تکلیف چهارم ریاضی مهندسی   استاد طالع

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$9u_{xx}=u_{tt}$

$$BC\begin{cases} u_x(0,t)=0 \\ u_x(x,t)=3 \end{cases} \qquad IC\begin{cases} u(x,0)=0 \\ u_t(x,0)=\cos 4x+\sin 2x \end{cases}$$

شرایط تعیین

$$w(x,t)=x(0)+\frac{x^2}{2\pi}(3-0)$$

$$w(x,t)=0+\frac{x^2}{2\pi}\times 3$$

$$u_{tt}=w_{tt}+v_{tt} \longrightarrow u_{tt}=0+v_{tt}=v_{tt}$$

$$u_{xx}=w_{xx}+v_{xx}$$

$$u_{xx}=\frac{3}{\pi}+v_{xx}$$

$$IC:\ u(x,0)=v(x,0)+w(x,0)=v(x,0)+\frac{3}{2\pi}x^2$$

$$0=v(x,0)+\frac{3}{2\pi}x^2 \longrightarrow v(x,0)=-\frac{3}{2\pi}x^2$$

$$u_t(x,0)=v_t(x,0)+w_t(x,0)=v_t(x,0)+0$$

$$v_t(x,0)=\cos(4x)+\sin(2x) \longrightarrow$$ جایگذاری در معادله ابتدایی

$$۹\left(\frac{۳}{\pi} + u_{xx}\right) = u_{tt} \longrightarrow \frac{۲۷}{\pi} + ۹u_{xx} = u_{tt}$$

$$BC\begin{cases} u_x(0,t) = 0 \\ u_x(x,t) = 0 \end{cases} \qquad IC\begin{cases} u(x,0) = \frac{-۳}{۲\pi}x^{۲} \\ u_t(x,0) = \cos ۳x + \sin ۲x \end{cases}$$

شرایط نیومن :

$$u(x,t) = \sum_{0}^{\infty} T_n(t)\cos\left(\frac{n\pi x}{\pi}\right)$$

$$= \sum_{0}^{\infty} T_n(t)\cos(nx)$$

$$u(x,t) = X(x)\,T(t)$$

$$u_x(0,t) = X_x(0)T(t) \to X_x(0) = 0$$

$$u_x(x,t) = X_x(x)\,T(t) \to X_x(x) = 0$$

$$u(x,t) = \frac{1}{۲}A_0 t + B_0 + \sum_{n=1}^{\infty} T_n(t)\cos(nx)$$

جاگذاری

$$\frac{۲۷}{\pi} + ۹u_{xx} = u_{tt}$$

$$\frac{۲۷}{\pi} + ۹\left(-n^{۲}\sum_{n=1}^{\infty} T_n(t)\cos(nx)\right) = \sum_{n=1}^{\infty} T_n''\cos(nx)$$

$$u_x = -n\sum_{n=1}^{\infty} T_n(t)\sin(nx) \qquad u_{xx} = -n^{۲}\sum_{n=1}^{\infty} T_n(t)\cos(nx)$$

$$u_{tt} = \sum_{n=1}^{\infty} T_n''\cos(nx) \qquad u_t = \sum T_n(t)\cos(nx)$$

$$\frac{۲۷}{\pi} = \sum_{n=1}^{\infty}\left(۹n^{۲}T_n(t) + T_n''(t)\right)\cos(nx)$$

$$A(n) = \frac{2}{\pi} \int_0^{\pi} \frac{27}{\pi} \cos nx = 0$$

$$9n^2 T_n(t) + T_n''(t) = 0 \qquad r^2 + 9n^2 = 0 \to r = \pm 3in$$

$$T_n(t) = A_n \cos(3nt) + B_n \sin(3nt)$$

$$u(x,t) = \frac{1}{2} A_0 t + B_0 + \sum_{n=1}^{\infty} \left(A_n \cos(3nt) + B_n \sin(3nt)\right)\cos(nx)$$

$$u(x,0) + \frac{3}{2\pi} x^2 = \sum_{n=1}^{\infty} A_n \cos(nx) + B_0$$ شرایط اولیه

