Simulation And Analysis Of Thermionic Emission

by

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Certificate

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This is to certify that the project report entitled **"Simulation and Analysis of Thermionic Emission"** submitted as Project 2 at the Department of Physics and Astronomy, University of Kansas, Lawrence, for the Computational Physics course (PHSX 815) is carried out by Mr. Mohammad Ful Hossain Seikh under my instruction and guidance.

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Abstract

Study of Thermionic Emission using Poisson Distribution will help us understand the dependence of probability of emission on its rate. Besides the discussion on the hypotheses testing, this report gives a statistical analysis of the data obtained using them. It further discusses the probabilities obtained by selecting the rate paramters from a Gaussian distribution.

By studying and comparing the histograms and plots, it can be concluded that given the confidence level, $(1-\alpha)\times 100\%$, the power of the test, β, and critical value of rate parameter, λ_c depend on the distribution and its rate. This project report can serve as a good study material for the beginners of Python3.

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Chapter 1

Introduction

Poisson Distribution [1] (named after the French mathematician Sim$e$on Denis Poisson) is a discrete probability distribution that gives the probability of a given number of events occurring in a fixed interval of time (or space) if these events occur with a known constant mean rate and independently of the time since the last event. For example: the number of meteorites greater than 5 meters diameter that strike Earth in a year, the number of patients arriving in an emergency room between 1 and 2 pm, the rate of decay of radioactive nuclei, the number of laser photons hitting a detector in a particular time interval, the rate of thermionic emission of a particular metal surface and so on.

For this project, I shall discuss the case of thermionic emission from a particular metal surface. Suppose, from the theoretical studies, we know that the rate of emission of electrons is λ_1 but in the experiments it is found to be λ_2. We can develop two hypotheses using these two rates in the Poisson distribution. To make it a little more realistic, we can think of those rates to be dynamic i.e. rates (Fig. 1.1) are the pieces of another distribution (here I have used Gaussian distribution in my code [2]).

The code that I wrote for this purpose has a function named normal() which has a form:

normal $(\mathrm{x},\, \mu,\, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

with a number of random points n = 1000. The function is then used to get the distribution of rates (λ_1: $\mu_1 = 10.0$, $\sigma_1 = 1.5$ and λ_2: $\mu_2 = 20.0$, $\sigma_2 = 5.0$) and among those rates, two numbers are chosen randomly and used in Poisson ditribution which has the form:

Poisson $(\mathrm{k},\, \lambda(x, \mu, \sigma)) = \frac{\lambda^k e^{-\lambda}}{k!}$

The histograms for Poisson Distribution are shown in Fig 1.2.

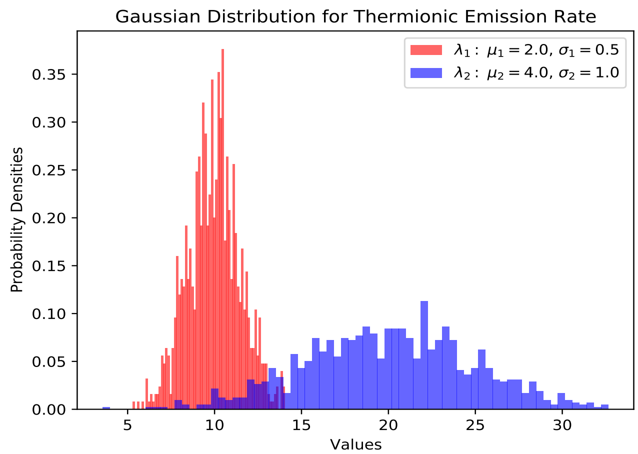

Figure 1.1: An example of histograms of Gaussian Distribution of thermionic emission rates. Red one is for rate 1, λ_1 and Blue is for rate 2, λ_2.

