w(1-w)(Z_{t-1}) + w(1-w)^2(Z_{t-2}) + w(1-w)^3(Z_{t-3}) + \cdots$$

Since

$$\hat{Z}_t = w(Z_{t-1}) + w(1-w)(Z_{t-2}) + w(1-w)^2(Z_{t-3}) + w(1-w)^3(Z_{t-4}) + \cdots$$

$$(1-w)\hat{Z}_t = w(1-w)(Z_{t-1}) + w(1-w)^2(Z_{t-2}) + w(1-w)^3(Z_{t-3}) + \cdots$$

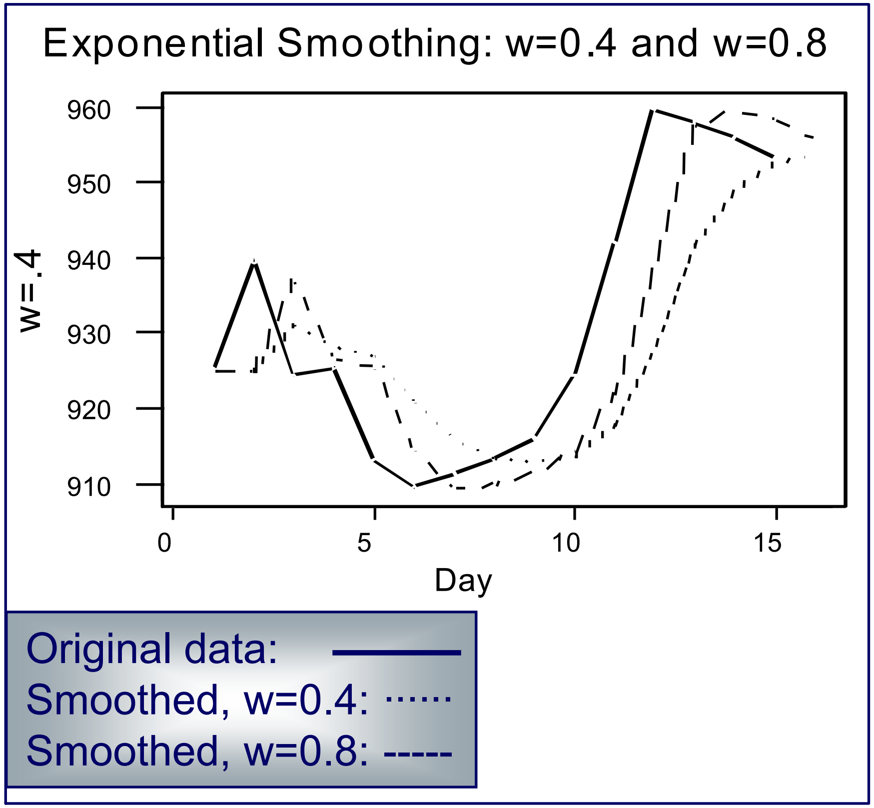
So

$$\mathbf{\hat{Z}_{t+1} = w(Z_t) + (1-w)(\hat{Z}_t)}$$

$$\mathbf{\hat{Z}_{t+1} = Z_t + (1-w)(\hat{Z}_t - Z_t)}$$

Example 12-4

Day	Z	w=.4	w=.8
1	925	925.000	925.000
2	940	925.000	925.000
3	924	931.000	937.000
4	925	928.200	926.600
5	912	926.920	925.320
6	908	920.952	914.664
7	910	915.771	909.333
8	912	913.463	909.867
9	915	912.878	911.573
10	924	913.727	914.315
11	943	917.836	922.063
12	962	927.902	938.813
13	960	941.541	957.363
14	958	948.925	959.473
15	955	952.555	958.295
16	*	953.533	955.659



