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تمرین ۹-۱

1) $y=\dfrac{\sec x}{c_1+c_2\operatorname{tg}x} \longrightarrow yc_1+c_2y\operatorname{tg}x=\sec x$ ① $\xrightarrow{\frac{d}{dx}}$

$c_1y'+c_2y'\operatorname{tg}x+c_2y\sec^2x=\operatorname{tg}x\sec x \longrightarrow y'(c_1+c_2\operatorname{tg}x)=$

$\sec x(\operatorname{tg}x-c_2y\sec x) \xrightarrow{\times y} y'(c_1y+c_2y\operatorname{tg}x)=$ ②

$y\sec x(\operatorname{tg}x-c_2y\sec x) \xrightarrow{①,②} \dfrac{y}{y'}\cancel{\sec x}(\operatorname{tg}x-c_2y\sec x)=$

$\cancel{\sec x}$ ($c_1$ حذف) $\longrightarrow y'=y\operatorname{tg}x-c_2y^2\sec x$ ③ $\xrightarrow{\frac{d}{dx}} y''=y'\operatorname{tg}x+y\sec^2x$

$-2c_2yy'\sec x-c_2y^2\sec x\operatorname{tg}x \longrightarrow (-c_2y\sec x)(2y'+y\operatorname{tg}x)=$

$y''-y'\operatorname{tg}x-y\sec^2x \longrightarrow -c_2y\sec x=\dfrac{y(y''-y'\operatorname{tg}x-y\sec^2x)}{2y'+y\operatorname{tg}x}$ ④ $\xrightarrow{③,④}$

$y'-y\operatorname{tg}x=\dfrac{yy''-yy'\operatorname{tg}x-y^2\sec^2x}{2y'+y\operatorname{tg}x} \longrightarrow 2y'^2-2yy'\operatorname{tg}x+\cancel{yy'\operatorname{tg}x}$

$-y^2\operatorname{tg}^2x=yy''-\cancel{yy'\operatorname{tg}x}-y^2\sec^2x \longrightarrow yy''=2y'^2-y^2(\underbrace{\operatorname{tg}^2x-\sec^2x}_{})$

$\dfrac{\sin^2x}{\cos^2x}-\dfrac{1}{\cos^2x}=\dfrac{\sin^2x-1}{\cos^2x}=-1$

$\Rightarrow \boxed{yy''-2y'^2-y^2=0}$

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$(2)\quad y = c_1x + c_2\sin x + x^2 \;\; ①\;\longrightarrow\; y' = c_1 + c_2\cos x + 2x \;\; ②\;\longrightarrow\; y'' = -c_2\sin x + 2 \longrightarrow$

$c_1x = xy' - xc_2\cos x - 2x^2$

$c_2\sin x = 2 - y'' \;\; ③\quad \xrightarrow{①,③}\quad y = c_1x + 2 - y'' + x^2 \;\; ④$

$\longrightarrow c_1x = xy' - x\left(\dfrac{2-y''}{\sin x}\right)\cos x - 2x^2 \;\; ⑤\quad \xrightarrow{④,⑤}\quad y = xy' - x\left(\dfrac{2-y''}{\sin x}\right)\cos x$

$-2x^2 + x^2 + 2 - y'' \longrightarrow y\sin x = y'x\sin x - 2x\cos x + y''x\cos x - x^2\sin x$

$+(2-y'')\sin x \longrightarrow \boxed{y''(x\cos x - \sin x) + y'(x\sin x) - y\sin x = x^2\sin x + 2x\cos x - 2\sin x}$

بخش ۱-۱۰

$(2)\quad y = \ln(\mathrm{tg}\,x + c) \longrightarrow y' = \dfrac{1+\mathrm{tg}^2x}{\mathrm{tg}\,x + c} \longrightarrow \mathrm{tg}\,x + c = \dfrac{1+\mathrm{tg}^2x}{y'} \Rightarrow$

$e^y = \dfrac{1+\mathrm{tg}^2x}{y'} \xrightarrow{y' \to -\frac{1}{y'}} y' = \dfrac{-e^y}{1+\mathrm{tg}^2x} \longrightarrow \dfrac{dy}{dx} = \dfrac{-e^y}{\frac{1}{\cos^2x}} \longrightarrow -dy\,e^{-y} = dx\cos^2x$

$\xrightarrow{\int} e^{-y} = \dfrac{1}{2}\left(x + \dfrac{1}{2}\sin 2x + c\right) \longrightarrow \boxed{e^{-y} = \dfrac{x}{2} + \dfrac{\sin 2x}{4} + k}$

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۴) $r = c + c\cos\theta \xrightarrow{\frac{d}{d\theta}} \frac{dr}{d\theta} = -c\sin\theta \rightarrow c = \frac{-dr}{d\theta \cdot \sin\theta} \rightarrow$

