

Section A

(a)

(a) Input: $[1.2, 0.8, 2.0]$

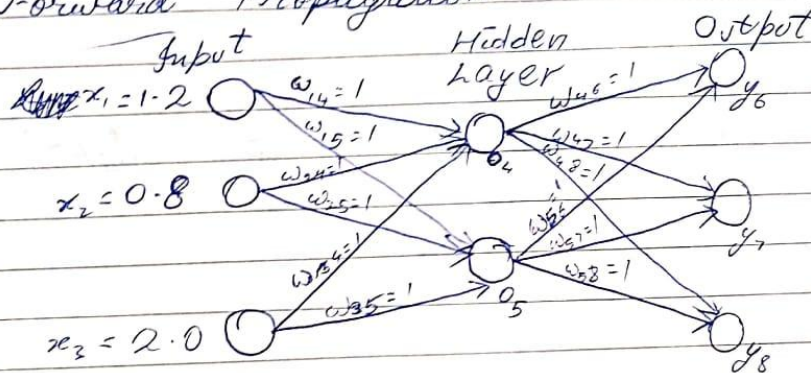
Loss: MSE loss

Activation Function: ReLU $\{\max(0, x)\}$ where $x \geq 0$

Learning rate: $\alpha = 0.01$

Output: $[3.0, 2.5, 4.0]$

Forward Propagation -



Let the initial weights be $(1, 1, 1, 1, 1, 1)$ & bias be 0's.

$$\text{Thus, } o'_4 = w_{14}x_1 + w_{24}x_2 + w_{34}x_3 = 4$$

$$o_4 = \text{ReLU}(4) = 4$$

$$o'_5 = w_{15}x_1 + w_{25}x_2 + w_{35}x_3 = 4$$

$$o_5 = \text{ReLU}(5) = 5$$

$$y_8 = \cancel{4.6} + w_{46} \cdot 4 + w_{56} \cdot 5 = 9$$

$$y_7 = w_{47} \cdot 4 + w_{57} \cdot 5 = 9$$

$$y_8 = w_{48} \cdot 4 + w_{58} \cdot 5 = 9$$

Computing the MSE loss -

$$L = \frac{1}{3} [(3-9)^2 + (2.5-9)^2 + (4-9)^2]$$

$$= \frac{1}{3} [36 + 42.25 + 25]$$

$$\approx 34.42$$

Back Propagation (Updating the weights) -

$$w_{i_{new}} = w_{i_{old}} - \alpha \frac{\partial L}{\partial w_i}$$

For hidden-output layer \Rightarrow

$$\frac{\delta L}{\delta w_{46}} = -2(\hat{y}_{out_1} - y_4) o_4 = -2(-6)4 = 48$$

$$\frac{\delta L}{\delta w_{47}} = -2(\hat{y}_{out_2} - y_7) o_4 = -2(-6.5)4 = 52$$

$$\frac{\delta L}{\delta w_{48}} = -2(\hat{y}_{out_3} - y_8) o_4 = -2(-5)4 = 40$$

$$\frac{\delta L}{\delta w_{56}} = -2(\hat{y}_{out_1} - y_6) o_5 = -2(-6)5 = 60$$

$$\frac{\delta L}{\delta w_{57}} = -2(\hat{y}_{out_2} - y_7) o_5 = -2(-6.5)5 = 65$$

$$\frac{\delta L}{\delta w_{58}} = -2(\hat{y}_{out_3} - y_8) o_5 = -2(-5)5 = 50$$

$$\frac{\delta L}{\delta w_{46}} w_{46}^{new} = w_{46} - \alpha \frac{\delta L}{\delta w_{46}} = 1 - 0.01 \times 48 = 0.52$$

$$w_{47}^{new} = 1 - 0.01 \times 52 = 0.48$$

$$w_{48}^{new} = 1 - 0.01 \times 40 = 0.6$$

$$w_{56}^{new} = 1 - 0.01 \times 60 = 0.4$$

$$w_{57}^{new} = 1 - 0.01 \times 65 = 0.35$$

$$w_{58}^{new} = 1 - 0.01 \times 50 = 0.5$$

For input-hidden layer \Rightarrow

$$\frac{\delta L}{\delta w_{14}} = -2(\hat{y}_{out_1} - y_6) w_{46} \cdot x_1 = -2(-6) 0.52 \times 1.2 = 7.49$$

$$\frac{\delta L}{\delta w_{15}} = -2(\hat{y}_{out_2} - y_7) w_{47} \cdot x_2 = -2(-6.5) 0.48 \times 0.8 = 5$$

$$\frac{\delta L}{\delta w_{24}} = -2(\hat{y}_{out_3} - y_8) w_{48} \cdot x_3 = -2(-5) 0.6 \times 2 = 12$$

$$\frac{\delta L}{\delta w_{25}} = -2(\hat{y}_{out_1} - y_6) w_{56} \cdot x_1 = -2(-6) 0.4 \times 1.2 = 5.76$$

