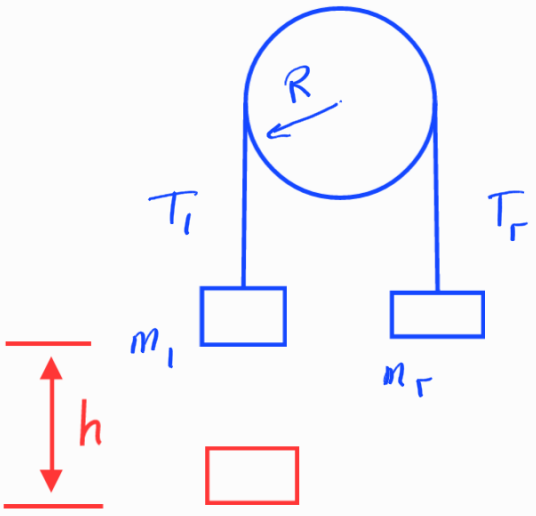
۱- فرض کنیم $m_1 > m_2$



$$\begin{cases} T_1 - m_1 g = m_1 a_1 \\ T_2 - m_2 g = m_2 a_2 \\ T_1 R - T_2 R = I\alpha \\ -a_1 = a_2 = R\alpha \end{cases}$$

الف) $h = -\frac{1}{2} a_1 t^2 \Rightarrow a_1 = \frac{-2h}{t^2}$

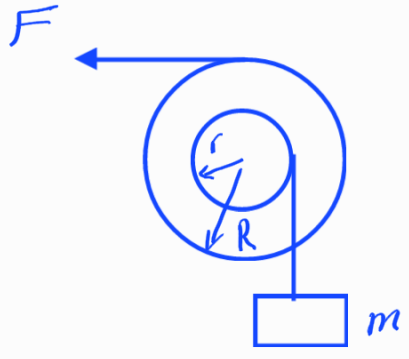
ب) $T_1 = m_1 (g + a_1) = m_1 \left(g - \frac{2h}{t^2}\right)$

پ) $T_2 = m_2 (g + a_2) = m_2 \left(g + \frac{2h}{t^2}\right)$

ت) $\alpha = -\frac{a_1}{R} = \frac{2h}{Rt^2}$

ث) $I = (T_1 - T_2) R / \alpha$

۲-

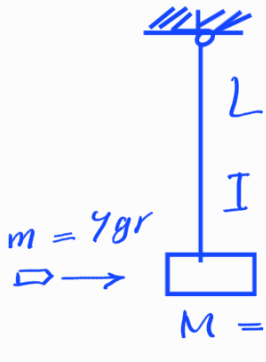


$$\begin{cases} T - mg = ma & (1) \\ FR - Tr = I\alpha & (2) \end{cases}$$

(1) $\Rightarrow T = m(g + a)$

(2) $\Rightarrow I = \frac{FR - Tr}{\alpha}$

۴-

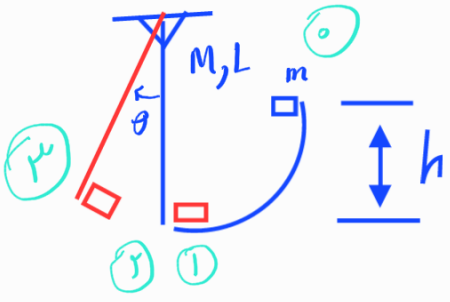


$$L_i = L_f \Rightarrow mvL = (I + mL^2 + ML^2)\omega$$

$$(0/009)(0/9)V = (0/09 + (0/009)(0/9)^2 + (0/15)(0/9)^2)(\tfrac{V}{0/9})$$

$$\Rightarrow V = 20/2\ m/s$$

۵-



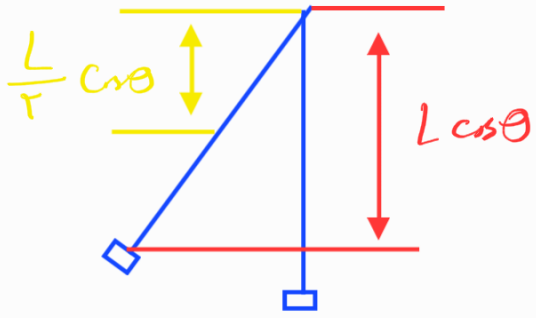
$$\begin{cases} E_0 = E_1 \\ L_1 = L_2 \\ E_2 = E_3 \end{cases}$$

