Final Exam: Impact and Penetration

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Problem 1

A protective wall is struck by a rigid ogive projectile traveling 1,000 m/s. Provide a safe and economic wall design. Material and projectile properties provided in 1.

Table 1: Material and projectile properties.

Material properties.

Material Type	ρ_0 Ton/m^3	Yield strength σ_y or f'_c, MPa	Unit cost \$/Ton
Steel(A572 Grade 60)(METLINE INDUSTRIES, 2024)	7.850	400	1,800
Concrete(UHPC*)(Akhnoukh and Buckhalter, 2021)	2.500	90	1,200

* Ultra high performance concrete.

Projectile properties.

Material Type	mass, kg	Length, m	s, m	d, m	μ_m
Steel	0.2	0.05	0.1	0.04	0.2

Solution

The objective here is to prevent perforation in an economic way. There are multiple solutions, due to the variety of methods available.

Concrete panel

Forrestal et al. (1994, 1996); Frew et al. (2006, 1998) proposed the following formula for projectile depth of penetration[1],

$$\begin{aligned} h_\text{pen} &= \frac{(L+0.5k'd)}{2N}\left(\frac{\rho_p}{\rho_0}\right)\ln\left[1+\frac{N\rho_0 V_1^2}{Sf_\text{c}}\right]+2d \\ &= \frac{2M}{\pi d^2\rho_0 N}\ln\left[1+\frac{N\rho_0 V_1^2}{Sf_\text{c}}\right]+2d \end{aligned} \tag{1}$$

[1]DOP.

$$k' = \left(4\psi^2 - \frac{4\psi}{3} + \frac{1}{3}\right)(4\psi - 1)^{0.5} - 4\psi^2(2\psi - 1)\sin^{-1}\left[\frac{(4\psi-1)^{0.5}}{2\psi}\right] \quad (2)$$

$$N = \frac{8\psi - 1}{24\psi^2}; \quad V_1^2 = \frac{V_0^2 - (2d/(L+0.5k'd))\,(Sf_c/\rho_p)}{1 + N\,(2d/(L+0.0k'd))\,(\rho_0/\rho_p)}; \quad S = 82.6 \times \left(f_c \times 10^{-6}\right)^{-0.544} \quad (3)$$

Forrestal et al. (2003) modified the above. The proposed formula are only applicable for ogive-nosed projectiles while neglecting friction between projectile and target medium.

$$\begin{aligned}
\psi &= \frac{0.1}{0.04} \to N = \frac{8(2.5) - 1}{24(2.5)^2} \to = 0.1267 \\
S &= 82.6 \times \left(90 \times 10^6 \times 10^{-6}\right)^{-0.544} \to = 7.143 \\
k' &= \left(4(2.5)^2 - \frac{4(2.5)}{3} + \frac{1}{3}\right)(4(2.5) - 1)^{0.5} - 4(2.5)^2(2(2.5) - 1)\sin^{-1}\left[\frac{(4(2.5)-1)^{0.5}}{2(2.5)}\right] \\
&= 1.650 \\
V_1^2 &= \left[(1000)^2 - \left(\frac{2(0.04)}{0.05 + 0.5(1.650)(0.04)}\right)\left(\frac{(7.430)\,(90 \times 10^6)}{7850}\right)\right] \\
&\times \left[1 + (0.1267)\left(\frac{20.04)}{0.05 + 0.5(1.650)(0.04)}\right)\left(\frac{2500}{7850}\right)\right]^{-1} \\
&= 886000 \to V_1 = 941.6 \text{ m/s} \\
h_{\text{pen}} &= \frac{2(0.2)}{\pi(0.04)^2(2500)(0.1267)} \ln\left[1 + \frac{(0.1267)(2500)(941.6)^2}{(7.143)\,(90 \times 10^6)}\right] + 2(0.04) \\
&= 0.1711 \to \text{ or } 17.11 \text{ cm}
\end{aligned}$$