ضریب سری فوریه کسینوسی — ضریب سری فوریه ثابت

$$B_0 = \frac{1}{\pi} \int_0^{\pi} -\frac{3}{2\pi} x^2 dx = \frac{1}{\pi} \left. -\frac{x^3}{2\pi} \right]_0^{\pi} = -\frac{\pi}{2}$$

$$A_n = \frac{1}{\pi} \int_0^{\pi} -\frac{3}{2\pi} x^2 \cos nx =$$

$$-\frac{3}{2\pi^2} \left[\frac{(n^2x^2 - 2)\sin(nx) + 2nx\cos(nx)}{n^3} \right]_0^{\pi}$$

$$= -\frac{3}{2\pi} \times \frac{-2\pi n}{n^3} = \frac{3}{n^2}$$

$$u_t(x,0) \to u_t = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} -3n A_n \sin(3nt) + 3n B_n \cos(3nt)$$

$$u_t(x,0) \to \frac{1}{2} A_0 + \sum_{n=1}^{\infty} 3n B_n \cos(3nt) = \cos 3x + \sin 2x$$

$$\frac{1}{2} A_0 = \frac{1}{\pi} \int_0^{\pi} \cos 3x + \sin 2x \to A_0 = 0$$

$$B_n = \frac{1}{3n} \int_0^{\pi} (\cos 3x + \sin 2x) \cos(nx) =$$

$$\frac{1}{3n} \times \frac{-(18-2n^2)+2n^2-18}{(n^2-9)(n^2-4)} = \frac{\overset{4(n^2-9)}{4n^2-36}}{(n^2-9)(n^2-4)} \times \frac{1}{3n}$$

$$\boxed{r(x,t) = -\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{3}{\pi n^2} \cos(3nt) + \frac{1}{(n^2-4)} \times \frac{1}{3n} \times \sin(3nt) \times \cos(nx)}$$

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(۲) شرایط مرزی دیریکله :

$$u(x,t) = \sum_{n=1}^{\infty} G_n(t) \sin\left(\frac{n}{2}x\right), \quad L=2\pi$$

$$\xrightarrow{\text{جایگذاری}} \frac{1}{4} \sum_{n=1}^{\infty} G_n \sin\left(\frac{n}{2}x\right) = -\sum_{n=1}^{\infty} \frac{n^2}{4} G_n(t) \sin\left(\frac{n}{2}x\right)$$

$$\Rightarrow \sum_{n=1}^{\infty} \left[\frac{1}{4} G_n(t) + \frac{n^2}{4} G_n(t)\right] \sin\left(\frac{n}{2}x\right) = 0 \Rightarrow$$

$$G_n(t) + \frac{n^2}{4} G_n(t) = 0 \xrightarrow{\text{معادله مشخصه}} \lambda + \frac{n^2}{4} = 0 \rightarrow \lambda = -\frac{n^2}{4}$$

$$G_n(t) = b_n e^{-\frac{n^2}{4}t} \Rightarrow u(x,t) = \sum_{n=1}^{\infty} b_n e^{-\frac{n^2}{4}t} \sin\left(\frac{n}{2}x\right)$$

$$u(x,0) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n}{2}x\right) = \delta\left(x-\frac{1}{2}\right) \Rightarrow$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} \delta\left(x-\frac{1}{2}\right) \sin\left(\frac{n}{2}x\right)$$ شرایط اولیه

$$\Rightarrow b_n = \frac{1}{\pi} \sin\left(\frac{n}{4}\right) \Rightarrow$$

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} \frac{1}{\pi} \sin\left(\frac{n}{4}\right) e^{-\frac{n^2}{4}t} \sin\left(\frac{n}{2}x\right)}$$

۳) شرایط مرزی دیریکله

$$w(x,t) = C + Dx$$

$$u(x,t) = v(x,t) + C + Dx$$

$$\begin{cases} u(0,t) = \overbrace{v(0,t)}^{0} + C \longrightarrow C = 0 \\ u(2\pi,t) = v(2\pi,t) + 2\pi D = 1 \longrightarrow D = \frac{1}{2\pi} \end{cases} \Longrightarrow w(x,t) = \frac{x}{2\pi}$$

$$u(x,t) = v(x,0) + \frac{x}{2\pi} \xrightarrow{\text{جایگذاری}} v_t(x,t) = \varepsilon v_{xx}(x,t) + \Pi\left(\frac{x-\pi}{2\pi}\right)$$

$$IC: \; u(x,0) = v(x,0) + \frac{x}{2\pi} = \Pi\left(\frac{x}{2\pi}\right) + \frac{x}{2\pi} \Longrightarrow$$

$$v(x,0) = \Pi\left(\frac{x}{2\pi}\right) \qquad \begin{cases} v(0,t) = 0 \\ v(2\pi,t) = 0 \end{cases}$$

$$v(x,t) = \sum_{n=1}^{\infty} G_n(t) \sin\left(\frac{n}{2}x\right) \; ; \; L = 2\pi$$ شرایط مرزی دیریکله:

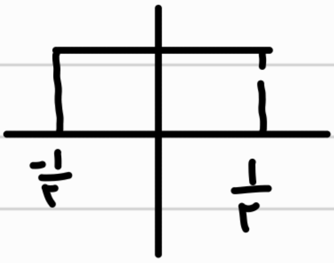
$$\xrightarrow{\text{جایگذاری}} \sum_{n=1}^{\infty} G_n'(t) \sin\left(\frac{n}{2}x\right) = -\sum_{n=1}^{\infty} G_n(t)\, n^2 \sin\left(\frac{n}{2}x\right) + \Pi\left(\frac{x-\pi}{2\pi}\right)$$

$$\Longrightarrow \sum_{n=1}^{\infty} \left[G_n'(t) + n^2 G_n(t) \right] \sin\left(\frac{n}{2}x\right) = \Pi\left(\frac{x-\pi}{2\pi}\right)$$

$$G_n'(t) + n^2 G_n(t) = \frac{1}{\pi} \int_0^{2\pi} \Pi\left(\frac{x-\pi}{2\pi}\right) \sin\left(\frac{n}{2}x\right) dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} \sin\left(\frac{n}{2}x\right) dx = 2\left(\frac{1-(-1)^n}{n\pi}\right)$$

معادله مشخصه: $\lambda + n^2 = 0 \Longrightarrow -n^2 = \lambda \Longrightarrow G_n^h(t) = Be^{-nt}$ جواب عمومی



$$G_n^p(t) = A \Longrightarrow n^2 A = 2 \times \frac{1-(-1)^n}{n\pi} \Longrightarrow A = 2 \times \frac{1-(-1)^n}{n^3\pi}$$

جواب خصوصی

$$G_n(t) = Be^{-n^2 t} + 2\,\frac{1-(-1)^n}{n^3\pi} \Rightarrow v(x,t) = \sum_{n=1}^{\infty}\left[Be^{-n^2 t} + 2\,\frac{1-(-1)^n}{n^3\pi}\right]\sin\left(\frac{n}{2}x\right)$$

$$v(x,0) = \sum_{n=1}^{\infty}\left[B + 2\,\frac{1-(-1)^n}{n^3\pi}\right]\sin\left(\frac{n}{2}x\right) = \pi\left(\frac{x}{2\pi}\right)$$

$$B + 2\,\frac{1-(-1)^n}{n^3\pi} = \frac{1}{\pi}\int_0^{2\pi}\pi\left(\frac{x}{2\pi}\right)\sin\left(\frac{n}{2}x\right)dx = \frac{1}{\pi}\int_0^{\pi}\sin\left(\frac{n}{2}x\right)dx = 0$$

$$\Rightarrow B = -2\,\frac{1-(-1)^n}{n^3\pi}$$

$$v(x,t) = \sum_{n=1}^{\infty} 2\,\frac{1-(-1)^n}{n^3\pi}\left[1-e^{-n^2 t}\right]\sin\left(\frac{n}{2}x\right)$$

$$\boxed{u(x,t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^3}\left[1-e^{-n^2 t}\right]\sin\left(\frac{n}{2}x\right) + \frac{x}{4\pi}}$$

5

$$u_{tt} = u_{xx} \qquad 0<x<1 \qquad t>0$$

$$\begin{cases} u_x(0,t) = t-4 & u(1,t) = 7t \\ u(x,0) = 4-4x & u_t(x,0) = 8(x-1) \end{cases}$$

