مبانی رایانش نرم

فازی: محاسبات (اعداد) و روابط

هادی ویسی

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فهرست

- محاسبات فازی
  - اعداد فازی
  - عملگرهای حسابی
  - لاتیس
- روابط
  - روابط کلاسیک
  - خواص روابط کلاسیک
  - روابط فازی
  - خواص روابط فازی

اعداد فازی . . .

- مفهوم اعداد فازی
  - اعدادی که به یک عدد حقیقی نزدیک هستند
  - اعدادی که اطراف یک بازه از اعداد حقیقی هستند

- کاربردها
  - کنترل، تصمیم‌گیری، بهینه‌سازی، استدلال تقریبی

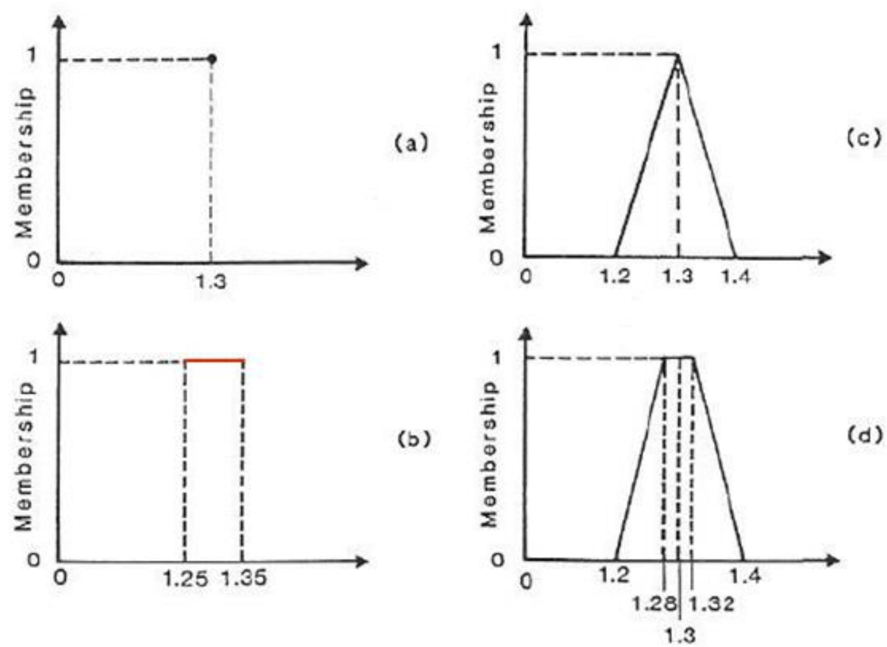
- تعریف
  - مجموعه‌های فازی روی R با تابع عضویت $A{:}\mathbf{R}\rightarrow[0,1]$
  - هر عدد فازی یک مجموعه فازی محدب است

اعداد فازی . . .

- ویژگی‌های مورد نیاز برای عدد فازی A

(i) A must be a normal fuzzy set;
(ii) ${}^{\alpha}A$ must be a closed interval for every $\alpha \in (0, 1]$;
(iii) the support of A, ${}^{0+}A$, must be bounded.

- مثال
  - عدد کریسپ و فازی
  - بازه کریسپ و فازی



اعداد فازی . . .

