۲۱) $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 2^{u}\,2\,du = 2\int 2^{u}\,du = 2\times\frac{2^{u}}{\ln 2}$

$= \frac{2^{u+1}}{\ln 2} = \frac{2^{\sqrt{x}+1}}{\ln 2}$

۴۹) $\int_{\pi/4}^{\pi/3} \frac{\sqrt{\tan\theta}}{\sin 2\theta}\,d\theta = \int \frac{\sqrt{\tan\theta}}{2\sin\theta\cos\theta}\,d\theta = \int \frac{\sqrt{\tan\theta}}{2\tan\theta}\sec^2\theta =$

$\frac{1}{2}\int \frac{\sec^2\theta}{\sqrt{\tan\theta}}\,d\theta = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\times 2\sqrt{u} = \sqrt{u} = \sqrt{\tan\theta}\Big|_{\pi/4}^{\pi/3}$

$= \sqrt{\sqrt{3}} - 1 = \sqrt[4]{3} - 1$

۲) $\int \frac{1}{x^7 - x}\,dx = \int \frac{1}{x(x^6-1)}\,dx$ مسائل تمرین فصل ۷

$u = \frac{x^6-1}{x^6} \rightarrow du = \frac{6}{x^7}\,dx$

$\int \frac{1}{x\times u x^6}\,\frac{x^7}{6}\,dx = \frac{1}{6}\int \frac{1}{u}\,dx = \frac{1}{6}\ln|u| = \frac{1}{6}\ln\left|\frac{x^6-1}{x^6}\right|$

(13) $\int_0^{\pi/2}\frac{\sin x}{1+\cos x+\sin x} = \int_0^{\pi/2}\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\,dx$

$\frac{1}{2}\int_0^{\pi/2}\left(\frac{\sin x/2-\cos x/2}{\sin x/2+\cos x/2}+1\right)dx = -\frac{1}{2}\int_0^{\pi/2}\frac{-\sin x/2+\cos x/2}{\sin x/2+\cos x/2}+\frac{1}{2}\int_0^{\pi/2}dx$

$=\left(-\frac{1}{2}\ln\left|\sin\frac{x}{2}+\cos x/2\right|+\frac{1}{2}x\right)\Big|_0^{\pi/2} = -\frac{1}{2}\ln\sqrt{2}+\pi/4$

(27) $\int x\sec x\tan x\,dx = x\sec x-\int\sec x\,dx$ تمرینات مروری فصل ۷ استوارت

$= x\sec x-\ln|\tan x+\sec x|$

$x \to dx \qquad \sec x\tan x \to \sec x$

(28) $\int\frac{\sqrt[4]{x}+1}{\sqrt[4]{x}-1}\,dx = 4\int\frac{(u+2)(u+1)^2}{u}\,dx$

$4\int\frac{u^3+4u^2+5u+2}{u}\,dx = 4\left[\int u^2du+\int 4u\,du+\int 5\,du+\int\frac{2}{u}du\right]$

$=4\left(\frac{u^3}{3}+2u^2+5u+2\ln|u|\right) = u^3+8u^2+20u+8\ln u$

$=(\sqrt[4]{x}-1)^3+8(\sqrt[4]{x}-1)^2+20(\sqrt[4]{x}-1)+8\ln(\sqrt[4]{x}-1)$

(32) $\int_0^{\pi/4}\frac{x\sin x}{\cos^3 x}\,dx$

$I=\frac{x}{2\cos^2 x}-\frac{1}{2}\int\frac{1}{\cos^2 x}\,dx=\frac{x}{2\cos^2 x}-\frac{1}{2}\int\sec^2 x\,dx$

$=\frac{x}{2\cos^2 x}-\frac{1}{2}\tan\Big|_0^{\pi/4}=\left(\frac{\pi/4}{2\times\frac{1}{2}}-\frac{1}{2}\times 1\right)-(0-0)=\pi/4-1/2$

(۳۲) $I = \int \frac{x^2}{(x^2+c)^{3/2}}\,dx$ $\qquad u\,;\,dv$

$$\int \frac{a^2\tan^2\theta}{(a^2\sec^2\theta)^{3/2}}\,a\sec^2\theta\,d\theta = \int \frac{a^2\tan^2\theta\,a\sec^2\theta\,d\theta}{a^3\sec^3\theta} =$$

$$\int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int \frac{\sec^2\theta-1}{\sec\theta}\,d\theta = \int(\sec\theta-\cos\theta)\,d\theta =$$

$$\boxed{\ln|\sec\theta+\tan\theta| - \sin\theta + C}$$

تمرینات بخش ۹-۴ جزوه،

(۱) $\int_{-1}^{1}\frac{1-x}{3^x+3^{-x}}\,dx = \frac{2}{\ln 3}\left(\tan^{-1}3-\pi/4\right)$

$$I = \underbrace{\int_{-1}^{1}\frac{1}{3^x+3^{-x}}}_{\text{زوج}} - \underbrace{\int_{-1}^{1}\frac{x}{3^x+3^{-x}}}_{\text{فرد}} = 2\int_0^1\frac{1}{3^x+3^{-x}}\,dx$$

