تمرینات بخش ۱-۵ جواد

(۱) الف)

$S_t = (1+\frac{\pi}{4})\times 2$

$S_1 = \int_0^1 (1-t^2)\,dt \to t - \frac{t^3}{3}\Big|_0^1 = \frac{2}{3}$

$S_2 = \int_{-\frac{\pi}{4}}^{0} \sec^2 t\,dt = \tan t\Big|_{-\frac{\pi}{4}}^{0} = 1 \Rightarrow S_{کل} = 2 + \frac{2\pi}{4} - \frac{5}{3} = \frac{\pi}{2} + \frac{1}{3}$

ب) $\int_0^{\frac{1.118}{\sqrt{1.118^2+1}} = t} \left(\frac{x}{\sqrt{x^2+1}}\right) - (x^4 - x)\,dx = \frac{1}{2}\sqrt{x^2+1} - \frac{x^5}{5} + \frac{x^2}{2}\Big|_0^5$

$\simeq \frac{785}{1000}$

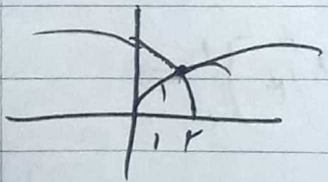
(۲) ۱) $I = \int_{-2}^{2} \sqrt{9 - \frac{9}{4}x^2} = \int_{-2}^{2} \frac{3}{2}\sqrt{4-x^2}\,dx = \frac{3}{2}\int_{-2}^{+2}\sqrt{4-x^2}\,dx \xrightarrow{\text{نیم دایره}}$

$\frac{3}{2}\times\frac{\pi 2^2}{2} = 3\pi$ (I)

۲) $\frac{3}{2}\int_{-2}^{+2}\sqrt{4-x^2}\,dx = \frac{3}{4}\left(\sqrt{4-x^2}\,x + 4\sin^{-1}\left(\frac{x}{2}\right)\right)\Big|_{-2}^{+2} = 3\pi$

تمرینات بخش ۱-۵ استوارت

(۲ ج) $x = y^4 \Rightarrow y = \sqrt[4]{x} = \sqrt{2-x}\,,\ y = 0$



$I = \int_0^1 \left((2-y^2) - y^4\right) dy = 2 - \frac{1}{3} - \frac{1}{5} = \frac{22}{15}$

$\left(2y - \frac{y^3}{3}\right) - \frac{y^5}{5}\Big|_0^1$

$\Rightarrow I = \frac{22}{15}$

تمرینات بخش ۲-۵ استوارت

(14) $A = \pi(f^2(x) - g^2(x)) = \pi(1+\cos x)^2 - \pi(1+\sin x)^2$

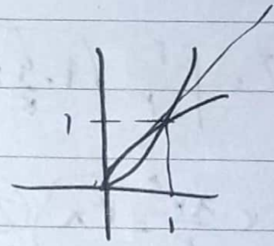
$\Rightarrow \pi(\cos 2x + 2\cos x - 2\sin x) \Rightarrow V = \int_0^{\pi/4} \pi[\cos 2x + 2\cos x - 2\sin x]\,dx = \pi\left[\frac{\sin 2x}{2} + 2\sin x + 2\cos x\right]_0^{\pi/4} \Rightarrow$

$V = \left(\frac{4\sqrt{2}-3}{2}\right)\pi$

تمرینات بخش ۳-۵ استوارت

(5) $y_1 = \sqrt{x}\ ,\ y_2 = x^2 \Rightarrow \sqrt{x} = x^2 \Rightarrow x = x^4$

$x^4 - x = 0 \Rightarrow x(x^3-1) = 0 \Rightarrow x = 0, 1$



$\xrightarrow{\text{حالت ۱}} A(y) = \pi(R^2 - r^2) \Rightarrow A(y) = \pi(y - y^4)$

$\Rightarrow V = \int A(y) = \int_0^1 \pi(y-y^4) = \pi\int_0^1 (y-y^4) = \pi\left(\frac{y^2}{2} - \frac{y^5}{5}\right)\Big|_0^1$