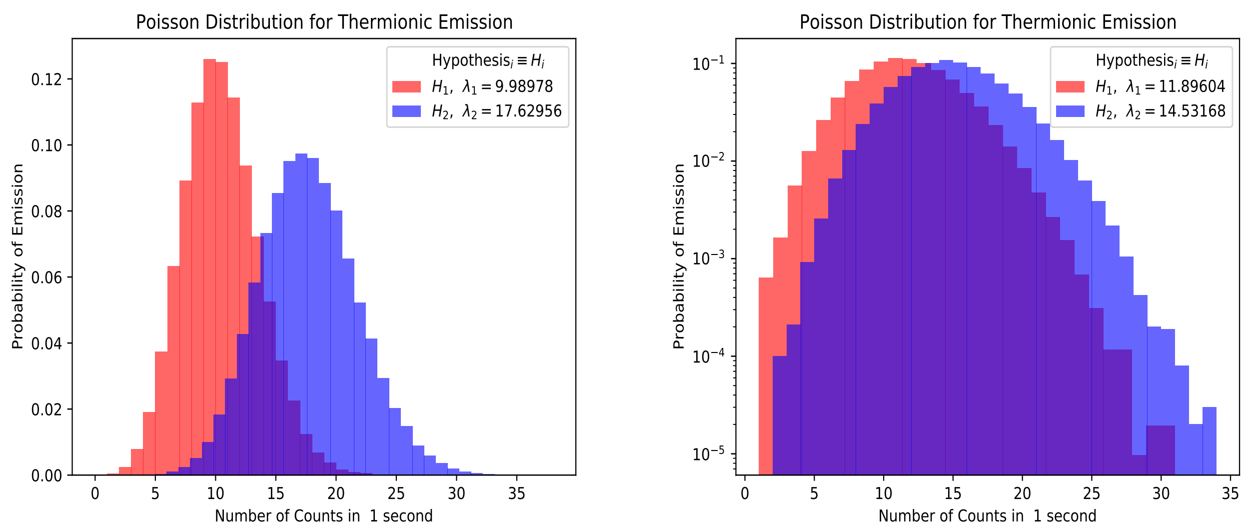

Figure 1.2: Histograms of Poisson Distribution for thermionic emission (**Left**: Normal scale and **Right**: Log scale). Code is given in [2]

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Chapter 2

Code and Algorithm Analysis

For this project, I have written 3 different codes: code for the distribution of rate and distribution of thermionic emission ([2]), likelihood of the two hypotheses ([3]) and log likelihood ratio for two different hypotheses ([3]). Of course the number of codes could be reduced to one but I separted the codes for clarity and better understanding.

```
 emission_number = 100000
mu, sigma, n = 2.0, 0.5, 1000
def normal(x, mu, sigma):
    return (2.0*np.pi*sigma**2.0)**-0.5 * np.exp(-0.5 * (x-mu)**2 / sigma**2.0)

emission_constant1 = np.random.normal(10.0, 1.5, 1000)
rate1 = np.random.choice(emission_constant1)
dist1 = random.poisson(rate1, emission_number)
np.savetxt("dist1.txt", dist1, fmt = '%u')


emission_constant2 = np.random.normal(20.0, 5.0, 1000)
rate2 = np.random.choice(emission_constant2)
dist2 = random.poisson(rate2, emission_number)
np.savetxt("dist2.txt", dist2, fmt = '%u')
```

Figure 2.1: Algorithm for Poisson distribution which takes arguments from Gaussian distribution. Complete code is in ([2]).

Above figure (Fig. 2.1) is a piece of code for the Poisson distribution which takes arguments from Gaussian distribution. Here emission_constant i (i = 1, 2) are the distribution of rates i.e.

$$\lambda(x) = \lambda(x|\mu, \sigma)$$

is a normal distribution and dist i (i = 1, 2) are thermionic emission given by Poisson distribution:

$$P(k) = P(k|\lambda) = P(k|\lambda(x|\mu, \sigma))$$

```
k = 0
Liklihood1, Liklihood2 = 0, 0
H1_LLR = []
H2_LLR = []

probability1 = []
probability2 = []

for k in range(0, len(hypothesis1)):

    Liklihood1 = (np.exp( - rate1))*(rate1**hypothesis1[k])/np.math.factorial(hypothesis1[k])
    probability1.append(Liklihood1)
    Liklihood2 = (np.exp( - rate2))*(rate2**hypothesis1[k])/np.math.factorial(hypothesis1[k])
    probability2.append(Liklihood1)

    H1_LLR.append(np.log10(Liklihood1/Liklihood2))
    H2_LLR.append(np.log10(Liklihood2/Liklihood1))

w1 = np.ones_like(H1_LLR) / len(H1_LLR)
w2 = np.ones_like(H2_LLR) / len(H2_LLR)
```

Figure 2.2: Algorithm for Log Likelihood Ratio (LLR) for two hypotheses. Complete code is in ([3]).

