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COLLEGE OF ENGINEERING

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INTELLIGENT SYSTEM

ASSIGNMENT#3

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2 QUESTION #1

2.1 CONVOLUTIONAL NEURAL NETWORK IN CLASSIFICATION(CNN)

In this part we intend to design CNN using keras library, we have used Cifar10 dataset to analyse data.

This dataset includes 60000 pictures which classified in 10 classes.

In first part of this analyse we are supposed to report the error and accuracy in each epoch and also plot cost function of evaluation and test data based on epoch in time domain.

Note that the minimum value for epoch must be around 10 cycles.

Also Note that I make several functions for each part of investigation to have dipper insight on functionality of each architecture.

First of all, we load Cifar10 dataset and consider 20 percent of data for evaluation and also 80 percent of that for training model.

```
import sys
from matplotlib import pyplot
from keras.datasets import cifar10
from tensorflow.keras.utils import to_categorical
import numpy as np
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Conv2D
from tensorflow.keras.layers import MaxPooling2D
from tensorflow.keras.lavers import Dense
from tensorflow.keras.layers import Flatten
from tensorflow.keras.optimizers import SGD
from tensorflow.keras.optimizers import Adam
from tensorflow.keras.layers import Dropout
# load train and test dataset
def load_dataset():
 # load dataset
 (trainX, trainY), (testX, testY) = cifar10.load_data()
 # one hot encode target values
 trainY = to categorical(trainY)
 testY = to categorical(testY)
 return trainX, trainY, testX, testY
trainX, trainy, testX, testy = load dataset()
train_dataX = trainX[0 : round(trainX.shape[0] * 0.8)]
train_dataY = trainy[0 : round(trainX.shape[0] * 0.8)]
eval_dataX = trainX[round(trainX.shape[0] * 0.8):-1]
eval_dataY = trainy[round(trainX.shape[0] * 0.8):-1]
```

In the next step we want to show a picture from each classes to get more familiar with this dataset:

```
kinds = []
indexes = []
i = 0
while len(kinds) < 10:
    current_cat = np.where(train_dataY[i] == 1)[0][0]
    if kinds.count(current_cat) == 0:
        kinds.append(current_cat)
        indexes.append(i)
    i = i+1

for i in range(len(indexes)):|
    pyplot.imshow(train_dataX[indexes[i]])
    pyplot.show()</pre>
```

And as result of above code we have:

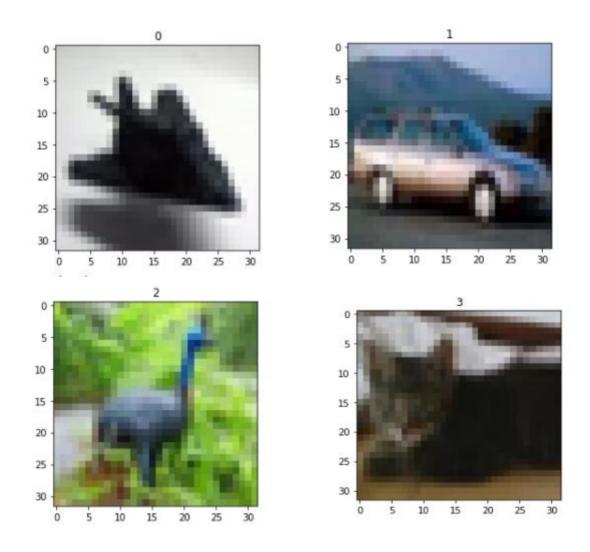


Figure 1: Representation of Dataset



Figure 2: Representation of Dataset

Then, we are going to use appropriate prepressing methods on our data and describe why those type of methods are used.

1.

you can see further I have written a function called prep-pixels which normalize RGB pictures and change those data in interval of 0 to 1.

Note that with normalize data with dividing by 255 which is the maximum of each RGB channel.

```
def prep_pixels(train, test):
    # convert from integers to floats
    train_norm = train.astype('float32')
    test_norm = test.astype('float32')
    # normalize to range 0-1
    train_norm = train_norm / 255.0
    test_norm = test_norm / 255.0
    # return normalized images
    return train_norm, test_norm
```

Furthermore, we also function to

2.

we have used reshape() as of the second pre-processing tool as you can see below:

```
# return normalized images
train_norm = np.reshape(train_norm,(len(train),32,32,3))
validation_norm = np.reshape(validation_norm,(len(validation),32,32,3))
test_norm = np.reshape(test_norm,(len(test),32,32,3))
```

3.

And finally using to_categorical() function which implement one hot encode target values considered as of the third pre-processing which we do on data.

```
# one hot encode target values
trainY = to_categorical(trainY)
testY = to_categorical(testY)
return trainX, trainY, testX, testY
```

Afterward, we are going to design model according to HW description.

We have used **softmax** as of activation function in the final layer because in this problem we face with a classification task and also we have coded each output to one-hot form so in these cases softmax helps us to normalize output of final layer and also showing probability of classification to a specific group in one-hot implementation can give us a better insight.

Note that in this implementation the batch size value is 64 and also I have used 20 epochs in my analyses.

2.2 INCREASING HIDDEN LAYER

In this part we intend to analyse the effect of increasing number of hidden layer on accuracy.

So in this part I have written different function with different number of hidden layer and analyse accuracy of each of them.

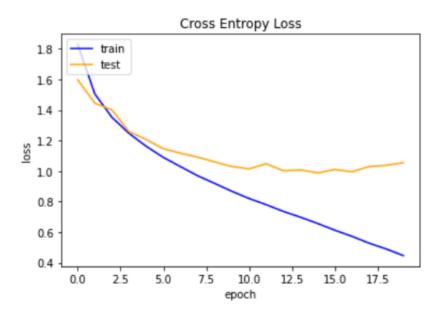
You can see my implementation below:

```
def define_model_baseline1():
   model = Sequentia1()
   model.add(Conv20(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same', input_shape=(32, 32, 3)))
   model.add(Conv20(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(MaxPooling2D((2, 2)))
   model.add(Flatten())
   model.add(Dense(128, activation='relu', kernel_initializer='he_uniform'))
   model.add(Dense(10, activation='softmax'))
   # compile model
opt = SGD(learning_rate=0.001, momentum=0.9)
   model.compile(optimizer=opt, loss='categorical_crossentropy', metrics=['accuracy'])
 def define model baseline2():
   model = Sequential()
   model.add(conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same', input_shape=(32, 32, 3)))
model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
model.add(MaxPooling2D((2, 2)))
model.add(Conv2D(64, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
model.add(Conv2D(64, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(MaxPooling2D((2, 2)))
    model.add(Flatten())
   model.add(Dense(128, activation='relu', kernel_initializer='he_uniform'))
   model.add(Dense(10, activation='softmax'))
    # compile model
   opt = SGD(lr=0.001, momentum=0.9)
   model.compile(optimizer=opt, loss='categorical_crossentropy', metrics=['accuracy'])
return model
def define model baseline3():
  model = Sequential()
   model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same', input_shape=(32, 32, 3)))
   model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(MaxPooling2D((2, 2)))
   model.add(Conv2D(64, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(Conv2D(64, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(MaxPooling2D((2, 2)))
  model.add(Conv2D(128, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
model.add(Conv2D(128, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(MaxPooling2D((2, 2)))
   model.add(Flatten())
   model.add(Dense(128, activation='relu', kernel_initializer='he_uniform'))
   model.add(Dense(10, activation='softmax'))
   # compile model
  opt = SGD(1r=0.001, momentum=0.9)
  model.compile(optimizer=opt, loss='categorical_crossentropy', metrics=['accuracy'])
  return model
```