$$\frac{\delta L}{\delta w_{34}} = -2(\hat{y}_{out_2} - y_7) w_{57} \cdot x_2 = -2(-6.5) 0.35 \times 0.8 = 3.64$$

$$\frac{\delta L}{\delta w_{35}} = -2(\hat{y}_{out_3} - y_8) w_{58} \cdot x_3 = -2(-5) 0.5 \times 2 = 10$$

$$w_{14}^{new} = w_{14} - \alpha \frac{\delta L}{\delta w_{14}} = 1 - 0.01 \times 7.49 = 0.93$$

$$w_{15}^{new} = 1 - 0.01 \times 5 = 0.95$$

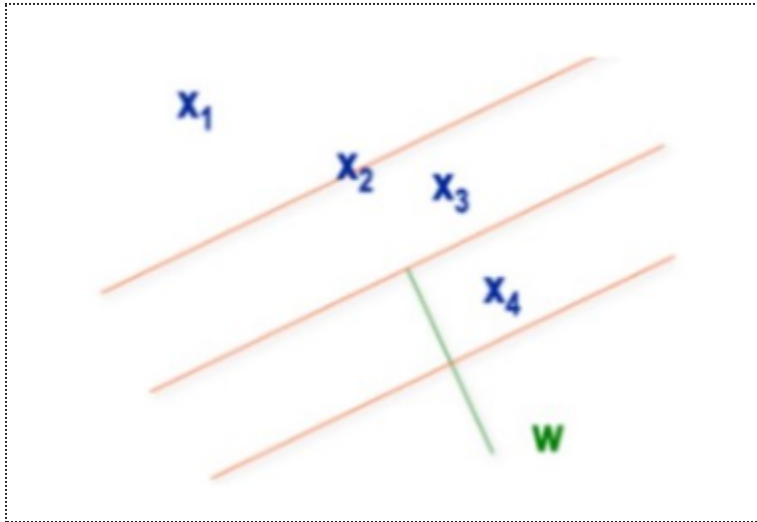
$$w_{24}^{new} = 1 - 0.01 \times 12 = 0.88$$

$$w_{25}^{new} = 1 - 0.01 \times 5.76 = 0.94$$

$$w_{34}^{new} = 1 - 0.01 \times 3.64 = 0.96$$

$$w_{35}^{new} = 1 - 0.01 \times 10 = 0.9$$

(b)



(b) (i) Point x_1 is on the correct side of the margin -
The α_i is typically greater than zero
but less than C . ($0 \leq \alpha_i < C$)

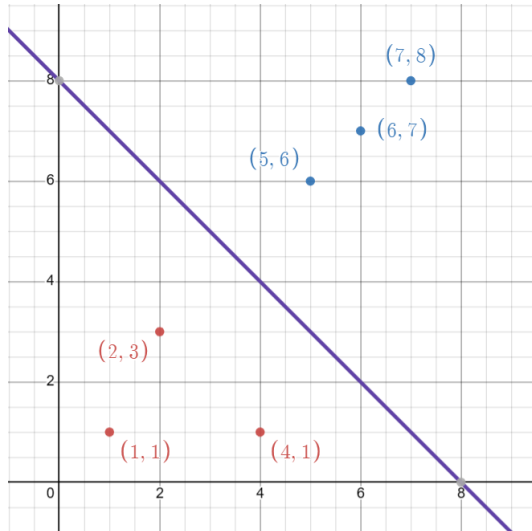
(ii) Point x_2 is on the margin -
The α_i parameter ^{b/w 0} is equal to C . because
~~it~~

(iii) Point x_3 is on the wrong side of margin -
The α_i parameter is equal to C .

(iv) Point x_4 is on the wrong side of the decision
hyperplane -
The α_i parameter is typically greater
than C .

(c)

(a) The red points denote the positive class while the blue points denote the negative class.



Through plotting the points we can clearly see that the data is linearly separable and one such decision boundary would be $x+y-8 = 0$;

(b) We can clearly see that the points (2,3), (4,1), and (5,6) will be used to construct the optimal hyperplane. Thus one support vector would pass through (2,3), and (4,1) and the other support vector would pass through the (5,6) point.

(b) ~~Dist~~ Eq'n of support vectors passing through (2,3) & (4,1) \rightarrow

$$\frac{y-1}{x-4} = \frac{2}{-2} = -1$$

$$\Rightarrow x+y=5$$

Hence, line parallel to it & passing through (5,6) would be - ~~6x~~,

$$x+y=c$$

$$\Rightarrow 5+6=c$$

$$\Rightarrow x+y=11$$

Hence support vectors are - $x+y=5$ & $x+y=11$.

Now, the optimal hyperplane, would be -

$$x+y=k \quad \text{where } k = \frac{5+11}{2} = 8$$

Hence, $x+y=8$ is the optimal ~~hyperplane~~ margin or the decision boundary.

(c) The support vectors as previously identified are - $(2,3)$, $(4,1)$ & $(5,6)$ while their equations are - $x+y=5$ & $x+y=11$.

