تمرین سری ۶ کتاب استوارت

۵۲) $xe^y + ye^x = 1$

$e^y + xy'e^y + y'e^x + ye^x = 0 \Rightarrow e^0 + 0 + y' + e^1 = 0 \Rightarrow e + y' + 1 = 0$

$y' = -1 - e \Rightarrow y - 1 = (-1-e)(x-0)$

۶۸) $f'(x) = 2xe^{-x/2} + x^2 (-1/2) e^{-x/2} = 0$

$1/2\, x \left(4e^{-1/2} + (-xe^{-x/2})\right) = 0 \Rightarrow x = 0$

$f(0) = 0$ Min

$f(4) = 16 \times \frac{1}{e^2} = \frac{16}{e^2} = \left(\frac{4}{e}\right)^2 \Rightarrow e^{-x/2}(4-x) = 0$

max $x = 4$

۹۹) $f'(x) = 1 + e^x$

$\frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(0)} = \boxed{1/2}$

تمرین ۳.۶ استوارت

۴۸) $\ln(x^2-1) = S \Rightarrow 2\sqrt{e^2 - 1} = x$

$e^{2x} - 3e^x + 2 = 0$

$e^x = t \quad t^2 - 3t + 2 = 0$

$t = 1, 2$

$e^x = 1 \quad e^x = 2$

$\ln 1 = x \quad \ln 2 = x$

۴۶) $Q(t) = Q_0(1 - e^{-t/a})$

تمرینات ۸-۴

۳۶) $$\lim_{x\to 0^+} \frac{x^x-1}{\ln x + x - 1} = \frac{x^x(\ln x+1)}{\frac{1}{x}+1} = \frac{x^{x+1}(\ln x+1)}{\frac{1+x}{x^{x+1}}}$$

$$\frac{\frac{1}{x}}{x^{x+1} - x^{x+1}\left(\ln x + \frac{x+1}{x}\right)} = \frac{1}{x^{x+2}\left(1-\ln x - \frac{x+1}{x}\right)} = \boxed{0}$$

$$(x^{x+1})' \Rightarrow \ln y = (x+1)\ln x \to e^{(x+1)\ln x} = y \to y' = \underbrace{e^{(x+1)\ln x}}_{x^{x+1}} \times \left(\ln x + \frac{x+1}{x}\right)$$

۴۵)

تمرین ۸-۴

$$\ln A = \lim_{x\to\infty} \frac{\ln 2}{1+\ln x} \times \ln x \longrightarrow \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{\ln 2 / x}{1/x} = \boxed{\ln 2}$$

۵۳) $$\lim_{x\to 0} \cot x - \frac{1}{x} = \frac{x\cos x - \sin x}{x \sin x} = \frac{x(1-\frac{x^2}{2}) - x}{x^2} = \frac{1-\frac{x^2}{2}-1}{x} = 0$$

۶۰) $$\lim_{x\to\infty} x^{(\ln 2)/(1+\ln x)} \Rightarrow \ln y = \frac{(\ln 2)}{1+\ln x}\ln x$$

$$y = e^{\frac{\ln 2}{1+\ln x}\ln x} = \boxed{2}$$

تمرینات ۲-۵ جزوه

1) a) $\frac{e^x - e^{-x}}{e^x + e^{-x}} < 1 \Rightarrow \frac{e^x + e^{-x} - 2e^{-x}}{e^x + e^{-x}} < 1$

$\Rightarrow 1 - \frac{2e^{-x}}{e^x + e^{-x}} < 1$ ✓

$-1 < \frac{\sinh x}{\cosh x} \Rightarrow \frac{-(e^x + e^{-x}) + 2e^x}{e^x + e^{-x}} = -1 + \frac{2e^x}{e^x + e^{-x}}$ ✓

b) $\frac{1}{2}x^2 + 1 \leq \cosh x \qquad x^2 + 2 \leq e^x + e^{-x} \xrightarrow{\ '\ } 2x \leq e^x - e^{-x}$

$x \leq \sinh x \rightarrow 1 \leq \cosh x$ ✓

2) a) $\operatorname{sech}^{-1} x = \operatorname{Ln}\left(\frac{1 + \sqrt{1 - x^2}}{x}\right) = \cosh^{-1}\left(\frac{1}{x}\right), \quad 0 < x \leq 1$