- انواع پایه اعداد فازی

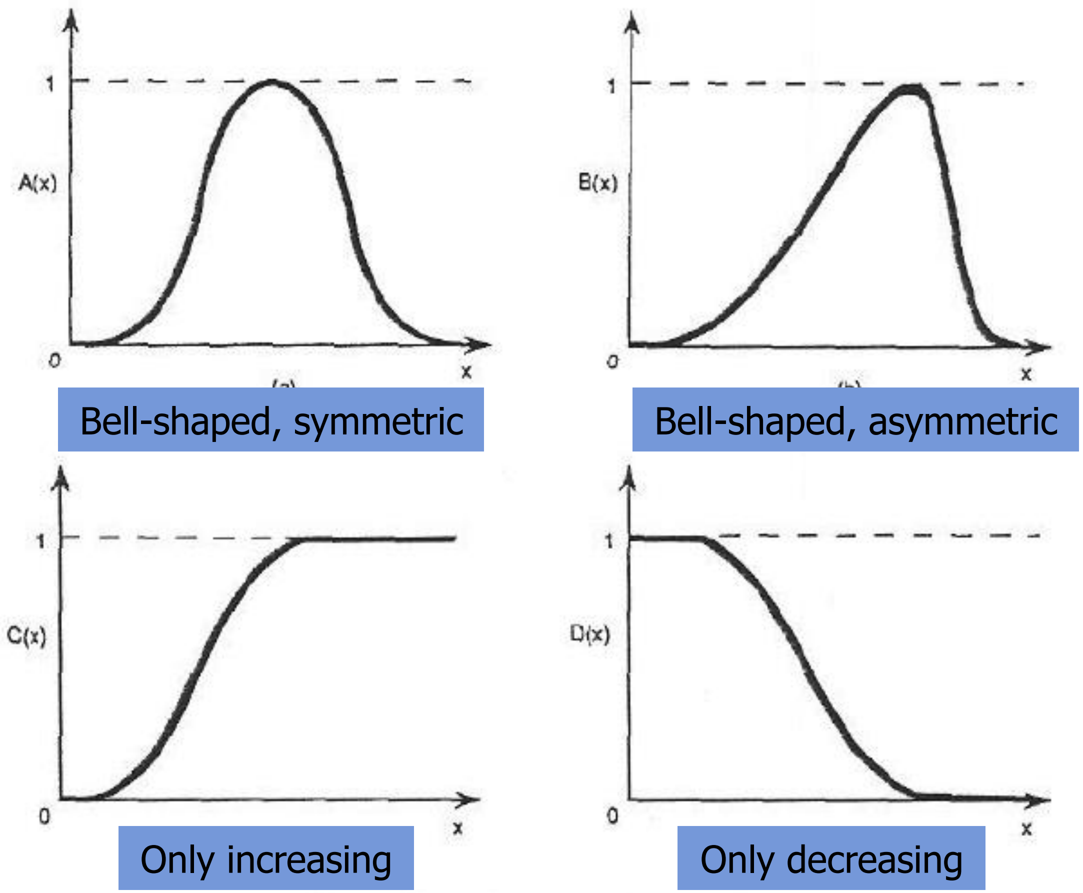


Figure 4.2 Basic types of fuzzy numbers.

اعداد فازی . . .

- قضیه: فرم کلی اعداد فازی
  - توابع عضویت می‌تواند قطعه–قطعه باشد

Theorem 4.1. Let $A \in \mathcal{F}(\mathbb{R})$. Then, A is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \varnothing$ such that

$$A(x) = \begin{cases} 1 & \text{for } x \in [a, b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty), \end{cases} \tag{4.1}$$

where l is a function from $(-\infty, a)$ to $[0, 1]$ that is monotonic increasing, continuous from the right, and such that $l(x) = 0$ for $x \in (-\infty, \omega_1)$; r is a function from (b, ∞) to $[0, 1]$ that is monotonic decreasing, continuous from the left, and such that $r(x) = 0$ for $x \in (\omega_2, \infty)$.

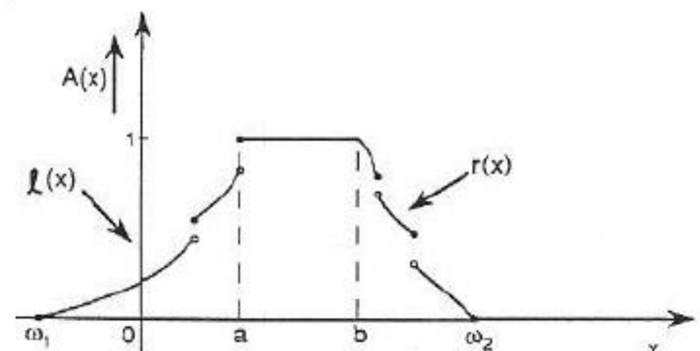


Figure 4.3 General fuzzy number A expressed in the form (4.1).

اعداد فازی . . .

- مثال . . .

$$A(x) = \begin{cases} 1 & \text{for } x \in [a, b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty), \end{cases}$$

