تمرینات بخش ۴، ۳، ۲ دوم:

سوال سوم

4) $(e^{-2\sqrt{x}} - y)\,dx - \sqrt{x}\,dy = 0$

حل می‌کنیم عامل انتگرال‌ساز تابعی از x است:

$$\frac{\mu_x}{\mu} = \frac{M_y - N_x}{N} = \frac{1-\frac{1}{2\sqrt{x}}}{\sqrt{x}} \Rightarrow \ln\mu = \int\frac{1}{\sqrt{x}} - \frac{1}{2}\int\frac{1}{x} \Rightarrow$$

$$\ln\mu = 2\sqrt{x} - \frac{1}{2}\ln x \Rightarrow \mu = \frac{1}{\sqrt{x}}e^{2\sqrt{x}} \Rightarrow$$

$$\left(\frac{1}{\sqrt{x}} - \frac{e^{2\sqrt{x}}}{\sqrt{x}}y\right)dx - e^{2\sqrt{x}}\,dy = 0 \Rightarrow u = -e^{2\sqrt{x}}y + A(x) \Rightarrow A'(x) = \frac{1}{\sqrt{x}}$$

$$\Rightarrow A(x) = 2\sqrt{x} \Rightarrow 2\sqrt{x} - e^{2\sqrt{x}}y = c \xrightarrow{y(1)=1} c = -1 \Rightarrow \boxed{e^{2\sqrt{x}}y - 2\sqrt{x} = 1}$$

5) $(4y^3 - 2x)\frac{dy}{dx} + y = 0 \Rightarrow (4y^3 - 2x)\,dy + y\,dx = 0$

حل می‌کنیم μ تابعی از y است:

$$\frac{\mu_y}{\mu} = \frac{1+2}{-y} \Rightarrow \ln\mu = -3\int\frac{1}{y}dy = -3\ln y \Rightarrow \mu = e^{\ln y^{-3}} \Rightarrow \mu(y) = y^{-3}$$

$$\Rightarrow (4 - 2y^{-3}x)\,dy + y^{-2}\,dx = 0 \Rightarrow u = y^{-2}x + A(y) \Rightarrow A'(y) = 4 \Rightarrow A(y) = 4y$$

$$\Rightarrow y^{-2}x + 4y = c \xrightarrow{y(1)=1} c = 5 \Rightarrow y^{-2}x + 4y = 5 \Rightarrow x = (5-4y)y^2$$

6) $y(1+x^2y)\,dx + x\,dy = 0$ | حول تابع z را داده‌ایم: $z = xy \Rightarrow z_x = y,\ z_y = x$

$$\Rightarrow \frac{\mu_z}{\mu} = \frac{1+2x^2y-1}{xy - xy(1+x^2y)} = \frac{2x^2y}{xy(-x^2y)} = \frac{-2}{xy} = \frac{-2}{z} \Rightarrow$$

$\ln\mu(z) = -2\ln z \Rightarrow \mu = z^{-2} \Rightarrow \frac{1}{x^2y^2} = x^{-2}y^{-2} \Rightarrow$

$(x^{-2}y^{-1} + x^2)dx + x^{-1}y^{-2}dy = 0 \Rightarrow$

$A(x) = \frac{x^3}{3} \Rightarrow A'(x) = x^2 \Rightarrow u = -x^{-1}y^{-1} + A(x)$ $\Rightarrow -x^{-1}y^{-1} + \frac{x^3}{3} = \frac{C}{3} \Rightarrow$

$\frac{x^3 - C}{3} = \frac{1}{xy} \Rightarrow y = \frac{3}{x^4 - Cx}$

7) $(2xy\ln y)dx + (x^2 + y^2\sqrt{1+y^2})dy = 0$ عامل انتگرال‌ساز μ تابعی از y

$\frac{\mu_y}{\mu} = \frac{2x(\ln y + 1)}{-2xy \times 2xy} = \frac{-1}{y} \Rightarrow \ln\mu = \ln\frac{1}{y} \Rightarrow \mu = \frac{1}{y} \Rightarrow$

$(2x\ln y)dx + (\frac{x^2}{y} + y\sqrt{1+y^2})dy = 0$

$u = x^2\ln y + A(y) \Rightarrow A'(y) = y\sqrt{1+y^2} \Rightarrow A(y) = \frac{1}{3}(\sqrt{1+y^2})^3 \Rightarrow$

$x^2\ln y + \frac{(1+y^2)^{\frac{3}{2}}}{3} = C$ ✓

9) $(3xy + y^2) + (x^2 + xy)\frac{dy}{dx} = 0 \Rightarrow (3xy + y^2)dx + (x^2 + xy)dy \Rightarrow$

$y(3x + y)dx + x(x + y)dy = 0$

به عنوان عامل انتگرال‌ساز $\mu(x,y)$ به صورت $x^\alpha y^\beta$ است

$\Rightarrow x^\alpha y^{\beta+1}(3x + y)dx + x^{\alpha+1}y^\beta(x + y)dy = 0$

$3x^{\alpha+1}(1+\beta)y^\beta + x^\alpha y^{\beta+1}(\beta+2) = (\alpha+1)y^{\beta+1}x^\alpha + y^\beta(\alpha+2)x^{\alpha+1}$

