

my library for ICPC

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repo: [git@github.com:kmyk/competitive-programming-library.git](https://github.com/kmyk/competitive-programming-library)

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1 misc

1.1 environment.sh

```
1 #!/bin/bash
2
3 cat <<EOF > ~/.vimrc
4 syntax on
5 set smartindent
6 set tabstop=4
7 set shiftwidth=4
8 set expandtab
9 set relativenumber
10 EOF
11
12 setxkbmap -option ctrl:swapcaps
13
14 alias e=vim
15 alias cxx='CXX_Std=c++14_Wall_02'
16 alias cxxo='CXX_Std=c++14_Wall_03_mtune=native_march=native'
17 alias cxxg='CXX_Std=c++14_Wall_gu-fsanitize=undefined_D_GLIBCXX_DEBUG'
18
19 judge() {
20     for f in test/*.in ; do
21         echo $f
```

```

22         diff <(. /a.out < $f) ${f%.in}.out
23     done
24 }
25 print() {
26     for x in {A..Z} ; do
27         url=http://255.255.255.255/${x}_ja.html
28         if [[ $(curl -s $url -o /dev/null -w '%{http_code}') = 200 ]] ; then
29             google-chrome --headless --disable-gpu --print-to-pdf $url
30             lpr output.pdf
31         fi
32     done
33 }

```

1.2 template.cpp

```

1  :%! sh -c "'cat'"
2  #!/bin/sh
3  cat <<EOF
4  /**
5   * @file .cpp
6   * @author 'git config user.name'
7   * @date 'date +%a. %d, %Y'
8   */
9  #include <bits/stdc++.h>
10 #include <algorithm>
11 #include <array>
12 #include <cassert>
13 #include <climits>
14 #include <cmath>
15 #include <cstdio>
16 #include <functional>
17 #include <iostream>
18 #include <map>
19 #include <numeric>
20 #include <queue>
21 #include <set>
22 #include <tuple>
23 #include <unordered_map>
24 #include <unordered_set>
25 #include <vector>
26 #define REP(i, n) for (int i = 0; (i) < int(n); ++ (i))
27 #define REP3(i, m, n) for (int i = (m); (i) < int(n); ++ (i))
28 #define REP_R(i, n) for (int i = int(n) - 1; (i) >= 0; -- (i))
29 #define REP3R(i, m, n) for (int i = int(n) - 1; (i) >= int(m); -- (i))
30 #define ALL(x) begin(x), end(x)
31 #define dump(x) cerr << #x "=\n" << x << endl
32 #define unittest_name_helper(counter) unittest_ ## counter
33 #define unittest_name(counter) unittest_name_helper(counter)
34 #define unittest __attribute__((constructor)) void unittest_name(__COUNTER__) ()
35 using ll = long long;
36 using namespace std;
37 template <class T> using reversed_priority_queue = priority_queue<T, vector<T>, greater<T> >;
38 template <class T> inline void chmax(T & a, T const & b) { a = max(a, b); }
39 template <class T> inline void chmin(T & a, T const & b) { a = min(a, b); }
40 template <typename X, typename T> auto vectors(X x, T a) { return vector<T>(x, a); }
41 template <typename X, typename Y, typename... Zs> auto vectors(X x, Y y, Z z, Zs... zs) { auto cont = vectors(y, z, zs...); return vector<decltype(cont)>(x, cont); }
42 template <typename T> ostream & operator << (ostream & out, vector<T> const & xs) { REP (i, int(xs.size()) - 1) out << xs[i] << ' '; if (not xs.empty()) out << xs.back(); return out; }
43 void graphviz(vector<vector<int> > const & g, bool is_digraph = false, string const & name = "graph") { ofstream ofs(name + ".dot"); ofs << (is_digraph ? "di" : "") << "graphgraph_name{" << endl; ofs << "uuuugraph[bgcolor=\n\"#00000000\n\"] << endl; ofs << "uuuunode[shape=\ncircle,ustyle=\nfilled,\nfillcolor=\n\"#ffffff\n\"] << endl; REP (i, g.size()) for (int j : g[i]) if (is_digraph or i <= j) ofs << "uuuu" << i << (is_digraph ? "u->u" : "u-u") << j << endl; ofs << "}" << endl; ofs.close(); system(("dot\u-Tupng\u" + name + ".dot\u" + name + ".png").c_str()); } /* {{f}} */
44 const int dy[] = { -1, 1, 0, 0 };
45 const int dx[] = { 0, 0, 1, -1 };
46 bool is_on_field(int y, int x, int h, int w) { return 0 <= y and y < h and 0 <= x and x < w; }
47 int main() {
48     int n; scanf("%d", &n);
49     vector<ll> a(n); REP (i, n) scanf("%lld", &a[i]);
50     vector<vector<int> > f = vectors(h, w, int());
51     REP (y, h) REP (x, w) scanf("%d", &f[y][x]);
52     printf("%lld\n", ans);
53     return 0;
54 }
55 EOF

```

2 data structure

2.1 data-structure/binary-indexed-tree.inc.cpp

```

1  template <typename Monoid>
2  struct binary_indexed_tree { // on monoid
3      typedef typename Monoid::underlying_type underlying_type;
4      vector<underlying_type> data;
5      Monoid mon;
6      binary_indexed_tree(size_t n, Monoid const & a_mon = Monoid()) : mon(a_mon) {
7          data.resize(n, mon.unit());
8      }
9      void point_append(size_t i, underlying_type z) { // data[i] += z
10         for (size_t j = i + 1; j <= data.size(); j += j & -j) data[j - 1] = mon.append(data[j - 1], z);
11     }
12     underlying_type initial_range_concat(size_t i) { // sum [0, i)
13         underlying_type acc = mon.unit();
14         for (size_t j = i; 0 < j; j -= j & -j) acc = mon.append(data[j - 1], acc);
15         return acc;
16     }
17 };
18
19 unittest {
20     binary_indexed_tree<plus_t> bit(8);
21     bit.point_append(3, 4);
22     bit.point_append(4, 3);
23     bit.point_append(7, 1);
24     assert (bit.initial_range_concat(3) == 0);
25     assert (bit.initial_range_concat(5) == 7);
26     assert (bit.initial_range_concat(8) == 8);
27     bit.point_append(4, 2);
28     assert (bit.initial_range_concat(3) == 0);
29     assert (bit.initial_range_concat(5) == 9);
30     assert (bit.initial_range_concat(8) == 10);
31 }

```

2.2 data-structure/segment-tree.inc.cpp

```

1  /**
2   * @brief a segment tree, or a fenwick tree
3   * @tparam Monoid (commutativity is not required)

```

```

4  */
5  template <class Monoid>
6  struct segment_tree {
7      typedef typename Monoid::underlying_type underlying_type;
8      int n;
9      vector<underlying_type> a;
10     const Monoid mon;
11     segment_tree() = default;
12     segment_tree(int a_n, underlying_type initial_value = Monoid().unit(), Monoid const & a_mon = Monoid()) : mon(a_mon) {
13         n = 1; while (n < a_n) n *= 2;
14         a.resize(2 * n - 1, mon.unit());
15         fill(a.begin() + (n - 1), a.begin() + ((n - 1) + a_n), initial_value); // set initial values
16         REP_R (i, n - 1) a[i] = mon.append(a[2 * i + 1], a[2 * i + 2]); // propagate initial values
17     }
18     void point_set(int i, underlying_type z) { // 0-based
19         a[i + n - 1] = z;
20         for (i = (i + n) / 2; i > 0; i /= 2) { // 1-based
21             a[i - 1] = mon.append(a[2 * i - 1], a[2 * i]);
22         }
23     }
24     underlying_type range_concat(int l, int r) { // 0-based, [l, r)
25         underlying_type lacc = mon.unit(), racc = mon.unit();
26         for (l += n, r += n; l < r; l /= 2, r /= 2) { // 1-based loop, 2x faster than recursion
27             if (l % 2 == 1) lacc = mon.append(lacc, a[(l++) - 1]);
28             if (r % 2 == 1) racc = mon.append(a[(-- r) - 1], racc);
29         }
30         return mon.append(lacc, racc);
31     }
32 };
33 struct plus_monoid {
34     typedef int underlying_type;
35     int unit() const { return 0; }
36     int append(int a, int b) const { return a + b; }
37 };
38 template <int mod>
39 struct modplus_monoid {
40     typedef int underlying_type;
41     int unit() const { return 0; }
42     int append(int a, int b) const { int c = a + b; return c < mod ? c : c - mod; }
43 };
44 struct max_monoid {
45     typedef int underlying_type;
46     int unit() const { return 0; }
47     int append(int a, int b) const { return max(a, b); }
48 };
49 struct min_monoid {
50     typedef int underlying_type;
51     int unit() const { return INT_MAX; }
52     int append(int a, int b) const { return min(a, b); }
53 };

