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$$f(x)\begin{cases} 1 & -\pi < x < 0 \\ \cos^2(2x) = \dfrac{\cos(4x)+1}{2} & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{T}\int f(x)dx = \frac{1}{2\pi}\left(\int_{-\pi}^{0} 1\,dx + \int_0^{\pi}\left(\frac{\cos(4x)+1}{2}\right)dx\right) =$$

$$\frac{1}{2\pi}\left(\pi + \frac{\pi}{2}\right) = \frac{3\pi}{2}\times\frac{1}{2\pi} = \boxed{\frac{3}{4}}$$

$$a_n = \frac{1\times 2}{2\pi}\int f(x)\cos(nx)dx = \frac{1}{\pi}\left(\underbrace{\int_{-\pi}^{0}\cos(nx)dx}_{0} + \int_0^{\pi}\right.$$

$$\left.\left(\frac{\cos x(\cos(4x)+1)}{2}dx\right) = \frac{1}{\pi}\left(\int_0^{\pi}\frac{\cos(nx)\cos(4x)}{2}dx\right)\underbrace{\int_0^{\pi}\frac{\cos(nx)}{2}dx}_{0}\right)$$

$$= \frac{1}{2\pi}\left(\frac{\sin(n-4)x}{n-4}\right) + \frac{\sin(n+4)x}{n+4}\Bigg)\Bigg|_0^{\pi} = \frac{1}{2\pi}\times\frac{n\sin(n\pi)}{n^2-16} = 0$$

$$b_n = \frac{1\times 2}{2\pi}\int f(x)\sin(nx)dx = \frac{1}{\pi}\left(\int_{-\pi}^{0}\sin(nx)dx + \int_0^{\pi}\sin(nx)\frac{\cos(4x)+1}{2}dx\right.$$

$$= \frac{1}{\pi}\left(\frac{-\cos(nx)}{n}\Bigg|_{-\pi}^{0} + \frac{1}{4}\left(\frac{-\cos(n-4)x}{n-4} - \frac{2\cos(nx)}{n} -\right.\right.$$

$$\left.\left.\frac{\cos(n+4)x}{n+4}\right)\Bigg|_{0}^{\pi}\right)$$

$$\frac{1}{\pi}\left(\frac{\cos(n\pi - 1}{n} + \frac{-(n^2-8)(\cos(n\pi)-1)}{n(n^2-16)}\right) = \frac{-8(\cos(n\pi)-1)}{n(n^2-16)}$$

$$a_0 = \frac{3}{4} \qquad a_n = \frac{n\sin(n\pi)}{2\pi(n^2-16)} = 0 \qquad b_n = \frac{-8(\cos(n\pi)-1)}{n(n^2-16)}$$

if $n = 2k \Longrightarrow b_n = 0$

$n = 2k + 1 \qquad b_n = \dfrac{-16}{n(n^2 - 16)}$

$$f(x) = \frac{3}{4} + \sum_{n=1}^{\infty} \frac{-16}{(2n-1)\left((2n-1)^2 - 16\right)} \sin(nx)$$