$3^x = u$

$du = \ln 3\; 3^x\,dx$

$$2\int_0^1\frac{1}{u+\frac{1}{u}}\,\frac{du}{u\ln 3} = \frac{2}{\ln 3}\int_0^1\frac{1}{u^2+1}\,du$$

$$= \frac{2}{\ln 3}\tan^{-1}u = \frac{2}{\ln 3}\tan^{-1}(3^x)\Big|_0^1$$

$$= \frac{2}{\ln 3}\left(\tan^{-1}3-\pi/4\right)$$

(۲) $\int_{\pi}^{5\pi/4}\frac{\sin 2x}{\cos^4 x}\,dx = \int_{\pi}^{5\pi/4}\frac{\sin 2x}{\cos^4x+\sin^4x} = 2\int_{\pi}^{5\pi/4}\frac{\sin 2x}{2-\sin^2 2x}\,dx$

$$= 2\int_0^{\pi/2}\frac{\sin u}{2-\sin^2 u}\,\frac{du}{2} = \int_0^{\frac{\pi}{2}}\frac{\cos u}{1+\sin^2 u} \xrightarrow{\sin u = t} \int_0^1\frac{dt}{1+t^2}$$

$$= \tan^{-1}t = \pi/4$$

۵) $\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^2\,dx}{(9+4x^2)^{3/2}}$ $\qquad u = \frac{3}{2}\tan\theta$

$$\int \frac{x^2\,dx}{8(\frac{9}{4}+x^2)^{3/2}} = \int \frac{\frac{27}{8}\tan^2(\theta)\times\frac{3}{2}\sec^2\theta}{8\times(\frac{9}{4}+\frac{9}{4}\tan^2\theta)^{3/2}}\,d\theta = \int \frac{27}{16}\,\frac{\tan^2\theta\sec^2\theta}{27\sec^3\theta}\,d\theta$$

$$= \frac{3}{16}\int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \frac{3}{16}\int_0^{\frac{\pi}{3}} \sin\theta\tan\theta\,d\theta = \frac{3}{16}\int \sec\theta\tan\theta\,d\theta$$

$$-\frac{3}{16}\int \sin\theta\,d\theta = \frac{3}{16}\sec\theta\Big|_0^{\pi/3} + \frac{3}{16}\cos\theta\Big|_0^{\pi/3}$$

$$= \frac{3}{16}(2-1) + \frac{3}{16}\left(\frac{1}{2}-1\right) = \boxed{\frac{3}{32}}$$

۸) $$\int \frac{-x\,dx}{\sqrt{3-2x-x^2}} = \int \frac{x\,dx}{\sqrt{4-(x+1)^2}} = \int \frac{(u-1)\,du}{\sqrt{4-u^2}}$$

$$= \int \frac{u\,du}{\sqrt{4-u^2}} - \int \frac{du}{\sqrt{4-u^2}} = -\sqrt{4-u^2} - \sin^{-1}\left(\frac{u}{2}\right) + C$$

$$\boxed{= -\sqrt{3-2x-x^2} - \sin^{-1}\left(\frac{x+1}{2}\right) + C}$$

۱۴) $$\int_{\sqrt{2}/3}^{2/3} \frac{dx}{x^5\sqrt{9x^2-1}} = \int \frac{-\frac{1}{3}\sec\theta\tan\theta}{\frac{1}{3^5}\sec^5\theta\sqrt{\frac{1}{9}(\sec^2-1)}}\,d\theta$$ تمرین ۷-۳ استوارت

$$= \int \frac{\frac{1}{3}\sec\theta\tan\theta}{3^{-6}\sec^5\theta\tan\theta}\,d\theta = \int_{\frac{\pi}{4}}^{\pi/3} \frac{3^5}{\sec^4\theta}\,d\theta = 3^5\int \frac{1}{\sec^4\theta}\,d\theta$$

$$= 3^5\left(\frac{\cos^3\theta\sin\theta}{4} + \frac{3}{4}\int\cos^2\theta\,d\theta\right) = 3^5\left(\frac{\cos^3\theta\sin\theta}{4} + \frac{3}{4}\left(\frac{\cos\theta\sin\theta}{2} + \frac{\theta}{2}\right)\right)$$

$$= 3^5\left(\frac{7\sqrt{3}}{64} + \frac{\pi}{32} + \frac{1}{4}\right)$$

بخش ۷-۲ انتگرال

(۳۴) $\int \frac{\sin\theta}{\cos^3\theta}\, d\theta = \int \frac{\sin\theta}{\cos\theta \cos^2\theta}\, d\theta$

$= \int \frac{1}{\cos^2\theta} \tan\theta \, d\theta = \int \sec^2\theta \tan\theta \, d\theta$

$I = \int \sec^2\theta \tan\theta \, d\theta = \int u\,du = \frac{u^2}{2} + C = \frac{\sec^2\theta}{2} + C$