```

2.3 data-structure/dual-segment-tree.inc.cpp

```

1  template <class OperatorMonoid>
2  struct dual_segment_tree {
3      typedef typename OperatorMonoid::underlying_type operator_type;
4      typedef typename OperatorMonoid::target_type underlying_type;
5      int n;
6      vector<operator_type> f;
7      vector<underlying_type> a;
8      const OperatorMonoid op;
9      dual_segment_tree() = default;
10     dual_segment_tree(int a_n, underlying_type initial_value, OperatorMonoid const & a_op = OperatorMonoid()) : op(a_op) {
11         n = 1; while (n < a_n) n *= 2;
12         a.resize(n, initial_value);
13         f.resize(n-1, op.unit());
14     }
15     underlying_type point_get(int i) { // 0-based
16         underlying_type acc = a[i];
17         for (i = (i+n)/2; i > 0; i /= 2) { // 1-based
18             acc = op.apply(f[i-1], acc);
19         }
20         return acc;
21     }
22     void range_apply(int l, int r, operator_type z) { // 0-based, [l, r)
23         assert (0 <= l and l <= r and r <= n);
24         range_apply(0, 0, n, l, r, z);
25     }
26     void range_apply(int i, int il, int ir, int l, int r, operator_type z) {
27         if (l <= il and ir <= r) { // 0-based
28             if (i < f.size()) {
29                 f[i] = op.append(z, f[i]);
30             } else {
31                 a[i-n+1] = op.apply(z, a[i-n+1]);
32             }
33         } else if (ir <= l or r <= il) {
34             // nop
35         } else {
36             range_apply(2*i+1, il, (il+ir)/2, 0, n, f[i]);
37             range_apply(2*i+2, (il+ir)/2, ir, 0, n, f[i]);
38             f[i] = op.unit();
39             range_apply(2*i+1, il, (il+ir)/2, l, r, z);
40             range_apply(2*i+2, (il+ir)/2, ir, l, r, z);
41         }
42     }
43     // fast methods
44     inline underlying_type point_get(int i) {
45         return a[i + n - 1];
46     }
47     inline void point_set_primitive(int i, underlying_type z) {
48         a[i + n - 1] = z;
49     }
50     void point_set_commit() {
51         REP_R (i, n - 1) a[i] = mon.append(a[2 * i + 1], a[2 * i + 2]);
52     }
53 };
54
55 struct plus_operator_monoid {
56     typedef int underlying_type;
57     typedef int target_type;
58     int unit() const { return 0; }
59     int append(int a, int b) const { return a + b; }
60     int apply(int a, int b) const { return a + b; }
61 };
62 struct max_operator_monoid {
63     typedef int underlying_type;
64     typedef int target_type;
65     int unit() const { return INT_MIN; }
66     int append(int a, int b) const { return max(a, b); }
67     int apply(int a, int b) const { return max(a, b); }
68 };
69 struct min_operator_monoid {
70     typedef int underlying_type;
71     typedef int target_type;
72     int unit() const { return INT_MAX; }

```

```

73     int append(int a, int b) const { return min(a, b); }
74     int apply(int a, int b) const { return min(a, b); }
75 };
76
77 unittest {
78     dual_segment_tree<min_operator_monoid> segtree(12, 100);
79     segtree.range_apply(2, 7, 50);
80     segtree.range_apply(5, 9, 30);
81     segtree.range_apply(1, 11, 80);
82     assert (segtree.point_get( 0) == 100);
83     assert (segtree.point_get( 1) == 80);
84     assert (segtree.point_get( 2) == 50);
85     assert (segtree.point_get( 3) == 50);
86     assert (segtree.point_get( 4) == 50);
87     assert (segtree.point_get( 5) == 30);
88     assert (segtree.point_get( 6) == 30);
89     assert (segtree.point_get( 7) == 30);
90     assert (segtree.point_get( 8) == 30);
91     assert (segtree.point_get( 9) == 80);
92     assert (segtree.point_get(10) == 80);
93     assert (segtree.point_get(11) == 100);
94 }
95
96 template <int MOD>
97 struct linear_operator_monoid {
98     typedef pair<int, int> underlying_type;
99     typedef int target_type;
100     linear_operator_monoid() = default;
101     underlying_type unit() const {
102         return make_pair(1, 0);
103     }
104     underlying_type append(underlying_type g, underlying_type f) const {
105         target_type fst = g.first *(11) f.first % MOD;
106         target_type snd = (g.second + g.first *(11) f.second) % MOD;
107         return make_pair(fst, snd);
108     }
109     target_type apply(underlying_type f, target_type x) const {
110         return (f.first *(11) x + f.second) % MOD;
111     }
112 };

```

2.4 data-structure/lazy-propagation-segment-tree.inc.cpp

```

1  /**
2   * @note lazy_propagation_segment_tree<max_monoid, plus_operator_monoid> is the starry sky tree
3   * @note verified https://www.hackerrank.com/contests/world-codesprint-12/challenges/factorial-array/submissions/code/1304452669
4   * @note verified https://www.hackerrank.com/contests/world-codesprint-12/challenges/animal-transport/submissions/code/1304454860
5   */
6  template <class Monoid, class OperatorMonoid>
7  struct lazy_propagation_segment_tree { // on monoids
8      static_assert (is_same<typename Monoid::underlying_type, typename OperatorMonoid::target_type>::value, "");
9      typedef typename Monoid::underlying_type underlying_type;
10     typedef typename OperatorMonoid::underlying_type operator_type;
11     const Monoid mon;
12     const OperatorMonoid op;
13     int n;
14     vector<underlying_type> a;
15     vector<operator_type> f;
16     lazy_propagation_segment_tree() = default;
17     lazy_propagation_segment_tree(int a_n, underlying_type initial_value = Monoid().unit(), Monoid const & a_mon = Monoid(), OperatorMonoid const & a_op = OperatorMonoid()
18         ) : mon(a_mon), op(a_op) {
19         n = 1; while (n <= a_n) n *= 2;
20         a.resize(2 * n - 1, mon.unit());
21         fill(a.begin() + (n - 1), a.begin() + ((n - 1) + a_n), initial_value); // set initial values
22         REP_R (i, n - 1) a[i] = mon.append(a[2 * i + 1], a[2 * i + 2]); // propagate initial values
23         f.resize(max(0, (2 * n - 1) - n), op.identity());
24     }
25     void point_set(int i, underlying_type z) {
26         assert (0 <= i and i < n);
27         point_set(0, 0, n, i, z);
28     }
29     void point_set(int i, int il, int ir, int j, underlying_type z) {
30         if (i == n + j - 1) { // 0-based
31             a[i] = z;
32         } else if (ir <= j or j+1 <= il) {
33             // nop
34         } else {
35             range_apply(2 * i + 1, il, (il + ir) / 2, 0, n, f[i]);
36             range_apply(2 * i + 2, (il + ir) / 2, ir, 0, n, f[i]);
37             f[i] = op.identity();
38             point_set(2 * i + 1, il, (il + ir) / 2, j, z);
39             point_set(2 * i + 2, (il + ir) / 2, ir, j, z);
40             a[i] = mon.append(a[2 * i + 1], a[2 * i + 2]);
41         }
42     }
43     void range_apply(int l, int r, operator_type z) {
44         assert (0 <= l and l <= r and r <= n);
45         range_apply(0, 0, n, l, r, z);
46     }
47     void range_apply(int i, int il, int ir, int l, int r, operator_type z) {
48         if (l <= il and ir <= r) { // 0-based
49             a[i] = op.apply(z, a[i]);
50             if (i < f.size()) f[i] = op.compose(z, f[i]);
51         } else if (ir <= l or r <= il) {
52             // nop
53         } else {
54             range_apply(2 * i + 1, il, (il + ir) / 2, 0, n, f[i]);
55             range_apply(2 * i + 2, (il + ir) / 2, ir, 0, n, f[i]);
56             f[i] = op.identity();
57             range_apply(2 * i + 1, il, (il + ir) / 2, l, r, z);
58             range_apply(2 * i + 2, (il + ir) / 2, ir, l, r, z);
59             a[i] = mon.append(a[2 * i + 1], a[2 * i + 2]);
60         }
61     }
62     underlying_type range_concat(int l, int r) {
63         assert (0 <= l and l <= r and r <= n);
64         return range_concat(0, 0, n, l, r);
65     }
66     underlying_type range_concat(int i, int il, int ir, int l, int r) {
67         if (l <= il and ir <= r) { // 0-based
68             return a[i];
69         } else if (ir <= l or r <= il) {
70             return mon.unit();
71         } else {
72             return op.apply(f[i], mon.append(
73                 range_concat(2 * i + 1, il, (il + ir) / 2, l, r),
74                 range_concat(2 * i + 2, (il + ir) / 2, ir, l, r)));
75         }
76     }
77 };
78
79 struct max_monoid {
80     typedef int underlying_type;
81     int unit() const { return 0; }

