HW#4 EGM6341

Due: ~~2/10/2023~~ 2/13/2023
Assigned: 2/3/2023

From the textbook by Atkinson:

pp 496-503:
#6a, #21, #23

pp 574-583:
#2, #10b, #14, #18, ~~#33~~

pp 496-503:
Problem #6a

Let

$$f_n(x) = det\begin{bmatrix} x & 1 & 0 & & \dots & 0 \\ 1 & x & 1 & 0 & \dots & 0 \\ 0 & 1 & x & 1 & & \\ & & & \dots & & \\ & & 0 & 1 & x & 1 \\ 0 & \dots & 0 & 0 & 1 & x \end{bmatrix}$$

with the matrix of order n. Also define $f_0(x)=1$.

a) Show $f_{n+1}(x) = xf_n(x) - f_{n-1}(x)$, $n \geq 1$

Problem #21

Probe the following: for $x \in \mathbf{C}^n$

(a) $\|x\|_\infty \leq \|x\|_1 \leq n\|x\|_\infty$
(b) $\|x\|_\infty \leq \|x\|_2 \leq \sqrt{n}\|x\|_\infty$
(c) $\|x\|_2 \leq \|x\|_1 \leq \sqrt{n}\|x\|_2$

Problem #23

Show

$$\lim_{p\to\infty}\left[\sum_{j=1}^{n}|x_j|^p\right]^{1/p} = \operatorname*{Max}_{1\leq i\leq n}|x_i|, \quad x \in C^n$$

pp 574-583:
Problem #2

Consider the linear system

$$6x_1 + 2x_2 + 2x_3 = -2$$
$$2x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3 = 1$$

$$x_1 + 2x_2 - x_3 = 0$$

and verify its solution is

x_1=2.6, x_2=-3.8, x_3=-5.0.

a) Using four-digit floating-point decimal arithmetic with rounding, solve the preceding system by Gaussian elimination without pivoting.
b) Repeat part (a), using partial pivoting. In performing the arithmetic operations, remember to round to four significant digits after each operation, just as would be done on a computer.

Problem #10

Using the Choleski method, calculate the decomposition A = LL^T for

b)

$$A = \begin{bmatrix} 15 & -18 & 15 & -3 \\ -18 & 24 & -18 & 4 \\ 15 & -18 & 18 & -3 \\ -3 & 4 & -3 & 1 \end{bmatrix}$$

$$\text{(that is, } A = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{12} & b_{22} & 0 & 0 \\ b_{13} & b_{23} & b_{33} & 0 \\ b_{14} & b_{24} & b_{34} & b_{44} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & b_{33} & b_{34} \\ 0 & 0 & 0 & b_{44} \end{bmatrix}\text{)}$$

Problem #14. Using the algorithm (8.3.23-24) for solving tridiagonal systems, solve Ax=b with

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}, \underline{b} = \begin{bmatrix} 3 \\ -2 \\ 2 \\ -2 \\ 1 \end{bmatrix}$$

Check the hypotheses and conclusions of Theorem 8.2 are satisfied by this example.

Problem #18 Find the condition number $A = \begin{bmatrix} 100 & 99 \\ 99 & 98 \end{bmatrix}$

~~Problem #33 (20 pts) Implement programs for iteratively solving the discretization (8.8.5) of Poisson's equation on the unit square. To have a situation for which you have a true solution of the linear system, choose Poisson equations in which there is no discretization error in going to (8.8.5). This will be true if the truncation errors in (8.8.3) are identically zero, as, for example, with u(x,y) =~~ ~~x^2y^2.~~

(a) ~~Solve (8.8.5) w/ Jacobi's method. Observe the actual error in each iterate as well as~~ $\cancel{\|x - x^{(\nu)}\|_\infty}$~~. Estimate the constant c of (8.7.2), and compute the estimated error bound of (8.7.5). Compare with the true iteration error.~~

(b) ~~Repeat with the Gauss-Seidel method. Also compare the iteration rate c with that predicted by (8.8.13).~~

(c) ~~Implement the SOR method, using the optimal acceleration parameter~~ $\cancel{\omega^*}$ ~~from (8.8.15).~~