(۴۵) $\int_{\pi/6}^{\pi/3} \csc^3 x \, dx = \int_{\pi/6}^{\pi/3} \csc u \csc^2 u \, du = -\csc u \cot x \Big|_{\pi/6}^{\pi/3} -$

$\int_{\pi/6}^{\pi/3} \csc x \cot^2 x \, dx = -\frac{2}{3} + 2\sqrt{3} - \int_{\pi/6}^{\pi/3} \csc \cdot (\csc^2 x - 1) =$

$-\frac{2}{3} \times 2\sqrt{3} - \int_{\pi/6}^{\pi/3} \csc^3 x + \int_{\pi/6}^{\pi/3} \csc x =$

$-\frac{2}{3} \times 2\sqrt{3} - L + \ln|\csc x - \cot x| \Big|_{\pi/6}^{\pi/3} = -\frac{2}{3} + 2\sqrt{3} - L + \ln\frac{1}{\sqrt{3}} - \ln|2-\sqrt{3}|$

$\Rightarrow 2L = -\frac{2}{3} + 2\sqrt{3} - \frac{\ln 3}{2} - \ln(2-\sqrt{3}) \Rightarrow L = -\frac{1}{3} + \sqrt{3} - \frac{\ln 3}{4} - \frac{\ln(2-\sqrt{3})}{2}$

تمرینات تکمیلی ۷-۲ جزوه (۱)

1) $\int \sqrt{\frac{1+x}{1-x}} \frac{dx}{1-x} = 2\sqrt{\frac{1+x}{1-x}} - 2\tan^{-1}\sqrt{\frac{1+x}{1-x}} + C$

$L = \int \frac{\sqrt{u}}{1-x} \frac{(1-x)^2}{2} du = \int \frac{\sqrt{u}}{u+1} du = \int \frac{t}{t^2+1} 2t\,dt =$

$2\int \frac{t^2}{t^2+1} dt = \boxed{2\sqrt{\frac{1+x}{1-x}} - 2\tan^{-1}\sqrt{\frac{1+x}{1-x}} + C}$

(۷۴) $f(0)=1$ $\qquad f'(0)=?$

$$\int \frac{f(x)}{x^2(x+1)^3} = \cancel{\frac{A}{x}} + \frac{B}{x^2} + \cancel{\frac{C}{x+1}} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3}$$

$$\frac{ax^2+bx+1}{x^2(x+1)^3} =$$

باید A و C صفر =

$$\Rightarrow \frac{ax^2+bx+1}{x^2(x+1)^3} = \frac{B}{x^2} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3} \Rightarrow$$

$$ax^2+bx+1 = B(x+1)^3 + Dx^2(x+1) + Ex^2 = (B+D)x^3 + (3B+D+E)x^2 + (3x+1)B$$

$B+D=0$

$3B+D+E=a$

$3B=b \Rightarrow b=3 \Rightarrow f'(0)=3$

$B=1$

(۲) $\displaystyle\int_0^{\pi/2} \frac{dx}{1+\sin x\cos x} = \ln 2$ $\qquad$ تمرین تیپ ۲-۴ جزوه ①

$u=\tan\frac{x}{2}$ $\quad dx=\frac{2du}{1+u^2}$ $\quad x=\frac{\pi}{2} \to u=1$ $\quad x=0 \to u=0$

$$\int_0^1 \frac{\frac{2du}{1+u^2}}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}} = \int_0^1 \frac{2du}{1+u^2+2u+1-u^2} = \int_0^1 \frac{2du}{2+2u} = \int_0^1 \frac{du}{1+u}$$

$$= \ln|1+u|\Big|_0^1 = \ln 2 - \ln 1 = \ln 2$$

① تمرینات بخش ۴-۵ جزوه

۲) $\int \frac{dx}{x^4+4}$

$$\frac{1}{x^4+4} = \frac{1}{x^4+4x^2+4-4x^2} = \frac{1}{(x^2+2)^2-4x^2} = \frac{1}{8}\,\frac{x+2}{x^2+2x+2} - \frac{1}{8}\,\frac{x-2}{x^2-2x+2}$$

$$= \int \frac{1}{x^4+4}\,dx = \frac{1}{8}\int \frac{x+2}{x^2+2x+2} - \frac{1}{8}\int \frac{x-2}{x^2-2x+2} \xrightarrow{x+1=u}$$

$$\frac{1}{8}\int \frac{u+1}{u^2+1} - \frac{1}{8}\int \frac{u-2}{u^2+2u-2} = \frac{\ln u^2+1}{16} + \frac{1}{8}\tan^{-1}(u) - \frac{1}{8}\times$$

$$\int \frac{x-2}{x^2+2x+2}\,dx = \frac{\ln(x^2+2x+2)}{16} + \frac{1}{8}\tan^{-1}(x+1) - \frac{\ln(x^2-2x+2)}{16}$$

$$\boxed{+\frac{\tan^{-1}(x-1)}{8}}$$

۴) $\int \frac{dx}{x(x^2+1)(x^2+2)^2} = \frac{x}{x^2(x^2+1)(x^2+1)^2} \xrightarrow{u=x^2} \frac{\sqrt{a}}{u(u+1)(u+1)^2}$

