

Sparse Matrix Kernels

In order to solve the sparse systems for *TSOPF_RS_b678_c2* and *torso1* matrices, we first use naive implementation of the lower triangular solver and then improve it by taking advantage of Algorithm 3.2 described in “A survey of direct methods for sparse linear systems”, page 10. The Sparse Solver finds the set $Reach_L(B)$ and solves the equation just for these nonzero entries of vector x . It does so using *topologicalSort()*.

First, we convert the matrix market format to CSC using *csc_formatter.h* and then pass the matrices to the solver.

Correctness of sparse solver approved by comparing the answer to the naive solver using $\|xS - xD\|_{\infty} < \epsilon$, where xS is the solution of Sparse Solver, and xD is the solution of the Dense Solver. You can see the test for *TSOPF_RS_b678_c2*:

```
107     auto xS : vector<double> = lSolve(A, b, SPARSE);
108     auto xD : vector<double> = lSolve(A, b, SPARSE: !SPARSE);
109
110     double max = -DBL_MAX;
111     for (int i = 0; i < xD.size(); i++)
112         max = std::max(abs(lcpp_x: xD[i] - xS[i]), max);
113     cout << max;
```

main

SparseMatrixKernels x

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4.33681e-19

Process finished with exit code 0

The execution time for each solver is measured using *high_resolution_clock*. You can see a test for *TSOPF_RS_b678_c2*:

```
93     auto start : time_point<...> = high_resolution_clock::now();
94     auto xS : vector<double> = lSolve(A, b, SPARSE: !SPARSE);
95     auto stop : time_point<...> = high_resolution_clock::now();
96     auto duration : duration<...> = duration_cast<microseconds>(fd: stop - start);
97     cout << duration.count() << endl;
```

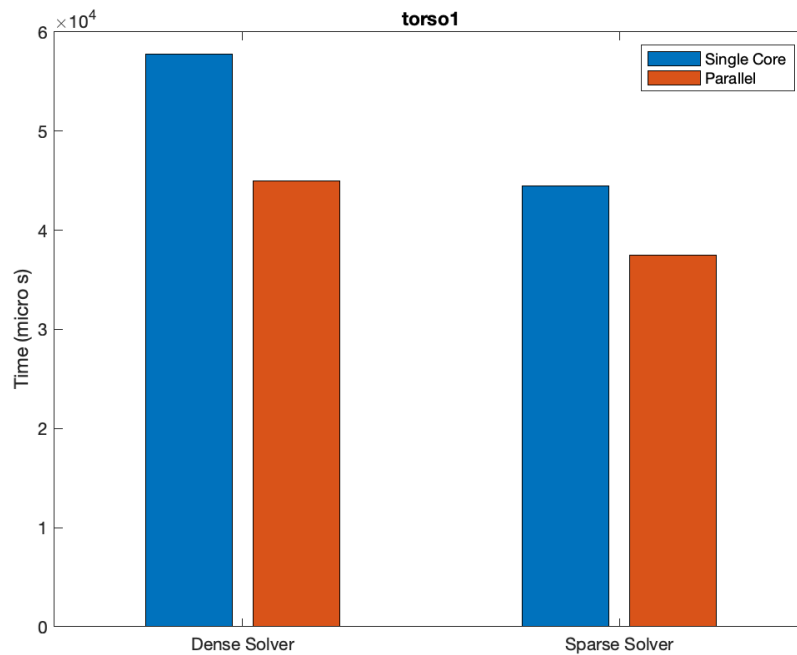
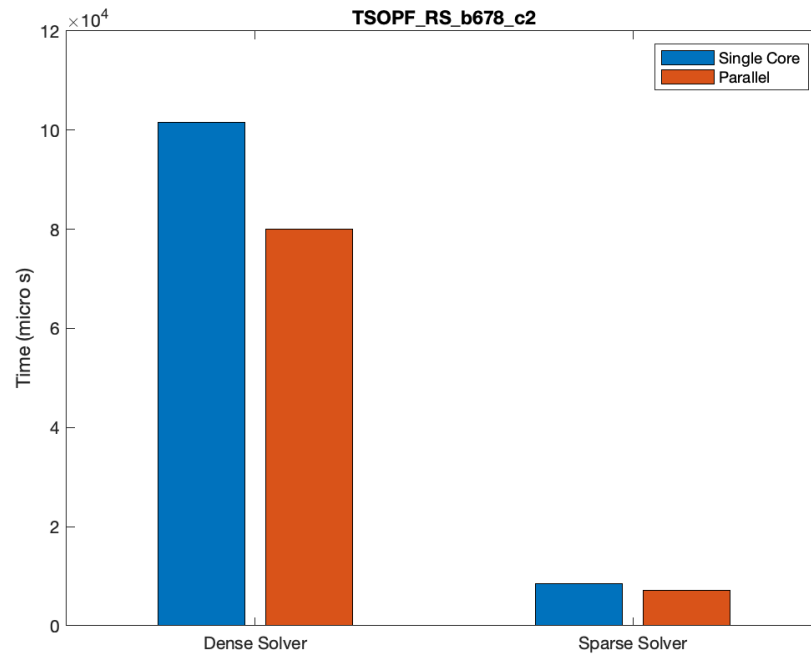
main

SparseMatrixKernels x

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103588

All tests are done on MacBook Pro, Quad-Core Intel Core i5, 8 GB Memory. Now we compare running times for variation of these methods for two systems:



The effect of parallelism for Dense Solver is more than Sparse Solver. That is because the Sparse Solver uses *topologicalSort*, which OpenMP could not optimize since it has to preserve the order of steps.

Also, for ill condition systems, we provide another useful test that measures the relative error of two answer vectors. You can see an example here:

```
7      auto xS : vector<double> = lSolve(A, b, SPARSE);
8      auto xD : vector<double> = lSolve(A, b, SPARSE: !SPARSE);
9
10     double max = -DBL_MAX;
11     for (int i = 0; i < xD.size(); i++)
12         max = std::max(abs( lcpp_x: xD[i] - xS[i]) / std::max(abs( lcpp_x: xD[i]), abs( lcpp_x: xS[i])), max);
13     cout << max;
```

main

SparseMatrixKernels x

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8.86562e-16

All codes are written in a clean and well-commented way, which you can follow the algorithms and steps easily.