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Cairo University
Faculty of Engineering
Aerospace Engineering Dept.

Report (1)
AER 3120 - Aerodynamics (Boundary Layer)
3^{rd} Year, 2^{nd} Semester 2021/2022

Solving the potential flow around an arbitrary airfoil using panel method

By:

Mohammad Khaled Gamal Ali
Sec:2, BN:14

Submitted to:

Dr Hesham M. Elbanna

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Abstract

In this project we will calculate the pressure coefficient for a given number of airfoils [NACA0006, NACA0012, NACA0018] at $U_\infty = 50\frac{m}{s},\ \alpha = 0^\circ$ using a source vortex panel method.

Introduction

The development of airfoil theory to predict lift and pressure estimates for a given airfoil has gone through several stages. The first successful airfoil theory, which based on conformal transformation, was developed by Joukowski. He represented a potential flow by a complex potential and maps the complex potential flow around the circle in Z plane to the corresponding flow around the airfoil in the Z plane, which makes it possible to use the results for the cylinder with circulation to calculate the flow around an airfoil. However, it can only apply to a particular family of airfoil shape and all members of this family have a cusped trailing edge, which is inconsistent with the practical situation that has trailing edges with finite angles.

The second airfoil theory is the thin airfoil theory. In thin airfoil theory, the airfoil is replaced with its mean camber line. The flow pattern is built up by placing a bound vortex sheet on the camber line and adjusting its strength so that the camber line becomes a streamline of the flow. Within this framework, the theory adequately predicts lift and moment for thin airfoil. Nevertheless, its drawback is also obvious – it cannot be applied to arbitrarily thick airfoils because of the ignored thickness effects.

With the advent of digital computers offers the attractive alternative of a numerical rather than an analytical solution, a new method in aerodynamic design is widely used nowadays – the panel method. It relies on the distribution of singularities on discrete segments of the airfoil surface. By satisfying no penetration condition and Kutta condition, a system of linear algebraic equations to be solved for the unknown singularity-strength is created, with which, the lift coefficients and pressure distribution can be easily predicted. Panel method can be applied to airfoil section with any thickness and camber.

Methodology

For incompressible, inviscid and irrotational flow, the vector velocity can be represented as the gradient of a scalar velocity potential, $\vec{V} = \nabla\phi$, and the resulting flow is referred to as potential flow. According to continuity equation $\nabla \cdot \vec{V} = 0$, velocity potential satisfies the Laplace's equation $\nabla^2\phi = 0$.It is a liner partial differential equation and can be solved subject to no penetration boundary condition that no flow can cross the surface of the object.

Panel Geometry

$$XC_a = \frac{X_{Bi}+X_{Bi+1}}{2}$$

$$YC_a = \frac{Y_{Bi}+Y_{Bi+1}}{2}$$

$$\overline{S_a} = \sqrt{(X_{Bi+1} - X_{Bi})^2 + (Y_{Bi+1} - Y_{Bi})^2}$$

ϕ_a: Angle from +ve X-axis to inside the panel

δ_a: Angle from +ve X-axis to the outward normal vector

β_a: Angle between the free stream velocity and the outward normal vector

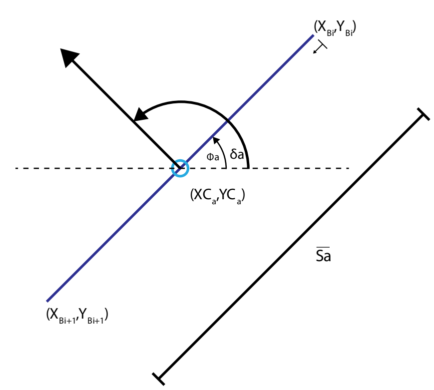


Figure 1: Panel Geometry

Source/ Vortex Method

For the normal velocity:

$$V_{n,i} = V_\infty Cos(\beta_i) + \sum_{j=1}^{N} \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial n_i} \ln(r_{ij})\, ds_j + \sum_{j=1}^{N} \frac{-\gamma}{2\pi} \int_j \frac{\partial \theta_{ij}}{\partial n_i} ds_j = 0$$

At $j = i$

$$\sum_{j=i}^{N} \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial n_i} \ln(r_{ij})\, ds_j = \frac{\lambda_j}{2}$$

$$\sum_{j=i}^{N} \frac{-\gamma}{2\pi} \int_j \frac{\partial \theta_{ij}}{\partial n_i} ds_j = 0$$

Let $\int_j \frac{\partial}{\partial n_i} \ln(r_{ij})\, ds_j = I_{ij}$, $\int_j \frac{\partial \theta_{ij}}{\partial n_i} ds_j = K_{ij}$

$$\therefore V_{n,i} = V_\infty Cos(\beta_i) + \frac{\lambda_j}{2} + \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\lambda_j}{2\pi} I_{ij} + \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{-\gamma}{2\pi} K_{ij} = 0 \text{ [multiplying by } 2\pi]$$

$$\pi\lambda_j + \sum_{\substack{j=1 \\ j\neq i}}^{N} \lambda_j\, I_{ij} + \sum_{\substack{j=1 \\ j\neq i}}^{N} -\gamma\, K_{ij} = -2\pi V_\infty Cos(\beta_i)$$

Therefore, the system of equation will be [for let's say 3 panels]:

$$\begin{bmatrix} \pi & I_{12} & I_{13} & -(K_{12}+K_{13}) \\ I_{21} & \pi & I_{23} & -(K_{21}+K_{23}) \\ I_{31} & I_{32} & \pi & -(K_{31}+K_{31}) \\ \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \gamma \end{bmatrix} = \begin{bmatrix} -2\pi V_\infty Cos(\beta_1) \\ -2\pi V_\infty Cos(\beta_2) \\ -2\pi V_\infty Cos(\beta_3) \\ \dots \end{bmatrix} \quad \begin{matrix} N\ Unkowns \\ N-1\ equations \end{matrix}$$

For the last equation we will apply the Kutta condition[1].

We will approximate the Kutta condition by setting the first panel and the last panel velocity to be equal

$$V_{t,N} = -V_{t,1} \to V_{t,N} + V_{t,1} = 0$$

Similar to the normal velocity the tangential velocity eq is:

$$V_{t,1} = V_\infty Sin(\beta_1) + \frac{\gamma_1}{2} + \sum_{j=2}^{N} \frac{\lambda_j}{2\pi} J_{1j} + \sum_{j=2}^{N} \frac{-\gamma}{2\pi} L_{1j}$$

$$V_{t,N} = V_\infty Sin(\beta_N) + \frac{\gamma_N}{2} + \sum_{j=1}^{N} \frac{\lambda_j}{2\pi} J_{Nj} + \sum_{j=1}^{N} \frac{-\gamma}{2\pi} L_{Nj}$$

Let $\int_j \frac{\partial}{\partial t_i} \ln(r_{ij})\, ds_j = J_{ij}$, $\int_j \frac{\partial \theta_{ij}}{\partial t_i} ds_j = L_{ij}$

Therefore

$$V_{t,N} + V_{t,1} = V_\infty Sin(\beta_1) + \frac{\gamma_1}{2} + \sum_{j=2}^{N} \frac{\lambda_j}{2\pi} J_{1j} + \sum_{j=2}^{N} \frac{-\gamma}{2\pi} L_{1j} + V_\infty Sin(\beta_N) + \frac{\gamma_N}{2} + \sum_{j=1}^{N} \frac{\lambda_j}{2\pi} J_{Nj} + \sum_{j=1}^{N} \frac{-\gamma}{2\pi} L_{Nj} = 0$$

[1] In fluid flow around a body with a sharp corner, the Kutta condition refers to the flow pattern in which fluid approaches the corner from above and below, meets at the corner, and then flows away from the body. None of the fluid flows around the sharp corner.

$$\sum_{\substack{j=1\\ j\neq i\\ j\neq N}}^{N} \lambda_j \left(J_{1j} + J_{Nj}\right) + \gamma \left[\sum_{\substack{j=1\\ j\neq i\\ j\neq N}}^{N} -\left(L_{1j} + L_{Nj}\right) + 2\pi\right] = -2\pi V_\infty\left(Sin(\beta_1) + Sin(\beta_N)\right)$$