```

```

82     int append(int a, int b) const { return max(a, b); }
83 };
84 struct plus_operator_monoid {
85     typedef int underlying_type;
86     typedef int target_type;
87     int identity() const { return 0; }
88     int apply(underlying_type a, target_type b) const { return a + b; }
89     int compose(underlying_type a, underlying_type b) const { return a + b; }
90 };
91 typedef lazy_propagation_segment_tree<max_monoid, plus_operator_monoid> starry_sky_tree;
92
93 struct min_monoid {
94     typedef int underlying_type;
95     int unit() const { return INT_MAX; }
96     int append(int a, int b) const { return min(a, b); }
97 };
98 struct plus_with_int_max_operator_monoid {
99     typedef int underlying_type;
100    typedef int target_type;
101    int identity() const { return 0; }
102    int apply(underlying_type a, target_type b) const { return b == INT_MAX ? INT_MAX : a + b; }
103    int compose(underlying_type a, underlying_type b) const { return a + b; }
104 };
105
106 unittest {
107     lazy_propagation_segment_tree<min_monoid, plus_with_int_max_operator_monoid> segtree(9);
108     segtree.point_set(2, 2);
109     segtree.point_set(3, 3);
110     segtree.point_set(4, 4);
111     segtree.point_set(6, 6);
112     assert (segtree.range_concat(2, 3) == 2);
113     assert (segtree.range_concat(5, 8) == 6);
114     segtree.range_apply(1, 4, 9);
115     assert (segtree.range_concat(3, 6) == 4);
116     assert (segtree.range_concat(0, 3) == 11);
117 }
118
119 template <int N>
120 struct count_monoid {
121     typedef array<int, N> underlying_type;
122     underlying_type unit() const { return underlying_type(); }
123     underlying_type append(underlying_type a, underlying_type b) const {
124         underlying_type c = {};
125         REP (i, N) c[i] = a[i] + b[i];
126         return c;
127     }
128 };
129 template <int N>
130 struct increment_operator_monoid {
131     typedef int underlying_type;
132     typedef array<int, N> target_type;
133     underlying_type identity() const { return 0; }
134     target_type apply(underlying_type a, target_type b) const {
135         if (a == 0) return b;
136         target_type c = {};
137         REP (i, N - a) c[i + a] = b[i];
138         return c;
139     }
140     underlying_type compose(underlying_type a, underlying_type b) const { return a + b; }
141 };

```

2.5 data-structure/dynamic-segment-tree.inc.cpp

```

1  /**
2   * @note verified http://arc054.contest.atcoder.jp/submissions/1335245
3   */
4  template <class Monoid>
5  struct dynamic_segment_tree { // on monoid
6      typedef Monoid monoid_type;
7      typedef typename Monoid::type underlying_type;
8      struct node_t {
9          int left, right; // indices on pool
10         underlying_type value;
11     };
12     deque<node_t> pool;
13     int root; // index
14     int width; // of the tree
15     int size; // the number of leaves
16     Monoid mon;
17     dynamic_segment_tree(Monoid const & a_mon = Monoid()) : mon(a_mon) {
18         node_t node = { -1, -1, mon.unit() };
19         pool.push_back(node);
20         root = 0;
21         width = 1;
22         size = 1;
23     }
24 protected:
25     int create_node(int parent, bool is_right) {
26         // make a new node
27         int i = pool.size();
28         node_t node = { -1, -1, mon.unit() };
29         pool.push_back(node);
30         // link from the parent
31         assert (parent != -1);
32         int & ptr = is_right ? pool[parent].right : pool[parent].left;
33         assert (ptr == -1);
34         ptr = i;
35         return i;
36     }
37     int get_value(int i) {
38         return i == -1 ? mon.unit() : pool[i].value;
39     }
40 public:
41     void point_set(int i, underlying_type z) {
42         assert (0 <= i);
43         while (width <= i) {
44             node_t node = { root, -1, pool[root].value };
45             root = pool.size();
46             pool.push_back(node);
47             width *= 2;
48         }
49         point_set(root, -1, false, 0, width, i, z);
50     }
51     void point_set(int i, int parent, bool is_right, int il, int ir, int j, underlying_type z) {
52         if (il == j and ir == j+1) { // 0-based
53             if (i == -1) {
54                 i = create_node(parent, is_right);
55                 size += 1;
56             }
57             pool[i].value = z;
58         } else if (ir <= j or j+1 <= il) {
59             // nop
60         } else {
61             if (i == -1) i = create_node(parent, is_right);
62             point_set(pool[i].left, i, false, il, (il+ir)/2, j, z);

```

```

63     point_set(pool[i].right, i, true, (il+ir)/2, ir, j, z);
64     pool[i].value = mon.append(get_value(pool[i].left), get_value(pool[i].right));
65 }
66 }
67 underlying_type range_concat(int l, int r) {
68     assert(0 <= l and l <= r);
69     if (width <= 1) return mon.unit();
70     return range_concat(root, 0, width, l, min(width, r));
71 }
72 underlying_type range_concat(int i, int il, int ir, int l, int r) {
73     if (i == -1) return mon.unit();
74     if (l <= il and ir <= r) { // 0-based
75         return pool[i].value;
76     } else if (ir <= l or r <= il) {
77         return mon.unit();
78     } else {
79         return mon.append(
80             range_concat(pool[i].left, il, (il+ir)/2, l, r),
81             range_concat(pool[i].right, (il+ir)/2, ir, l, r));
82     }
83 }
84 template <class Func>
85 void traverse_leaves(Func func) {
86     return traverse_leaves(root, 0, width, func);
87 }
88 template <class Func>
89 void traverse_leaves(int i, int il, int ir, Func func) {
90     if (i == -1) return;
91     if (ir - il == 1) {
92         func(il, pool[i].value);
93     } else {
94         traverse_leaves(pool[i].left, il, (il+ir)/2, func);
95         traverse_leaves(pool[i].right, (il+ir)/2, ir, func);
96     }
97 }
98 };

```

2.6 data-structure/union-find-tree.inc.cpp

```

1 struct union_find_tree {
2     vector<int> data;
3     union_find_tree() = default;
4     explicit union_find_tree(size_t n) : data(n, -1) {}
5     bool is_root(int i) { return data[i] < 0; }
6     int find_root(int i) { return is_root(i) ? i : (data[i] = find_root(data[i])); }
7     int tree_size(int i) { return -data[find_root(i)]; }
8     int unite_trees(int i, int j) {
9         i = find_root(i); j = find_root(j);
10        if (i != j) {
11            if (tree_size(i) < tree_size(j)) swap(i, j);
12            data[i] += data[j];
13            data[j] = i;
14        }
15        return i;
16    }
17    bool is_same(int i, int j) { return find_root(i) == find_root(j); }
18 };

