مورخه ۲-۱۱ ۲-۱۱-۶ فصل ۱۱

1- $2^{x} = e^{x\ln 2}$

$e^{t} = 1 + t + \frac{t^{2}}{2!} + \dots = \sum_{n}^{\infty} \frac{t^{n}}{n!} \Rightarrow e^{x\ln 2} = \sum_{n=0}^{\infty} \frac{(\ln 2\, x)^{n}}{n!} =$

$\sum_{n=0}^{\infty} \frac{(\ln 2)^{n}}{n!} x^{n} \Rightarrow a_n = \frac{(\ln 2)^{n}}{n!}, \quad \frac{1}{R} = \lim_{n\to\infty} \frac{\frac{(\ln 2)^{n+1}}{(n+1)!}}{\frac{(\ln 2)^{n}}{n!}} =$

$\lim_{n\to\infty} \frac{\ln 2}{n+1} = 0 \rightarrow R = \infty$

سوال دوم

1. $f(x) = x^{4} - 3x^{2} + 1 \Rightarrow f(1) = -1$

$f'(x) = 4x^{3} - 6x \rightarrow f'(1) = -2, \quad f''(x) = 12x^{2} - 6 \Rightarrow f''(1) = 6$

$f'''(x) = 24x \rightarrow f'''(1) = 24 \Rightarrow f^{(4)}(x) = 24 \Rightarrow f^{(4)}(1) = 24$

$f^{(5)}(x) = \dots = f^{(n)}(x) = 0 \quad \forall n \geq 5.$

$f(x) = -1(x-1)^{0} - \frac{2}{1!}(x-1)^{1} + \frac{6}{2!}(x-1)^{2} + \frac{24}{3!}(x-1)^{3} +$

$\frac{24}{4!}(x-1)^{4} + \sum_{n=5}^{\infty} \frac{0}{n!} x^{n} = $

سری تابع را بنویسید ... در $x$ قرار دهید است $\lim_{x\to\infty} e^{x} = \infty$

$f(x) = -1 - 2(x-1) + 3(x-1)^{2} + 4(x-1)^{3} + (x-1)^{4}$

$$f(x)=e^{2x}\Rightarrow f'(x)=2e^{2x}\Rightarrow f''(x)=4e^{2x}\Rightarrow f^{n}(x)=2^{n}e^{2x}$$

$$\Rightarrow f^{n}(2)=2^{n}e^{4}\Rightarrow e^{2x}=\sum_{n=0}^{\infty}\frac{2^{n}e^{4}}{n!}(x-2)^{n}\Rightarrow L=\frac{2^{n}e^{4}}{n!}$$

$$\Rightarrow \lim\left(\frac{a_{n+1}}{a_n}\right)=0=\frac{1}{R}=0\longrightarrow R=\infty$$

[illegible]

$$f^{n}(3)=\frac{(-1)^{n}n!}{2^{n}(n+1)}\Rightarrow f(x)=\sum_{n=0}^{\infty}\frac{f^{n}(3)(x-3)^{n}}{n!}\Rightarrow$$

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^{n}}{2^{n}(n+1)}(x-3)^{n}\rightarrow L_n\frac{(-1)^{n}}{2^{n}(n+1)}\Rightarrow$$

$$\boxed{\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{\frac{1}{2^{n+1}(n+2)}}{\frac{1}{2^{n}(n+1)}}=\frac{1}{2}\rightarrow R=2}$$

$$|x-3|<2\longrightarrow 1<x<5$$

15) $f(x)=x\cos x=x\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{2n}}{(2n)!}=\sum_{n=0}^{\infty}\frac{(-1)^{n}x^{2n+1}}{(2n)!}\rightarrow$

$$a_n=\frac{(-1)^{n}}{(2n)!}\rightarrow L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=0$$

