محمد امانلو
استاد طاهری
تکلیف دوم ریاضیات مهندسی

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$$f(x)=\begin{cases} 0 & |x|>\pi \\ x & |x|<\pi \end{cases}$$

تابع $f(x)$ فرد است پس ضریب کسینوسی انتگرال فوریه آن برابر صفر است. و فقط ضریب سینوسی دارد.

$$B_n(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\underbrace{f(x)}_{\text{فرد}}\sin(\omega x)\,dx=\frac{2}{\pi}\int_{0}^{\infty}f(x)\sin(wx)\,dx=\frac{2}{\pi}\int_{0}^{\pi}x\sin(wx)\,dx=\frac{2}{\pi}\left(\frac{\sin(\omega x)}{w^2}-\frac{x\cos(wx)}{w}\right)\Big|_0^{\pi}=\frac{-2\pi\cos(\pi w)}{\pi w}=$$

$$\frac{-2\cos(\pi w)}{w}$$

$$f(x)=\int_0^{\infty}\frac{-2\cos(\pi w)}{w}\sin(xw)\,dw$$

$$f(x)\begin{cases} 0 & |x| > 1 \\ \sinh(x) & |x| < 1 \end{cases}$$

$f(x)$ یک تابع فرد است. پس ضریب کسینوسی نیست و فقط ضریب سینوسی است

$$B_n(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(wx)\,dx = \frac{2}{\pi}\int_0^{\infty} f(x)\sin(wx)\,dx = \frac{2}{\pi}$$

$$\int_0^1 \sinh(x)\,\sin(wx)\,dx$$

این انتگرال را از طریق جزء به جزء حل می‌کنیم.

$$\frac{2}{\pi}\left(\frac{\cosh(x)\sin(wx)}{w^2+1} - \frac{w\sinh(x)\cos}{w^2+1}\right)\Big|_0^1 = \frac{2}{\pi}\left(\frac{\cosh(1)\sin(w)}{w^2-1} - \right.$$

$$\left.\frac{w\sinh(1)\cos(w)}{w^2+1}\right) = B(w)$$

$$f(x) = \int_0^{\infty} \frac{2}{\pi}\left(\frac{\cosh(1)\sin(w) - w\sinh(1)\cos(w)}{w^2+1}\right.\sin(wx)dx$$

$$f(x)\begin{cases} \sin x & 0 < x < \pi \\ 0 & \text{در غیر این صورت} \quad \text{otherwise} \end{cases}$$

$$I = \int_0^{\infty} \frac{\cos^2\left(\frac{\pi x}{2}\right)}{1-x^2}\,dx = 0$$

$$A_n = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(wx)\,dx = \frac{1}{\pi}\int_0^{\pi} \sin x\cos(wx)\,dx =$$

$$\frac{1}{\pi}\int_0^{\pi} \sin x \cos(wx)\,dx = \frac{1}{\pi}\int_0^{\pi} \frac{\sin((1-w)x)+\sin((1+w)x)}{2}$$

$$= \frac{-1}{2\pi}\left(\frac{\cos((1-w)x)}{1-w} + \frac{\cos(1+w)x}{1+w}\right)\Big|_0^{\pi} =$$

$$\frac{-1}{2\pi}\left(\left(\frac{-\cos w\pi}{1-w} + \frac{-\cos(\pi w)}{1+w}\right) - \left(\frac{1}{1-w}+\frac{1}{1+w}\right)\right) =$$

$$\frac{1}{2\pi}\left(\frac{2\cos w\pi}{1-w^2} + \frac{2}{1-w^2}\right) = \frac{1 \cdot 2}{2\pi}\left(\frac{\cos w\pi + 1}{1-w^2}\right) =$$

$$\frac{\cos(w\pi)+1}{\pi(1-w^2)}$$

$$B_n = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(wx)\,dx = \frac{1}{\pi}\int_0^{\pi} \sin x \sin(wx)\,dx =$$

$$\frac{1}{2\pi}\int_0^{\pi} (\cos(1-w)x - \cos(1+w)\,dx = \frac{1}{2\pi}\left(\frac{\sin(1-w)x}{1-w}\right.$$

$$\left. - \frac{\sin(1+w)x}{1+w}\right)\Big|_0^{\pi} = 0$$

$$f(x)\int_0^{\infty}\frac{\cos(wx)+1}{\pi(1-w^2)}\cos(wx)\,dw=\int_0^{\infty}\frac{2\cos^2\left(\frac{w\pi}{2}\right)}{\pi(1-w^2)}\cos(wx)\,dw$$

به جای x ، صفر قرار میدهیم . $f(0)=0$

$$0=\int_0^{\infty}\frac{2\cos^2\left(\frac{w\pi}{2}\right)}{\pi(1-w^2)}\,dw=\int_0^{\infty}\frac{\cos^2\left(\frac{w\pi}{2}\right)}{(1-w^2)}\,dw=0$$

