University of Tehran

College of Engineering

School of Electrical and Computer Engineering (ECE)

School of Mechanical Engineering (ME)

Mechatronics & Robotics

Homework 1

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Deadline: 12 March 2024 (22 esfand), 23:59

List of Problems

Problem 1: (Fixed-based Robots) 5 points

Describe parallel and serial robots and their advantages and disadvantages.

Problem 2: (Mobile-based Robots) 5 points

You have seen NAO in your tour in the human and robot interaction lab. (Fig-1)

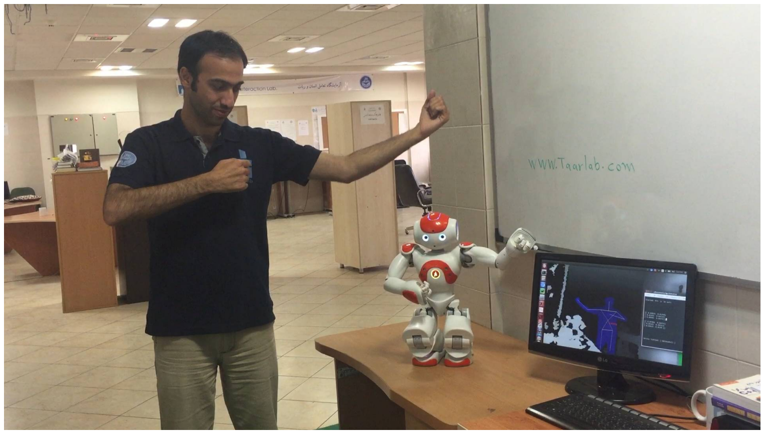

Figure 1: NAO and one of the members of human and robot interaction lab - immitation project

NAO is a humanoid robot developed by SoftBank Robotics. It stands at around 58 cm tall and is equipped with a variety of sensors, cameras, microphones, and other hardware components. NAO's design allows it to perform a wide range of tasks, making it suitable for various applications such as education, research, and entertainment.

Additionally, NAO's capabilities extend beyond education to research and development. Its sensors and actuators allow researchers to explore topics such as human-robot interaction, artificial intelligence, and machine learning. NAO has also been used in healthcare settings, assisting therapists in rehabilitation exercises and providing companionship to patients.

Before answering to the questions below, watch this video to get to know NAO and its abilities better!

Now answer to the questions below, using the internet and the NAO humanoid documentations.

1. Discuss the key features and capabilities of the Nao humanoid robot and how they contribute to its utility and versatility in various robotics applications.

2. Write down how many degrees of freedom does NAO humanoid have?

3. How do series and parallel joint configurations in the Nao humanoid robot contribute to its functionality? Compare and contrast the types of joints used in Nao's design, highlighting their advantages and applications in robotics.

Problem 3: (Mechanical joints) 5 points

Given the uploaded video in the group and the elearn system, please explain which joints of the human body mentioned in the video correspond to which mechanical joints. Explain your answers by stating the shape of each joint and its mechanical equivalent.

Figure 2: Human body

Problem 4: (Mathematical background) 8 points

1. Let $\mathbf{u}$ and $\mathbf{v}$ be any 3-dimensional vectors, and define $\mathbf{T}$ as

$$\mathbf{T} \equiv \mathbf{1} + \mathbf{u}\mathbf{v}^T$$

The (unit) eigenvectors of $\mathbf{T}$ are denoted by $\mathbf{w}_1, \mathbf{w}_2$, and $\mathbf{w}_3$. Show that, say, $\mathbf{w}_1$ and $\mathbf{w}_2$ are any unit vectors perpendicular to $\mathbf{v}$ and different from each other, whereas $\mathbf{w}_3 = \mathbf{u}/\|\mathbf{u}\|$. Also show that the corresponding eigenvalues, denoted by λ_1, λ_2, and λ_3, associated with $\mathbf{w}_1, \mathbf{w}_2$, and $\mathbf{w}_3$, respectively, are given as

$$\lambda_1 = \lambda_2 = 1, \quad \lambda_3 = 1 + \mathbf{u} \cdot \mathbf{v}$$

2. Show that if $\mathbf{u}$ and $\mathbf{v}$ are any 3-dimensional vectors, then

$$\det\left(\mathbf{1} + \mathbf{u}\mathbf{v}^T\right) = 1 + \mathbf{u} \cdot \mathbf{v}$$