$\operatorname{sech}^{-1} x = y \qquad \operatorname{sech} y = x \qquad \cosh^{-1} x = \operatorname{Ln}(x + \sqrt{x^2 - 1})$

$\frac{1}{\cosh} = x \qquad \cosh^{-1}\left(\frac{1}{x}\right) = y = \operatorname{sech}^{-1}(x)$

$\cosh^{-1}\left(\frac{1}{x}\right) = \operatorname{Ln}\left(\frac{1}{x} + \frac{\sqrt{1 - x^2}}{x}\right)$

b) $\tanh^{-1} x = \frac{1}{2} \operatorname{Ln}\left(\frac{1 + x}{1 - x}\right) \qquad |x| < 1$

$\tanh^{-1} x = y \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad ye^x + ye^{-x} = e^x - e^{-x}$

$e^x(y - 1) = e^{-x}(-1 - y)$

$e^{2x} = \frac{y + 1}{1 - y} \rightarrow y^{-1} = \frac{1}{2} \operatorname{Ln} \frac{x + 1}{1 - x}$

۴) $\frac{3}{2}e^{x} - \frac{3}{2}e^{-x} + 0.9e^{x} + 0.9e^{-x} = -\frac{9}{5}$

$\frac{24}{10}e^{x} - \frac{6}{10}e^{-x} = -\frac{9}{5}$ $\qquad e^{x} = t$

$12t - \frac{3}{t} = -9 \longrightarrow t = \frac{1}{4} \longrightarrow e = -\ln 4$

تمرینات ۷-۲

۱۸) $\dfrac{\sinh x + \cosh x}{\cosh x - \sinh x} = \dfrac{e^{x}}{e^{-x}} = e^{2x}$

۲۰) $\sinh x = \frac{12}{13}\cosh x \qquad \cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x - \frac{144}{169}\cosh^2 x = 1 \Rightarrow \cosh x = \frac{13}{5}$

$\sinh x = \frac{12}{5}$

۵۲) $\sec\theta + \tan\theta = x$

$\sec\theta = \cosh x = \dfrac{e^{x} + e^{-x}}{2}$

$\dfrac{1+\sin\theta}{\cos\theta} = e^{x} \Rightarrow e^{-x} = \dfrac{\cos\theta}{1+\sin\theta}$

$\Rightarrow \frac{1}{2}\left(\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta + 1}\right) =$

$\frac{1}{2}\,\dfrac{1+\sin^2\theta + 2\sin\theta + \cos^2\theta}{\cos\theta(\sin\theta+1)} = \dfrac{1+\sin\theta}{(1+\sin\theta)\cos\theta} = \boxed{\sec\theta \checkmark}$

تمرینات کتاب ۸-۶ جزوه

۱)

۱) $\lim \frac{2}{2\sqrt{2x+1}} - \frac{-4}{2\sqrt{1-4x}} = \frac{1}{\sqrt{2x+1}} + \frac{2}{\sqrt{1-4x}} = \boxed{3}$

۴) $\lim \sqrt{\frac{-t^2+t+1}{t^2}} - \sqrt{\frac{-t+1}{t^2}} = \frac{-2t+1}{2\sqrt{-t^2+t+1}} - \frac{-1}{2\sqrt{-t+1}} = \frac{1}{2}+\frac{1}{2} = \boxed{1}$

۷) $\lim_{x\to 0^+} \frac{e^x-1-x}{x(e^x-1)} = \frac{0}{0} = \frac{e^x-1}{e^x(x+1)-1} = \frac{e^x}{e^x+x+1 \cdot e^x} = \boxed{\frac{1}{2}}$

۲) $\frac{f'(x)}{f'(x)+xf''(x)} = 3 \Rightarrow f'(x), f''(x) \to +\infty$ غلط

$\frac{1}{3} = \frac{f'(x)+xf''(x)}{f'(x)} = 1 + \frac{f''(x)x}{f'(x)} = \frac{-2}{3} = \boxed{\frac{1}{3}}$

تکالیف ۸-۶

۳۴) $\frac{x^x-1}{\ln x + x - 1} = \frac{0}{\infty} = \boxed{0}$

۵۲) $\lim_{x\to 0} \frac{1}{\tan x} - \frac{1}{x} = \lim_{x\to 0} \frac{x-\tan x}{x\tan x} = \frac{1-(1+\tan^2 x)}{\tan x + x(1+\tan^2 x)} =$