۱۵a) $\int \frac{3x+1}{4x^2+12x-7}\,dx = \int \frac{2x+3-2}{(2x+3)^2-16}$ تمرینات ۴-۷ استوارت

$$= \int \frac{2x+3}{\bigcirc} - \int \frac{2dx}{\bigcirc} = \frac{1}{4}\int \frac{8x+12}{4x^2+12x-7} - \int \frac{2dx}{(2x+3)^2-4^2}$$

$$= \frac{1}{4}\ln|4x^2+12x-7| - \int \frac{du}{u^2-4^2} = \frac{1}{4}\ln|4x^2+12x-7| - \frac{1}{8}\ln\left|\frac{u-4}{u+4}\right|$$

$$= \frac{1}{4}\ln|4x^2+12x-7| - \frac{1}{8}\ln\left|\frac{2x-1}{2x+7}\right| + C$$

$I_n=\int_0^1 \underbrace{(\arccos x)^n}_{u}\, dx$ (۱۴)

$\cos u = x \Rightarrow dx = -\sin u\, du \Rightarrow I_n = \int_0^1 u^n \cdot -\sin u\, du$

$= \int_0^1 -u^n \sin u\, du$

$I_n = u^n \cos u - n\int_0^1 u^{n-1}\cos u\, du = u^n \cos u - n\left[u^{n-1}\sin u - (n-1)\int \sin u\, u^{n-2}\, du\right]$

$= u^n \cos u - \int u n^{n-1}\cos u\, du = u^n \cos u - u n^{n-1}\sin u + n(n-1)\int_0^1 \sin u\, u^{n-2}\, du$

$= n(n-1)(-I_n) = 0 + n\left(\frac{\pi}{2}\right)^{n-1} - \boxed{n(n-1)I_n}$

(۴۸) $\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1}\sin x + \frac{n-1}{n}\int \cos^{n-2} x\, dx$ تمرین ۷-۱ اشتباه

$= \int \cos^n x\, dx = \int \underbrace{\cos^{n-1}x}_{u}\, \underbrace{\cos x\, dx}_{dv} = \sin x \cos^{n-1} x - (n-1)\int -\sin^2 x \cos^{n-2} x$

$= \int \cos^n dx = \sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\, dx - (n-1)\int \cos^n x\, dx$

$1+(n-1)\int \cos^n dx = \sin x \cos^{n-1} x + (n-1)\cos^{n-2} x\, dx$

$\int \cos^n x\, dx = \frac{1}{n}\sin u \cos^{n-1} u + \frac{n-1}{n}\int \cos^{n-2} x\, dx$

(۵۱) $\int x^4 e^x dx = \int x^4 e^u dx = x^4 e^u - 4\int u^3 e^x dx =$

$x^4 e^x - 4\left[x^3 e^x - 3\int x^2 e^x dx\right]$

$= x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24\int x e^x = u e^u - 4x^3 e^u + 12x^2 e^u - 24(xe^x - \int e^x)$

$= e^x(x^4 + 4x^3 + 12x^2 - 24x + 24) + C$

$$\int_1^2 \underbrace{x}_{u}\underbrace{f''(x)\,dx}_{dv} \qquad dv = f''(x)\,dx \Rightarrow v = f'(x)$$

$$u = x \Rightarrow du = dx$$

$$xf'(x) - \int f'(x)\,dx = xf'(x) - f(x)$$

$$= 2f'(2) - f(2) - f'(1) + f(1) = \boxed{2}$$

(49)

تمرینات بخش ۲-۲ جزوه، (1)

$$2)\ I_n = \int_{x^n} \frac{dx}{\sqrt{ax+b}} , \quad \int \frac{1}{x^n} = \frac{dx}{\underbrace{\sqrt{ax+b}}_{v}}$$

$$= \frac{1}{x^n} \times \frac{2\sqrt{ax+b}}{a} - \int \frac{-2n}{x^{n+1}} \frac{\sqrt{ax+b}}{a} = \frac{2\sqrt{ax+b}}{ax^n} + \frac{2n(ax+b)}{a}$$

$$= \int \frac{1}{\sqrt{ax+b}\,x^{n+1}} = \frac{2\sqrt{ax+b}}{ax^n} + \frac{2n(ax+b)}{a} I_{n+1}$$

$$I_{n+1} = \frac{aI_n - \frac{2\sqrt{ax+b}}{x^n}}{2n(ax+b)} = \frac{ax^n I_n - 2\sqrt{ax+b}}{2n(ax+b)} \Rightarrow$$

$$I_n = \frac{ax^{n-1} I_{n-1} - 2\sqrt{ax+b}}{2(n-1)(ax+b)}$$

$$3)\ I_n = \int \frac{dx}{(1+\ln x)^n} = \frac{x}{(1+\ln x)^n} - \int x \frac{-n}{x(\ln x+1)^{n+1}} =$$

