۴: $4y'' - 4y' + y = 0 \rightarrow (4D^2 - D + 1)y = 0$

$4r^2 - r + 1 = 0 \rightarrow (2r-1)^2 = 0 \rightarrow r_1 = r_2 = \frac{1}{2} \rightarrow$ مكرر

$y = c_1 e^{\frac{x}{2}} + c_2 x e^{\frac{x}{2}}$

$y(0) = 4 \rightarrow c_1 + c_2{}_{(0)} = 4 \rightarrow c_1 = 4$

$y' = \frac{1}{2} c_1 e^{\frac{x}{2}} + \frac{c_2}{2} x e^{\frac{x}{2}} + c_2 e^{\frac{x}{2}} \rightarrow y'(0) = -1$

$\rightarrow \begin{cases} \frac{1}{2} c_1 + c_2 = -1 \\[4pt] c_1 = 4 \end{cases} \rightarrow c_2 = -3$

$$\boxed{y = 4 e^{\frac{x}{2}} - 3 e^{\frac{x}{2}}}$$

۵: $y'' + 2y' + 4y = 0 \rightarrow (D^2 + 2D + 4)y = 0$

$\rightarrow r^2 + 2r + 4 = 0 \rightarrow r_1, r_2 = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2} = \frac{-2 \pm \sqrt{-12}}{2}$

$r_1 = \frac{-2 + 2\sqrt{3}\,i}{2} = -1 + i\sqrt{3} \quad , \quad r_2 = -1 - i\sqrt{3}$

$y = e^{-x}\left[c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x \right] \rightarrow y(0) = 1 \rightarrow \boxed{c_1 = 1}$

$y' = -e^{-x}\left[c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x \right] + e^{-x}\left[-\sqrt{3}c_1 \sin\sqrt{3}x + \sqrt{3}c_2 \cos\sqrt{3}x \right]$

$y'(0) = 0 \rightarrow 0 = -c_1 + \sqrt{3}c_2 \rightarrow \sqrt{3}c_2 = 1 \rightarrow c_2 = \frac{1}{\sqrt{3}}$

(۳)