Number Uncertainties In β^+ Decay Of Carbon10 Nuclei

by

Mohammad Ful Hossain Seikh

class instructor

Professor Christopher Rogan

Submitted as Project 4
for the course of Computational Physics (PHSX 815)

at the

Department of Physics and Astronomy
UNIVERSITY OF KANSAS, LAWRENCE, KS

May 2021

Certificate

Dr. Christopher Rogan
Professor of Physics and Astronomy

This is to certify that the project report entitled **"Number Uncertainties In β^+ Decay Of Carbon10 Nuclei"** submitted as Project 4 at the Department of Physics and Astronomy, University of Kansas, Lawrence, for the Computational Physics course (PHSX 815) is carried out by Mr. Mohammad Ful Hossain Seikh under my instruction and guidance.

@ The Department of Physics and Astronomy
University of Kansas, Lawrence
Kansas, 66045, USA

Certified by . . . .

Christopher Rogan
Class Instructor

Author . . . .

Mohammad Ful Hossain Seikh
First Year Graduate Student

Abstract

Study of radioactive beta decay in proton rich ^{10}C nuclei will give us a good understanding of uncertainties in the number of radioactive nuceli and its effect on the decay distributions. Besides the discussion on two hypotheses; rate of pure ^{10}C nuclei and mixed (^{10}C & ^{19}Ne) nuclei, this report gives a statistical analysis of the data obtained using them. It further discusses the differences between theoretical and simulated decay distributions.

By studying and comparing the scatter plots, histograms and plots, it can be concluded that the number uncertainties between theory and simulation do not depend on the number of radioactive nuclei taken. This project report can serve as a good study material for the beginners of Matplotlib and Numpy libraries in Python.

Mohammad Ful Hossain Seikh
Department of Physics and Astronomy, KU
fulhossain@ku.edu

Acknowledgment

I would like to express my profound gratitude to Professor Christopher Rogan for his wonderful lectures in the course. The report presented here would not have been possible without the help of my dear friend, Aafiya who suggested me important changes in a few codes that I wrote. Special thanks to my friend and TA of this course, Margaret Lazarovits who gave a few lectrures on Machine Learning and AI ethics. I would also like to extend my sincere thanks to the donors of Carl and Catherine Chaffee Award which supported me financially for the session 2020 - 2021.

Mohammad Ful Hossain Seikh
Department of Physics and Astronomy, KU

Contents

List of Figures

Chapter 1

Introduction

The fundamental law of radioactive decay is based on the fact that the decay, i.e. the transition of a parent nucleus to a daughter nucleus is a purely statistical process. The decay probability is a property of an atomic nucleus and remains equal in time. Mathematically this law is expressed as:

$$-\frac{dN}{dt} = \lambda N \tag{1.1}$$

where N is the number of undecayed radioactive nuclei and λ is the decay constant defined as the probability of decay per nucleus per unit of time. These can be written as:

$$N = N_0 e^{-\lambda t}, \qquad \lambda = \frac{ln(2)}{t_{1/2}} \tag{1.2}$$

where N_0 is the total number of radioactive nuclei at time t = 0 (before the decay initiates) and $t_{1/2}$ is the half life of it (time at which number of radioactive nuclei become $N_0/2$). As the parent nuclei decays, daughter nuclei forms and the expression for the number of daughter nuclei is:

$$N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 \left(1 - e^{-\lambda t}\right) \tag{1.3}$$

For this project, I have considered the radioactive positron decay of proton rich ^{10}C nuclei given by the following:

$$^{10}C \rightarrow {}^{10}B + e^{+} + \nu_e \tag{1.4}$$

This isotope of carbon is not natural and is available in trace amount. Back in 1962, the half lives of ^{10}C and ^{19}Ne were not well known [1]. Only two measurements of the half-life of carbon-10 have been reported; the first of 8.8 sec [2] was in error due to an impurity in the boron powder used; the second, due to Sherr et al. [3], was 19.1 $\pm$ 0.8 sec. In the case of neon-19, values ranging from 17.4$\pm$0.2 sec [4] up to 20.3$\pm$0.5 sec. have been reported.