$r = \frac{-dr}{d\theta \cdot \sin\theta}(1+\cos\theta) \xrightarrow{\frac{dr}{d\theta} \rightarrow -r^{۲}\frac{d\theta}{dr}} r = \frac{r^{۲}\, d\theta}{dr \cdot \sin\theta}(1+\cos\theta) \longrightarrow$

$\frac{dr}{r} = d\theta\left(\frac{1+\cos\theta}{\sin\theta}\right) \xrightarrow{\int} Ln\, r = Ln\, tg\frac{\theta}{۲} + Ln \sin\theta + c \longrightarrow$

$r = \frac{\sin\frac{\theta}{۲}}{\cos\frac{\theta}{۲}} \times \sin\frac{\theta}{۲} \cdot \cancel{\cos\frac{\theta}{۲}} \cdot c \rightarrow r = c(\underbrace{۲\sin^{۲}\frac{\theta}{۲}}_{1-\cos\theta}) \longrightarrow \boxed{r = c(1-\cos\theta)}$

۵) $\frac{dx}{dt} = ۰,۱kx + ۰,۰۱kx^{۲} \rightarrow \frac{dx}{dt} - \underbrace{۰,۱k}_{f(t)}x = \underbrace{۰,۰۱k}_{g(t)}x^{۲} \rightarrow u = x^{-1} \rightarrow$

$\frac{u'}{-1} - ۰,۱u = ۰,۰۱k \rightarrow u' + ۰,۱u = -۰,۰۱k \rightarrow \mu(t) = e^{\int ۰,۱dt} = e^{۰,۱t}$

$\rightarrow u = \frac{1}{e^{۰,۱t}}\left(\int -۰,۰۱ke^{۰,۱t} + C\right) \rightarrow u = \frac{1}{e^{۰,۱t}}(-۰,۱ke^{۰,۱t} + C) \longrightarrow$

$x^{-1} = -۰,۱k + \frac{C}{e^{۰,۱t}} \xrightarrow[(۱,۴)]{(۰,۱۰)} \begin{cases} ۰,۱ = -۰,۱k + C \\ \frac{1}{۴} = -۰,۱k + \frac{C}{e^{۰,۱}} \end{cases} \rightarrow \frac{-1}{۱۵} = C\left(\underbrace{1 - \frac{1}{e^{۰,۱}}}_{\frac{e^{۰,۱}-1}{e^{۰,۱}}}\right)$

$\longrightarrow C = \frac{e^{۰,۱}}{۱۵(1-e^{۰,۱})} \rightarrow x^{-1} = -۰,۱k + \frac{e^{۰,۱}}{e^{۰,۱t}(1-e^{۰,۱})\,۱۵} \rightarrow k = \frac{۵e^{۰,۱} - ۲}{۳(1-e^{۰,۱})}$

$\rightarrow x = \frac{۵(\frac{۳}{۴})^{t}}{1-۰,۵(۳/۴)^{t}} \rightarrow t = ۲ \rightarrow \boxed{x = ۳,۹۱}$

$x = ۲ \rightarrow \boxed{t = ۳,۸۲}$

۸۰ درصد از [illegible] بعد ۲ [illegible]

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6) $5\frac{dQ}{dt} + 50Q = 100 \rightarrow \frac{dQ}{dt} + \underbrace{10Q}_{P(t)} = \underbrace{20}_{g(t)} \rightarrow \mu(t) = e^{\int 10\,dt}$

$= e^{10t} \rightarrow Q = e^{-10t}\left(\int 20 e^{10t}\,dt + C\right) \rightarrow Q = e^{-10t}\left(2e^{10t} + C\right) \rightarrow$

$Q = 2 + Ce^{-10t} \xrightarrow{(0,4)} C = 2 \rightarrow \boxed{Q = 2 + 2e^{-10t}} \xrightarrow{\frac{d}{dt}} \boxed{I = -20e^{-10t}}$

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7) $\frac{dI}{dt} = I\frac{dI}{dQ}, \quad L\frac{dI}{dt} + \frac{Q}{C} = 0 \rightarrow LI\frac{dI}{dQ} + \frac{Q}{C} = 0$

$(LI)\,dI = -\frac{Q}{C}\,dQ \xrightarrow{\int} \frac{L}{2}I^2 = -\frac{Q^2}{2C}\Big|_{Q_0}^{Q} \rightarrow I^2 = \frac{1}{LC}\left(Q_0^2 - Q^2\right)$

$\rightarrow \boxed{I = \pm\frac{1}{\sqrt{LC}}\sqrt{Q_0^2 - Q^2}} \rightarrow \frac{dQ}{dt} = I \rightarrow Q = \int I\,dt =$

$\int \pm\frac{1}{\sqrt{LC}}\sqrt{Q_0^2 - Q^2}\,dt \rightarrow \boxed{Q = Q_0\cos\left(\frac{t}{\sqrt{LC}}\right)} \xrightarrow{I = \frac{dQ}{dt}}$

