جزوه ۱-۵

1) $f(x) = f(a) + f'(x)(x-a) \rightarrow (1+x)^k = \boxed{1+kx}$

$f'(x) = k(1+x)^{k-1} \xrightarrow{f'(0)} f'(0) = k$

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2) $f(x) \approx f(a) + f'(a)(x-a)$

$f(x) = \sqrt[3]{x} \rightarrow 3 + \frac{1}{27}(x-27)$

$\hookrightarrow x = 25 \rightarrow \boxed{3 + \frac{-2}{27}}$

$f(x+dx) = f(a) + f'(a)\,dx$

$f(x) = \sqrt[3]{x} \xrightarrow{a=27} 3 + \frac{1}{27}dx \longrightarrow \boxed{3 - \frac{2}{27}}$

3- $\frac{du}{dy} = 24xy - xy^2 \rightarrow 18x + 8 \Rightarrow -4 = 18x + 8 + \frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{-1}{24}x$ (I)

$r^2 = 169 - (13-y)^2 \Rightarrow r^2 = 26y - y^2 \Rightarrow r = \sqrt{26y - y^2}$ (II)

$\frac{dr}{dy} = \frac{26-2y}{2\sqrt{26y-y^2}} = \frac{13-y}{\sqrt{\;}} = \frac{5}{12} \rightsquigarrow \frac{5}{12} \cdot \frac{-1}{24x} = \frac{-5}{288x}$

4- $x^2 + 32 = y^2 \rightarrow 2x\frac{dx}{dy} = 2y\frac{dy}{dt} = \boxed{20} \Rightarrow \boxed{u_x = 2.5}$

parsnote

تمرینات کتب ۱-۵ جزوه

۱) $(1+x)^K = 1 + Kx$

$L(x) = f(a) + f'(x)(x-a) \to \quad K(1+x)^{K-1}(x-a) + f(a)$

$f(a+dx) = f(a) + f'(a)dx$

$K(1)(1)x + 1 = \boxed{Kx+1}$, $a \to 0$

۲) خطی‌سازی: $\sqrt[3]{x} = f(a) + f'(a)(x-a)$

$\xrightarrow{a \to 27} 3 + \frac{1}{3}\frac{1}{\sqrt[3]{27^2}}(x-27)$

$= 3 + \frac{1}{27}(x-27)$

$x \to 25$: $\boxed{3 + \frac{-2}{27}}$

دیفرانسیل: $f(a+dx) = f(a) + f'(a)dx$

$f(27-2) = f(27) + f'(27) \times 2$

$= \boxed{3 - \frac{2}{27}}$

۳) $g(a) = f(a) + f'(x)(x-a)$

$g(0.125) = 0.9702 + \frac{1}{5}\sqrt[5]{(\frac{2+x}{2-x})^4} \times \frac{-2-x-2+x}{(2+x)^2} \times (x-0.125)$

$f(a+dx) = f(a) + f'(a)dx$

$\frac{|f''(c)|}{2}\Delta x$

تمرینات بخش ۲-۱ ← ۱۶۲ سولمات

~~۳۸~~) $f(x) = \frac{1}{x-1} \quad x>1 \quad a=2$

a) $f'(x) = \frac{-1}{(x-1)^2} < 0 \longrightarrow$ نزولی

b) $(f^{-1})' = \frac{1}{f' f^{-1}}$

c) $y = \frac{1}{x-1} \Rightarrow \frac{1}{y} = x-1 \Rightarrow \frac{1}{y} + 1 = x \Rightarrow \frac{1+y}{y} = x$

$\Rightarrow \frac{1+x}{x} = f^{-1}(x)$

d) $(f^{-1})' = \frac{1(x) - (1+x)}{x^2} = \frac{-1}{x^2} = -\frac{1}{4} \qquad R = y>1$

$D \quad x>0$

۴۲) $(f^{-1})' = \frac{1}{f' f^{-1}} \Rightarrow f(x) = \sqrt{x^3+x^2+x+1} \qquad a = (2, \sqrt{15})$

$f'(x) = \frac{3x^2+2x+1}{2\sqrt{x^3+x^2+x+1}} = \frac{45 + 2\sqrt{15}+1}{2\sqrt{15\sqrt{15}+15+\sqrt{15}+1}}$

$\frac{\sqrt{15}+23}{\sqrt{16\sqrt{15}+16}}$

۴۴) $g(x) = \frac{1}{f^{-1}(x)} \qquad f(3)=2 \qquad f'(3) = \frac{1}{9} \qquad g'(2) = ?$

