Cairo University
Faculty of Engineering
Aerospace Engineering Department

Digital Control Project: Liquid Level Control

AER 4410 Digital Control Applications
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Givens:

BN:49 corresponds to

$\boxed{t_s = 15 + \left(\frac{49-25}{10}\right) = 17.4sec}$

$\boxed{\rho_{water} = 1gm/cm^3\,, g = 981cm/\sec^2}$

$\boxed{Tank\ \ cross\ section = 15 \times 15cm^2}$

$\boxed{V_{out} = 0.4\,Volts}$

Q1

Treating the pump as a gain ($\boldsymbol{K_p \frac{cm^3}{sec}/Volts}$), determine this gain and the pressure resistance in the outlet pipe.

Solution:

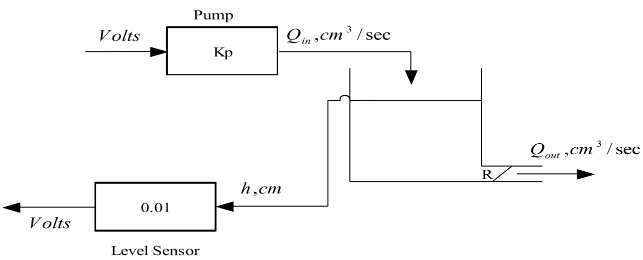

Figure 1: The Plant Block Diagram

Modeling the plant transfer function:

$$
\begin{aligned}
Q_{in} &= K_p V_{in} \\
Q_{out} &= \frac{\rho g h}{R} \\
Q_{in} - Q_{out} &= A\,\dot{h} \\
K_p V_{in} - \frac{\rho g h}{R} &= Ash
\end{aligned}
$$

$$TF = \frac{h}{K_p V_{in}} = \frac{1}{As + \frac{\rho g}{R}} = \frac{1/A}{s + \frac{\rho g}{AR}}$$

But $h = \frac{V_{sensor}}{K_{sensor}}$

$$\therefore TF = \frac{V_{sensor}}{V_{in}} = \frac{0.01K_p/A}{s + \frac{\rho g}{AR}} \equiv \frac{K}{s + \frac{1}{\tau}}$$

Also $t_s = 4\tau \to 17.4 = 4 \times \frac{AR}{\rho g} \to 17.4 = 4 \times \frac{15\times15\times R}{1\times981} \to \boxed{R = 18.966}$

To calculate the pump gain we apply the final value theorem:

$$
\begin{aligned}
V_{out}\big|_{final} &= \lim_{s\to0} s \times TF \times V_{in} \\
&= \frac{0.01K_p/\cancel{A}}{\cancel{s} + \frac{\rho g}{\cancel{A}R}} \times \cancel{s} \times \frac{\cancel{1}}{\cancel{s}} \times V_{in}
\end{aligned}
$$

$$\therefore 0.4 = \frac{0.01RK_p}{\rho g} \times 10$$

$$\to \boxed{K_p = \frac{0.4 \times 981}{10 \times 0.01 \times 18.966} = 206.89655}$$

Q2

It's required to design a signal conditioning unit for the level sensor in a level range $\mathbf{0 \to 50}$ cm.

Solution:

Sensor sensitivity $= \ 10mV$

Height range $0 \to 50\ cm$

Mapping the height range to the sensor we get

$$\begin{array}{ccc} 0 & \xrightarrow{\hspace{3em} Desired\ Height \hspace{3em}} & 50cm \\ & \boxed{\downarrow Sensitvity = 10mv \downarrow} & \\ 0 & \xrightarrow{\hspace{3em} Sensor\ Readigns \hspace{3em}} & 0.5V \\ & \boxed{\downarrow Gain = 10 \downarrow} & \\ 0 & \xrightarrow{Desired\ Microcontroller\ range} & 5V \end{array}$$

Therefore, we will need to multiply the sensor signal by a value of 10 using Op-Amps