$V = \frac{3\pi}{10}$

$\xrightarrow[\text{استوانه‌ای}]{\text{حالت ۲}}$ $V = \int_0^1 2\pi x(\sqrt{x} - x^2)\,dx \Rightarrow 2\pi\left[\frac{2}{5}x^{5/2} - \frac{x^4}{4}\right]_0^1 \Rightarrow$

$V = \frac{3\pi}{10}$

۲۰) $x = y^2+1,\ x = 2 \Rightarrow y = -2 \Rightarrow r = (y+2) \Rightarrow$

$2-(y^2+1) = 1-y^2 \Rightarrow V = \int_{-1}^{1} 2\pi (y+2)(1-y^2)\,dy \Rightarrow$

$$V = 2\pi\left(2y - \frac{2y^3}{3} + \frac{y^2}{2} - \frac{y^4}{4}\right) = 2\pi\left(4-\frac{4}{3}\right) = \boxed{\frac{16\pi}{3}}$$

تمرینات بخش ۲.۵ جزوه

۲) $\int_{-2.5}^{+2.5} \frac{-4h}{a^2} x^2 + h\,dx \Rightarrow \frac{-4h}{3a^2}x^3 + hx \Big|_{-2.5}^{2.5} \Rightarrow$

$dV = \frac{ah}{3}dx = \frac{4}{3} h y\, dx \Rightarrow dV = \frac{4h}{3}\sqrt{R^2-x^2}\,dx$

$\Rightarrow V = \frac{4}{3}\int_{-R}^{R} \sqrt{R^2-x^2}\,dx = \frac{\pi h R^2 \times 2}{3}$

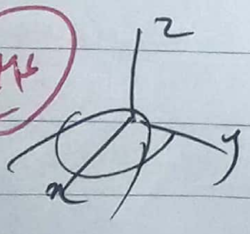
۷) $dV = \pi\left((3-x)^2-(x^2+1)^2\right)dx \Rightarrow V = \pi\int_{-1}^{1} 8 - x^4 - x^2 - 6x\,dx$

$\Rightarrow V = \pi\left(8x - x^5/5 - x^3/3 - 3x^2\right)dx \Rightarrow V = \pi\left(\frac{-60+3/32+5/8-45/4}{15}\right)$

$-\left(\frac{-240+96+40-180}{15}\right) + \left(\frac{120-3-5-45}{15}\right) - \left(\frac{-60+3/32+5/8-45/4}{15}\right)$

$= \boxed{\frac{117}{5}\pi}$

۳۴) روش سطح مقطع با [illegible]



$a = 2-2x^2 \Rightarrow dV = (2-2x^2)^2 dx \Rightarrow$

$V = \int_{-1}^{1} (2-2x^2)^2 dx = \int_{-1}^{1} (4x^4 - 8x^2 + 4)dx$

$\Rightarrow V = \left(\frac{4}{5}x^5 - \frac{8}{3}x^3 + 4x\right) = \frac{64}{15}$

$2-x^2 = x^2 \Rightarrow \boxed{x = \pm 1}$

سوالات اضافی فصل ۵

۸) $y=\int_1^x \sqrt{\sqrt{t}-1}\,dt \Rightarrow y'=\sqrt{\sqrt{x}-1} \Rightarrow$

$y'^2=\sqrt{x}-1 \Rightarrow dS=2\pi x\,dL$

$\Rightarrow S=2\pi\int_1^{16} x^{5/4}\,dx=2\pi\left(\frac{4}{9}x^{9/4}\right)\Big|_1^{16} \Rightarrow S=\frac{4088\pi}{9}$

تمرینات بخش ۳-۵ جزوه

۱۱) $y=\left(\frac{x}{2}\right)^{2/3} \quad x\in[0,2] \Rightarrow \frac{x^{2/3}}{2^{2/3}}=y \Rightarrow$

$x=2y^{3/2} \Rightarrow x'=3\sqrt{y} \Rightarrow x=0 \Rightarrow y=0$

$x=2 \Rightarrow y=1$

$L=\int_{f(a)}^{f(b)} \sqrt{1+x'^2}\,dy=\int_0^1 \sqrt{1+9y}\,dy=\frac{1}{9}\times\frac{2}{3}(1+9y)^{3/2}\Big|_0^1$