16) $f(x)=\frac{1}{x}\quad a=-2\rightarrow f(x)=x^{-1}\Rightarrow f'(x)=-1x^{-2}\Rightarrow$

$$f''(x)=2x^{-3}\Rightarrow f'''(x)=-6x^{-4}\Rightarrow f^{n}(x)=n!(-1)^{n}x^{-(n+1)}\Rightarrow$$

$$f^{n}(-2)=(-1)^{n}(n!)(-2)^{-(n+1)}=-n!\,2^{-(n+1)}\Rightarrow$$

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{n}(x)}{n!}(x+2)^{n}=\sum_{n=0}^{\infty}\frac{-n!\,x\,2^{-(n+1)}}{n!}(x+2)^{n}$$

$$\sum_{n=0}^{\infty} 3^{-(n+1)} (x+2)^n = \sum_{n=0}^{\infty} \frac{1}{3^{n+1}} (x-(-2))^n \to a_n = \frac{1}{3^{n+1}} \Rightarrow$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{1}{3^{n+2}}}{\frac{1}{3^{n+1}}} = \frac{1}{3} \Rightarrow \boxed{R=3}$$

۴۸ $(1-x)^{\frac{3}{2}}$ و $(1+t)^{\frac{3}{2}} = \sum_{n=0}^{\infty} \binom{\frac{3}{2}}{n} t^n \Rightarrow$

$$(1-x)^{\frac{3}{2}} = \sum_{n=0}^{\infty} \binom{\frac{3}{2}}{n} (-x)^n \to (1-x)^{\frac{3}{2}} = 1 - \frac{3}{2}x + \left(\frac{-1}{18}\right)x^2 \ldots$$

۴۹ $f(x) = x^2(\ln(1+x^2)) = x^2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot x^{2n}}{n} =$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n+2}}{n} \quad |x|<1$$

$$\ln(1+t) = t - \frac{t^2}{2} + \frac{t^3}{3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} t^n}{n} \to \ln(1+x^2) =$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n}$$

۹۹ $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

$$\frac{1}{\sqrt[10]{e}} = e^{-\frac{1}{10}} = 1 - \frac{1}{10} + \frac{1}{200} - \frac{1}{6000} + \frac{1}{240000} = 0/90483$$

سری متناوب و خطای [illegible]

$$\boxed{e^{-\frac{1}{10}} = 0/904837418}$$

(5) $\int \arctan(x^2)\,dx \qquad \arctan(t) = t - \frac{t^3}{3} + \frac{t^5}{5} - \cdots$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \to \arctan(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2n+1} =$$

$$x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \cdots$$

$$\int \arctan(x^2)\,dx \; \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2n+1} = \frac{x^3}{3} - \frac{x^7}{3 \cdot 7} + \frac{x^{11}}{5 \cdot 11} - \cdots$$

$$\Rightarrow \int \arctan x^2\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(2n+1)(4n+3)}$$

(69) $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^{2n}(2n)!} = \sum_{n=0}^{\infty} \left(\frac{\pi}{4}\right)^{2n} \frac{(-1)^n}{(2n)!} = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

(70) $$\frac{1}{1 \times 2} - \frac{1}{3 \times 2^3} + \frac{1}{5 \times 2^5} - \cdots \quad \arctan\left(\frac{1}{2}\right) = 0.463647$$

تمرین کتاب ۱۱-۵ حل شود

$$f'(x) = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} (-x^2)^n \quad |x^2| < 1$$

$$\Rightarrow |x| < 1$$

$$= 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + \frac{5}{16}x^6 + \cdots \qquad \int f'(x)\,dx =$$

$$\sin^{-1}(x) = x + \frac{x^3}{6} + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \cdots = \sum_{0}^{\infty} \frac{(-1)^n x^{2n+1} \binom{-\frac{1}{2}}{n}}{2n+1}$$

$|x| \leq 1$

سری اختلافی در ۱- همگراست.

