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تکلیف سوم ریاضیات مهندسی

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a) $f(t) = ۴\sin(۳t) + ۲\cos(۹t) \longrightarrow L(f(t)) = F(s) = ۴ \times \frac{۳}{s^۲+۹} + ۲ \times \frac{s}{s^۲+۸۱}$

$$\boxed{F(s) = \frac{۱۲}{s^۲+۹} + \frac{۲s}{s^۲+۸۱}}$$

b) $f(t) = ۳t^۲ + ۵t + ۲ \longrightarrow L(f(t)) = F(s) = ۳ \times \frac{۲!}{s^۳} + \frac{۵}{s^۲} + \frac{۲}{s}$

$$\boxed{F(s) = \frac{۶}{s^۳} + \frac{۵}{s^۲} + \frac{۲}{s}}$$

c) $f(t) = \delta(t-۲)\cos t \longrightarrow L(f(t)) = \boxed{F(s) = e^{-۲s} \frac{s}{s^۲+۱}}$

d) $f(t) = t^۲ e^{-t}\cos(t) \longrightarrow L(e^{-t}\cos t) = \frac{s+۱}{(s+۱)^۲+۱}$

$L(t^۲ e^{-t}\cos(t)) = (-۱)^۲ F^{(۲)}(s) = F^{(n)}(s)$

$$F'(s) = \frac{(s+۱)^۲+۱ - ۲(s+۱)(s+۱)}{((s+۱)^۲+۱)^۲}$$

e) $f(t) = \frac{\sin(۴t)}{t} = \int_s^\infty \frac{a}{s^۲+a^۲} ds = \tan^{-۱}\frac{s}{a} \Big]_s^\infty$

$= \frac{\pi}{۲} - \tan^{-۱}\frac{s}{۴}$

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$$f(t)=\sin(at)-at\cos(at) \longrightarrow L(f(t))=F(s) \longrightarrow$$

$$F(s)=\frac{a}{s^2+a^2}-a\left(-\frac{(1)(s^2+a^2)-2s(s)}{(s^2+a^2)^2}\right) \longrightarrow$$

$$F(s)=\frac{a}{s^2+a^2}+a\left(\frac{a^2-s^2}{(s^2+a^2)^2}\right)$$

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② تبدیل لاپلاس معکوس:

a) $$F(s)=\frac{s-1}{s^2-2s+5}=\frac{s}{s^2-2s+5}-\frac{1}{s^2-2s+5} \longrightarrow$$

$$F(s)=\frac{s}{(s-1)^2+4}-\frac{1}{(s-1)^2+4} \longrightarrow L^{-1}(F(s))=f(t) \longrightarrow$$

$$f(t)=e^{t}\cos 2t+e^{t}\sin 2t-e^{t}\sin 2t=\boxed{e^{t}\cos 2t}$$

b) $$F(s)=\frac{1+e^{-s}}{s^2}=\frac{1}{s^2}+\frac{e^{-s}}{s^2}=t+(t-1)u(t)$$

c) $$F(s)=\frac{(s+2)(s+4)}{s(s+1)(s+3)} \longrightarrow \frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+3}$$

$$A(s^2+4s+3)+B(s^2+3s)+C(s^2+s)\equiv s^2+6s+8$$

$$\begin{cases} A+B+C=1 \\ 4A+3B+C=6 \\ 3A=8 \longrightarrow A=\frac{8}{3} \end{cases} \longrightarrow \begin{cases} B+C=-\frac{5}{3} \\ 3B+C=-\frac{14}{3} \end{cases} \longrightarrow$$

$$\begin{cases} 3B+3C=-5 \\ 9B+3C=-14 \end{cases}$$

$$6B=-9 \longrightarrow \boxed{B=\frac{-3}{2}} \qquad \boxed{C=\frac{-1}{6}}$$

$$\boxed{f(t)=\frac{8}{3}+e^{-t}\left(-\frac{3}{2}\right)-\frac{1}{6}e^{-3t}}$$

d) $F(s) = \dfrac{s+4}{(s+1)^2(s+2)} = \dfrac{s}{(s+1)^2(s+2)} + \dfrac{3}{(s+1)^2(s+2)}$

$L^{-1}(F(s)) = f(t) \longrightarrow f(t) = (e^{-t})\cos t - e^{-2t}\sin t + 2e^{-2t} +$

e) $F(s) = \dfrac{s^2-2}{(s^2+3)^2} \qquad \dfrac{As+B}{(s^2+3)^2} + \dfrac{C}{s^2+3} \rightsquigarrow$

$As^3 + 3As + Bs^2 + 3B + C(s^4 + 6s^2 + 9) \equiv s^2 - 2$

$Cs^4 + 6Cs^2 + 9C + As^3 + 3As + Bs^2 + 3B \equiv s^2 - 2$

$F(s) = \left(\dfrac{s}{s^2+3}\right)^2 - \dfrac{2}{(s^2+3)^2} \longrightarrow f(t) = \dfrac{1}{\sqrt{3}} \sin^2 \sqrt{3}t - 2t\sin\sqrt{3}$

f) $F(s) = \dfrac{s+1}{(s^2+s+1)(s+1)} \longrightarrow \dfrac{As+B}{s^2+s+1} + \dfrac{C}{s+1}$

