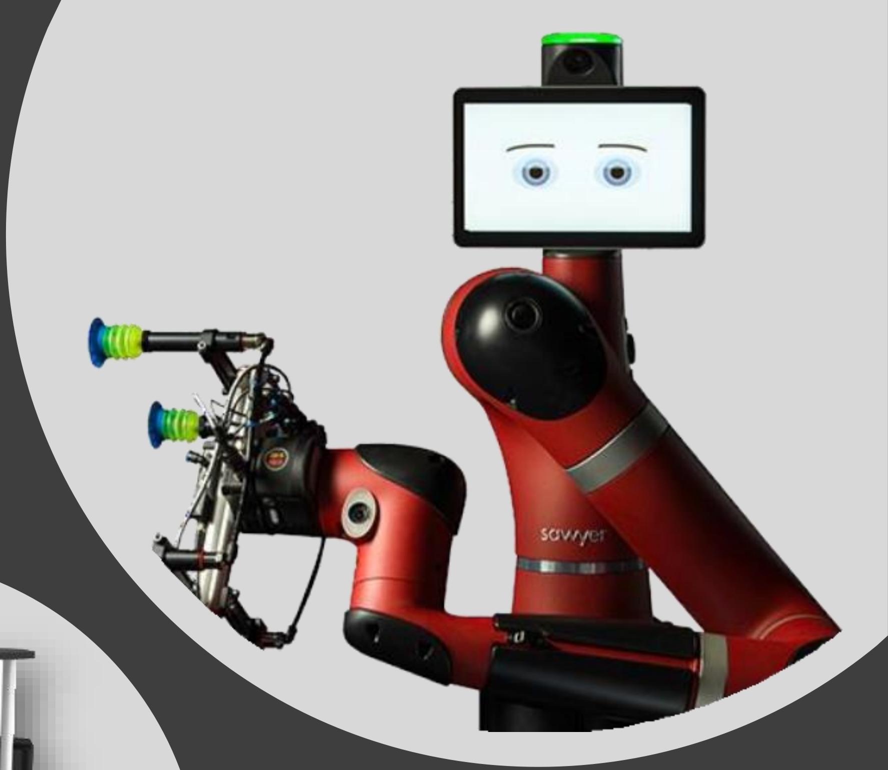
ROBOTICS CORNER

# Mobile Robot Kinematics

# Motivating Questions

- What is a *model*?
- Why do we need a model?
- What is *kinematics*?
- How to model the kinematics of a mobile robot?