$$g'(x)=\frac{1}{\sqrt{1+x^2}}=(1+x^2)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}x^{2n}=$$

$$(1-\frac{x^2}{2}+\frac{3x^4}{8}-\frac{x^6}{16}+\cdots$$

$$\int g'(x)\,dx=\sinh^{-1}(x)=x-\frac{x^3}{6}+\frac{3x^5}{40}-\frac{5x^7}{112}+\cdots=$$

$$\sum_{n=0}^{\infty}\frac{\binom{-\frac{1}{2}}{n}x^{2n+1}}{2n+1}$$

۲) $$f(x)=\sin^2 x=\frac{1-\cos 2x}{2}=\frac{1-\sum_{n=0}^{\infty}\frac{(-1)^n(2x)^{2n}}{(2n)!}}{2}=$$

$$\frac{\sum_{n=1}^{\infty}\frac{(-1)(2x)^{2n}}{(2n)!}}{2}=\sum=\frac{\frac{(-1)^{n+1}\,2^{2n}x^{2n}}{(2n)!}}{2}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{2n-1}x^{2n}}{2n!}=|a_n|=\frac{2^{2n-1}}{(2n)!},\ \lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=0$$

$$\Rightarrow \boxed{R=\infty}$$

۵) $$f(x)=\tan x=\left(\ln(-\cos x)\right)'=\left(\ln(1-\cos x-1)\right)'=$$

$$\left(\ln\left(1+\frac{-2x^2}{2!}+\frac{x^4}{4!}+\cdots\right)\right)'=\left(-2+\frac{x^2}{2!}+\cdots\right)-\frac{\left(-2+\frac{x^2}{2!}-\frac{x^4}{4!}\cdots\right)^2}{2}+$$

$$\left(\frac{\left(-2+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right)^3}{3}\cdots\right)'=\frac{2x}{2!}-\frac{4x^3}{4!}+\cdots$$

(۲) $f(x) = \sin^2(x) = \dfrac{1-\cos 2x}{2} = \dfrac{1 - \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}}{2} =$

$-\dfrac{\sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}}{2} = \sum_{n=1}^{\infty} \dfrac{\frac{(-1)^{n+1} 2^{2n} x^{2n}}{(2n)!}}{2} =$

$\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1} (2)^{2n-1} x^{2n}}{(2n)!}$ , $|a_n| = \dfrac{2^{2n-1}}{(2n)!} \Rightarrow \lim_{n\to\infty} \dfrac{a_{n+1}}{a_n} = 0 \Rightarrow \boxed{R = \infty}$

(۴) ~~$f(x) = \tan x = (\ln(-\cos x))'$~~

$f(x) = \tan x = \dfrac{\sin x}{\cos x} = \dfrac{x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots} = \sum (c_n x^n)$ , $b_n = \dfrac{(-1)^n}{(2n)!}$

$\Rightarrow x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} \sum_{i=1}^{n} c_i b_{n-i} \cdot x^n \Rightarrow$

$a_1 = b_0 c_1 + c_0 b_1 \Rightarrow a_1 = 1 = 1 c_1 + 0 \Rightarrow \boxed{c_1 = 1}$

$a_3 = b_0 c_3 + \cancel{b_1 c_2} + b_2 c_1 + \cancel{b_3 c_0} \Rightarrow 1 c_3 + (-\frac{1}{2})(1) = -\frac{1}{6}$

$c_3 = \frac{1}{3} \rightarrow a_5 = b_0 c_5 + \cancel{b_1 c_4} + b_1 c_3 + \cancel{b_2 c_2} + b_2 c_1 + \cancel{b_5 c_0}$

$\Rightarrow a_5 = c_5 + (-\frac{1}{2} \cdot \frac{1}{3}) + \frac{1}{24} \cdot 1 = \frac{1}{120} \Rightarrow c_5 = \frac{1}{6} + \frac{1}{120} - \frac{5}{120} = \frac{2}{15}$

$a_n = c_n + b_n c_0 \rightarrow \tan x = x + \frac{1}{3} x^3 + \frac{2}{15} x^5 + \cdots$

(۶) $f(x) = \dfrac{e^x}{\cos x} = \sum_{n=0}^{\infty} c_n x^n = \dfrac{\sum_{n=0}^{\infty} \frac{1}{n!} x^n}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots} = \sum_{n=0}^{\infty} c_n x^n$