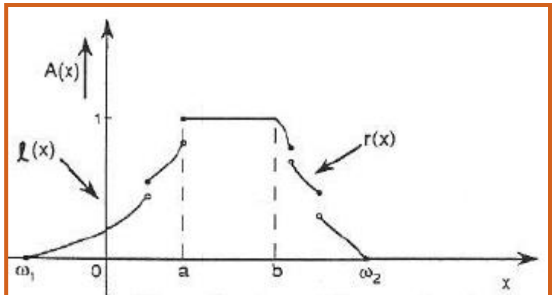
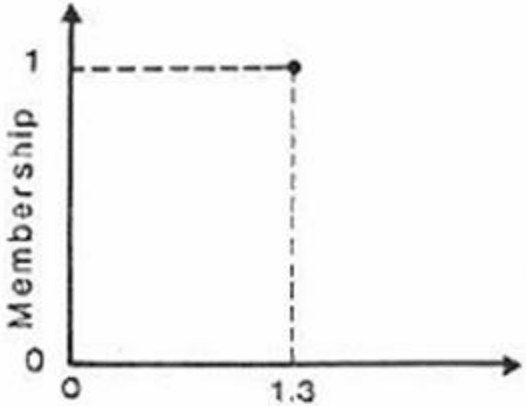


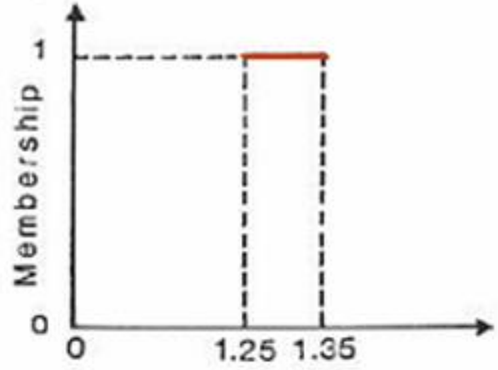
Figure 4.3 General fuzzy number A expressed in the form (4.1).

(a) $\omega_1 = a = b = \omega_2 = 1.3$, $l(x) = 0$ for all $x \in (-\infty, 1.3)$, $r(x) = 0$ for all $x \in (1.3, \infty)$.

(b) $\omega_1 = a = 1.25$, $b = \omega_2 = 1.35$, $l(x) = 0$ for all $x \in (-\infty, 1.25)$, $r(x) = 0$ for all $x \in (1.35, \infty)$.



(a)



(b)

اعداد فازی . . .

- مثال

$$A(x) = \begin{cases} 1 & \text{for } x \in [a, b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty), \end{cases}$$

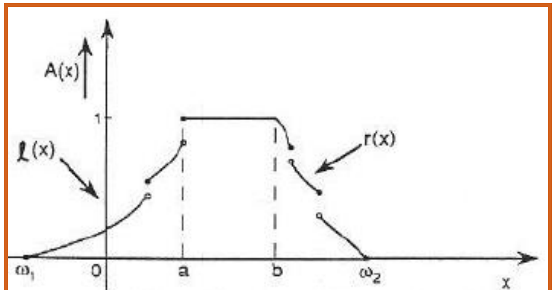
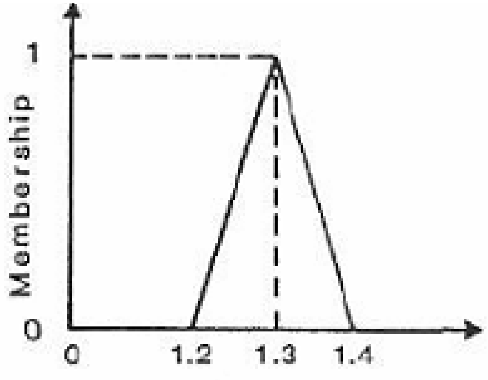


Figure 4.3 General fuzzy number A expressed in the form (4.1).

(c) $a = b = 1.3, \omega_1 = 1.2, \omega_2 = 1.4,$

$$l(x) = \begin{cases} 0 & \text{for } x \in (-\infty, 1.2) \\ 10(x-1.3)+1 & \text{for } x \in [1.2, 1.3), \end{cases}$$

$$r(x) = \begin{cases} 10(1.3-x)+1 & \text{for } x \in (1.3, 1.4] \\ 0 & \text{for } x \in (1.4, \infty). \end{cases}$$

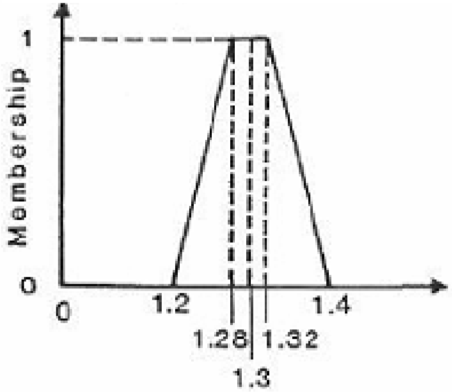


(c)

(d) $a = 1.28, b = 1.32, \omega_1 = 1.2, \omega_2 = 1.4,$

$$l(x) = \begin{cases} 0 & \text{for } x \in (-\infty, 1.2) \\ 12.5(x-1.28)+1 & \text{for } x \in [1.2, 1.28), \end{cases}$$

$$r(x) = \begin{cases} 12.5(1.32-x)+1 & \text{for } x \in (1.32, 1.4] \\ 0 & \text{for } x \in (1.4, \infty). \end{cases}$$



(d)

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اعداد فازی . . .