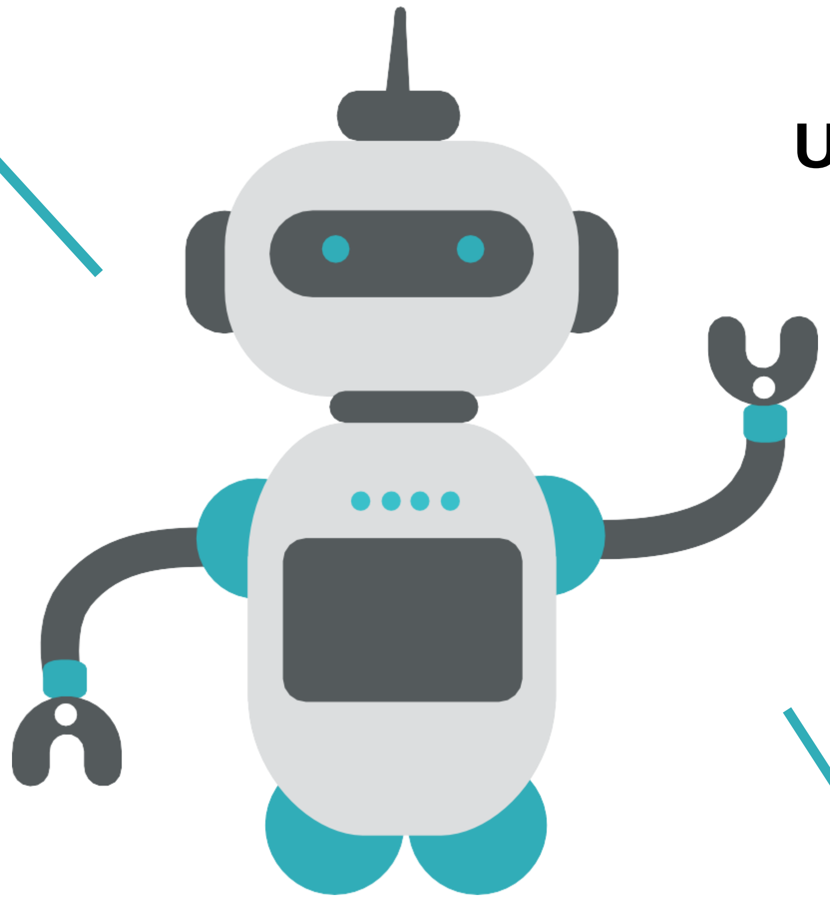



# Model is:

a simplified, mathematical representation of a real / physical system which is known as Mathematical model.
 it also refers to **the process of creating a mathematical representation of a real-world scenario to make a prediction or provide insight**.

# Is model useful?

*“All models are wrong, but some are useful.”*
**George Box**



**Usage:**

- Describe the behaviour of the system
- Estimate the future behaviour of the system
- Give an insight regarding the property of the system
- Prepare for a better control strategy

# Model example


Horizontal Motion $(a_x = 0)$

$$x = x_0 + v_x t$$

$$v_x = v_{0x} = v_x = \text{velocity is a constant.}$$

Vertical Motion $\left(\text{assuming positive is up } a_y = -g = -9.80\text{m/s}^2\right)$

$$y = y_0 + v_{0y} t - \tfrac{1}{2}\text{gt}^2$$

$$v_y = v_{0y} - \text{gt}$$

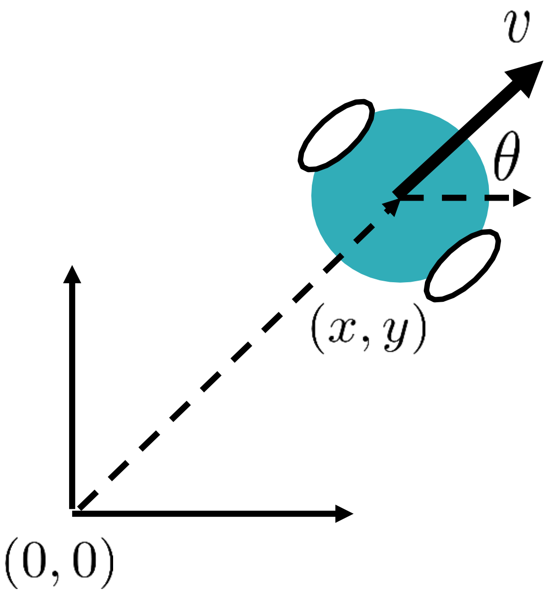
# What is Kinematics?

- Studying the **MOTION** of an object without considering its **CAUSES**

- **Motion**
Speed, Velocity , Acceleration

**Causes**:
Torque, Force

# Kinematics model of a mobile robot




**Output:**

Position $(x, y)$

Orientation / heading $(\theta)$

**Input:**

Linear speed $v$

Angular speed $\omega$

| Position | Orientation | Velocity |
|---|---|---|
| $\Delta x = v_x \Delta t$<br>$\Delta y = v_y \Delta t$ | $\Delta \theta = \omega \Delta t$ | $v_x = v \cos \theta$<br>$v_y = v \sin \theta$ |

# Rigid Body transformation

To start with basics of robotics we should first know what is a frame in 2D/3D world. A frame is nothing, but a coordinate axis attached to a body as shown in below figures.

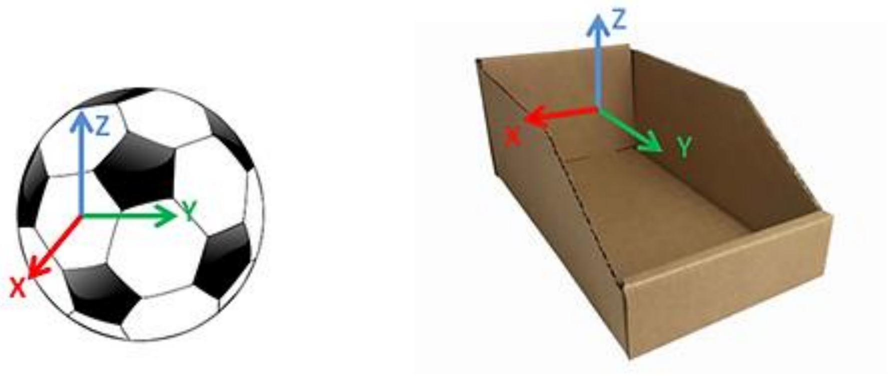



**Fig. 1** 3D Frames attached to objects


**Fig. 2** 2D Frames attached to objects

# Transformations

***Key Point:* Key Point: Generally, color code is followed for frame representation i.e., Red for X axis, Green for Y axis and  Blue for Z axis. RGB for XYZ**