Later the dead-time corrected data were fit and yielded a half-life of 19.3009 $\pm$ 0017 sec [5]. Considering the lifetime of [5], the daughter and parent nuclei are plotted in the following Figure 1.1.

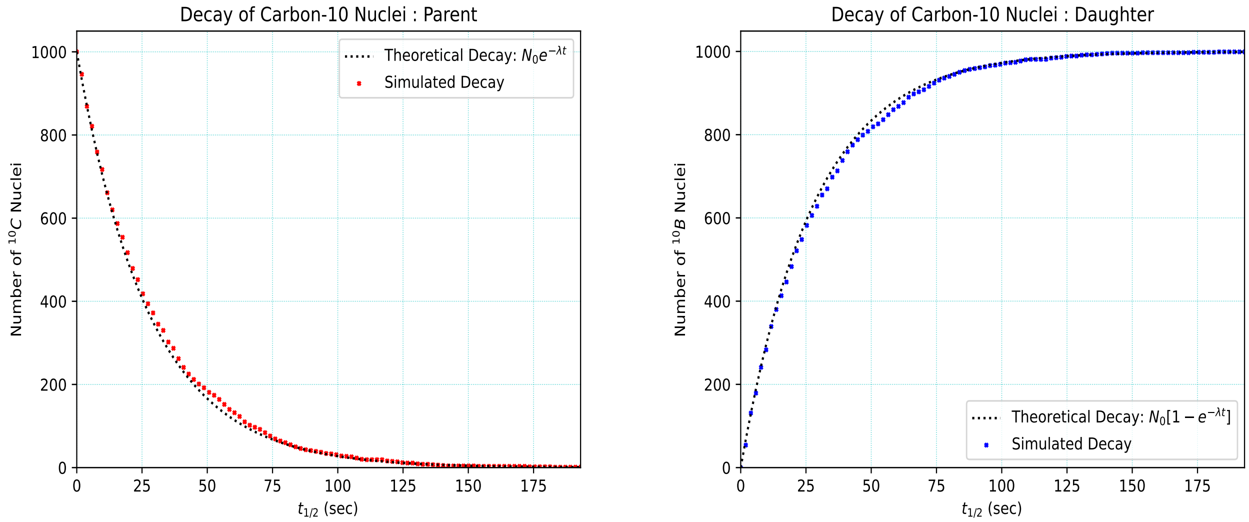

Figure 1.1: Simulated (crossed points) and Theoretical (dashed line) decays of ^{10}C nuclei. Left: Parent Nuclei & Right: Daughter Nuclei. Code is given in [6].

Chapter 2

Code and Algorithm Analysis

For this project, I have written three codes [6][7][8] which give the daughter and parent nuclei decay distributions, errors in simulated and theoretical counts, likelihoods and log likelihood ratio of the two hypetheses.

For the algorithm, random numbers (say z) between 0 and 1 are created for the simulation of radioactive decay. The transformation $x = -\frac{1}{\lambda}\log(1-z)$ gives us $z = 1 - e^{-\lambda x}$. Then in the defined function, $-\frac{1}{\lambda}\log(1-z)$ is returned which is later used for different decay rates.

```
def my_ran(rate):
    z = random.random()
    return -np.log(1 - z)/rate


N0 = 1000
nuclei_life = []
t_half = 19.3009
rate = np.log(2)/t_half

for n in range(N0):
    nuclei_life.append(my_ran(rate))

nuclei_life = np.sort(np.array(nuclei_life))
n_half_lives = 10
times = np.linspace(0.0, n_half_lives*t_half, 100)

N_decayed = []
N_undecayed = []

for t in times:
    try: q = np.argwhere(nuclei_life <= t)[-1][0]
    except: q = 0
    N_decayed.append(q)
N_undecayed = N0 - np.array(N_decayed)

residue_daughter = abs(np.array(N_decayed) - np.array(N0 - N0*np.exp(-times*rate)))
residue_parent = abs(np.array(N_undecayed) - np.array(N0*np.exp(-times*rate)))
```

Figure 2.1: Algorithm for the calculation of simulated and theoretical decays of parent and daughter ^{10}C nuclei. Number errors are given by residues. Complete code is given in [6][7].