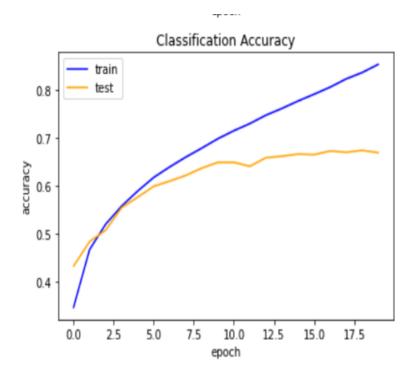
In this part I have tested three state of adding hidden layer and found that the best Accuracy and F1score is for CNN with 2 hidden layers so we choose 2 for number of hidden layers.

Further you can see the result of 1 hidden layer CNN for 20 epochs!

```
Epoch 1/20
       :===========] - 10s 16ms/step - loss: 1.8284 - accuracy: 0.3467 - val_loss: 1.5973 - val_accuracy: 0.4325
Fnoch 2/20
625/625 [===
     Epoch 3/20
625/625 [============] - 10s 15ms/step - loss: 1.3511 - accuracy: 0.5199 - val loss: 1.3998 - val accuracy: 0.5075
625/625 [============] - 10s 15ms/step - loss: 1.2459 - accuracy: 0.5578 - val loss: 1.2571 - val accuracy: 0.5545
Epoch 5/20
Fnoch 6/20
     ============================ - 9s 15ms/step - loss: 1.0890 - accuracy: 0.6176 - val_loss: 1.1455 - val_accuracy: 0.5991
Epoch 7/20
Epoch 8/20
Epoch 9/20
Epoch 10/20
625/625 [===
       Epoch 11/20
Epoch 12/20
     625/625 [====
Epoch 13/20
625/625 [===========] - 9s 14ms/step - loss: 0.7354 - accuracy: 0.7473 - val_loss: 1.0017 - val_accuracy: 0.6587
Epoch 14/20
625/625 [============] - 8s 13ms/step - loss: 0.6975 - accuracy: 0.7615 - val loss: 1.0065 - val accuracy: 0.6618
Epoch 15/20
Epoch 16/20
625/625 [=====
      Epoch 17/20
       ===========] - 8s 13ms/step - loss: 0.5725 - accuracy: 0.8058 - val_loss: 0.9949 - val_accuracy: 0.6728
Fnoch 18/20
Epoch 19/20
Epoch 20/20
> 59.450
```

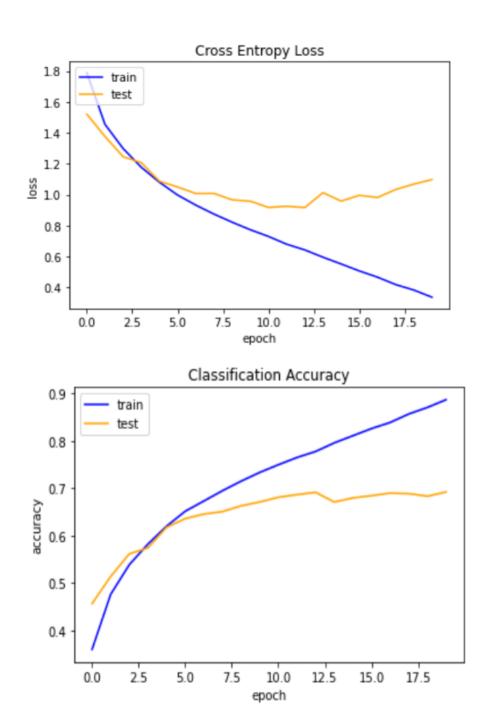


Enoch 1/20



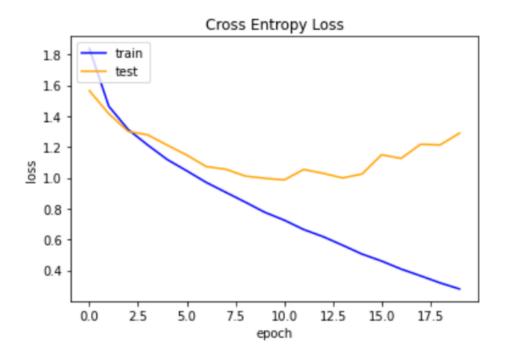
Further you can see the result of 2 hidden layer CNN for 20 epochs!

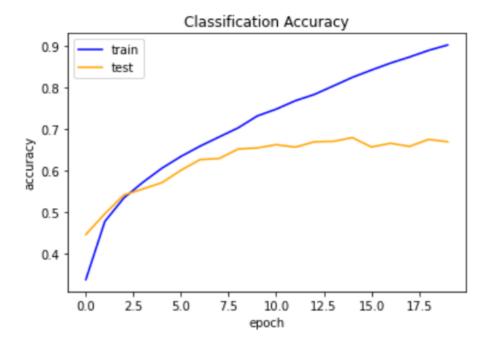
```
=========] - 14s 20ms/step - loss: 1.7880 - accuracy: 0.3601 - val_loss: 1.5207 - val_accuracy: 0.4565
Epoch 2/20
        Epoch 3/20
              ====] - 11s 17ms/step - loss: 1.2981 - accuracy: 0.5388 - val_loss: 1.2453 - val_accuracy: 0.5617
Epoch 4/20
625/625 [====
                - 11s 17ms/step - loss: 1.1765 - accuracy: 0.5823 - val_loss: 1.2066 - val_accuracy: 0.5742
Epoch 5/20
                - 11s 17ms/step - loss: 1.0799 - accuracy: 0.6199 - val_loss: 1.0866 - val_accuracy: 0.6177
Epoch 6/20
625/625 [==
                - 10s 17ms/step - loss: 0.9971 - accuracy: 0.6514 - val_loss: 1.0493 - val_accuracy: 0.6359
Epoch 7/20
625/625 [===
        ==========] - 11s 17ms/step - loss: 0.9320 - accuracy: 0.6730 - val_loss: 1.0072 - val_accuracy: 0.6455
Epoch 8/20
Epoch 9/20
625/625 [===
        Epoch 10/20
625/625 [===:
        :=========] - 10s 17ms/step - loss: 0.7728 - accuracy: 0.7333 - val_loss: 0.9575 - val_accuracy: 0.6710
Epoch 11/20
625/625 [============] - 11s 17ms/step - loss: 0.6792 - accuracy: 0.7648 - val_loss: 0.9252 - val_accuracy: 0.6865
     625/625 [====
Epoch 14/20
Epoch 16/20
625/625 [===:
      Epoch 17/20
625/625 [===
        Epoch 18/20
        Epoch 19/20
625/625 [====
      Epoch 20/20
> 60.660
```