$\boxed{I = -\frac{Q_0}{\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right)}$

۸) $-\pi r^2\,dh = a v\,dt \to -\pi r^2\,dh = ak\sqrt{2gh}\,dt \to$

$-\pi \frac{R^2 h^2}{H^2}\,dh = ak\sqrt{2gh}\,dt \to \frac{-\pi R^2}{akH^2\sqrt{2g}}\, h^{\frac{3}{2}}\,dh = dt \xrightarrow{\int}$

$T = \frac{\pi R^2 H^{\cancel{\frac{5}{2}}\,\frac{1}{2}}}{\frac{5}{2} \cancel{H^2}\, ak\sqrt{2g}} \to \boxed{T = \frac{\pi R^2}{5ak}\sqrt{\frac{2H}{g}}}$

بخش ۲-۲ تا ۴-۲

۱) $y_1(x) \qquad y_2(x) \qquad y_3(x)$

$$\begin{cases} 1+x^3 & 1 & 3+x^3 \\ 1 & 1+x^3 & 3+x^3 \end{cases}$$

$[-1,0] \to$
$(0,1]$

$$\begin{cases} c_1 y_1(x) & c_2 y_2(x) & c_3 y_3(x) \\ c_1 + c_1 x^3 & c_2 & 3c_3 + c_3 x^3 \\ c_1 & c_2 + c_2 x^3 & 3c_3 + c_3 x^3 \end{cases}$$

[illegible] $\longrightarrow$

$$\begin{cases} c_1 + c_1x^3 + c_2 + 3c_3 + c_3x^3 = 0 \to \underbrace{c_3 = -c_1 \qquad c_2 = -2c_3}_{} \quad [-1,0] \\ c_1 + c_2 + 3c_3 \to \underbrace{(c_2 + c_3)x^3 = 0}_{} \quad (0,1] \quad \boxed{c_2 = c_3 = c_1 = 0} \end{cases}$$

$\longrightarrow c_2 = -c_3$

$\boxed{\text{مستقل}}$

$[-1,0]$

$$W(x) = \begin{vmatrix} 1+x^3 & 1 & 3+x^3 \\ 3x^2 & 0 & 3x^2 \\ 6x & 0 & 6x \end{vmatrix}$$

دو ستون ضریب همند $\longrightarrow \boxed{W_{(x)} = 0}$

برای (۰,۱] هم صفر میشود

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۲) $y'' - \underbrace{\frac{4}{x}}_{p(x)} y' + \underbrace{\frac{(x^2+6)}{x^2}}_{q(x)} y = 0$

$y=0$ یک جواب این معادله است، درست است زیرا

$p(x)$ و $q(x)$ در $0$ ناپیوسته است اما چون $y$ و $y'$ در

$0$ پیوسته است پس قضیه وجود و یکتایی برقرار است

و میدونیم یک جواب $x^2\sin x$ است و صفر نیز یک جواب ⇐ دلیل → وجود غیر صفر → $\{x^2\sin x , 0\}$

← جواب یکتا: $\boxed{x^2\sin x}$

۳) $y_1$ و $y_2$ جواب: ترکیب خطی هم جواب:

$c_1y_1 + c_2y_2 = y_3 \rightarrow c_1 = c_2 = 1 \longrightarrow \boxed{y_3 = y_1 + y_2}$

$c_1 = 1 , c_2 = -1 \longrightarrow \boxed{y_3 = y_1 - y_2}$ یا $\boxed{y_3 = y_2 - y_1}$ یا برعکس

و در ریشه‌های این مخرج

۴) دو ریشه متوالی $y_1$ : $a , b$

$\frac{y_2}{y_1} \xrightarrow{\frac{d}{dx}} \frac{y_2'y_1 - y_1'y_2}{(y_1)^2} \rightarrow \exists x \in (a,b) \longrightarrow$

$y_2'(x)\,y_1(x) - y_1'(x)\,y_2(x) = 0 \Rightarrow W(x) = 0$ که این نباید برقرار باشد ⇐

$\boxed{y_1(x) = 0}$ فقط این می‌ماند

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v) $p y_1' + q y_1 = -y_1''$

$p y_2' + q y_2 = -y_2'' \longrightarrow \begin{cases} p e^x + q e^x = -e^x \\ p + q(x+1) = 0 \end{cases} \longrightarrow$

$$\begin{cases} p + q = -1 \\ \underbrace{p + qx + q}_{-1} = 0 \longrightarrow qx = 1 \longrightarrow q = \frac{1}{x} \end{cases}$$

$$\frac{1}{x} + p = -1 \longrightarrow p = \frac{x+1}{-x} \longrightarrow \boxed{y'' - \frac{x+1}{x} y' + \frac{y}{x} = 0}$$