Above figure (Fig. 2.2) is a piece of code for the calculation of Log Likelihood Ratio (LLR) for two hypotheses. It is given as:

$$\mathsf{H1\_LLR} = \log\left(\frac{\mathcal{L}_{H_1}}{\mathcal{L}_{H_2}}\right) \qquad \text{and} \qquad \mathsf{H2\_LLR} = \log\left(\frac{\mathcal{L}_{H_2}}{\mathcal{L}_{H_1}}\right)$$

Also, wi ($\mathsf{i} = 1, 2$) are the weights of the LLRs for hypothesis 1 and hypothesis 2.

Chapter 3

Output Interpretation

3.1 Likelihoods

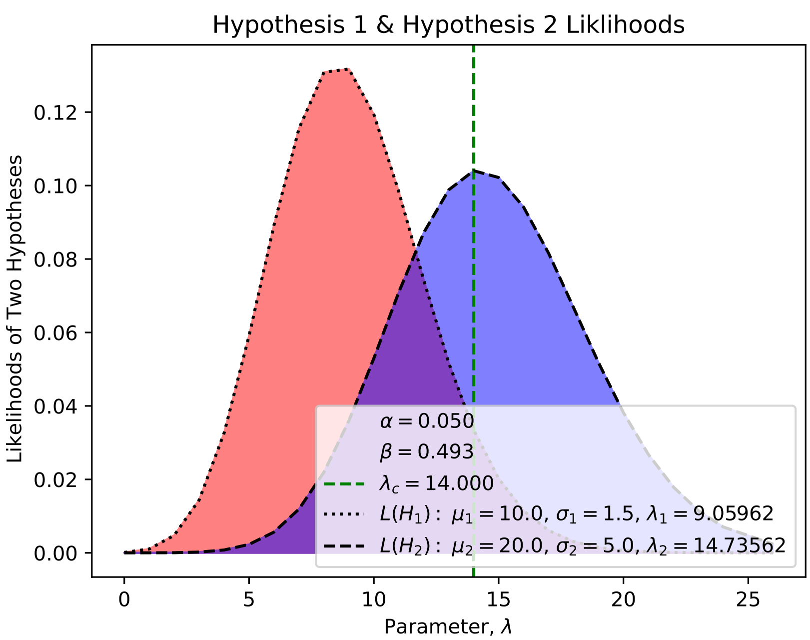

Figure 3.1: Likelihoods of Hypothesis 1 and Hypothesis 2. Complete code is in ([3]).

The confidence level for the test is 95% and the power of the test is $\beta = 0.493$. Also, we can see that the critical value is $\lambda_c = 14.0$, rate for hypothesis 1 (rate given by theory), $\lambda_1 = 9.05962$ and rate for hypothesis 2 (rate calculated in experiments), $\lambda_2 = 14.73562$. As $\lambda_c < \lambda_2$, hypothesis 2 can be rejected with 95% CL.

A few other likelihood plots are shown in Fig: 3.2 below:

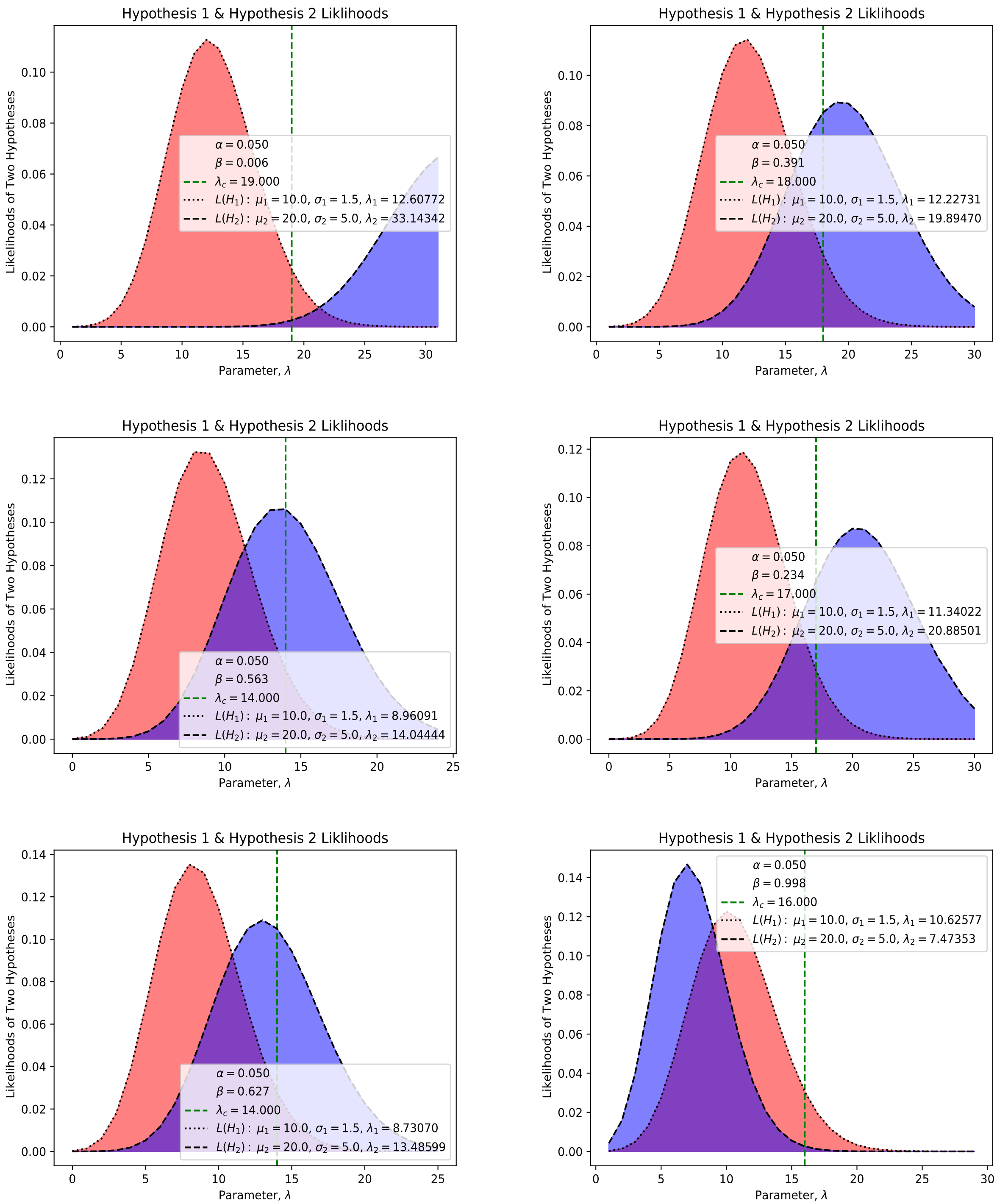

Figure 3.2: Likelihoods of Hypothesis 1 and Hypothesis 2. It can be noticed that the last row plots actually favor both the hypotheses. Complete code is in ([3]).

