Model Predictive Control: Overview

Presenter:

Mohammadhadi Alizadeh

Summer 2022

Model Predictive Control (MPC): Overview

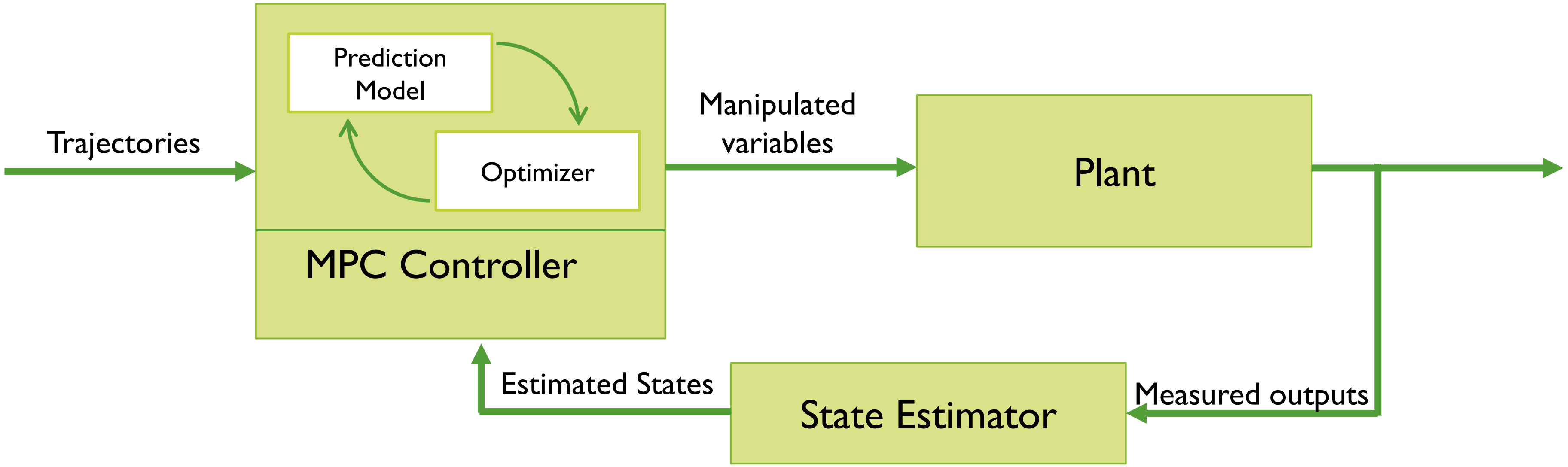

Fig5. MPC loop[1].

1- Reproduced From Mathworks.