~~(d) Let a be a point in $x+y=8$ line.
Therefore, $a=(a, 8-a)$
Calculating its perpendicular distance with $x+y=11$, we~~

(d) Distance would be $d_1 = \frac{|c_2 - c_1|}{\sqrt{1+m^2}} = \frac{|11-8|}{\sqrt{1+(-1)^2}}$

$$\Rightarrow d_1 = \frac{3\sqrt{2}}{2} \quad \text{b/w } x+y=8 \text{ \& } x+y=11$$

Distance b/w $x+y=8$ & $x+y=5 \Rightarrow d_2 = \frac{|5-8|}{\sqrt{1+(-1)^2}} = \frac{3}{\sqrt{2}}$

$$\Rightarrow d_2 = \frac{3\sqrt{2}}{2}$$

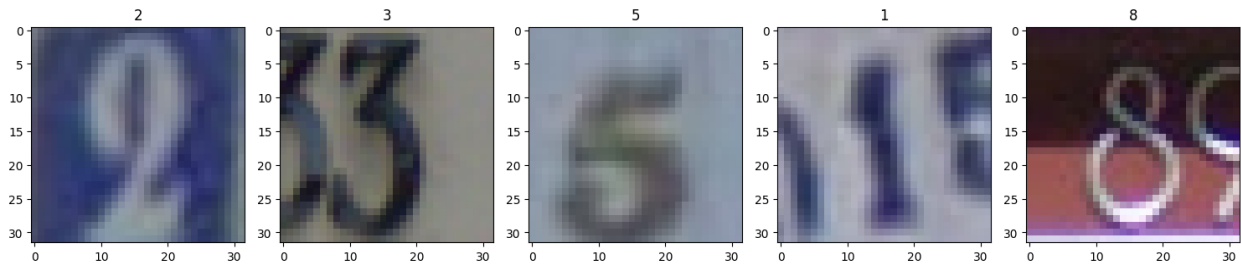
(e) The optimal margin would not change if any point other than the support vectors would be removed. If we remove any of the support vectors, then the optimal margin

would change, with the exception of one of (2,3) or (4,1) being removed. If both are removed then the optimal margin would change.

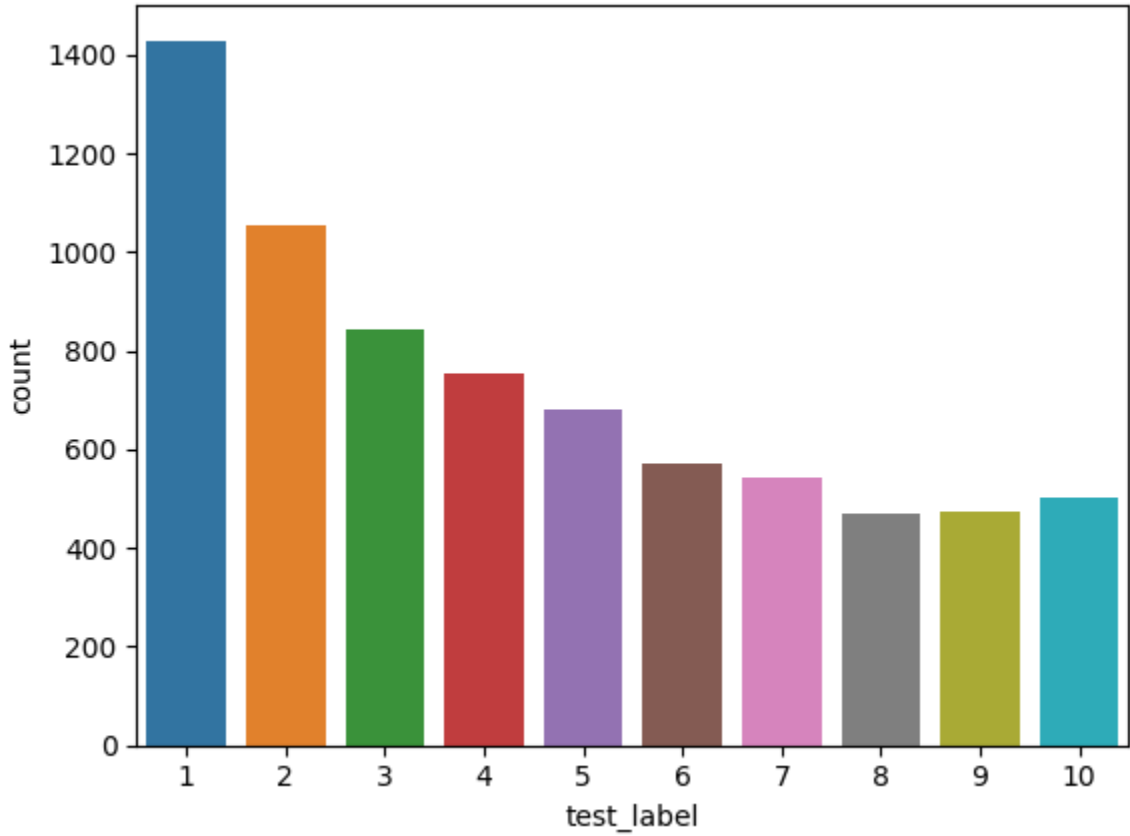
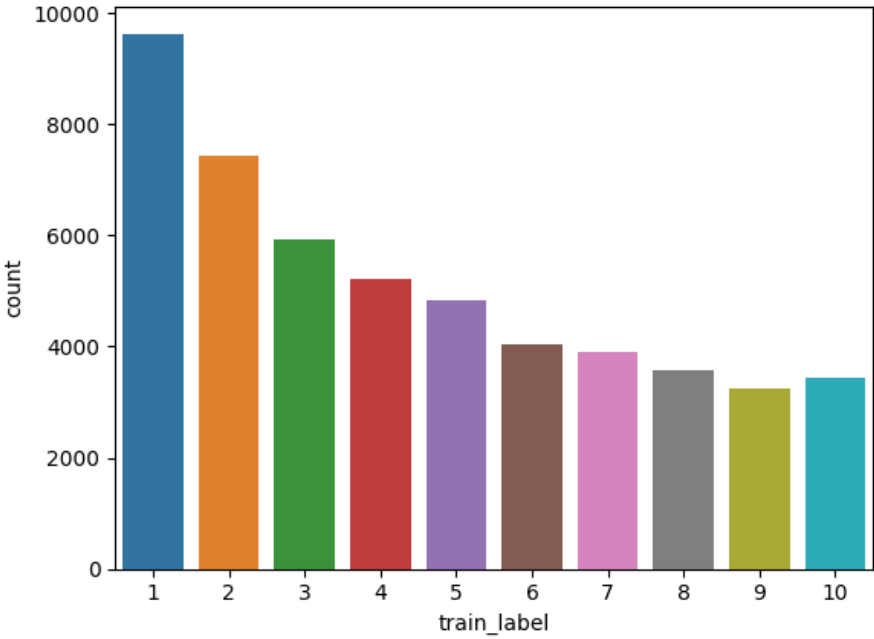
Section B

