# Tutorial - 4

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Discrete

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1.

→ Given,

The height of students is a continuous random variable because it can take any value within a range (eg; 150cm, 170cm, 170.032cm) etc. Given if we measure height with infinite precision, it will be continuous.

2.

→ soln:-

Mean,

~~Formula~~

The mean of a discrete random variable is, $\mu = E[x] = \Sigma x_i p(x = x_i)$

$\mu = (1)(0.01) + (2)(0.03) + (3)(0.25) + (4)(0.35) + (5)(0.20) + (6)(0.10) + (7)(0.04) + (8)(0.02)$

$\therefore \mu = 4.26$

Then,

variance is,

$var(x) = E(x^2) - (E(x))^2$

or $E(x^2) = 1^2(0.01) + 2^2(0.03) + 3^2(0.25) + 4^2(0.35) + 5^2(0.20) + 6^2(0.10) + 7^2(0.04) + 8^2(0.02)$

or $E[x^2] = 19.82$

$\therefore var(x) = 19.82 - (4.26)^2 = 1.6724$

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$\therefore$ $SD = \sqrt{var(x)}$
$= \sqrt{1.6724}$
$= 1.293$ Ans

3. soln:-
Given,
Outcomes for a six-sided dice are 1, 2, 3, 4, 5 & 6 each with probability of 1/6.

So, Expected value:-
$E(x) = \frac{1}{6}(1+2+3+4+5+6)$
$= \frac{21}{6}$
$= 3.5$

~~Variance~~ $(E[x^2] = \frac{1}{6}[1^2+2^2+3^2+4^2+5^2+6^2]$
$= \frac{1}{6} \times 91$
$= 15.1667$

$\therefore$ variance $(var(x)) = E(x^2) - (E(x))^2$
$= 15.1667 - 3.5^2$
$= 2.9167$

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Now,

$\because SD(\sigma) = \sqrt{var(x)}$

$= \sqrt{2.9167}$

$= 1.7078$

$\simeq 1.71$ Ans

4. Soln:-

Given

In this game, the player wins if all three coin show the same face (either head or false). Then the possible outcomes are:-

3 HEADS: $P(3H) = 1/8$

3 TAILS: $P(3T) = 1/8$

So,

The total probability of winning is:

$P(win) = P(3H) + P(3T)$

$= \frac{1}{8} + \frac{1}{8}$

$= 0.25$ Ans

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5. Soln:-

Given,

$F(x) = kx^2$ for $x = 1, 2, 3$.

To satisfy the conditions of PMF, the sum must equal to 1.

So,

$k(1^2) + k(2^2) + k(3^2) = 1$

or $k(1+4+9) = 1$

or $k \times 14 = 1$

or $k = \frac{1}{14}$

$\therefore k \approx 0.0714$ &

6. Soln:-

Given,

PMF,

$F(y) = k \times 0.25^y$ for $y = 1, 2, 3$

The sum of probabilities must equal to 1;

$k \times (0.25^1 + 0.25^2 + 0.25^3) = 1$

or $k \times 0.328 = 1$

or $k = \frac{1}{0.328}$

$\therefore k \approx 3.05$

$\therefore k = 3$ (approx.)

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7. a) soln:-
Given,
The sum of all probability must be equal to 1.
Then,
$\therefore PMF = \cancel{k+2k+} 9k + 10k^2$
or, $1 = 9k + 10k^2$
or, $9k + 10k^2 - 1 = 0$
or, $10k^2 + 10k - k - 1 = 0$
or, $10k(k+1) - 1(k+1) = 0$
or, $\cancel{10k)} (10k-1)(k+1) = 0$

So,
$\therefore k = \frac{1}{10}$

b) soln:-
The sum of all probability must equal to 1

b) soln:-
$P(x \leq 6)$ is the sum of the probabilities from $x=1$ to $x=6$
i.e $P(x \leq 6) = k + 2k + 2k + 3k + k + 2k^2$
using $k = 1/10$
or, $P(x \leq 6) = \frac{1}{10} + \frac{2 \times 1}{10} + \frac{2 \times 1}{10} + \frac{3 \times 1}{10} + \frac{1}{10} + 2\left(\frac{1}{10}\right)^2$

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$\therefore P(x \leq 6) = 0.83$

$P(3 < x \leq 6)$ is the sum of the probability for $x = 4, 5, 6$

$P(3 < x \leq 6) = 3k + k + 2k^2$
using $k = 1/10$

$P(3 < x \leq 6) = \frac{3 \times 1}{10} + \frac{1}{10} + 2\left(\frac{1}{10}\right)^2$

$\therefore P(3 < x \leq 6) = 0.42$

8. Soln:-
Given,
Ram is tossing an unfair coin repeatedly untill he observes a head for the first time where $P(H) = p$. The no. of tosses untill the first head is observed follow a geometric with parameter $p$.

probability mass function (PMF) for the geometric distribution is:
$P(Y = y) = (1-p)^{y-1} \cdot p$ for $y = 1, 2, 3$

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Hence, The distribution of $Y$ is geometric with parameter $p$.