So, the system of equation matrix will be

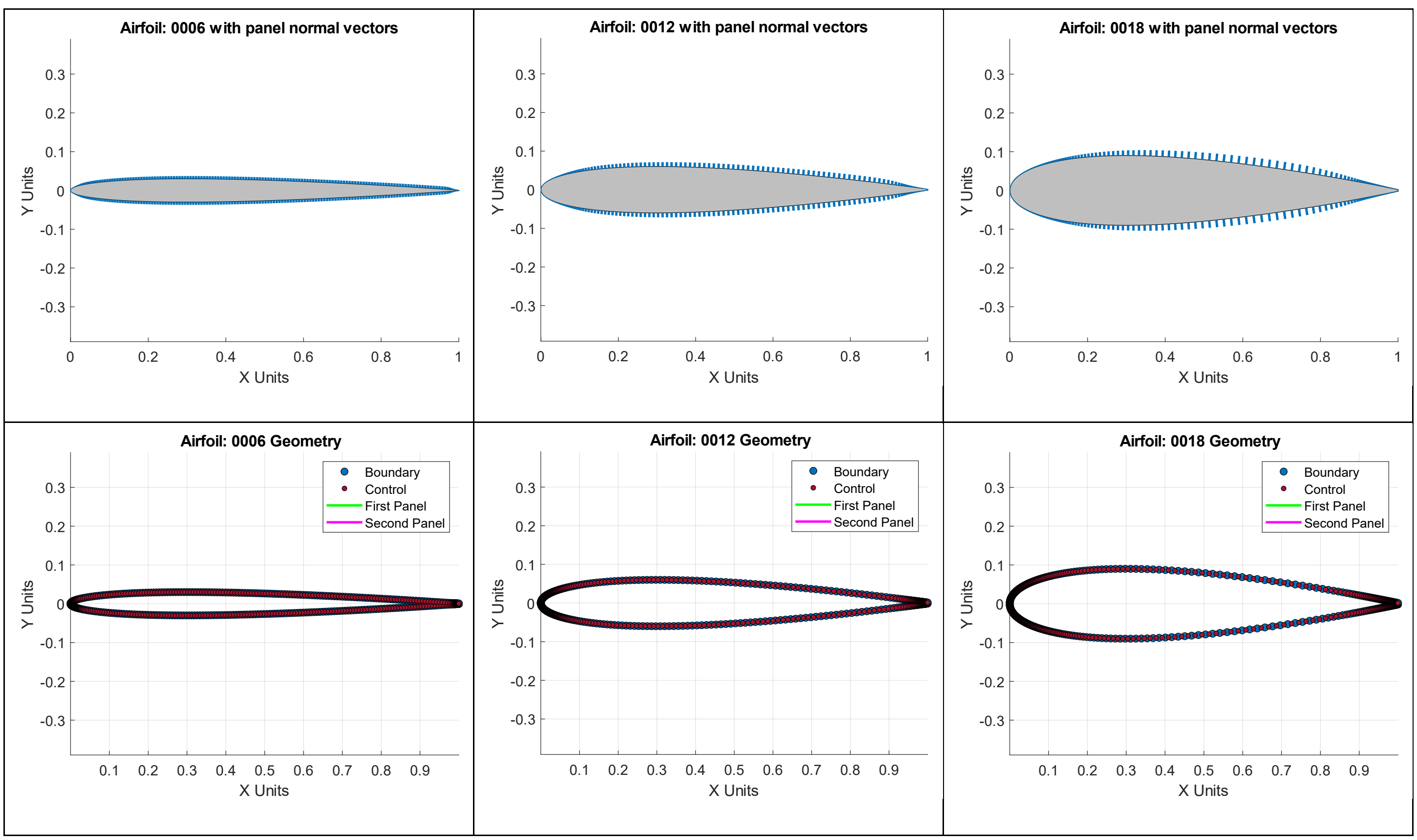
$$\begin{bmatrix} \pi & I_{12} & I_{13} & -(K_{12}+K_{13}) \\ I_{21} & \pi & I_{23} & -(K_{21}+K_{23}) \\ I_{31} & I_{32} & \pi & -(K_{31}+K_{31}) \\ J_{31} & (J_{13}+J_{32}) & J_{31} & -(L_{12}+L_{13}+L_{31}+L_{32})+2\pi \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \gamma \end{bmatrix} = \begin{bmatrix} -2\pi V_\infty Cos(\beta_1) \\ -2\pi V_\infty Cos(\beta_2) \\ -2\pi V_\infty Cos(\beta_3) \\ -2\pi V_\infty\left(Sin(\beta_1)+Sin(\beta_N)\right) \end{bmatrix}$$

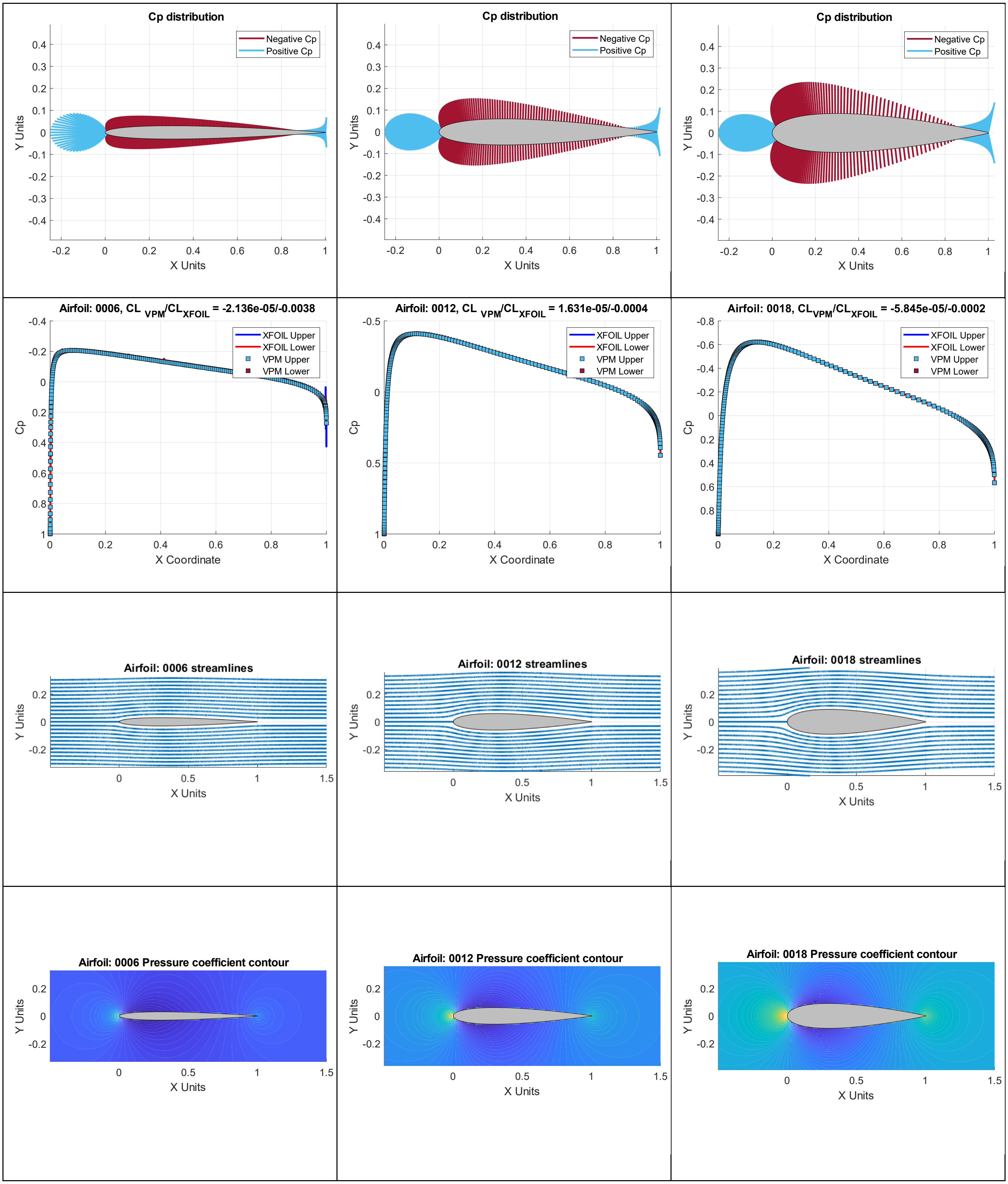
Results at $[U_\infty = 50\ m/s,\ \alpha = 0^\circ]$