$$\frac{x}{(1+\ln x)^n} + n\int \frac{1}{(1+\ln x)^{n+1}} = \frac{x}{(1+\ln x)^n} + nI_{n+1}$$

$$I_{n+1} = \frac{I_n}{n} - \frac{x}{n(1+\ln x)^n} \longrightarrow I_n = \frac{I_{n-1}}{n-1} + \frac{x}{(n-1)(1+\ln x)^{n-1}}$$

$$\int_a^b (x-a)(x-b) f''(x)\,dx$$ ٤)

$$I = \int_a^b \underbrace{(x-a)(x-b)}_{u}\, \underbrace{f''(x)\,dx}_{dv} = (x-a)(x-b)f'(x) - \int_a^b f'(x)(2x-(a+b))\,dx$$

u' $\quad$ $f'(x)$

$$= -\int_a^b f'(x)(2x-(a+b))\,dx = -(2x-(a+b))f(x)\Big|_a^b - 2\int_a^b f(x)\,dx$$

$$= 2\int_a^b f(x)\,dx$$ تمرینات الشورت بخش ۷-۱ کتاب

$$\int x \tan^2 x\,dx$$ ۱۳)

$1 \quad \tan x - x \Rightarrow (\tan x - x)x - \int \tan x - x = I$

$\int 1+\tan^2 = \tan \rightarrow \int 1 + \int \tan^2 = \tan$

$x + \bigcirc = \tan \rightarrow \tan x - x$

$I = (\tan x\, x - x^2) - \int \tan x - \int x = \ldots$

$x(\tan x) - x^2 - \ln(\cos x) - \frac{1}{2}x^2$

$$\int_0^1 \frac{r^3}{\sqrt{r^2+4}}\,dr \xrightarrow[dt=2r\,dr]{4+r^2=t} \int_4^5 \frac{t-4}{2\sqrt{t}}\,dt = \frac{1}{2}\int_4^5 \left(\sqrt{t} - \frac{4}{\sqrt{t}}\right)$$ ۱۴)

$$= \frac{1}{2}\int_4^5 \sqrt{t} - \frac{1}{2}\int_4^5 \frac{4}{\sqrt{t}} = \frac{1}{2}\left(\frac{t^{3/2}}{3/2} - 4\frac{t^{1/2}}{1/2}\right) =$$

$$\boxed{\frac{16}{3} - \frac{7}{3}\sqrt{5}}$$

$$7)\ \int (\sin^{-1}x)^2\,dx = \int \underbrace{u^2}_{2u}\ \underbrace{\cos u\,du}_{\sin u} \rightarrow \sin u\, u^2 - \int 2u\sin u\,du$$

$$\int \underbrace{2u}_{2}\ \underbrace{\sin u\,du}_{-\cos u} = -2u\cos u + \int 2\cos u = -2u\cos u + 2\sin u$$

$$u^2\sin u + 2u\cos u - 2\sin u = u^2\sin(\arcsin x) + 2\arcsin x\cos u - 2\sin u$$

$$x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\,\arcsin x - 2x$$

$$13)\ \int_{-\pi}^{\pi} x\cos(x+|x|)\,dx = -\frac{\pi^2}{2}$$

$$x>0\quad \int_0^{\pi} \underbrace{x}_{1}\ \underbrace{\cos(2x)}_{\frac{1}{2}\sin(2x)} \qquad \frac{1}{2}\sin(2x)\,x - \frac{1}{2}\int \sin(2x)$$

$$\frac{x}{2}\sin 2x - \cos 2x + \frac{1}{2}x^2 \Big|_{-\pi}^{\pi} = \boxed{\frac{-\pi^2}{2}}$$

$$\int \sqrt{a^2-x^2}\,dx = \int \frac{a^2-x^2}{\sqrt{a^2-x^2}} \rightarrow \int \frac{a^2}{\sqrt{a^2-x^2}} - \int \frac{x^2}{\sqrt{a^2-x^2}} = \boxed{a^2\sin^{-1}\left(\frac{x}{a}\right) - \frac{a^2\arcsin\left(\frac{x}{a}\right) - x\sqrt{a^2-x^2}}{2}}$$

2

یا

$$x = a\sin u$$

$$\int \sqrt{a^2 - a^2\sin^2 u} = \int a\sqrt{1-\sin^2 u}\times a\cos u \quad \int a^2\cos^2 x$$

$$= a^2\int \cos^2 u = a^2\int \cos^2 u\,du = \int a^2\,\frac{1-\cos 2u}{2} =$$

$$\boxed{a^2\int \frac{1}{2} - \frac{\cos 2u}{2} = a^2\int \frac{du}{2} - \frac{a^2}{2}\int \cos 2u = \frac{a^2}{2}u - \frac{\sin 2u}{4}}$$

$$a^2\sin^{-1}\left(\frac{x}{a}\right) - \frac{a^2\sin^{-1}\left(\frac{x}{a}\right) - x\sqrt{a^2-x^2}}{2}$$