Further you can see the result of 3 hidden layer CNN for 20 epochs!

```
Fnoch 1/20
        =============] - 16s 23ms/step - loss: 1.8393 - accuracy: 0.3383 - val_loss: 1.5652 - val_accuracy: 0.4465
Fnoch 2/20
625/625 [===========] - 14s 22ms/step - loss: 1.4637 - accuracy: 0.4780 - val loss: 1.4151 - val accuracy: 0.4964
625/625 [===========] - 14s 22ms/step - loss: 1.1182 - accuracy: 0.6057 - val loss: 1.2116 - val accuracy: 0.5711
Epoch 6/20
625/625 [==
         Epoch 7/20
          =========] - 14s 22ms/step - loss: 0.9701 - accuracy: 0.6586 - val_loss: 1.0728 - val_accuracy: 0.6264
625/625 [===
Epoch 8/20
Enoch 9/20
625/625 [===:
       Fpoch 10/20
625/625 [===
         Epoch 11/20
625/625 [===========] - 14s 22ms/step - loss: 0.7255 - accuracy: 0.7473 - val_loss: 0.9862 - val_accuracy: 0.6622
625/625 [=====
         Epoch 13/20
625/625 [===
          ============] - 14s 22ms/step - loss: 0.6184 - accuracy: 0.7826 - val loss: 1.0291 - val accuracy: 0.6691
Epoch 14/20
625/625 [===:
Epoch 15/20
           ==========] - 14s 22ms/step - loss: 0.5627 - accuracy: 0.8030 - val_loss: 0.9991 - val_accuracy: 0.6701
625/625 [===
            ==========] - 14s 22ms/step - loss: 0.5053 - accuracy: 0.8237 - val_loss: 1.0249 - val_accuracy: 0.6790
Epoch 16/20
625/625 [======
Epoch 17/20
          ============] - 14s 22ms/step - loss: 0.4603 - accuracy: 0.8411 - val_loss: 1.1496 - val_accuracy: 0.6565
625/625 [===
          ===========] - 13s 22ms/step - loss: 0.4084 - accuracy: 0.8578 - val_loss: 1.1257 - val_accuracy: 0.6655
Epoch 18/20
625/625 [====
         ===============] - 13s 22ms/step - loss: 0.3643 - accuracy: 0.8724 - val_loss: 1.2173 - val_accuracy: 0.6580
Epoch 19/20
         Epoch 20/20
```





2.3 RELU ACTIVATION FUNCTION VS TANH ACTIVATION FUNCTION

In this part we intend to analyse the effect of replacing Tanh instead of ReLU activation function on accuracy.

You can see my implementation below:

```
def define_model_baseline1_tanh():
    model = Sequential()
    model.add(Conv2D(32, (3, 3), activation='tanh', kernel_initializer='he_uniform', padding='same', input_shape=(32, 32, 3)))
    model.add(Conv2D(32, (3, 3), activation='tanh', kernel_initializer='he_uniform', padding='same'))
    model.add(MaxPooling2D((2, 2)))
    model.add(Flatten())
    model.add(Dense(128, activation='tanh', kernel_initializer='he_uniform'))
    model.add(Dense(10, activation='softmax'))
    # compile model
    opt = SGD(learning_rate=0.001, momentum=0.9)
    model.compile(optimizer=opt, loss='categorical_crossentropy', metrics=['accuracy'])
    return model
```

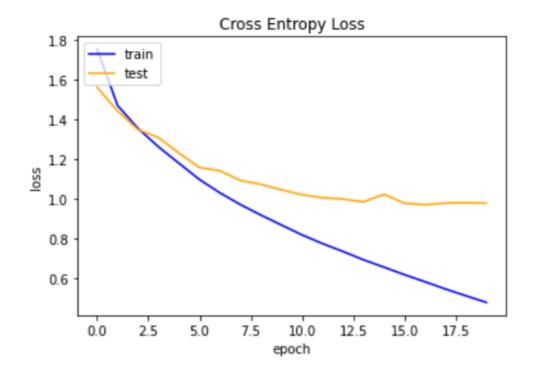
Results have been showed that the ReLU activation function acts pretty much better in comparison with tanh activation function!

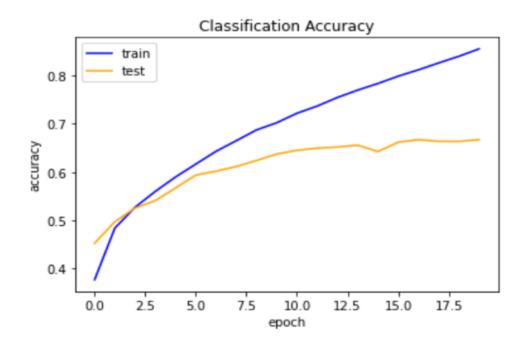
Further you can see the result of 1 hidden layer CNN for 20 epochs with Tanh!

Note that the ReLU baseline has been showed in previous part so I have ignored to show the results of that in this part!

