HW#6 EGM6341 Spring 2023

Assigned: 2/20/2023
Due: 2/27/2023

A1

The following is the built-in humps function that MATLAB uses to demonstrate some of its numerical capabilities:

$$f(x) = \frac{1}{(x-0.3)^2 + 0.01} + \frac{1}{(x-0.9)^2 + 0.04} - 6$$

The humps function exhibits both flat and steep regions over a relatively short x range. Here are some values that have been generated at intervals of 0.1 over the range from x = 0 to 1:

x	0	0.1	0.2	0.3	0.4	0.5
f(x)	5.176	15.471	45.887	96.5	47.448	19

x	0.6	0.7	0.8	0.9	1	
f(x)	11.692	12.382	17.846	21.703	16	

Fit these data with a cubic spline with not-a-knot end Conditions

1. #35 p185-194, Atkinson

Consider finding a cubic spline interpolating function for the data

x	0	1	2	2.5	3	4
y	1.4	0.6	1.0	0.65	0.6	1.0

Use the "not-a-knot" condition to obtain boundary conditions supplementing (3.7.19). Graph the resulting function $s(x)$. Compare it to the use of piecewise linear interpolation, connecting the successive points (x_i, y_i) by line segments.

2. For the data given below,

x	0.1	0.2	0.4	0.6	0.9	1.3	1.5	1.7	1.8
y	0.75	1.25	1.45	1.25	0.85	0.55	0.35	0.28	0.18

consider regression using the following nonlinear model:

$$y = \alpha x e^{\beta x}$$

You can linearize this function by first defining z as

$$z = y/x$$

then taking logarithm

$$\text{Ln}(z) = \ln(\alpha) + \beta x$$

Now you are ready to fit the data using the above linear function.
Your job: determine α and β.
Develop a plot of your fit along with the data.

3. (Least square fit) In many applications, one wishes to correlate a dependent variable z to two or more independent variables, x and y. The simplest model is a linear fit

$$z = a_0 + a_1 x + a_2 y$$

Defining $z_i - (a_0 + a_1 x_i + a_2 y_i)$ as the residual, one can determine the "best" values of the coefficients (a_0, a_1, a_2) by minimizing the sum of the square of the residuals,

$$E_2 = \sum_{i=1}^{n} (z_i - a_0 - a_1 x_i - a_2 y_i)^2$$

where n is the total number of the data available for fitting.
a) Derive a 3x3 system of equations for (a_0, a_1, a_2).
b) The following data was given. Find the linear fit for the data

x	*y*	*z*
0	0	5
2	1	10
2.5	2	9
1	3	0
4	6	3
7	2	27

4. #4, p.239, Atkinson

Graph the errors of the Taylor series approximations $p_n(x)$ to $f(x) = \sin[(\pi x/2)]$ on $-1 \le x \le 1$, for $n = 1, 3, 5$. Note the behavior of the error both near the origin and near the endpoints.

5. #5, p.240, Atkinson

Let $f(x)$ be three times continuously differentiable on $[-\alpha, \alpha]$ for some $\alpha > 0$, and consider approximating it by the rational function.

$$R(x) = \frac{a + bx}{1 + cx}$$

To generalize the idea of the Taylor series, choose the constants *a*, *b*, and *c* so that

$$R^{(j)}(0) = f^{(j)}(0), \quad j = 0,1,2$$

Is it always possible to find such an approximation $R(x)$? The function $R(x)$ is an example of a *Pade approximation* to *f(x)*. See Baker (1975) and Brezinski (1980).