HW 11

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1\. BVP $y'' - (1 - x/4)\, y = x \rightarrow \boxed{b(x) = 0}$ & $c(x) = -(1 - x/4)$

y values are given: $y(0) = 1$ & $y(2) = 5$

x	0	0.5	1	1.5	2
i	1	2	3	4	5

(i) $\Delta x = 0.5 = h$

$$\left(1 - \frac{h}{2} b_i\right) y_{i-1} - (2 - c_i h^2)\, y_i + \left(1 + \frac{h}{2} b_i\right) y_{i+1} = d_i h^2$$

$\boxed{i = 2} \rightarrow (1 - 0)(1) - \left(2 + \frac{7}{8}\left(\frac{1}{4}\right)\right) y_2 + (1)\, y_3 = (0.5)\,\frac{1}{4}$

$\boxed{i = 3} \rightarrow y_2 - \left(2 + \frac{3}{4}\left(\frac{1}{4}\right)\right) y_3 + (1)\, y_4 = (1)\,\frac{1}{4}$

$\boxed{i = 4} \rightarrow y_3 - \left(2 + \frac{5}{6}\left(\frac{1}{4}\right)\right) y_4 + (1)(5) = (1.5)\,\frac{1}{4}$

$$\begin{bmatrix} -2 - \frac{7}{32} & 1 & 0 \\ 1 & -2 - \frac{3}{16} & 1 \\ 0 & 1 & -2 - \frac{5}{24} \end{bmatrix} \begin{Bmatrix} y_2 \\ y_3 \\ y_4 \end{Bmatrix} = \begin{Bmatrix} -0.875000000000000 \\ 0.250000000000000 \\ -4.625000000000000 \end{Bmatrix} \rightarrow \begin{Bmatrix} y_2 \\ y_3 \\ y_4 \end{Bmatrix} = \begin{Bmatrix} 1.204870471513754 \\ 1.798306358671143 \\ 2.978924688079370 \end{Bmatrix}$$

(ii) $\Delta x = 0.1$ wrote the matlab code below for this:

```
format long
a=0;
b=2;
h=0.5;
x=a:h:b;
y0=1
yn=5
%% making the tridiagonal matrix
for i=2:length(x)-1
 A(i-1)=-(2-c(x(i))*h^2);
 B(i-1)=1-(h/2)*bb(x(i));
 C(i-1)=1+(h/2)*bb(x(i)) ;
end
AA=zeros(length(x)-2);
for i=1:length(A)
 AA(i,i)=A(i);
end
for i=1:length(A)-1
 AA(i,i+1)=C(i);
 AA(i+1,i)=B(i+1);
end
%% making the d vector:
for i=2:length(x)-1
 d(i-1)=DD(x(i))*(h^2);
end
%% applying BC:
d(1)=d(1)-B(1)*y0;
d(end)=d(end)-C(end)*yn;
d=d'
yy=linsolve(AA,d)
function y=c(x)
y=-(1-(x./4));
end
function y=bb(x)
y=0;
end
function y=DD(x)
y=x;
end
```

triadiagonal matrix for $\Delta x = 0.1$

$$
A = \left[\begin{array}{ccccccccccccccccccc}
-2.00975 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -2.0095 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -2.00925 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -2.009 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -2.00875 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -2.0085 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -2.00825 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.008 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.00775 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.0075 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.00725 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.007 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.00675 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.0065 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.00625 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.006 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.00575 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.0055 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.00525
\end{array}\right]
$$

$$
Ay = d \rightarrow y = \left\{\begin{array}{c} y_2 \\ \vdots \\ y_{20} \end{array}\right\} = \left\{\begin{array}{c}
1.016054958087338 \\
1.043016452016027 \\
1.081886602238868 \\
1.133764203532420 \\
1.199845682657763 \\
1.281425811506361 \\
1.379898059752764 \\
1.496754466992127 \\
1.633584909967426 \\
1.792075635994974 \\
1.974006929292483 \\
2.181249772827363 \\
2.415761364772035 \\
2.679579345928917 \\
2.974814592834338 \\
3.303642430944973 \\
3.668292123641276 \\
4.071034496048517 \\
4.514167558184025
\end{array}\right\}
$$