Chen and Li (2002); Li and Chen (2003) extended the equations presented by Forrestal et al. (1993, 1996) to a dimensionless form applicable to arbitrary-nosed projectiles. According to Chen and Li (2002); Li and Chen (2003) maximum penetration depth in a thick plate is

$$\frac{h_{\text{pen}}}{d} = \frac{2}{\pi} N \ln\left(1 + \frac{I_0}{N}\right) \quad (4)$$

$$I_0 = \frac{MV_0^2}{N_1 S f_c d^3}; \quad N = \frac{M}{N_2 \rho_0 d^3}; \quad S = 72.0 \times \left(f_c \times 10^{-6}\right)^{-0.5} \quad (5)$$

where I_0 and N are defined as the impact and geometry functions, N_1 and N_2 account for projectile nose geometry as well as friction. For an arbitrary-nosed projectile, N_1 and N_2 are obtained through integrating the projectile nose profile function along the nose length. Explicit expressions of N_1 and N_2 for ogive, conical, blunt, truncated ogive, hemispherical, and flat projectile noses are given Chen and Li (2002). For ogive nosed projectiles we have

$$
\begin{aligned}
N_1 &= 1 + 4\mu_{\mathrm{m}}\psi^2 \left[\left(\frac{\pi}{2} - \phi_0 \right) - \frac{\sin 2\phi_0}{2} \right] \\
N_2 &= N^* + \mu_{\mathrm{m}}\psi^2 \left[\left(\frac{\pi}{2} - \phi_0 \right) - \frac{1}{3} \left(2\sin 2\phi_0 + \frac{\sin 4\phi_0}{4} \right) \right] \\
N^* &= \frac{1}{3\psi} - \frac{1}{24\psi^2}, \quad 0 < N^* \leq \frac{1}{2} \\
\phi_0 &= \sin^{-1}\left(1 - \frac{1}{2\psi} \right), \quad \text{where } \psi \geq \frac{1}{2}
\end{aligned}
$$

Inclusion of sliding friction coefficient makes this formula quite advantageous.

$$
\begin{aligned}
\phi_0 &= \sin^{-1}\left(1 - \frac{1}{2\psi} \right) \rightarrow = 0.9273 \text{ or } 53.13^\circ \\
N^* &= \frac{1}{3(2.5)} - \frac{1}{24(2.5)^2} \rightarrow = 0.1267 \\
N_1 &= 1 + 4(0.2)(2.5)^2 \left[\left(\frac{\pi}{2} - (0.9273) \right) - \frac{\sin 2(0.9273)}{2} \right] \rightarrow = 1.817 \\
N_2 &= (0.1267) + 0.2(2.5)^2 \left[\left(\frac{\pi}{2} - (0.9273) \right) - \frac{1}{3} \left(2\sin 2(0.9273) + \frac{\sin 4(0.9273)}{4} \right) \right] \\
&= 0.1871 \\
S &= 72.0 \times \left(\left(90 \times 10^6\right) \times 10^{-6} \right)^{-0.5} \rightarrow = 7.589 \\
I_0 &= \frac{(0.2)(1000)^2}{(1.8175)(7.589)\left(90 \times 10^6\right)(0.04)^3} \rightarrow = 2.5172 \\
N &= \frac{0.2}{(0.1871)(2500)(0.04)^3} \rightarrow = 6.681 \\
h_{\text{pen}} &= (0.04) \left(\frac{2}{\pi}(6.681) \ln\left(1 + \frac{(2.5172)}{(6.681)} \right) \right) \\
&= 0.05440 \text{ or } 5.440 \text{ cm}
\end{aligned}
$$

while with no friction

$$
\begin{aligned}
N_1 &= 1 \\
N_2 &= N^* \rightarrow = 0.1267 \\
I_0 &= \frac{(0.2)(1000)^2}{(1)(7.590)\left(90 \times 10^6\right)(0.04)^3} \rightarrow = 4.575 \\
N &= \frac{0.2}{(0.1267)(2500)(0.04)^3} \rightarrow = 9.866 \\
h_{\text{pen}} &= (0.04) \left(\frac{2}{\pi}(9.866) \ln\left(1 + \frac{(4.575)}{(9.866)} \right) \right) \\
&= 0.09571 \text{ or } 9.571 \text{ cm}
\end{aligned}
$$