$\Rightarrow 3(1+\beta) = \alpha + 2$, $\beta + 2 = \alpha + 1$ $\Rightarrow 3\alpha = \alpha + 2 \Rightarrow \alpha = 1$, $\beta = 0 \Rightarrow \mu(x,y) = x \Rightarrow$

$(3x^2y + y^2x)\,dx + (x^3 + x^2y)\,dy = 0$ ✓

$u = x^3y + \frac{x^2y^2}{2} + A(y) \Rightarrow A(y) = 0 \Rightarrow x^3y + \frac{x^2y^2}{2} = c$ ✓

سؤال دوم:

$2xy(1+xy^2)\,dy - dx = 0 \quad , \quad z = 2\ln x y^2$

$\frac{\mu_z}{\mu} = \frac{0 - 2y - 4y^3x}{+(2xy + 2x^2y^3)\cdot -\frac{2}{x} + 1(-2y^2)} = -1 \Rightarrow \ln\mu(z) = -\int dz \Rightarrow -z = \ln\mu$

$\Rightarrow \mu = e^{-z} = e^{y^2 - \ln x^2} = \frac{e^{y^2}}{x^2} \Rightarrow \left(\frac{2y\,e^{y^2}}{x} + 2y^3e^{y^2}\right)dy - \frac{e^{y^2}}{x^2}\,dx = 0$

$u = \frac{e^{y^2}}{x} + A(y) \Rightarrow A'(y) = 2y^3e^{y^2} \Rightarrow A(y) = y^2e^{y^2} - e^{y^2} \Rightarrow$

$\frac{e^{y^2}}{x} + e^{y^2}(y^2-1) = c_1 \Rightarrow \frac{e^{y^2}}{x} = c_1 - e^{y^2}(y^2-1) \Rightarrow x = \frac{e^{y^2}}{c_1 + e^{y^2}(1-y^2)}$

$\Rightarrow x = \frac{1}{c_1e^{y^2} + 1 - y^2} \xrightarrow{\times c_1 = -c} x = \frac{1}{1 - y^2 - ce^{-y^2}}$ ✓

سؤال چهارم:

$(3xy + y^2)\,dx + (x^2 + xy)\,dy$

$\mu(x,y) = \frac{1}{xy(2x+y)} \Rightarrow \left(\frac{3}{2x+y} + \frac{y}{x(2x+y)}\right)dx + \left(\frac{x}{y(2x+y)} + \frac{1}{2x+y}\right)dy \Rightarrow$

$\downarrow$

$u = \frac{1}{2}\left(\ln(2x+y) + \ln(x) + A(y)\right) \Rightarrow A'(y) =$

$\frac{1}{2}y = \ln\sqrt{y} \Rightarrow x\sqrt{2x+y}\,\sqrt{y} = c_1 \Rightarrow x^2(2x+y)y = c_2 \Rightarrow 2x^3y + x^2y^2 = c_2$

$\Rightarrow x^3y + \frac{x^2y^2}{2} = \frac{c}{2}$ ✓

تمرین ریاضی ۴-۱

سؤال اول:

1) $y' + 3y = 8 \Rightarrow \mu(x)y' + 3\mu(x)y = 8\mu(x)$

$\mu'(x) = 3\mu(x) \Rightarrow \mu(x) = e^{3x} \Rightarrow (e^{3x}y)' = 8e^{3x} \Rightarrow e^{3x}y = \frac{8}{3}e^{3x} + C$

if $y(0) = 2 \Rightarrow C = -\frac{2}{3} \Rightarrow y = (8 - 2e^{-3x})/3 = (2(4 - e^{-3x}))/3$ ✓

if $y(0) = 4 \Rightarrow C = \frac{4}{3} \Rightarrow y = \frac{8 + 4e^{-3x}}{3} = 4(2 + e^{-3x})/3$ ✓

3) $\tan(x)y' + y = 3x\sec x \Rightarrow y' + y\cot x = \frac{3x}{\cos x} \times \frac{\cos x}{\sin x} \Rightarrow$

$\mu(x)y' + \cot(x)\mu(x)y = 3x\csc(x)\mu(x) \Rightarrow$

$\mu'(x) = \cot(x)\mu(x) \Rightarrow \frac{\mu'(x)}{\mu(x)} = \cot(x) \Rightarrow \ln\mu = \int \cot(x)dx \Rightarrow$

$\mu(x) = \sin x \Rightarrow (\sin(x)y)' = 3x \Rightarrow y\sin(x) = \frac{3}{2}x^2 + C \Rightarrow y = \frac{1}{\sin x}\left(\frac{3}{2}x^2 + C\right)$

سؤال سوم:

2) $(3\tan(x) - 2\cos(y))\sec^2 x\,dx + \tan x(\sin y)dy = 0$

$\tan x = t \Rightarrow dt = \sec^2 x\,dx,\ \cos y = u \Rightarrow du = -\sin y\,dy \Rightarrow$

