In the name of God

Mechatronics

HW#2

Solution

Written by

Navid Razaghi

Note:

$$c_i \equiv \cos\theta_i$$

$$s_i \equiv \sin\theta_i$$

problem 1 solution

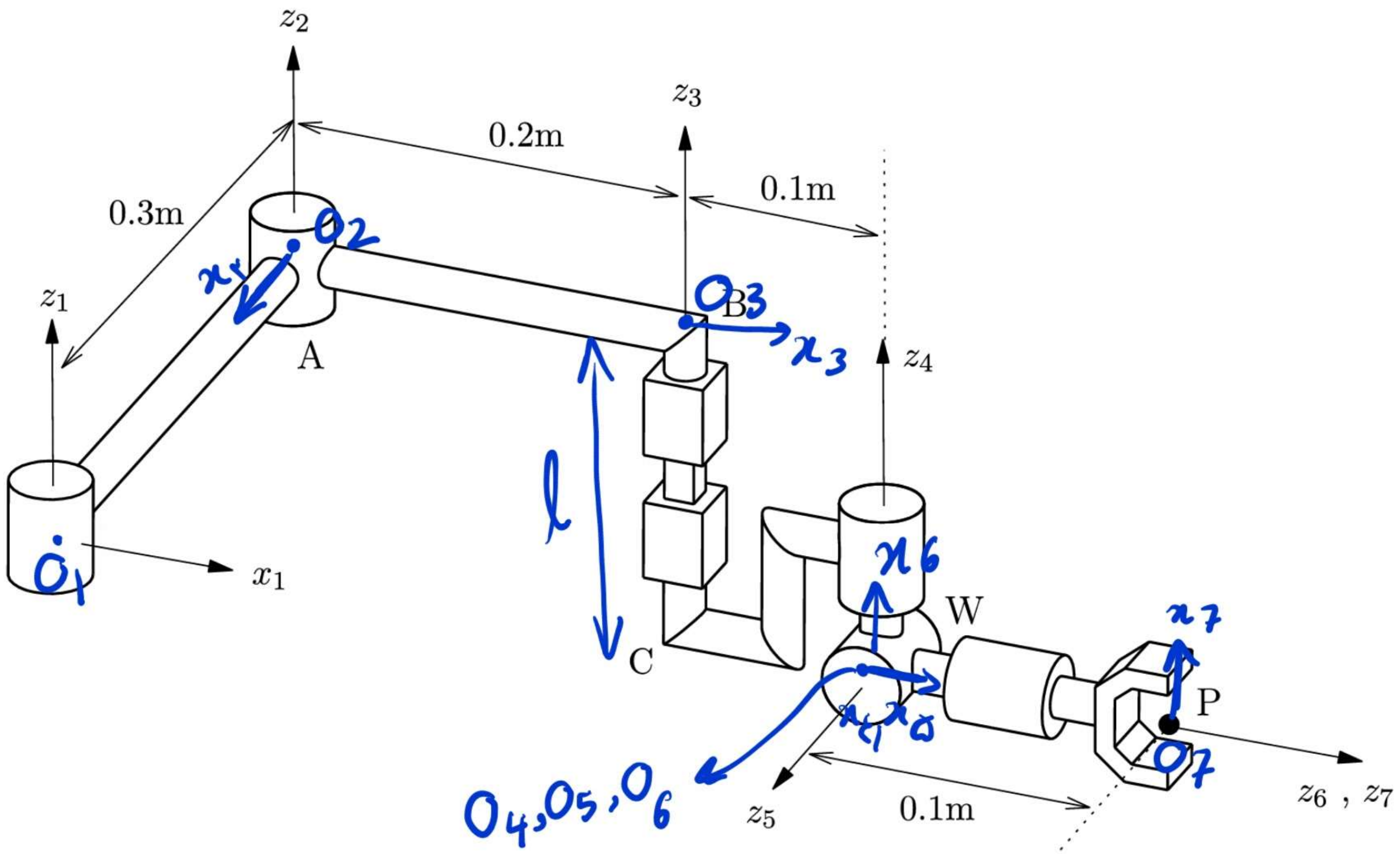


Figure 1: 6-DOF robot

D-H Parameters

i	a_i	b_i	α_i	θ_i
1	0.3	0	0	θ_1
2	0.2	0	0	θ_2
3	0.1	$-l$	0	0
4	0	0	$+\pi/2$	θ_4
5	0	0	$+\pi/2$	θ_5
6	0	0.1	0	θ_6