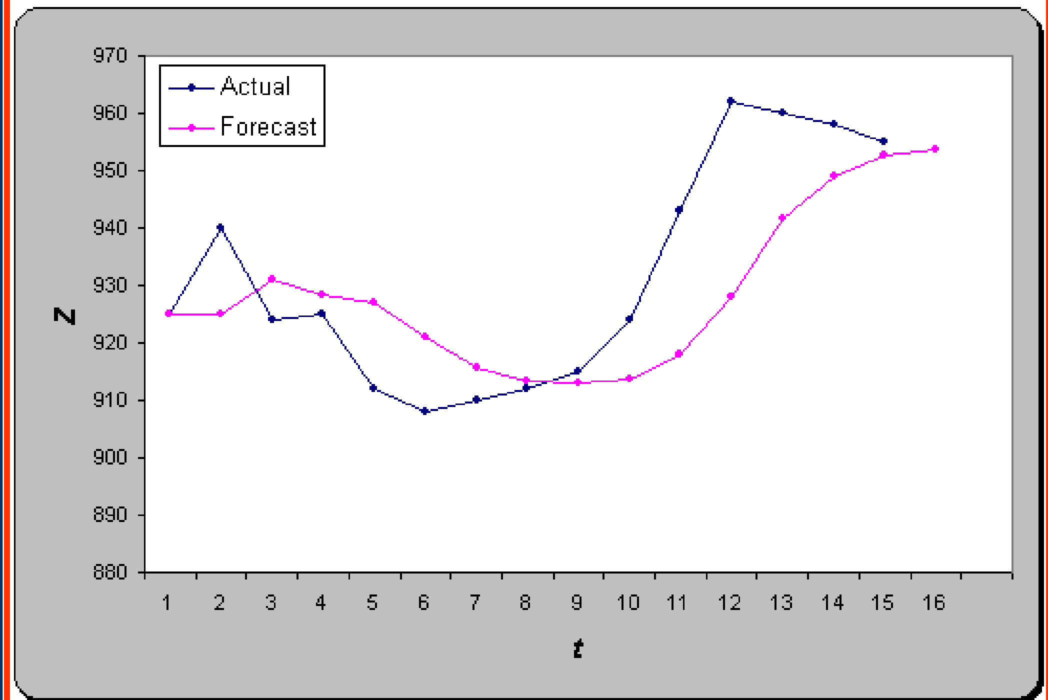
Example 12-4 – Using the Template

Exponential Smoothing

	MAE	MAPE	MSE
	11.3034	1.20%	216.931

w = 0.4

t	Z_t	Forecast	|Error|	%Error	$Error^2$
1	925	925			
2	940	925			
3	924	931	7	0.76%	49
4	925	928.2	3.2	0.35%	10.24
5	912	926.92	14.92	1.64%	222.606
6	908	920.952	12.952	1.43%	167.754
7	910	915.771	5.7712	0.63%	33.3067
8	912	913.463	1.46272	0.16%	2.13955
9	915	912.878	2.12237	0.23%	4.50445
10	924	913.727	10.2734	1.11%	105.543
11	943	917.836	25.1641	2.67%	633.23
12	962	927.902	34.0984	3.54%	1162.7
13	960	941.541	18.4591	1.92%	340.737
14	958	948.925	9.07544	0.95%	82.3635
15	955	952.555	2.44526	0.26%	5.9793
16		953.533			



12-6 Index Numbers

An ***index number*** is a number that measures the *relative* change in a set of measurements over time. For example: the Dow Jones Industrial Average (DJIA), the Consumer Price Index (CPI), the New York Stock Exchange (NYSE) Index.

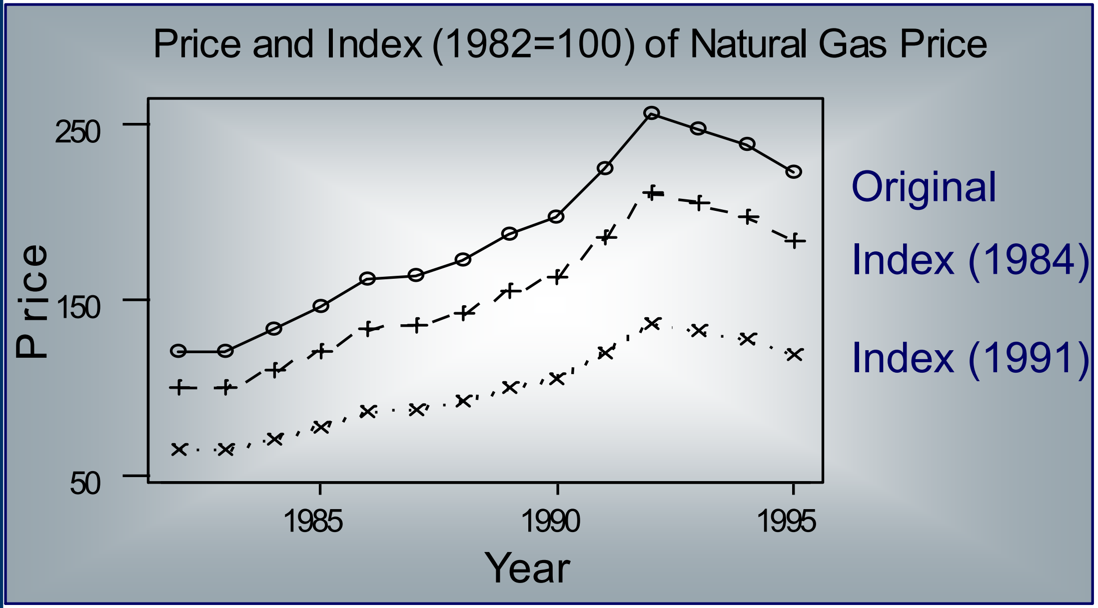
$$\text{Index number in period } i\text{:} \quad = 100 \frac{\text{Value in period } i}{\text{Value in base period}}$$

