تمرین کامپیوتری

Optimization with Matlab

1 Introduction

In this homework we compare several approximate least-squares and linear programming solutions for the illumination problem. The homework also serves as a review of least-squares and linear programming.

We will use the Matlab command `x=A\b` to solve a least-squares problem

$$\text{minimize } \|Ax - b\|_2^2$$

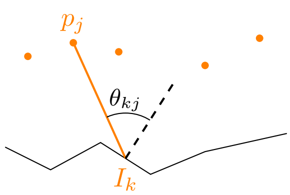
To solve the linear program

$$\begin{aligned} &\text{minimize } c^T x \\ &\text{subject to } Ax \leq b \end{aligned}$$

we use `x=linprog(c,A,b)`. The command linprog is defined in the Matlab Optimization Toolbox.

2 The Illumination problem

m lamps illuminating n, small flat patches



intensity I_k at patch k depends linearly on lamp powers p_j:

$$I_k = \sum_{j=1}^{m} a_{kj} p_j = a_k^T p \tag{1}$$

$$a_{kj} = r_{kj}^{-2} \max\{\cos\theta_{kj}, 0\} \tag{2}$$

problem: achieve desired illumination I_d with bounded lamp powers

$$\begin{aligned} &\text{minimize } \max_{k=1,\ldots,n} |\log I_k - \log I_d| \\ &\text{subject to } 0 \leq p_j \leq p_{\max},\ j = 1,\ldots,m \end{aligned}$$

with variable $p \in \mathcal{R}^m$. The data matrix A (with rows a_k^T) is available in `hw1data.m` on the course website (download the file and type `A=hw1data` in Matlab), and was generated using (2) , for the geometry shown in figure 1. There are 10 lamps ($m = 10$) and 20 patches ($n = 20$). We take $I_d = 1$ and $p_{\max} = 1$.

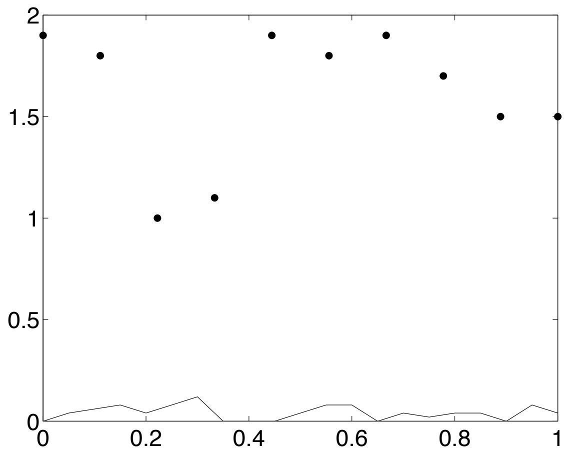


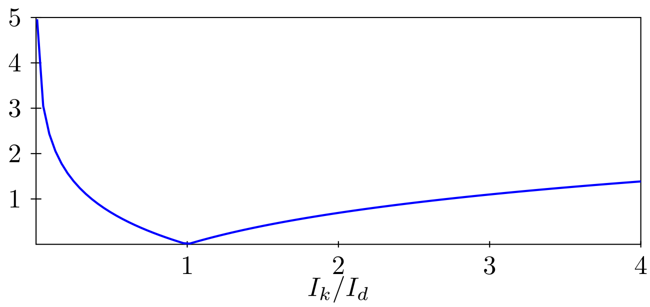
Figure 1: Patch and lamp geometry

Why is it difficult to solve?

Main reason: The objective function

$$|\log I_k - \log I_d| = |log\frac{I_k}{I_d}|$$

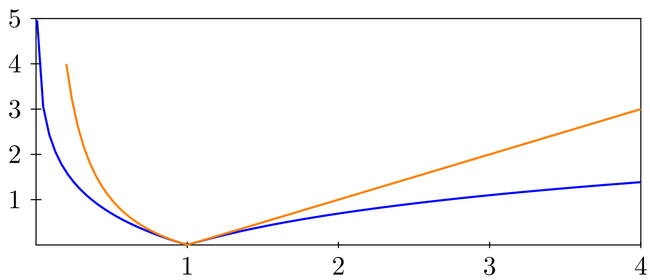
is not convex



As mentioned in the lecture, the problem (with $I_d = 1$ and $p_{\max} = 1$) is equivalent to

$$\text{minimize } f_0(p) = \max_{k=1,\dots,n} h(a_k^T p)$$
$$\text{subject to } 0 \leq p_j \leq p_{\max},\ j = 1, \dots, m$$

where $h(u) = \max\{u, 1/u\}$. The function h is nonlinear, nondifferentiable, and convex.