3.2 Log Likelihood Ratio (LLR)

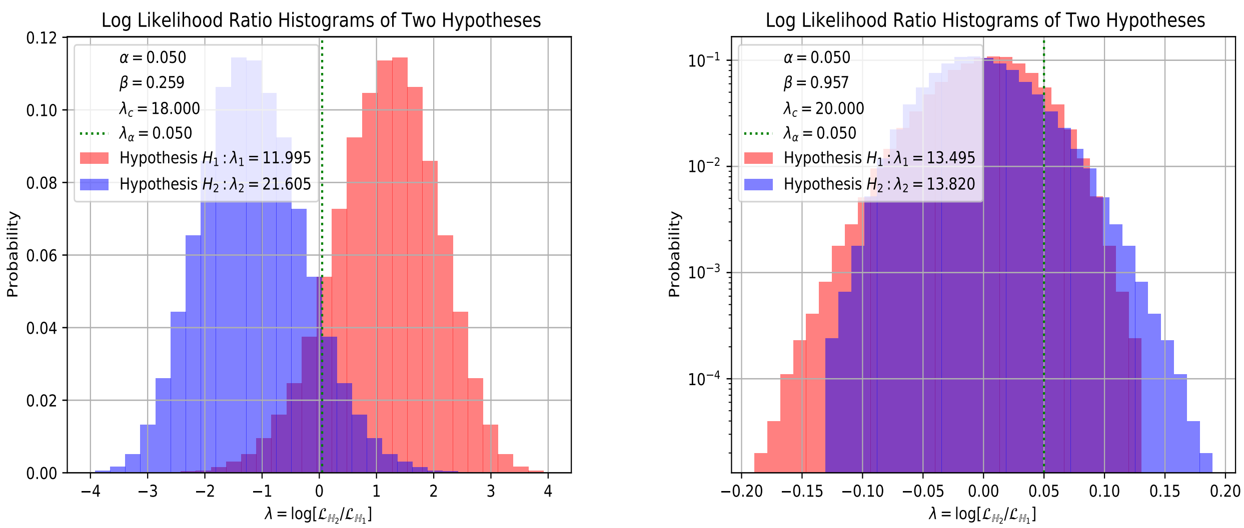

Figure 3.3: Histograms of Log Likelihood Ratio of two hypotheses (**Left**: Normal scale and **Right**: Log scale). Code is given in [3]
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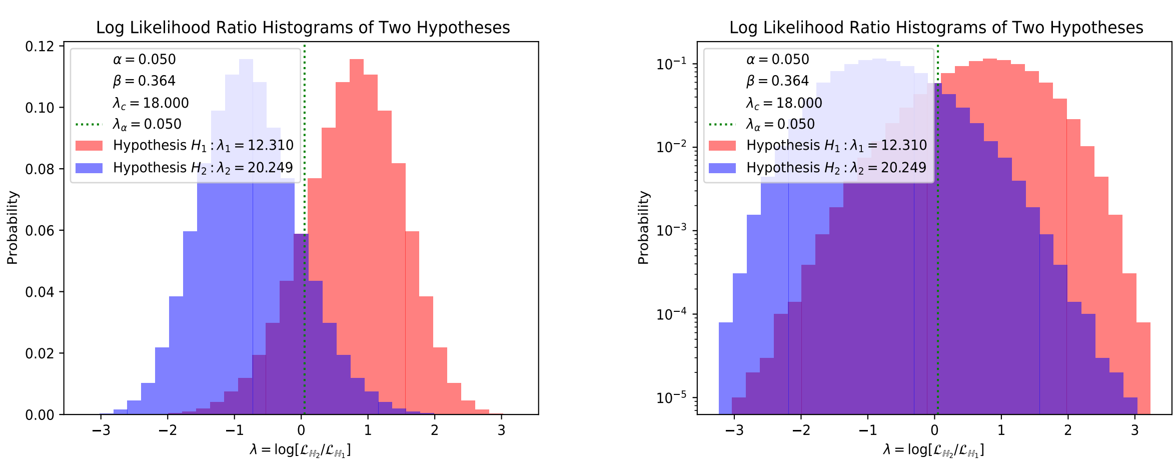
In the above figure (Fig: 3.3), the left LLR plots supports hypothesis 1 whereas the right one supports both the hypotheses with 95% CL. Using more values of rate from the Normal distribution, we can show a few more LLR plots in normal scale and in log scale below (Fig: 3.4): beginfigure[H]