As you can see further the accuracy for tanh is 37% which is very low in comparison to ReLU!

```
Epoch 1/20
      625/625 [==
Epoch 2/20
      ===========] - 9s 14ms/step - loss: 1.4683 - accuracy: 0.4836 - val_loss: 1.4407 - val_accuracy: 0.4969
Epoch 3/20
     625/625 [===
625/625 [===:
     ============================== - 8s 14ms/step - loss: 1.2598 - accuracy: 0.5598 - val_loss: 1.3067 - val_accuracy: 0.5408
Epoch 5/20
625/625 [====
     Epoch 6/20
      ============] - 9s 14ms/step - loss: 1.0958 - accuracy: 0.6165 - val_loss: 1.1560 - val_accuracy: 0.5936
Epoch 7/20
     625/625 [===
Epoch 9/20
     Epoch 10/20
625/625 [===
     :============================= ] - 9s 14ms/step - loss: 0.8670 - accuracy: 0.7024 - val_loss: 1.0442 - val_accuracy: 0.6374
Fnoch 11/20
625/625 [============] - 9s 14ms/step - loss: 0.8174 - accuracy: 0.7221 - val_loss: 1.0193 - val_accuracy: 0.6452
Epoch 12/20
625/625 [===
     Epoch 13/20
625/625 [====
     Fnoch 14/20
Epoch 15/20
     Epoch 16/20
      ============================== - 9s 14ms/step - loss: 0.6174 - accuracy: 0.7990 - val_loss: 0.9769 - val_accuracy: 0.6619
Epoch 17/20
625/625 [===
      Epoch 18/20
     625/625 [===
Epoch 19/20
625/625 [===
       Epoch 20/20
> 37.540
```





2.4 ADAM VS SGD

Enoch 1/20

In this part we intend to analyse the effect of replacing ADAM solver instead of SGD activation function in minimizing cost function on accuracy.

You can see my implementation below:

```
def define_model_baseline1_dropout(dropout_percent):
   model = Sequentia1()
   model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same', input_shape=(32, 32, 3)))
   model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(MaxPooling2D((2, 2)))
   model.add(Dropout(dropout_percent))
   model.add(Flatten())
   model.add(Dense(128, activation='relu', kernel_initializer='he_uniform'))
   model.add(Dense(128, activation='relu', kernel_initializer='he_uniform'))
   model.add(Dense(10, activation='softmax'))
# compile model
   opt = SGD(learning_rate=0.001, momentum=0.9)
   model.compile(optimizer=opt, loss='categorical_crossentropy', metrics=['accuracy'])
   return model
```

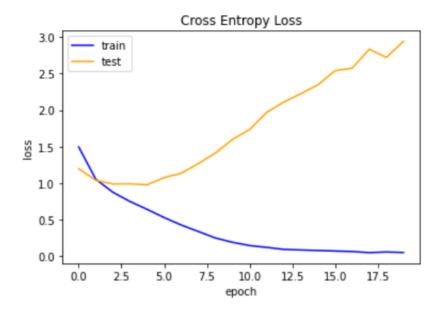
Results have been showed that the ADAM solver acts pretty much better in comparison with SGD activation function!

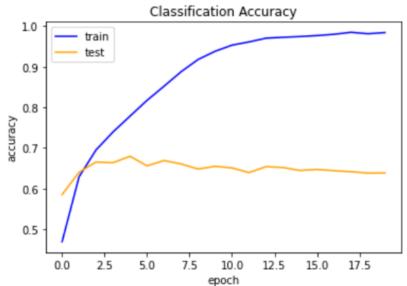
Further you can see the result of 1 hidden layer CNN for 20 epochs with ADAM!

Note that the SGD baseline has been showed in previous part so I have ignored to show the results of that in this part!

As you can see further the accuracy for ADAM is 56% which is very low in comparison to SGD!

```
625/625 [===
   =============] - 10s 16ms/step - loss: 1.4960 - accuracy: 0.4695 - val_loss: 1.1975 - val_accuracy: 0.5855
Epoch 2/20
625/625 [==:
   625/625 [====
   Epoch 4/20
625/625 [===
   Epoch 5/20
625/625 [====
   Epoch 6/20
    Fnoch 7/20
Epoch 8/20
Epoch 9/20
Epoch 10/20
625/625 [=====
   Epoch 11/20
    625/625 [===:
Epoch 12/20
   ============================== ] - 9s 14ms/step - loss: 0.1207 - accuracy: 0.9608 - val_loss: 1.9671 - val_accuracy: 0.6395
Enoch 13/20
625/625 [====
    Epoch 14/20
625/625 [====
   Epoch 15/20
    625/625 [===
Epoch 16/20
    Epoch 17/20
625/625 [====
   Epoch 18/20
   625/625 [===
Epoch 19/20
625/625 [===========] - 8s 13ms/step - loss: 0.0578 - accuracy: 0.9810 - val loss: 2.7204 - val accuracy: 0.6384
Epoch 20/20
```





2.5 EFFECT OF ADDING DROPOUT LAYER

In this part we intend to analyse the effect of adding Dropout layer in on accuracy.

You can see my implementation below:

```
def define_model_baseline1_dropout(dropout_percent):
   model = Sequential()
   model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same', input_shape=(32, 32, 3)))
   model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', padding='same'))
   model.add(MaxPooling2D((2, 2)))
   model.add(Dropout(dropout_percent))
   model.add(Flatten())
   model.add(Dense(128, activation='relu', kernel_initializer='he_uniform'))
   model.add(Dropout(dropout_percent))
   model.add(Dense(10, activation='softmax'))
   # compile model
   opt = SGD(learning_rate=0.001, momentum=0.9)
   model.compile(optimizer=opt, loss='categorical_crossentropy', metrics=['accuracy'])
   return model
```

I have used four values as of input argument which abbreviated below:

```
[0.1 0.2 0.25 0.3]
```

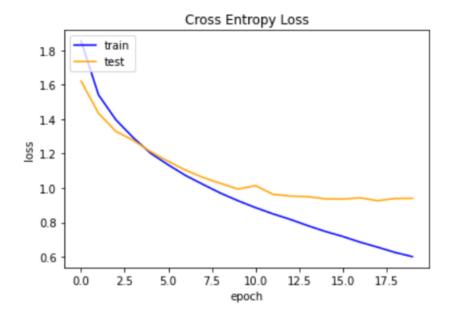
Results have been showed that the best values for dropout percent is **0.1** and with testing based on this value we have obtained best accuracy.

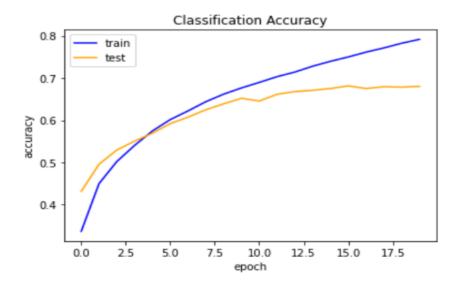
Further you can see the result of 1 hidden layer CNN for 20 epochs with different <u>dropout</u> percent parameters based on above values.

As you can see further the accuracy for **0.1** dropout percent is **54%** which is very considerable in comparison with each other!