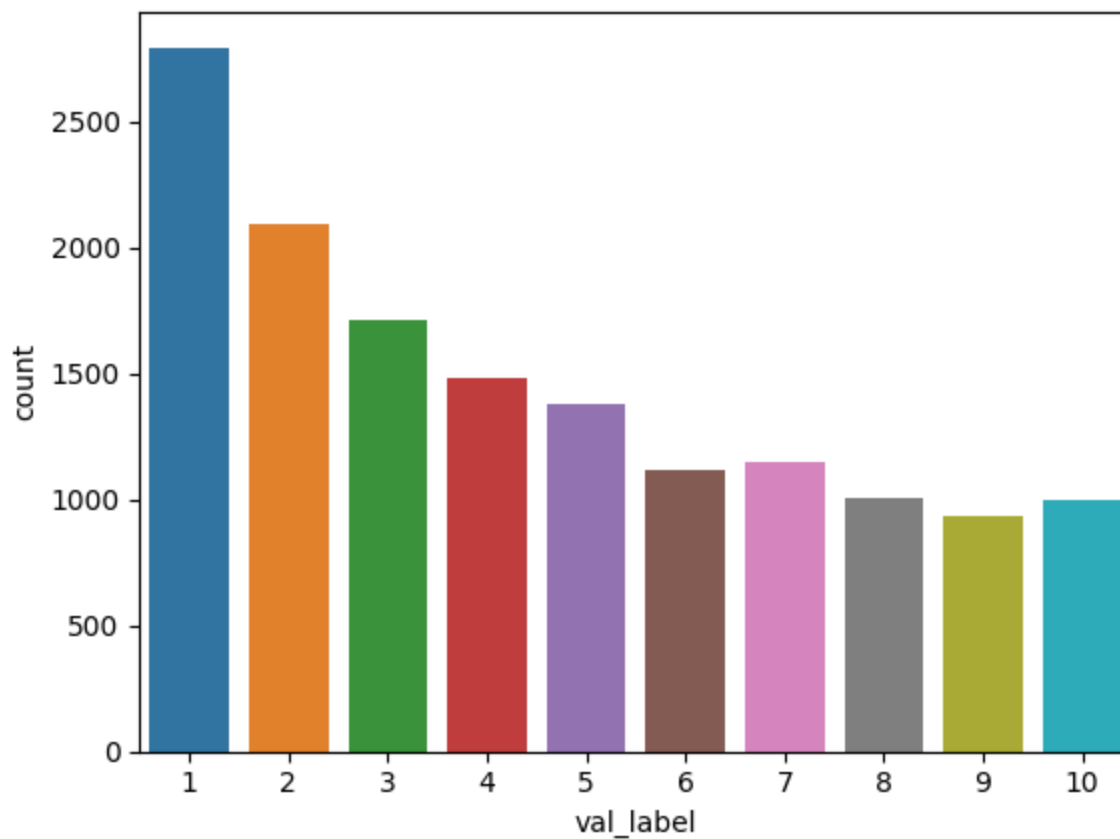
Section C

5 Unique Images -

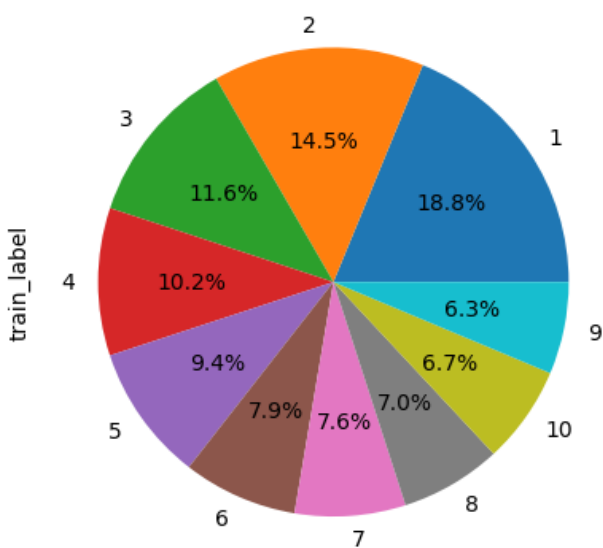