2

BVP $y'' - (1 - x/4)\, y = x \rightarrow \boxed{b(x) = 0}$ & $c(x) = -(1 - x/4)$

y is given only at $x=0$: $y(0) = 1$

$y'(2) = 2$

→ in this case we will make a fictitious point at x_{n+1}:

→ centeral difference

$$\frac{y_{n+1} - y_{n-1}}{2h} = y'(2) = 2 \rightarrow \boxed{y_{n+1} = y_{n-1} + 2h\, y_n'}$$

→ we can plug it in eq below

$$\left(1 - \frac{h}{2} b_i\right) y_{i-1} - \left(2 - c_i h^2\right) y_i + \left(1 + \frac{h}{2} b_i\right) y_{i+1} = d_i h^2$$

$$\rightarrow \left(1 - \frac{h}{2} b_n\right) y_{n-1} - \left(2 - c_n h^2\right) y_n + \left(1 + \frac{h}{2} b_n\right)\left(y_{n-1} + 2h\, y_n'\right) = d_n h^2$$

$$\rightarrow \boxed{2 y_{n-1} - \left(2 - c_n h^2\right) y_n = d_n h^2 - 2h\, y_n'}$$

I implemented this with The matlab code below:

B.C

```
format long
a=0;
b=2;
h=0.1;
x=a:h:b;
y0=1
dyn=2
```

making tridiagonal matrix

```
%% making the tridiagonal matrix
for i=2:length(x)
 A(i-1)=-(2-c(x(i))*h^2);
 B(i-1)=1-(h/2)*bb(x(i));
 C(i-1)=1+(h/2)*bb(x(i)) ;
end
AA=zeros(length(x)-1);
for i=1:length(A)
 AA(i,i)=A(i);
end
for i=1:length(A)-1
 AA(i,i+1)=C(i);
 AA(i+1,i)=B(i+1);
end
```

making vector d →

```
%% making the d vector:
for i=2:length(x)
 d(i-1)=DD(x(i))*(h^2);
end
```

applying B.C →

```
%% applying BC:
d(1)=d(1)-B(1)*y0;
d(end)=d(end)-2*h*dyn;
AA(end,end-1)= AA(end,end-1)+C(end)
AA
d=d'
yy=linsolve(AA,d)
function y=c(x)
y=-(1-(x./4));
end
function y=bb(x)
y=0;
end
function y=DD(x)
y=x;
end
```

the tridiagonal matrix :

$A =$

-2.00975	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-2.0095	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	-2.00925	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	-2.009	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	-2.00875	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	-2.0085	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	-2.00825	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	-2.008	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	-2.00775	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	-2.0075	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	-2.00725	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	-2.007	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	-2.00675	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	-2.0065	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2.00625	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2.006	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2.00575	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2.0055	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2.00525	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2.005

$A * y = d$ → Solving it →

$$y = \begin{Bmatrix} y_2 \\ \vdots \\ y_{21} \end{Bmatrix} = \begin{Bmatrix} 0.901256654240559 \\ 0.812300560859963 \\ 0.733061322807537 \\ 0.663602901991080 \\ 0.604116907292544 \\ 0.554916935532817 \\ 0.516433757725119 \\ 0.489211158418653 \\ 0.473902248379537 \\ 0.471266080765362 \\ 0.482164408756927 \\ 0.507558428711980 \\ 0.548505357668017 \\ 0.606154697788313 \\ 0.681744043444233 \\ 0.776594289371679 \\ 0.892104101035355 \\ 1.029743511279985 \\ 1.191046510836654 \\ 1.377602504575216 \end{Bmatrix}$$

3

$$\partial u/\partial t = 0.5 \frac{\partial^2 u}{\partial x^2} \qquad 0 \leq x \leq 1 \qquad \underline{u(t,0) = 0} \ \& \ \underline{u(t,1) = 1} \qquad \Delta x = 0.05$$