```

2.7 data-structure/treap.inc.cpp

```

1 #include <random>
2 #include <memory>
3
4 // https://www.hackerrank.com/contests/zalando-codesprint/challenges/give-me-the-order/submissions/code/6004391
5 template <typename T>
6 struct treap {
7     typedef T value_type;
8     typedef double key_type;
9     value_type v;
10    key_type k;
11    shared_ptr<treap> l, r;
12    size_t m_size;
13    treap(value_type v)
14        : v(v)
15        , k(generate())
16        , l()
17        , r()
18        , m_size(1) {
19    }
20    static size_t size(shared_ptr<treap> const & t) {
21        return t ? t->m_size : 0;
22    }
23    static shared_ptr<treap> merge(shared_ptr<treap> const & a, shared_ptr<treap> const & b) { // destructive
24        if (not a) return b;
25        if (not b) return a;
26        if (a->k > b->k) {
27            a->r = merge(a->r, b);
28            return update(a);
29        } else {
30            b->l = merge(a, b->l);
31            return update(b);
32        }
33    }
34    static pair<shared_ptr<treap>, shared_ptr<treap> > split(shared_ptr<treap> const & t, size_t i) { // [0, i) [i, n), destructive
35        if (not t) return { shared_ptr<treap>(), shared_ptr<treap>() };
36        if (i <= size(t->l)) {
37            shared_ptr<treap> u; tie(u, t->l) = split(t->l, i);
38            return { u, update(t) };
39        } else {
40            shared_ptr<treap> u; tie(t->r, u) = split(t->r, i - size(t->l) - 1);
41            return { update(t), u };
42        }
43    }
44    static shared_ptr<treap> insert(shared_ptr<treap> const & t, size_t i, value_type v) { // destructive
45        shared_ptr<treap> l, r; tie(l, r) = split(t, i);
46        shared_ptr<treap> u = make_shared<treap>(v);
47        return merge(merge(l, u), r);
48    }
49    static pair<shared_ptr<treap>, shared_ptr<treap> > erase(shared_ptr<treap> const & t, size_t i) { // (t \ t_i, t_i), destructive
50        shared_ptr<treap> l, u, r;
51        tie(l, r) = split(t, i + 1);
52        tie(l, u) = split(l, i);
53        return { merge(l, r), u };
54    }
55 private:
56    static shared_ptr<treap> update(shared_ptr<treap> const & t) {
57        if (t) {
58            t->m_size = 1 + size(t->l) + size(t->r);
59        }
60        return t;
61    }
62    static key_type generate() {
63        static random_device device;

```

```

64     static default_random_engine engine(device());
65     static uniform_real_distribution<double> dist;
66     return dist(engine);
67 }
68 };

```

2.8 data-structure/sparse-table.inc.cpp

```

1  /**
2   * @brief sparse table on a semilattice
3   * @note a semilattice is a commutative idempotent semigroup
4   * @note for convenience, it requires the unit
5   * @note space:  $O(N \log N)$ 
6   * @note time:  $O(N \log N)$  for construction;  $O(1)$  for query
7   */
8  template <class Semilattice>
9  struct sparse_table {
10     typedef typename Semilattice::underlying_type underlying_type;
11     vector<vector<underlying_type> > table;
12     Semilattice lat;
13     sparse_table() = default;
14     sparse_table(vector<underlying_type> const & data, Semilattice const & a_lat = Semilattice())
15         : lat(a_lat) {
16         int n = data.size();
17         int log_n = 32 - __builtin_clz(n);
18         table.resize(log_n, vector<underlying_type>(n));
19         table[0] = data;
20         REP (k, log_n - 1) {
21             REP (i, n) {
22                 table[k + 1][i] = i + (1ll << k) < n ?
23                     lat.append(table[k][i], table[k][i + (1ll << k)]) :
24                     table[k][i];
25             }
26         }
27     }
28     underlying_type range_concat(int l, int r) const {
29         if (l == r) return lat.unit(); // if there is no unit, remove this line
30         assert (0 <= l and l < r and r <= table[0].size());
31         int k = 31 - __builtin_clz(r - l); // log2
32         return lat.append(table[k][l], table[k][r - (1ll << k)]);
33     }
34 };
35 struct max_semilattice {
36     typedef int underlying_type;
37     int unit() const { return INT_MIN; }
38     int append(int a, int b) const { return max(a, b); }
39 };
40 struct gcd_semilattice {
41     typedef int underlying_type;
42     int unit() const { return 0; }
43     int append(int a, int b) const { return gcd(a, b); }
44 };
45
46 unittest {
47     random_device device;
48     default_random_engine gen(device());
49     int n = uniform_int_distribution<int>(1, 500)(gen);
50     vector<int> data(n);
51     REP (i, n) data[i] = uniform_int_distribution<int>(- 1000000, 1000000)(gen);
52     sparse_table<max_semilattice> table(data);
53     auto range_concat = [&](int l, int r) {
54         int acc = INT_MIN;
55         REP3 (i, l, r) chmax(acc, data[i]);
56         return acc;
57     };
58     REP (iteration, 500) {
59         int l = uniform_int_distribution<int>(0, n - 1)(gen);
60         int r = uniform_int_distribution<int>(l + 1, n)(gen);
61         assert (table.range_concat(l, r) == range_concat(l, r));
62     }
63 }

```

2.9 data-structure/sliding-window.inc.cpp

```

1  /**
2   * @brief the sliding window minimum algorithm
3   * @note to get maximums, use greater<T>
4   * @note verified http://poj.org/problem?id=2823
5   * @note verified http://cf16-tournament-round3-open.contest.atcoder.jp/tasks/asaporo-d
6   */
7  template <typename T, class Compare = less<T> >
8  struct sliding_window {
9     deque<pair<int, T> > data;
10     function<bool (T const &, T const &)> compare;
11     sliding_window(Compare const & a_compare = Compare()) : compare(a_compare) {}
12     T front() { return data.front().second; } //  $O(1)$ , minimum
13     void push_back(int i, T a) { while (not data.empty() and compare(a, data.back().second)) data.pop_back(); data.emplace_back(i, a); } //  $O(1)$  amortized.
14     void pop_front(int i) { if (data.front().first == i) data.pop_front(); }
15     void push_front(int i, T a) { if (data.empty() or not compare(data.front().second, a)) data.emplace_front(i, a); }
16 };

```

3 graph

3.1 graph/ford-fulkerson.inc.cpp

```

1  struct edge_t { int to, cap, rev; };
2  int maximum_flow_destructive(int s, int t, vector<vector<edge_t> > & g) { // ford fulkerson,  $O(EF)$ 
3      int n = g.size();
4      vector<bool> used(n);
5      function<int (int, int)> dfs = [&](int i, int f) {
6          if (i == t) return f;
7          used[i] = true;
8          for (edge_t & e : g[i]) {
9              if (used[e.to] or e.cap <= 0) continue;
10             int nf = dfs(e.to, min(f, e.cap));
11             if (nf > 0) {
12                 e.cap -= nf;
13                 g[e.to][e.rev].cap += nf;
14                 return nf;
15             }
16         }
17         return 0;
18     };
19     int result = 0;
20     while (true) {
21         used.clear(); used.resize(n);
22         int f = dfs(s, numeric_limits<int>::max());

```

```

23     if (f == 0) break;
24     result += f;
25 }
26 return result;
27 }
28 void add_edge(vector<vector<edge_t> > &g, int from, int to, int cap) {
29     g[from].push_back((edge_t) { to, cap, int(g[to].size() ) });
30     g[to].push_back((edge_t) { from, 0, int(g[from].size() - 1) });
31 }
32 int maximum_flow(int s, int t, vector<vector<edge_t> > g /* adjacency list */) { // ford fulkerson, O(FE)
33     return maximum_flow_destructive(s, t, g);
34 }
35
36 vector<pair<int,int> > perfect_bipartite_matching(set<int> const &a, set<int> const &b, vector<vector<int> > const &g /* adjacency list */) { // O(V + FE)
37     assert (a.size() + b.size() <= g.size());
38     int n = g.size();
39     int src = n;
40     int dst = n + 1;
41     vector<vector<edge_t> > h(n + 2);
42     auto add_edge = [&](int from, int to, int cap) {
43         h[from].push_back((edge_t) { to, cap, int(h[to].size() ) });
44         h[to].push_back((edge_t) { from, 0, int(h[from].size() - 1) });
45     };
46     repeat (i,n) {
47         if (a.count(i)) {
48             add_edge(src, i, 1);
49             for (int j : g[i]) if (b.count(j)) {
50                 add_edge(i, j, 1); // collect edges e : a -> b, from g
51             }
52         }
53         if (b.count(i)) {
54             add_edge(i, dst, 1);
55         }
56     }
57     maximum_flow_destructive(src, dst, h);
58     vector<pair<int,int> > ans;
59     for (int from : a) {
60         for (edge_t e : h[from]) if (b.count(e.to) and e.cap == 0) {
61             ans.emplace_back(from, e.to);
62         }
63     }
64     return ans;
65 }