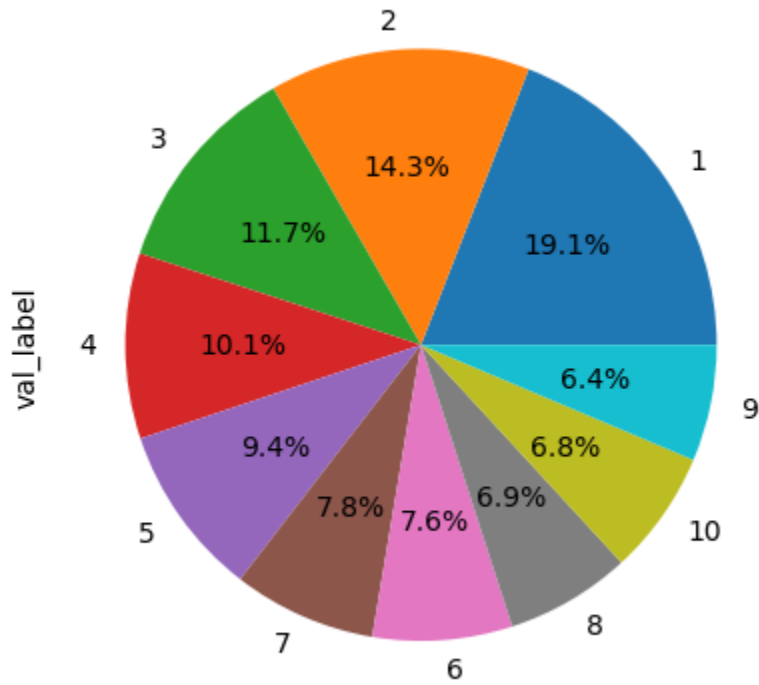
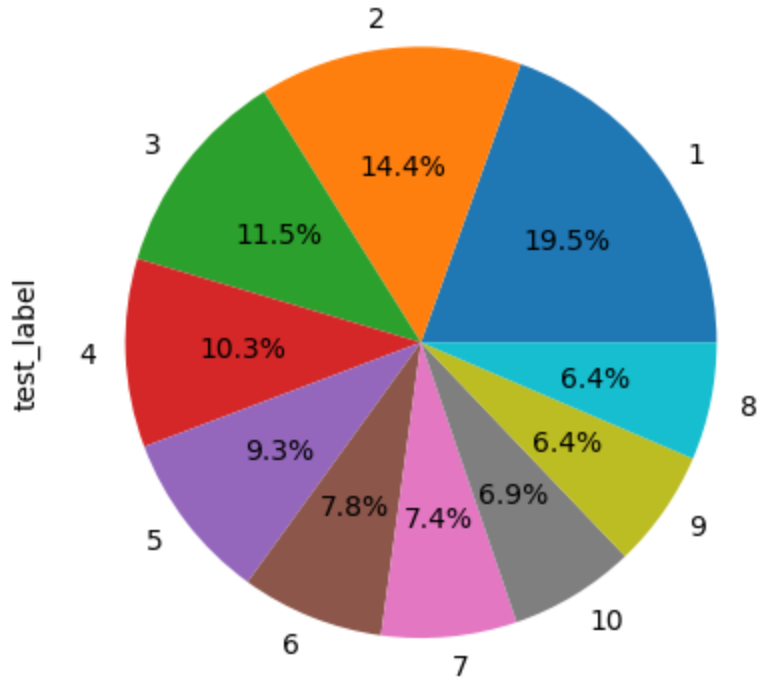
Distribution of class labels -