Table 1:CL, CM, and velocities comparison

Airfoil	C_L		C_M		Velocities at				
	SVP code	XFOIL	SVP code	XFOIL	X=0.2	X=0.4	X=0.6	X=0.8	X=1
NACA0006	0.0000	-0.0038	0.0000	0.0009	54.4795	53.2676	52.0202	50.6324	42.6867
NACA0012	0.0000	-0.0004	0.0000	0.0001	58.8949	56.5240	53.9473	51.1309	37.2108
NACA0018	0.0000	-0.0002	0.0000	0.0000	63.2762	59.7557	55.7870	51.5052	32.8747

Table 2: MATLAB Code results




Cp distribution
Negative Cp
Positive Cp
Y Units
X Units
Airfoil: 0006, CL_{VPM}/CL_{XFOIL} = -2.136e-05/-0.0038
Airfoil: 0012, CL_{VPM}/CL_{XFOIL} = 1.631e-05/-0.0004
Airfoil: 0018, CL_{VPM}/CL_{XFOIL} = -5.845e-05/-0.0002
XFOIL Upper
XFOIL Lower
VPM Upper
VPM Lower
Cp
X Coordinate
Airfoil: 0006 streamlines
Airfoil: 0012 streamlines
Airfoil: 0018 streamlines
Airfoil: 0006 Pressure coefficient contour
Airfoil: 0012 Pressure coefficient contour
Airfoil: 0018 Pressure coefficient contour

References

[1] J. Anderson, Aircraft Performance & Design, McGraw-Hill Education, 1999.

Appendix A [MATLAB Codes]

Main Code

```
% SOURCE/VORTEX PANEL METHOD - SINGLE AIRFOIL
% Written by: JoshTheEngineer
% GitHub    : www.github.com/jte0419
% Notes     : This code is not optimized, but is instead written in such a way
%             that it is easy to follow along with my YouTube video derivations
%
% Functions Needed:
% - XFOIL.m
% - COMPUTE_IJ_SPM.m
% - COMPUTE_KL_VPM.m
% - STREAMLINE_SPM.m
% - STREAMLINE_VPM.m
% - COMPUTE_CIRCULATION.m
%
% Programs Needed:
% - xfoil.exe
%
% Folder Needed:
% - Airfoil_DAT_Selig: folder containing all Selig-format airfoils

clear;
clc;
```

Givens

```
% Flag to specify creating or loading airfoil
flagAirfoil.XFoilCreate = 1;                % Create specified NACA airfoil in XFOIL
flagAirfoil.XFoilLoad   = 0;                % Load Selig-format airfoil from directory

% User-defined knowns
Vinf = 50;                                  % Freestream velocity [] (just leave this at 1)
AoA  = 0;                                   % Angle of attack [deg]
NACA = '0018';                              % NACA airfoil to load [####(#)]

% Plotting flags
flagPlot = [1;          % Airfoil with panel normal vectors
```

```
        1;      % Geometry boundary pts, control pts, first panel, second panel
        1;      % Cp vectors at airfoil surface panels
        1;      % Pressure coefficient comparison (XFOIL vs. SPVP)
        1;      % Airfoil streamlines
        1];     % Pressure coefficient contour
```

XFOIL - CREATE/LOAD AIRFOIL

```
% PPAR menu options
PPAR.N  = '400';                            % "Number of panel nodes"
PPAR.P  = '4';                              % "Panel bunching parameter"
PPAR.T  = '1';                              % "TE/LE panel density ratios"
PPAR.R  = '1';                              % "Refined area/LE panel density ratio"
PPAR.XT = '1 1';                            % "Top side refined area x/c limits"
PPAR.XB = '1 1';                            % "Bottom side refined area x/c limits"

% Call XFOIL function to obtain the following:
% - Airfoil coordinates
% - Pressure coefficient along airfoil surface
% - Lift, drag, and moment coefficients
[xFoilResults,success] = XFOIL(NACA,PPAR,AoA,flagAirfoil);      % Get XFOIL results for prescribed airfoil
if (success == 0)                           % If user canceled airfoil dialog box
  return;                                   % Exit the program
end

% Separate out results from XFOIL function results
afName  = xFoilResults.afName;              % Airfoil name
xFoilX  = xFoilResults.X;                   % X-coordinate for Cp result
xFoilY  = xFoilResults.Y;                   % Y-coordinate for Cp result
xFoilCP = xFoilResults.CP;                  % Pressure coefficient
XB      = xFoilResults.XB;                  % Boundary point X-coordinate
YB      = xFoilResults.YB;                  % Boundary point Y-coordinate
xFoilCL = xFoilResults.CL;                  % Lift coefficient
xFoilCD = xFoilResults.CD;                  % Drag coefficient
xFoilCM = xFoilResults.CM;                  % Moment coefficient

% Number of boundary points and panels
numPts = length(XB);                        % Number of boundary points
numPan = numPts - 1;                        % Number of panels (control points)
```

CHECK PANEL DIRECTIONS - FLIP IF NECESSARY

```
% Check for direction of points
edge = zeros(numPan,1);                     % Initialize edge value array
for i = 1:1:numPan                          % Loop over all panels
  edge(i) = (XB(i+1)-XB(i))*(YB(i+1)+YB(i));            % Compute edge values
end
sumEdge = sum(edge);                        % Sum all edge values

% If panels are CCW, flip them (don't if CW)
```

```
if (sumEdge < 0)                                    % If panels are CCW
    XB = flipud(XB);                                % Flip the X-data array
    YB = flipud(YB);                                % Flip the Y-data array
end
```

PANEL METHOD GEOMETRY

```
% Initialize variables
XC   = zeros(numPan,1);                             % Initialize control point X-coordinate array
YC   = zeros(numPan,1);                             % Initialize control point Y-coordinate array
S    = zeros(numPan,1);                             % Initialize panel length array
phiD = zeros(numPan,1);                             % Initialize panel orientation angle array [deg]

% Find geometric quantities of the airfoil
for i = 1:1:numPan                                  % Loop over all panels
    XC(i)   = 0.5*(XB(i)+XB(i+1));                  % X-value of control point
    YC(i)   = 0.5*(YB(i)+YB(i+1));                  % Y-value of control point
    dx      = XB(i+1)-XB(i);                        % Change in X between boundary points
    dy      = YB(i+1)-YB(i);                        % Change in Y between boundary points
    S(i)    = (dx^2 + dy^2)^0.5;                    % Length of the panel
    phiD(i) = atan2d(dy,dx);                        % Angle of the panel (positive X-axis to inside face) [deg]
    if (phiD(i) < 0)                                % Make all panel angles positive [deg]
        phiD(i) = phiD(i) + 360;
    end
end

% Compute angle of panel normal w.r.t horizontal and include AoA
deltaD             = phiD + 90;                     % Angle from positive X-axis to outward normal vector [deg]
betaD              = deltaD - AoA;                  % Angle between freestream vector and outward normal vector [deg]
betaD(betaD > 360) = betaD(betaD > 360) - 360;      % Make all panel angles between 0 and 360 [deg]

% Convert angles from [deg] to [rad]
phi  = phiD.*(pi/180);                              % Convert from [deg] to [rad]
beta = betaD.*(pi/180);                             % Convert from [deg] to [rad]
```