$(3t - 2u)dt - t\,du = 0$

عامل انتگرال‌ساز μ فقط برحسب t

$\frac{\mu'}{\mu(t)} = \frac{M_u - N_t}{N} = \frac{-2+1}{-t} = \frac{1}{t} \Rightarrow \ln\mu = \ln t \Rightarrow \mu = t \Rightarrow$

$(3t^2 + 2ut)dt - t^2du = 0 \quad dw = 0 \Rightarrow w = \text{const}$

$w = t^3 - t^2u + A(u) \Rightarrow A(u) = 0 \Rightarrow t^3 - t^2u = C_1 \Rightarrow$

$t^3 + C = t^2u \Rightarrow \tan^3 x + C = \tan^2 x\cos(y)$ ✓

3) $2x e^{2y} y' = 3x^4 + e^{2y}$, $e^{2y} = u \Rightarrow u'_x = 2y'_x e^{2y} \Rightarrow$

$xu' = 3x^4 + u \Rightarrow xu' - u = 3x^4 \Rightarrow u' - \frac{u}{x} = 3x^3 \Rightarrow$

$\mu u' - \frac{\mu}{x} u = 3x^3 \mu \Rightarrow \mu' = \frac{-\mu}{x} \Rightarrow \frac{\mu'}{\mu} = -\frac{1}{x} \Rightarrow \ln\mu = \ln\frac{1}{x} \Rightarrow \mu = \frac{1}{x}$

$\Rightarrow \left(\frac{u}{x}\right)' = 3x^2 \Rightarrow \frac{u}{x} = x^3 + C \Rightarrow u = x^4 + Cx \Rightarrow e^{2y} = x^4 + Cx \Rightarrow$

$2\ln y = \ln(x^4 + Cx) \Rightarrow y = \frac{1}{2}\ln(x^4 + Cx) \Rightarrow y = \ln\sqrt{x^4 + Cx} \Rightarrow y = \ln\sqrt{x^4 + Cx}$ ✓

سوال چهارم :

3) $y' = \frac{xy^2 - \sin x\cos x}{y(1-x^2)} \Rightarrow yy' - \frac{xy^2}{(1-x^2)} = -\frac{\sin x\cos x}{(1-x^2)} \Rightarrow u = y^2$

$\Rightarrow u' = 2yy' \Rightarrow \frac{1}{2}u' + \frac{x}{x^2-1}u = \frac{\sin x\cos x}{x^2-1} \Rightarrow u' + \frac{2x}{x^2-1} = \frac{2\sin x\cos x}{x^2-1} \Rightarrow$

$\mu u' + \frac{2\mu x u}{x^2-1} = \frac{\mu \sin 2x}{x^2-1}$

$\mu' = \frac{2x\mu}{x^2-1} \Rightarrow \frac{\mu'}{\mu} = \frac{2x}{x^2-1} \Rightarrow \ln\mu = \ln(x^2-1) \Rightarrow \mu = x^2 - 1 \Rightarrow$

$((x^2-1)u)' = \frac{(x^2-1)(\sin 2x)}{x^2-1} = \sin 2x$

$(x^2-1)u = -\frac{1}{2}\cos 2x + C_1 \xrightarrow{x^2-1} (1-x^2)u = \frac{1}{2}(1-2\sin^2 x) + C_2 \Rightarrow$

$(1-x^2)u + \sin^2 x = C \Rightarrow (1-x^2)y^2 + \sin^2 x = C \xrightarrow{y(0)=2} C = 4 \Rightarrow$

$y^2(1-x^2) + \sin^2 x = 4$ ✓

موضوع ______ تاریخ / /

4) $(x^4\ln x - 2xy^3)dx + 3x^2y^2dy = 0,\ y(2)=1 \Rightarrow$

$3x^2y^2y' - 2xy^3 = -x^4\ln x \Rightarrow y^3 = u \Rightarrow u' = 3y^2y' \Rightarrow$

$x^2u' - 2xu = -x^4\ln x \Rightarrow u' - \frac{2}{x}u = -x^2\ln x \Rightarrow \mu u' - \frac{2\mu}{x}u = \mu x^2\ln x$

$\Rightarrow \mu' = \frac{-2\mu}{x} \Rightarrow \ln\mu = \ln\frac{1}{x^2} \Rightarrow \mu = \frac{1}{x^2}$ ✓ $\Rightarrow \left(\frac{u}{x^2}\right)' = -\ln x \Rightarrow \frac{u}{x^2} =$

$x - x\ln x + C \Rightarrow y^3 = x^3 - x^3\ln x + Cx^2 \xrightarrow{y(2)=1} C = \frac{-7+8\ln 2}{4} \Rightarrow$

$y^3 + x^3\ln x - x^3 + \frac{-8\ln 2+7}{4}x^2 = 0 \Rightarrow 4y^3 + 4x^3(\ln x - 1) - 8x^2\ln 2 + 7x^2 = 0$ ✓