3 Homework

You are asked to compute suboptimal feasible solutions p using the following five methods, and to calculate their objective values $f_0(p)$. The exact solution is

$$p = (1, 0.2023, 0, 0, 1, 0, 1, 0.1882, 0, 1)$$

with $f_0(p) = 1.4297$.

1. **Equal lamp powers**. Take $p_j = \gamma$ for $j = 1, \ldots, m$, and plot $f_0(p)$ versus γ over the interval $[0, 1]$. Graphically determine the optimal value of γ.

2. **Least-squares with rounding**. Solve the least-squares problem

$$\text{minimize } \sum_{k=1}^{n} (a_k^T p - 1)^2$$

If the solution has negative coefficients, set them to zero; if some coefficients are greater than 1, set them to 1.

3. **Weighted least-squares**. Solve the weighted least-squares problem

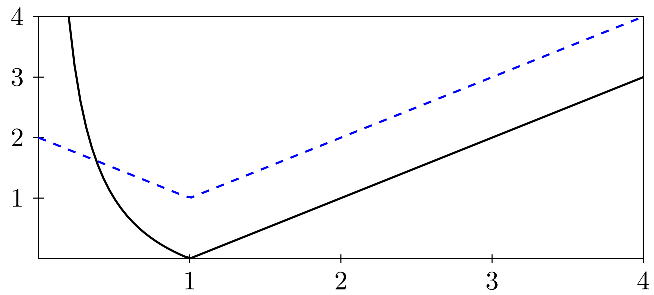
$$\text{minimize } \sum_{k=1}^{n} (a_k^T p - 1)^2 + \sum_{j=1}^{m} (p_j - 0.5)^2$$

If the solution has negative coefficients, set them to zero; if some coefficients are greater than 1, set them to 1.

4. **Chebyshev approximation**. Solve the problem

$$\begin{aligned} &\text{minimize } f_0(p) = \max_{k=1,\ldots,n} |a_k^T p - 1| \\ &\text{subject to } 0 \le p_j \le p_{\max}, j = 1, \ldots, m \end{aligned}$$

using linear programming. We can think of this problem as obtained by approximating the nonlinear function $h(u)$ by a piecewise-linear function $|u - 1| + 1$. As shown in the figure below, this is a good approximation around $u = 1$.

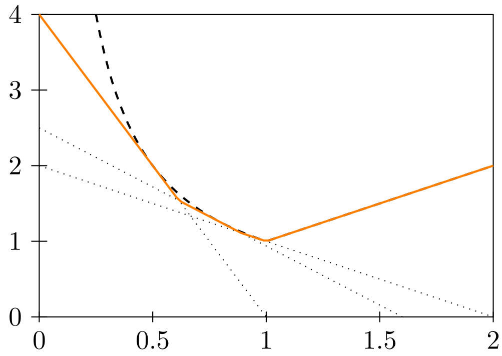


5. **Piecewise-linear approximation**. We can improve the accuracy of the previous method by using a piecewise-linear approximation of h with more than two segments. To construct a piecewise-linear approximation of $1/u$, we take the pointwise maximum of the first-order approximations

$$h(u) \approx 1/\hat{u} - (1/\hat{u}^2)(u - \hat{u}) = 2/\hat{u} - u/\hat{u}^2$$

at a number of different points $\hat{u}$. This is shown in the figure below, for $\hat{u} = 0.5, 0.8, 1$. In other words,

$$h_{\text{pwl}} = \max \left\{ u, \frac{2}{0.5} - \frac{1}{0.5^2} u, \frac{2}{0.8} - \frac{1}{0.8^2} u, 2 - u \right\}$$



Solve the problem

$$\begin{aligned} &\text{minimize } f_0(p) = \max_{k=1,\dots,n} h_{\mathrm{pwl}}(a_k^T p) \\ &\text{subject to } 0 \leq p_j \leq p_{\max}, j = 1, \dots, m \end{aligned}$$

using linear programmming