Chen and Li (2002) states that the formula is valid if the penetration depth is larger than the projectile diameter and the projectile nose length while projectile remains rigid without noticeable deformation and damage.

Steel panel

The formula proposed by Chen and Li (2002) has a good agreement with tests on metal, concrete and soil samples with variable nose shapes and velocities. From Li and Chen (2003),

$$
\begin{aligned}
\frac{X}{d} =& \frac{2}{\pi} N \ln\left(1 + \frac{I}{N}\right) \\
I =& \frac{\lambda \Phi_{\mathrm{J}}}{A N_1}; \quad N = \frac{\lambda}{B N_2}; \quad \lambda = \frac{M}{\rho_0 d^3}; \quad \Phi_{\mathrm{J}} = \frac{\rho_0 V_{\mathrm{i}}^2}{\sigma_{\mathrm{y}}}
\end{aligned}
$$

Based on Forrestal and Luk (1988), Li and Chen (2003) suggests the following for an elastic, perfectly plastic material, where γ[2] is the Poisson's ratio:

$$
A = \frac{2}{3}\left\{1 + \ln\left[\frac{E}{3(1-\gamma)\sigma_y}\right]\right\}
$$

And for incompressible materials $B = 1.5$. Note that parameters ϕ_0, N^*, N_1, N_2 just depend on the penetrator and not the target material. So considering sliding friction,

$$
\phi_0 = 0.9273 \text{ or } 53.130°; \quad N^* = 0.1267; \quad N_1 = 1.8175; \quad N_2 = 0.1871;
$$

and

$$
\begin{aligned}
A =& \frac{2}{3}\left\{1 + \ln\left[\frac{200 \times 10^9}{3(1-0.30)400 \times 10^6}\right]\right\} \rightarrow = 4.315 \\
\lambda =& \frac{0.2}{7850(0.04)^3} \rightarrow = 0.3981 \\
\Phi_{\mathrm{J}} =& \frac{(7850)(1000)^2}{400 \times 10^6} \rightarrow = 19.62 \\
I =& \frac{(0.3981)(19.62)}{(4.315)(1.817)} \rightarrow = 0.9962 \\
N =& \frac{(0.3981)}{(1.5)(0.1871)} \rightarrow = 1.418 \\
X =& (0.04)\left(\frac{2}{\pi}(1.418)\ln\left(1 + \frac{(0.9962)}{(1.418)}\right)\right) \rightarrow = 0.01922 \text{ or } 1.922 \text{ cm}
\end{aligned}
$$

While without friction

$$
\phi_0 = 0.9273 \text{ or } 53.13°; \quad N^* = N_2 \rightarrow = 0.1267; \quad N_1 = 1; \quad A = 4.315;
$$

[2] About 0.30 for most steel grades.

Table 2: Summary of results and costs based on table 1.

Panel type	X with(without) friction cm	Design cm(in)	Cost $\$/\mathrm{m}^2$
Concrete(Forrestal et al., 1994, 1996; Frew et al., 2006, 1998)	-(17.11)	30(12)	900.0
Concrete(Chen and Li, 2002; Li and Chen, 2003)	5.440(9.571)	16(7)	480.0
Steel(Chen and Li, 2002; Li and Chen, 2003)	1.922(3.323)	6(2.5)	270.0

$$\lambda = 0.3981; \quad \Phi_{\mathrm{J}} = 19.62$$

then

$$\begin{aligned} I =& \frac{(0.3981)(19.625)}{(4.315)(1)} \rightarrow= 1.811 \\ N =& \frac{(0.3981)}{(1.5)(0.1267)} \rightarrow= 2.095 \\ X =& (0.04)\left(\frac{2}{\pi}(2.095)\ln\left(1+\frac{1.811}{2.095}\right)\right) \rightarrow= 0.03323 \text{ or } 3.323\,\mathrm{cm} \end{aligned}$$

Final design

Results from earlier discussions are presented in table 2.