قبل از برخورد: $L_1 = mvL \qquad v = \sqrt{2gh}$

بعد از برخورد: $L_2 = mv'L + I\omega \qquad v' = L\omega \qquad I = \frac{1}{3}ML^2$

$$L_1 = L_2 \Rightarrow mv = mL\omega + \frac{1}{3}ML\omega$$

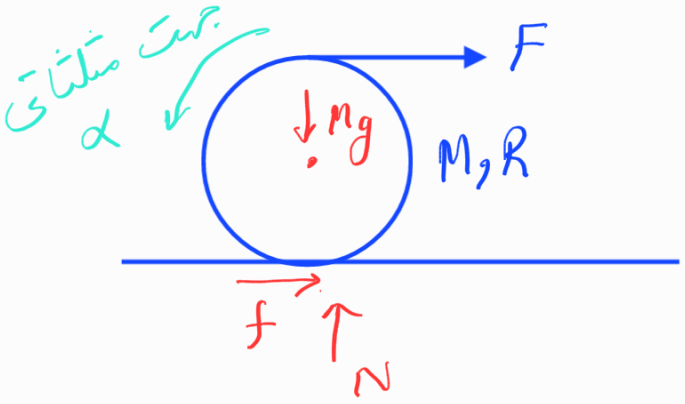
$E_2 = E_3$:



$$E_2 = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 = \left(\frac{m}{2} + \frac{M}{6}\right)L^2\omega^2$$

$$E_3 = mg(L - L\cos\theta) + Mg\left(\frac{L}{2} - \frac{L}{2}\cos\theta\right)$$

$$= gL(1 - \cos\theta)\left(m + \frac{M}{2}\right)$$



-۵

$$F_{net,x} = ma_x$$

$$\tau_z = I_{com,z}\,\alpha$$

$$a_z = -R\alpha \quad (1)$$

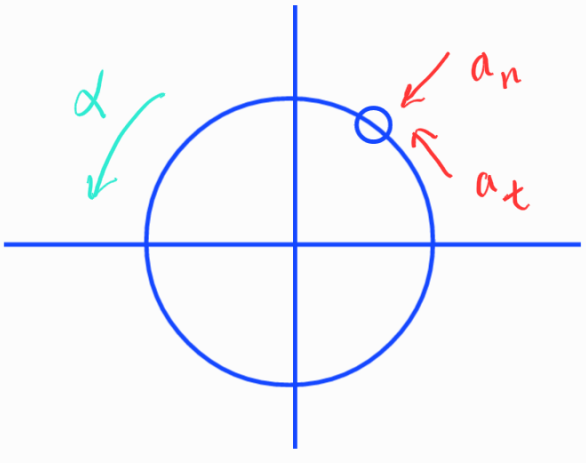
$$x: \quad F + f = Ma_x \quad (2)$$

$$z: \quad -FR + fR = \frac{1}{2}MR^2\alpha \Rightarrow -F + f = \frac{1}{2}MR\alpha \quad (3)$$

$$(1), (3) \Rightarrow F - f = \frac{1}{2}Ma_x \quad (4)$$

$$(2) + (4) \Rightarrow 2F = \frac{3}{2}Ma_x \Rightarrow a_x = \frac{4}{3}\frac{F}{M}$$

$$\Rightarrow \alpha = -\frac{4}{3}\frac{F}{MR} \Rightarrow f = +\frac{1}{3}F$$



-۶

$$\alpha = ct$$

$$c = 0.1 \text{ rad/s}^3$$

$$R = 10\text{ m}$$

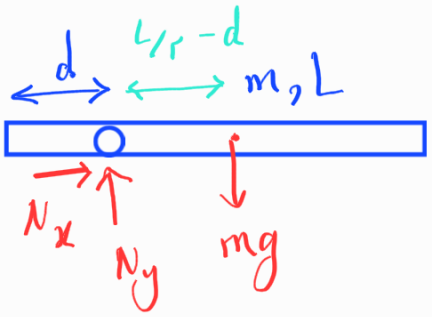
$$a_t = R\alpha = Rct \qquad a_n = \frac{V^2}{R}$$