$a_0 = b_0 c_0 \rightarrow 1 = 1 c_0 \rightarrow c_0 = 1$

$C_1 \Rightarrow c_1 = 1$

$a_2 = b_0 c_2 + b_1 c_1 + b_2 c_0 \Rightarrow \frac{1}{2} = 1\times c_2 - \frac{1}{2}\times 1 \Rightarrow c_2 = 1$

$a_3 = b_0 c_3 + b_1 c_2 + \cancel{b_2 c_1} + \cancel{b_3 c_0} \to \frac{1}{6} = c_3 + -\frac{1}{2} \Rightarrow$

$c_3 = \frac{4}{6} = \frac{2}{3}$ $\qquad a_4 = b_0 \cancel{c_4} + b_1 c_3 + \cancel{b_3 c_1} + b_4 c_0$

$\frac{1}{24} = c_4 - \frac{1}{2}\times 1 + \frac{1}{24} \to c_4 = \frac{1}{2}$

$f(x) = 1 + x + x^2 + \frac{2}{3}x^3 + \frac{1}{2}x^4 + \dots$

۵) $\frac{\sin x}{1+\cos x} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots}{1+\cos x} = \sum \frac{x^n}{2n}$

تابع فرد است ضرایب زوج صفر است

$a_1 = b_0 c_1 + b_1 c_0 \Rightarrow 1 = 2 c_1 = c_1 = \frac{1}{2}$

$a_3 = b_0 c_3 + \cancel{b_1 c_2} + b_2 c_1 + \cancel{b_3 c_0} \Rightarrow -\frac{1}{6} = c_3 + c_3 \to c_3 = -\frac{19}{6}$

$a_5 = b_0 c_5 + b_1 c_4 + b_2 c_3 + b_3 c_2 + b_4 c_1 + b_5 c_0 = \to$

$c_5 + 2\times\left(-\frac{19}{6}\right) = \frac{1}{120} \to c_5 = \frac{19}{3} + \frac{1}{120} = \frac{1141}{120}, \frac{1129}{120}$

۴) $f'(x) = \frac{1}{\sqrt{1+x}} = (1+x)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} x^n = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3$

$+ \frac{35}{128}x^4 - \frac{63}{256}x^5 + \dots \qquad -1 < x < 1$

د)

$f(x)=\sqrt[4]{1-x}=(1+(-x))^{\frac{1}{4}}$

$\sum_{n=0}^{\infty}\binom{\frac{1}{4}}{n}x^n \Rightarrow R=1$

$1-\frac{1}{4}x-\sum^{\infty}\frac{x^n(3\times7\times\ldots(4n-5))}{4^n}$

کافیست نقطه‌ای صورت را در x ضرب کنیم تا شعاع همگرایی

$e = f(x)=\frac{x}{\sqrt{4+x^2}}=\frac{x}{2}\left(1+\frac{x^2}{4}\right)^{-\frac{1}{2}}$

$f(x)=x\cdot\frac{1}{2x}\left(1+\left(\frac{x}{2}\right)^2\right)^{-\frac{1}{2}}\Rightarrow f(x)=\frac{x}{2}\sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}\left(\frac{x}{2}\right)^{2n}=$

$\sum\frac{(-1)^n(2n-1)!!}{2^{3n+1}n!}x^{2n+1}=\frac{x}{2}-\frac{x^3}{16}+\frac{3x^5}{256}-\ldots$

۷) $f(x)=(x+8)^{-\frac{1}{2}}\xrightarrow{t=x-1}f(t)=(9+t)^{-\frac{1}{2}}=\frac{1}{3}\left(1+\frac{t}{9}\right)^{-\frac{1}{2}}\Rightarrow$

$f(t)=\frac{1}{3}\sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}\left(\frac{t}{9}\right)^n=\frac{1}{3}-\frac{1}{54}t+\frac{1}{648}t^2-\frac{5}{34992}t^3+\ldots$

تمرینات بخش ۶-۱۱ جزوه

ب) $\tan^{-1}(x)=u\rightarrow\sqrt[3]{1+\tan^{-1}x}=(1+u)^{\frac{1}{3}}=\sum_{n=0}^{\infty}\binom{\frac{1}{3}}{n}x^n$