$g'(x) = \frac{-{f^{-1}(x)}'}{(f^{-1}(x))^2} = \frac{-1}{9 \times f'(3)} = \boxed{-1}$

خرده‌شب ۱-۲

۱- $f(x^3+x)=x$

$(3x^2+1)f'(x^3+x)=1 \Rightarrow f'(x^3+x)=\frac{1}{3x^2+1}$

$f'(0)=1 \qquad x^3+x=2 \Rightarrow \boxed{f'(2)=1/4}$

$(x-1)(x^2+x+2)$

۲- $\forall x>0 \quad f(x)=\frac{1}{x} \qquad fog(x)=x$

مشتق‌پذیر ⇐ صعودی و پیوسته ⇐ $g'(x) \times f'(g(x)) = 1$

$\boxed{g'(x)=\frac{1}{f'(g(x))}=\frac{1}{1/g}=g(x)}$ ✓

تمرینات تحویلی ۲-۳

۱- $f(x) = (1+a^x)^{1/x}, \quad a>0, \quad x>0$

$$f(x) = e^{\frac{1}{x}\ln(1+a^x)} \Rightarrow f'(x) = \left(\frac{1}{x}\right)'(1+a^x)^{1/x}\,\mathrm{Ln}(1+a^x)$$

$$f'(x) = \underbrace{\frac{-1}{x^2}}_{-}\ \underbrace{(1+a^x)^{1/x}}_{+}\ \underbrace{\mathrm{Ln}(1+a^x)}_{+}$$ نزولی!

۲- $f(x) = \begin{cases} x^{x\ln x} & x>0 \\ x^x & x\leqslant 0 \end{cases}$

$$\left.\begin{aligned} &\lim_{x\to 0^+} f(x) = x^{\cdot\ln 0} = \boxed{1} \\ &\lim_{x\to 0^-} f(x) = x^0 = \boxed{1} \Rightarrow f(0)=1 \end{aligned}\right\} \Rightarrow$$ پیوسته

$$f(x) = \begin{cases} e^{x\ln x \ln x}, & x>0 \\ e^{x\ln x} & x<0 \end{cases} \Rightarrow f'_{0^+}(x) \neq f'_{0^-}(x)$$

۳-

۴- $f(x) = x^2 \cos x (1+x^4)^{-7}$

$f'(x) = 2x\left[\cos x (1+x^4)^{-7}\right] + x^2 \left(\cos x (1+x^4)^{-7}\right)'$

$-\sin x (1+x^4)^{-7} - 7(1+x^4)^{-8} \times 4x^3 \cos x$

$2x \cos x (1+x^4)^{-7} - x^2 \sin x (1+x^4)^{-7} - 28x^5 \cos x (1+x^4)^{-8}$

۷- $\lim_{x\to 0} (1-2x)^{1/x} = e^{2-} \lim_{x\to 0} \left[(1-2x)^{-1/2x}\right)^{-2} = e^{-2}$

$\ln (1-2x)^{1/x}$

$= \lim_{x\to 0} \ln \frac{1-2x}{x} = \boxed{-2} \Rightarrow$

$(1-2x)^{1/x} = e^{-2}$

۸- $f(x) = x^m (x-1)^n$ $\quad m,n \in \mathbb{Z}$ $\quad f'(x) = \left(\frac{m}{x} - \frac{n}{1-x}\right) f(x)$

$m,n > 1$

$\ln f(x) = \ln x^m + \ln (x-1)^n = m \ln x + n \ln (x-1)$

$f'(x) = 0 \quad \frac{m}{x} = \frac{n}{1-x} \rightarrow m - mx = nx$

$m = (m+n)x \Rightarrow x = \frac{m}{m+n} \Rightarrow 0 < x < 1$

$\frac{f'(x)}{f(x)} = \frac{m}{x} + \frac{n}{x-1} = \frac{m}{x} - \frac{n}{1-x} \Rightarrow f'(x) = \left(\frac{m}{x} - \frac{n}{1-x}\right) f(x)$ ✓

زوج $\Rightarrow$ $(x-1) > 0$ $\quad f(x) > 0 \rightarrow$ max

فرد $\Rightarrow$ $(x-1) < 0$ $\quad f(x) < 0 \rightarrow$ min

تمرین فصل ۴-۲

۷) a) $\cot^{-1} x = \begin{cases} \sin^{-1} \dfrac{1}{\sqrt{x^2+1}} & x>0 \\ \pi - \sin^{-1}\left(\dfrac{1}{\sqrt{x^2+1}}\right) & x<0 \end{cases}$

$$\frac{1}{\sqrt{1+x^2}} = -\frac{\sqrt{1+x^2}}{1+x^2} = \boxed{\frac{-x}{\sqrt{(1+x^2)^3}}}$$

b) $\sin^{-1}\frac{\sqrt{5}}{5} = \alpha \qquad \sin\alpha = \frac{\sqrt{5}}{5} \qquad \cos\alpha = \frac{2}{\sqrt{5}} \qquad \alpha + \beta = \pi/4$