The alogorithm works in such a way that at any point of time, the number of decayed and undecayed nuclei sums out to be N_0 both in the case of simulation as well as theory. The residues in daughter and parent nuclei are equal and is given as:

$$R = N_d - N_0\left(1 - e^{-\lambda t}\right) = N_u - N_0 e^{-\lambda t} \tag{2.1}$$

where R is the residue in both parent and daughter nuclei, N_u and N_d are number of nuclei undecayed and decayed respectively. For the two different hypotheses, it is considered that the first hypothesis has rate λ_1 (pure ^{10}C nuclei) and the second one has amixed rate of:

$$\lambda_{mix} = 0.8\lambda_1 + 0.2\lambda_2 \tag{2.2}$$

with 80% of ^{10}C and 20% of ^{19}Ne impurity. For the calculation of Log Likelihood Ratio (LLR) for two hypotheses shown in code [8], the following was used:

$$\mathsf{H1\_LLR} = \log\left(\frac{\mathcal{L}_{H_1}}{\mathcal{L}_{H_2}}\right) \quad \text{and} \quad \mathsf{H2\_LLR} = \log\left(\frac{\mathcal{L}_{H_2}}{\mathcal{L}_{H_1}}\right)$$

Chapter 3

Output Interpretation

3.1 Uncertainties in Radioactive Decay

There can be various sources of uncertainties in the positron decay of ^{10}C nuclei. For the half-life measurement, there can be conservative systematic uncertainty, unceratinties limited by data, and impurity uncertainty [5]. For this project, only impurity and statistical uncertainties are considered. The statistical uncertainty arises due to difference in simulated and theoretical outputs whereas impurity uncertainty is introduced by the presence of different radioactive elements in the sample; for example, presence of 20% ^{19}Ne nuclei in the sample changes the rate a little bit which in turn effects the distribution.

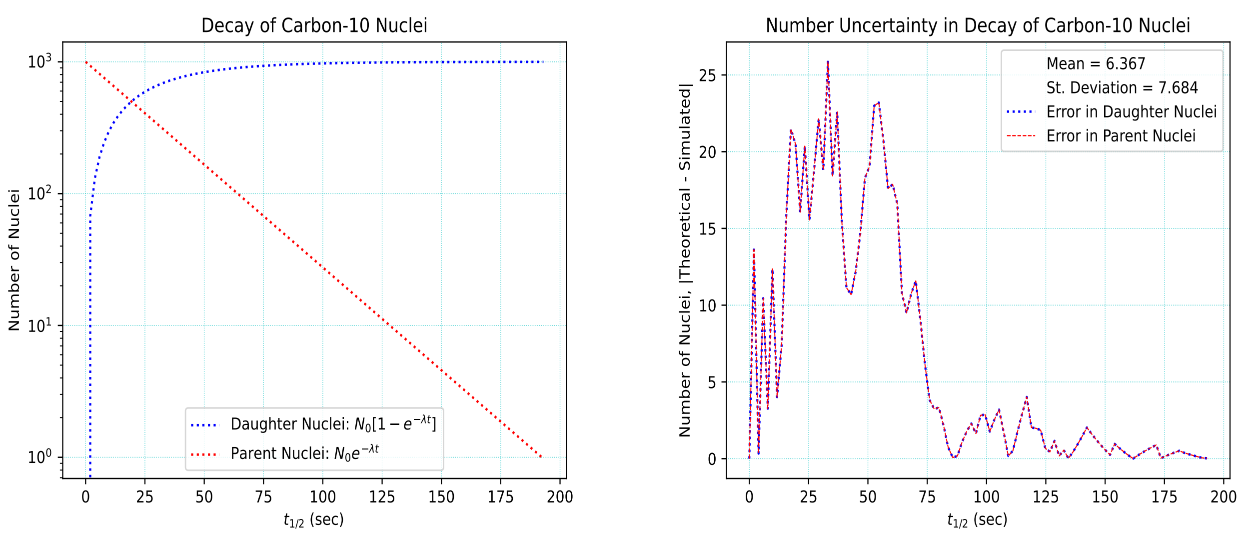

Figure 3.1: Left: (Log scale) decay of ^{10}C nuceli, Right: Coincided residues for parent and daughter nuclei. Code is given in [6].