$1+\frac{1}{3}u-\frac{1}{9}u^2+\frac{5}{81}u^3-\frac{10}{243}u^4+\ldots=$

$\tan^{-1}x=\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)=f(x)=1+\frac{1}{3}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)$

$-\frac{1}{9}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)^2+\frac{5}{81}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right)^3+\ldots$

لازم است که توان جملات سری را به ترتیب زیر بنویسیم

$1+\frac{x}{3}-\frac{x^2}{9}-\frac{4}{81}x^3+\ldots$ $\quad x^3=-\frac{4}{81}$

۲)

$$A = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{n}{(n+1)!} \quad \sum_{n=1}^{\infty} \frac{1}{(n+1)!} + \frac{1}{2!}$$

$$\Rightarrow A = \sum_{n=1}^{\infty} \frac{1}{n!} - \frac{1}{(n+1)!} = \boxed{\frac{1}{2} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \cdots - 1}$$

۴) $A = \sum_{n=1}^{\infty} \frac{2^n}{n!} = \sum_{n=0}^{\infty} \frac{2^n}{n!} - 1 = e^2 - 1$

$$x = 2 \Rightarrow \sum_{n=0} \frac{x^n}{n!} = e^x = e^2$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(n+2)}{n!} x^n = \sum_{n=0} \frac{2x^n}{n!} + \sum_{n=0} \frac{n}{n!} x^n = 2e^x + \sum_{n=1} \frac{x^n}{(n-1)!} =$$

$$2e^x + x \sum_{n=1} \frac{x^{n-1}}{(n-1)!} = \sum_{n=1} \frac{(x^{n-1})}{(n-1)!} = \left(\sum_{n=0} \frac{x^n}{n!}\right)' = (e^x)' = e^x$$

$$\Rightarrow f(x) = 2e^x + xe^x = (2+x)e^x \checkmark$$

۵) $f(x) = \frac{1}{x^2 + 2x + 5} = \frac{1}{(1+x)^2 + 4}$

$$t = x+1, \; f(x) = \frac{1}{t^2+4} \Rightarrow \frac{1}{\frac{t^2}{4} + 1} \times \frac{1}{4} = \frac{1}{4} \sum_{n=0}^{\infty} (-1)^n \left(\frac{t^2}{4}\right)^n =$$

$$\frac{1}{4} \sum \frac{(-1)^n t^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{4^{n+1}}$$

ضریب جمله عمومی سری $x^{99}$ برابر $c_{99}$ است بنابراین

$$\left.\begin{array}{l} \frac{f_n (-1)^{\frac{n}{2}}}{4^{\frac{n}{2}+1}} = C_{99} = \frac{-1}{4^{24}} \\ C_n = \frac{f^{(n)}(-1)}{n!}, \; C_{99} = \frac{f^{(99)}}{99!} \end{array}\right\} \Rightarrow f^{(99)}_{-1} = \frac{-99!}{4^{24}}$$

(۶)

۲. $\int_0^{0/5} x^2 \tan^{-1}x\,dx = \int_0^{0/5} x^2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right)dx =$

$\int \left(x^3 - \frac{x^5}{3} + \frac{x^7}{5} + \frac{x}{7} + \dots\right)dx$

$I = \left(\frac{x^5}{5} - \frac{x^7}{21} + \frac{x^9}{45} - \frac{x^{11}}{77} + \dots\right)\Big]_0^{0/5} = \frac{(0/5)^5}{5} - \frac{(0/5)^7}{21} + \frac{(0/5)^9}{45} - \frac{(0/5)^{11}}{77}$

چون سری متناوب است خطای حاصل به جمله اول کمتر از جمله چهارم است.