$a_4 = a_5 = b_5 = 0 \rightarrow$ decoupled Robot

$\vec{a_i}$'s & Q_i's

$$\vec{a}_1 = \begin{bmatrix} 0.3\cos\theta_1 \\ 0.3\sin\theta_1 \\ 0 \end{bmatrix} ; \quad \vec{a}_2 = \begin{bmatrix} 0.2\cos\theta_2 \\ 0.2\sin\theta_2 \\ 0 \end{bmatrix} ; \quad \vec{a}_3 = \begin{bmatrix} 0.1 \\ 0 \\ -l \end{bmatrix}$$

$$\vec{a}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad \vec{a}_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad \vec{a}_6 = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad Q_2 = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad Q_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_4 = \begin{bmatrix} c_4 & 0 & s_4 \\ s_4 & 0 & -c_4 \\ 0 & 1 & 0 \end{bmatrix} ; \quad Q_5 = \begin{bmatrix} c_5 & 0 & s_5 \\ s_5 & 0 & -c_5 \\ 0 & 1 & 0 \end{bmatrix} ; \quad Q_6 = \begin{bmatrix} c_6 & -s_6 & 0 \\ s_6 & c_6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

wrist point

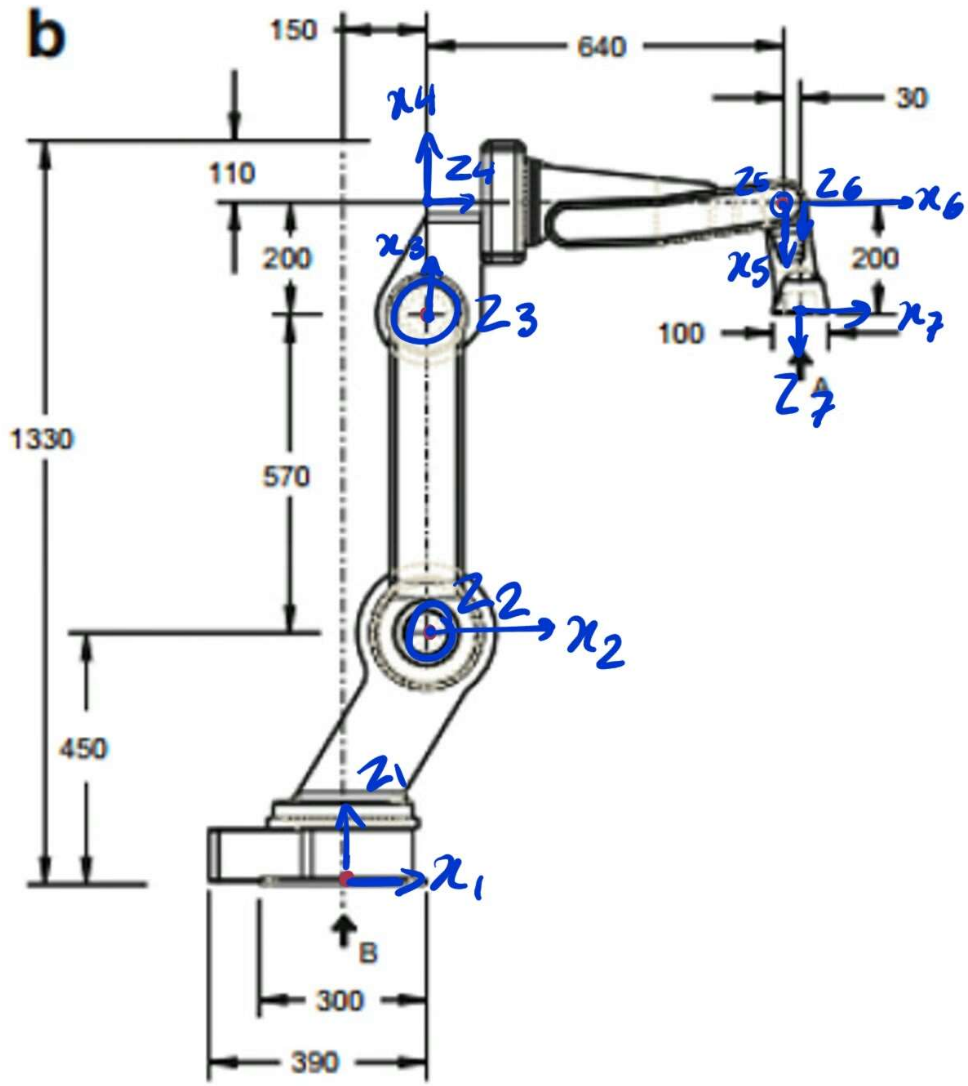
we must find O_4 :

$$\vec{a}_1 + \underset{\sim}{Q}_1 \vec{a}_2 + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \vec{a}_3 = \begin{bmatrix} 0.3\,c_1 \\ 0.3\,s_1 \\ 0 \end{bmatrix} +$$

$$\begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2\,c_2 \\ 0.2\,s_2 \\ 0 \end{bmatrix} +$$

$$\begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0 \\ -l \end{bmatrix} = \cdots$$