$=\boxed{10^{3/2}\times\frac{2}{27}-\frac{2}{27}}$

۱۲) $L=4\int_\alpha^\beta \sqrt{x_t'^2+y_t'^2}\,dt \Rightarrow 4\int_0^{\pi/2}\sqrt{3\sin^2 t\cos t-3\cos^2 t\sin t}$

تمرینات بخش ۱-۸ استوارت

۱۰) $x=\frac{y^4}{8}+\frac{1}{4y^2} \Rightarrow x'=\frac{y^3}{2}-\frac{1}{2y^3} \Rightarrow$

$L=\int_1^2 \sqrt{1+\left(\frac{y^3}{2}-\frac{1}{2y^3}\right)^2}\,dy \Rightarrow L=\int_1^2 \frac{\sqrt{y^3+y^{-3}}}{2}=$

$\frac{1}{2}\left(\frac{y^4}{4}-\frac{1}{2}y^{-2}\right)\Big|_1^2=\boxed{\frac{33}{16}}$

(۴۱) $y=\int_1^x \sqrt{t^3-1}\,dt \quad 1\leq x\leq 4 \Rightarrow dy=\sqrt{x^3-1}$

$y'^2=x^3-1 \Rightarrow 1+y'^2=x^3 \Rightarrow x^{3/2}=\sqrt{1+y'^2} \Rightarrow L=\int_a^b \sqrt{1+y'^2}$

$\Rightarrow L=\int_1^4 x^{3/2}dx=\frac{2}{5}x^{5/2}\Big|_1^4=\frac{62}{5}$

تمرینات بخش ۵-۴ جزوه ،

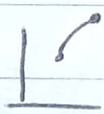
(۱) $x^{2/3}+y^{2/3}=a^{2/3} \quad a>0 \quad , \quad a^{2/3}(\sin^2 t+\cos^2 t)=a^{2/3}$

$x^{2/3}=a^{2/3}\times\sin^2 t \; , \; y^{2/3}=a^{2/3}\times\cos^2 t \Rightarrow x=\sqrt{a^2\sin^6 t} \; , \; y=\sqrt{a^2\cos^6 t}$

$L=\int_\alpha^\beta \sqrt{9a^2\sin^4\cos^2+9a^2\cos^4\sin^2 t} \Rightarrow$

$L=\int_\alpha^\beta \sqrt{9a^2\sin^2\cos^2 t\times 1} \Rightarrow L=\int_\alpha^\beta \frac{\sqrt{9}}{2}\sin 2t \Rightarrow L=-\frac{\sqrt{9}}{4}\cos 2t\Big|_0^{a^{2/3}}$

$\Rightarrow L=\frac{12\pi}{5}a^2$

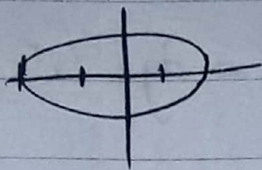
(۲) $y=2\sqrt{x} \quad 1\leq x\leq 2$  $P=2\pi\int_1^2 y\,dL \Rightarrow P=2\pi\int_1^2 y\sqrt{1+y'^2}\,dx$

$\Rightarrow \frac{1}{\sqrt{x}}=y' \quad \Rightarrow P=2\pi\int_1^2 2\sqrt{x}\times\sqrt{1+\frac{1}{x}}\,dx \Rightarrow P=2\pi\int_1^2 2\sqrt{x+1}$

$\Rightarrow P=2\pi\times 2\left[\frac{2}{3}(x+1)^{3/2}\right]_1^2 \Rightarrow P=\frac{8\pi}{3}(\sqrt{27}-\sqrt{8})$