۳- $\int_0^x f(t)\,dt = [f^2(x)] \to f(x) = [f^2(x)]' \to f(x) = 2f(x)\,f'(x)$

$\boxed{\Rightarrow f'(x) = \frac{1}{2}}$

۲) $\int \sin(\ln x)\,dx = \sin(u)$ ترتیب تشخیص ۴_۳ جزء ،

$u = \ln x \to du = \frac{1}{x}dx \to x\,du = dx \Big\} \to dx = e^u du$

$x = e^u$

$\int \sin(u) e^u du = e^u \sin u - \int \cos u\, e^u = e^u \sin u - (e^u \cos u + \int e^u \sin u)\,du$

$+\cos(u) \quad e^u \qquad -\sin \quad e^u$

~~ترتیب تشخیص ۴_۳ جزء~~

$I = e^u \sin u - e^u \cos u - I \Rightarrow 2I = e^u \sin u - e^u \cos u$

$u = \ln x$

$e^{\ln x}\sin(\ln x) - e^{\ln x}\cos(\ln x) = \boxed{\dfrac{x\sin(\ln x) - x\cos \ln x}{2}}$

۴) $\int \left(\frac{\ln x}{x}\right)^2 dx = -\frac{\ln^2 x}{x} - 2\frac{\ln x}{x} - \frac{2}{x}$

$2\left(\frac{\ln x}{x}\right)\left(\frac{1-\ln x}{x}\right) \quad x$

$\frac{\ln^2 x}{x} - \int 2\left(\frac{\ln x}{x}\right)\left(\frac{1-\ln x}{x}\right) = \frac{\ln^2 x}{x} - \int \frac{2\ln x - \ln^2 x}{x^2}$

$\boxed{= -\frac{\ln^2 x}{x} - \frac{2\ln x}{x} - \frac{2}{x}}$

۵) $\int \frac{\overset{u}{x e^{2x}}\,dx}{(1+2x)^2} = \frac{e^{2x}}{8x+4}$

$2xe^{2x} + e^{2x} \qquad -\frac{1}{2}\frac{1}{2x+1} \to \frac{-xe^{2x}}{2(2x+1)} - \int \frac{-(2x e^{2x} + e^{2x})\,dx}{4x+2}$

$\int \frac{-e^{2x}}{2}dx = -\frac{1}{4}\int e^u = -\frac{1}{4}e^u = -\frac{e^{2x}}{4} = \frac{e^{2x}}{8x+4}$

مسائل اضافی فصل ۴

۲- $y = 4x - x^3 \rightarrow \int_a^{a+1} 4x - x^3 = S_{max} \Rightarrow 2x^2 - \frac{x^4}{4} \Big|_a^{a+1} = S_{max}$

$2(a+1)^2 - \frac{(a+1)^4}{4} - (2a^2 - \frac{a^4}{4}) = 2(2a+1) + \frac{a^4}{4} - \frac{(a+1)^4}{4}$ مشتق

$4 + a^3 - (a+1)^3 = 0 \Rightarrow 4 + a^3 - (a^3 + 1 + 3a^2 + 3a) = 0$

$a = -2 \rightarrow \int_{-2}^{-1} (4x - x^3) = 2x^2 - \frac{x^4}{4} \Big|_{-2}^{-1} = (2 - \frac{1}{4}) - (\ldots) = \boxed{\frac{9}{4}}$

$a = 0 \rightarrow \int_0^1 (4x - x^3) = 2x^2 - \frac{x^4}{4} \Big|_0^1 = 2 - \frac{1}{4} = \boxed{\frac{7}{4}}$

۵۲- $\sin t = -\sin(-t) \Rightarrow \sin$ یک تابع فرد است

۷۸- $\displaystyle\int \frac{\sin x}{1+\cos^2 x} = \int \frac{-du}{1+u^2} = -\int \frac{1}{1+u^2}\,du = -\arctan u$

$= -\arctan(\cos x) + C$

۸۴- $\displaystyle\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx = \quad \arcsin x = u \Rightarrow du = \frac{1}{\sqrt{1-x^2}}$

$\displaystyle = \int_0^{1/2} u\,du = \frac{1}{2}u^2\Big|_0^{1/2} = \frac{1}{2}\big(\underbrace{\sin^{-1} x}_{\frac{\pi}{6}}\big)^2 = \frac{1}{2}\left(\frac{\pi^2}{36} - 0\right) = \boxed{\frac{\pi^2}{72}}$

بخش مرور مطالب فصل ۴ کتاب استوارت

۴۹ $\displaystyle\int_0^2 f(x)\,dx = 6$

$\displaystyle\int_0^{\pi/2} f(2\sin\theta)\cos\theta\,d\theta = \int_0^2 f(u)\frac{du}{2} = \frac{1}{2}\times 6 = \boxed{3}$

۵۳- $\displaystyle 2\int_b^a f(x) f'(x)\,dx = [F(b)]^2 - [F(a)]^2$

$= u = f(x) \Rightarrow du = f'(x)\,dx \Rightarrow \frac{1}{2}\times 2\,\cancel{\ }\, F^2\Big|_b^a$