Now, why do we need to attach a frame to a body or what is the use of these frames?

Well, the answer is simple, to locate a object's position and orientation with respect to other. An example is illustrated in fig 3.

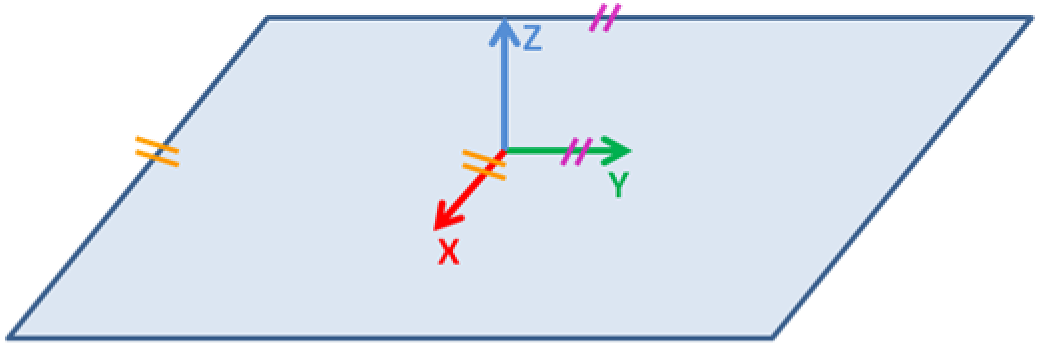
But how are we going to determine position and orientation of the objects using frames?

To answer this question, we need to know about 2D/3D **Transformations**.

# What is Transformation?

Transformation is simply the change of position and orientation of a frame attached to a body with respect to a frame attached to another body. Transformations in a planar space is known as 2D transformation and transformations in a spatial world is known as 3D transformation

**Translation**: Change in position
**Rotation**: Change in orientation
**Transformation**: Translation + Rotation



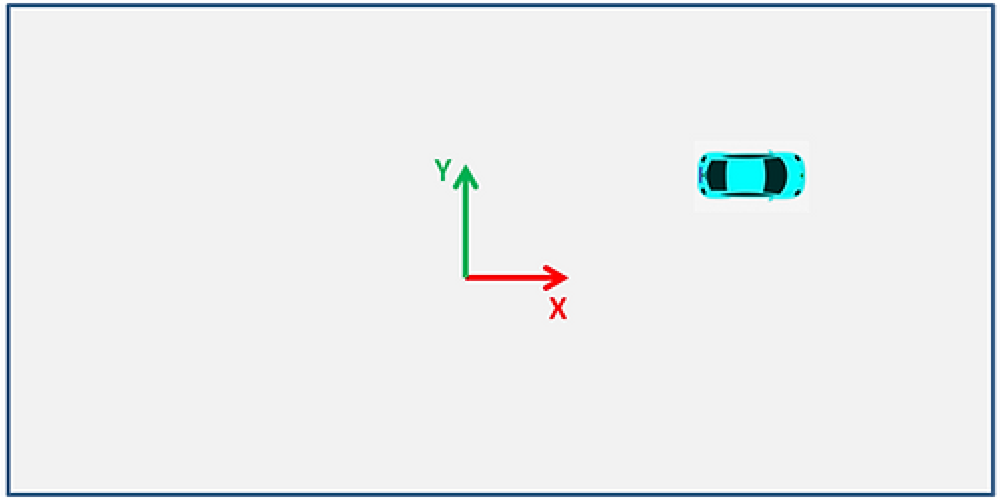

**2D Transformations:**

Let's start with the easy one first, 2D Transformation. We all know that 2D is nothing but a plane which is parallel to two axes and perpendicular to the third axis