بنابراین $\Rightarrow E < \frac{(0/5)^{11}}{77} \approx 24 \times 10^{-6} \approx 5 \times 10^{-6}$

$\Rightarrow I \approx 0/00625 - 0/000372 + 0/0000434 \approx 0/0059224$

(۷)

$\int_0^1 \frac{\ln(1+x)}{x}dx = \int_0^1 \frac{\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} - \dots\right)}{x}dx =$

$\int_0^1 \left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots\right)dx \Rightarrow$

$= \int_0^1 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{n-1}}{n}dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n^2}\Big]_0^1 =$

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n^2}\Big]_0^1 = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \dots$

۲

$$\sum_{n=0}^{\infty} \frac{\cos n\theta}{n!} = \frac{1}{2}\sum_{n=0}^{\infty} \frac{(e^{i\theta})^n + (e^{-i\theta})^n}{n!} = \frac{1}{2}\left(e^{e^{i\theta}} + e^{e^{-i\theta}}\right)$$

$$= \frac{1}{2}\left(e^{\cos\theta + i\sin\theta} + e^{\cos\theta - i\sin\theta}\right) = \frac{1}{2} e^{\cos\theta}(\cos\sin\theta + i\sin\sin\theta) +$$

$$e^{\cos\theta}(\cos\theta - i\sin\theta) = \frac{1}{2}\cos\theta(2\cos\sin\theta) = e^{\cos\theta}\cos\sin\theta \checkmark$$

$$g(\theta) = \sum_{n=0}^{\infty} \frac{\sin n\theta}{n!} = \sum_{n=0}^{\infty} \frac{(e^{i\theta})^n - (e^{-i\theta})^n}{n!} \Rightarrow$$

$$\frac{1}{2i} e^{\cos\theta}(\cos\sin\theta + i\sin\sin\theta - \cos\sin\theta + i\sin\sin\theta) = \frac{1}{2i}\left(e^{\cos\theta}(2i\sin\sin\theta)\right)$$

$$\boxed{e^{\cos\theta}\sin(\sin\theta)}$$

تمرین ۱۱_۱۱ کتاب

۴ $$e^{-x}\sin x = \underbrace{\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \dots\right)}_{a}\underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)}_{b}$$

$c_0 = a_0 b_0 = 0 \quad c_1 = b_1 a_0 + a_0 b_1 = 1 \quad c_2 = a_0 b_2 + a_2 b_0 + a_1 b_1 + b_2 a_0 = -1$

$c_3 = a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0 = -\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$

$T_3 = 0 + x - x^2 + \frac{x^3}{3}, \dots$

۲۹) $۲x^{\circ} = \frac{۲\pi x}{۱۸}$ Rad $\rightarrow \sin(x) = x - \frac{x^۳}{۶} + \frac{x^۵}{۱۲۰} - \frac{x^۷}{۵۰۴۰} + \dots$

۰/۶۶۳۲۲ − ۰/۰۴۸۶۲ + ۰/۰۰۱۰۶۹ = ۰/۶۱۵۲۶۹

$\frac{۳۸x^۷}{۵۰۴۰} < ۱۰^{-۵}$     عبارت بالا بیش از $۱۰^{-۵}$ نمی‌شود تا رقم اعشار است

۲۶) $V = ۲\pi k_e \sigma \left[\sqrt{d^۲ + R^۲} - d\right] = ۲\pi k_e \sigma d \left(\sqrt{1 + \frac{R^۲}{d^۲}} - 1\right)$

$\sqrt{1+t} = 1 + \frac{1}{۲}t + \dots$     بسط مک‌لورن $\frac{R}{d}$ پیدا می‌کنیم

با ترکیب حذف جملات در نتیجه با جای‌گذاری

$V = \frac{۲\pi k_e \sigma R^۲}{d^۲}$

تمرین حل مسائل

۳۲) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n} = \sum_{n=1}^{\infty} \frac{۲}{n(\sqrt{n+1} + \sqrt{n-1})}$

$b_n = \frac{۲}{n\sqrt{n}} \rightarrow \lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{۲}{n(\sqrt{n+1}+\sqrt{n-1})}}{\frac{۲}{n\sqrt{n}}} = \frac{1}{۲} \Rightarrow$

$b_n = \frac{۲}{n^{3/2}} \Rightarrow c = \sum a_n$ (همگرا)     پس $a_n$ همگرا است