Plant


Plant
First Principle Models
OpenModelica
OMMatlab
Control Algorithm
Matlab

Plant

- Compiles
- Complex to debug

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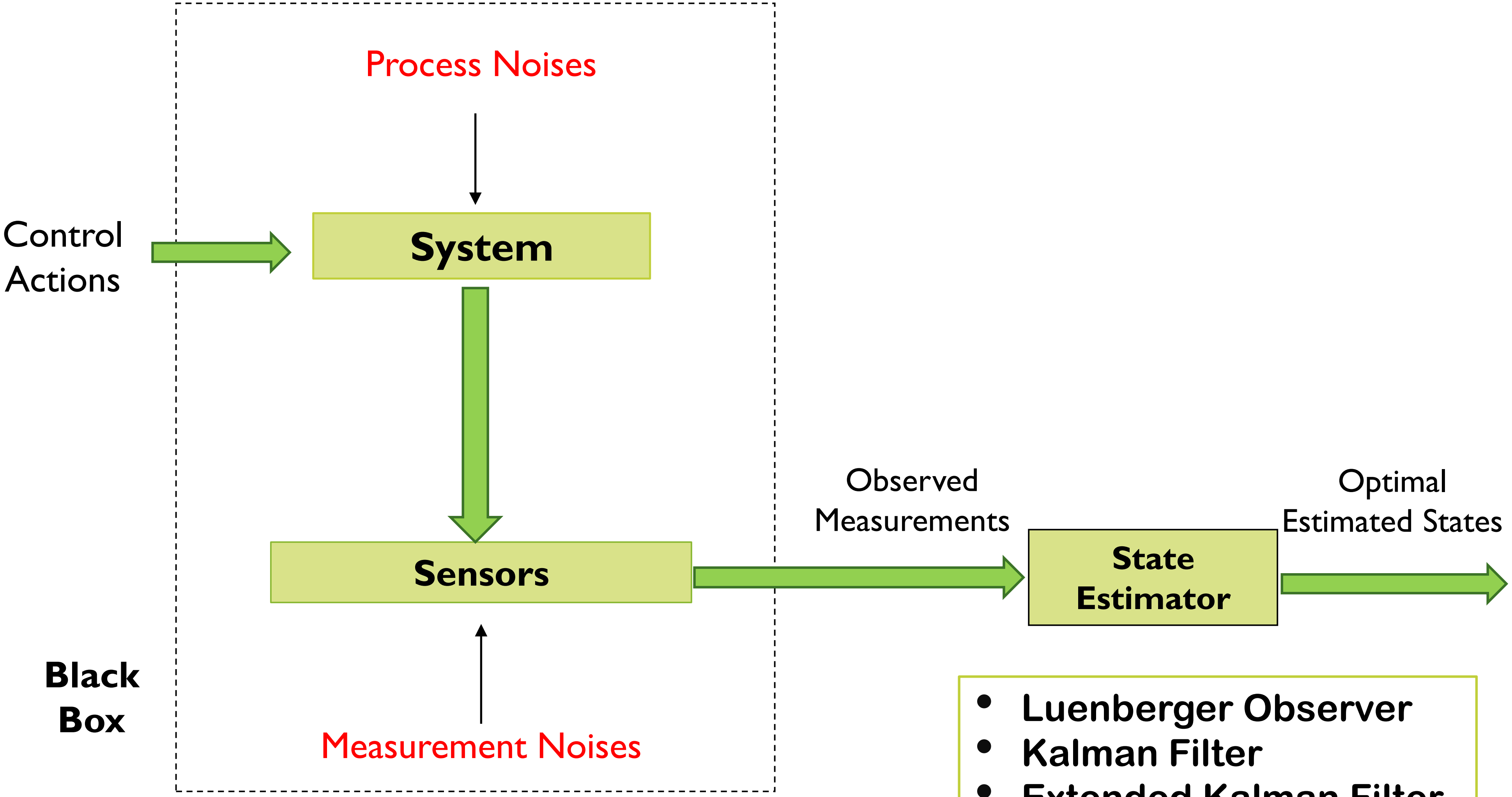
Might be Slow

State Estimation: Overview



- Luenberger Observer
- Kalman Filter
- Extended Kalman Filter

State Estimation: Extended Kalman Filter[1]

- **Nonlinear state transition and measurement**

$$x_k = f(x_{k-1}, u_{k-1}) + w_{k-1}$$
$$z_k = h(x_k) + v_k$$

1-Terejanu, Ga. 2003. *"Extended Kalman Filter Tutorial." Technical Report: Extended Kalman Filter Tutorial.*

State Estimation: Extended Kalman Filter[1]

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$$F_{k-1} = \frac{\partial f}{\partial x}\Big|_{\hat{x}^+_{k-1}, u_{k-1}} \longrightarrow \text{State Transition Matrix for covariance and gain calculations}$$

$$H_k = \frac{\partial h}{\partial x}\Big|_{\hat{x}^-_k} \longrightarrow \text{Sensor space transformation Matrix}$$

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- **Predict**

$$\hat{x}^-_k = f(\hat{x}^+_{k-1}, u_{k-1})$$
$$P^-_k = F_{k-1} P^+_{k-1} F^T_{k-1} + Q$$

- **Update**

$$\tilde{y}_k = z_k - h(\hat{x}^-_{k-1})$$
$$K_k = P^-_k H^T_k (R + H_k P^-_k H^T_k)^{-1}$$
$$\hat{x}^+_k = \hat{x}^-_k + K_k \tilde{y}_k$$
$$P^+_k = (I - K_k H_k) P^-_k$$

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State Estimation: Extended Kalman Filter[1]