$=\frac{8\pi}{3}(3\sqrt{3}-2\sqrt{2})$

۳) $x^2+4y^2=4$



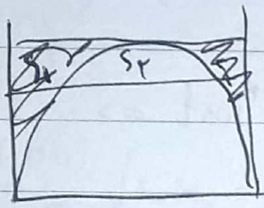
$$\left.\begin{array}{l} x^2=4\sin^2\theta \\ 4y^2=4\cos^2\theta \end{array}\right\} \Rightarrow \begin{array}{l} x=2\sin\theta \Rightarrow x'=2\cos\theta \\ y=\cos\theta \Rightarrow y'=-\sin\theta \end{array} \Rightarrow P=\frac{8\pi^2}{3/4}+2\pi$$

۸) درسطح مطلب فصل ۸ استثنا

$$y=\int_1^x \sqrt{\sqrt{t}-1}\,dt \Rightarrow dy=\sqrt{\sqrt{x}-1} \Rightarrow$$

$$y'^2=\sqrt{x}-1 \Rightarrow P=2\pi\int y\,dL=\int_1^{16} y\sqrt{\sqrt{x}+1-1}\times dy=2\pi\int_1^{16} x^{1/4}$$

۱۶) مسائل اضافی فصل ۸



$$S_1=S_{\text{ذوزنقه}}-S_{\text{قطاع}}$$

$$S_1=\frac{(2-\cos\theta)\sin\theta}{2}-\frac{\theta\pi}{180}$$

$$S_2=\pi\frac{(180-2\theta)}{360}-\frac{\sin(90-\theta)\times\cos(90-\theta)}{2}\times 2 \Rightarrow$$

$$S_2=\frac{180-2\theta}{360}\pi-\frac{\sin 2\theta}{2}$$

$$S_T=\sin\theta-\frac{\sin 2\theta}{2}+\sin\theta-\frac{\sin 2\theta}{2}+\frac{(180-2\theta)\pi}{360}-\frac{\sin 2\theta}{2}-\frac{2\pi\theta}{180}$$

تمرینات بخش ۱_۶ جزوه

1

الف) $r = 1 + 2r\cos\theta \Rightarrow \sqrt{x^2 + y^2} = 1 + 2x$

$\Rightarrow x^2 + y^2 = 4x^2 + 4x \Rightarrow y^2 = 3x^2 + 4x + 1$

$\Rightarrow y = \sqrt{3x^2 + 4x + 1}$

ب) $x^4 + y^4 + 2x^2y^2 + 2x^3 + 2xy^2 - y^2 = 0 \Rightarrow r^4\cos^4\theta + r^4\sin^4\theta + 2r^4\sin^2\cos^2\theta$

$+ 2r^3\cos\theta + 2r\cos\theta + r^2\sin^2\theta - r^2\sin^2\theta$

$\Rightarrow r^4(\cos^4\theta + \sin^4\theta + 2\sin^2\cos^2) + 2r^3\cos\theta(\cos^2 + \sin^2\theta) - r^2\sin^2\theta$

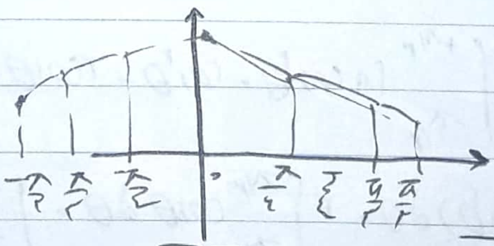
$= r^4 + 2r^3\cos\theta - r^2\sin^2\theta = 0 \Rightarrow r^4 + 2r^3\cos\theta - r^2 + r^2\cos^2\theta = 0$

$\Rightarrow r^2 + 2r\cos\theta - \cos^2\theta - r \Rightarrow \Delta = b^2 - 4ac = 4(\cos^2 + \sin^2) = 4$

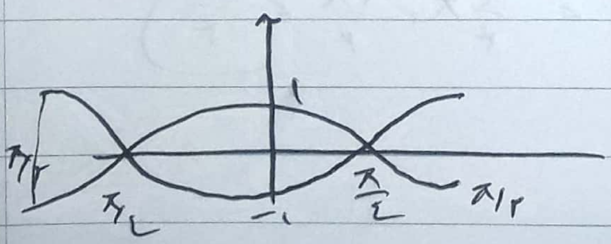
$\Rightarrow r = \dfrac{-2\cos\theta \pm 2}{2} \Rightarrow \boxed{r = 1 - \cos\theta}$