به جای w ، x قرار میدهیم

$$\int_0^{\infty}\frac{\cos^2\left(\frac{x\pi}{2}\right)}{(1-x^2)}\,dx=0$$

$$f(x)=\begin{cases}1-x^2 & |x|<1\\ 0 & |x|>1\end{cases}\qquad I=\int\frac{(x\cos x-\sin x)^2}{x^6}\,dx=\frac{\pi}{15}$$

$f(x)$ زوج است ، سینوس نمیدهد و انتگرال سینوسی نیست

$$A(w)=\frac{1}{\pi}\int_{-\infty}^{\infty}f(x)\cos(wx)\,dx=\frac{1}{\pi}\times 2\int_0^{\infty}f(x)\cos(wx)\,dx=$$

$$\frac{2}{\pi}\int_0^1 f(x)\cos(wx)=\frac{2}{\pi}\int_0^1(1-x^2)\cos(wx)\,dx=$$

$$\frac{2}{\pi}\left(\int_0^1\cos(wx)\,dx-\int x^2\cos(wx)\,dx\right)=\frac{2}{\pi}$$

$$\left(\frac{\sin(wx)}{w}\Big|_0^1 - \left(\frac{x^2\sin(wx)}{w} + \frac{2x\cos(wx)}{w^2} - \frac{2\sin(wx)}{w^3}\right)\Big|_0^1\right) = \frac{2}{\pi}\left(\frac{\sin(w)}{w} - \frac{\sin(w)}{w^2} - \frac{2\sin(w)}{w^3}\right)$$

$$= \frac{2}{\pi}\left(-2\left(\frac{\cos(w)}{w^2} - \frac{\sin(w)}{w^3}\right)\right) = \frac{4}{\pi}\left(\frac{\sin(w)}{w^3} - \frac{\cos(w)}{w^2}\right)$$

$$f(x) = \int_0^\infty \frac{4}{\pi}\left(\frac{\sin(w) - w\cos w}{w^3}\right)\cos(wx)\,dx$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty}(1-x^2)^2\,dx = \int_0^\infty A^2(w) + B^2(w)\,dx = \int_0^\infty$$ * انتگرال فوریه

$$\frac{16}{\pi^2}\,\frac{(\sin w - w\cos w^2)}{w^6}\,dw$$

$$\Rightarrow \frac{16}{\pi\,15} = \int_0^\infty \frac{16}{\pi^2}\left(\frac{w\cos w - \sin w}{w^6}\right)^2 dw$$. به‌جای w ، x می‌گذاریم

$$\frac{\pi}{15} = \int_0^\infty \frac{x\cos x - \sin x^2}{x^6}\,dx$$

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$$F\left(\frac{x}{b^2+x^2}\right)$$

ما میدانیم : $F\left(e^{-a|t|}\right) = \dfrac{2a}{a^2+w^2} \Rightarrow$

$$F\left(\frac{1}{a^2+x^2}\right) = \frac{\pi}{a} \times e^{-a|w|}$$

$$F(x\,f(t)) = i\,\frac{d\,f(w)}{dw}$$

$$F\left(\frac{x}{b^2+x^2}\right) = i\left(\frac{\pi}{b}\,e^{-b|w|}\right)' \left[\begin{array}{ll} -\pi i e^{-bw} & w \geqslant 0 \\ \pi i e^{bw} & w < 0 \end{array}\right.$$

$$F\left(\frac{x}{b^2+x^2}\right) = -\operatorname{sgn}(w)\, i\pi\, e^{-b|w|}$$

$$F^{-1}\left(\frac{1}{b+i\alpha^2}\right)$$

ما میدانیم : $F(\delta(t)) = 1$

$$F\left(\frac{\delta(t)}{b+i\alpha^2}\right) = \frac{1}{(b+ia^2)}$$

بنابراین : $F^{-1}\left(\dfrac{1}{(b+ia)^2}\right) = \dfrac{\delta(t)}{(b+i\alpha)^2}$

$$F^{-1}\left(\frac{1}{(b+ia)^2}\right)$$

$$F\left(u(t)e^{-bt}\right) \Rightarrow \frac{1}{b+i\alpha}$$

$$F\left(u(t)\,te^{-bt}\right) = i\cdot\left(\frac{1}{b+i\alpha}\right)'$$

$$F\left(u(t)\,te^{-bt}\right) = \frac{i \times i \times -1}{(b+i\alpha)^2} = \frac{1}{(b+i\alpha)^2}$$

بنابراین $F^{-1}\left(\frac{1}{(b+i\alpha)^2}\right) = u(t)\, t e^{-bt}$

$f(\alpha) = \frac{1}{(i\alpha+4)(i\alpha-4)}$

$F\left(u(t) e^{-dt}\right) = \frac{1}{d+ia}$

$F^{-1}\left(\frac{1}{d+ia}\right) = u(t) e$

$f(a) = \frac{1}{8}\left(\frac{1}{ia-4} - \frac{1}{ia+4}\right) =$

$\frac{1}{8} F^{-1}\left(\frac{1}{ia-4} - \frac{1}{ia+4}\right) = \frac{1}{8}\left(F^{-1}\left(\frac{1}{ia-4}\right) - F^{-1}\left(\frac{1}{ia+4}\right)\right)$