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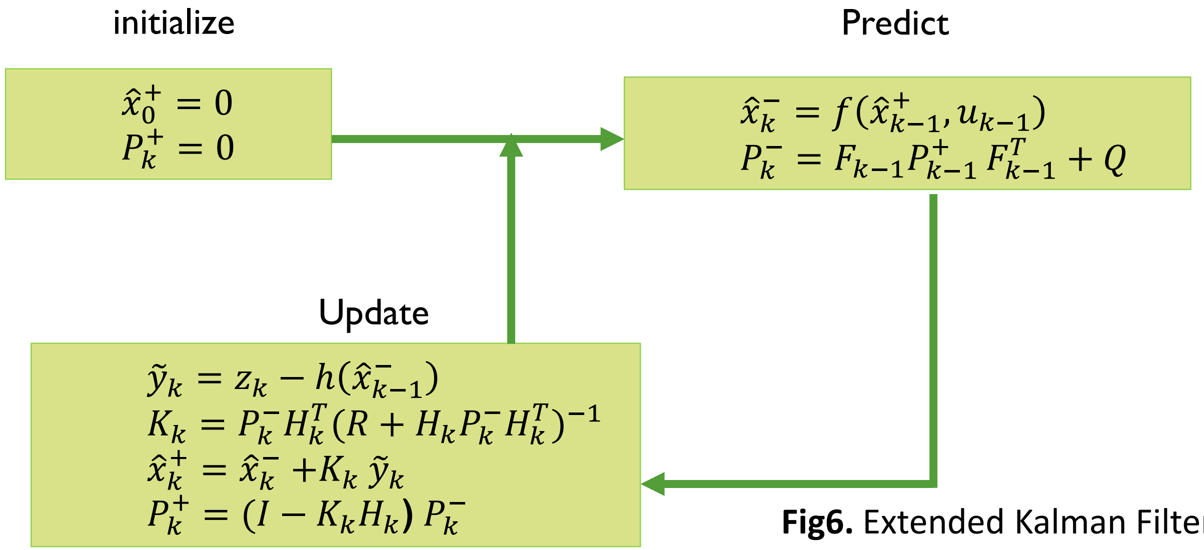

Fig6. Extended Kalman Filter loop.

1-Terejanu, Ga. 2003. *"Extended Kalman Filter Tutorial." Technical Report: Extended Kalman Filter Tutorial.*

Prediction Model[1]

- **Step-test identification methods**
  - **Cannot be used for a SwoMV system**

$\dot{x} = \mathrm{f}(x, \mathrm{u})$
y= $g(x, \mathrm{u})$
q= $h(x, \mathrm{u})$

x is the state variable
u is the process input
y is the output
q is the vector of quality

1-Mhaskar, Prashant, Abhinav Garg, and Brandon Corbett. n.d. Modeling and Control of Batch Processes.

Prediction Model[1]

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$\rightarrow$

x is the state variable
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- **Linearizing the nonlinear model**

Prediction Model[1]

- **Step-test identification methods**
  - **Cannot be used for a SwoMV system**

$\dot{x} = \mathrm{f}(x, \mathrm{u})$
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- **Linearizing the nonlinear model**
- **Subspace identification methods**

$\tilde{x}(\mathrm{k}+1)$= $\mathrm{A}\tilde{x}(\mathrm{k}) + \mathrm{Bu}(\mathrm{k})$
$\mathrm{y}(\mathrm{k}) = \mathrm{C}\tilde{x}(\mathrm{k}) + \mathrm{Du}(\mathrm{k})$
q= $G\tilde{x}(\mathrm{k}) + \mathrm{Fu}(\mathrm{k})$

$\tilde{x}$ is the subspace state
u is the process input
y is the output
q is the vector of quality

1-Mhaskar, Prashant, Abhinav Garg, and Brandon Corbett. n.d. *Modeling and Control of Batch Processes.*

Dynamic Optimization: Single Shooting Method[1]

$$min_u \; S(t_f)$$

With respect to:

Dynamic Model
Initial Conditions
Path constraints
Control(input) constraints
Terminal Constraints

1- *Chachuat, Benoit. 2016. "OPTIMIZATION From Theory to Practice IC-32 : Spring Term 2009," . August.*