عبارت همگن با شرایط مرزی ناهمگن

$$w(x,t) = (x-L)\,a(t) + b(t)$$

$$w(x,t) = (x-1)(t-4) + 7t = xt - 4x - t + 4 + 7t$$

$$= xt + 6t - 4x + 4$$

$$u(x,t) = v(x,t) + w(x,t)$$

$$\left.\begin{array}{l} u_{tt} = v_{tt} + 0 \\ u_{xx} = v_{xx} + 0 \end{array}\right\} \rightarrow u_{tt} = v_{xx}$$

$$u(x,0) = v(x,0) + w(x,0) \rightarrow v(x,0) = \underbrace{4 - 4x - (4 - 4x)}_{4-4x-4+4x} = 0$$

$$u_t(x,0) = v_t(x,0) + w_t(x,0) \rightarrow v_t(x,0) = -1(x-1) - x - 4$$

$$BC: \begin{cases} v(1,t) = u(1,t) - w(1,t) = Vt - (Vt) = 0 \\ v_x(0,t) = u_x(0,t) - w_x(0,t) = \underbrace{t - 4 - (t - 4)}_{t-4-t+4} = 0 \end{cases}$$

$$\Longrightarrow v_{tt} = v_{xx} \quad (\text{همگن})$$

$$IC: \begin{cases} v(x,0) = 0 \\ v_t(x,0) = -1(x-1) - x - 4 \end{cases} \qquad BC: \begin{cases} v_x(0,t) = 0 \\ v(1,t) = 0 \end{cases} \quad \text{همگن}$$

$$v(x,t) = F(x)G(t) \longrightarrow v_{tt} = F(x)\ddot{G}(t)$$

$$v_{xx} = F(x)'' G(t) \Longrightarrow F(x)\ddot{G}(t) = F(x)'' G(t) \Longrightarrow$$

$$\frac{\ddot{G}(t)}{G(t)} = \frac{(F(x))''}{F(x)} = -k^2$$

$$G_n(t) = B_n \cos(\lambda_n t) + C_n \sin(\lambda_n t)$$

$$v(x,t) = \sum_{n=1}^{\infty} G_n(t) \cos\left(\frac{2n-1}{2}\pi x\right), \quad L=1$$

شرایط مرزی ترکیبی

$$\Rightarrow \sum_{n=1}^{\infty} \left[\ddot{G}_n(t) + \left(\frac{2n-1}{2}\pi\right)^2 G_n(t) \right] \cos\left(\frac{2n-1}{2}\pi x\right) = 0$$

$$\Rightarrow \ddot{G}_n(t) + \left(\frac{2n-1}{2}\pi\right)^2 G_n(t) = 0 \xrightarrow[\text{مشخصه}]{\text{معادله}} \lambda^2 + \left(\frac{2n-1}{2}\pi\right)^2 = 0$$

$$\longrightarrow \lambda = \pm i \frac{2n-1}{2}\pi$$

$$G_n(t) = c_n \cos\left(\frac{2n-1}{2}\pi t\right) + d_n \sin\left(\frac{2n-1}{2}\pi t\right)$$

$$v(x,t) = \sum_{n=1}^{\infty} \left[c_n \cos\left(\frac{2n-1}{2}\pi t\right) + d_n \sin\left(\frac{2n-1}{2}\pi t\right) \right] \cos\left(\frac{2n-1}{2}\pi x\right)$$

$$v(x,0) = 0 \longrightarrow \sum_{n=1}^{\infty} c_n \cos\left(\frac{2n-1}{2}\pi x\right) = 0 \to c_n = 0$$

$$v_t(x,0) = \sum_{n=1}^{\infty} \frac{2n-1}{2}\pi d_n \cos\left(\frac{2n-1}{2}\pi x\right) = \delta(x-1) - x - 6$$

$$\frac{2n-1}{2}\pi d_n = 2\int_0^1 \left[\delta(x-1) - x - 6 \right] \cos\left(\frac{2n-1}{2}\pi x\right) dx$$

$$\frac{2n-1}{2}\pi d_n = 2\int_0^1 -6 \cos\left(\frac{2n-1}{2}\pi x\right) dx = \frac{-24}{\pi} \frac{(-1)^n}{1-2n}$$

$$\Rightarrow d_n = \left(\frac{2}{\pi(2n-1)}\right)^2 (-1)^n \times (12)$$

$$v(x,t) = 12 \sum_{n=1}^{\infty} \left(\frac{2}{\pi(2n-1)}\right)^2 (-1)^n \sin\left(\frac{2n-1}{2}\pi t\right) \cos\left(\frac{2n-1}{2}\pi x\right)$$

$$\longrightarrow u(x,t) = 12 \sum_{n=1}^{\infty} \left(\frac{2}{\pi(2n-1)}\right)^2 (-1)^n \sin\left(\frac{2n-1}{2}\pi t\right) \cos\left(\frac{2n-1}{2}\pi x\right) + xt - 6x + 6t + 6$$