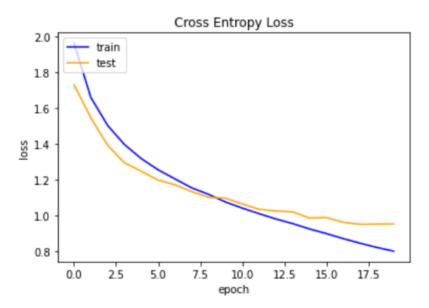
```
Epoch 1/20
     :=========] - 11s 17ms/step - loss: 1.8543 - accuracy: 0.3368 - val_loss: 1.6215 - val_accuracy: 0.4315
Enoch 2/20
625/625 [===
   Epoch 3/20
625/625 [===
    Epoch 5/20
Fnoch 6/20
   Epoch 7/20
625/625 [===
   625/625 [===========] - 10s 15ms/step - loss: 1.0202 - accuracy: 0.6440 - val_loss: 1.0608 - val_accuracy: 0.6249
Epoch 9/20
    Epoch 10/20
```

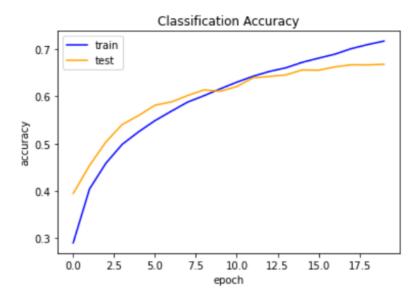
```
Epoch 11/20
 625/625 [===
                                                                                    =] - 9s 15ms/step - loss: 0.8858 - accuracy: 0.6896 - val_loss: 1.0137 - val_accuracy: 0.6455
Epoch 12/20
                                                                                          - 9s 14ms/step - loss: 0.8496 - accuracy: 0.7032 - val_loss: 0.9634 - val_accuracy: 0.6616
 Epoch 13/20
 625/625 [===
                                                                                           - 9s 15ms/step - loss: 0.8172 - accuracy: 0.7142 - val_loss: 0.9528 - val_accuracy: 0.6679
 Epoch 14/20
 625/625 [===
                                                                                          - 10s 15ms/step - loss: 0.7809 - accuracy: 0.7280 - val_loss: 0.9496 - val_accuracy: 0.6710
 Epoch 15/20
 625/625 [======
                                         Epoch 16/20
                                625/625 [====
 Epoch 18/20
 625/625 [====
                                           ==========] - 9s 15ms/step - loss: 0.6560 - accuracy: 0.7714 - val_loss: 0.9259 - val_accuracy: 0.6795
 Epoch 19/20
 625/625 [====
                                              ==========] - 8s 13ms/step - loss: 0.6253 - accuracy: 0.7824 - val_loss: 0.9388 - val_accuracy: 0.6785
Epoch 20/20
20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20 20/20
```



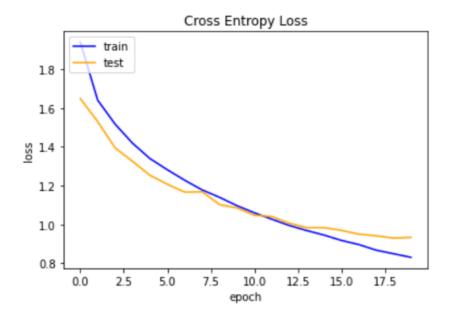


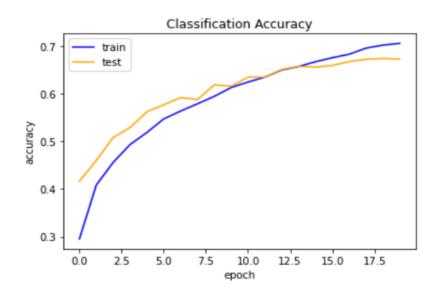
```
- Y E
Epoch 1/20
        ============================== ] - 10s 15ms/step - loss: 1.9636 - accuracy: 0.2904 - val_loss: 1.7307 - val_accuracy: 0.3947
Epoch 2/20
Epoch 3/20
625/625 [==:
        Epoch 4/20
625/625 [====
      Epoch 5/20
625/625 [============] - 9s 15ms/step - loss: 1.3185 - accuracy: 0.5253 - val loss: 1.2478 - val accuracy: 0.5597
Epoch 6/20
Epoch 7/20
625/625 [====
        Epoch 8/20
625/625 [============] - 9s 14ms/step - loss: 1.1552 - accuracy: 0.5881 - val loss: 1.1343 - val accuracy: 0.6020
Epoch 9/20
625/625 [====
      ============================= ] - 10s 15ms/step - loss: 1.1193 - accuracy: 0.6016 - val_loss: 1.1029 - val_accuracy: 0.6138
Epoch 10/20
Epoch 11/20
625/625 [===:
       ================================ - 9s 15ms/step - loss: 1.0427 - accuracy: 0.6302 - val loss: 1.0652 - val accuracy: 0.6210
Epoch 12/20
625/625 [===
        Enoch 13/20
       625/625 [=====
Epoch 14/20
Epoch 15/20
625/625 [============] - 10s 15ms/step - loss: 0.9263 - accuracy: 0.6721 - val loss: 0.9871 - val accuracy: 0.6560
Epoch 16/20
625/625 [============] - 9s 15ms/step - loss: 0.9005 - accuracy: 0.6809 - val loss: 0.9902 - val accuracy: 0.6555
Epoch 17/20
625/625 [====
           Epoch 18/20
625/625 [===
         :===========] - 10s 16ms/step - loss: 0.8468 - accuracy: 0.7011 - val_loss: 0.9521 - val_accuracy: 0.6670
Epoch 19/20
625/625 [===:
         ===========] - 10s 16ms/step - loss: 0.8229 - accuracy: 0.7096 - val_loss: 0.9532 - val_accuracy: 0.6667
Enoch 20/20
> 50.490
```