```

3.2 graph/dinic.inc.cpp

```

1 // https://kimiyuki.net/blog/2016/01/16/arc-031-d/
2 double maximum_flow(int s, int t, vector<vector<double> > const &capacity /* adjacency matrix */) { // dinic, O(V^2E)
3     int n = capacity.size();
4     vector<vector<double> > flow(n, vector<double>(n));
5     auto residue = [&](int i, int j) { return capacity[i][j] - flow[i][j]; };
6     vector<vector<int> > g(n); repeat (i,n) repeat (j,n) if (capacity[i][j] or capacity[j][i]) g[i].push_back(j); // adjacency list
7     double result = 0;
8     while (true) {
9         vector<int> level(n, -1); level[s] = 0;
10        queue<int> q; q.push(s);
11        for (int d = n; not q.empty() and level[q.front()] < d; ) {
12            int i = q.front(); q.pop();
13            if (i == t) d = level[i];
14            for (int j : g[i]) if (level[j] == -1 and residue(i,j) > 0) {
15                level[j] = level[i] + 1;
16                q.push(j);
17            }
18        }
19        vector<bool> finished(n);
20        function<double (int, double)> augmenting_path = [&](int i, double cur) -> double {
21            if (i == t or cur == 0) return cur;
22            if (finished[i]) return 0;
23            finished[i] = true;
24            for (int j : g[i]) if (level[i] < level[j]) {
25                double f = augmenting_path(j, min(cur, residue(i,j)));
26                if (f > 0) {
27                    flow[i][j] += f;
28                    flow[j][i] -= f;
29                    finished[i] = false;
30                    return f;
31                }
32            }
33            return 0;
34        };
35        bool cont = false;
36        while (true) {
37            double f = augmenting_path(s, numeric_limits<double>::max());
38            if (f == 0) break;
39            result += f;
40            cont = true;
41        }
42        if (not cont) break;
43    }
44    return result;
45 }
46
47 // https://kimiyuki.net/blog/2017/10/22/kupc-2017-h/
48 uint64_t pack(int i, int j) {
49     return (uint64_t(i) << 32) | j;
50 }
51 ll maximum_flow(int s, int t, int n, unordered_map<uint64_t, ll> &capacity /* adjacency matrix */) { // dinic, O(V^2E)
52     auto residue = [&](int i, int j) { auto key = pack(i, j); return capacity.count(key) ? capacity[key] : 0; };
53     vector<vector<int> > g(n); repeat (i,n) repeat (j,n) if (residue(i, j) or residue(j, i)) g[i].push_back(j); // adjacency list
54     ll result = 0;
55     while (true) {
56         vector<int> level(n, -1); level[s] = 0;
57         queue<int> q; q.push(s);
58         for (int d = n; not q.empty() and level[q.front()] < d; ) {
59             int i = q.front(); q.pop();
60             if (i == t) d = level[i];
61             for (int j : g[i]) if (level[j] == -1 and residue(i,j) > 0) {
62                 level[j] = level[i] + 1;
63                 q.push(j);
64             }
65         }
66         vector<bool> finished(n);
67         function<ll (int, ll)> augmenting_path = [&](int i, ll cur) -> ll {
68             if (i == t or cur == 0) return cur;
69             if (finished[i]) return 0;
70             finished[i] = true;
71             for (int j : g[i]) if (level[i] < level[j]) {
72                 ll f = augmenting_path(j, min(cur, residue(i,j)));
73                 if (f > 0) {
74                     capacity[pack(i, j)] -= f;
75                     capacity[pack(j, i)] += f;
76                     finished[i] = false;
77                     return f;
78                 }
79             }
80         }

```



```

80         return 0;
81     };
82     bool cont = false;
83     while (true) {
84         ll f = augmenting_path(s, numeric_limits<ll>::max());
85         if (f == 0) break;
86         result += f;
87         cont = true;
88     }
89     if (not cont) break;
90 }
91 return result;
92 }

```

3.3 graph/minimum-cost-flow.inc.cpp

```

1  template <class T>
2  struct edge { int to; T cap, cost; int rev; };
3  template <class T>
4  void add_edge(vector<vector<edge<T> > > & graph, int from, int to, T cap, T cost) {
5      graph[from].push_back((edge<T>) { to, cap, cost, int(graph[to].size()) });
6      graph[to].push_back((edge<T>) { from, 0, -cost, int(graph[from].size()) - 1 });
7  }
8  /**
9   * @brief minimum-cost flow with primal-dual method
10   * @note mainly  $O(V^2UC)$  for  $U$  is the sum of capacities and  $C$  is the sum of costs. and additional  $O(VE)$  if negative edges exist
11   */
12  template <class T>
13  T min_cost_flow_destructive(int src, int dst, T flow, vector<vector<edge<T> > > & graph) {
14      T result = 0;
15      vector<T> potential(graph.size());
16      if (0 < flow) { // initialize potential when negative edges exist (slow). you can remove this if unnecessary
17          fill(ALL(potential), numeric_limits<T>::max());
18          potential[src] = 0;
19          while (true) { // Bellman-Ford algorithm
20              bool updated = false;
21              REP (e_from, graph.size()) for (auto & e : graph[e_from]) if (e.cap) {
22                  if (potential[e_from] == numeric_limits<T>::max()) continue;
23                  if (potential[e.to] > potential[e_from] + e.cost) {
24                      potential[e.to] = potential[e_from] + e.cost; // min
25                      updated = true;
26                  }
27              }
28              if (not updated) break;
29          }
30      }
31      while (0 < flow) {
32          // update potential using dijkstra
33          vector<T> distance(graph.size(), numeric_limits<T>::max()); // minimum distance
34          vector<int> prev_v(graph.size()); // constitute a single-linked-list represents the flow-path
35          vector<int> prev_e(graph.size());
36          { // initialize distance and prev_v, e
37              reversed_priority_queue<pair<T, int> > que; // distance * vertex
38              distance[src] = 0;
39              que.emplace(0, src);
40              while (not que.empty()) { // Dijkstra's algorithm
41                  T d; int v; tie(d, v) = que.top(); que.pop();
42                  if (potential[v] == numeric_limits<T>::max()) continue; // for unreachable nodes
43                  if (distance[v] < d) continue;
44                  // look round the vertex
45                  REP (e_index, graph[v].size()) {
46                      // consider updating
47                      edge<T> e = graph[v][e_index];
48                      int w = e.to;
49                      if (potential[w] == numeric_limits<T>::max()) continue;
50                      T d1 = distance[v] + e.cost + potential[v] - potential[w]; // updated distance
51                      if (0 < e.cap and d1 < distance[e.to]) {
52                          distance[w] = d1;
53                          prev_v[w] = v;
54                          prev_e[w] = e_index;
55                          que.emplace(d1, w);
56                      }
57                  }
58              }
59          }
60          if (distance[dst] == numeric_limits<T>::max()) return -1; // no such flow
61          REP (v, graph.size()) {
62              if (potential[v] == numeric_limits<T>::max()) continue;
63              potential[v] += distance[v];
64          }
65          // finish updating the potential
66          // let flow on the src->dst minimum path
67          T delta = flow; // capacity of the path
68          for (int v = dst; v != src; v = prev_v[v]) {
69              chmin(delta, graph[prev_v[v]][prev_e[v]].cap);
70          }
71          flow -= delta;
72          result += delta * potential[dst];
73          for (int v = dst; v != src; v = prev_v[v]) {
74              edge<T> & e = graph[prev_v[v]][prev_e[v]]; // reference
75              e.cap -= delta;
76              graph[v][e.rev].cap += delta;
77          }
78      }
79      return result;
80  }