$u_{xx} = u_t \qquad x>0 \quad t>0$ ②

$$\begin{cases} u(0,t) = e^{-4t} \\ u(x,0) = x\sin(x) \end{cases} \qquad u(x,t) = F(x)G(t)$$

$$\frac{\dot{G}}{G(t)} = \frac{F''(x)}{F(x)} = k \quad \begin{cases} ODE(x) & \frac{F''(x)}{F(x)} = k \\ ODE(t) & \frac{\dot{G}}{G(t)} = k \end{cases}$$

$$u(0,t) = F(0)\,G(t) = e^{-4t} \rightarrow F(0) = 1$$

$$K<0 \longrightarrow \dot{G}(t) = kG(t) \longrightarrow G(t) = Ae^{kt} \qquad k = -\omega^2$$

$$G_\omega(t) = Ae^{-\omega^2 t}$$

$$ODE: \; F''(x) + \omega^2 F(x) = 0 \longrightarrow$$

$$F_\omega(x) = A(\omega)\cos(\omega x) + B(\omega)\sin(\omega x)$$

$$F_\omega(0) = 1 = A(\omega) \longrightarrow F_\omega(x) = \cos(\omega x) + B(\omega) + \sin(\omega x)$$

$$u_\omega(x,t) = F_\omega(t)\cos(t) = \left(\cos(\omega x) + B(\omega)\sin(\omega x)\right) Ae^{-\omega^2 t}$$

$$u(x,0) = F(x)G(0) = A\int_0^\infty \left[\cos(\omega x) + B(\omega)\sin(\omega x)\right] d\omega$$

$$= x\sin x \Longrightarrow A = 1 \rightarrow u_\omega(x,t) = F_\omega(x)\,G_\omega(t)$$

$$= \left(\left(\cos(\omega x) + B(\omega)\sin(\omega x)\right)e^{-\omega^2 t}\right.$$

$$u(x,t) = \int_0^\infty \left[\cos(\omega x) e^{-\omega^2 t} + B(\omega)\sin(\omega x) e^{-\omega^2 t}\right] d\omega$$

$$\int_0^\infty \left[\cos(\omega x) + B(\omega)\sin(\omega x)\right] dx = x \sin c(x) = \sin x$$

$$B(\omega) = \frac{2}{\pi}\int_0^\infty \sin x \sin(\omega x)\, dx = \frac{1}{2\pi}\int_0^\infty \left[\cos(x-\omega x) - \cos(x+\omega x)\right] dx$$

$$= \frac{1}{\pi}\left(\frac{\sin(x-\omega x)}{1-\omega} - \frac{\sin(x+\omega x)}{1+\omega}\right)\Big]_0^\infty$$

4

$$w(x,t) = C + Dx \to u(x,t) = v(x,t) + C + Dx$$

شرایط مرزی دیریکله :

$$\underbrace{v(0,t)}_{0} + C = u(0,t) = 4 \Rightarrow C = 4$$

$$u(2,t) = v(2,t) + 4 + 2D = 7 \to D = \frac{3}{2}$$

$$u(x,t) = v(x,t) + 4 + \frac{3}{2}x \longrightarrow v_{xx}(x,t) - v_{tt}(x,t) = 7xt$$

$$\begin{cases} u_t(x,0) = v_t(x,0) = 2 \longrightarrow v_t(x,0) = 2 \\ u(x,0) = v(x,0) + 4 + \frac{3}{2}x = x^2 + \frac{3}{2}x \longrightarrow v(x,0) = x^2 - 4 \end{cases}$$

$$\begin{cases} v(2,t) = 0 \\ v(0,t) = 0 \end{cases}$$

$$v(x,t) = \sum_{n=1}^{\infty} G_n(t)\sin\left(\frac{n\pi}{2}x\right) \quad L = 2$$