COMPUTE SOURCE AND VORTEX PANEL STRENGTHS

```
% Geometric integrals for SPM and VPM (normal [I,K] and tangential [J,L])
% - Refs [2], [3], [6], and [7]
[I,J] = COMPUTE_IJ_SPM(XC,YC,XB,YB,phi,S);          % Call COMPUTE_IJ_SPM function (Refs [2] and [3])
[K,L] = COMPUTE_KL_VPM(XC,YC,XB,YB,phi,S);          % Call COMPUTE_KL_VPM function (Refs [6] and [7])

% Populate A matrix
% - Simpler option:
A = I + pi*eye(numPan,numPan);
% A = zeros(numPan,numPan);                         % Initialize the A matrix
% for i = 1:1:numPan                                % Loop over all i panels
%     for j = 1:1:numPan                            % Loop over all j panels
%         if (j == i)                               % If the panels are the same
%             A(i,j) = pi;                          % Set A equal to pi
```

```
%         else                                                    % If panels are not the same
%             A(i,j) = I(i,j);                                    % Set A equal to I
%         end
%     end
% end
% Right column of A matrix
for i = 1:1:numPan                                                % Loop over all i panels (rows)
    A(i,numPan+1) = -sum(K(i,:));                                 % Add gamma term to right-most column of A matrix
end

% Bottom row of A matrix (Kutta condition)
for j = 1:1:numPan                                                % Loop over all j panels (columns)
    A(numPan+1,j) = (J(1,j) + J(numPan,j));                       % Source contribution of Kutta condition equation
end
A(numPan+1,numPan+1) = -sum(L(1,:) + L(numPan,:)) + 2*pi;         % Vortex contribution of Kutta condition equation

% Populate b array
% - Simpler option:
b = -Vinf*2*pi*cos(beta);
% b = zeros(numPan,1);                                            % Initialize the b array
% for i = 1:1:numPan                                              % Loop over all i panels (rows)
%     b(i) = -Vinf*2*pi*cos(beta(i));                             % Compute RHS array
% end

% Last element of b array (Kutta condition)
b(numPan+1) = -Vinf*2*pi*(sin(beta(1)) + sin(beta(numPan)));      % RHS of Kutta condition equation

% Compute result array
resArr = A\b;                                                     % Solve system of equations for all source strengths and single vortex strength

% Separate lambda and gamma values from result array
lambda = resArr(1:end-1);                                         % All panel source strenths
gamma  = resArr(end);                                             % Constant vortex strength
```

COMPUTE PANEL VELOCITIES AND PRESSURE COEFFICIENTS

```
% Compute velocities on each panel
Vt = zeros(numPan,1);                                             % Initialize tangential velocity
Cp = zeros(numPan,1);                                             % Initialize pressure coefficient
for i = 1:1:numPan
    term1 = Vinf*sin(beta(i));                                    % Uniform flow term
    term2 = (1/(2*pi))*sum(lambda.*J(i,:)');                      % Source panel terms when j is not equal to i
    term3 = gamma/2;                                              % Vortex panel term when j is equal to i
    term4 = -(gamma/(2*pi))*sum(L(i,:));                          % Vortex panel terms when j is not equal to i

    Vt(i) = term1 + term2 + term3 + term4;                        % Compute tangential velocity on panel i
    Cp(i) = 1-(Vt(i)/Vinf)^2;                                     % Compute pressure coefficient on panel i
end
```

COMPUTE LIFT AND MOMENT

```
% Compute normal and axial force coefficients
CN = -Cp.*S.*sin(beta);                                    % Normal force coefficient []
CA = -Cp.*S.*cos(beta);                                    % Axial force coefficient []

% Compute lift and moment coefficients
CL = sum(CN.*cosd(AoA)) - sum(CA.*sind(AoA));                              % Decompose axial and normal to lift coefficient []
CM = sum(Cp.*(XC-0.25).*S.*cos(phi));                          % Moment coefficient []

% Print the results to the Command Window
fprintf('======= RESULTS =======\n');
fprintf('Lift Coefficient (CL)\n');
fprintf('\tSPVP : %2.4f\n',CL);                                % From this SPVP code
fprintf('\tK-J  : %g\n',2*sum(gamma.*S));                       % From Kutta-Joukowski lift equation
fprintf('\tXFOIL: %2.4f\n',xFoilCL);                            % From XFOIL program
fprintf('Moment Coefficient (CM)\n');
fprintf('\tSPVP : %2.4f\n',CM);                                % From this SPVP code
fprintf('\tXFOIL: %2.4f\n',xFoilCM);                            % From XFOIL program
```

COMPUTE STREAMLINES

```
if (flagPlot(5) == 1 || flagPlot(6) == 1)
   % Grid parameters
   nGridX = 100;                                        % X-grid for streamlines and contours
   nGridY = 100;                                        % Y-grid for streamlines and contours
   xVals  = [min(XB)-0.5 max(XB)+0.5];                          % X-grid extents [min, max]
   yVals  = [min(YB)-0.3 max(YB)+0.3];                          % Y-grid extents [min, max]

   % Streamline parameters
   stepsize = 0.01;                                     % Step size for streamline propagation
   maxVert  = nGridX*nGridY*100;                                % Maximum vertices
   slPct    = 25;                                       % Percentage of streamlines of the grid
   Ysl      = linspace(yVals(1),yVals(2),floor((slPct/100)*nGridY))';     % Create array of Y streamline starting points

   % Generate the grid points
   Xgrid   = linspace(xVals(1),xVals(2),nGridX)';                  % X-values in evenly spaced grid
   Ygrid   = linspace(yVals(1),yVals(2),nGridY)';                  % Y-values in evenly spaced grid
   [XX,YY] = meshgrid(Xgrid,Ygrid);                             % Create meshgrid from X and Y grid arrays

   % Initialize velocities
   Vx = zeros(nGridX,nGridY);                                 % Initialize X velocity matrix
   Vy = zeros(nGridX,nGridY);                                 % Initialize Y velocity matrix

   % Solve for grid point X and Y velocities
   for m = 1:1:nGridX
      for n = 1:1:nGridY
         XP      = XX(m,n);                               % Current iteration's X grid point
         YP      = YY(m,n);                               % Current iteration's Y grid point
         [Mx,My] = STREAMLINE_SPM(XP,YP,XB,YB,phi,S);                 % Compute Mx and My geometric integrals (Ref [4])
         [Nx,Ny] = STREAMLINE_VPM(XP,YP,XB,YB,phi,S);                 % Compute Nx and Ny geometric integrals (Ref [8])
```

```
            [in,on] = inpolygon(XP,YP,XB,YB);
            if (in == 1 || on == 1)                                    % If the grid point is in or on the airfoil
                Vx(m,n) = 0;                                           % Set X-velocity equal to zero
                Vy(m,n) = 0;                                           % Set Y-velocity equal to zero
            else                                                       % If the grid point is outside the airfoil
                Vx(m,n) = Vinf*cosd(AoA) + sum(lambda.*Mx./(2*pi)) + ...    % Compute X-velocity
                        sum(-gamma.*Nx./(2*pi));
                Vy(m,n) = Vinf*sind(AoA) + sum(lambda.*My./(2*pi)) + ...    % Compute Y-velocity
                        sum(-gamma.*Ny./(2*pi));
            end
        end
    end

    % Compute grid point velocity magnitude and pressure coefficient
    Vxy  = sqrt(Vx.^2 + Vy.^2);                                        % Compute magnitude of velocity vector []
    CpXY = 1-(Vxy./Vinf).^2;                                           % Pressure coefficient []
end
```