→ convergence requirement of explicit method :

$$0 \leq 4r \leq 2 \rightarrow r = \nu \Delta t / (\Delta x)^2$$

$\nu \geq 0$ → material propertise $\boxed{\nu = 0.5}$

$\Delta t \geq 0$ → we cannot go back in time

$$\nu \Delta t / \Delta x^2 \leq \frac{1}{2} \rightarrow \frac{(0.5)}{(0.05)^2} \Delta t \leq 1/2 \rightarrow \boxed{\Delta t \leq 0.0025} \rightarrow \boxed{\text{I choose } \Delta t = 0.001\,s}$$

$$\rightarrow u_m^{n+1} = r u_{m+1}^n + (1-2r) u_m^n + r u_{m-1}^n$$

Implemented this approach with Matlab code below

ν →

x domain →

end time & time step size →

x step size →

B.Cs →

r value →

setting up solution matrix & applying initial & boundry values

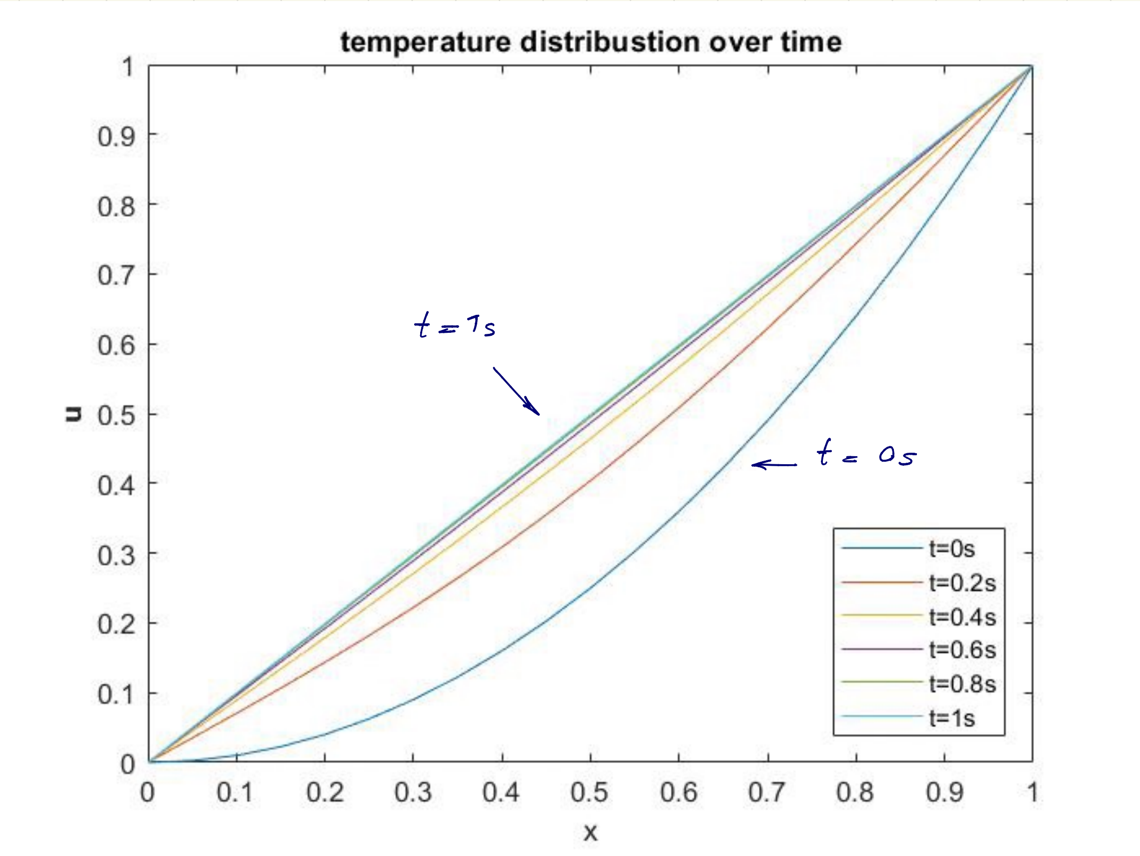
applying explicit method →

plotting results

```
format long
v=0.5;
a=0;
b=1;
tf=1;
dt=0.001;
h=0.05;
x=a:h:b;
t=0:dt:tf;
u0=0;
u1=1;
r=v*dt/h^2;
m=length(x);
n=length(t);
u=zeros(n,m);
%% applying BC & initial conditions
u(1,:)=DD(x(1:end));
u(:,1)=u0;
u(:,m)=u1;
for j=1:n-1
for i=2:m-1
 u(j+1,i)=r*u(j,i+1)+(1-2*r)*u(j,i)+r*u(j,i-1);
end
end
for i=1:6
plot (x,u(200*(i-1)+1,:))
hold on
end
hold on
xlabel('x');
ylabel('u','FontWeight','b');
legend('t=0s',...
 't=0.2s','t=0.4s',...
 't=0.6s','t=0.8s',"t=1s");
title("temperature distribustion over time");
t=table(u(1,:)',u(201,:)',u(401,:)',u(601,:)',u(801,:)',u(1
001,:)','VariableNames',["t=0s","t=0.2s","t=0.4s","t=0.6s",
"t=0.8s","t=1.0s"])
function y=DD(x)
y=x.^2;
end
```

→ $\Delta t = 0.001$ but I printed the results every 0.2s

t=0s	t=0.2s	t=0.4s	t=0.6s	t=0.8s	t=1.0s
0	0	0	0	0	0
0.0025	0.0349623165607591	0.0443978269119256	0.0479128727802807	0.049222426721985	0.0497103098474432