Problem 5: (Yaw, pitch and roll) 10 points

Given a fixed-axis rotation scenario where a disk has been rotated along the x, y, and z-axes respectively, the provided rotation matrix represents the resulting rotation. Your task is to determine the yaw, pitch, and roll angles based on this final rotation matrix.

$$\mathbf{Q} = \begin{bmatrix} \frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{4} + \frac{3}{8} & \frac{1}{4} + \frac{3\sqrt{3}}{8} \\ \frac{1}{4} & \frac{3}{4} + \frac{\sqrt{3}}{8} & \frac{-\sqrt{3}}{4} + \frac{3}{8} \\ \frac{-\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} \end{bmatrix}$$

Problem 6: (Plücker line) 12 points

1. Given two points P and Q in 3-dimensional space with coordinates (1,2,3) and (4,5,6) respectively, derive two Plücker line representations for the line passing through these points.
2. Find the transformation matrix for transforming the first representation to the second one.
3. Assess the accuracy of your response from the preceding section by placing two representations and the matrix you found in the transforming equation.

Problem 7: (Rotation) 8 points

Figure 3 shows a plate which needs to be upgraded from an initial configuration ('flat on the surface')the lower surface) has a final configuration (resting on an inclined plane) by a manipulator. Working in a reference system i,j, k (shown in the figure), determine:

1. The matrix representation of the rotation undergone by the plate during this operation.

   Hint: You can imagine the rotation along the z-axis happened first, then y-axis.(ZY) But its okay to consider any other rotations, just provide the order you considered in your report.

2. The unit vector along the axis of rotation and the angle of rotation.

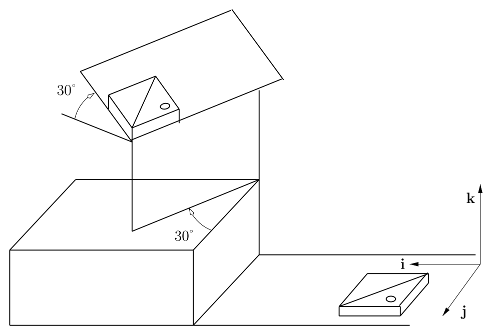

Figure 3: Question 7 system

Problem 8: (Rotation of a Rigid Body) 12 points

A rigid body rotates around a fixed point. The rotation matrix Q giving the orientation of the body at all times is given by

$$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} c^2 - \frac{1}{3}s^2 & \frac{2}{3}s^2 - \frac{2\sqrt{3}}{3}sc & \frac{2}{3}s^2 + \frac{2\sqrt{3}}{3}sc \\ \frac{2}{3}s^2 + \frac{2\sqrt{3}}{3}sc & c^2 - \frac{1}{3}s^2 & \frac{2}{3}s^2 - \frac{2\sqrt{3}}{3}sc \\ \frac{2}{3}s^2 - \frac{2\sqrt{3}}{3}sc & \frac{2}{3}s^2 + \frac{2\sqrt{3}}{3}sc & c^2 - \frac{1}{3}s^2 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\phi = 2K\pi$

t_3 $\phi = \pi, 3\pi, \ldots$

$$s \equiv \sin\left(\frac{\alpha t}{2}\right), \quad c \equiv \cos\left(\frac{\alpha t}{2}\right)$$

t_2

where α is a constant value and t stands for time (s).

1. Find a General Expression for the Quadratic Invariants of Rotation, as a Function of time.
2. Give a physical interpretation for the rotation in question.
3. Calculate the quadratic invariants of rotation at instants:

$$t_1 = \frac{\pi}{2\alpha}, t_2 = \frac{2\pi}{\alpha} \text{ et } t_3 = \frac{\pi}{\alpha}.$$

Problem 9: (Rotation matrix properties) 8 points

Given the following 3×3 matrix:

$$\mathbf{R} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}$$

1. Show that it is a rotation matrix
2. Determine a unit vector that defines the axis of rotation and the angle (in degrees) of rotation.
3. What are the Euler parameters representing R?

Problem 10: (Aircraft rotation calculations) 15 + 5 points

In order to determine the orientation of an aircraft during a flight, telemetry devices located on the nose of the aircraft and at the tips of the wings are used. points A, B and C in Figure-4. The positions of the sensors are given in the figure, as well as the reference mark attached to the plane.