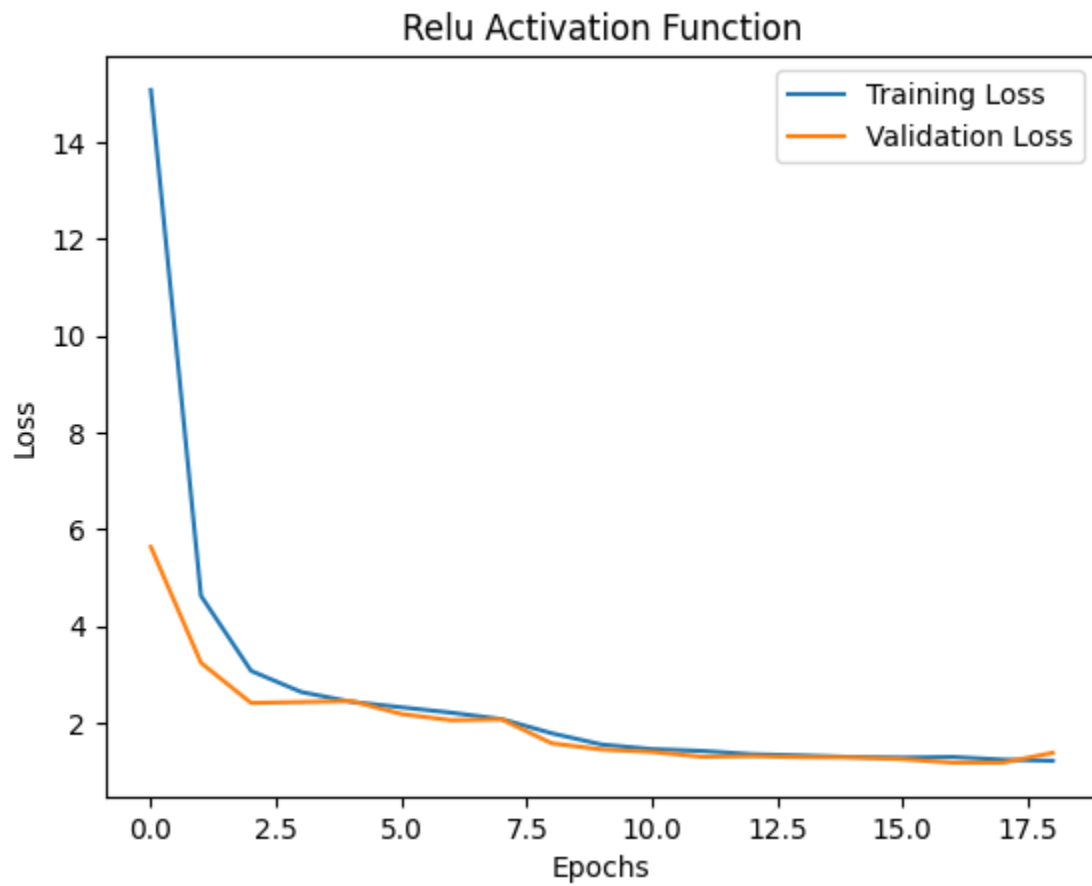


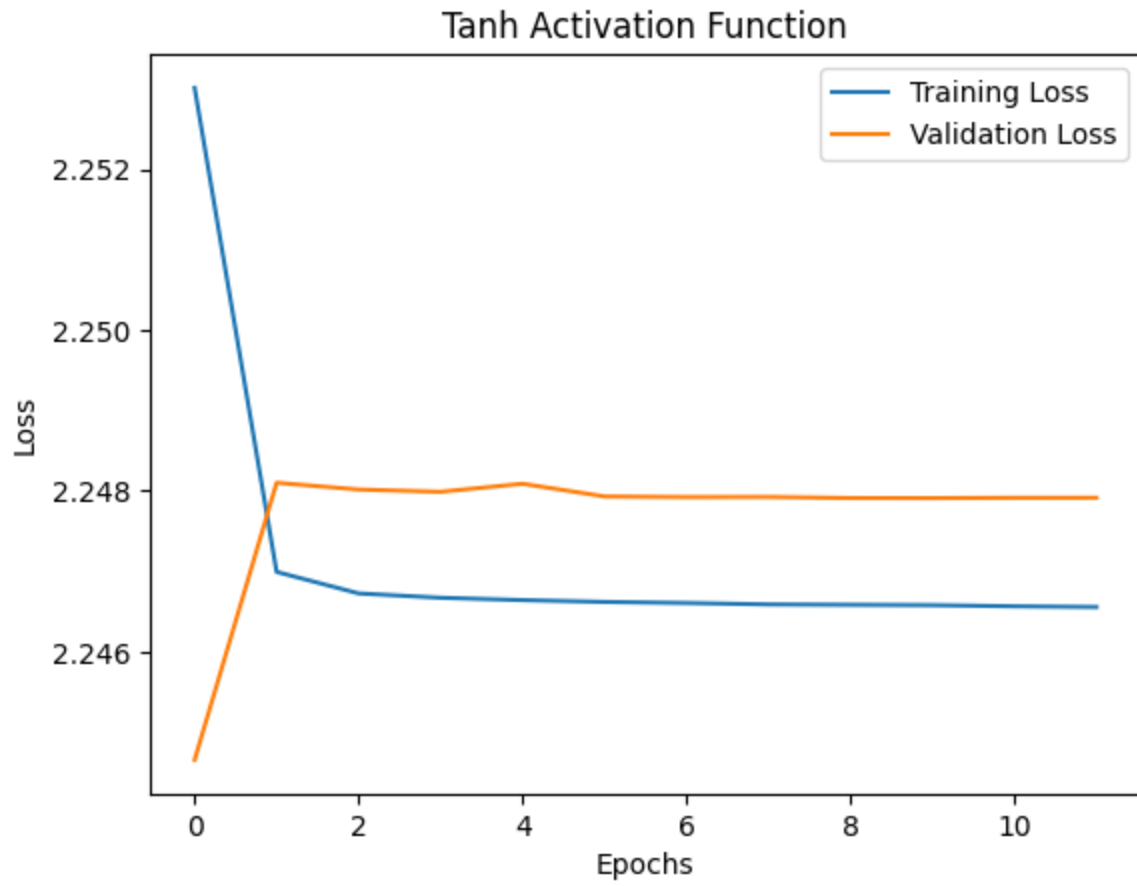
Pie Chart for the class labels distribution -