Problem 2
sol.:



D-H parameters

i	a_i	b_i	α_i	θ_i
1	150	450	$+\frac{\pi}{2}$	θ_1
2	570	0	0	θ_2
3	200	0	$+\frac{\pi}{2}$	θ_3
4	0	640	$+\frac{\pi}{2}$	θ_4
5	30	0	$+\pi/2$	θ_5
6	0	200	0	θ_6

* All joints are revolute joint.*

Is decoupled (wrist) or not?

In Wrist robot we have:

$$a_4 = a_5 = b_5 = 0$$

but in this robot:

$$a_5 = 30 \neq 0$$

So this robot is not decoupled!

Calculating $\vec{a_i}$'s

$$\vec{a_i} = \begin{bmatrix} a_i \cos\theta_i \\ a_i \sin\theta_i \\ b_i \end{bmatrix}$$

$$\Rightarrow \vec{a_1} = \begin{bmatrix} 150 \cos\theta_1 \\ 150 \sin\theta_1 \\ 450 \end{bmatrix} ; \vec{a_2} = \begin{bmatrix} 570 \cos\theta_2 \\ 570 \sin\theta_2 \\ 0 \end{bmatrix} ; \vec{a_3} = \begin{bmatrix} 200 \cos\theta_3 \\ 200 \sin\theta_3 \\ 0 \end{bmatrix}$$

$$\vec{a_4} = \begin{bmatrix} 0 \\ 0 \\ 640 \end{bmatrix} ; \vec{a_5} = \begin{bmatrix} 30 \cos\theta_5 \\ 30 \sin\theta_5 \\ 0 \end{bmatrix} ; \vec{a_6} = \begin{bmatrix} 0 \\ 0 \\ 200 \end{bmatrix}$$

Calculating Q_i

$$Q_i = \begin{bmatrix} c_i & 0 & s_i \\ s_i & 0 & -c_i \\ 0 & 1 & 0 \end{bmatrix} \qquad i = 1, 3, 4, 5$$

$$Q_j = \begin{bmatrix} c_j & -s_j & 0 \\ s_j & c_j & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad j = 2, 6$$

