Date : / / معادلات 12 محمد امانلو

۸۱۰۱۰۰۰۸۴

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تمرینات بخش ۲-۵

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۱۹-۵-۱) $\mathcal{L}^{-1}\left(\frac{1}{s(s^2+a^2)}\right)$

$\frac{A}{S}+\frac{BS+C}{S^2+a^2}$ $\qquad AS^2+Aa^2$

$\qquad Bs^2+Cs$

$A+B=0 \longrightarrow B=\frac{-1}{a^2}$

$C=0$

$Aa^2=1 \longrightarrow A=\frac{1}{a^2}$

$\mathcal{L}^{-1}\left(\frac{1}{a^2}\left(\frac{1}{s-0}\right)\right)+\mathcal{L}^{-1}\left(\frac{\frac{-1}{a^2}s}{s^2+a^2}\right)=\frac{1}{a^2}e^{0t}-\frac{1}{a^2}\cos at=$

$\boxed{\frac{1-\cos at}{a^2}}$

۲۵-۵-۱) $\mathcal{L}^{-1}\left(\frac{2s}{(s^2-1)^2}\right)$ $\quad \frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{C}{s-1}+\frac{D}{(s-1)^2}$

$$\left.\begin{array}{l} AS-A(s^2-1)=As^3-As^2-As+A \\ B(s^2-2s+1)=Bs^2-2Bs+1 \\ Cs+C(s^2-1)=Cs^3+Cs^2-Cs-C \\ D(s^2+2s+1)=Ds^2+2Ds+D \end{array}\right\} \begin{array}{ll} b=\frac{1}{2} & A=0 \\ B=\frac{-1}{2} & C=0 \end{array}$$

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$$\mathcal{L}^{-1}\left(\qquad \right)=\mathcal{L}^{-1}\left(\frac{1}{2}\left(\frac{1}{(S-1)^2}-\frac{1}{(S+1)^3}\right.\right.$$

$$-\frac{1}{2}\left(e^{t}t-e^{-t}t\right)=\boxed{t\sinh t}$$

$$1)\ \mathcal{L}^{-1}\left(\frac{S}{S^2-2S+3}\right)=\mathcal{L}^{-1}\left(\frac{S-1+1}{S^2-2S+1+2}\right) \qquad -٢$$

$$\rightsquigarrow \frac{(S-1)+1}{(S-1)^2+2} \xrightarrow{\mathcal{L}^{-1}} \mathcal{L}^{-1}\left(\frac{(S-1)}{(S-1)^2+2}\right)+\frac{1}{\sqrt{2}}\mathcal{L}^{-1}\left(\frac{\sqrt{2}}{(S-1)^2+2}\right.$$

$$\longrightarrow \mathcal{L}^{-1}\left(\frac{S}{S^2-2S+3}\right)=\boxed{e^{t}\cos\sqrt{2}t+\frac{e^{t}\sin\sqrt{2}t}{\sqrt{2}}}$$

$$2)\ \mathcal{L}^{-1}\left(\frac{8S}{S^4+64}\right)\longrightarrow\frac{8S}{S^4+64}=\frac{8S}{(S^2+8)^2-16S}=\frac{8S}{(S^2+8+4S)(S^2+8-4S)}$$

$$\frac{AS+B}{S^2+8+4S}+\frac{CS+D}{S^2+8-4S}\longrightarrow A+C=0 \qquad -4A+B+4C+D=0$$

$$8A-4B+8C+4D=8$$

$$\longrightarrow A=C=0,\ D=1,\ B=-1 \qquad 8B+8D=0$$

$$\mathcal{L}^{-1}\left(\frac{-1}{(S+2)^2+4}\right)+\mathcal{L}^{-1}\left(\frac{1}{(S-2)^2+4}\right)=\frac{-1}{2}e^{-2t}\sin 2t+\frac{1}{2}e^{2t}\sin 2t$$

$$=\sin 2t\left(\frac{e^{2t}-e^{-2t}}{2}\right)=\boxed{\sin 2t\sinh 2t}$$

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$3)\ \left(\ln\left(\frac{s-a}{s-b}\right) \Rightarrow \ln\left(\frac{s-a}{s-b}\right) = \ln(s-a) = \ln(s-b)\right.$ ۲-

مشتق لگاریتم $= \frac{1}{s-a} - \frac{1}{s-b} = F'(s)$

$\xrightarrow{\mathcal{L}^{-1}} e^{4t} - e^{bt} \xrightarrow{\div(-t)} \frac{e^{bt} - e^{4t}}{t}$

$4)\ \mathcal{L}^{-1}(\cot g^{-1}(s+3) \Rightarrow \mathcal{L}^{-1}\left(\frac{-1}{(s+3)^2+1}\right) = F'(s) - e^{-3t}\sin t$