$$\omega = \frac{1}{2}ct^2 \qquad V = R\omega$$

$$t = 4\text{ s} \quad \alpha = 0.4 \text{ rad/s}^2 \quad \omega = 0.8 \text{ rad/s} \quad V = 8 \text{ m/s}$$

$$a_t = 4 \text{ m/s}^2 \qquad a_n = 6.4 \text{ m/s}^2$$

$$a = \sqrt{a_n^2 + a_t^2} \simeq 7.55 \text{ m/s}^2$$



-V

$$\tau_A = I_A \alpha$$

$$I_A = \frac{1}{12} mL^2 + m\left(\frac{L}{2} - d\right)^2$$

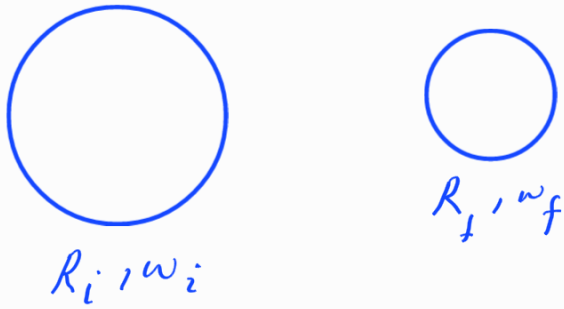
$$\tau_A = -mg\underbrace{\left(\frac{L}{2} - d\right)}_{x}$$

$$\rightarrow \alpha = \frac{\tau_A}{I_A} = -g\,\frac{\frac{L}{2} - d}{\frac{1}{12}L^2 + \left(\frac{L}{2} - d\right)^2} = \frac{-gx}{\frac{1}{12}L^2 + x^2}$$

$$\frac{d\alpha}{dx} = -g\,\frac{\left(\frac{1}{12}L^2 + x^2\right) - x(2x)}{\left(\frac{1}{12}L^2 + x^2\right)} = 0$$

$$\Rightarrow \frac{L^2}{12} = x^2 \Rightarrow x = \frac{L}{2\sqrt{3}} \qquad d = \frac{L}{2} - x$$

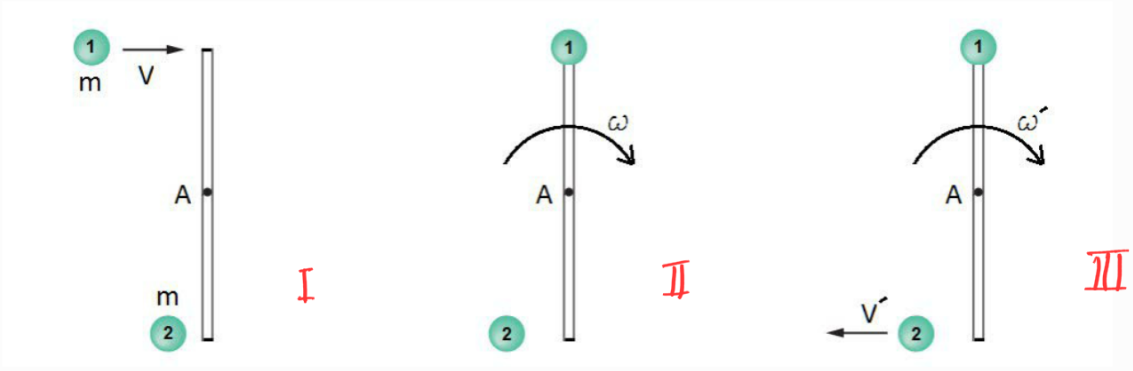
-A



$$\tau_{ext} = 0 \Rightarrow L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f \quad (1)$$

$$\Rightarrow \frac{2}{5} m R_i^2 \omega_i = \frac{2}{5} m R_f^2 \omega_f \Rightarrow \frac{\omega_f}{\omega_i} = \left(\frac{R_i}{R_f}\right)^2 = 2^2$$

$$\frac{K_f}{K_i} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} \overset{(1)}{=} \frac{\omega_f}{\omega_i} = 2^2$$