Suprisingly UHPC proves more expensive. Design thickness suggestions were made accounting for 25% increase due to formula deviations and including a 1.2 safety factor which rises the margin to about 50% in order to prevent spalling and scabbing as suggested by Krauthammer (2008).

Problem 2

A 200 kg cased cylindrical charge is set off 45 m away from the concrete face of a two layered blast wall. Design this wall for the largest primary fragments, and also calculate wall ballistic thickness. The PETN charge is encased in a mild steel casing 40 cm long with $d_i/t_c = 10$.

Solution

The main assumption is that the fragment is stopped by penetrating the steel layer thus peforating the concrete layer.

The specific weight and Gurney energy constant for PETN are respectively 0.0635 $\mathrm{lb/in}^3$ and 9,600 ft/sec based on UFC3-340-02 (2014) while specific weight of mild steel casing is assumed to be about 490 $\mathrm{lb/ft}^3$.

Total weight of cased charge equals the sum total of charge and casing

Table 3: Given.

Item	Value
Charge weight, kg(lbs)	200(441)
Distance, m(ft)	45(150)
Casing length, cm(in)	40(16)

weights. Thus considering a cylindrical shape

$$\begin{aligned}
W_{\text{Total}} =& W_{ACT} + W_c \rightarrow \rho_{\text{PETN}} V_{ACT} + \rho_c V_c \\
=& l \left(\rho_{\text{PETN}} \left(\frac{\pi}{4}\right) d_i^2 + \rho_c \left(\frac{\pi}{4}\right) \left(d_i + t_c^2 - d_i^2\right) \right) \\
=& t_c^2 l \left(\frac{\pi}{4}\right) \left(100\rho_{\text{PETN}} + 44\rho_c\right) \\
\rightarrow t_c =& 0.115(1.38)\,\text{ft(in)} \rightarrow d_i = 1.15(13.8)\,\text{ft(in)} \\
W_{ACT} =& 16(0.0635) \left(\frac{\pi}{4}\right) (13.8)^2, W_c = 290\,\text{lbs} \\
W =& 1.2 W_{ACT} \rightarrow = 181\,\text{lbs}
\end{aligned}$$

Initial velocity of primary fragments resulting from a higher-order detonation of a cylindrical casing with evenly distributed explosives is expressed as(UFC3-340-02, 2014)

$$v_o = \sqrt{2E' \left(\frac{W/W_c}{1 + 0.5W/W_c}\right)} = 9600 \sqrt{\frac{181/290}{1 + 0.5\,(181/290)}} \rightarrow = 6621\,\text{ft/s}$$

largest fragment weight w_f is calculated by setting N_f equal to 1,

$$\begin{aligned}
N_f =& \frac{8 W_c e^{-{w_f}^{1/2}/M_A}}{M_A^2} \rightarrow = 1 \\
w_f =& \left(M_A \ln\left[\frac{8W_c}{N_f M_A^2}\right] \right)^2, M_A = B t_c^{5/6} d_i^{1/3} \left(1 + \frac{t_c}{d_i}\right) \\
\xrightarrow[\text{UFC3-340-02 (2014)}]{\text{From Table 2-7}} B =& 0.248 \\
M_A =& 0.248\,(1.38)^{5/6}\,(13.8)^{1/3}\,(1 + 0.1) \rightarrow = 0.8558 \\
w_f =& \left(0.8558 \ln\left[\frac{8(290)}{(1)(0.8558)^2}\right] \right)^2 \rightarrow = 47.5\,\text{oz}
\end{aligned}$$