$\xrightarrow{\div(-t)} \frac{e^{-3t}\sin t}{t}$

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$2)\ y'' + 3y' + 2y = \sin 2t \quad y(0) = 2 \quad y'(0) = -1$ ۳-

$\mathcal{L}(y') = sF(s) - f(0) \qquad \mathcal{L}(y'') = s^2F(s) - sf(0) - f'(0)$

$\Rightarrow s^2F(s) - 2s + 1 + 3sF(s) - 6 + 2F(s) = \dfrac{2}{s^2+4}$

$(s^2+3s+2)F(s) = \dfrac{3}{s^2+4} + 2s + 5$

$F(s) = \dfrac{2}{(s^2+4)(s+1)(s+2)}$ ① $+ \dfrac{3s}{(s+1)(s+2)}$ ② $+ \dfrac{5}{(s+1)(s+2)}$ ③

① $\dfrac{As+B}{s^2+4} + \dfrac{C}{s+1} + \dfrac{D}{s+2} \longrightarrow As^3 + 3As^2 + 2As$

$Bs^2 + 3Bs + 2B$

$Cs + 2C(s^2+4) = \qquad Cs^3 + 2Cs^2 + 4sC + 8C$

$Ds + D(s^2+4) = \qquad Ds^3 + Ds^2 + 4Ds + 4D$

$A + C + D = 0$

$3A + B + 2C + D = 0$ $\Big\} \longrightarrow A + \dfrac{4C + 1 - 5C}{4} = 0$

$C + 2D - B = 0$

$2A + 3B + 4C + 4D = 0$

$2B + 8C + 4D = 2 \longrightarrow B + 4C + 2D = 1$ $\Big\}$ $4D + 5C = 1$

$D = \dfrac{1-5C}{4}$

$A = \dfrac{C-1}{4} \qquad \dfrac{3C-3}{4} + B + \dfrac{8C}{4} + \dfrac{1-5C}{4} = 0$

$\longrightarrow \dfrac{6C-2}{4} + B = 0 \longrightarrow B = \dfrac{2-6C}{4}$

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$2-6C+16C+2-10C=4 \quad \times$

$2C-2+6-18C+16C+4-20C=0$

$20C=8 \longrightarrow C=\frac{8}{20}=\frac{2}{5}=\boxed{0,4=C}$

$A=\frac{-0,6}{4}=-\boxed{0,15} \qquad B=\frac{2-2,4}{4}=\boxed{-0,1} \qquad \boxed{D=-0,25}$

$\longrightarrow \frac{-0,15\,s}{s^2+4}+\frac{-0,1\,(2)}{(2)\,s^2+4}+\frac{0,4}{s+1}-\frac{0,25}{s+2} \xrightarrow{\mathcal{L}^{-1}}$

$-0,15\cos 2t-0,05\sin 2t+0,4e^{-t}-0,25e^{-2t}$

② $2s=As+A+Bs+2B \longrightarrow \left.\begin{array}{l} 2B+A=0 \\ 2B+2A=2\times 2 \end{array}\right\} \begin{array}{l} B=-2 \\ A=4 \end{array}$

$\xrightarrow{\mathcal{L}^{-1}} -2e^{-t}+4e^{-2t}$

③ $As+A+Bs+2B=5 \longrightarrow A+B=0 \longrightarrow A=-B$

$A+2B=5 \longrightarrow B=5 \quad A=-5$

$\xrightarrow{\mathcal{L}^{-1}} \cancel{5e^{-t}-5e^{-2t}}$

①+②+③ $\longrightarrow \frac{3}{20}\cos 2t-\frac{1}{20}\sin 2t+\frac{17}{5}e^{-t}-\frac{5}{4}e^{-2t}=y$

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③ $ty'' + (1-2t)y' - 2y = 0$, $y(0)=1$, $y'(0)=2$

$(-1)' \frac{d}{ds}(\mathcal{L}(y'')) - 2(-1)'\frac{d}{ds}(\mathcal{L}(y')) + \mathcal{L}(y') - 2\mathcal{L}(y) = 0$

$\frac{-d}{ds}(s^2 Y(s) - sy(0) - y'(0)) - \frac{2D}{ds}(sY(s) - y(0))$

$+ sY(s) - y(0) - 2Y(s) = 0$

$= -(2sY(s) + s^2 Y'(s)) - 2(Y(s) + sY'(s)) + sY(s) - 1 - 2Y(s) = 0$

$(-s^2 + 2s^{+1})Y's + (\quad -s' \quad)Y(s) - 1$

$\frac{dY}{Y} = \frac{s}{-s^2+2s} ds \longrightarrow \ln Y = \ln[-(2-s)] + C_1$

$Y = (s) = \frac{C_2}{2-s} \longrightarrow \mathcal{L}^{-1}: C_2 e^{+2t}$

$y(0) = 1 \longrightarrow C_2 e^0 = 1 \longrightarrow C_2 = 1$

$y = e^{2t}$

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$y'' + 2y' + y = t$, $y(0) = -3$ $y(1) = -1$, $y(0) = b$