دوران ساعتگرد را مثبت در نظر می‌گیریم

$$L_I = L_{II} : \quad mv\frac{L}{2} = \left(I + m\left(\frac{L}{2}\right)^2\right)\omega \qquad I = \frac{1}{12}ML^2$$

(ذره + میله)

$$\Rightarrow \omega = \frac{2mv}{\left(\frac{1}{2}M+m\right)L} \quad (1) \qquad I_t$$

$$L_{II} = L_{III} : \quad I_t\omega = I_t\omega' + mv'\frac{L}{2} \qquad I_t = \left(\frac{M}{12}+\frac{m}{4}\right)L^2$$

$$\left(\frac{M}{12}+\frac{m}{4}\right)L^2(\omega-\omega') = mv'\frac{L}{2} \Rightarrow \omega-\omega' = \frac{2mv'}{\left(\frac{M}{2}+m\right)L} \quad (2)$$

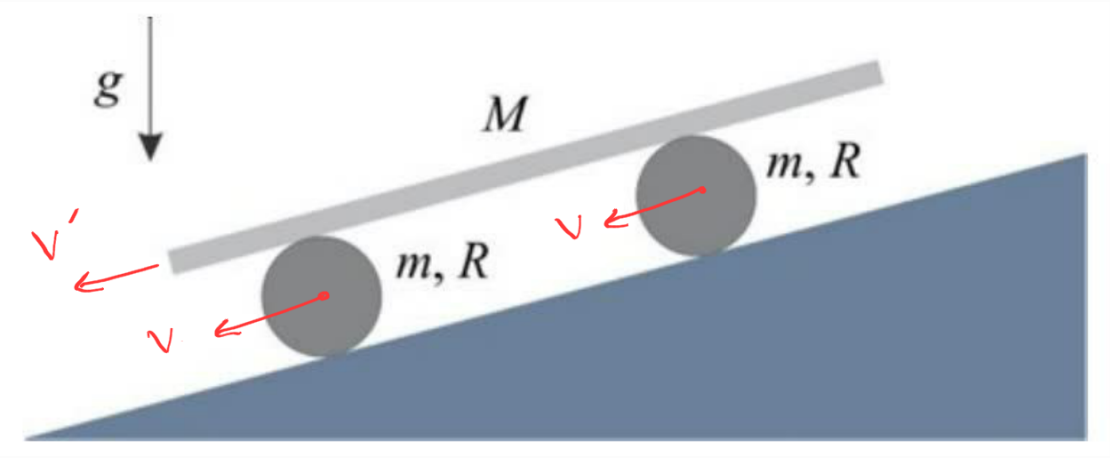
$$E_{II} = E_{III} : \quad \frac{1}{2}I_t\omega^2 = \frac{1}{2}I_t\omega'^2 + \frac{1}{2}mv'^2$$

$$\Rightarrow \left(\frac{M}{12}+\frac{m}{4}\right)L^2(\omega^2-\omega'^2) = mv'^2 \quad (3)$$

$$\frac{(3)}{(2)} \Rightarrow \omega+\omega' = \frac{2v'}{L} \Rightarrow v' = \frac{L}{2}(\omega+\omega') \quad (4)$$

$$(2),(4) \Rightarrow \omega' = \frac{M}{M+4m}\omega \Rightarrow v' = \frac{2M+4m}{M+4m}\frac{L}{2}\omega \quad (5)$$

$$(1),(5) \Rightarrow v' = \frac{2M+4m}{M+4m} \times \frac{2m}{M+2m}V$$



توجه کنیم که سرعت و جابجایی تخته ۲ برابر سرعت و جابجایی مرکز استوانه‌ها است:

$$h' = 2h \qquad V' = 2V$$

$$V = R\omega$$

$$\Delta K + \Delta U = 0$$

$$\Delta K = 2\left[\frac{1}{2}mV^2 + \frac{1}{2}I\omega^2\right] + \frac{1}{2}MV'^2$$

$$= mV^2 + \frac{1}{2}mR^2\omega^2 + 2MV^2$$

$$= \left(\frac{3}{2}m + 2M\right)V^2$$

$$\Delta U = -2(mgh) - Mgh' = -2gh(m+M)$$

$$\Rightarrow V^2 = \frac{m+M}{\frac{3}{2}m+2M}(2gh)$$