0.01	0.0702950234519221	0.0889335979329586	0.095877137559491	0.0984639998786925	0.0994277528278687
0.0225	0.10635936620696	0.133741860527131	0.143942920894986	0.147743394456079	0.149159286432852
0.04	0.143498531294001	0.178950452505719	0.192157849620344	0.197078354151201	0.198911521196026
0.0625	0.182029186061132	0.224677354234759	0.240565878114659	0.246485254463507	0.248690557919809
0.09	0.222233683316871	0.271027783567152	0.289206205999849	0.295978699475693	0.298501837453394
0.1225	0.264353122062939	0.318091605031695	0.338112312915873	0.345571162252719	0.348350006720994
0.16	0.308581432370571	0.365941117073035	0.387311134140878	0.395272677713553	0.398238804299137
0.2025	0.355060625356242	0.414629271827397	0.436822397355265	0.445090595538162	0.448170968360508
0.25	0.403877319729744	0.464188371268386	0.486658136880716	0.495029399194001	0.498148169251051
0.3025	0.455060625356242	0.514629271827397	0.536822397355265	0.545090595538162	0.548170968360508
0.36	0.508581432370571	0.565941117073035	0.587311134140879	0.595272677713553	0.598238804299136
0.4225	0.564353122062939	0.618091605031695	0.638112312915873	0.645571162252719	0.648350006720994
0.49	0.622233683316871	0.671027783567152	0.689206205999849	0.695978699475693	0.698501837453394
0.5625	0.682029186061132	0.724677354234759	0.740565878114659	0.746485254463507	0.748690557919809
0.64	0.743498531294001	0.778950452505718	0.792157849620344	0.797078354151201	0.798911521196026
0.7225	0.80635936620696	0.833741860527131	0.843942920894986	0.847743394456079	0.849159286432852
0.81	0.870295023451922	0.888933597932959	0.895877137559491	0.898463999878693	0.899427752827869
0.9025	0.934962316560759	0.944397826911926	0.947912872780281	0.949222426721985	0.949710309847443
1	1	1	1	1	1



