Time Complexity of H-Tree

Since the time to create an H-Branch is constant, we can assume it to be = 1.

$$T(0) = 1$$

$$T(1) = 4 + T(0) = 4 + 1 = 5$$

$$T(2) = 4^2 + T(1) = 4^2 + 4 + 1 = 21$$

$$T(3) = 4^3 + T(2) = 4^3 + 4^2 + 4 + 1 = 85$$

$$T(4) = 4^4 + T(3) = 4^4 + 4^3 + 4^2 + 4 + 1 = 341$$

$$T(5) = 4^5 + T(4) = 4^5 + 4^4 + 4^3 + 4^2 + 4 + 1 = 1365$$

$$T(n) = 4^n + T(n-1) = 4^n + 4^{n-1} + 4^{n-2} + ... + 4^1 + 4^0$$

$$= 4^n(1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + 0) = 4^n * C$$

$$T(n) = T(4^n * C) = O(2^n)$$

So, the method has $O(2^n)$ (exponential) time complexity.