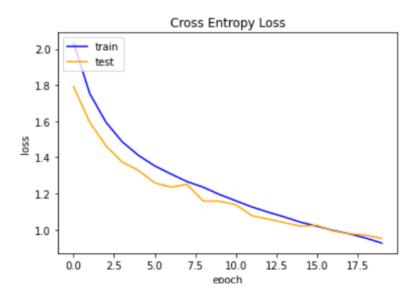


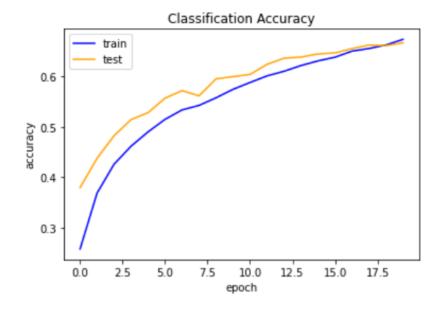
```
Epoch 1/20
       625/625 [====
Epoch 2/20
625/625 [===
       Epoch 3/20
625/625 [===
                     - 9s 15ms/step - loss: 1.5173 - accuracy: 0.4557 - val_loss: 1.3947 - val_accuracy: 0.5079
Epoch 4/20
625/625 [===
          625/625 [=============] - 9s 15ms/step - loss: 1.3399 - accuracy: 0.5185 - val loss: 1.2537 - val accuracy: 0.5620
Epoch 6/20
625/625 [==:
         Fnoch 7/20
625/625 [====
       Epoch 8/20
625/625 [=====
       Epoch 9/20
625/625 [===
         Epoch 10/20
625/625 [===
           ==========] - 9s 15ms/step - loss: 1.0970 - accuracy: 0.6129 - val_loss: 1.0854 - val_accuracy: 0.6157
Epoch 11/20
Epoch 12/20
625/625 [===
         ===============] - 10s 16ms/step - loss: 1.0274 - accuracy: 0.6346 - val_loss: 1.0411 - val_accuracy: 0.6345
Epoch 13/20
Epoch 14/20
625/625 [===
         Epoch 15/20
625/625 [===
             :========] - 9s 14ms/step - loss: 0.9456 - accuracy: 0.6669 - val_loss: 0.9838 - val_accuracy: 0.6557
Epoch 16/20
           =========] - 9s 15ms/step - loss: 0.9180 - accuracy: 0.6754 - val_loss: 0.9698 - val_accuracy: 0.6591
625/625 [===
Epoch 17/20
       625/625 [====
Epoch 18/20
625/625 [===
          ===========] - 9s 15ms/step - loss: 0.8678 - accuracy: 0.6955 - val_loss: 0.9418 - val_accuracy: 0.6721
Epoch 19/20
625/625 [===
           :==========] - 8s 14ms/step - loss: 0.8492 - accuracy: 0.7019 - val_loss: 0.9297 - val_accuracy: 0.6739
Epoch 20/20
625/625 [============================ ] - 8s 14ms/step - loss: 0.8309 - accuracy: 0.7055 - val_loss: 0.9339 - val_accuracy: 0.6726
> 51.910
```





```
Epoch 1/20
Epoch 2/20
    625/625 [==
Fnoch 3/20
Epoch 4/20
625/625 [===
    Epoch 5/20
625/625 [===========] - 9s 15ms/step - loss: 1.4112 - accuracy: 0.4902 - val_loss: 1.3277 - val_accuracy: 0.5283
Fnoch 6/20
625/625 [====
    Epoch 7/20
625/625 [===
     Enoch 8/20
Epoch 9/20
Epoch 10/20
625/625 [===
     :==========] - 9s 15ms/step - loss: 1.1944 - accuracy: 0.5745 - val_loss: 1.1572 - val_accuracy: 0.5998
Fnoch 11/20
Epoch 12/20
Epoch 13/20
625/625 [====
     Fnoch 14/20
Enoch 14/20
625/625 [============================] - 10s 15ms/step - loss: 1.0701 - accuracy: 0.6217 - val_loss: 1.0388 - val_accuracy: 0.6386
Epoch 15/20
625/625 [============] - 9s 15ms/step - loss: 1.0410 - accuracy: 0.6309 - val loss: 1.0192 - val accuracy: 0.6448
Epoch 16/20
625/625 [====
     Epoch 17/20
Epoch 18/20
625/625 [===:
     Fnoch 19/20
625/625 [============] - 9s 14ms/step - loss: 0.9536 - accuracy: 0.6625 - val_loss: 0.9692 - val_accuracy: 0.6613
Epoch 20/20
> 49,410
```



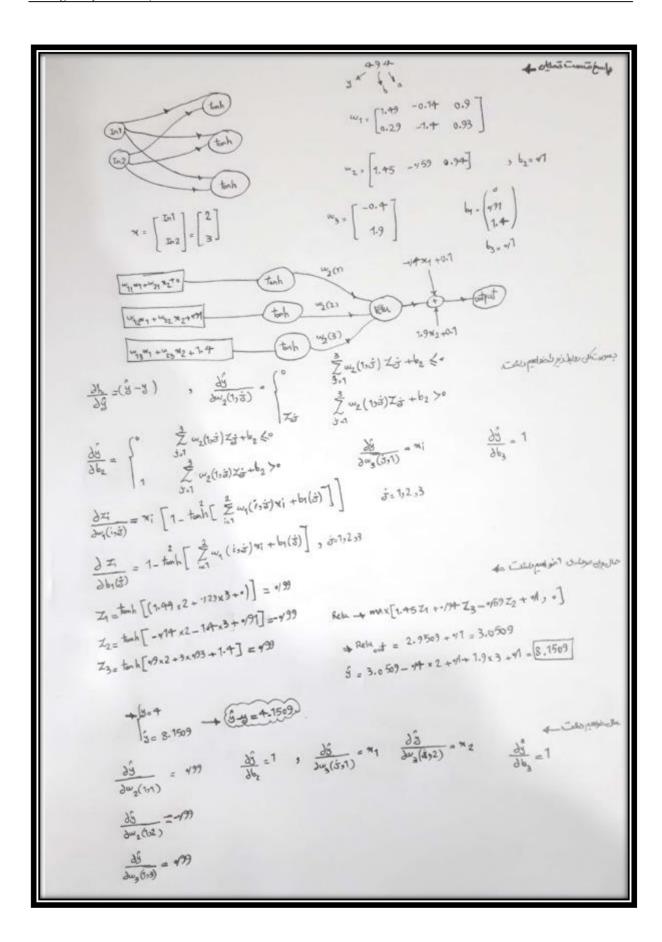


Here you can find all of my code implementation using Python if there is any issue with that please let me know to provide more detailed information. (be in touch <u>here!</u>)

3 QUESTION #2

3.1 FULLY CONNECTED NEURAL NETWORK (THEORETICAL)

In this part we intend to repeat algorithm for two separated cycles for both forwarding and back-propagating, you can see more detailed information below where I have done all of step for a theoretical approach.