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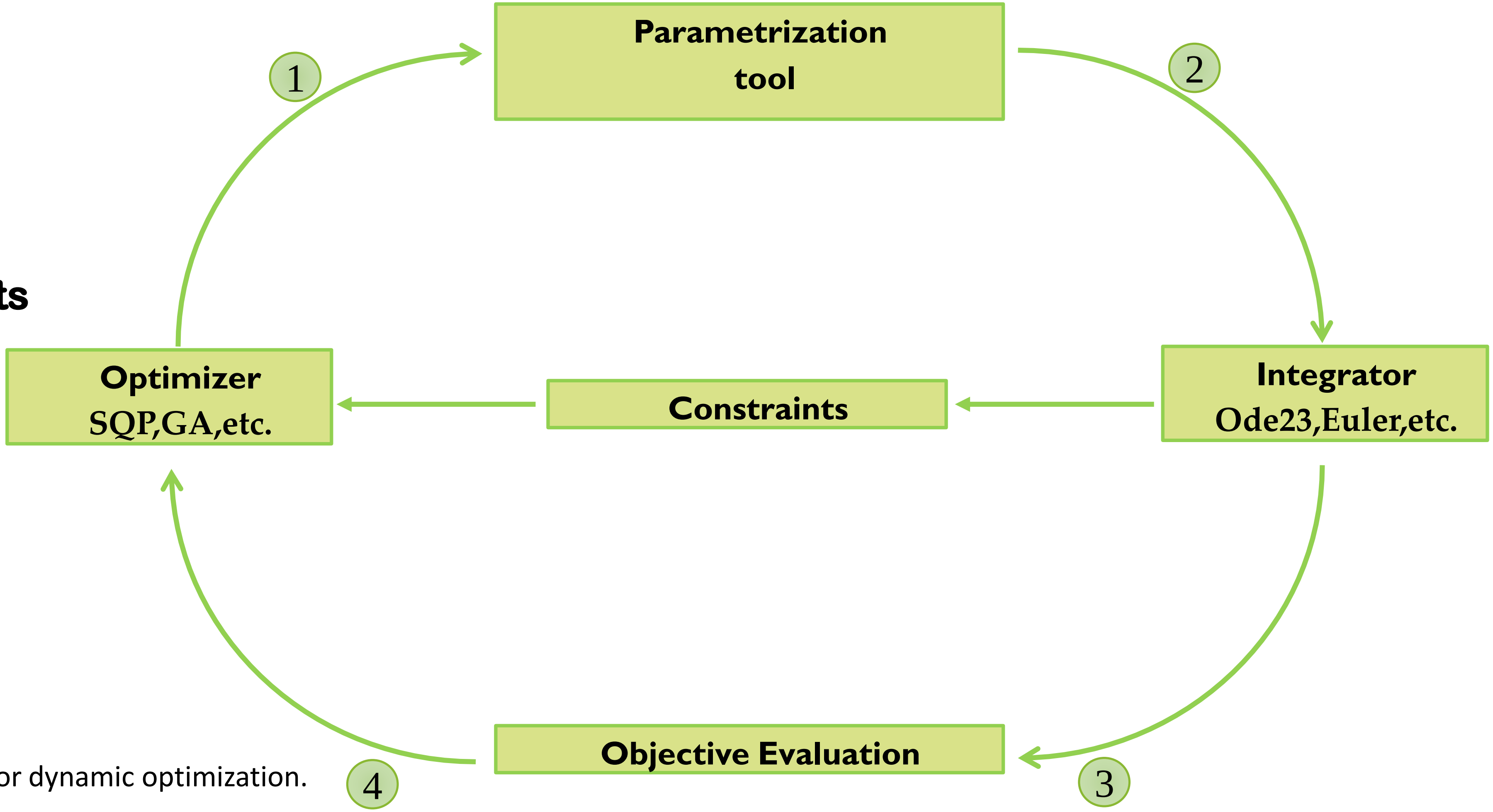

Fig7. Single shooting algorithm for dynamic optimization.

1- *Chachuat, Benoit. 2016. "OPTIMIZATION From Theory to Practice IC-32 : Spring Term 2009," . August*.