PLOTTING

```
% FIGURE: Airfoil with panel normal vectors
if (flagPlot(1) == 1)
    figure(1);                                                  % Create figure
    cla; hold on; grid off;                                     % Get ready for plotting
    set(gcf,'Color','White');                                   % Set color to white
    set(gca,'FontSize',12);                                     % Set font size
    fill(XB,YB,[0.75 0.75 0.75]);                               % Plot airfoil
    for i = 1:1:numPan                                          % Loop over all panels
        X(1) = XC(i);                                           % Set X start of panel orientation vector
        X(2) = XC(i) + S(i)*cosd(betaD(i)+AoA);                 % Set X end of panel orientation vector
        Y(1) = YC(i);                                           % Set Y start of panel orientation vector
        Y(2) = YC(i) + S(i)*sind(betaD(i)+AoA);                 % Set Y end of panel orientation vector
        plot(X,Y,"Color",[0 0.4470 0.7410],'LineWidth',2);      % Plot panel normal vector
    end
    xlabel('X Units');                                          % Set X-label
    ylabel('Y Units');                                          % Set Y-label
    xlim('auto');                                               % Set X-axis limits to auto
    ylim('auto');                                               % Set Y-axis limits to auto
    title(['Airfoil: ' xFoilResults.afName ...                  % Title
        ' with panel normal vectors'])
    axis equal;                                                 % Set axes equal
    zoom reset;                                                 % Reset zoom
end
% FIGURE: Geometry with the following indicated:
% - Boundary pts, control pts, first panel, second panel
if (flagPlot(2) == 1)
    figure(2);                                                  % Create figure
    cla; hold on; grid on;                                      % Get ready for plotting
    set(gcf,'Color','White');                                   % Set color to white
    set(gca,'FontSize',12);                                     % Set font size
    plot(XB,YB,'Color',[0 0.4470 0.7410],'LineWidth',3);        % Plot airfoil panels
```

```
    p1 = plot([XB(1) XB(2)],[YB(1) YB(2)],'g-','LineWidth',2);              % Plot first panel
    p2 = plot([XB(2) XB(3)],[YB(2) YB(3)],'m-','LineWidth',2);              % Plot second panel
    pB = plot(XB,YB,'ko','MarkerFaceColor',[0 0.4470 0.7410]);                           % Plot boundary points (black circles)
    pC = plot(XC,YC,'ko','MarkerFaceColor',[0.6350 0.0780 0.1840],'MarkerSize',4);                            % Plot control points (red circles)
    legend([pB,pC,p1,p2],...                                                % Show legend
           {'Boundary','Control','First Panel','Second Panel'});
    xlabel('X Units');                                                  % Set X-label
    ylabel('Y Units');                                                  % Set Y-label
    xlim('auto');                                                       % Set X-axis limits to auto
    ylim('auto');                                                       % Set Y-axis limits to auto
    title(['Airfoil: ' xFoilResults.afName ...
        ' Geometry'])
    axis equal;                                                         % Set axes equal
    zoom reset;                                                         % Reset zoom
end
% FIGURE: Cp vectors at airfoil control points
if (flagPlot(3) == 1)
    figure(3);                                                          % Create figure
    cla; hold on; grid on;                                              % Get ready for plotting
    set(gcf,'Color','White');                                           % Set color to white
    set(gca,'FontSize',12);                                             % Set font size
    Cps = abs(Cp*0.25);                                                 % Scale and make positive all Cp values
    for i = 1:1:length(Cps)                                             % Loop over all panels
        X(1) = XC(i);                                                   % Control point X-coordinate
        X(2) = XC(i) + Cps(i)*cosd(betaD(i)+AoA);                       % Ending X-value based on Cp magnitude
        Y(1) = YC(i);                                                   % Control point Y-coordinate
        Y(2) = YC(i) + Cps(i)*sind(betaD(i)+AoA);                       % Ending Y-value based on Cp magnitude

        if (Cp(i) < 0)                                                  % If pressure coefficient is negative
            p{1} = plot(X,Y,'Color',[0.6350 0.0780 0.1840],'LineWidth',2);                                 % Plot as a red line
        elseif (Cp(i) >= 0)                                             % If pressure coefficient is zero or positive
            p{2} = plot(X,Y,'Color',[0.3010 0.7450 0.9330],'LineWidth',2);                                 % Plot as a blue line
        end
    end
    fill(XB,YB,[0.75 0.75 0.75]);                                       % Plot the airfoil as black polygon
    legend([p{1},p{2}],{'Negative Cp','Positive Cp'});                  % Show legend
    xlabel('X Units');                                                  % Set X-label
    ylabel('Y Units');                                                  % Set Y-label
    xlim('auto');                                                       % Set X-axis limits to auto
    ylim('auto');                                                       % Set Y-axis limits to auto
    title('Cp distribution')
    axis equal;                                                         % Set axes equal
    zoom reset;                                                         % Reset zoom
end
% FIGURE: Pressure coefficient comparison (XFOIL vs. VPM)
if (flagPlot(4) == 1)
    figure(4);                                                          % Create figure
    cla; hold on; grid on;                                              % Get ready for plotting
    set(gcf,'Color','White');                                           % Set color to white
    set(gca,'FontSize',12);                                             % Set font size
    midIndX = floor(length(xFoilCP)/2);                                 % Airfoil middle index for XFOIL data
    midIndS = floor(length(Cp)/2);                                      % Airfoil middle index for SPM data
```

```
    pXu = plot(xFoilX(1:midIndX),xFoilCP(1:midIndX),'b-','LineWidth',2);    % Plot Cp for upper surface of airfoil from XFOIL
    pXl = plot(xFoilX(midIndX+1:end),xFoilCP(midIndX+1:end),'r-',...        % Plot Cp for lower surface of airfoil from XFOIL
                'LineWidth',2);
    pVl = plot(XC(1:midIndS),Cp(1:midIndS),'ks','MarkerFaceColor',[0.6350 0.0780 0.1840]);     % Plot Cp for upper surface of airfoil from SPM
    pVu = plot(XC(midIndS+1:end),Cp(midIndS+1:end),'ks',...                 % Plot Cp for lower surface of airfoil from SPM
                'MarkerFaceColor',[0.3010 0.7450 0.9330]);
    legend([pXu,pXl,pVu,pVl],...                                            % Show legend
           {'XFOIL Upper','XFOIL Lower','VPM Upper','VPM Lower'});
    xlabel('X Coordinate');                                                 % Set X-label
    ylabel('Cp');                                                           % Set Y-label
    xlim([0 1]);                                                            % Set X-axis limits
    ylim('auto');                                                           % Set Y-axis limits to auto
    set(gca,'Ydir','reverse')                                               % Reverse direction of Y-axis
    title(['Airfoil: ' xFoilResults.afName ...                              % Title
           ', CL_{VPM}/CL_{XFOIL} = ' ...
           num2str((2*sum(gamma.*S)),4) '/' num2str(xFoilCL,4)]);
    zoom reset;                                                             % Reset zoom
end
% FIGURE: Airfoil streamlines
if (flagPlot(5) == 1)
    figure(5);                                                              % Create figure
    cla; hold on; grid on;                                                  % Get ready for plotting
    set(gcf,'Color','White');                                               % Set color to white
    set(gca,'FontSize',12);                                                 % Set font size
    for i = 1:1:length(Ysl)                                                 % Loop over all Y streamline starting points
        sl = streamline(XX,YY,Vx,Vy,xVals(1),Ysl(i),[stepsize,maxVert]);    % Plot the streamline
        set(sl,'Color',[0 0.4470 0.7410],'LineWidth',2);                    % Set streamline line width
    end
    fill(XB,YB,[0.75 0.75 0.75]);                                           % Plot airfoil as black polygon
    xlabel('X Units');                                                      % Set X-label
    ylabel('Y Units');                                                      % Set Y-label
    xlim(xVals);                                                            % Set X-axis limits
    axis equal;                                                             % Set axes equal
    ylim(yVals);                                                            % Set Y-axis limits
    title(['Airfoil: ' xFoilResults.afName ...
        ' streamlines'])
    zoom reset;                                                             % Reset zoom
end

% FIGURE: Pressure coefficient contour
if (flagPlot(6) == 1)
    figure(6);                                                              % Create figure
    cla; hold on; grid on;                                                  % Get ready for plotting
    set(gcf,'Color','White');                                               % Set color to white
    set(gca,'FontSize',12);                                                 % Set font size
    contourf(XX,YY,CpXY,100,'EdgeColor','none');                            % Plot Cp contour
    fill(XB,YB,[0.75 0.75 0.75]);                                           % Plot airfoil as black polygon
    xlabel('X Units');                                                      % Set X-label
    ylabel('Y Units');                                                      % Set Y-label
    xlim(xVals);                                                            % Set X-axis limits
    axis equal;                                                             % Set axes equal
    ylim(yVals);                                                            % Set Y-axis limits
```

```
    title(['Airfoil: ' xFoilResults.afName ...
        ' Pressure coefficient contour'])
    zoom reset;                                         % Reset zoom
end
```

```
======= RESULTS =======
Lift Coefficient (CL)
        SPVP: -0.0000
        K-J: -5.76594e-05
        XFOIL: -0.0009
Moment Coefficient (CM)
        SPVP: 0.0000
        XFOIL: 0.0002
======= Velocities at specified X coordinates =======
    0.2000    0.4000    0.6000    0.8000    0.9999

   54.4795   53.2676   52.0202   50.6324   42.6867
```

```
======= RESULTS [NACA 0012] =======
Lift Coefficient (CL)
        SPVP: -0.0000
        K-J: -2.32096e-06
        XFOIL: -0.0009
Moment Coefficient (CM)
        SPVP: 0.0000
        XFOIL: 0.0002
======= Velocities at specified X coordinates =======
    0.2000    0.4000    0.6000    0.8000    0.9998

   58.8949   56.5240   53.9473   51.1309   37.2108
```

```
======= RESULTS [NACA 0018] =======
Lift Coefficient (CL)
        SPVP: 0.0000
        K-J : 7.62499e-05
        XFOIL: 0.0002
Moment Coefficient (CM)
        SPVP: -0.0000
        XFOIL: -0.0000
======= Velocities at specified X coordinates =======
    0.2000    0.4000    0.6000    0.8000    0.9997

   63.2762   59.7557   55.7870   51.5052   32.8747
```