$\mathcal{L}(y') = SF(S) - f(0)$ $\mathcal{L}(y'') = S^2F(S) - Sf(0) - f'(0)$

$\mathcal{L}(t) = \frac{-e^{-st}}{s} b - \frac{e^{-st}}{s^2} = \Big|_0^{\infty} 0 - 0 + 0 + \frac{1}{s^2} - \frac{1}{s^2}$

$S^2F(S) + 3S - b + 2SF(S) + 3 + F(S) = \frac{1}{S^2}$

$(S^2 + 2S + 1)F(S) = \frac{1}{S^2} - 3S - 3 + b$

$F(S) = \frac{1}{S^2(S+1)^2} - \frac{3S}{(S+1)^2} + \frac{b}{(S+1)^2}$ ① ② ③

① $1 = \frac{A}{S} + \frac{B}{S^2} + \frac{C}{S+1} + \frac{D}{(S+1)^2}$

$AS(S^2 + 2S + 1) = AS^3 + 2AS^2 + AS$

$BS^2 + 2BC + B$

$CS^3 + CS^2$

$CS^2(S+1) = DS^2$

$\begin{cases} A + C = 0 \\ B + C + 2A + D = 0 \\ B + A = 0 \end{cases}$

$\boxed{B = 1}$ $\boxed{A = -2}$ $\boxed{C = 0}$

$\frac{1}{S^2} \xrightarrow{\mathcal{L}^{-1}} \frac{1}{S} \longrightarrow \int_0^t 1\,dt = t$

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$\frac{2}{s+1} \xrightarrow{\mathcal{L}^{-1}} 2e^{-t}$ $\frac{1}{(s+1)^2+0} \xrightarrow{\mathcal{L}^{-1}} te^{-t}$

$\Rightarrow 2e^{t} + \frac{e^{-t}}{t} + t - 2$

② $\frac{-3s}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$

$\begin{cases} B + As + A = -3s \\ A = -3 \quad B = 3 \\ \Rightarrow -3(e^{-t}) + t3e^{-t} \end{cases}$

③ $\frac{b}{(s+1)^2} \xrightarrow{\mathcal{L}^{-1}} tbe^{-t}$ $t - 2 + e^{-t}((\frac{4+B}{t}) - 1)$

$\Rightarrow y = ① + ② + ③ = -e^{-t} + t(4+b)e^{-t} + t - 2$

$y(1) = -1 \rightarrow -e^{-1} + (4+b)e^{-1} + 1 - 2 = -1$

$\hookrightarrow 1 = 4 + B \rightarrow b = -3$

$\hookrightarrow y = t - 2 + e^{-t}(t-1)$

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$3 \int_0^\infty e^{-t} \frac{\sin t}{t} dt \qquad \mathcal{L}(f(t)) = \int_0^\infty f(t) e^{-st} dx$

$s = -1$

$f(t) = \frac{\sin^2 t}{t} \longrightarrow \int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt = \mathcal{L}\left(\frac{\sin^2 t}{t}\right)$

$\cancel{f(t)}\, 5a \longrightarrow \int_1^\infty \mathcal{L}(\sin^2 u)\, du = \int_1^\infty \mathcal{L}\left(\frac{1}{2} - \frac{1}{2}\cos 2u\right) du$

$\longrightarrow \int_1^\infty \frac{1}{2u} \quad \frac{-1u}{2u^2+4} du = \left(\frac{1}{2}\ln u - \frac{1}{4}\ln u^2 + 4\right)\Big|_1^\infty$

$\frac{1}{4}\left(\ln \frac{u^2}{u^2+4}\right)\Big|_1^\infty = 0 - \frac{1}{4}\ln\frac{1}{5} = \ln\frac{C}{4}$

$4) \int_0^\infty t e^{-2t} \sin t\, dx = \mathcal{L}(t \sin t)\Big|_{s=2} \quad \mathcal{L}(t^n f(t)) = -1^n F^n(s)$

$n = 1 \qquad f(t) = \sin t \longrightarrow (-1)^1 F'(s)$

$F(s) = \mathcal{L}(\sin t) = \frac{1}{s+1} = F'(s) = \frac{-2s}{(s^2+1)^2}$

$F'(2) = \frac{-4}{25} \Rightarrow \mathcal{L}(t \sin t) = -1 \times \frac{-4}{25} = \boxed{\frac{4}{25}}$

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