2

الف) $r = 2\cos\theta + 1$



ب) $r^2 = \cos 2\theta \qquad r = 0 \Rightarrow \sqrt{\cos 2\theta} \Rightarrow \theta = \pm \pi/4$



5) $(x^2+y^2)^{3/2} = 9xy \Rightarrow (r^2\sin^2\theta + r^2\cos^2\theta)^{3/2} = 9r^2\sin\theta\cos\theta$

$\Rightarrow r^3 = 9r^2\sin\theta\cos\theta \Rightarrow \sin\theta\cos\theta = \frac{r}{9} \Rightarrow r = \frac{9}{2}\sin 2\theta$

$x = r\cos\theta \Rightarrow x = \frac{9}{2}\sin 2\theta\cos\theta \Rightarrow \frac{dx}{d\theta} = 0 \Rightarrow 2\cos 2\theta \times \cos\theta$

$+\sin\theta\sin 2\theta = 0 \Rightarrow 1 - 2\sin^2\theta = \sin^2\theta \Rightarrow \sin\theta = \frac{\sqrt{3}}{3}$

$\Rightarrow x = 9\sin\theta(1-\sin^2\theta) = \frac{9\sqrt{3}}{3} \times \frac{2}{3} = \boxed{2\sqrt{3}}$

1) تمرینات بخش ۶-۳

$r = 3\cos\theta \quad r = 1+\cos\theta$

$1+\cos\theta = 3\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \pm\pi/3$

$A = \frac{9\pi}{4} - \frac{1}{2}\int_{-\pi/3}^{+\pi/3}(9\cos^2\theta - \cos^2\theta - 2\cos\theta - 1)\,d\theta \Rightarrow A = \frac{9\pi}{4}$

$-\frac{1}{2}\int(8\cos^2\theta)\,d\theta - 2\int_{-\pi/3}^{\pi/3}\cos\theta\,d\theta - \int_{-\pi/3}^{\pi/3}d\theta =$

$\frac{9}{4}\pi - \frac{1}{2}(2\sin 2\alpha - 2\sin\alpha + 3\alpha) = \frac{9}{4}\pi + \frac{2\pi}{3} = \boxed{\frac{5\pi}{4}}$

۴) $r = \sin^{۲}\left(\frac{\theta}{۲}\right) \quad \theta \in [۰, \pi] \quad L = \int_{b}^{a} \sqrt{r^{۲} + r'^{۲}}\, d\theta \Rightarrow$

$L = \int_{۰}^{\pi} \sqrt{\frac{1-\cos\theta}{۲}} = \int_{۰}^{\pi} \sin\frac{\theta}{۲} = -۲\cos\frac{\theta}{۲}\Big|_{۰}^{\pi} = ۲ \Rightarrow$

$L = ۲$

۵) $dp = ۲\pi y\, dL \quad , r = a(1 + \cos\theta) \quad L = \int_{a}^{b} \sqrt{r^{۲} + r_{\theta}'^{۲}}\, d\theta$

$\Rightarrow L = \int_{a}^{b} \sqrt{a^{۲} + a\cos^{۲}\theta + a^{۲}\sin^{۲}\theta}\, d\theta = \int_{a}^{b} a\sqrt{۲}\, d\theta$

$\Rightarrow P = ۲\pi a \int_{۰}^{۲\pi} (\sin\theta + \sin\theta\cos\theta)\sqrt{۲}\, a\, d\theta = ۲\pi a^{۲} \int_{۰}^{۲\pi} (\sin\theta + \sin\theta\cos\theta)$

$\Rightarrow ۲\cos\frac{\theta}{۲}\, d\theta = ۴\pi a \int_{۰}^{۲\pi} (\sin\theta \cos\frac{\theta}{۲})\, d\theta + \frac{1}{۲}\int_{۰}^{۲\pi} \sin\theta \cos\frac{\theta}{۲}$

$= \frac{۳۲\pi a^{۲}}{۵}$