Getting velocities at required points [0.2, 0.4, 0.6, 0.8]

```
X_req=[0.2, 0.4, 0.6, 0.8, XC(1)];
v_index=zeros(1,length(X_req));
v_req=zeros(1,length(X_req));
for i = 1:1:length(X_req)

v_index(i)=find(XC>=X_req(i) & XC<X_req(i)+0.02,1,'last' );
v_req(i)=Vt(v_index(i)-1)+((X_req(i)-XC(v_index(i)-1))/(XC(v_index(i))-XC(v_index(i)-1)))*(Vt(v_index(i))-
Vt(v_index(i)-1));
end
fprintf('======= Velocities at specified X coordinates =======\n');
disp(X_req)
disp(v_req)
```

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XFOIL Code

```
function [xFoilResults,success] = XFOIL(NACA,PPAR,AoA,flagAirfoil)
```

```
% PURPOSE
% - Create or load airfoil based on flagAirfoil
% - Save and read airfoil coordinates
% - Save and read airfoil pressure coefficient
% - Save and read airfoil lift, drag, and moment coefficients
%
% INPUTS
% - NACA        : Four-digit NACA airfoil designation
% - PPAR        : Paneling variables used in XFOIL PPAR menu
% - AoA         : Angle of attack [deg]
% - flagAirfoil : Flag for loading/creating airfoil
%
% OUTPUTS
% - xFoilResults : Structure containing all results
% - success      : Flag indicating whether solution was a success
```

CALL XFOIL FROM MATLAB

```
xFoilResults = [];                                    % Initialize results structure

if (flagAirfoil.XFoilCreate == 1)                     % If the user wants XFOIL to create a NACA airfoil
    airfoilName        = NACA;                        % Set the airfoilName to the input NACA airfoil
    xFoilResults.afName = airfoilName;                % Send the airfoil name back from this function
    success            = 1;                           % This will be successful
elseif (flagAirfoil.XFoilLoad == 1)                   % If the user wnats to load a DAT file airfoil
    [flnm,~,success]   = uigetfile('./Airfoil_DAT_Selig/*.dat',...    % User input of airfoil file to load
                                   'Select Airfoil File');
    airfoilName        = flnm(1:end-4);               % Set the airfoilName based on loaded file
    xFoilResults.afName = airfoilName;                % Send the airfoil name back from this function
    if (success == 0)                                 % If the user exited dialog box without selecting airfoil
        return;                                       % Exit the function
    else                                              % If user selected an airfoil
        success = 1;                                  % This will be successful
    end
end

% Save-to file names
saveFlnm    = ['Save_' airfoilName '.txt'];           % Airfoil coordinates save-to file
saveFlnmCp  = ['Save_' airfoilName '_Cp.txt'];        % Airfoil Cp save-to file
saveFlnmPol = ['Save_' airfoilName '_Pol.txt'];       % Airfoil polar save-to file

% Delete files if they exist
if (exist(saveFlnm,'file'))                           % If airfoil coordinate file exists
    delete(saveFlnm);                                 % Delete it
```

```
end
if (exist(saveFlnmCp,'file'))                                   % If airfoil Cp file exists
    delete(saveFlnmCp);                                         % Delete it
end
if (exist(saveFlnmPol,'file'))                                  % If airfoil polar file exists
    delete(saveFlnmPol);                                        % Delete it
end

% Create the airfoil
fid = fopen('xfoil_input.inp','w');                             % Create an XFoil input file, and make it write-able
if (flagAirfoil.XFoilLoad == 1)                                 % If user wants to load DAT airfoil file
    fprintf(fid,['LOAD ' './Airfoil_DAT_Selig/' flnm '\n']);    % Load selected airfoil
elseif (flagAirfoil.XFoilCreate == 1)                           % If user wants to specify a 4-digit airfoil
    fprintf(fid,['NACA ' NACA '\n']);                           % Specify NACA airfoil
end
fprintf(fid,'PPAR\n');                                          % Enter the PPAR (paneling) menu
fprintf(fid,['N ' PPAR.N '\n']);                                % Define "Number of panel nodes"
fprintf(fid,['P ' PPAR.P '\n']);                                % Define "Panel bunching paramter"
fprintf(fid,['T ' PPAR.T '\n']);                                % Define "TE/LE panel density ratios"
fprintf(fid,['R ' PPAR.R '\n']);                                % Define "Refined area/LE panel density ratio"
fprintf(fid,['XT ' PPAR.XT '\n']);                              % Define "Top side refined area x/c limits"
fprintf(fid,['XB ' PPAR.XB '\n']);                              % Define "Bottom side refined area x/c limits"
fprintf(fid,'\n');                                              % Apply all changes
fprintf(fid,'\n');                                              % Back out to XFOIL menu

% Save the airfoil data points
fprintf(fid,['PSAV ' saveFlnm '\n']);                           % Save the airfoil coordinate file

% Get Cp and polar data
fprintf(fid,'OPER\n');                                          % Enter OPER menu
fprintf(fid,'Pacc 1 \n');                                       % Begin polar accumulation
fprintf(fid,'\n\n');                                            % Don't enter save or dump file names
fprintf(fid,['Alfa ' num2str(AoA) '\n']);                       % Set angle of attack
fprintf(fid,['CPWR ' saveFlnmCp '\n']);                         % Write the Cp file
fprintf(fid,'PWRT\n');                                          % Save the polar data
fprintf(fid,[saveFlnmPol '\n']);                                % Save polar data to this file
if (exist(saveFlnmPol) ~= 0)                                    % If saveFlnmPol already exists
    fprintf(fid,'y \n');                                        % Overwrite existing file
end

fclose(fid);                                                    % Close the input file

cmd = 'xfoil.exe < xfoil_input.inp';                            % Write command to run XFoil

[~,~] = system(cmd);                                            % Run XFoil with the input file

fclose all;                                                     % Close all files

% Delete the XFoil input file
if (exist('xfoil_input.inp','file'))                            % If the input file exists
```

```
    delete('xfoil_input.inp');                                 % Delete the file
end
```

```
Not enough input arguments.

Error in XFOIL (line 23)
if (flagAirfoil.XFoilCreate == 1)                              % If the user wants XFOIL to create a NACA airfoil
```

READ CP DATA

```
fidCP = fopen(saveFlnmCp);                                     % Open the airfoil Cp file
dataBuffer = textscan(fidCP,'%f %f %f','HeaderLines',3,...          % Read in X, Y, and Cp data
                      'CollectOutput',1,...
                      'Delimiter','');
fclose(fidCP);                                                 % Close the file
delete(saveFlnmCp);                                            % Delete the file

% Save airfoil Cp data to function solution variable
xFoilResults.X  = dataBuffer{1,1}(:,1);                        % X-data points
xFoilResults.Y  = dataBuffer{1,1}(:,2);                        % Y-data points
xFoilResults.CP = dataBuffer{1,1}(:,3);                        % Cp data
```

