HW#9 EGM6341 Spring 2023

Due: 4/5/2023

In problem #14, compute the convergence ratio of integration using:

R(n) = [I(n/2) – I(n/4)] / [I(n) – I(n/2)]

where n is the number of intervals.

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#14 Apply COMPOSITE 4-point Gauss-Legendre quadrature (see nodes and weights below) to the integrals $I = \int_a^b f(x)dx$ in Problem 1. Compare the results with those for trapezoidal and Simpson rules. Use n=2, 4, 8, 16, 32, …

a) $I = \int_0^1 exp(-x^2)dx;$ b) $I = \int_0^1 x^{2.5}\,dx$

c) $I = \int_{-4}^4 \frac{1}{1+x^2}dx;$ d) $I = \int_0^{2\pi} \frac{1}{2+cos(x)}dx$

e) $I = \int_0^{\pi} e^x\,cos(4x)\,dx$

Gauss-Legendre quadrature nodes & weights for

$I = \int_{-1}^{1} f(t)\; dt \sim \sum_{i=1}^{n} w_i\, f(t_i)$

n=4:

t_i	w_i
-0.861136311594052575224	0.347854845137454538573731
-0.339981043584856264802	0.652145154862546142626
0.339981043584856264802	0.652145154862546142626
0.861136311594052575224	0.347854845137454538573731

#A1

Use a change of variable, $x=\xi^q$, to remove the singularity in the following integral,

$$I = \int_0^1 \frac{e^x}{x^{\sqrt{2}-1}}dx.$$

After applying the change of variable, use Matlab "integral" function OR composite Simpson 1/3-rule (*with sufficient number of intervals to ensure accuracy*) to obtain the final result for *I*. Use format long and keep all decimals in reporting your results.

#A2 (this is modified from prob. 37 on p. 328 of Atkinson's book)

Consider evaluating $I = \int_0^{\infty} \frac{sin(x)}{x}dx$ (exact value = π/2).

a) Use Matlab function "integral": I=integral(f,0,inf).
Is the result satisfactory comparing with the exact value? Why
b) Use the 256-nodes Gauss-Laguerre quadrature (click the link below to obtain the nodes and weights). You need to put the integral in the proper form as in (5.6.11)
https://ufl.instructure.com/files/77009187/download?download_frd=1
Is the result satisfactory? Why

c) Let $I = \int_0^{\infty} \frac{sin(x)}{x} dx = \int_0^{b} \frac{sin(x)}{x} dx + \int_b^{\infty} \frac{sin(x)}{x} dx \equiv I_1 + I_2$.

Use Matlab function "integral": I=integral(f,0,b) for b=1000, for I_1 and and use the following asymptotic integral for b»1 (see a similar problem in HW1, prob. A2) for I_2:

$$\int_b^{\infty} \frac{sin(x)}{x} dx \sim \left(\frac{1}{b} - \frac{2}{b^3} + \ldots\right) cos\, b + \left(\frac{1}{b^2} - \frac{6}{b^4} + \ldots\right) sin\, b$$

Determine the error for final value of I.

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Prob. #6

Write a computer program to solve $y' = f(x, y)$, $y(x_0) = y_0$, using Euler's methods. Write it to be used with an arbitrary f, step size h, and interval $[x_0, b]$. Using the program, solve $y' = x^2 - y$, $y(0) = 1$, for $0 \le x \le 4$, with step sizes of h = 0.25, 0.125 and 0.0625 in succession. For each value of *h*, print the true solution, approximate solution, error, and relative error at the nodes *x* = 0, .25, .50, .75, ... , 4.00. The true solution is $Y(x) = x^2 - 2x + 2 - e^{-x}$. Analyze your output and supply written comments on it. Analysis of output is as important as obtaining it.

a) Use Euler's explicit method.
b) Use Euler's modified method (predictor-corrector method).