$\cancel{As^2} + \cancel{As} + \cancel{Bs} + B + \cancel{Cs^2} + \cancel{Cs} + C \equiv s+1 \longrightarrow$

$A+C=0$

$A+B+C=1 \rightsquigarrow \begin{cases} A=0 \\ C=0 \\ B=1 \end{cases} \qquad F(s) = \dfrac{1}{s^2+s+1}$

$B+C=1$

$F(s) = \dfrac{1}{\left(s+\frac{1}{2}\right)^2 - \frac{1}{4} + 1} = \dfrac{1}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} \longrightarrow$

$f(t) = \dfrac{1}{\frac{\sqrt{3}}{2}} \left(e^{-\frac{1}{2}t}\right) \sin \dfrac{\sqrt{3}}{2} t$

معادلات دیفرانسیل زیر را به کمک تبدیل لاپلاس حل کنید

a) $\ddot{y} + 2\dot{y} + y = \delta(t-2) \qquad y(0)=0 \quad \dot{y}(0)=0$

$$s^2Y(s) + 2sY(s) + Y(s) = e^{-2s} \longrightarrow Y(s)(s^2+2s+1) = e^{-2s}$$

$$Y(s) = \frac{e^{-2s}}{s^2+2s+1} = \frac{e^{-2s}}{(s+1)^2} \longrightarrow$$

$$u_2(t)e^{-t}\sin t$$

b) $\ddot{y} + 4y = \sin(t) - u_{2\pi}(t)\sin(t-2\pi) \quad , y(0)=0 \quad \dot{y}(0)=0$

$$L\{\ddot{y}\} + L(4y) = L(\sin t - u_{2\pi}(t)\sin(t-2\pi)) \longrightarrow$$

$$s^2Y(s) + 4Y(s) = \frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1} \longrightarrow$$

$$Y(s)(s^2+4) = \frac{1-e^{-2\pi s}}{s^2+1} \longrightarrow$$

$$Y(s) = \frac{1-e^{-2\pi s}}{(s^2+1)(s^2+4)} = \frac{1}{(s^2+1)(s^2+4)} - \frac{e^{-2\pi s}}{(s^2+1)(s^2+4)}$$

$$\frac{As+B}{s^2+1} + \frac{Cs+d}{s^2+4} \longrightarrow \cancel{As^3} + \cancel{4As} + Bs^2 + 4B + \cancel{Cs^3} + ds^2 + \cancel{Cs} + d$$

$$\begin{cases} C+A=0 \longrightarrow C=-A \longrightarrow C=0 \\ B+d=0 \longrightarrow B=-d \\ 4A+C=0 \longrightarrow 3A=0 \longrightarrow A=0 \qquad d=-\frac{1}{3} \\ 4B+d=1 \longrightarrow 3B=1 \longrightarrow B=\frac{1}{3} \end{cases}$$

$$y(t) = \frac{1}{3}\sin t + \frac{-1}{3}\sin 2t - u_{2\pi}(t)\left(\frac{1}{3}\sin(t-2\pi) - \frac{1}{3}\sin 2(t-2\pi)\right)$$

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c) $\ddot{y}+۵\dot{y}+۴y=te^{-t} \quad y(۰)=۱ \quad \dot{y}(۰)=۰$

$$s^{۲}Y(s)+\cancel{sY(s)}+۵sY(s)+۴Y(s)=\frac{۱}{(s+۱)^{۲}} \longrightarrow$$

$$Y(s)\left(s^{۲}+۵s+۴\right)=\frac{۱}{(s+۱)^{۲}} \longrightarrow$$

$$Y(s)=\frac{۱}{(s+۱)^{۲}(s^{۲}+۵s+۴)}=\frac{۱}{(s+۱)^{۲}(s^{۲}+۵s+۴)}$$

$$f(t)=y(t)=-te^{-t}\sin\frac{\sqrt{۵}}{۲}t-t^{۲}\sin\frac{\sqrt{۵}}{۲}t$$

$$\frac{X(s)}{F(s)}=\frac{s+۳}{s^{۳}+۱۱s^{۲}+۱۲s+۱۸} \longrightarrow$$

$$\frac{X(s)}{F(s)}=\frac{s}{s^{۳}+۱۱s^{۲}+۱۲s+۱۸}+\frac{۳}{s^{۳}+۱۱s^{۲}+۱۲s+۱۸}$$

$$a_n\frac{d^ny}{dt^n}+\cdots+a_1\frac{dy}{dt}+a_0y(t)=b_m\frac{d^mu}{dt^m}+\cdots$$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{b_ms^m+b_{m-1}s^{m-1}+\cdots+b_1s+b_0}{a_ns^n+a_{n-1}s^{n-1}+\cdots+a_1s+a_0}$$

$$\frac{x(t)}{F(t)}=۱۱\dddot{y}(t)-۴\ddot{y}(t)-۱۸(\dot{y}(t))-۳y(t)+۹$$