3.2 Number Independent Number Uncertainty

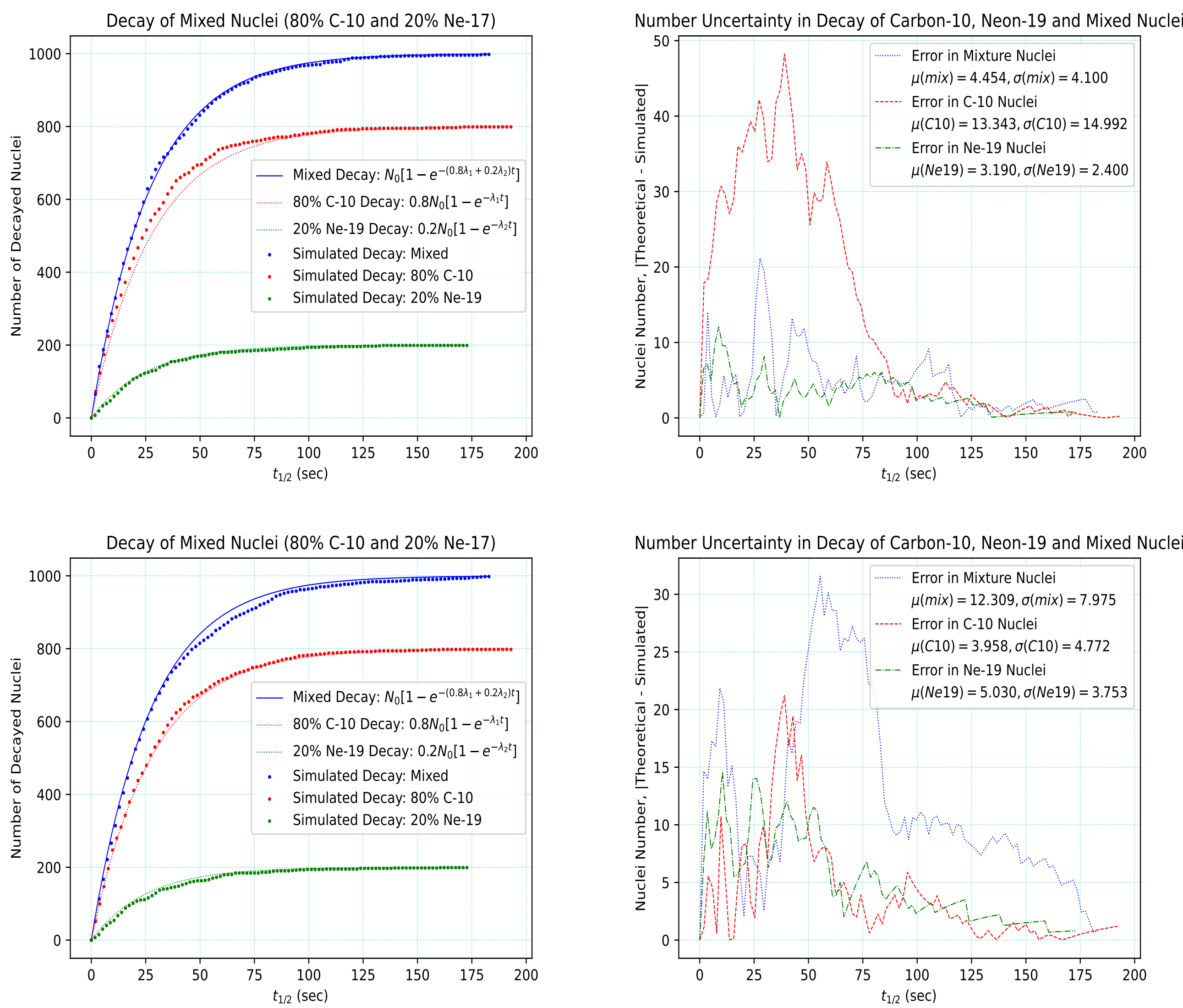

Figure 3.2: Number uncertainties in the decay of ^{10}C, ^{19}Ne, and mixed nuclei. Code is given in [7].

It can be seen in the above Figure 3.2 that the unceraintties in the simulated and theoretical calculation do not depend on the number of radioactive nuclei. For example, in the top two plots, ^{10}C has the maximum standard deviation evenif it has 800 nuclei whereas in the bottom two plots the mixture of 1000 nuclei has maximum uncertainties. This is because the simulation has