9. Soln.:

Given,

The sum of two dice can take values from 2 to 12. We are asked to find $P(Y < 11)$.

The total number of possible outcome when rolling two dice is $6 \times 6 = 36$

The number of outcomes where the sum is less than or equal to 11 is 35 (only one outcome, sum is 12 is excluded)

So, Probability $P(Y < 11) = \frac{35}{36} = 0.972$

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10. Soln:-
Given that 1 in 10 people struck by lighting die, we want to model survival.

let, $x$ be a Bernoulli Random variable where,
$p(survival) = 0.9$
$p(death) = 0.1$

The expected value is:
$$E[x] = p(survival) = 0.9$$

11. Soln:-
Given,
1 in 20 children in USA have food allergy and we select random sample of 25 children. we want to find the probability that exactly 3 children have a food allergy.

using Binomial Probability,
$n = 25$ = no. of trials.
probability of success $(p) = 1/20 = 0.05$
no. of success $(k) = 3$

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using formula,
$P(X=K) = \binom{n}{K} p^K (1-p)^{n-K}$

substituting the values,
$P(X=3) = \binom{25}{3} (0.05)^3 (0.95)^{22}$

$P(X=3) \approx 0.093$

12. soln:-
Given,
For a Binomial distribution,
Mean$(\mu) = np = 25 \times 0.05 = 1.25$
variable$(\sigma) = np(1-p) = 25 \times 0.05 \times 0.95$
$= 1.1875$

Standard Deviation$(\sigma) = \sqrt{1.1875}$
$= 1.09$

13. soln:-
Given,
probability of computer crashing is 5%
or $p=0.05$, with $n=25$ computers.

a. Exactly 3 crashes
using Binomial probability, with $K=3$.
$P(X=3) = \binom{25}{3} (0.05)^3 (0.95)^{22}$
$\approx 0.093$

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b. At most 3 crashes.
we sum the probabilities of 0, 1, 2, and 3 crashes.

$$P(x \leq 3) = \sum_{k=0} \binom{25}{k} (0.05)^k (0.95)^{25-k}$$

$$\approx 0.966$$

14. soln:-
P(success in each trial) = 1/5

= 0.2 and we are asked to find the probabilities for 0 success, 3 success and 2 failures in 6 trials.

a. P(0 success):
$n=6$, $k=0$ and $p=0.2$

$$P(X=0) = \binom{6}{0} (0.2)^0 (0.8)^6$$

$$\approx 0.262$$

b. for $K=3$,

$$P(X=3) = \binom{6}{3} (0.2)^3 (0.8)^3$$

$$P(X=3) = \frac{6!}{3!(6-3)!} (0.2)^3 (0.8)^3$$

$$\approx 0.0819$$

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c. Since there are 6 trials, 2 failures correspond to 4 success ∵ failures = n - success.
So,

$$P(X=4) = \binom{6}{4} (0.2)^4 (0.8)^2$$

$$= \frac{6!}{4!\,2!} \times 0.2^4 \times 0.8^2$$

$$\approx 0.01536$$

15. Soln:-
Given,
P(crash on any single motorcycle ride) = 0.005
P(exactly 3 crashes in first 1000 rides) = ?

Now, using Binomial distribution,
where, $n = 1000$, $k = 3$ and $p = 0.005$

~~Binomial~~ using formula,

$$P(X=3) = \binom{1000}{3} (0.005)^3 (0.995)^{997}$$

using approx.

$$P(X=3) \approx \binom{1000}{3} (0.005)^3 (0.995)^{997}$$

$$\approx 0.1403$$

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16. Soln:-

Given,

$p = 0.005$ $n = 1000$

$P(x \geq 10) = ?$

we can use the poisson approximation for the binomial distribution instead of calculating each probability of $x = 1, 2, 3, \ldots$ because 'n' is large and 'p' is small

For poisson approx.

$\lambda = np = 1000 \times 0.005$

$= 5$

poisson PMF is.

$$P(X=K) = \frac{\lambda e^{-\lambda}}{K!}$$

To find the $P(x \geq 10)$, we first find $(P x < 10)$ by summing the probabilities for $x$ 0, 1, 2, ....., 9 and then substract this from 1.

$P(x \geq 10) = 1 - p(x < 10)$

or, $p(x \geq 10) = 1 - 0.9682$

$\therefore P(x \geq 10) = 0.318$

Hence, probability of having 10 or more crashes in the first 1000 rides is approx. 0.031.