$\sin^{-1}\frac{\sqrt{10}}{10} = \beta \qquad \sin\beta = \frac{\sqrt{10}}{10} \qquad \cos\beta = \frac{3}{\sqrt{10}}$

$\sin(\alpha+\beta) = \frac{\sqrt{2}}{2} \rightarrow \frac{\sqrt{5}}{5} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{\sqrt{10}}{10} = \frac{\sqrt{2}}{2}$ ✓

۵) $y = \csc^{-1}(u) \rightarrow u = \csc y \Rightarrow u = \frac{1}{\sin y} \rightarrow \sin y = \frac{1}{u}$

$y = \sin^{-1}\left(\frac{1}{u}\right) \Rightarrow y' = \sin^{-1}\left(\frac{1}{u}\right)' = \dfrac{-u'/u^2}{\sqrt{1-1/x^2}} = \dfrac{-u'/u^2}{\frac{\sqrt{u^2-1}}{u}} = \dfrac{-u'}{\sqrt{u^2-1}\cdot u}$

تمرین ۳-۶ کتاب استوارت

۵۴) $xe^y + ye^x = 1$

$e^y + xy'e^y + y'e^x + ye^x = 0 \Rightarrow e^1 + 0 + y' + e^0 = 0 \Rightarrow e + y' + 1 = 0$

$y' = -1-e \Rightarrow y - 1 = (-1-e)(x-0)$

۴۸) $f'(x) = 2xe^{-x/2} + x^2(-1/2)e^{-x/2} = 0$

$1/2\, x\left(4e^{-x/2} + (-xe^{-x/2})\right) = 0 \Rightarrow x = 0$

$f(0) = 0$ Min

$f(4) = 16 \times \frac{1}{e^2} = \frac{16}{e^2} = \left(\frac{4}{e}\right)^2 \Rightarrow e^{-x/2}(4-x) = 0$

max $x = 4$

۵۵) $f'(x) = 1 + e^x$

$\frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(0)} = \boxed{1/2}$

تمرین ۳-۷ استوارت

۲۸) $\ln(x^2-1) = S \Rightarrow 2\sqrt{e^2-1} = x$

$e^{2x} - 3e^x + 2 = 0$

$e^x = t \quad t^2 - 3t + 2 = 0$

$t = 1, 2$

$e^x = 1 \quad e^x = 2$

$\ln 1 = x \quad \ln 2 = x$

۴۶) $Q(t) = Q_0(1 - e^{-t/a})$

تمرینات بخش ۱-۲ ← ۵۰ تا ۵۰ سلسلۀ

۱۸) $f(x)=\frac{1}{x-1}$ $x>1$ $a=۲$

a) $f'(x)=\frac{-1}{(x-1)^2}<0 \rightarrow$ نزولی

b) $(f^{-1})'=\frac{1}{f'f^{-1}}$

c) $y=\frac{1}{x-1} \Rightarrow \frac{1}{y}=x-1 \Rightarrow \frac{1}{y}+1=x \Rightarrow \frac{1+y}{y}=x$

$\Rightarrow \frac{1+x}{x}=f^{-1}(x)$

d) $(f^{-1})'=\frac{1(x)-(1+x)}{x^2}=\frac{-1}{x^2}=-1/_{C}$ $R: y>1$

$D: x>0$

۴۲) $(f^{-1})'=\frac{1}{f'f^{-1}} \Rightarrow f(x)=\sqrt{x^3+x^2+x+1}$ $a=(۲,\sqrt{۱۵})$

$f'(x)=\frac{۳x^2+۲x+1}{۲\sqrt{x^3+x^2+x+1}}=\frac{۴۵+۲\sqrt{۱۵}+1}{۲\sqrt{۱۵\sqrt{۱۵}+۱۵+\sqrt{۱۵}+1}}$

$\frac{\sqrt{۱۵}+۲۳}{\sqrt{۱۶\sqrt{۱۵}+۱۶}}$

۴۴) $g(x)=\frac{1}{f^{-1}(x)}$ $f(۳)=۲$ $f'(۳)=۱/۹$ $g'(۲)=؟$

$g'(x)=\frac{-f^{-1}(x)'}{(f^{-1}(x))^2}=\frac{-1}{۹\times f'(۳)}=-1$