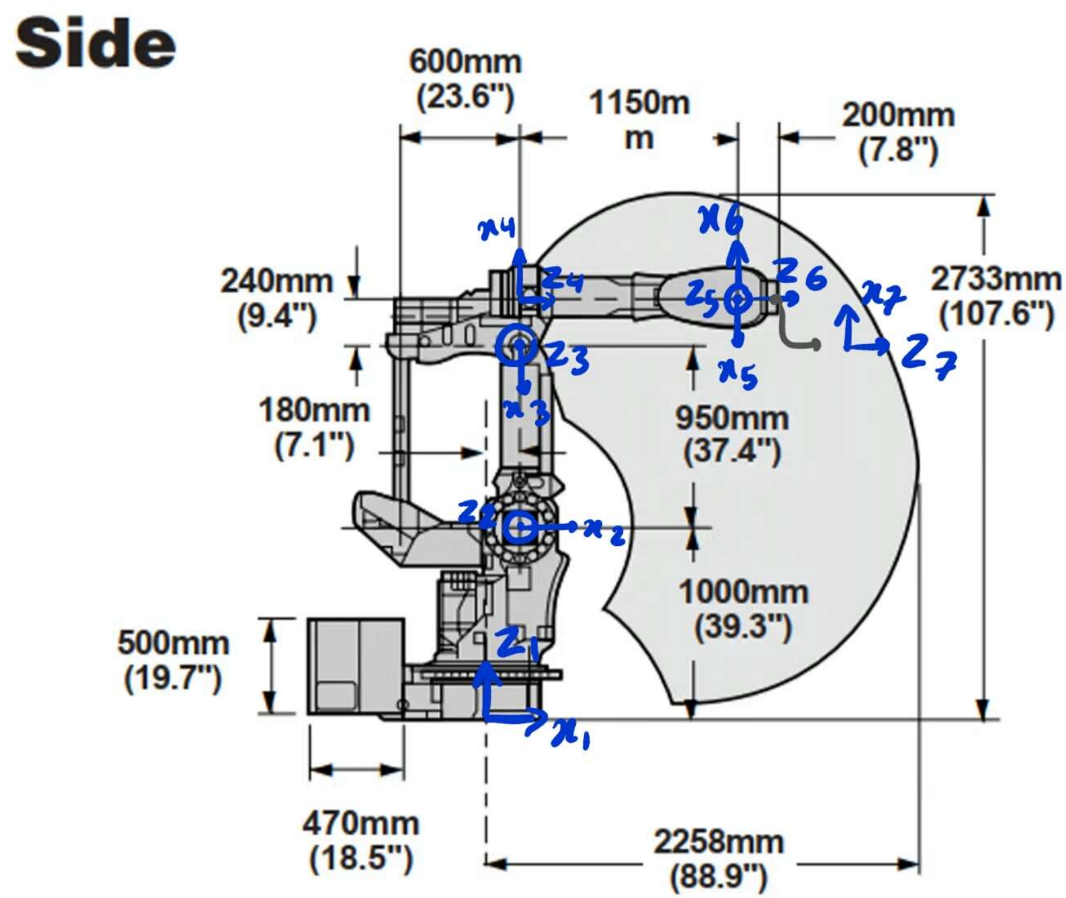
FKP

$$\vec{P} = \vec{a_1} + \underset{\sim}{Q}_1 \vec{a_2} + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \vec{a_3} + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \vec{a_4}$$

$$+ \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \vec{a_5} + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \underset{\sim}{Q}_5 \vec{a_6}$$

$$\underset{\sim}{Q} = \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \underset{\sim}{Q}_5 \underset{\sim}{Q}_6$$

problem 3 sol:



Side view

D-H parameters

i	a_i	b_i	α_i	θ_i
1	180	1000	$+\frac{\pi}{2}$	θ_1
2	950	0	0	θ_2
3	240	0	$+\frac{\pi}{2}$	θ_3
4	0	1150	$+\frac{\pi}{2}$	θ_4
5	0	0	$+\frac{\pi}{2}$	θ_5
6	0	200	0	θ_6

* All joints are revolute joint.*

Calculating $\vec{a_i}$'s

$$\vec{a_i} = \begin{bmatrix} a_i \cos\theta_i \\ a_i \sin\theta_i \\ b_i \end{bmatrix}$$

$$\Rightarrow \vec{a_1} = \begin{bmatrix} 180\, c_1 \\ 180\, s_1 \\ 1000 \end{bmatrix} ; \quad \vec{a_2} = \begin{bmatrix} 950\, c_2 \\ 950\, s_2 \\ 0 \end{bmatrix} ; \quad \vec{a_3} = \begin{bmatrix} 240\, c_3 \\ 240\, s_3 \\ 0 \end{bmatrix}$$

$$\vec{a_4} = \begin{bmatrix} 0 \\ 0 \\ 1150 \end{bmatrix} ; \quad \vec{a_5} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad \vec{a_6} = \begin{bmatrix} 0 \\ 0 \\ 200 \end{bmatrix}$$

Calculating Q_i

$$Q_i = \begin{bmatrix} c_i & 0 & s_i \\ s_i & 0 & -c_i \\ 0 & 1 & 0 \end{bmatrix}, \quad i = 1, 3, 4, 5$$

$$Q_j = \begin{bmatrix} c_j & -s_j & 0 \\ s_j & c_j & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad j = 2, 6$$

FKP:

$$\vec{P} = \vec{a}_1 + \underset{\sim}{Q}_1 \vec{a}_2 + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \vec{a}_3 + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \vec{a}_4$$

$$\begin{bmatrix}0\\0\\0\end{bmatrix} + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \vec{a}_5 + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \underset{\sim}{Q}_5 \vec{a}_6$$

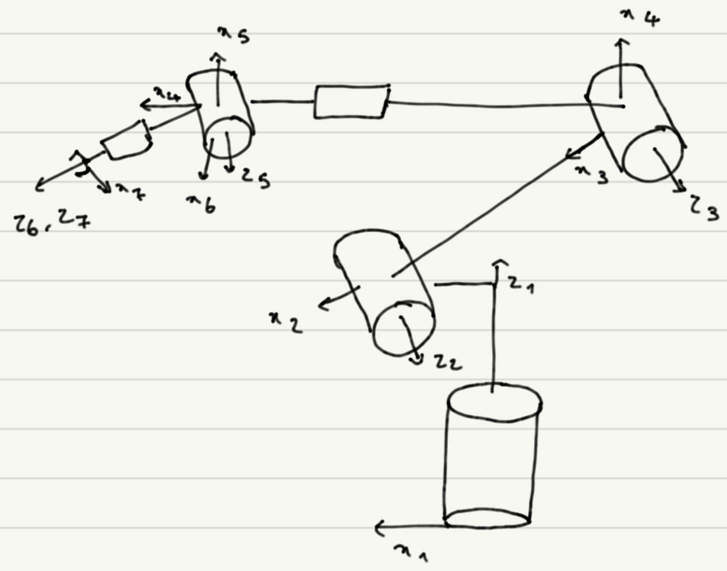
$$\underset{\sim}{Q} = \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \underset{\sim}{Q}_5 \underset{\sim}{Q}_6$$