UFC3-340-02 (2014) expresses striking velocity as

$$V_s = V_o e^{-12 k_V R_f}, k_V = 0.5(A/w_f)\rho_a C_D;$$

Assuming $A/w_f = 0.78/w_f^{1/3}$ for a random mild steel fragment (in^2/oz), $\rho_a = 0.00071$ oz/in^3 and $C_D = 1.2$ for primary fragments, thus for a cylindrical casing

$$
\begin{aligned}
d =& \sqrt{\left(\frac{4}{\pi}\right) 0.78 w_f^{2/3}} \rightarrow = 3.61\,\text{in} \rightarrow A = 10.24\,\text{in}^2 \\
k_V =& \frac{1}{2}\left(\frac{10.24}{47.5}\right)(0.00071)(1.2) \rightarrow = 9.184 \times 10^{-5} \\
V_s =& (6621) e^{-12(9.184\times 10^{-5})(150)} \rightarrow = 5612\,\text{ft/s}\,\text{(Case I)}
\end{aligned}
$$

Striking velocity is determinable assuming values like shape factor N and fragment nose tangent ogive caliber radius n for an alternate fragment form. Assuming $N = 1$ and $n = 1.5$, according to Figure 2-244b in UFC3-340-02 (2014)

$$
\begin{aligned}
V_f =& 1.2 d^3, \rho_f = 4.6\,\text{oz/in}^3 \rightarrow w_f = 5.52 d^3\,\text{oz} \rightarrow d = 2.05\,\text{in} \rightarrow A = 3.30\,\text{in}^2; \\
k_V =& \frac{1}{2}\left(\frac{3.30}{47.5}\right)(0.00071)(1.2) \rightarrow = 2.960 \times 10^{-5} \\
V_s =& (6621) e^{-12(2.960\times 10^{-5})(150)} \rightarrow = 6277\,\text{ft/s}\,\text{(Case II)}
\end{aligned}
$$

Perforating the concrete layer the fragment is embeded penetrating in the steel layer

$$
\begin{aligned}
X_f =& \begin{cases} 4.0 \times 10^{-3}\sqrt{K\,N\,D} d^{1.1} v_s^{0.9}, & X_f \leq 2d \\ 4.0 \times 10^{-6} K\,N\,D d^{1.2} v_s^{1.8} + d, & X_f > 2d \end{cases} \\
K =& \sqrt{12.91}\sqrt{f_c'} \rightarrow \frac{12.91}{\sqrt{13053}} = 0.1130
\end{aligned}
$$

Penetration depth is larger than $2d$ for both cases.

Case I

$$
\begin{aligned}
N =& 1, D = 47.5/(2.05)^3 \rightarrow = 5.51 \\
X_f =& 4.0 \times 10^{-6}(0.113)(1)(5.51)(2.05)^{1.2}(6277)^{1.8} + 2.05 \rightarrow = 42.45\,\text{in} \\
\xrightarrow[\text{for } f_c']{\text{Correcting}} X_f' =& X_f\sqrt{\frac{4,000}{f_c'}} \rightarrow (42.45)\sqrt{\frac{4000}{13053}} = 23.5\,\text{in}
\end{aligned}
$$