```

3.4 graph/two-edge-connected-components.inc.cpp

```

1  /**
2   * @brief 2-edge-connected components decomposition
3   * @param g an adjacent list of the simple undirected graph
4   * @note  $O(V + E)$ 
5   */
6  pair<int, vector<int> > decompose_to_two_edge_connected_components(vector<vector<int> > const & g) {
7      int n = g.size();
8      vector<int> imos(n); { // imos[i] == 0 iff the edge i -> parent is a bridge
9          vector<char> used(n); // 0: unused ; 1: exists on stack ; 2: removed from stack
10         function<void (int, int)> go = [&](int i, int parent) {
11             used[i] = 1;
12             for (int j : g[i]) if (j != parent) {
13                 if (used[j] == 0) {
14                     go(j, i);
15                     imos[i] += imos[j];
16                 } else if (used[j] == 1) {
17                     imos[i] += 1;
18                     imos[j] -= 1;
19                 }
20             }
21             used[i] = 2;
22         };
23         repeat (i, n) if (used[i] == 0) {
24             go(i, -1);

```

```

25     }
26 }
27 int size = 0;
28 vector<int> component_of(n, -1); {
29     function<void (int)> go = [&](int i) {
30         for (int j : g[i]) if (component_of[j] == -1) {
31             component_of[j] = imos[j] == 0 ? size++ : component_of[i];
32             go(j);
33         }
34     };
35     repeat (i, n) if (component_of[i] == -1) {
36         component_of[i] = size++;
37         go(i);
38     }
39 }
40 return { size, move(component_of) };
41 }

```

4 modulus

4.1 modulus/powmod.inc.cpp

```

1  /**
2   * @param m must be a positive integer
3   * @note O(log y)
4   */
5  ll powmod(ll x, ll y, ll m) {
6      assert (0 <= x and x < m);
7      assert (0 <= y);
8      ll z = 1;
9      for (ll i = 1; i <= y; i <= 1) {
10         if (y & i) z = z * x % m;
11         x = x * x % m;
12     }
13     return z;
14 }
15 /**
16 * @param p must be a prime
17 * @note O(log p)
18 */
19 ll modinv(ll x, ll p) {
20     assert (x % p != 0);
21     return powmod(x, p - 2, p);
22 }

```

4.2 modulus/extgcd.inc.cpp

```

1  /**
2   * @brief extended gcd
3   * @description for given a and b, find x, y and gcd(a, b) such that ax + by = 1
4   * @note O(log n)
5   * @see https://topcoder.g.hatena.ne.jp/spaghetti_source/20130126/1359171466
6   */
7  tuple<ll, ll, ll> extgcd(ll a, ll b) {
8      ll x = 0, y = 1;
9      for (ll u = 1, v = 0; a; ) {
10         ll q = b / a;
11         swap(x -= q * u, u);
12         swap(y -= q * v, v);
13         swap(b -= q * a, a);
14     }
15     return make_tuple(x, y, d);
16 }
17 unittest {
18     random_device device;
19     default_random_engine gen(device());
20     REP (iteration, 1000) {
21         ll a = uniform_int_distribution<ll>(1, 10000)(gen);
22         ll b = uniform_int_distribution<ll>(1, 10000)(gen);
23         ll x, y, d; tie(x, y, d) = extgcd(a, b);
24         assert (a * x + b * y == 1);
25         assert (d == __gcd(a, b));
26     }
27 }
28 /**
29 * @note recursive version (slow)
30 */
31 pair<int, int> extgcd_recursive(int a, int b) {
32     if (b == 0) return { 1, 0 };
33     int na, nb; tie(na, nb) = extgcd(b, a % b);
34     return { nb, na - a/b * nb };
35 }
36 /**
37 * @note x and n must be relatively prime
38 * @note O(log n)
39 */
40 ll modinv(ll x, ll n) {
41     assert (1 <= x and x < n);
42     ll y = get<0>(extgcd(x, n)) % n;
43     return y >= 0 ? y : y + n;
44 }
45 }
46

```

5 number

5.1 number/gcd.inc.cpp

```

1  /**
2   * @note if arguments are negative, the result may be negative
3   */
4  template <typename T>
5  T gcd(T a, T b) {
6      while (a) {
7         b %= a;
8         swap(a, b);
9     }
10     return b;
11 }
12 template <typename T>
13 T lcm(T a, T b) {
14     return a / gcd(a, b) * b;
15 }
16
17 unittest {

```

```

18     assert (gcd(0, 0) == 0);
19     assert (gcd(42, 0) == 42);
20     assert (gcd(0, 42) == 42);
21     assert (gcd(3, -12) == 3);
22     assert (gcd(-3, 12) == -3);
23     assert (gcd(7, -12) == -1);
24     assert (gcd(-7, 12) == 1);
25     assert (gcd(-9, -12) == -3);
26     assert (gcd(-1, -1) == -1);
27 }

```

5.2 number/primes.inc.cpp

```

1  /**
2   * @brief enumerate primes in [2, n) with  $O(n \log \log n)$ 
3   */
4  vector<bool> sieve_of_eratosthenes(int n) {
5      vector<bool> is_prime(n, true);
6      is_prime[0] = is_prime[1] = false;
7      for (int i = 2; i * (ll) i < n; ++ i)
8          if (is_prime[i])
9              for (int k = 2 * i; k < n; k += i)
10                 is_prime[k] = false;
11      return is_prime;
12 }
13 vector<int> list_primes(int n) {
14     auto is_prime = sieve_of_eratosthenes(n);
15     vector<int> primes;
16     for (int i = 2; i < n; ++ i)
17         if (is_prime[i])
18             primes.push_back(i);
19     return primes;
20 }
21
22 /**
23  * @note the last number of primes must be  $\geq \sqrt{n}$ 
24  */
25 map<ll, int> prime_factorize(ll n, vector<int> const & primes) {
26     map<ll, int> result;
27     for (int p : primes) {
28         if (n < p * (ll) p) break;
29         while (n % p == 0) {
30             result[p] += 1;
31             n /= p;
32         }
33     }
34     if (n != 1) result[n] += 1;
35     return result;
36 }
37
38 /**
39  * @note if  $n < 10^9$ ,  $d(n) < 1200 + a$ 
40  */
41 vector<ll> list_factors(ll n, vector<int> const & primes) {
42     vector<ll> result;
43     result.push_back(1);
44     for (auto it : prime_factorize(n, primes)) {
45         ll p; int k; tie(p, k) = it;
46         int size = result.size();
47         REP (y, k) {
48             REP (x, size) {
49                 result.push_back(result[y * size + x] * p);
50             }
51         }
52     }
53     return result;
54 }
55
56 vector<ll> list_prime_factors(ll n, vector<int> const & primes) {
57     vector<ll> result;
58     for (int p : primes) {
59         if (n < p * (ll) p) break;
60         while (n % p == 0) {
61             result.push_back(p);
62             n /= p;
63         }
64     }
65     if (n != 1) result.push_back(n);
66     return result;
67 }
68
69 /**
70  * @brief fully factorize all numbers in [0, n) with  $O(n \log \log n)$ 
71  */
72 vector<vector<int>> extended_sieve_of_eratosthenes(int n) {
73     vector<vector<int>> prime_factors(n + 1);
74     REP3 (i, 2, n) {
75         if (prime_factors[i].empty()) {
76             for (int k = i; k < n; k += i) {
77                 prime_factors[k].push_back(i);
78             }
79         }
80     }
81     return prime_factors;
82 }