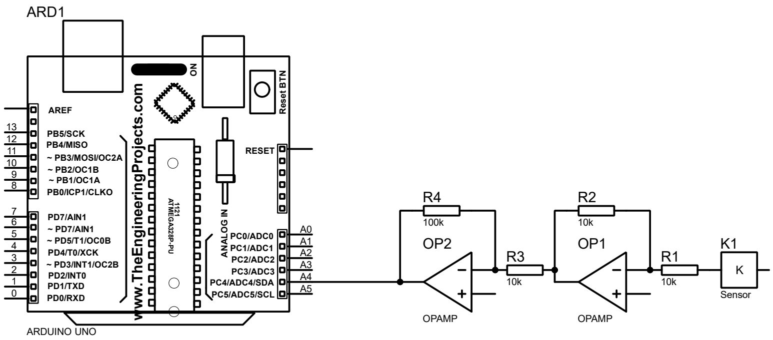

Figure 2: The Signal Conditioner Schematic

Q3

To discretize this system with 0.1 sec sampling period, is this sampling period suitable? if not, select a suitable sampling period.

Solution:

To check whether this sampling time is suitable or not for our system [1st order system] the sampling time should satisfy this condition

$$t_s \leq \frac{\tau}{10}, where\ t_s = 4\tau$$

For our system $t_s = 17.4 = 4\tau \to \tau = 4.35 \to \frac{\tau}{10} = 0.435$

Which means $\boxed{T_s = 0.1 \leq 0.435 \therefore T_s = 0.1\ is\ a\ suitable\ sampling\ time}$

Q4

Design a digital controller to achieve 5 sec settling time, 5 % overshoot, and 5 % steady state error.

Solution:

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \rightarrow 0.05 = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \rightarrow$$

$$\boxed{\zeta = 0.6901}$$

$$t_s = \frac{4}{\zeta\omega_n} \rightarrow 5 = \frac{4}{0.6901\omega_n} \rightarrow$$

$$\boxed{\omega_n = 1.15924}$$

$$\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2} = 0.83895}$$

$$\boxed{Z = e^{-\zeta\omega_n T_s} e^{i\omega_d T_s} = 0.91987 \pm 0.077354i}$$

$$T(Z) = \frac{a_0 Z^2 + a_1 Z + a_2}{(Z - 0.91987 + 0.077354i)(Z - 0.91987 - 0.077354i)}$$

$$T(Z) = \frac{\cancel{a_0 Z^2 + a_1 Z} + a_2}{Z^2 - 1.83974Z + 0.852144}$$

From the steady state error, we can get the value of a_2

$$E_{ss}(Z) = \lim_{Z\rightarrow 1}(1 - Z^{-1})R(Z)(1 - T(Z)) = \lim_{Z\rightarrow 1}\cancel{(1 - Z^{-1})} \times \frac{1}{\cancel{1 - Z^{-1}}}(1 - T(Z))$$

$$1 - T(1) = 0.05 \rightarrow \frac{a_2}{1 - 1.83974 + 0.852144} = 0.95 \rightarrow \boxed{a_2 = 0.0117838}$$

$$\boxed{\therefore T(Z)|_{desired} = \frac{0.0117838}{Z^2 - 1.83974Z + 0.852144}}$$

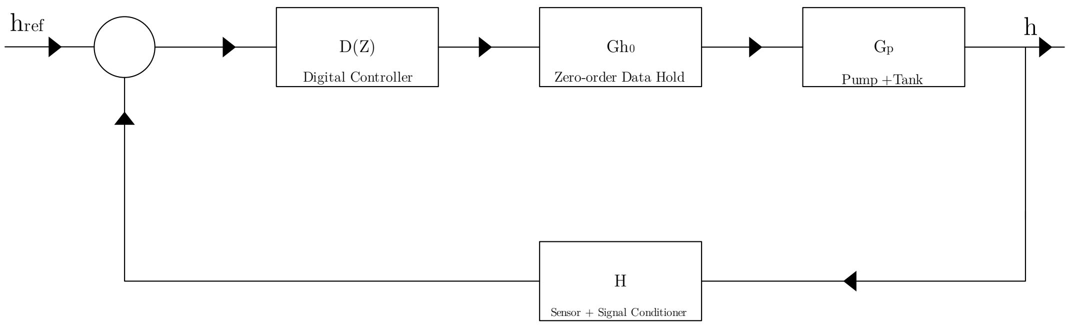

Figure 3: The Closed Loop System Block Diagram

$$T(z)|_{CL} = \frac{D(z)Gh_0 G_p(Z)}{1 + D(z)Gh_0 G_p H(Z)}$$

Where:

$$Gh_0 = \frac{1 - e^{-Ts}}{s} = \frac{1 - e^{-0.1s}}{s}$$

$$G_p = \frac{K_p K_{amp}/A}{s + \frac{\rho g}{AR}} = \frac{206.89655/225}{s + \frac{981}{225 \times 18.966}} = \frac{0.91954}{s + 0.229885}$$

$$Z[Gh_0 G_p(s)] = Z\left[(1 - e^{-0.1s})\left(\frac{0.91954}{s^2 + 0.229885s}\right)\right] \rightarrow (1 - Z^{-1})Z\left[\left(\frac{0.91954}{s^2 + 0.229885s}\right)\right]$$

From Z transform table:

$$Z\left[\frac{a}{s^2+as}\right] = \frac{(1-e^{-aT})Z^{-1}}{(1-Z^{-1})(1-e^{-aT}Z^{-1})}$$

$$(1-Z^{-1})Z\left[\left(\frac{0.91954}{s^2+0.229885s}\right)\right] = \frac{0.91954}{0.229885}(1-Z^{-1})Z\left[\left(\frac{0.229885}{s^2+0.229885s}\right)\right]$$

$$4(1-Z^{-1})Z\left[\left(\frac{0.229885}{s^2+0.229885s}\right)\right] = \frac{0.0909Z^{-1}}{(1-0.022726Z^{-1})} = \frac{0.0909Z^{-1}}{1-0.9772737Z^{-1}}$$

$$\boxed{\therefore Z\left[Gh_0G_p(s)\right] = \frac{0.0909}{\mathrm{Z}-0.9772737}}$$

$$T(z)|_{CL} = \frac{D(z)Gh_0G_p(Z)}{1+D(z)Gh_0G_pH(Z)} = T(Z)|_{desired}$$

$$\left(Gh_0G_p(Z) - Gh_0G_pH(Z)T(Z)|_{desired}\right) = T(Z)|_{desired}$$

$$\therefore D(Z) = \frac{T(Z)|_{desired}}{Gh_0G_p(Z) - Gh_0G_pH(Z)T(Z)|_{desired}}$$

$$D(Z) = \frac{\dfrac{0.0117838}{Z^2-1.83974Z+0.852144}}{\dfrac{0.0909}{\mathrm{Z}-0.9772737} - \dfrac{0.00909}{\mathrm{Z}-0.9772737}\times\dfrac{0.0117838}{Z^2-1.83974Z+0.852144}}$$

$$D(Z) = \frac{0.0117838\mathrm{Z}-0.01151599782606}{0.0909Z^2-0.167232366Z+0.0773527755}$$

$$\boxed{\therefore D(Z) = \frac{0.0117838\mathrm{Z}^{-1}-0.01151599782606Z^{-2}}{0.0909-0.167232366Z^{-1}+0.0773527755Z^{-2}}}$$

The control action implementation will be:

$$D(Z) = \frac{U(Z)}{E(Z)}$$

$0.0909\boldsymbol{u(k)} = 0.0117838\boldsymbol{e(k-1)} - 0.01151599782606\boldsymbol{e(k-2)} +$
$0.167232366\boldsymbol{u(k-1)} - 0.0773527755\boldsymbol{u(k-2)}$

$$\boxed{\boldsymbol{u(k)} = 0.129635\boldsymbol{e(k-1)} - 0.126689\boldsymbol{e(k-2)} + 1.83974\boldsymbol{u(k-1)} - 0.85097\boldsymbol{u(k-2)}}$$