READ AIRFOIL COORDINATES

```
fidAirfoil = fopen(saveFlnm);                                  % Open the airfoil file

dataBuffer = textscan(fidAirfoil,'%f %f','CollectOutput',1,...      % Read the XB and YB data from the data file
                      'Delimiter','','HeaderLines',0);

XB = dataBuffer{1}(:,1);                                       % Boundary point X-coordinate
YB = dataBuffer{1}(:,2);                                       % Boundary point Y-coordinate
fclose(fidAirfoil);                                            % Close the airfoil file
delete(saveFlnm);                                              % Delete the airfoil file

% Save airfoil boundary points to function solution variable
xFoilResults.XB = XB;                                          % Airfoil boundary X-points
xFoilResults.YB = YB;                                          % Airfoil boundary Y-points
```

READ POLAR DATA

```
% Load and read polar file (Save_Polar.txt)
fidPolar = fopen(saveFlnmPol);                                 % Open polar data file for reading

dataBuffer = textscan(fidPolar,'%f %f %f %f %f %f %f','CollectOutput',1,... % Read data from file
                      'Delimiter','','HeaderLines',12);
fclose(fidPolar);                                              % Close the file
delete(saveFlnmPol);                                           % Delete the file

% Extract polar data from buffer into function solution variable
xFoilResults.CL = dataBuffer{1,1}(2);                          % Lift coefficient
```

```
xFoilResults.CD = dataBuffer{1,1}(3);                              % Drag coefficient
xFoilResults.CM = dataBuffer{1,1}(5);                              % Moment coefficient
```

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Streamlines calculations [Source method]

```
function [Mx,My] = STREAMLINE_SPM(XP,YP,XB,YB,phi,S)

% FUNCTION - COMPUTE Mx AND My GEOMETRIC INTEGRALS FOR SOURCE PANEL METHOD
% Written by: JoshTheEngineer
% Updated   : 04/28/20 - Updated E value error handling to match Python
%
% PURPOSE
% - Compute the geometric integral at point P due to source panels
% - Source panel strengths are constant, but can change from panel to panel
% - Geometric integral for X-direction: Mx(pj)
% - Geometric integral for Y-direction: My(pj)
%
% REFERENCE
% - [1]: Streamline Geometric Integral SPM, Mx(pj) and My(pj)
%           Link: https://www.youtube.com/watch?v=BnPZjGCatcg
% INPUTS
% - XP     : X-coordinate of computation point, P
% - YP     : Y-coordinate of computation point, P
% - XB     : X-coordinate of boundary points
% - YB     : Y-coordinate of boundary points
% - phi    : Angle between positive X-axis and interior of panel
% - S      : Length of panel
%
% OUTPUTS
% - Mx     : Value of X-direction geometric integral (Ref [1])
% - My     : Value of Y-direction geometric integral (Ref [1])

% Number of panels
numPan = length(XB)-1;                                          % Number of panels/control points

% Initialize arrays
Mx = zeros(numPan,1);                                           % Initialize Mx integral array
My = zeros(numPan,1);                                           % Initialize My integral array

% Compute Mx and My
for j = 1:1:numPan                                              % Loop over the j panels
    % Compute intermediate values
    A  = -(XP-XB(j))*cos(phi(j))-(YP-YB(j))*sin(phi(j));        % A term
    B  = (XP-XB(j))^2+(YP-YB(j))^2;                             % B term
    Cx = -cos(phi(j));                                          % C term (X-direction)
    Dx = XP - XB(j);                                            % D term (X-direction)
    Cy = -sin(phi(j));                                          % C term (Y-direction)
    Dy = YP - YB(j);                                            % D term (Y-direction)
```

```
    E  = sqrt(B-A^2);                                      % E term
    if (~isreal(E))
        E = 0;
    end

    % Compute Mx, Ref [1]
    term1 = 0.5*Cx*log((S(j)^2+2*A*S(j)+B)/B);                      % First term in Mx equation
    term2 = ((Dx-A*Cx)/E)*(atan2((S(j)+A),E) - atan2(A,E));              % Second term in Mx equation
    Mx(j) = term1 + term2;                                  % X-direction geometric integral

    % Compute My, Ref [1]
    term1 = 0.5*Cy*log((S(j)^2+2*A*S(j)+B)/B);                      % First term in My equation
    term2 = ((Dy-A*Cy)/E)*(atan2((S(j)+A),E) - atan2(A,E));              % Second term in My equation
    My(j) = term1 + term2;                                  % Y-direction geometric integral

    % Zero out any NANs, INFs, or imaginary numbers
    if (isnan(Mx(j)) || isinf(Mx(j)) || ~isreal(Mx(j)))
        Mx(j) = 0;
    end
    if (isnan(My(j)) || isinf(My(j)) || ~isreal(My(j)))
        My(j) = 0;
    end
end
```

```
Not enough input arguments.

Error in STREAMLINE_SPM (line 31)
numPan = length(XB)-1;                                  % Number of panels/control points
```

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Streamlines calculations Vortex method

```
function [Nx,Ny] = STREAMLINE_VPM(XP,YP,XB,YB,phi,S)

% FUNCTION - COMPUTE Nx AND Ny GEOMETRIC INTEGRALS FOR VORTEX PANEL METHOD
% Written by: JoshTheEngineer
% YouTube   : www.youtube.com/joshtheengineer
% Website   : www.joshtheengineer.com
% Started   : 01/23/19
% Updated   : 01/23/19 - Started code
%
% PURPOSE
% - Compute the integral expression for constant strength vortex panels
% - Vortex panel strengths are constant, but can change from panel to panel
% - Geometric integral for X-velocity: Nx(pj)
% - Geometric integral for Y-velocity: Ny(pj)
%
% REFERENCES
```

```
% - [1]: Streamline Geometric Integral VPM, Nx(pj) and Ny(pj)
%          Link: https://www.youtube.com/watch?v=TBwBnW87hso
% INPUTS
% - XP     : X-coordinate of computation point, P
% - YP     : Y-coordinate of computation point, P
% - XB     : X-coordinate of boundary points
% - YB     : Y-coordinate of boundary points
% - phi    : Angle between positive X-axis and interior of panel
% - S      : Length of panel
%
% OUTPUTS
% - Nx     : Value of X-direction geometric integral
% - Ny     : Value of Y-direction geometric integral

% Number of panels
numPan = length(XB)-1;                                      % Number of panels (control points)

% Initialize arrays
Nx = zeros(numPan,1);                                       % Initialize Nx integral array
Ny = zeros(numPan,1);                                       % Initialize Ny integral array

% Compute Nx and Ny
for j = 1:1:numPan                                          % Loop over all panels
   % Compute intermediate values
   A  = -(XP-XB(j))*cos(phi(j))-(YP-YB(j))*sin(phi(j));     % A term
   B  = (XP-XB(j))^2+(YP-YB(j))^2;                          % B term
   Cx = sin(phi(j));                                        % Cx term (X-direction)
   Dx = -(YP-YB(j));                                        % Dx term (X-direction)
   Cy = -cos(phi(j));                                       % Cy term (Y-direction)
   Dy = XP-XB(j);                                           % Dy term (Y-direction)
   E  = sqrt(B-A^2);                                        % E term
   if (~isreal(E))
      E = 0;
   end

   % Compute Nx
   term1 = 0.5*Cx*log((S(j)^2+2*A*S(j)+B)/B);                       % First term in Nx equation
   term2 = ((Dx-A*Cx)/E)*(atan2((S(j)+A),E) - atan2(A,E));          % Second term in Nx equation
   Nx(j) = term1 + term2;                                           % Compute Nx integral

   % Compute Ny
   term1 = 0.5*Cy*log((S(j)^2+2*A*S(j)+B)/B);                       % First term in Ny equation
   term2 = ((Dy-A*Cy)/E)*(atan2((S(j)+A),E) - atan2(A,E));          % Second term in Ny equation
   Ny(j) = term1 + term2;                                           % Compute Ny integral

          % Zero out any NANs, INFs, or imaginary numbers
   if (isnan(Nx(j)) || isinf(Nx(j)) || ~isreal(Nx(j)))
      Nx(j) = 0;
   end
   if (isnan(Ny(j)) || isinf(Ny(j)) || ~isreal(Ny(j)))
      Ny(j) = 0;
```

```
    end
end
```

```
Not enough input arguments.

Error in STREAMLINE_VPM (line 34)
numPan = length(XB)-1;                              % Number of panels (control points)
```