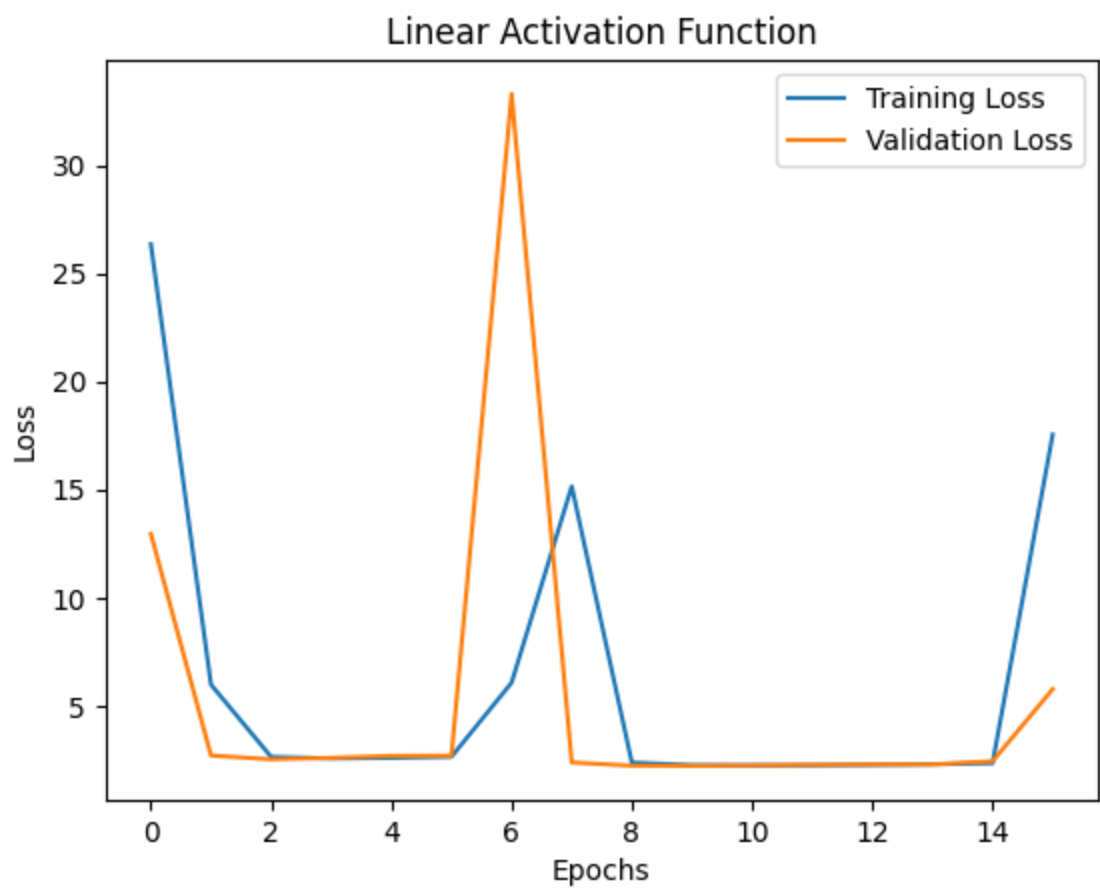


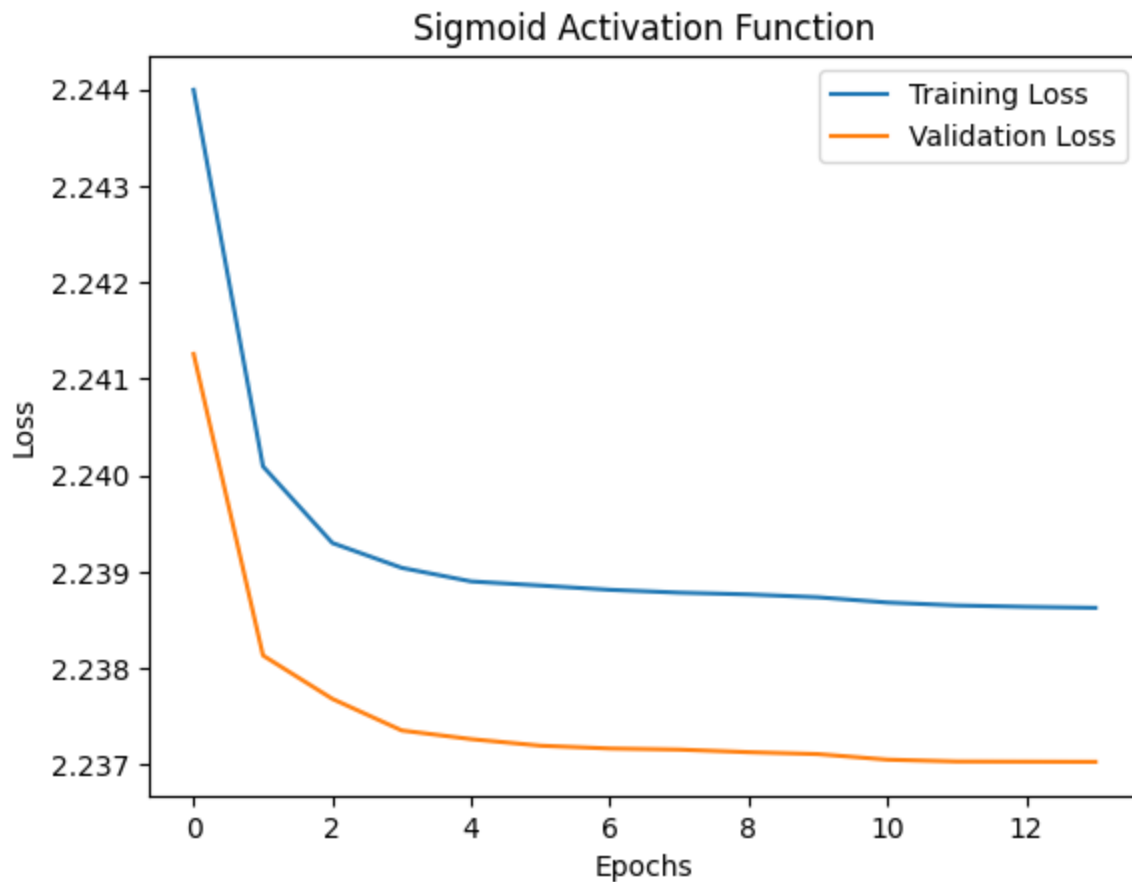


Activation Function Graphs for training loss and validation loss vs epochs -









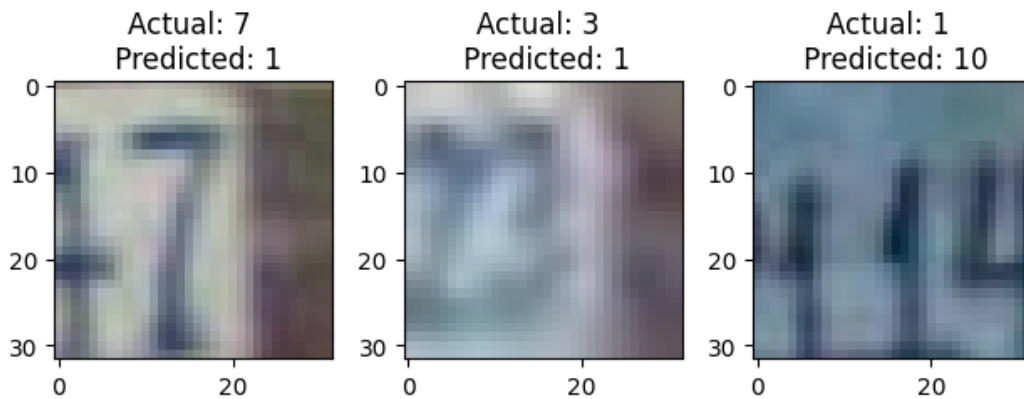
We get a decent accuracy after experimenting with different parameters for the MLPClassifier ~ 70%.

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20% | 40/200 [17:57<1:11:49, 26.94s/it]
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Stopping early ...
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Accuracy with hidden layer size (256, 128) on training set with activation function relu: 0.6924651924651924
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Misclassifications -



Misclassifications occur due to several reasons such as:

- The model might not have learned the distinguishing features of the classes well enough during training.
- The misclassified image might contain noise or other elements that confuse the model.
- The model might be overfitting to the training data, causing it to perform poorly on unseen data.