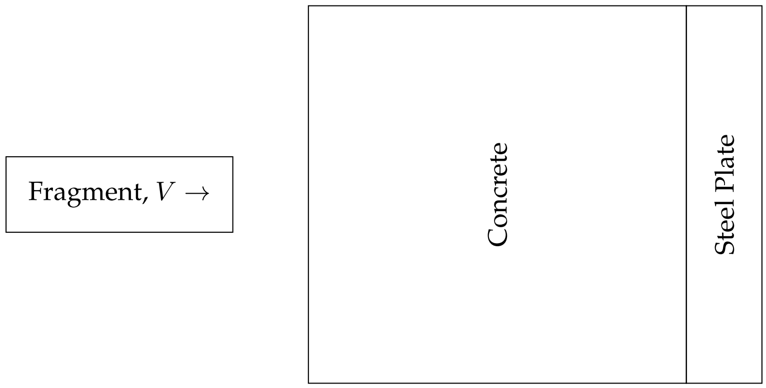
An armor piercing fragment woul produce this level of penetration and having a mild steel casing sanctions another correction based off of Table 4-16 from UFC3-340-02 (2014).

$$
\begin{aligned}
X_f' =& k X_f \rightarrow X_{\text{pen}} \text{ or } X_f = 0.7 \times (23.5) \rightarrow = 16.5\,\text{in} \\
T_{pf} =& 1.13 X_f d^{1/10} + 1.311 d \rightarrow = 1.13(16.5)(2.05)^{1/10} + 1.311(2.05) = 22.35\,\text{in}
\end{aligned}
$$

Considering $X_f > 2d$ UFC3-340-02 (2014) expresses fragment residual velocity after perforating a concrete wall T_c thick as

$$
V_r/V_s = \left[1 - \frac{T_c}{T_{pf}}\right]^{0.555}, X_f > 2d \tag{6}
$$

Figure 1: Wall layout.



In addition UFC3-340-02 (2014) provides the follwoing for mild steel plate penetration that in our case the striking velocity would equal the concrete wall fragment residual velocity. Note that resulting thickness values are ballistic limits and design should include certain factors of safety.

$$x = 0.21 w_f^{0.33} V_s^{1.22} \tag{7}$$

Given that $T_C/T_s = 5$ and $v_{r\,\text{(Concrete)}} = v_{s\text{(Steel)}}$ and that the striking velocity in the above formula is expressed in kft/sec after substituting eq. (6) into eq. (7) with known parameters the resulting equation is solved for T_s,

$$V_r = 6277 \left[1 - \frac{5T_s}{22.35}\right]^{0.555} \rightarrow x = 0.21 w_f^{0.33} V_s^{1.22}$$
$$x = 0.21(47.5)^{0.33} \left(6.277 \left[1 - \frac{5T_s}{22.35}\right]^{0.555}\right)^{1.22} \rightarrow = 3.13\,\text{in}$$
$$\text{and thus } \rightarrow T_c = 15.65\,\text{in}$$

The wall would end up $T_T = T_c + T_s = 18.8$ in thick.

Case II

Following the same previous steps,

$$
\begin{aligned}
N =& 1, D = 47.5/(3.61)^3 \rightarrow = 1.01 \\
X_f =& 4.0 \times 10^{-6}(0.113)(1)(1.01)(3.61)^{1.2}(5612)^{1.8} + 3.61 \rightarrow = 15.55 \text{ in} \\
X'_f =& X_f \sqrt{\frac{4000}{f'_c}} \rightarrow 15.55\sqrt{\frac{4000}{13053}} = 8.61 \text{ in} \\
X_{\text{Pen}} \text{ or } X_f =& 0.7(8.61) = 6.03 \text{ in} \\
T_{pf} =& 1.13(6.03)(3.61)^{0.1} + 1.311(3.61) \rightarrow = 12.48 \text{ in} \\
v_r =& 5612\left[1 - \left(\frac{5T_s}{12.48}\right)\right]^{0.555} \quad \text{for } X_f < 2d \\
T_s =& 0.21(47.5)^{0.33}\left[5.612\left[1 - \frac{5T_s}{12.48}\right]^{0.555}\right]^{1.22} \rightarrow = 2.02 \text{ in} \rightarrow T_c = 10.1 \text{ in}
\end{aligned}
$$

And in this case the wall will end up at least $T_T = 12.12$ in thick.

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