5) $y' = \frac{1}{x+\cos y} \Rightarrow \frac{1}{x'} = \frac{1}{x+\cos y} \Rightarrow x' - x = \cos y \Rightarrow$

$\mu x' - \mu x = \cos y\,\mu \Rightarrow \mu'(y) = -\mu(y) \Rightarrow \mu(y) = e^{-y} \Rightarrow$

$(e^{-y}x)' = (\cos y\, e^{-y}) \Rightarrow e^{-y}x = \frac{\sin y\, e^{-y}}{2} - \frac{\cos y\, e^{-y}}{2} + C_1 \Rightarrow$

$x = \frac{\sin y}{2} - \frac{\cos y}{2} + Ce^{y}$ ✓

تمرین

$2xy(1+xy^2)y' = 1 \Rightarrow y' = \frac{1}{2xy+2x^2y^3} \Rightarrow y' = \frac{1}{x'} \Rightarrow$

$x' = 2xy + 2x^2y^3 \Rightarrow x' + (-2y)x = (2y^3)x^2 \Rightarrow x^{-2}x' + (-2y)x^{-1} = 2y^3 \Rightarrow$

$x^{-1} = u \Rightarrow u' = -x^{-2}x' \Rightarrow -u' - 2yu = 2y^3 \Rightarrow u' + 2yu = -2y^3 \Rightarrow$

$\mu u' + (2\mu y)u = -2\mu y^3 \Rightarrow \mu' = 2\mu y \Rightarrow \ln\mu = y^2 \Rightarrow \mu = (e^{y^2}) \Rightarrow (e^{y^2}u)' =$

$-2e^{y^2}y^3 \Rightarrow e^{y^2}u = C_1 + e^{y^2} - y^2e^{y^2} \Rightarrow u = C_1e^{-y^2} + 1 - y^2 \Rightarrow u = \frac{1}{x} \Rightarrow x = \frac{1}{C_1e^{-y^2}+1-y^2}$

$C_1 = -C$

$\Rightarrow x = \frac{1}{1-y^2-Ce^{-y^2}}$ ✓

تمرینات بخش ۱-۵:

سوال اول:

$y' + 2\tan x \sec x = y^2 \sin x \qquad y_1 = \sec x$

$y = y_1 + \frac{1}{u} \Rightarrow y_1' - \frac{u'}{u^2} = \frac{-\sin x}{u^2} + \frac{2\sin x\, y_1}{u} - \sin x\, y_1^2 + 2\tan x \sec x$

$\Rightarrow \frac{-u'}{u^2} = \frac{-\sin x}{u^2} - \frac{2\sin x . \sec x}{u} \Rightarrow u' = \sin x + 2\tan x\, u \Rightarrow$

$u' - 2\tan x\, u = \sin x \Rightarrow \mu u' - 2\mu \tan x\, u = \sin x\, \mu$

$\mu' = -2\mu \tan x \Rightarrow \ln \mu = 2\ln \cos \Rightarrow \mu = \cos^2 \Rightarrow (\cos^2 x\, u)' = \sin x \cos^2 x \Rightarrow$

$\cos^2 x\, u = -\frac{1}{3}\cos^3 x + C \Rightarrow u = -\frac{1}{3}\cos + \frac{C}{\cos^2}$

$y = y_1 + \frac{1}{u} = \sec x + \frac{1}{-\frac{1}{3}\cos + \frac{C}{\cos^2}} = \sec x + \frac{3\cos^2}{-\cos^3 x + 3C}$ ✓

سوال دوم:

$y = \frac{-u'}{a_2(x)u} \Rightarrow$ ریکاتی $y' = a_2(x)y^2 + a_1(x)y + a_0(x)$

$\xrightarrow{(2)} y' = \frac{-u''(a_2(x)u) + (a_2'(x)u + u' \times a_2(x))u'}{(a_2(x)u)^2}$

$\xrightarrow{(1)} y' = \frac{(u')^2}{a_2^2(x)u^2} \times a_2(x) + \frac{a_1(x)(-u')}{a_2(x)u} + a_0(x)$

$\underline{(1),(2)} \Rightarrow -u''(a_2(x)u) + \frac{a_2'}{a(x)}uu' + (u'^2 \cancel{a_2(x)}) = \cancel{a_2(x)u'^2} +$

$a_0 a_2 u(u') \ldots + a_0(x)(a_2(x)u)^2 \Rightarrow -u''(a_2(x)u) + G(x)uu' =$

$P_1(x)(uu') + h(x)u^2 \xrightarrow{\div u} h(x)u'' + (\cos x) - P_1(x)u' - h(x)u = 0$

در تعیین $h(x)$ ها، معادلهٔ دیفرانسیل به صورت روبه‌رو است: $u''(x) + P(x)u' + Q(x)u = 0$

طبق جزوه ☐

سوال سوم:

1-16) $x(y')^2 + 2xy' - y = 0 \Rightarrow x = \frac{y}{y'^2 + 2y'} \Rightarrow x = \frac{y}{p^2+2p}$