# 2D Transformation

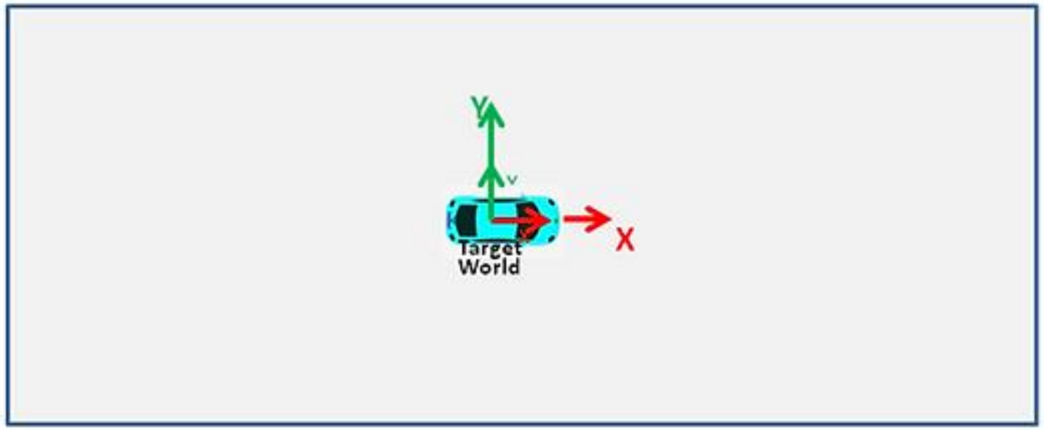
**Q:** Let us say you have a toy car on the floor as shown in fig 5. Now how many variables you need, to position this car on the floor?

As the name says 2D transform, so you have guessed 2 variables x,y then you are absolutely wrong. If you guessed 3 variables as x, y and theta then Congratulations for the correct answer.



# Frames

- Now let's see how to determine the position and orientation of a body using frames and 2D Transformation in a plane using above car example.
- Wait-a-minute, from which point are we trying to locate the car or with respect to what..?
- For that we have to first attach a fixed frame to the floor. This frame is called the "**reference frame**". Also we have to attach a frame to car.
- Let us name this reference frame attached to the floor as "**world**" frame and frame attached to car as "**target**" frame.



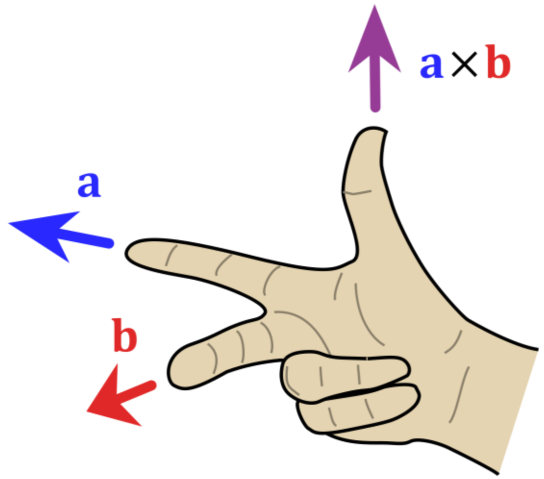
# World frame

- Let's put the world frame in the center of the floor with X and Y axes parallel to the floor and Z axis coming out of the plane (Use right hand rule to determine third axis).
- Let us assume initially both target frame and world frame are in the same position and orientation as shown in the fig 6.
- Therefore, the position of target frame (car) with respect to the world frame (observation point) is 0m in X direction and 0m in Y direction (0m,0m). Also the angle of the target frame with respect to fixed to world frame is 0 degrees. Hence the 2D transformation is as below,

$T_{world\text{-}target}$ = (x,y,theta)= (0m,0m,0 degree)