- Is this robot decoupled? Solve the IKP for this manipulator. Find the number of answers to the IKP.

$a_4 = a_5 = b_5 = 0 \Rightarrow$ This robot is decoupled ✓

سوال ۳ – FANUC S-420i

i	a_i	b_i	α_i	θ_i
1	180	1000	$\pi/2$	θ_1
2	950	0	0	θ_2
3	240	0	$\pi/2$	θ_3
4	0	1150	$\pi/2$	θ_4
5	0	0	$\pi/2$	θ_5
6	0	200	0	θ_6



P =

$$\begin{pmatrix} 180\cos(\theta_1) + 950\cos(\theta_1)\cos(\theta_2) + 200\sin(\theta_5)\ (\sin(\theta_1)\sin(\theta_4) - \cos(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))) - 200\cos(\theta_5)\ (\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2)) - 240\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 240\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \cdots \\ 180\sin(\theta_1) + 950\cos(\theta_2)\sin(\theta_1) - 200\sin(\theta_5)\ (\cos(\theta_1)\sin(\theta_4) + \cos(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1))) - 200\cos(\theta_5)\ (\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2)) + 1150\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + 1150\cos(\theta_3)\sin(\theta_1)\sin(\theta \cdots \\ 950\sin(\theta_2) - 1150\cos(\theta_2)\cos(\theta_3) + 240\cos(\theta_2)\sin(\theta_3) + 240\cos(\theta_3)\sin(\theta_2) + 1150\sin(\theta_2)\sin(\theta_3) + 200\cos(\theta_5)\ (\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)) + 200\cos(\theta_4)\sin(\theta_5)\ (\cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\,\mathrm{si} \cdots \end{pmatrix}$$

پاسخ کامل از طریق فایل متلب در دسترس است و عکس قرار داده شده بخشی از پاسخ است.

$\vec{a}_i = \underset{\sim}{Q}_i \vec{b}_i$ $\quad$ $b_i = \begin{bmatrix} a_i \\ b_i \sin\alpha_i \\ b_i \cos\alpha_i \end{bmatrix}$ $\quad$ $\vec{u}_i = \underset{\sim}{Q}_i \vec{e}$

$\vec{e} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$u_i = \begin{bmatrix} \sin\alpha_i \sin\theta_i \\ -\sin\alpha_i \cos\theta_i \\ \cos\alpha_i \end{bmatrix}$

تا قسمت ج داریم

$\vec{C} = \vec{a}_1 + \underset{\sim}{Q}_1 \vec{a}_2 + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \vec{a}_3 + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \vec{a}_4$

$\Rightarrow \quad Q_1(\vec{C} - \vec{a}_1) = \underset{\sim}{Q}_2(\vec{b}_2 + \underset{\sim}{Q}_3 \vec{b}_3 + \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \vec{b}_4)$ (1)

$\underset{\sim}{Q}_4 \vec{b}_4 = \vec{a}_4 = \begin{bmatrix} 0 \\ 0 \\ b_4 \end{bmatrix} \rightarrow a_4 = 0$

$\vec{a}_4 = b_4 \vec{e}$

$\underset{\sim}{Q}_3 Q_4 b_4 = b_4 \underbrace{\underset{\sim}{Q}_3 \vec{e}}^{\vec{u}_3} = b_4 \vec{u}_3$

$\Rightarrow \underset{\sim}{Q}_2 (\vec{b}_2 + \underset{\sim}{Q}_3 \vec{b}_3 + b_4 \vec{u}_3) = \underset{\sim}{Q}_1^T \vec{C} - \vec{b}_1$ (2)

$[\vec{C}] = [\vec{P}]_1 + [\vec{a}_5]_1 - [\vec{a}_6]_1$

$\vec{C} = \vec{P} - \underbrace{\underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 [\vec{a}_5]_5}_{\vec{0}} - \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \underset{\sim}{Q}_5 [\vec{a}_6]_6$