از دو طرف نسبت به y مشتق می گیریم

$$\frac{1}{p} = \frac{1(p^2+2p) - (2p\frac{dp}{dy} + 2\frac{dp}{dy})y}{(p(p+2))^2}$$

$$\Rightarrow p(p+2)^2 = p^2+2p - ((2p+2)dp)\frac{y}{dy} \Rightarrow p^3+3p^2+2p = (-2p-2)dp\frac{y}{dy}$$

$$\Rightarrow \frac{dy}{y} = \frac{-2p-2}{p(p^2+3p+2)}dp = \frac{-2\cancel{(p+1)}}{p\cancel{(p+1)}(p+2)} = \frac{-2}{p(p+2)}dp$$

$$\ln cy = \ln\frac{p+2}{p} \Rightarrow \left\{\begin{array}{l} cy = \frac{p+2}{p} \\ ① \ x = \frac{y}{p^2+2p} \end{array}\right\} \Rightarrow (yc)^2 = 4cx \checkmark$$

$$② \left\{\begin{array}{l} p = -1 \\ x = \frac{y}{p^2+2p} \end{array}\right\} \Rightarrow x = \frac{y}{-1} \Rightarrow y = -x \Rightarrow x+y=0 \checkmark$$

1-17: $y = xy' - 2y'^3 \Rightarrow y = px - 2p^3 \Rightarrow x = \frac{y+2p^3}{p} = \frac{y}{p} + 2p^2$

نسبت به y مشتق می گیریم:

$$\frac{1}{p} = \frac{p - \frac{dp}{dy}y}{p^2} + 4\frac{dp}{dy} \Rightarrow$$

$$p = p - \frac{dp}{dy}y + 4p^3\frac{dp}{dy} \Rightarrow (4p^3 - y)\frac{dp}{dy} = 0 \Rightarrow$$

$$① \left\{\begin{array}{l} \frac{dp}{dy} = 0 \Rightarrow p = c \\ y = px - 2p^3 \Rightarrow y = cx - 2c^3 \end{array}\right.$$

$$② \left\{\begin{array}{l} 4p^3 - y = 0 \Rightarrow y = 4p^3 \\ y = px - 2p^3 \end{array}\right\} \Rightarrow y = \frac{x^3}{4} - \frac{2x^3}{16} = \frac{x^3}{8} \checkmark$$

موضوع

تاریخ / /

سوال چهارم:

2) $y'^2 - 2yy' + e^x = 0 \qquad y'^2 = 4e^x \Rightarrow y = 4e^{\frac{1}{2}x}$

$y = \frac{y'^2 + e^x}{2} \Rightarrow y = \frac{p + e^x}{p} \xrightarrow[\text{مشتق گرفتیم}]{\text{از طرفین نسبت به}} p\frac{dp}{dx} = \frac{e^x p - \frac{dp}{dx} e^x}{2} \Rightarrow$

$p^3 = p^2\frac{dp}{dx} + e^x p - \frac{dp}{dx}e^x \Rightarrow (p^2 - e^x)dp = p(p^2 - e^x)dx$

① $p^2 - e^x = 0$, ② $\Rightarrow dp = p\,dx \Rightarrow \ln p = x + c \Rightarrow ce^x = p$

① $\left\{\begin{array}{l} e^x = p^2 \\ y = p + \frac{e^x}{p} \end{array}\right\} \Rightarrow y^2 = p^2 + 2e^x + \frac{e^{2x}}{p^2} \Rightarrow y^2 = 4e^x$ ✓

② $\left\{\begin{array}{l} ce^x = p \\ y = p + \frac{e^x}{p} \end{array}\right\} \Rightarrow ce^x + \frac{1}{c} = y$ ✓

3) $y = 2xy' + y^2y'^3 \xrightarrow{p = y'} y - p^3y^2 = 2px \Rightarrow x = \frac{y - p^3y^2}{2p} \Rightarrow x = \frac{1}{2}\left(\frac{y}{p} - p^2y^2\right)$

$\xrightarrow[\text{نسبت به } y]{\text{مشتق گیری از طرفین}} \frac{1}{p} = \frac{1}{2}\left(\left(\frac{p - \frac{dp}{dy}y}{p^2}\right) - \left(2p\frac{dp}{dy}y^2 + 2yp^2\right)\right) \xrightarrow{\times p^2}$

$p = \frac{p}{2} - \frac{y}{2}\frac{dp}{dy} - py^2\frac{dp}{dy} - yp^4 \Rightarrow (2p - p)dy = -y\,dp - 2p^3y^2\,dp - 2yp^4\,dy \Rightarrow (p + 2yp^4)dy = (-y - 2p^3y^2)dp \Rightarrow p(1 + 2yp^3)dy =$

$(-y)(1 + 2p^3y)dy \Rightarrow p\,dy = -y\,dp \Rightarrow \ln p = \ln\frac{1}{y} + c \Rightarrow p = \frac{c}{y}$ یا

$1 + 2p^3y = 0 \Rightarrow p = -\frac{1}{2}y \sim \frac{c}{y}$

$$\begin{cases} p = \frac{c}{y} \\ x = \frac{1}{2}\left(\frac{y}{p} - p^2 y^2\right) \end{cases} \Rightarrow 2x = \frac{y^2}{c} - c^2 \Rightarrow 2cx + c^3 = y^2 \checkmark$$