$= \frac{1}{8}\left(u(t) e^{+4t} - u(t) e^{-4t}\right) = \frac{u(t)}{8}\left(e^{4t} - e^{-4t}\right)$

$F^{+}\left(\frac{1}{w^2+8w+32}\right) = F^{-1}\left(\frac{1}{(w+4-4i)(w+4+4i)}\right)$

$F^{-1}\left(\frac{i}{8}\left(\frac{1}{w+(4+4i)} - \frac{1}{w+4-4i}\right)\right) = \frac{i}{8}\left(F^{-1}\left(\frac{1}{w+4+4i}\right)\right.$

$\left. - F^{-1}\left(\frac{1}{w+4-4i}\right)\right)$ $F^{-1}\left(\frac{1}{w}\right) = \frac{sgn(t)}{-2i} \Rightarrow F^{-1}\left(\frac{1}{w-b}\right)$

$$= \frac{sgn(t)}{-2i} e^{ibt} \quad ٢$$

$$١, ٢ \Rightarrow = \frac{i}{8} F^{-1}\left(\frac{1}{w+4+4i}\right) - \frac{i}{8} F^{-1}\left(\frac{1}{w+4+4i}\right)$$

$$= \frac{i}{8}\left(\frac{sgn(t)}{-2i} e^{t(4-4i)} - \frac{sgn(t)}{-2i} e^{t(-4-4i)}\right)$$

$$= \frac{sgn(t)}{16}\left(e^{(-4-4i)t} - e^{(4-4i)t}\right)$$

$$F^{-1}\left(\frac{1}{w^2+6w+21.25}\right) = F^{-1}\left(\frac{i}{14}\left(\frac{1}{w+3+7i} - \frac{1}{w+3-7i}\right)\right) = \frac{i}{14}\left(F^{-1}\left(\frac{1}{w+3+7i}\right) - F^{-1}\left(\frac{1}{w^2+3-7i}\right)\right) \quad ١$$

$$F^{-1}\left(\frac{1}{w}\right) = \frac{sgn(t)}{-2i} \qquad F^{-1}\left(\frac{1}{w-b}\right) = \frac{sgn(t)}{-2i} e^{ibt} \quad ٢$$

$$٢, ١ \Rightarrow \frac{i}{14}\left(\frac{sgn(t)}{-2i} e^{7-3i} - \frac{sgn(t)}{-2i} e^{-7-3i}\right)$$

$$= \frac{-i}{22i}\left(sgn(t)\, e^{7-3i} - sgn(t)\, e^{-7-3i} \right) = \frac{sgn(t)}{28}$$

$$\left(e^{-7-3i} - e^{7-3i} \right)$$

$$F^{-1}\left(e^{-(w)} \cos w \right) \quad f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-|w|} \cos w \, e^{jw}$$

$$dt \qquad f(t) = \frac{1}{2\pi} \int_{-\infty}^{0} e^{w} e^{jwt} \left(\frac{e^{jw} + e^{-jw}}{2} \right) dw +$$

$$\int_{0}^{\infty} \frac{e^{-w} e^{jwt} \left(e^{jw} + e^{-jw} \right)}{2} dw = \frac{1}{4\pi} \int_{-\infty}^{0}$$

$$e^{w(1+jt+j)} + e^{w(1+jt-j)} dw + \int_{0}^{\infty} e^{w(-1+j+t}$$

$$+ e^{w(-1+jt-j)} dw) = \frac{1}{4\pi} \left(\frac{e^{w(1+jt+j)}}{jt+j+1} + \right.$$

$$\frac{e^{w(1+jt-j)}}{1+jt-j} \int_{-\infty}^{0} + \frac{e^{w(1+jt+j)}}{-1+j\,t+j} + \frac{e^{w(-1+jt-j)}}{-1+jt-j}$$

$$\sum_{0}^{\infty} \Big)$$

$$\frac{1}{4\pi}\left(\left(\frac{1}{jt+j+1}+\frac{1}{jt-j+1}\right)+\left(0+0-\left(\frac{1}{jt+j-}\right.\right.\right.$$

$$\left.\left.\left.+\frac{1}{jt-j-1}\right)\right)\right)$$

$$\frac{1}{4\pi}\left(\frac{2(jt+1)}{(jt+1)^2+1} = \frac{2(jt-1)}{(jt-1)^2+1} = \frac{1}{2\pi}\left(\frac{2(2+t^2)}{t^4+4}\right)\right.$$

$$= \frac{1}{\pi} \times \frac{2+t^2}{4+t^4}$$