$$\frac{3i}{3\eta_{1}} = (3+3) \times 1 \times 1.45 \times x_{1} \times \left[1-\frac{1}{2}\right] = \sqrt{2.395}$$

$$\frac{3i}{31} = (3+3) \times 1 \times 1.45 \times x_{2} \times \left[1-\frac{1}{2}\right] = \sqrt{3.593}$$

$$\frac{3i}{31} = 4.1569 \times 1 \times \left[-\sqrt{93}\right] \times 2 \times \left[1-(\sqrt{9})^{2}\right] = -6.1947$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{93}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = -6.1462$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 2 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.553}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.553}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.553}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.553}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.553}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] = \sqrt{1.569}$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times 3 \times \left[1-(\sqrt{9})^{2}\right] \times \left[1-(\sqrt{9})^{2}\right] \times \left[1-(\sqrt{9})^{2}\right]$$

$$\frac{3i}{3\eta_{1}} = 4.1569 \times 1 \times \left[-\sqrt{99}\right] \times \left[1-(\sqrt{9})^{2}\right] \times \left[1-(\sqrt{9})^{2}$$

$$\frac{M}{\partial v_{11}} = 2.03925 \times 1 + 1.0201 \times 2 \times [1 - 693^{2}] = v/0.8433$$

$$\frac{M}{\partial v_{21}} = 2.03925 \times 1 + [-v/734] \times 2 \times [1 - v/3^{2}] = v/0.4733$$

$$\frac{M}{\partial v_{22}} = 2.03925 \times 1 \times [-v/734] \times 2 \times [1 - v/3^{2}] = v/0.4733$$

$$\frac{M}{\partial v_{22}} = 2.03925 \times 1 \times [-v/734] \times 2 \times [1 - v/3^{2}] = v/0.4234$$

$$\frac{M}{\partial v_{22}} = 2.03925 \times 1 \times [-v/3231] \times 2 \times [1 - v/3^{2}] = v/0.4234$$

$$\frac{M}{\partial v_{22}} = 2.03925 \times 1 \times [-v/3231] \times 3 \times [1 - v/3^{2}] = v/0.4434$$

$$\frac{M}{\partial v_{22}} = 2.03925 \times 1 \times [-v/3231] \times 3 \times [1 - v/3^{2}] = v/0.4434$$

$$\frac{M}{\partial v_{22}} = 2.03925 \times 1 \times [-v/3231] \times 3 \times [1 - v/3^{2}] = v/0.4434$$

$$\frac{M}{\partial v_{22}} = 2.03925 \times 7 \times 7 \times [-v/3325] \times (-v/735) \times (-v$$

3.2 RESEARCH

In this part we intend to find solutions for several questions which let us to have a dipper insight on concepts!

3.2.1 Different cost function

L1 and L2 are two loss functions in machine learning which are used to minimize the error.

L1 Loss function stands for Least Absolute Deviations. Also known as LAD.

L2 Loss function stands for Least Square Errors. Also known as LS.

L1 Loss Function is used to minimize the error which is the sum of the all the absolute differences between the true value and the predicted value.

$$L1LossFunction = \sum_{i=1}^{n} |y_{true} - y_{predicted}|$$

L2 Loss Function is used to minimize the error which is the sum of the all the squared differences between the true value and the predicted value.

$$L2LossFunction = \sum_{i=1}^{n} (y_{true} - y_{predicted})^{2}$$

Generally, L2 Loss Function is preferred in most of the cases. But when the outliers are present in the dataset, then the L2 Loss Function does not perform well. The reason behind this bad performance is that if the dataset is having outliers, then because of the consideration of the squared differences, it leads to the much larger error. Hence, L2 Loss Function is not useful here. Prefer L1 Loss Function as it is not affected by the outliers or remove the outliers and then use L2 Loss Function.

Huber_Loss:

Huber Loss is loss function that is used in robust regression. It is the solution to problems faced by L1 and L2 loss functions. It is less sensitive to outliers in data than the squared error loss. It's also differentiable at 0.

$$L_{\delta}(y,f(x)) = \left\{ egin{array}{ll} rac{1}{2}(y-f(x))^2 & ext{for} |y-f(x)| \leq \delta, \ \delta \, |y-f(x)| - rac{1}{2}\delta^2 & ext{otherwise.} \end{array}
ight.$$

The above function becomes quadratic when error value a is small and linear when a is large.

Reason for using Huber Loss?

One big problem with using MAE is its constantly large gradient when using gradient decent for training. This can lead to missing minima at the end of training using gradient descent. While with MSE, gradient decreases as the loss gets close to its minima, making it more precise.

Huber loss can be here, as it curves around the minima which decreases the gradient.

Compared with L2 loss, Huber Loss is less sensitive to outliers (because if the residual is too large, it is a piecewise function, loss is a linear function of the residual). Among them, δ is a set parameter, y represents the real value and f(x) represent the predicted value.

The Huber function is less sensitive to small errors than the $\ell 1$ norm, but becomes linear in the error for large errors.

3.2.2 Using evaluation data

This happening may occur when our model trains as well as it could be and means that we face with a perfect model which caused the error for validation data and trained data become as much as close it could be!

And also it caused when the complication of validation data is less that trained data and in these case we also face with this happening.

3.2.3 Gradient descent using Momentum

Gradient descent is an optimization algorithm that follows the negative gradient of an objective function in order to locate the minimum of the function.

A problem with gradient descent is that it can bounce around the search space on optimization problems that have large amounts of curvature or noisy gradients, and it can get stuck in flat spots in the search space that have no gradient.

Momentum is an extension to the gradient descent optimization algorithm that allows the search to build inertia in a direction in the search space and overcome the oscillations of noisy gradients and coast across flat spots of the search space.

The answer is Gradient descent using momentum as we know Gradient descent can be accelerated by using momentum from past updates to the search position.

It is designed to accelerate the optimization process, e.g. decrease the number of function evaluations required to reach the optima, or to improve the capability of the optimization algorithm, e.g. result in a better final result.

The momentum algorithm accumulates an exponentially decaying moving average of past gradients and continues to move in their direction.

Momentum has the effect of dampening down the change in the gradient and, in turn, the step size with each new point in the search space.

Momentum can increase speed when the cost surface is highly nonspherical because it damps the size of the steps along directions of high curvature thus yielding a larger effective learning rate along the directions of low curvature.

Momentum is most useful in optimization problems where the objective function has a large amount of curvature (e.g. changes a lot), meaning that the gradient may change a lot over relatively small regions of the search space.

It is also helpful when the gradient is estimated, such as from a simulation, and may be noisy, e.g. when the gradient has a high variance.

Finally, momentum is helpful when the search space is flat or nearly flat, e.g. zero gradient. The momentum allows the search to progress in the same direction as before the flat spot and helpfully cross the flat region.

3.3 APPROXIMATE SINE FUNCTION (NO PYTHON LIBRARIES)

Implementation in Python:

In this section we intend to implement a fully connected neural network to approximate a sine function in interval of [0,2pi].

$$f(x, y) = \sin(x + y); (x, y) \in [0, 2\pi] \times [0, 2\pi];$$

At the first step we are going to sampling from the [0,2pi] interval using random.uniform() function in python, afterward we make x1 and x2 vector as of our input neurons.

Afterward we use np.sin() function to obtain corresponding labels and finally we make dataset which have three rows:

First one normalize data values for x1-input

second one normalize data values for x2-input

and third one the corresponding labels for normalize data.