سوال پنجم:

$$y = xy'^2 + y'^4 \quad , y' = p \Rightarrow y = xp^2 + p^4 \xrightarrow[\text{نسبت به } x]{\text{مشتق‌گیری}} p = p^2 + 2px\frac{dp}{dx} + 4p^3\frac{dp}{dx} \Rightarrow$$

$$(p - p^2)\,dx = (2px + 4p^3)\,dp \Rightarrow p = 0 \text{ یا } (1-p)\,dx = (2x + 4p^2)\,dp \Rightarrow$$

$$(p-1)\,dx + (2x + 4p^2)\,dp = 0 \Rightarrow$$

عامل انتگرال‌ساز حتماً p است چون:

$$\frac{M_p - N_x}{-M} = \frac{\mu(p)}{\mu} = \frac{1-2}{-(p-1)} = \frac{1}{p-1} \Rightarrow \ln\mu = \ln(p-1) \Rightarrow \mu = p-1$$

$$\Rightarrow (p^2 - 2p + 1)\,dx + (2px - 2x + 4p^3 - 4p)\,dp = 0 \quad u = c \Rightarrow$$

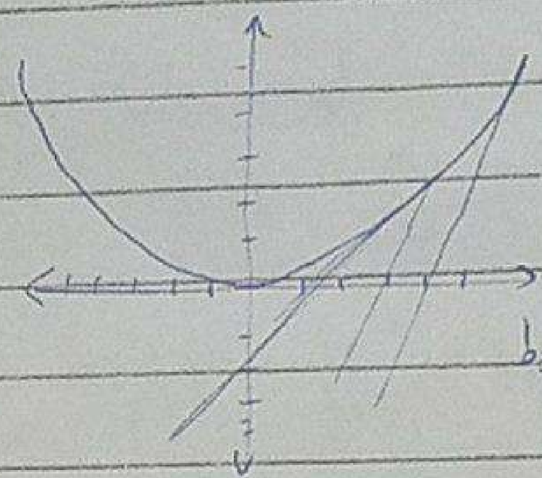
$$u = p^2x - 2px + x + A(p) \Rightarrow A'(p) = 4p^3 + 4p \Rightarrow A(p) = p^4 + \frac{4}{3}p^3$$

$$\Rightarrow \begin{cases} p^2x - 2px - x + p^4 + \frac{4}{3}p^3 = c \\ y = xp^2 + p^4 \checkmark \end{cases} \quad , \quad \begin{cases} p = 0 \\ y = xp^2 + p^4 \Rightarrow y = 0 \checkmark \end{cases}$$

$$, \begin{cases} p = 1 \Rightarrow y = x + 1 \checkmark \\ y = xp^2 + p^4 \end{cases}$$

موضوع .... تاریخ / /

سؤال ششم:



$cx-2c^2=y$

c	y
1	$x-2$
2	$2x-8$
3	$3x-18$
4	$4x-32$

با توجه به شکل مشاهده می‌کنیم که منحنی $\frac{x^2}{8}=y$ با این خطوط مماس است.

$*\ y_1=\frac{x^2}{8} \Rightarrow y_1'=\frac{2x}{8}=\frac{x}{4}$ $\qquad y_1'=y_2' \Rightarrow c=\frac{x}{4} \Rightarrow x=4c$

$**\ y_2=cx-2c^2 \Rightarrow y_2'=c$

$y_1=\frac{16c^2}{8}=2c^2 \qquad y_2=4c^2-2c^2=2c^2$

پس در نقطه‌ی تماس منحنی $\frac{x^2}{8}=y$ بر $y=cx-2c^2$ مماس است.

سؤال هفتم:

$x(y'+1)+e^{y'}=2x+y+2 \Rightarrow y=x(y'-1)+e^{y'}-2 \Rightarrow$

$y=x(p-1)+e^p-2 \Rightarrow p=1(p-1)+\frac{dp}{dx}x+\frac{dp}{dx}e^p \Rightarrow -dx+(x+e^p)dp=0$

$\Rightarrow \frac{\mu'(p)}{\mu(p)}=\frac{0-1}{+1} \Rightarrow \mu'(p)=-\mu(p) \Rightarrow \mu(p)=e^{-p} \Rightarrow$

$(-e^{-p})dx+(xe^{-p}+1)dp=0 \Rightarrow u=-xe^{-p}+A(p) \Rightarrow A'(p)=1 \Rightarrow A(p)=p \Rightarrow$

$-xe^{-p}+p=c$ $\qquad \begin{cases} y=x(p-1)+e^p-2 \Rightarrow y=e^p(p-c)(p-1)+e^p-2 \\ x=(p-c_1)e^p \end{cases}$

$\begin{cases} y=e^p(p+c)(p-1)+e^p-2 \\ x=(p+c)e^p \end{cases} \qquad \Leftarrow c=-c$ ✓

تمرین حل (تمرین تحویلی)