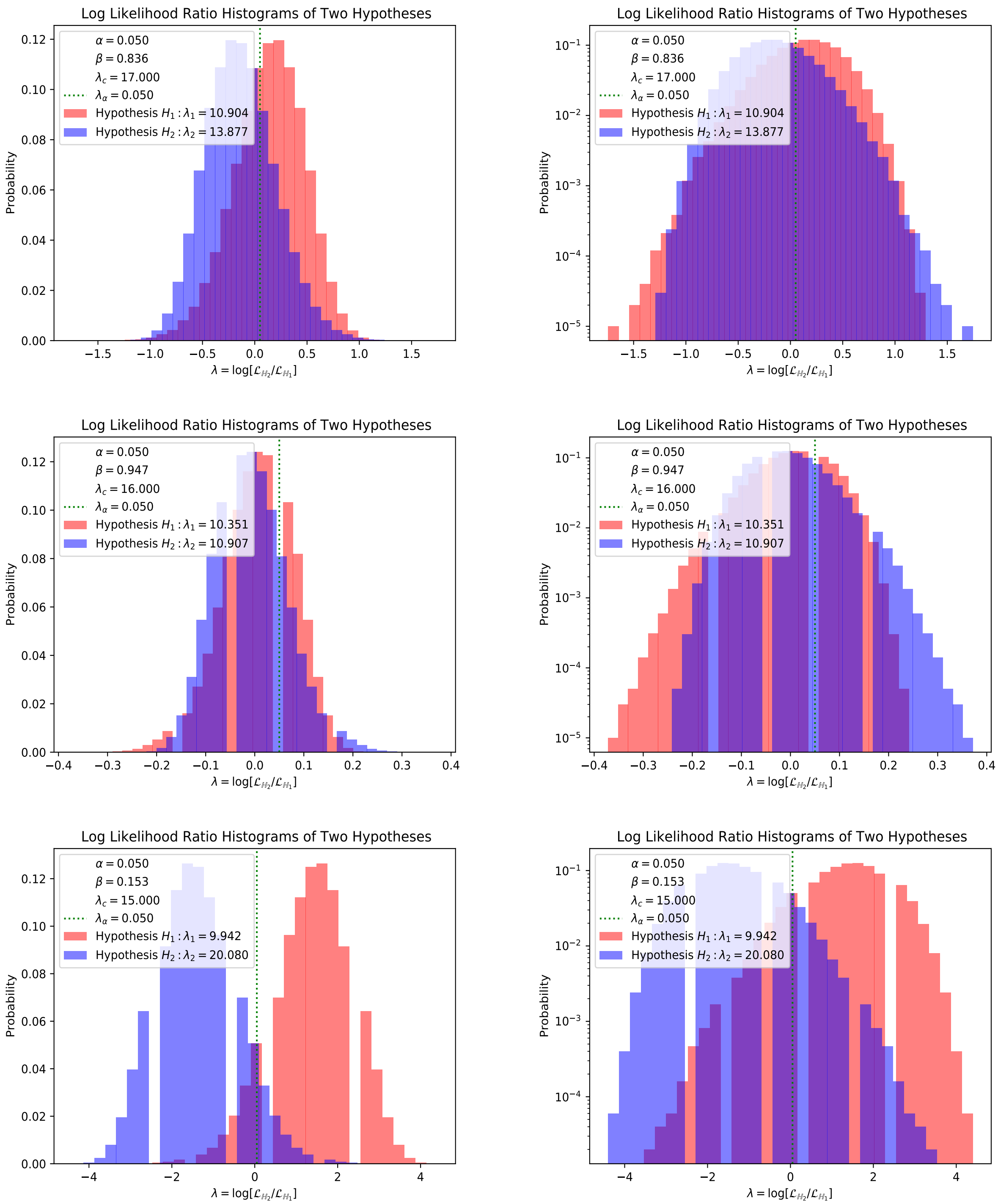


Figure 3.4: More Histograms of Log Likelihood Ratio (Left: Normal scale and Right: Log scale). The rates are randomly chosen from Gaussian distribution. Code is given in [3].

Bibliography

[1] https://en.wikipedia.org/wiki/Poisson_distribution

[2] https://github.com/Mohammad-Neutrino/PHSX815_Project2/blob/main/Thermionic_Emission.py

[3] https://github.com/Mohammad-Neutrino/PHSX815_Project2/blob/main/Liklihood.py

[4] https://github.com/Mohammad-Neutrino/PHSX815_Project2/blob/main/LLR.py