```

5.3 number/matrix.inc.cpp

```

1  template <typename T>
2  vector<vector<T>> operator * (vector<vector<T>> const & a, vector<vector<T>> const & b) {
3      int n = a.size();
4      vector<vector<T>> > c = vectors(n, n, T());
5      REP (y, n) REP (z, n) REP (x, n) c[y][x] += a[y][z] * b[z][x];
6      return c;
7  }
8  template <typename T>
9  vector<T> operator * (vector<vector<T>> const & a, vector<T> const & b) {
10     int n = a.size();
11     vector<T> c(n);
12     REP (y, n) REP (z, n) c[y] += a[y][z] * b[z];
13     return c;
14 }
15 template <typename T>
16 vector<vector<T>> > unit_matrix(int n) {
17     vector<vector<T>> > e = vectors(n, n, T());
18     REP (i, n) e[i][i] = 1;
19     return e;
20 }
21 template <typename T>
22 vector<vector<T>> > zero_matrix(int n) {
23     vector<vector<T>> > o = vectors(n, n, T());
24     return o;
25 }

```

```

26
27 template <typename T>
28 T determinant(vector<vector<T> > a) {
29     int n = a.size();
30     REP (z, n) { // make A upper triangular
31         if (a[z][z] == 0) { // swap rows to avoid zero-division
32             int x = z + 1;
33             for (; x < n; ++x) {
34                 if (a[x][z] != 0) {
35                     a[z].swap(a[x]);
36                     break;
37                 }
38             }
39             if (x == n) return 0; // A is singular
40         }
41         REP3 (y, z + 1, n) {
42             T k = a[y][z] / a[z][z];
43             REP3 (x, z + 1, n) {
44                 a[y][x] -= k * a[z][x]; // elim
45             }
46             a[y][z] = 0;
47         }
48     }
49     T acc = 1;
50     REP (z, n) acc *= a[z][z]; // product of the diagonal elems
51     return acc;
52 }
53
54 template <class T>
55 vector<vector<T> > small_matrix(vector<vector<T> > const & a) {
56     int n = a.size();
57     assert (n >= 1);
58     auto b = a;
59     b.resize(n - 1);
60     REP (y, n - 1) {
61         b[y].resize(n - 1);
62     }
63     return b;
64 }
65
66 template <typename T>
67 vector<T> gaussian_elimination(vector<vector<T> > f, vector<T> x) {
68     int n = x.size();
69     REP (y, n) {
70         int pivot = y;
71         while (pivot < n and abs(f[pivot][y]) < eps) ++ pivot;
72         assert (pivot < n);
73         swap(f[y], f[pivot]);
74         x[y] /= f[y][y];
75         REP3 (x, y + 1, n) f[y][x] /= f[y][y];
76         f[y][y] = 1;
77         REP (ny, n) if (ny != y) {
78             x[ny] -= f[ny][y] * x[y];
79             REP3 (x, y + 1, n) f[ny][x] -= f[ny][y] * f[y][x];
80             f[ny][y] = 0;
81         }
82     }
83     return x;
84 }
85
86 constexpr double eps = 1e-8;
87 template <typename T>
88 vector<vector<T> > inverse_matrix(vector<vector<T> > f) {
89     int n = f.size();
90     vector<vector<T> > g = unit_matrix<T>(n);
91     REP (y, n) {
92         int pivot = y;
93         while (pivot < n and abs(f[pivot][y]) < eps) ++ pivot;
94         assert (pivot < n);
95         swap(f[y], f[pivot]);
96         REP (x, n) g[y][x] /= f[y][y];
97         REP3 (x, y + 1, n) f[y][x] /= f[y][y];
98         f[y][y] = 1;
99         REP (ny, n) if (ny != y) {
100             REP (x, n) g[ny][x] -= f[ny][y] * g[y][x];
101             REP3 (x, y + 1, n) f[ny][x] -= f[ny][y] * f[y][x];
102             f[ny][y] = 0;
103         }
104     }
105     return g;
106 }
107
108 unittest {
109     vector<vector<double> > f { { 1, 2 }, { 3, 4 } };
110     auto g = f * inverse_matrix(f);
111     assert (abs(g[0][0] - 1) < eps);
112     assert (abs(g[0][1] ) < eps);
113     assert (abs(g[1][0] ) < eps);
114     assert (abs(g[1][1] - 1) < eps);
115 }
116
117 template <typename T>
118 vector<vector<T> > powmat(vector<vector<T> > x, ll y) {
119     int n = x.size();
120     auto z = unit_matrix<T>(n);
121     for (ll i = 1; i <= y; i <= 1) {
122         if (y & i) z = z * x;
123         x = x * x;
124     }
125     return z;
126 }
127
128 template <typename T>
129 vector<vector<T> > concat_matrix_vector(vector<vector<T> > const & f, vector<T> const & x) {
130     int h = f.size();
131     int w = f[0].size();
132     vector<vector<T> > fx(h);
133     REP (y, h) {
134         fx[y].resize(w + 1);
135         copy(whole(f[y]), fx[y].begin());
136         fx[y][w] = x[y];
137     }
138     return fx;
139 }

```

6 string

6.1 string/palindrome.inc.cpp

```

1 // http://snuke.hatenablog.com/entry/2014/12/02/235837
2 vector<int> manacher(string const & s) { // radiuses of odd palindromes, 0(N)
3     int n = s.length();
4     vector<int> r(n);
5     int i = 0, j = 0;

```

```

6     while (i < n) {
7         while (i-j >= 0 and i+j < n and s[i-j] == s[i+j]) ++ j;
8         r[i] = j;
9         int k = i;
10        while (i-k >= 0 and i+k < n and k+r[i-k] < j) {
11            r[i+k] = r[i-k];
12            ++ k;
13        }
14        i += k;
15        j -= k;
16    }
17    return r;
18 }
19 vector<int> odd_palindrome_length(string const & s) {
20     int n = s.length();
21     vector<int> r = manacher(s);
22     vector<int> l(n);
23     repeat (i,n) l[i-r[i]+1] = 2*r[i]-1;
24     repeat (i,n-1) setmax(l[i+1], l[i]-2);
25     return l;
26 }
27 vector<int> even_palindrome_length(string const & s) {
28     int n = s.length();
29     string t(2*n+1, '\0');
30     repeat (i,n) t[2*i+1] = s[i];
31     vector<int> r = manacher(t);
32     vector<int> l(n);
33     repeat (i,n) if (r[2*i+2] >= 3) l[i-r[2*i+2]/2+1] = r[2*i+2]-1;
34     repeat (i,n-1) setmax(l[i+1], l[i]-2);
35     return l;
36 }

```

7 utils

7.1 utils/binsearch.inc.cpp

```

1  /**
2   * @brief a flexible binary search
3   * @param[in] p a monotone predicate defined on [l, r)
4   * @return \min { x \in [l, r) \mid p(x) }, or r if it doesn't exist
5   */
6  template <typename UnaryPredicate>
7  int64_t binsearch(int64_t l, int64_t r, UnaryPredicate p) {
8      assert (l <= r);
9      -- l;
10     while (r - l > 1) {
11         int64_t m = l + (r - l) / 2; // to avoid overflow
12         (p(m) ? r : l) = m;
13     }
14     return r;
15 }
16
17 unittest {
18     for (int l : { 0, 1, 2, 3 }) {
19         for (int r : { 8, 9, 10, 11 }) {
20             assert (binsearch(l, r, [&](int n) { assert (l <= n and n < r); return true; }) == l);
21             assert (binsearch(l, r, [&](int n) { assert (l <= n and n < r); return false; }) == r);
22             REP3 (i, l, r + 1) {
23                 assert (binsearch(l, r, [&](int n) { assert (l <= n and n < r); return n >= i; }) == i);
24             }
25         }
26     }
27 }
28
29 /**
30 * @return \max { x \in [l, r] \mid p(x) }, or l if it doesn't exist
31 */
32 template <typename UnaryPredicate>
33 int64_t binsearch_max(int64_t l, int64_t r, UnaryPredicate p) {
34     assert (l <= r);
35     ++ r;
36     while (r - l > 1) {
37         int64_t m = l + (r - l) / 2; // to avoid overflow
38         (p(m) ? l : r) = m;
39     }
40     return l;
41 }
42
43 unittest {
44     for (int l : { 0, 1, 2, 3 }) {
45         for (int r : { 8, 9, 10, 11 }) {
46             assert (binsearch_max(l, r, [&](int n) { assert (l < n and n <= r); return false; }) == l);
47             assert (binsearch_max(l, r, [&](int n) { assert (l < n and n <= r); return true; }) == r);
48             REP3 (i, l, r + 1) {
49                 assert (binsearch_max(l, r, [&](int n) { assert (l < n and n <= r); return n <= i; }) == i);
50             }
51         }
52     }
53 }