Note that in this question I have used 80 percent of data for training and also I have used 20 percent of that for evaluation and rest of that for testing!

And also Note that my ReLU function gets a vector as of its input and give us a vector.

I implement ReLU in this way cuase of my implementation in sine_train() function in next part.

And also I sample from data in [0,2pi] interval because I asked from correspond TA and she said me that for reaching best accuracy go through this way!

Some extra notes:

I have used one layer neural network with ReLU activation function in each node and also I have used stochastic gradient descent approach according to HW description!

Further you can see all of my codes using forwarding and back-propagation to reach a reasonable y value!

Note that at the end code block I have used a matrix called data_plot which has two rows first one is x2 real values without normalizing which used for plotting because the interval must be 0,2pi and after normalization it would be something between 0,1 and it's not suitable for plotting!

And the normalize x2-value used for prediction because we train model with normalize data!

And unfortunately I have faced with a big problem!

I have checked whole algorithm more than once and everything was going true!

But I want to claim that the final result is just Mirror of correct answer!!

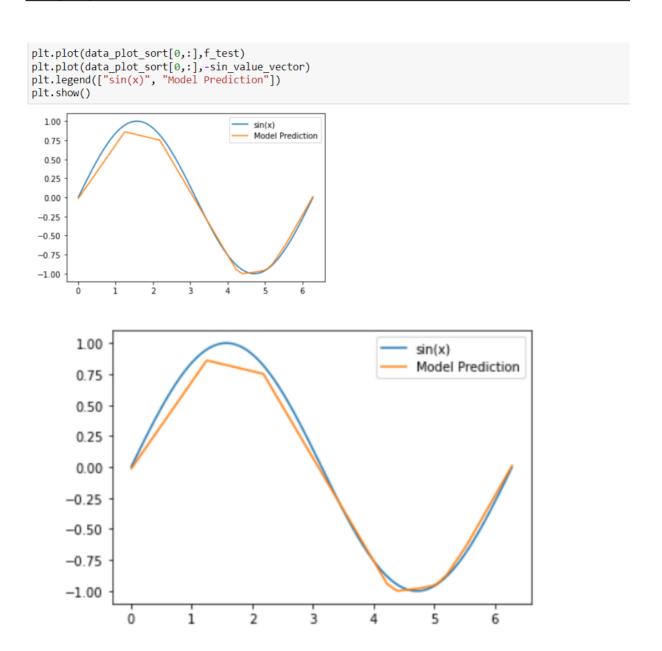
I talked with my friends and found that they were also struggling with same problem so I add a minus manually in my code which made result graphically correct but it's not still true

But I used my best of efforts to correct that by double checking algorithm several times but I failed in all of my tries :((

Here you can find all of my code implementation using Python if there is any issue with that please let me know to provide more detailed information. (be in touch **here!**)

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import random
np.random.seed(1000)
n=10000
y=np.zeros(n)
x1=np.random.uniform(0,2*(np.pi),n)
x2=np.random.uniform(0,2*(np.pi),n)
for i in range(n):
   y[i] = np.sin(x1[i]+x2[i])
x1_{mean} = sum(x1)/len(x1)
x2_{mean} = sum(x2)/len(x2)
x1_variance = np.std(x1)
x2_variance = np.std(x2)
x1_normalize = (x1-x1_mean)/x1_variance
x2_normalize = (x2-x2_mean)/x2_variance
data = np.array([x1_normalize,x2_normalize,y])
train_data = data[:,:int(0.6*n)]
validation_data = data[:,int(0.6*n):int(0.8*n)]
test_data = data[:,int(0.8*n):n]
def ReLU(x):
    data=np.zeros(np.size(x))
     for i in range(np.size(x)):
         data[i]=max(0,x[i])
    return data
def der_ReLU(x):
    data=np.zeros(np.size(x))
    for i in range(np.size(x)):
    data[i] = 1 if x[i]>0 else 0
    return data
def Sine_train(train_data):
     epoch=30
    hidden_layer=150
    input neurons=2
    output_neurons=1
    learning_rate=0.08
```

```
def Sine_train(train_data):
    epoch=30
    hidden_layer=150
    input_neurons=2
   output neurons=1
   learning_rate=0.08
   weights_in= 0.001 * np.random.randn(input_neurons,hidden_layer)
   bias_in=np.zeros(hidden_layer)
    weights_out= 0.001 * np.random.randn(hidden_layer)
    for i in range(epoch):
       for j in range(np.size(train_data[0,:])):
           x1 train=train data[0,j]
           x2_train=train_data[1,j
           y_train= train_data[2,j]
           #forward propogation
           hidden_layer_input= weights_in[0]*x1_train+weights_in[1]*x2_train+bias_in
           hidden_layer_output=ReLU(hidden_layer_input)
           predicted_output=np.dot(hidden_layer_output,weights_out)
           # #backward propogation
           Error=(predicted_output-y_train)
           delta_w2=hidden_layer_output*Error
delta_w1_1=weights_out*Error * der_ReLU(hidden_layer_input)*x1_train
delta_w1_2=weights_out*Error * der_ReLU(hidden_layer_input)*x2_train
           delta_b1=weights_out*Error * der_ReLU(hidden_layer_input)
           delta w1=np.zeros((2,hidden layer))
           delta_w1[0]=delta_w1_1
           delta_w1[1]=delta_w1_2
           weights_out-=delta_w2*learning_rate
           weights_in-=delta_w1*learning_rate
           bias_in-=delta_b1*learning_rate
    return weights_out,weights_in,bias_in
weights_out,weights_in,bias_in=Sine_train(train_data)
 data_plot = np.array([x2[int(0.8*np.size(x1)):np.size(x1)],x2_normalize[int(0.8*np.s
 data_plot_sort = data_plot[:, data_plot[0].argsort()]
 test_data=np.array([data_plot_sort[1,:],np.zeros(2000)])
 x1_test=test_data[0,:]
 x2_test=test_data[1,:]
p=np.size(x1_test)
sin_value_vector=np.zeros(p)
   _test=np.zeros(p)
 for i in range(p):
       x1_test=test_data[0,i]
      x2_test=test_data[1,i]
      hidden_layer_input= weights_in[0]*x1_test+weights_in[1]*x2_test+bias_in
      hidden_layer_output=ReLU(hidden_layer_input)
      sin_value_vector[i]=np.dot(hidden_layer_output,weights_out)
f_test[i] = np.sin(data_plot_sort[0,i])
 sin_value_vector
```



For running this part please have an accurate look on related directory and There you can easily find all of you need and also I would appreciate it if you would consider them

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5 REFERENCES

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