HW#8 EGM6341 Spring 2023

Due: 3/29/2023
Assigned: 3/21/2023

A1. The Simpson 1/3 rule for integration of

$$I_{exact} = \int_{x_0}^{x_2} f(x)dx \qquad \textbf{(1)}$$

is given by

$$I_{Simpson} = \mathbf{h(f_0 + 4f_1 + f_2)/3} \qquad \textbf{(2)}$$

a) Expand $f(x)$ using Taylor series near x_1 and integrate Eq. (1) analytically. If you keep enough terms in the expansion, you can take this results as almost exact (i.e. it is sufficiently accurate to be used to assess the error in (2)).
b) Expand $\mathbf{f_0} = f(x_1 - h)$ and $\mathbf{f_2} = f(x_1 + h)$ using Taylor series near x_1.
c) Evaluate the integration error: Error= $I_{Simpson} - I_{exact}$. Compare your result with the error given for Simpson's 1/3-rule by standard textbook.

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In problem #1-2, compute the convergence ratio of integration using:

R(n) = [I(n/2) – I(n/4)] / [I(n) – I(n/2)]

where n is the number of intervals.

#1 Write a program to evaluate $I = \int_a^b f(x)dx$ using the trapezoidal rule with n subdivisions, calling the result I(n). Use the program to calculate the following integrals with n=2,4,8,16,…,512

a) $I = \int_0^1 \exp(-x^2)dx$; b) $I = \int_0^1 x^{2.5}dx$

c) $I = \int_{-4}^{4} \frac{1}{1+x^2}dx$; d) $I = \int_0^{2\pi} \frac{1}{2+\cos(x)}dx$

e) $I = \int_0^{\pi} e^x \cos(4x)dx$

What is the order of convergence of I(n) for each case?

#2 Write a program to evaluate $I = \int_a^b f(x)dx$ using the Simpson's rule with n subdivisions, calling the result I_n to

a) $I = \int_0^1 \exp(-x^2)dx$; b) $I = \int_0^1 x^{2.5}dx$

c) $I = \int_{-4}^{4} \frac{1}{1+x^2}dx$; d) $I = \int_0^{2\pi} \frac{1}{2+\cos(x)}dx$

e) $I = \int_0^{\pi} e^x \cos(4x)dx$

#18 Derive one and two point Gaussian quadrature formulae for

$$I = \int_0^1 xf(x)dx = \sum_{j=1}^{n} w_j f(x_j)$$

#26 Consider the following table of approximate integrals In produced using Simpson's rule. Predict the order of convergence if I_n to I:

n	I_n
2	0.28451779686
4	0.28559254576
8	0.28570248748
16	0.28571317731
32	0.28571418363
64	0.28571427643

#31 Implement the algorithm Romberg of Section 5.4, and then apply it to the integrals of problem 1. Compare the results with those for the trapezoidal and Simpson rules.