2) $y' = \dfrac{y}{x^2 y^2 \ln y - x} \Rightarrow x' = x^2 y \ln y - \dfrac{x}{y} \Rightarrow \dfrac{x' + \frac{x}{y}}{x^2} = y \ln y$

$\Rightarrow x^{-2} = u \Rightarrow u' = -\dfrac{2x'}{x^3} \Rightarrow -\dfrac{1}{2}u' + \dfrac{u}{y} = y \ln y \Rightarrow u' - \dfrac{2u}{y} = -2y \ln y \Rightarrow$

$\mu' = \dfrac{-2\mu}{y} \Rightarrow \dfrac{\mu'}{\mu} = -\dfrac{2}{y} \Rightarrow \ln \mu = -2 \ln y \Rightarrow \mu = \dfrac{1}{y^2}$

$\left(\dfrac{1}{y^2}u\right)' = \dfrac{\ln y}{y} \Rightarrow \dfrac{u}{y^2} = c - (\ln y)^2 \Rightarrow \dfrac{1}{x^2 y^2} = c - (\ln y)^2 \Rightarrow x^2 y^2 = \dfrac{-1}{c - (\ln y)^2}$ ✓

3) $dy + (y \cot(x) - e^{\cos x})\,dx = 0 \Rightarrow y' + y \cot(x) = e^{\cos x} \Rightarrow \mu y' + \mu y \cot x =$
$\mu e^{\cos x}$, $\mu' = \cot x\, \mu \Rightarrow \ln \mu = \ln \sin x \Rightarrow \mu = \sin(x) \Rightarrow$
$(\sin(x) y)' = \sin(x) e^{\cos x} \Rightarrow \int \sin x\, e^{\cos x}\,dx = -e^{\cos x} + C \Rightarrow$
$\sin x\, y = -e^{\cos x} + C \Rightarrow y = \dfrac{-e^{\cos x} + C}{\sin x} \Rightarrow 1 = \dfrac{C-1}{1} \Rightarrow C = 2 \Rightarrow y = \dfrac{-e^{\cos x} + 2}{\sin x}$

4) $y' + x \sin 2y = 2x e^{-x^2} \cos^2 y \Rightarrow u = \tan y \Rightarrow u' = \sec^2 y\, y' \Rightarrow y' = u' \cos^2 y$
$u' \cos^2 y + x \dfrac{2u}{1+u^2} = 2x e^{-x^2} \Rightarrow u' + 2xu = 2x e^{-x^2} \Rightarrow \mu u' + 2\mu x u \Rightarrow$
$2x e^{-x^2} \mu \Rightarrow \mu' = 2\mu x \Rightarrow \ln \mu = x^2 \Rightarrow \mu = e^{x^2} \Rightarrow (e^{x^2} u)' = 2x \Rightarrow$
$e^{x^2} u = x^2 + C \Rightarrow \tan y = (x^2 + C) e^{-x^2}$ ✓

7) $y' + 1 = e^{-y}x \quad y = \ln u \Rightarrow \dfrac{u'}{u} = y' \Rightarrow \dfrac{u'}{u} + 1 = x \dfrac{1}{e^{\ln u}} \Rightarrow$
$u + u' = x \Rightarrow u' + e^x u = x e^x \Rightarrow (e^x u)' = x e^x \Rightarrow e^x u = x e^x - e^x + C \Rightarrow$
$u = x - 1 + Ce^{-x} \Rightarrow y = \ln u \Rightarrow y = \ln(x - 1 + Ce^{x})$

9) $\left(\frac{\ln x}{\ln y} + x^2y^3\right) y' + \left(\frac{y}{x}\ln(\ln(y)) + \frac{2}{3}xy^3\right) = 0 \to$

$\left(\frac{\ln x}{\ln y} + x^2y^3\right) dy + \left(\frac{y}{x}\ln(\ln y) + \frac{2}{3}xy^3\right) dx = 0$

[illegible]

$\frac{\mu'}{\mu} = \frac{\left(\frac{4}{3}xy^3 + \frac{1}{x\ln y} + \frac{\ln(\ln y)}{x} - \frac{1}{x\ln y} - 2xy^3\right)}{-\left(\frac{y}{x}\ln(\ln y) + \frac{2}{3}xy^3\right)} = \frac{-\frac{2}{3}xy^3 + \frac{\ln(\ln y)}{x}}{-y\left(\frac{\ln(\ln y)}{x} + \frac{2}{3}\dots\right)}$

$\Rightarrow \frac{-1}{y} = \ln\mu = -\ln y = \mu = \frac{-1}{y} \Rightarrow \left(\frac{\ln x}{y\ln y} + x^2y^2\right)dy + \left(\frac{1}{x}\ln(\ln y) + \frac{2}{3}xy^2\right)dx$

$\Rightarrow A_y = 0 \Rightarrow d_x = \dots \Rightarrow \ln x \ln(\ln y) + \frac{y^3x^2}{3} = c$