Example: Three connected tanks

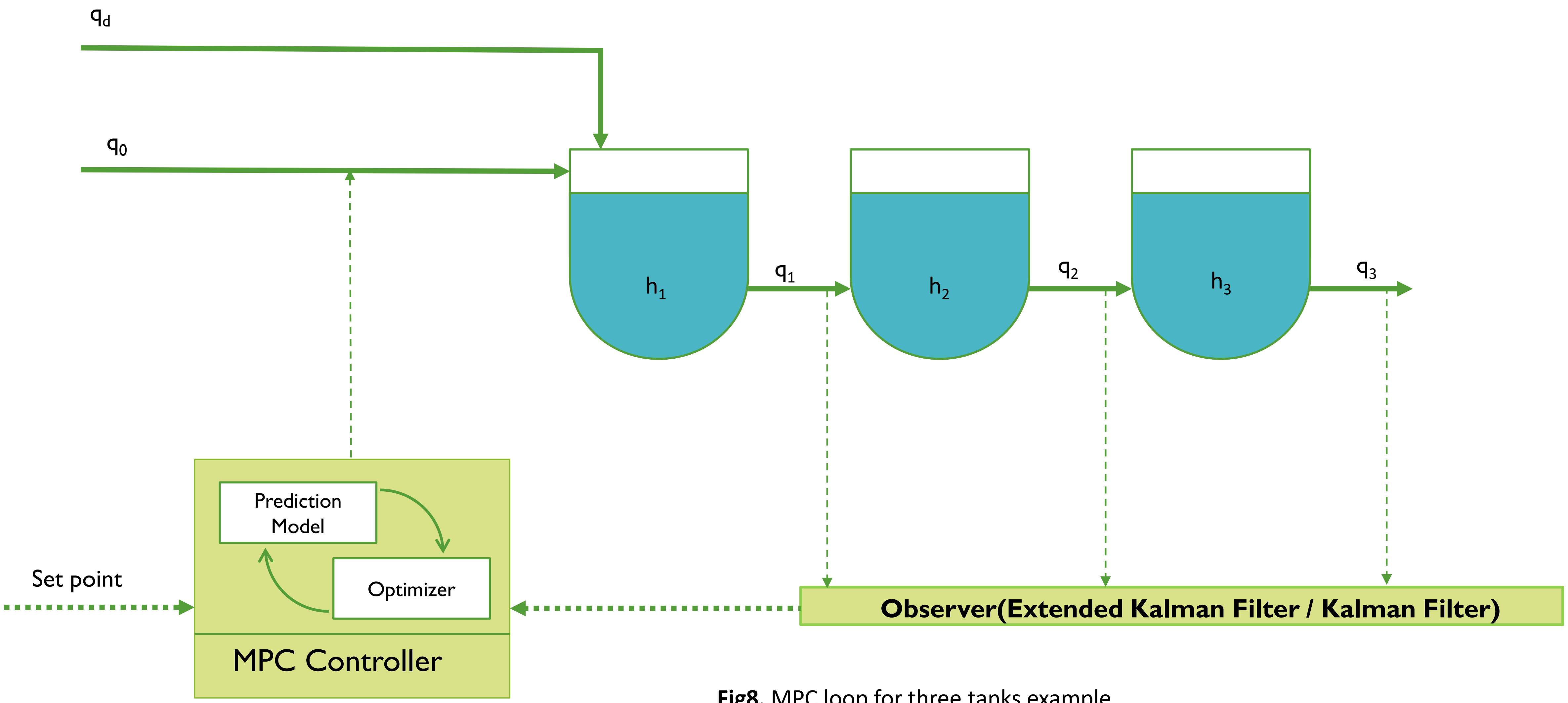

Fig8. MPC loop for three tanks example.