$\vec{C} = P - \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \underset{\sim}{Q}_5 \vec{a}_6$ ، $\underset{\sim}{Q} \underset{\sim}{Q}_6 = \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3 \underset{\sim}{Q}_4 \underset{\sim}{Q}_5$

$\vec{C} = \vec{P} - \underset{\sim}{Q} \underset{\sim}{Q}_6^T \vec{a}_6 \Rightarrow \vec{C} = \vec{P} - \underset{\sim}{Q} \vec{b}_6$

اندازه بردارها در دو طرف معادله 2 مساوی است.

$$r_1 = (\underset{\sim}{Q}_1^T \vec{C} - b_1)^T (\underset{\sim}{Q}_1 \vec{C} - \vec{b}_1) = \vec{C}^T \underset{\sim}{Q}_1 \underset{\sim}{Q}_1^T \vec{C} - \vec{C}^T Q_1 \vec{b}_1 - \vec{b}_1^T Q_1^T \vec{C}$$

$$+ \vec{b}_1^T \vec{b}_1 = \|C\|^2 - b_1^T \underset{\sim}{Q}_1^T \vec{C} + \|b_1\|^2$$

$$\|C\| = x_c^2 + y_c^2 + z_c^2$$

$$\vec{b}_i = \begin{bmatrix} a_i \\ b_i \sin\alpha_i \\ b_i \cos\alpha_i \end{bmatrix} \qquad \|b_i\|^2 = a_i^2 + b_i^2$$

$$r_1 = x_c^2 + y_c^2 + z_c^2 + a_1^2 + b_1^2 - 2 b_1^T Q_1^T \vec{C}$$

$$r_2 = (\vec{b}_2^T + \vec{b}_3^T \underset{\rightarrow}{Q}_3^T + \vec{u}_3^T b_4)\, \underset{\sim}{Q}_2^T \underset{\sim}{Q}_2 (\vec{b}_2 + \underset{\sim}{Q}_3 \vec{b}_3 + b_4 \vec{u}_3)$$

$$r_2 = \|b_2\|^2 + \|b_3\|^2 + b_4^2 + 2\vec{b}_2^T \underset{\sim}{Q}_3 \vec{b}_3 + 2b_4 \vec{b}_2^T \vec{u}_3 + 2b_4 u_3^T Q_3 \vec{b}_3$$

$$r_1 = r_2$$

$$\Longrightarrow \quad \overset{2a_1x_c}{A} \cos\theta_1 + \overset{2a_1y_c}{B} \sin\theta_1 + C\cos\theta_3 + D\sin\theta_3 + E = 0$$

$$C = 2a_2a_3 - 2b_2b_4\mu_2\mu_3$$

$$D = 2a_3b_2\mu_2 + 2a_2b_4\mu_3$$

3

$$E = a_2^2 + a_3^2 + b_2^2 + b_3^2 + b_4^2 - a_1^2 - x_c^2 - y_c^2 - (z_c - b_1)^2 + 2b_2b_3\lambda_2$$

$$+ 2b_2b_4\lambda_2\lambda_3 + 2b_3b_4\lambda_3$$

می توان سومین بخش رابطه 2 را بصورت زیر نوشت

$$F\cos\theta_1 + G\sin\theta_1 + H\cos\theta_3 + I\sin\theta_4 + J = 0 \qquad (4)$$

$F = y_c\mu_1 \quad G = -x_c\mu_1 \quad H = -b\mu_2\mu_3 \quad I = a_3\mu_2$

$J = b_2 + b_3\lambda_2 + b_4\lambda_2\lambda_3 - (z_c - b_1)\lambda_1$

از روابط ③ و ④ می توان نتیجه گرفت

$$\overset{\cos\theta_1}{C_1} = \frac{-G(C c_3 + D s_3 + E) + B(H C_3 + I s_3 + J)}{\Delta_1} \qquad (5)$$

$$\overset{\sin\theta_1}{S_1} = \frac{F(C C_3 + D S_3 + E) - A(H C_3 + I s_3 + J)}{\Delta_1}$$

$\Delta_1 = AG - F_B = -2a_1\mu_1(x_c^2 + y_c^2)$

با استفاده از رابطه 5 :

$$K\cos^2\theta_3 + L\sin^2\theta_3 + M\cos\theta_3\sin\theta_3 + N\cos\theta_3 + P\sin\theta_3 + Q = 0 \qquad (6)$$