```

7.2 utils/convex-hull-trick.inc.cpp

```

1  // http://d.hatena.ne.jp/sune2/20140310/1394440369
2  // http://techtips.hoge.blogspot.jp/2013/06/convex-hull-trickdequepop-back.html
3  // http://satanic0258.hatenablog.com/entry/2016/08/16/181331
4  // http://wcipeg.com/wiki/Convex_hull_trick
5  // verified: http://codeforces.com/contest/631/submission/31828502
6  struct line_t { ll a, b; };
7  bool operator < (line_t lhs, line_t rhs) { return make_pair(- lhs.a, lhs.b) < make_pair(- rhs.a, rhs.b); }
8  struct rational_t { ll num, den; };
9  rational_t make_rational(ll num, ll den = 1) {
10     if (den < 0) { num *= -1; den *= -1; }
11     return { num, den };
12 }
13 bool operator < (rational_t lhs, rational_t rhs) {
14     if (lhs.num == LLONG_MAX or rhs.num == - LLONG_MAX) return false;
15     if (lhs.num == - LLONG_MAX or rhs.num == LLONG_MAX) return true;
16     return lhs.num * rhs.den < rhs.num * lhs.den;
17 }
18 struct convex_hull_trick {
19     convex_hull_trick() {
20         lines.insert({ + LLONG_MAX, 0 }); // sentinels
21         lines.insert({ - LLONG_MAX, 0 });
22         cross.emplace(make_rational(- LLONG_MAX), (line_t) { - LLONG_MAX, 0 });
23     }
24     void add_line(ll a, ll b) {
25         auto it = lines.insert({ a, b }).first;
26         if (not is_required(*prev(it), { a, b }, *next(it))) {
27             lines.erase(it);
28             return;
29         }
30     }
31 }

```

```

29     }
30     cross.erase(cross_point(*prev(it), *next(it)));
31     { // remove right lines
32         auto ju = prev(it);
33         while (ju != lines.begin() and not is_required(*prev(ju), *ju, { a, b })) -- ju;
34         cross_erase(ju, prev(it));
35         it = lines.erase(++ ju, it);
36     }
37     { // remove left lines
38         auto ju = next(it);
39         while(next(ju) != lines.end() and not is_required({ a, b }, *ju, *next(ju))) ++ ju;
40         cross_erase(++ it, ju);
41         it = prev(lines.erase(it, ju));
42     }
43     cross.emplace(cross_point(*prev(it), *it), *it);
44     cross.emplace(cross_point(*it, *next(it)), *next(it));
45 }
46 ll get_min(ll x) const {
47     line_t f = prev(cross.lower_bound(make_rational(x)))->second;
48     return f.a * x + f.b;
49 }
50 private:
51     set<line_t> lines;
52     map<rational_t, line_t> cross;
53     template <typename Iterator>
54     void cross_erase(Iterator first, Iterator last) {
55         for (; first != last; ++ first) {
56             cross.erase(cross_point(*first, *next(first)));
57         }
58     }
59     rational_t cross_point(line_t f1, line_t f2) const {
60         if (f1.a == LLONG_MAX) return make_rational(- LLONG_MAX);
61         if (f2.a == - LLONG_MAX) return make_rational( LLONG_MAX);
62         return make_rational(f1.b - f2.b, f2.a - f1.a);
63     }
64     bool is_required(line_t f1, line_t f2, line_t f3) const {
65         if (f1.a == f2.a and f1.b <= f2.b) return false;
66         if (f1.a == LLONG_MAX or f3.a == - LLONG_MAX) return true;
67         return (f2.a - f1.a) * (f3.b - f2.b) < (f2.b - f1.b) * (f3.a - f2.a);
68     }
69 };
70 unittest {
71     default_random_engine gen;
72     repeat (iteration, 1000) {
73         vector<pair<int, int> > lines;
74         convex_hull_trick cht;
75         repeat (i, 100) {
76             int a = uniform_int_distribution<int>(- 30, 30)(gen);
77             int b = uniform_int_distribution<int>(- 30, 30)(gen);
78             lines.emplace_back(a, b);
79             cht.add_line(a, b);
80         }
81         repeat (i, 10) {
82             int x = uniform_int_distribution<int>(- 100, 100)(gen);
83             int y = INT_MAX;
84             for (auto line : lines) {
85                 int a, b; tie(a, b) = line;
86                 setmin(y, a * x + b);
87             }
88             assert (cht.get_min(x) == y);
89         }
90     }
91 }
92 struct inverted_convex_hull_trick {
93     convex_hull_trick data;
94     void add_line(ll a, ll b) { data.add_line(- a, - b); }
95     ll get_max(ll x) { return - data.get_min(x); }
96 };

```

7.3 utils/longest-increasing-subsequence.inc.cpp

```

1  template <typename T>
2  vector<T> longest_strict_increasing_subsequence(vector<T> const & xs) {
3      vector<T> l; // l[i] is the last element of the increasing subsequence whose length is i + 1
4      l.push_back(xs.front());
5      for (auto && x : xs) {
6          auto it = lower_bound(l.begin(), l.end(), x);
7          if (it == l.end()) {
8              l.push_back(x);
9          } else {
10             *it = x;
11         }
12     }
13     return l;
14 }
15
16 template <typename T>
17 vector<T> longest_weak_increasing_subsequence(vector<T> const & xs) {
18     vector<T> l;
19     for (auto && x : xs) {
20         auto it = upper_bound(l.begin(), l.end(), x);
21         if (it == l.end()) {
22             l.push_back(x);
23         } else {
24             *it = x;
25         }
26     }
27     return l;
28 }

```

7.4 utils/dice.inc.cpp

```

1  struct dice_t { // regular hezahedron group
2      //
3      //      \-----/
4      //      / \ C \ 4
5      //      / A \-----/ 2156
6      //      \ A / B / 3
7      //      \ / B / 21
8      //      v_B_/_/ ab bottom
9      int a, b; // in [1, 6]
10     int c() const {
11         static const int table[6][6] = {
12             { 0, 3, 5, 2, 4, 0 },
13             { 4, 0, 1, 6, 0, 3 },
14             { 2, 6, 0, 0, 1, 5 },
15             { 5, 1, 0, 0, 6, 2 },
16             { 3, 0, 6, 1, 0, 4 },
17             { 0, 4, 2, 5, 3, 0 },
18         };
19         assert (table[a-1][b-1] != 0);
20         return table[a-1][b-1];
21     }
22 };

```

```

22 };
23 dice_t rotate_up( dice_t dice) { return (dice_t) { dice.a, 7 - dice.c() }; }
24 dice_t rotate_right(dice_t dice) { return (dice_t) { 7 - dice.c(), dice.b }; }
25 dice_t rotate_down( dice_t dice) { return (dice_t) { dice.a, dice.c() }; }
26 dice_t rotate_left( dice_t dice) { return (dice_t) { dice.c(), dice.b }; }
27 bool operator == (dice_t x, dice_t y) { return x.a == y.a and x.b == y.b; }

```

7.5 utils/subset.inc.cpp

```

1  /**
2   * @sa https://kimiyaiki.net/blog/2017/07/16/enumerate-sets-with-bit-manipulation/
3   */
4
5  // for a set z, list y \subteq z, ascending order
6  for (int y = 0; ; y = (y - z) & z) {
7      ...
8      if (y == z) break;
9  }
10
11 // for a set z, list y \subteq z, descending order
12 for (int y = z; ; y = (y - 1) & z) {
13     ...
14     if (y == 0) break;
15 }
16
17 // for a set x and an ordinal n, list y s.t. x \subteq y \subteq n
18 or (int y = x; y < (1 << n); y = (y + 1) | x) {
19     ...
20 }
21
22 // for an ordinal n and integer k, list x \subteq n s.t. |x| = k
23 for (int x = (1 << k) - 1; x < (1 << n); ) {
24     ...
25     int t = x | (x - 1);
26     x = (t + 1) | (((~ t & - ~ t) - 1) >> (__builtin_ctz(x) + 1));
27 }

```