Three tanks example: simulation

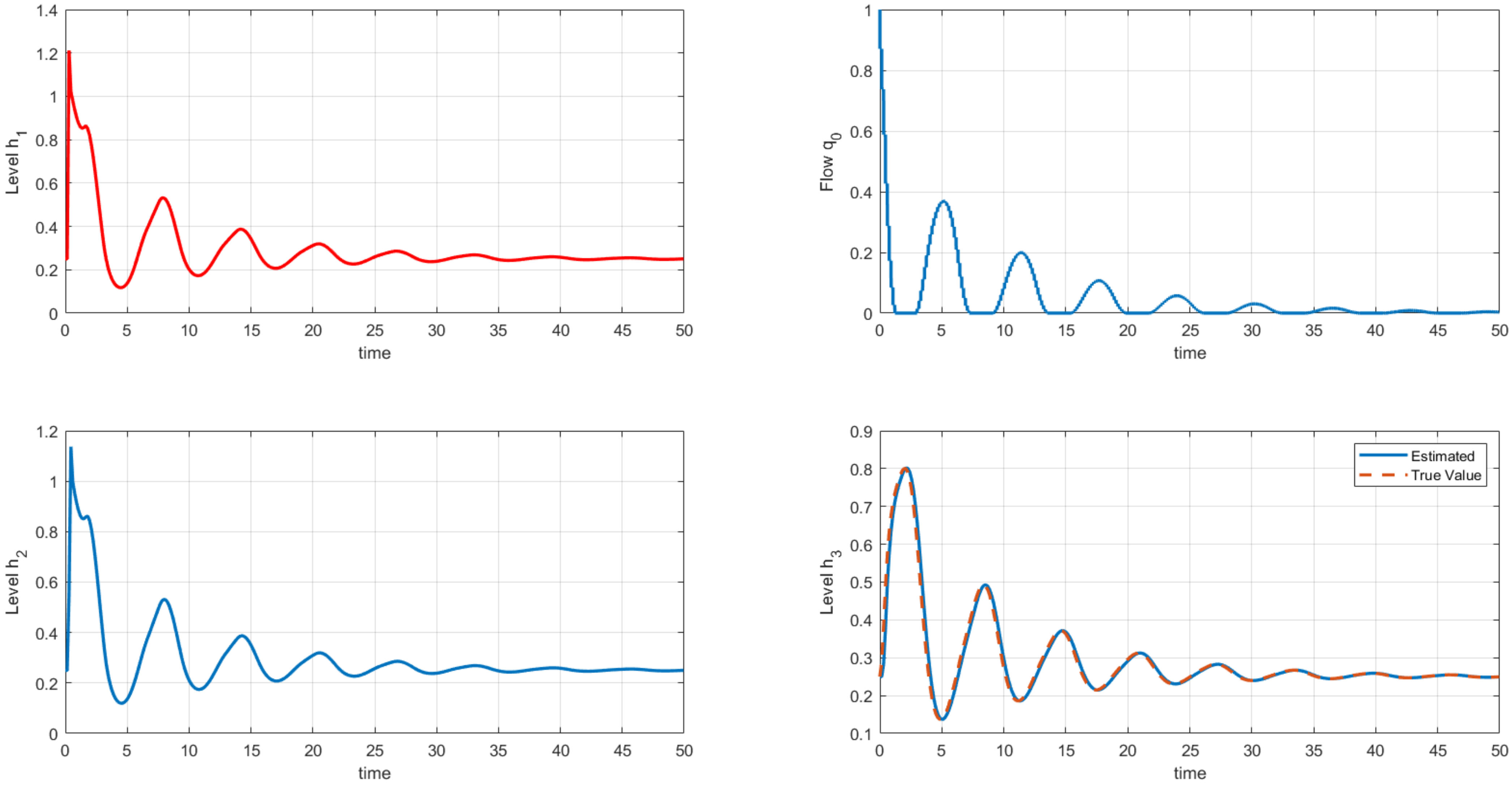

Fig9. Three tanks example simulation graphs with an EKF observer, a disturbance equation of q_d = 1+sin(t)*exp(-0.1*t), and SP=0.25.