been done using random numbers which can make the difference vary abruptly. Also, it is noticable that the shapes of these errors looks more like a Poisson fluctuation; rising in the beginning and falling with a long tail in the end.

3.3 Likelihood Ratio of Two Hypotheses

For this project, hypothesis 1 (H_1) is simply the decay rate of ^{10}C and hypothesis 2 (H_2) is a mixed decay rate (shown in Equation 2.2).

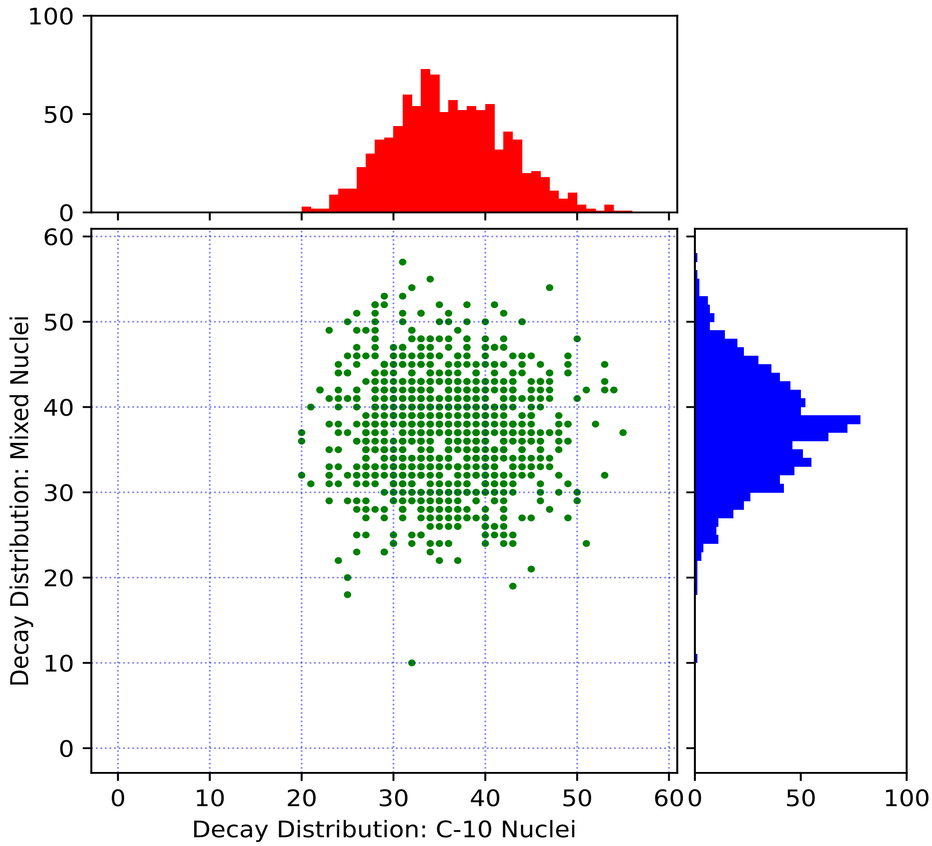

Figure 3.3: Scatter plot and poisson distribution histograms of two hypotheses. Complete code is given in [7].