10) $y' + e^{-x}y^2 + y = e^x$ $\quad y = \left(y_1 + \frac{1}{u}\right) \Rightarrow y_1' + \frac{-u'}{u^2} = +e^{-x}\left(y_1^2 + \frac{2y_1}{u} + \frac{1}{u^2}\right)$
$+ \left(y_1 + \frac{1}{u}\right) = e^x \Rightarrow \frac{-u'}{u^2} = \frac{2}{u} + \frac{e^{-x}}{u^2} + \frac{1}{u} \Rightarrow -u' = 3u + e^{-x} \Rightarrow$
$u' + 3u = -e^{-x} \Rightarrow \mu u' + 3\mu u = -e^{-x}\mu$, $\mu' = 3\mu \Rightarrow \mu = e^{3x} \Rightarrow$
$(e^{3x}u)' = -e^{2x} \Rightarrow e^{3x}u = -\frac{1}{2}e^{2x} + C \Rightarrow u = -\frac{1}{2}e^{-x} + Ce^{-3x} \Rightarrow y =$
$e^x + \frac{1}{-\frac{1}{2}e^{-x} + Ce^{-3x}} \Rightarrow \frac{1}{y - e^x} = -\frac{1}{2}e^{-x} + Ce^{-3e}$ ✓

12) $y = \frac{-1}{x^2y'} - xy' \to y = \frac{-1}{x^2p} - xp \xrightarrow[\text{نسبت به } x]{\text{مشتق گیری}}$

$p = \frac{2xp + \frac{dp}{dx}x^2}{(x^2p)^2} - p - \frac{dp}{dx}x \to x^3p^2 dp = 2p + x\frac{dp}{dx} - p x^3 \dots - x^4p^2\frac{dp}{dx}$

$\Rightarrow (2p(x^3) - 2p)dx = (x - x^4p^2)dp \to x(x^3p^2 - 1) + 2p(x^3p^2 - 1)dx = 0$

① $x^3p^2-1=0$ ② $x^3p^2-1\neq 0 \Rightarrow xdp+2pdx=0 \Rightarrow \frac{-dx}{x}=\frac{dp}{2p}$

$\Rightarrow \ln\frac{1}{x}=\ln c_1\sqrt{p} \Rightarrow \frac{1}{x}=c\sqrt{p} \Rightarrow x=\frac{c}{\sqrt{p}} \Rightarrow p=\frac{c}{x^2}$ ✓

① $\begin{cases} x^3p^2=1 \Rightarrow p=\sqrt[2]{\frac{1}{x^3}} \\ y=\frac{-1}{x^2p}-xp \Rightarrow y=\pm\left(\frac{\sqrt{x^3}}{x^2}+\frac{x}{\sqrt{x^3}}\right) \Rightarrow y=\pm\frac{2}{\sqrt{x}} \end{cases}$

② $\begin{cases} p=\frac{c}{x^2} \\ y=\frac{-1}{x^2p}-xp \end{cases} \Rightarrow y=\frac{-1}{c_1}-\frac{c_1}{x} \xrightarrow{c_1=-c} y=\frac{1}{c}+\frac{c}{x}$

14) $xy^2(xy'+y)=2 \Rightarrow \frac{2}{x^2y^2}=y'+\left(\frac{1}{x}\right)y \Rightarrow \frac{2}{x^2}=y^2y'+y^3\left(\frac{1}{x}\right)$

$\Rightarrow y^3=u \Rightarrow 3y^2y'=u' \Rightarrow \frac{2}{x^2}=\frac{u'}{3}+\frac{u}{x} \Rightarrow u'+\frac{3u}{x}=\frac{6}{x^2} \Rightarrow$

$u'\mu+\frac{3\mu}{x}u=\frac{6\mu}{x^2} \Rightarrow \mu'=\frac{3\mu}{x} \Rightarrow \frac{\mu'}{\mu}=\frac{3}{x} \Rightarrow \mu=x^3 \Rightarrow$

$(x^3u)'=6x \Rightarrow x^3u=3x^2+C \Rightarrow u=\frac{C}{x^3}+\frac{3}{x} \Rightarrow \frac{3x^2+C}{x^3}=y^3 \Rightarrow$

$y^3x^3=3x^2+C \Rightarrow y^3x^3-3x^2=C$

تاریخ / /

موضوع ________

$$۱۴)\ x\,dy - ۲y\,dx = (x-۲)e^{x} \Rightarrow \frac{\mu'}{\mu} = \frac{-۲-۱}{x} \Rightarrow \mu = \frac{1}{x^{۳}} \Rightarrow$$

$$\frac{dy}{x^{۲}} - \frac{۲y}{x^{۳}}dx = \frac{(x-۲)e^{x}}{x^{۳}}$$

$$\Rightarrow \left(\frac{y}{x^{۲}}\right)' = \frac{(x-۲)e^{x}}{x^{۳}} \Rightarrow \frac{y}{x^{۲}} = \int \frac{(x-۲)e^{x}}{x^{۳}}dx \Rightarrow \frac{y}{x^{۲}} = \frac{e^{x}}{x^{۲}} + C$$

$$\Rightarrow y = e^{x}x^{۲} + Cx^{۲}$$