4 stability

$$\frac{\rho - 1}{\Delta t} = \frac{\alpha}{(\Delta x)^2}\left[(1-\xi)\left(e^{ik\Delta x} - 2 + e^{-ik\Delta x}\right) + \xi\left(\rho e^{ik\Delta x} - 2\rho + \rho e^{-ik\Delta x}\right)\right]$$

$$\frac{\rho - 1}{\Delta t} = \frac{\alpha}{\Delta x^2}\left[(1-\xi)\left(2\cos(k\Delta x) - 2\right) + \xi\rho\left(2\cos(k\Delta x) - 2\right)\right]$$

$$\rho - 1 = \frac{\alpha\Delta t}{\Delta x^2}\left[(1-\xi)\left(2\cos(k\Delta x) - 2\right)\right] + \frac{2\alpha\Delta t}{\Delta x^2}\xi\rho\left(\cos(k\Delta x) - 1\right)$$

$$\rho\left(1 - \frac{2\alpha\Delta t}{\Delta x^2}\xi\left(-2\sin^2\left(\frac{k\Delta x}{2}\right)\right)\right) = 1 + \frac{\alpha\Delta t}{\Delta x^2}\left[(1-\xi)(4)\left(-\sin^2\left(\frac{k\Delta x}{2}\right)\right)\right]$$

$$\boxed{\rho = \frac{1 - \frac{4\alpha\Delta t}{\Delta x^2}[1-\xi]\sin^2(\theta)}{1 + \frac{4\alpha\Delta t}{\Delta x^2}[\xi]\sin^2(\theta)}}$$

$\rightarrow$ it is always stable $|\rho| < 1$

#4 (ii)

$$\frac{u^{n+1}_{m,j} - u^{n}_{m,j}}{\Delta t} = \frac{\alpha}{(\Delta x)^2}\left[(1-\xi)\left(u^{n}_{m+1,j} - 2u^{n}_{m,j} + u^{n}_{m-1,j}\right) + \xi\left(u^{n+1}_{m+1,j} - 2u^{n+1}_{m,j} + u^{n+1}_{m-1,j}\right)\right]$$

$$\rightarrow \text{LHS}: \quad \frac{1}{\Delta t}\left(u^{n}_{m} + \Delta t \left.\frac{\partial u}{\partial t}\right|^{n}_{m} + \frac{(\Delta t)^2}{2}\left.\frac{\partial^2 u}{\partial t^2}\right|^{n}_{m} + \frac{(\Delta t)^3}{3!}\left.\frac{\partial^3 u}{\partial t^3}\right|^{n}_{m} - u^{n}_{m}\right)$$

$$\boxed{\rightarrow \left.\frac{\partial u}{\partial t}\right|^{n}_{m} + \frac{\Delta t}{2}\left.\frac{\partial^2 u}{\partial t^2}\right|^{n}_{m} + \frac{(\Delta t)^2}{6}\left.\frac{\partial^3 u}{\partial t^3}\right|^{n}_{m} + \cdots} \quad \text{; LHS}$$

$$\rightarrow \frac{\alpha}{\Delta x^2}\Big[(1-\xi)\Big(u_m^n - \Delta x \frac{\partial u}{\partial x}\Big|_m^n + \boxed{\frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2}\Big|_m^n} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} - 2u_m^n + u_m^n + \Delta x \frac{\partial u}{\partial x}\Big|_m^n + \cdots$$

$$+ \boxed{\frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2}\Big|_m^n} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3}\Big|_m^n + \boxed{\frac{\Delta x^4}{4!}\frac{\partial^4 u}{\partial x^4}\Big|_m^n} + \boxed{\frac{\Delta x^4}{4!}\frac{\partial^4 u}{\partial x^4}\Big|_m^n}\Big]$$

$$+ \xi\Big(u_m^{n+1} - \Delta x \frac{\partial u}{\partial x}\Big|_m^{n+1} + \boxed{\frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2}\Big|_m^{n+1}} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3}\Big|_m^{n+1} - 2u_m^{n+1} + u_m^{n+1} + \Delta x \frac{\partial u}{\partial x}\Big|_m^{n+1} + \cdots$$