As the value of lambda is not very high, it would be difficult to get anything useful from the hypotheses testing. Therefore, λ_1 and λ_2 are multiplied by 1000 and then the results are noticable. It can be seen in the plots below (Figure 3.4) that both the decay rates are smaller than λ_c, critical rate parameter. Therefore, none of the hypotheses can be thrown away using this testing. The main reason for this is that there is no significant difference in the decay rates of the mixture of radiactive nuclei from pure ^{10}C nuclei.

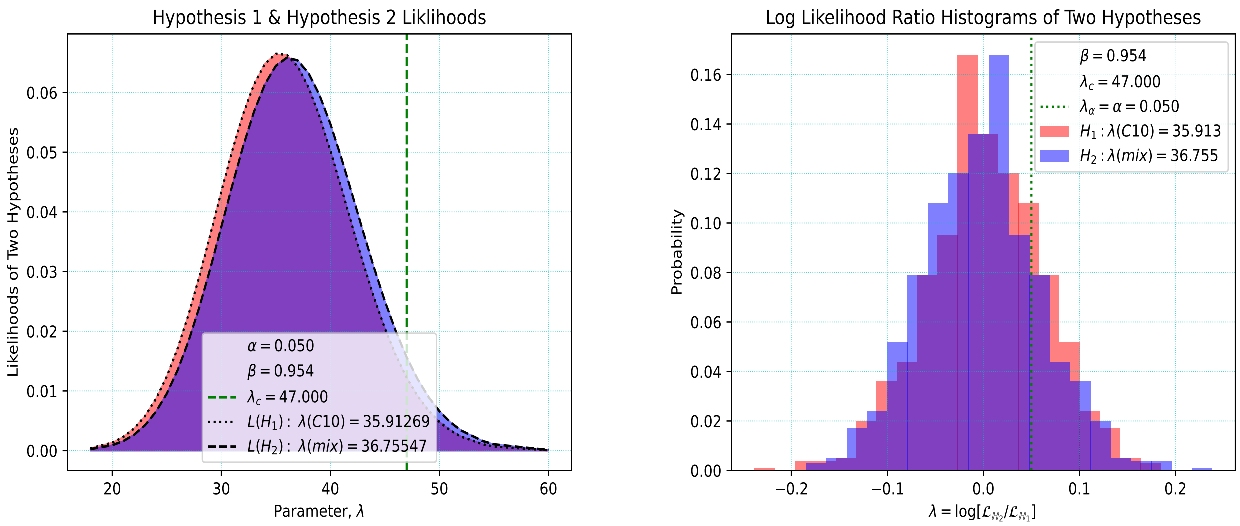

Figure 3.4: Likelihoods Left and Log Likelihood Ratio Right of two hypotheses. Complete code is given in [8].

Bibliography

[1] L. G. Earwaker, J. G. Jenkin & E. W. Titterton, ``Half Lives of Carbon-10 and Neon-19", Nature volume 195, pages271–272(1962).

[2] Delsasso, L. A., White, M. G., Barkas, W., & Creutz, E. C., Phys. Rev., 58, 586 (1940).

[3] Sherr, R., Meuther, H. R., & White, M. G., Phys. Rev., 75, 282 (1949).

[4] Herrmannsfeldt, W. B., Burman, R. L., Stahelin, P., Allen, J. S., & Braid, T. H., Bull. Amer. Phys. Soc., 4, 77 (1959).

[5] M. Dunlop, A. Laffoley, R. Dunlop, C. Andreoiu, & R. Austin, PoS INPC2016 (2016) 167.

[6] https://github.com/Mohammad-Neutrino/PHSX815_Project4/blob/Codes/decay.py.

[7] https://github.com/Mohammad-Neutrino/PHSX815_Project4/blob/Codes/decay_mixture.py.

[8] https://github.com/Mohammad-Neutrino/PHSX815_Project4/blob/Codes/decay_likelihood.py.