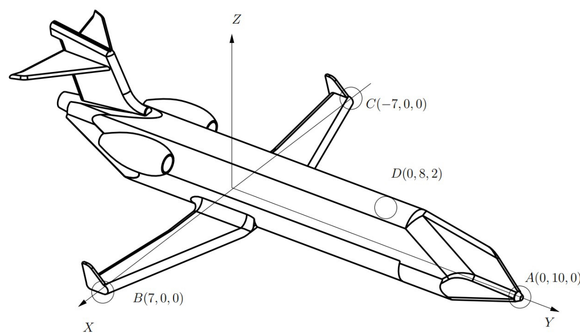

Figure 4: Plane coordinates

1. A few seconds after takeoff, we measure the position of points A, B and C and we find:
$$\mathbf{a} = \frac{1}{4}\begin{bmatrix} 390 \\ 4030 - 5\sqrt{3} \\ 1615 + 10\sqrt{3} \end{bmatrix}, \mathbf{b} = \frac{1}{8}\begin{bmatrix} 800 + 14\sqrt{3} \\ 8021 + 14\sqrt{3} \\ 3214 - 21\sqrt{3} \end{bmatrix} \quad \text{et} \quad \mathbf{c} = \frac{1}{8}\begin{bmatrix} 800 - 14\sqrt{3} \\ 7979 - 14\sqrt{3} \\ 3186 + 21\sqrt{3} \end{bmatrix}$$
From these data, determine the rotation matrix corresponding to the orientation of the plane (at this moment) in relation to the fixed reference frame, in which the data is provided.

2. The plane's ground control station uses a triplet of Euler angles to represent the orientation of the plane. The convention used corresponds to a first rotation of a angle ψ around the X axis, then an angle θ around the Y axis, then an angle ϕ around of the Z axis (each time in the local reference frame). Give the general expression of the matrix of global rotation as a function of angles ψ, θ and ϕ.

3. For the data provided above, give the value of the corresponding triplet of Euler angles to the orientation of the plane.

4. (Bonus) If we used a fourth beacon located at point D, what would it indicate when the beacons A, B and C indicate the data provided above?

Problem 11: (Rotation matrices) 12 + 10 points

Coding exercise

Write three functions for the questions below, then test your functions with the given rotation matrix as their input.

1. Write a function for calculating natural invariant parameters $(\vec{e}, \phi)$from the rotation matrix.
2. Write a function for calculating natural eulor parameters $(q_1, q_2, q_3, q_0)$from the rotation matrix.
3. Write a function for calculating natural euler rodrigues parameters $(\vec{r}, r_0)$from the rotation matrix.
4. (Bonus) In the context of robotics and rotation representations, a singularity refers to a special condition where a certain representation or computation becomes undefined or numerically unstable. Describe when does that happen in the euler rodrigues representation and why. Your task is to implement a function that converts Euler-Rodrigues parameters to a rotation matrix. However, in this advanced version, you need to handle singularities that may arise. Then test this function with two inputs:
   (a) One case where singularity happens.
   (b) One case where singularity does not happen.

Write the mathematics and logic behind your code in your report.

Homework Guidelines and Instructions

- The deadline for sending this exercise will be until the end of Tuesday, March 12.
- This time cannot be extended and you can use time grace if needed.
- The implementation must be in Python or Matlab programming language and your codes must be executable and uploaded along with the report.
- This exercise is done by one person.
- If any similarity is observed in the work report or implementation codes, this will be considered as fraud for the parties.
- Using ready-made codes without mentioning the source and without changing them will constitute cheating and your practice score will be considered zero.
- If you do not follow the format of the work report, you will not be awarded the grade of the report.
- Handwritten exercise delivery are acceptable as long as they are legible and clean.
- Please note that for the code exercise you MUST upload your code files and write a report for it as well. If one is missing, your answer won't be graded.
- A large part of your grade is related to the work report and problem solving process.
- Please upload the report, code file and other required attachments in the following format in the system: `HW1_[Lastname]_[StudentNumber].zip`
  For example, the: `HW1_Ezati_12345678.zip`
- If you have questions or doubts, you can contact the assistants through the following e-mail with the subject HW1-Mechatronics. Stay in touch educationally:
  - `r.yourdkhani@gmail.com` (Reihaneh Yourdkhani)
- Be happy and healthy