$\frac{-2\tan x(1+\tan^2 x)}{2+\tan^2 x} = \boxed{0}$

۶۴) $\ln A = \lim_{x\to 1} \tan(\frac{\pi}{2}x)(2-x) = \frac{\ln(2-x)}{\cot\frac{\pi x}{2}} = \frac{-\frac{1}{2-x}}{-\frac{\pi}{2}(1+\cot^2(\frac{\pi}{2}x))}$

$= \frac{2}{(2-x)\pi} = \boxed{\frac{2}{\pi}}$

۷۴) $\lim_{\frac{\pi}{2}^+} \frac{\cot x}{-\cos x} = \frac{1}{\sin x} = \boxed{1}$ H ع $\Rightarrow \boxed{1}$

$\lim_{x\to\frac{\pi}{2}} \frac{\cot x}{\cos x} \overset{H}{=} \frac{-1-\cot^2 x}{-\sin x} = \boxed{1}$

۵۷) $\dfrac{۳f'(۲+۳x) + ۵f'(۲+۵x)}{۱} = ۸f'(۲) = \boxed{۵۶}$

تمرینات اضافی + مرور مطالب فصل ۲

۴۰) $y = \tan^{-1}(u)$

$y' = \dfrac{u'}{۱+u^۲} \Rightarrow y' = \dfrac{(\arcsin\sqrt{x})'}{۱+\arcsin^۲\sqrt{x}}$

$y' = \dfrac{۱}{۲\sqrt{x-x^۲}\,(۱+\arcsin^۲\sqrt{x})}$

۵۸) $y' = \ln x + ۱ = ۲$

$y - y_۰ = m(x-x_۰) = ۲(x-e) = y-e \quad y = ۲x - e$

۷۲) $\dfrac{\sin x}{۲x+۱} = \dfrac{۰}{۱} = \boxed{۰}$

۱۱۶) $f' = ۱ + ۲x + e^x \qquad f(x) = ۱ ,\ x = ۰$

$\rightarrow \dfrac{۱}{f'(f^{-1}(۱))} = \boxed{\dfrac{۱}{۲}}$

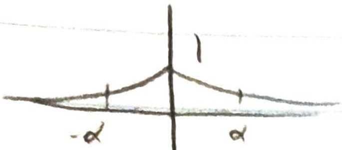
۱۱۴) $S = ae^{-a} \qquad S_{max} = ? \qquad S' = ۰ \qquad a'e^{-a} + a x - ae^{-a} = e^{-a} - ae^{-a}$

$= (۱-a)e^{-a} = \boxed{۰} \Rightarrow$

$S_{max} = ۱ \times e^{-۱} = \boxed{\dfrac{۱}{e}}$

1) $y = e^{-x^2}$ $y' = -2x\,e^{-x^2}$ $-2e^{-x^2} - 2x(-2x)e^{-x^2} = (-4x^2+2)e^{-x^2} = 0]$

$y'' = 0$



$\Rightarrow \boxed{\alpha = \frac{\sqrt{2}}{2}}$

4) $\frac{(\sinh x)'}{1+\sinh^2 x} = \frac{\cosh x}{\cosh^2 x} = \frac{1}{\cosh x}$ سمت راست ✓

سمت چپ $= \frac{\tanh' x}{\sqrt{1-\tanh^2 x}} = \frac{\cosh x}{\cosh^2 x} = \frac{1}{\cosh x}$ ✓

10) $\lim\limits_{x\to\infty} \left(-1+1+\frac{\ln(1+x^2)}{2\ln x}\right)^{x^2\ln x} = \lim\limits_{x\to\infty}\left(1+\frac{\ln\left(\frac{1+x^2}{x^2}\right)}{\ln x^2}\right)^{x^2\ln x}$

$= \left(\left(1+\frac{\ln\frac{1+x^2}{x^2}}{\ln x^2}\right)^{\frac{\ln x^2}{\ln(1+\frac{1}{x^2})}}\right)^{\left(\frac{\ln(1+\frac{1}{x^2})}{2\ln x}\right)x^2\ln x} = e^{\frac{x^2\ln(1+\frac{1}{x^2})}{2}} = e^{\frac{\ln(1+\frac{1}{x^2})^{x^2}}{2}}$

14) $e^{x+y-2} \geq xy$

$x+y-2 \geq \ln xy \iff x-1+y-1 \geq \ln x + \ln y$

$x-1 \geq \ln x \quad x>0$

$y-1 \geq \ln y \quad y>0$