$= F(a)^2 - F^2(b)$

$$\int \frac{5}{7} u^{7/2} \longrightarrow -2^{2/5} < u < 2^{2/5} \longrightarrow \int_{-2^{2/5}}^{2^{2/5}} \rightarrow 2 \times \frac{5}{7} \times 2 \times 2^{2/5} = \frac{20}{7} \times \sqrt[5]{4}$$

$$= \boxed{\frac{20}{7} \sqrt[5]{4}}$$

6

$$f(x) = e^{-x} + 2\int_0^x e^{-3t} f(x-t)\,dt \qquad \text{Ans: } e^{-x} + 2xe^{-x} = f(x)$$

$x - t = u$

$du = -dt$

$$e^{-3x}\int_0^x e^{3u} f(u)\,du \Rightarrow \int_0^x e^{-3t} f(x-t)\,dt =$$

$$= e^{-3x}\int_0^x e^{3t} f(t)\,dt \quad \boxed{\checkmark}$$

• تمرینات بخش ۴-۵ کتاب استوارت

$$12-\int \sec^2 2\theta \, d\theta \qquad u = 2\theta \Rightarrow du = 2d\theta \Rightarrow d\theta = \frac{1}{2}du$$

$$\int \frac{1}{\cos^2 2\theta} = \frac{1}{2}\int \sec^2 u \,\frac{1}{2}du = \boxed{\frac{\tan u}{2}}$$

$$25.\int \sqrt{x} \sin(1+x^{3/2})\,dx \rightarrow u = x^{3/2}+1 \Rightarrow du = \frac{3}{2}dx\sqrt{x} \Rightarrow$$

$$dx = \frac{2}{3}\frac{1}{\sqrt{x}}$$

$$\frac{2}{3}\int \sin u = \frac{2}{3}(-\cos u) = \frac{2}{3}(-\cos(1+x^{3/2}))$$

$$\boxed{= -\frac{2}{3}\cos(1+x^{3/2})}$$

4) $\int \sec^6 x\,dx = \sec^4 x \sec^2 x = \overset{(1+\tan^2)^2}{\dfrac{1}{\cos^4}} \times \overset{(1+\tan^2 x)}{\dfrac{1}{\cos^2}} =$

$u = \tan x \Rightarrow du = \underbrace{(1+\tan^2 x)}_{\sec^2}\,dx \Rightarrow dx = \dfrac{1}{\sec^2 x}\,du$

$\int \sec^2 x \times (1+\tan^2 x)\,dx = \int \sec^2 x \times (1+\tan^2 x)\dfrac{1}{\sec^2 x}\,du$

$= \int (1+\tan^2 x)^2\,du = \int (1+u^2)^2\,du = \int (1+u^4+2u^2)\,du$

$(u + \frac{1}{5}u^5 + \frac{2}{3}u^3) = \boxed{\tan x + \frac{1}{5}\tan^5 x + \frac{2}{3}\tan^3 x}$

(4)

$\int \dfrac{\sin x^n}{x}\,dx = \dfrac{1}{n}\int \dfrac{\sin t}{t}\,dt$ $\quad u = x^n \Rightarrow x = u^{1/n}$

$du = n x^{n-1}\,dx \Rightarrow dx = \frac{1}{n}x^{1-n}$

$\int \dfrac{\sin u}{u^{1/n}}\,\dfrac{1}{n}x^{1-n} = \int \dfrac{\sin u}{u^{1/n}} \times \dfrac{1}{n}u^{\frac{1-n}{n}}$

$\dfrac{1}{n}\int \sin u \times u^{\frac{-1}{n}} \times u^{\frac{1-n}{n}} = \dfrac{1}{n}\int \sin u \times u^{\frac{1-n-1}{n}} = \dfrac{1}{n}\int \sin u \times u^{-1}$

$= \boxed{\dfrac{1}{n}\int \dfrac{\sin u}{u}}$

(5)

$\int_{-2}^{2} \sqrt[5]{x^2}\,dx$

مستقیم:

$\int_{-2}^{2} x^{2/5}\,dx = \dfrac{5x^{7/5}}{7}\Big|_{-2}^{2} \longrightarrow \dfrac{5}{7}(+2)^{7/5} - \dfrac{5}{7}(-2)^{7/5}$

$\dfrac{10}{7} \times 2^{7/5} = \dfrac{20}{7} \times \sqrt[5]{4} = \boxed{\dfrac{20\sqrt[5]{4}}{7}}$

غیر مستقیم:

$x^{2/5} = u \Rightarrow du = \dfrac{2}{5}x^{-3/5}\,dx \Rightarrow \dfrac{5}{2}du\,x^{3/5} = \dfrac{5}{2}u^{\frac{5}{2}\times\frac{3}{5}},\ \dfrac{5}{2}u^{3/2}\,du$

$x = u^{5/2}$

$\int u \times \dfrac{5}{2}u^{3/2} = \int \dfrac{5}{2} \times u^{5/2} = \dfrac{5}{2} \times u^{7/2} \times \dfrac{2}{7} = \dfrac{5}{7}u^{7/2}$