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Computing the integrals I_{ij}, J_{ij}

```
function [I,J] = COMPUTE_IJ_SPM(XC,YC,XB,YB,phi,S)

% Written by: JoshTheEngineer
% Updated   : 04/28/20 - Updated E value error handling to match Python
%
% PURPOSE
% - Compute the integral expression for constant strength source panels
% - Source panel strengths are constant, but can change from panel to panel
% - Geometric integral for panel-normal    : I(ij)
% - Geometric integral for panel-tangential: J(ij)
%
% REFERENCES
% - [1]: Normal Geometric Integral SPM, I(ij)
%        Link: https://www.youtube.com/watch?v=76vPudNET6U
% - [2]: Tangential Geometric Integral SPM, J(ij)
%        Link: https://www.youtube.com/watch?v=JRHnOsueic8
%
% INPUTS
% - XC  : X-coordinate of control points
% - YC  : Y-coordinate of control points
% - XB  : X-coordinate of boundary points
% - YB  : Y-coordinate of boundary points
% - phi : Angle between positive X-axis and interior of panel
% - S   : Length of panel
%
% OUTPUTS
% - I   : Value of panel-normal integral (Eq. 3.163 in Anderson or Ref [1])
% - J   : Value of panel-tangential integral (Eq. 3.165 in Anderson or Ref [2])

% Number of panels
numPan = length(XC);                                % Number of panels/control points

% Initialize arrays
I = zeros(numPan,numPan);                           % Initialize I integral matrix
J = zeros(numPan,numPan);                           % Initialize J integral matrix

% Compute integral
```

```
for i = 1:1:numPan                                      % Loop over i panels
   for j = 1:1:numPan                                   % Loop over j panels
      if (j ~= i)                                   % If the i and j panels are not the same
         % Compute intermediate values
         A  = -(XC(i)-XB(j))*cos(phi(j))-(YC(i)-YB(j))*sin(phi(j));     % A term
         B  = (XC(i)-XB(j))^2+(YC(i)-YB(j))^2;                          % B term
         Cn = sin(phi(i)-phi(j));                       % C term (normal)
         Dn = -(XC(i)-XB(j))*sin(phi(i))+(YC(i)-YB(j))*cos(phi(i));     % D term (normal)
         Ct = -cos(phi(i)-phi(j));                      % C term (tangential)
         Dt = (XC(i)-XB(j))*cos(phi(i))+(YC(i)-YB(j))*sin(phi(i));      % D term (tangential)
         E  = sqrt(B-A^2);                             % E term
         if (~isreal(E))
            E = 0;
         end

         % Compute I (needed for normal velocity), Ref [1]
         term1  = 0.5*Cn*log((S(j)^2+2*A*S(j)+B)/B);                   % First term in I equation
         term2  = ((Dn-A*Cn)/E)*(atan2((S(j)+A),E) - atan2(A,E));       % Second term in I equation
         I(i,j) = term1 + term2;                        % Compute I integral

         % Compute J (needed for tangential velocity), Ref [2]
         term1  = 0.5*Ct*log((S(j)^2+2*A*S(j)+B)/B);                   % First term in J equation
         term2  = ((Dt-A*Ct)/E)*(atan2((S(j)+A),E) - atan2(A,E));       % Second term in J equation
         J(i,j) = term1 + term2;                        % Compute J integral
      end

      % Zero out any NANs, INFs, or imaginary numbers
      if (isnan(I(i,j)) || isinf(I(i,j)) || ~isreal(I(i,j)))
         I(i,j) = 0;
      end
      if (isnan(J(i,j)) || isinf(J(i,j)) || ~isreal(J(i,j)))
         J(i,j) = 0;
      end
   end
end
```

```
Not enough input arguments.

Error in COMPUTE_IJ_SPM (line 33)
numPan = length(XC);                                    % Number of panels/control points
```

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Computing the integrals K_{ij}, L_{ij}

```
function [K,L] = COMPUTE_KL_VPM(XC,YC,XB,YB,phi,S)

% FUNCTION - COMPUTE K AND L GEOMETRIC INTEGRALS FOR VORTEX PANEL METHOD
% Written by: JoshTheEngineer
```

```
% Started   : 01/23/19
% Updated   : 01/23/19 - Started code
%
% PUROSE
% - Compute the integral expression for constant strength vortex panels
% - Vortex panel strengths are constant, but can change from panel to panel
% - Geometric integral for panel-normal    : K(ij)
% - Geometric integral for panel-tangential: L(ij)
%
% REFERENCES
% - [1]: Normal Geometric Integral VPM, K(ij)
%           Link: https://www.youtube.com/watch?v=5lmIv2CUpoc
% - [2]: Tangential Geometric Integral VPM, L(ij)
%           Link: https://www.youtube.com/watch?v=IxWJzwIG_gY
%
% INPUTS
% - XC  : X-coordinate of control points
% - YC  : Y-coordinate of control points
% - XB  : X-coordinate of boundary points
% - YB  : Y-coordinate of boundary points
% - phi : Angle between positive X-axis and interior of panel
% - S   : Length of panel
%
% OUTPUTS
% - K   : Value of panel-normal integral (Ref [1])
% - L   : Value of panel-tangential integral (Ref [2])

% Number of panels
numPan = length(XC);                                        % Number of panels

% Initialize arrays
K = zeros(numPan,numPan);                                   % Initialize K integral matrix
L = zeros(numPan,numPan);                                   % Initialize L integral matrix

% Compute integral
for i = 1:1:numPan                                          % Loop over i panels
    for j = 1:1:numPan                                      % Loop over j panels
        if (j ~= i)                                         % If panel j is not the same as panel i
            A  = -(XC(i)-XB(j))*cos(phi(j))-(YC(i)-YB(j))*sin(phi(j));      % A term
            B  = (XC(i)-XB(j))^2+(YC(i)-YB(j))^2;                           % B term
            Cn = -cos(phi(i)-phi(j));                                       % C term (normal)
            Dn = (XC(i)-XB(j))*cos(phi(i))+(YC(i)-YB(j))*sin(phi(i));       % D term (normal)
            Ct = sin(phi(j)-phi(i));                                        % C term (tangential)
            Dt = (XC(i)-XB(j))*sin(phi(i))-(YC(i)-YB(j))*cos(phi(i));       % D term (tangential)
            E  = sqrt(B-A^2);                                               % E term
            if (~isreal(E))
                E = 0;
            end

            % Compute K
            term1  = 0.5*Cn*log((S(j)^2+2*A*S(j)+B)/B);                     % First term in K equation
            term2  = ((Dn-A*Cn)/E)*(atan2((S(j)+A),E)-atan2(A,E));          % Second term in K equation
```

```
            K(i,j) = term1 + term2;                                     % Compute K integral

            % Compute L
            term1  = 0.5*Ct*log((S(j)^2+2*A*S(j)+B)/B);                     % First term in L equation
            term2  = ((Dt-A*Ct)/E)*(atan2((S(j)+A),E)-atan2(A,E));          % Second term in L equation
            L(i,j) = term1 + term2;                                     % Compute L integral
        end

        % Zero out any NANs, INFs, or imaginary numbers
        if (isnan(K(i,j)) || isinf(K(i,j)) || ~isreal(K(i,j)))
            K(i,j) = 0;
        end
        if (isnan(L(i,j)) || isinf(L(i,j)) || ~isreal(L(i,j)))
            L(i,j) = 0;
        end
    end
end
```

```
Not enough input arguments.

Error in COMPUTE_KL_VPM (line 37)
numPan = length(XC);                                            % Number of panels
```

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