$K = 4a_1^2H^2 + \mu_1^2C^2 \quad , L = 4a_1^2I^2 + \mu_1^2 D^2$

$M = 2(4a_1 H I + \mu_1^2 C D) \quad , N = 2(4a_1^2 H J + \mu_1^2 C E)$

$P = 2(4a_1^2 I J + \mu_1^2 D E) \quad , Q = 4a_1^2 J^2 + \mu_1^2 E^2 - 4a_1^2\mu_1^2(x_c^2 + y_c^2)$

$\tan\left(\frac{\theta_3}{2}\right) = t \quad \rightarrow \sin\theta_3 = \frac{2t}{1+t^2} \quad , \cos\theta_3 = \frac{1-t^2}{1+t^2}$

با جایگذاری روابط تانژانت نصف کمان در (6) :

$$R t_3^4 + S t_3^3 + T t_3^3 + U t_3 + V = 0 \qquad (7)$$

$$R = 4a_1^2 (J-H)^2 + \mu_1^2 (E-C)^2 - 4(x_c^2 + y_c^2) a_1^2 \mu_1^2$$

$$S = 4\left(4a_1^2 I (J-H) + \mu_1^2 D (E-C)\right)$$

$$T = 2\left(4a_1^2 (J^2 - H^2 + 2I^2) + \mu_1^2 (E^2 - C^2 + 2D^2) - 4(x_c^2 + y_c^2) a_1^2 \mu_1^2\right)$$

$$U = 4\left[4a_1^2 I (H+J) + \mu_1^2 O (C+E)\right]$$

$$V = 4a_1^2 (J+H)^2 + \mu_1^2 (E+C)^2 - 4(x_c^2 + y_c^2) a_1^2 \mu_1^2$$

با توجه به اینکه ربات 3 لولا دارد ، θ_3 چهار جواب دارد و IKP چهار جواب دارد

Problem 4 sol.:

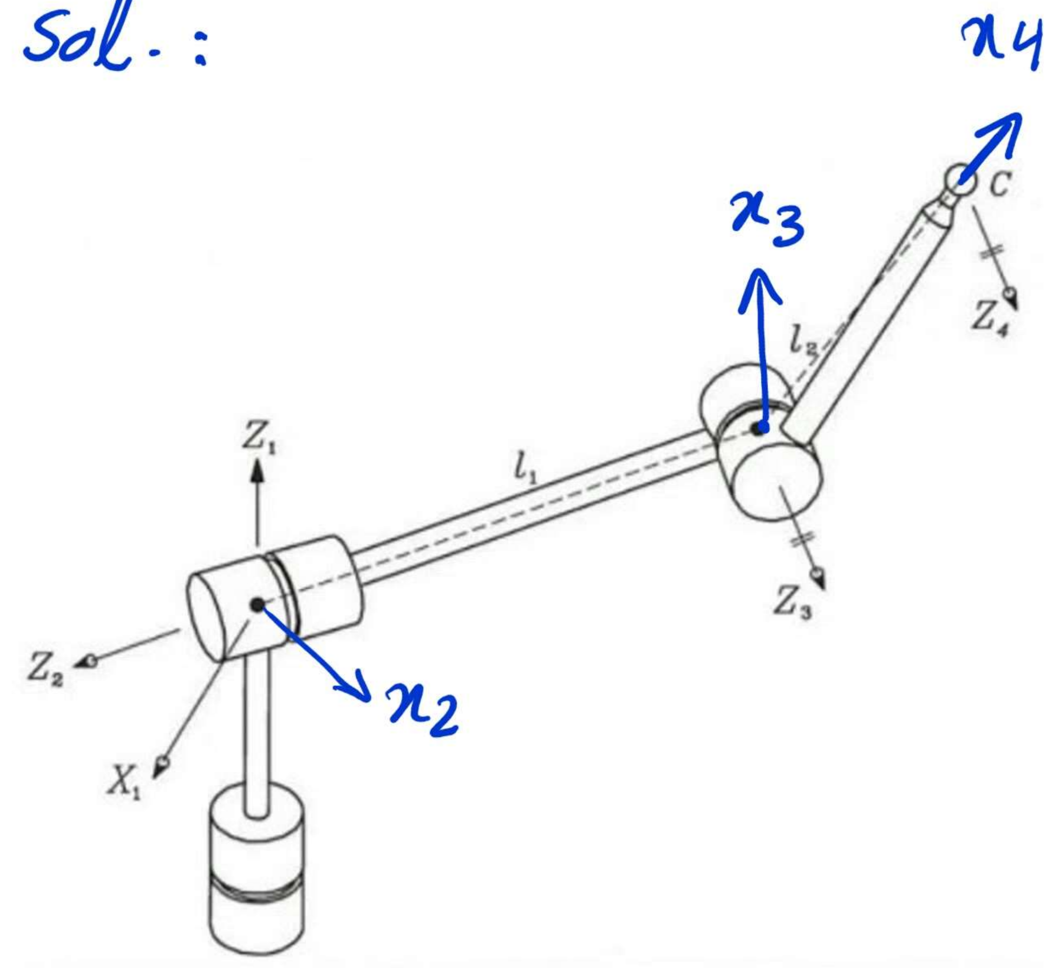


Figure 4: 3-DOF Robot

- Find the table of D-H parameters by drawing the axes corresponding to each joint.