# Right hand rule

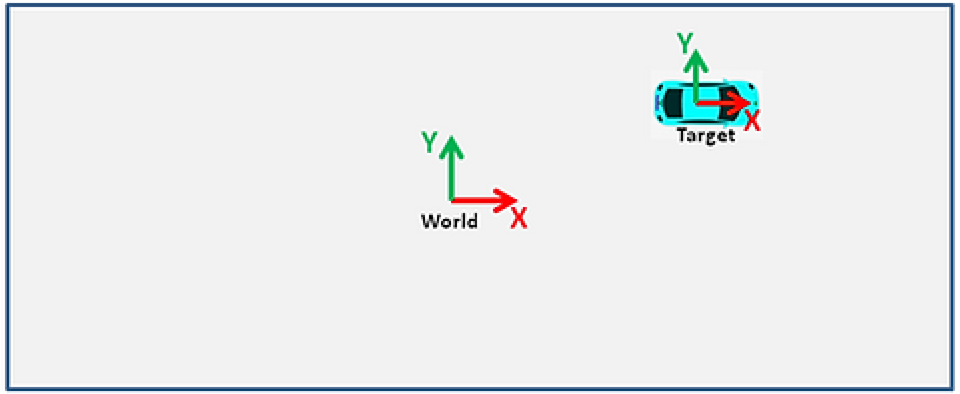
- **Coordinate System:** Imagine you have a three-dimensional coordinate system, with three mutually orthogonal axes - X, Y, and Z. The X-axis typically points to the right, the Y-axis points upward, and the Z-axis points out of the screen or paper toward you.
- **Thumb, Index Finger, and Middle Finger:** Using your right hand, align your thumb, index finger, and middle finger such that:
  - Your thumb points along the positive direction of the axis of rotation.
  - Your index finger points along the positive direction of the axis you want to describe the rotation or orientation about.
  - Your middle finger naturally points in the direction of the resulting rotation or orientation.



# Target Frame

- Now let's move the car to a new position from the reference frame. Let's say 1m in X direction and 1m in Y direction and we will not disturb the orientation of the car as shown in the below fig 7. And transformation would be:

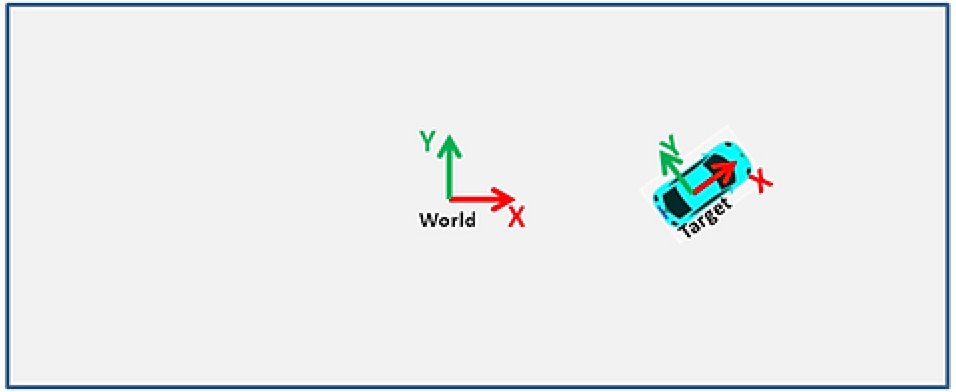
$T_{world\text{-}target}$ = (x,y,theta)= (1m,1m,0 degree)



# Transformation (world-target)

- Then let's move the car such that it is 1m far from reference frame along X direction and 0m along Y direction. This time I will rotate the car +45 degrees (use right hand thumb rule to determine +ve or -ve rotation) with respect to the reference frame. The position of the car will be as shown in fig 8 and the transformation is as below

$T_{world\text{-}target}$ = (x,y,theta)= (1m,0m,45 degrees)



# 2D Transformation matrix

- let's get into some real math. Now we will learn how to represent 2D transform in matrix form.
- Before that we need to learn about matrix representation of 2D Rotation.
- 2D Rotation matrix is a 2X2 matrix. It is given by

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Here θ is angle of rotation.

- Assume reference frame to be frame A and target frame to be B. Transformation of frame B with respect to frame A is given as below.

$$T_{AB} = \begin{bmatrix} R_{AB} & (P_{AB})^T \\ 0 & 1 \end{bmatrix}$$

Here PAB is a 1X2 matrix which represents the translation of frame B with respect to frame A.

$$P_{AB} = [x \quad y]$$

RAB is rotation matrix which is same as R matrix given above

As you may have noticed, we have added 0's and 1 in the last row of TAB matrix to simplify matrix operations.

Therefore, 2D transform matrix is a 3X3 matrix and its expanded form is as following

$$T_{AB} = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix}$$

Yes..!!! This is how 2D transform matrix looks like.

# Thank you