۳۴) $a_n = \sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n}}{\ln n} \Rightarrow \lim_{n\to\infty} a_n \;\; \frac{\sqrt{n}}{\ln n} \xrightarrow{H} \frac{\frac{1}{۲\sqrt{n}}}{\frac{1}{n}} =$

$\lim_{n\to\infty} \frac{n}{\sqrt{n}} = \infty$     پس واگراست

د) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{3^{2n}(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\sqrt{\pi}}{3}\right)^{2n}}{(2n)!} = \cos\left(\frac{\sqrt{\pi}}{3}\right)$

۴ه) $\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!} \Rightarrow \frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{R} \rightarrow$

$\lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+3)!}}{\frac{2^n}{(n+2)!}} \rightarrow R=\infty \Rightarrow (x-2)\in \mathbb{R} \rightarrow x\in \mathbb{R}$

تمرین اضافی

$\sin(\angle P_n A P_{n+1}) = \frac{|P_nP_{n+1}|}{|P_{n+1}A_1|} = \frac{|P_nP_{n+1}|}{\sqrt{|A_1P_n|^2+|P_nP_{n+1}|^2}}$

$\Rightarrow a_n = \frac{|P_nP_{n+1}|}{\sqrt{|A_1P_n|^2+|P_nP_{n+1}|^2}} = \frac{2^{n-1}}{\sqrt{(A_1P_n)^2+2^{2n-2}}}$

$\lim a_n = a_{n+1} \rightarrow \frac{2^n}{\sqrt{(A_1P_{n+1})^2+4^n}} = \frac{2^{n-1}}{\sqrt{(A_1P_n)^2+2^{2n-2}}}$

$\frac{4(A_1P_n)^2+4^n}{A_1P_2^{\,n}+2^{2n-2}+2^n} \Rightarrow 3A_1P_2^{\,2} = 2^{2n-1} \Rightarrow |A_1P| = \frac{2^{n-2}}{\sqrt{3}}$

$\Rightarrow \sin\theta = \frac{2^{n-1}}{\sqrt{\frac{2^{2n-2}}{3}+2^{2n-2}}} = \frac{2^{n-1}}{\sqrt{\frac{4\times 2^{2n-2}}{3}}} = \frac{\sqrt{3}\cdot 2^{n-1}}{\sqrt{2^{2n}}} = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$

(11) $\sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right) = \sum_{n=2}^{\infty} \ln\left(\frac{n^2-1}{n^2}\right) = \sum_{n=2}^{\infty} \left(\ln(n^2-1) - \ln n^2\right)$

$\sum_{n=2}^{\infty} \left(\ln(n-1) + \ln(n+1) - 2\ln(n)\right) = \ln 1 + \ln 3 - 2\ln 2 + \ln 2 + \ln 4 - 2\ln 3 + \ldots$

$\ln 3 + \ln 5 - 2\ln 4 \ldots$

میبینیم که از دومی عبارات ترم ها که مشترک اند حذف و به جز [illegible]

$\sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right) = \ln 1 - 2\ln 2 + \ln 2 = \ln 1 - \ln 2 = -\ln 2$

(12) $I = \int_0^1 x^{x}\,dx = \int_0^1 e^{\ln x \cdot x}\,dx = \int_0^1 \sum_{n=0}^{\infty} \frac{(x\ln x)^n}{n!}\,dx$

$\sum_{n=0}^{\infty} \int_0^1 \frac{(\ln x)^n x^n}{n!}\,dx$

$\int_0^1 \ln^n x \, x^n \,dx = \frac{\ln^n x \, x^{n+1}}{n+1}\Big]_0^1 - \int \frac{(\ln x)^{n-1} x^{n+1}}{n+1} \cdot \frac{n}{x} \, dx$

جز به جز را ادامه بدیم

$\frac{n!(-1)^n}{(n+1)^n}\int_0^1 x^n\,dx = \frac{n!(-1)^n}{(n+1)^{n+1}} x^{n+1}\Big]_0^1 = \frac{n!(-1)^n}{(n+1)^{n+1}}$

$I = \sum_{n=0}^{\infty} \frac{\frac{n!(-1)^n}{(n+1)^{n+1}}}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)^{n+1}} = \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^n}}$