i	a_i	b_i	α_i	θ_i
1	0	0	$+\pi/2$	θ_1
2	0	$-l_1$	$+\pi/2$	θ_2
3	l_2	0	0	θ_3

- According to the previous part, find the $\overrightarrow{a_i}$ vectors and Q_i matrices related to each link.

$$\vec{a_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad \vec{a_2} = \begin{bmatrix} 0 \\ 0 \\ -l_1 \end{bmatrix} ; \quad \vec{a_3} = \begin{bmatrix} l_2 c_3 \\ l_2 s_3 \\ 0 \end{bmatrix}$$

$$\underset{i=1,2}{\underset{\sim}{Q_i}} = \begin{bmatrix} c_i & 0 & s_i \\ s_i & 0 & -c_i \\ 0 & 1 & 0 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- According to the previous part, obtain the forward kinematics problem.

$$\vec{P} = \underbrace{\vec{a}_1}_{\begin{bmatrix}0\\0\\0\end{bmatrix}} + \underset{\sim}{Q}_1 \vec{a}_2 + \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \vec{a}_3$$

$$\underset{\sim}{Q} = \underset{\sim}{Q}_1 \underset{\sim}{Q}_2 \underset{\sim}{Q}_3$$

$$\vec{P} = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -l_1 \end{bmatrix} + \overbrace{\begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 \\ s_2 & 0 & -c_2 \\ 0 & 1 & 0 \end{bmatrix}}^{\begin{bmatrix} c_1c_2 & s_1 & c_1s_2 \\ s_1c_2 & -c_1 & s_1s_2 \\ s_2 & 0 & -c_2 \end{bmatrix}} \begin{bmatrix} l_2c_3 \\ l_2s_3 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{Q} = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 \\ s_2 & 0 & -c_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

MATLAB

$$\Longrightarrow \quad \vec{P} = \begin{bmatrix} l_2 s_1 s_3 - l_1 s_1 + l_2 c_1 c_2 c_3 \\ l_1 c_1 - l_2 c_1 s_3 + l_2 c_2 c_3 s_1 \\ l_2 c_3 s_2 \end{bmatrix}$$

$$Q = \begin{bmatrix} s_1 s_3 + c_1 c_2 c_3 & c_3 s_1 - c_1 c_2 s_3 & c_1 s_2 \\ c_2 c_3 s_1 - c_1 s_3 & -c_1 c_3 - c_2 s_1 s_3 & s_1 s_2 \\ c_3 s_2 & -s_2 s_3 & -c_2 \end{bmatrix}$$

- According to the above part, solve the inverse kinematics problem for θ_1 and θ_3 by referring to the two equations that are solved to obtain the value of these joints. (It means two equations in which there is no θ_2 term and it is possible to calculate the indicated values by forming a system of equations)

$$\begin{cases} x = l_2 s_1 s_3 - l_1 s_1 + l_2 c_1 \underline{\underline{c_2}} c_3 \\ y = l_1 c_1 - l_2 c_1 s_3 + l_2 \underline{\underline{c_2}} c_3 s_1 \\ z = l_2 c_3 s_2 \end{cases}$$

$$\frac{x - l_2 s_1 s_3 + l_1 s_1}{y - l_1 c_1 + l_2 c_1 s_3} = \frac{\cancel{l_2} c_1 \cancel{c_3} \cancel{c_2}}{\cancel{l_2} \cancel{c_3} s_1 \cancel{c_2}}$$

$$\begin{cases} \dfrac{x - l_2 s_1 s_3 + l_1 s_1}{y - l_1 c_1 + l_2 c_1 s_3} = \dfrac{c_1}{s_1} \\[2ex] \underbrace{\left(\dfrac{z}{l_2 c_3}\right)^2}_{} + \underbrace{\left(\dfrac{x - l_2 s_1 s_3 + l_1 s_1}{l_2 c_1 c_3}\right)^2}_{} = 1 \end{cases}$$

$$(\sin\theta_2)^2 + (\cos\theta_2)^2 = 1 \leftarrow \text{اتحاد مثلثاتی}$$