شرایط مرزی دیریکله

$$\sum_{n=1}^{\infty} G_n(t)\left(\frac{n\pi}{2}\right)^2 \sin\left(\frac{n\pi}{2}x\right) - \sum_{n=1}^{\infty} \ddot{G}_n(t)\sin\left(\frac{n\pi}{2}x\right) = Vxt$$

$$\Rightarrow -Vxt = \sum_{n=1}^{\infty}\left[\ddot{G}_n(t) + \left(\frac{n\pi}{2}\right)^2 G_n(t)\right]\sin\left(\frac{n\pi}{2}x\right) \longrightarrow$$ سری فوریه

$$\ddot{G}_n(t) + \left(\frac{n\pi}{2}\right)^2 G_n(t) = \int_0^2 (-Vxt)\sin\left(\frac{n\pi}{2}x\right)dx = -28\frac{t(-1)^n}{n\pi}$$

$$\longrightarrow \lambda^2 + \frac{n^2\pi^2}{4} = 0 \longrightarrow \lambda = \pm i\frac{n\pi}{2} \Rightarrow$$

$$G_n^h(t) = c_n\cos\left(\frac{n\pi t}{2}\right) + d_n\sin\left(\frac{n\pi t}{2}\right)$$ جواب عمومی

$$G_n^p(t) = At+B \Rightarrow \left(\frac{n\pi}{2}\right)^2(At+B) = -28\frac{(-1)^n}{n\pi}t \Rightarrow$$

$$A = -14(-1)^n\left(\frac{2}{n\pi}\right)^3, \quad B=0$$ جواب خصوصی

$$G_n(t) = G_n^p(t) + G_n^h(t) = c_n\cos\left(\frac{n\pi}{2}t\right) + d_n\sin\left(\frac{n\pi t}{2}\right) - 14(-1)^n\left(\frac{2}{n\pi}\right)^3 t$$

$$V(x,t) = \sum_{n=1}^{\infty}\left[c_n\cos\left(\frac{n\pi}{2}t\right) + d_n\sin\left(\frac{n\pi t}{2}\right) - 14(-1)^n\left(\frac{2}{n\pi}\right)^3 t\right]\sin\left(\frac{n\pi x}{2}\right)$$

$$V(x,0) = \sum_{n=1}^{\infty} c_n\sin\left(\frac{n\pi}{2}x\right) = x^2-4 \Rightarrow$$

$$c_n = \int_0^2 (x^2-4)\sin\left(\frac{n\pi}{2}x\right)$$

$$= 2\frac{\left(4\pi n x\sin\left(\frac{\pi n x}{2}\right) - \left(\pi^2 n^2 (x-2)(x+2) - 8\right)\cos\left(\frac{\pi n x}{2}\right)\right)}{n^3\pi^3}\Bigg|_0^2$$

$$= -8\frac{\left(n^2\pi^2 - 2(-1)^n + 2\right)}{n^3\pi^3}$$

$$v_t(x,0) = \sum_{n=1}^{\infty}\left[\frac{n\pi}{2} d_n - 14(-1)^n\left(\frac{2}{n\pi}\right)^3\right]\sin\left(\frac{n\pi x}{2}\right) = 2 \Longrightarrow$$

$$\frac{n\pi}{2} d_n - 14(-1)^n\left(\frac{2}{n\pi}\right)^3 = \frac{4}{\pi n}\left(1-(-1)^n\right) \Longrightarrow$$

$$d_n = 2\left(\frac{2}{n\pi}\right)^2\left(1-(-1)^n\right) + 14(-1)^n\left(\frac{2}{n\pi}\right)^4$$

$$v(x,t) = \sum_{n=1}^{\infty}\left[\frac{-8(n^2\pi^2 - 2(-1)^n + 2)}{n^2\pi^3}\cos\left(\frac{n\pi}{2}t\right) + \right.$$

$$\left.\left(2\left(\frac{2}{n\pi}\right)^2\left(1-(-1)^n\right) + 14(-1)^n\left(\frac{2}{n\pi}\right)^4\right)\sin\left(\frac{n\pi}{2}t\right) - 14(-1)^n\left(\frac{2}{n\pi}\right)^3 t\right]$$

$\sin\left(\frac{n\pi x}{2}\right)$

$$u(x,t) = \sum_{n=1}^{\infty}\left[\frac{-8(n^2\pi^2 - 2(-1)^n + 2)}{n^3\pi^3}\cos\left(\frac{n\pi}{2}t\right) + \left(2\left(\frac{2}{n\pi}\right)^2\left(1-(-1)^n\right) + 14(-1)^n\left(\frac{2}{n\pi}\right)^4\right)\right.$$

$\sin\left(\frac{n\pi}{2}t\right)$

$$\left. - 14(-1)^n\left(\frac{2}{n\pi}\right)^3 t\right]\sin\left(\frac{n\pi}{2}x\right)$$