$$\boxed{\frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2}\Big|_m^{n+1}} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3}\Big|_m^{n+1} + \boxed{\frac{\Delta x^4}{4!}\frac{\partial^4 u}{\partial x^4}\Big|_m^{n+1}} + \boxed{\frac{\Delta x^4}{4!}\frac{\partial^4 u}{\partial x^4}\Big|_m^{n+1}}$$

$$\rightarrow \frac{\alpha}{\Delta x^2}\Big[(1-\xi)\Big(\Delta x^2\frac{\partial^2 u}{\partial x^2}\Big|_m^n + \frac{\Delta x^4}{12}\frac{\partial^4 u}{\partial x^4}\Big|_m^n\Big) + \xi\Big(\Delta x^2\frac{\partial^2 u}{\partial x^2}\Big|_m^{n+1} + \frac{\Delta x^4}{12}\frac{\partial^4 u}{\partial x^4}\Big|_m^{n+1}\Big)\Big]$$

$$\rightarrow \frac{\alpha}{\Delta x^2}\Big[(1-\xi)\Big(\Delta x^2\frac{\partial^2 u}{\partial x^2}\Big|_m^n + \frac{\Delta x^4}{12}\frac{\partial^4 u}{\partial x^4}\Big|_m^n$$

$$\xi\Big(\Delta x^2\Big(\frac{\partial^2 u}{\partial x^2}\Big|_m^n - \Delta t\frac{\partial^3 u}{\partial x^2\partial t}\Big|_m^n + \Delta t^2\frac{\partial^4 u}{\partial x^2\partial t^2} + \frac{\Delta x^4}{12}\Big(\frac{\partial^4 u}{\partial x^4}\Big|_m^n - \Delta t\frac{\partial^5 u}{\partial x^4\partial t} + \frac{\Delta t^2}{2}\frac{\partial^6 u}{\partial x^4\partial t^2}\Big)$$

RHS : $$\frac{\alpha}{(\Delta x)^2}\Big[\Delta x^2\frac{\partial^2 u}{\partial x^2}\Big|_m^n - \xi\Delta x^2\frac{\partial^2 u}{\partial x^2}\Big|_m^n + \frac{\Delta x^4}{12}\frac{\partial^4 u}{\partial x^4}\Big|_m^n - \xi\frac{\Delta x^4}{12}\frac{\partial^4 u}{\partial x^4}\Big|_m^n$$

$$+\xi\Delta x^2\frac{\partial^2 u}{\partial x^2}\Big|_m^n - \xi\Delta x^2\Delta t\frac{\partial^3 u}{\partial x^2\partial t} + \xi\frac{\Delta x^4}{12}\Big(\frac{\partial^4 u}{\partial x^4}\Big) - \xi\frac{\Delta x^4\Delta t}{12}\frac{\partial^5 u}{\partial x^4\partial t} + \cdots$$

$$\boxed{\text{RHS : } \frac{\alpha}{(\Delta x)^2}\Big[\Delta x^2\frac{\partial^2 u}{\partial x^2}\Big|_m^n + \frac{\Delta x^4}{12}\frac{\partial^4 u}{\partial x^4}\Big|_m^n - \xi\Delta x^2\Delta t\frac{\partial^3 u}{\partial x^2\partial t} + \cdots}$$

LHS : $$\boxed{\rightarrow \frac{\partial u}{\partial t}\Big|_m^n + \frac{\Delta t}{2}\frac{\partial^2 u}{\partial t^2}\Big|_m^n + \frac{(\Delta t)^2}{6}\frac{\partial^3 u}{\partial t^3}\Big|_m^n + \cdots}$$ LHS

$$\frac{\partial u}{\partial t}\bigg|_m^n = \alpha \frac{\partial^2 u}{\partial x^2}\bigg|_m^n + \underbrace{\left[\frac{\Delta x^2 \alpha}{12} \frac{\partial^4 u}{\partial x^4} - \xi \Delta t \frac{\partial^3 u}{\partial x^2 \partial t}\bigg|_m^n - \frac{\Delta t}{2} \frac{\partial^2 u}{\partial t^2}\bigg|_m^n \right]}_{\text{Truncation error}}$$