تمرینات کتاب ۲-۴ جزوه

۱

۲) $\int_0^{13}\frac{dx}{\sqrt[3]{(1+2x)^2}} = 3\,\frac{1}{2}u^{1/3}\times 2 = 3u^{1/3}\times\frac{1}{2}$

$= 3\times 3\times\frac{1}{2} - 3\times\frac{1}{2} = 3\times\frac{1}{2}\times 2 = 3$

۳) $\int_0^1\frac{dx}{(1+\sqrt{x})^4} = \frac{1}{6} \rightarrow u = 1+\sqrt{x} \Rightarrow dx = 2\sqrt{x}\,du$

$\frac{2\sqrt{x}\,du}{u^4} = \frac{2(u-1)}{u^4}du = 2\int\frac{u-1}{u^4}du$

$= 2\int\frac{1}{u^3}-\frac{1}{u^4} = 2\int u^{-3}-u^{-4} = 2\left(-\frac{1}{2}u^{-2}+\frac{1}{3}u^{-3}\right)$

$2\,\frac{1}{3}u^{-3}-\frac{1}{2}u^{-2}\Big|_0^1 = 2\times\frac{1}{4}-\frac{1}{3} = \frac{1}{12}\times 2 = \frac{1}{6}$

۵) $\int\frac{6x^3+x^2-2x+1}{2x-1}dx = x^3+x^2+\frac{1}{2}\ln|2x-1|$

$u = 2x-1 \Rightarrow u+1 = 2x \Rightarrow \frac{u+1}{2} = x$

$du = 2dx \rightarrow \frac{1}{2}\left(\frac{6(u+1)^3}{8}+\frac{(u+1)^2}{4}-1\right)\times 2 = \frac{1}{8}\,\frac{3(u+1)^3+(u+1)^2-4}{u}$

$\frac{1}{8}\int\frac{3(u+1)^3}{u}+\frac{(u+1)^2}{u}-4 = 3u^2+10u+\frac{4}{u}+7$

$\frac{1}{8}\left(3\int u^2du+10\int u\,du+4\int\frac{1}{u}du+7\int 1\,du\right) =$

$\frac{1}{8}\left(3\times\frac{1}{3}u^3+10\times\frac{1}{2}u^2+4\ln(u)+7u\right) = u^3+5u^2+4\ln u+7u$

$(2x+1)^3+5(2x-1)^2+4\ln(2x-1)+7(2x-1) = \boxed{x^3+x^2+\frac{1}{2}\ln|2x-1|}$

تمرینات بخش ۴-۱ جزوه

۱

۱) $\int (\sqrt{x}+1)(x-\sqrt{x}+1)\,dx$

$$\int x\sqrt{x}+\cancel{x}-\cancel{x}+\cancel{\sqrt{x}}-\cancel{\sqrt{x}}+1 = x^{3/2}+1 \Rightarrow \boxed{\frac{2}{5}x^{5/2}+x}$$

$$= \boxed{\frac{2}{5}x^{2}\sqrt{x}+x}$$

۲) $$\int \sin^2 3x\,dx = \int \frac{1-\cos 2\times 3x}{2} = \int \frac{1}{2}-\frac{\cos 6x}{2}$$

$$\frac{1}{2}x-\frac{\sin 6x}{12}=$$

۴) $$\int \frac{2+3x^2}{x^2(1+x^2)} = \int \frac{2(1+x^2)+x^2}{x^2(1+x^2)} = \int \frac{2}{x^2}+\frac{1}{1+x^2} = \boxed{-\frac{2}{x}+\tan^{-1}x}$$

ص

۳) $$\int_0^{\pi/2} [4\sin^2 x]\,dx = \frac{3\pi}{4}$$

$0 \leq 4\sin^2 x < 1$ ، $1 \leq 4\sin^2 x < 2$

$2 \leq 4\sin^2 x < 3$ ، $3 \leq 4\sin^2 x < 4$ ، $\sin = 1$

$$\overset{①}{0} + \overset{②}{x|} + \overset{③}{2x|} + \overset{④}{3x|}$$

①، $[0, \frac{\pi}{6})$ $[\frac{\pi}{6}, \frac{\pi}{4})$ $(\frac{\pi}{4}, \frac{\pi}{3}]$

$$\frac{\pi}{4}-\frac{\pi}{6}+\frac{2\pi}{3}-\frac{2\pi}{4}+\frac{3\pi}{2}-\frac{3\pi}{3} = \frac{\pi}{4}-\frac{\pi}{6} =$$

$$\frac{\pi}{4}-\frac{\pi}{6}+\frac{2\pi}{3}-\frac{\pi}{2}+\frac{3\pi}{2}-\pi = \frac{6\pi}{24}-\frac{4\pi}{24}+\frac{16\pi}{24}-\frac{12\pi}{24}+\frac{36\pi}{24}-\frac{24\pi}{24}$$

$$\frac{18\pi}{24} = \boxed{\frac{3\pi}{4}}$$