Changing the base period of an index:

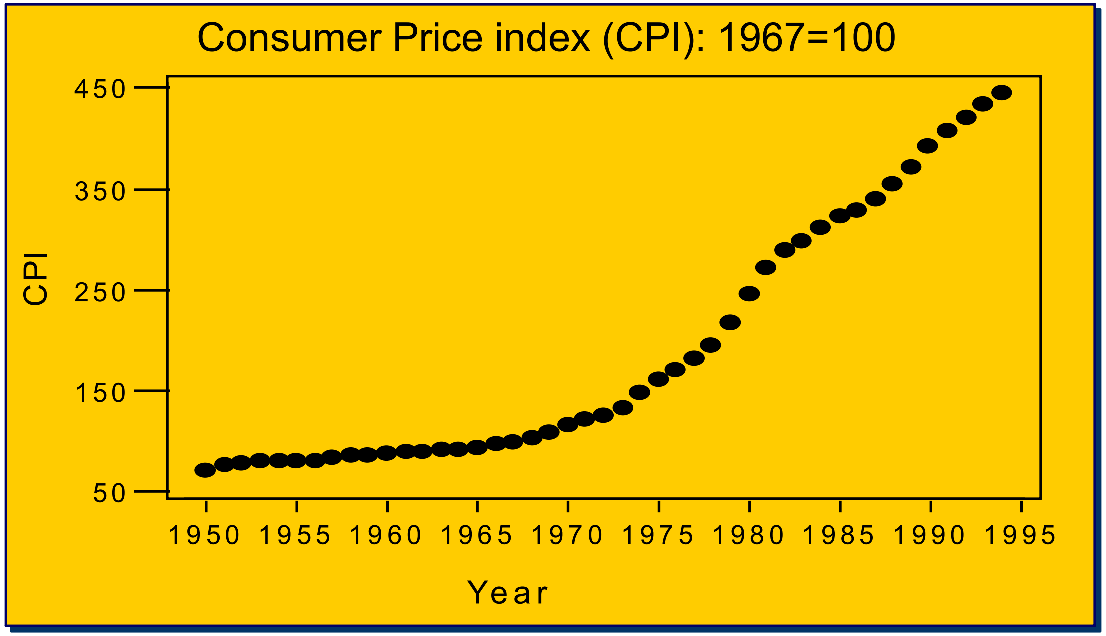
$$\text{New index value:} \quad = 100 \frac{\text{Old index value}}{\text{Index value of new base}}$$

Index Numbers: Example 12-5

Year	Price	Index 1984-Base	Index 1991-Base
1984	121	100.0	64.7
1985	121	100.0	64.7
1986	133	109.9	71.1
1987	146	120.7	78.1
1988	162	133.9	86.6
1989	164	135.5	87.7
1990	172	142.1	92.0
1991	187	154.5	100.0
1992	197	162.8	105.3
1993	224	185.1	119.8
1994	255	210.7	136.4
1995	247	204.1	132.1
1996	238	196.7	127.3
1997	222	183.5	118.7



Consumer Price Index – Example 12-6



Example 12-6:

$$\text{Adjusted Salary} = \frac{\text{Salary}}{\text{CPI}}100$$

Year	Salary	Adjusted Salary
1980	29500	11953.0
1981	31000	11380.3
1982	33600	11610.2
1983	35000	11729.2
1984	36700	11796.8
1985	38000	11793.9

Example 12-6: Using the Template

Price Index

BaseYear	1984	121	Base
	Year	**Price**	**Index**
	1984	121	100
	1985	121	100
	1986	133	109.92
	1987	146	120.66
	1988	162	133.88
	1989	164	135.54
	1990	172	142.15
	1991	187	154.55
	1992	197	162.81
	1993	224	185.12
	1994	255	210.74
	1995	247	204.13
	1996	238	196.69
	1997	222	183.47