Three tanks example: simulation

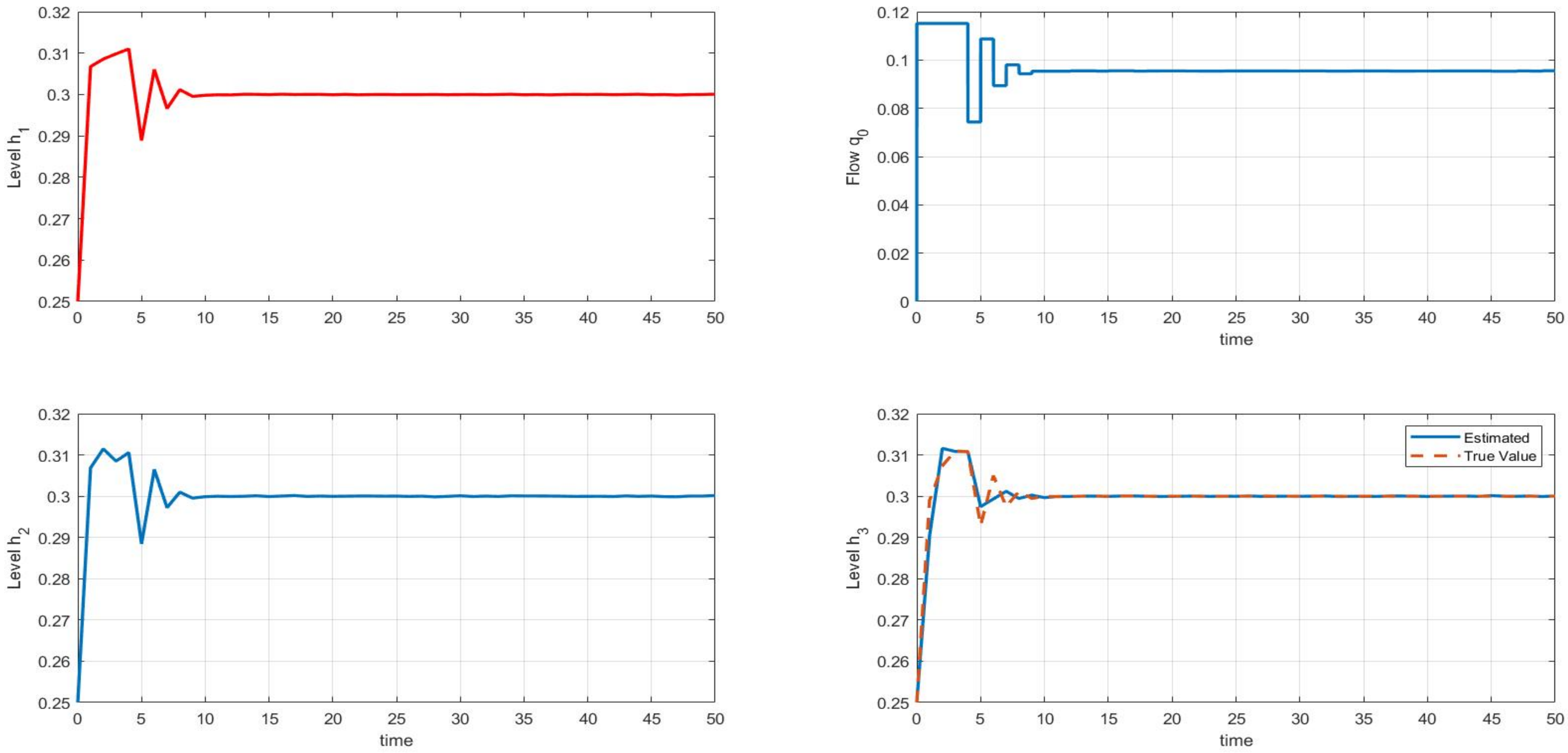

Fig11. Three tanks example simulation graphs with an EKF observer, a disturbance equation of q_d = 1, and SP=0.3.

- Meidanshahi, Vida, and Thomas A Adams Ii. 2016. “Integrated Design and Control of Semicontinuous Distillation Systems Utilizing Mixed Integer Dynamic Optimization.” *Computers and Chemical Engineering*.
- Phimister, James R, and Warren D Seider. 2000. “Semicontinuous , Middle-Vessel Distillation of Ternary Mixtures”
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- *Chachuat, Benoit. 2016. “OPTIMIZATION From Theory to Practice IC-32 : Spring Term 2009,” . August*.
- Garg, *Abhinav, and Prashant Mhaskar. 2017. “Subspace Identification-Based Modeling and Control of Batch Particulate Processes.” Industrial & Engineering Chemistry Research 56 (26): 7491–7502.*
- *Mhaskar, Prashant, Abhinav Garg, and Brandon Corbett. n.d. Modeling and Control of Batch Processes.*