- متغیر زبانی "Very Large"

$$a = 90, b = 100, \omega_1 = 77.5, \omega_2 = 100,$$

$$l(x) = \begin{cases} 0 & \text{for } x \in (-\infty, 77.5) \\ 0.08(x-90)+1 & \text{for } x \in [77.5, 90), \end{cases}$$

$$r(x) = 0 \quad \text{for } x \in (100, \infty).$$

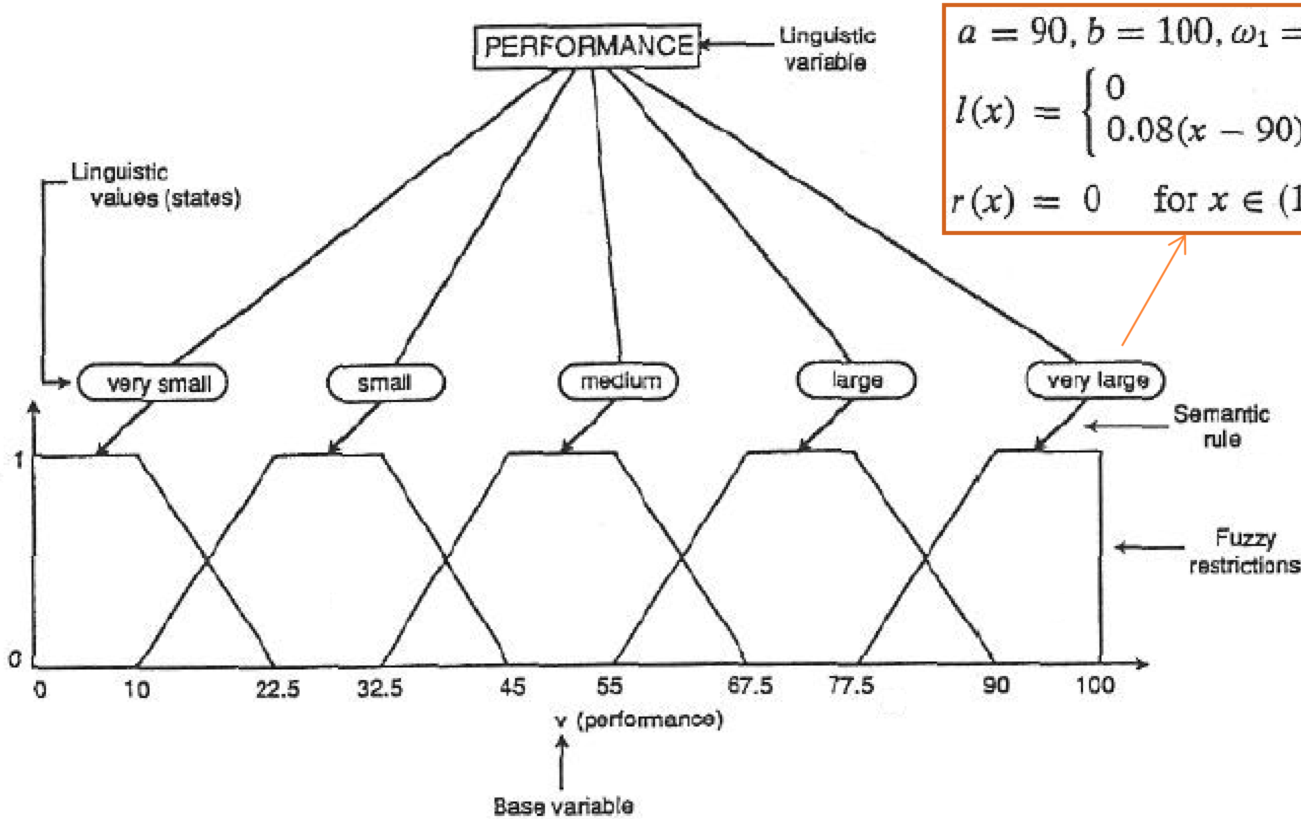
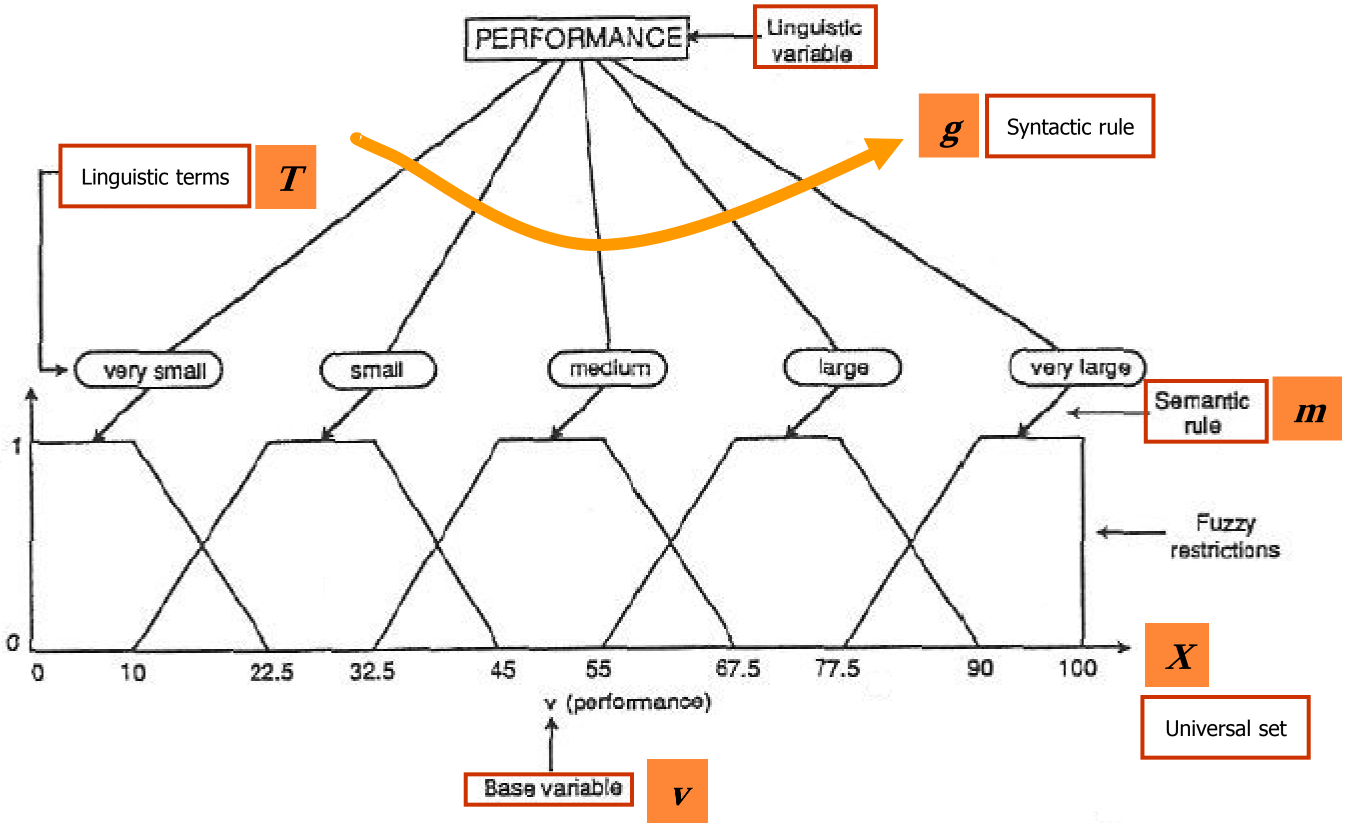


Figure 4.4 An example of a linguistic variable.

اعداد فازی . . .

- متغیر زبانی



(v, T, X, g, m)

v: the name of the variable (base variable)

T: the set of linguistic terms of v

X: universal set

g: syntactic rule for generating linguistic terms

m: semantic rule assigning to each linguistic term a meaning

اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- ویژگی‌های اعداد فازی
  - هر مجموعه فازی به صورت کامل و منحصر به فرد با برش‌های آلفای آن تعریف می‌شود
  - برش‌های آلفای هر عدد فازی، به ازای هر مقدار آلفا در بازه $[0,1]$، بازه‌های بسته‌ای از اعداد حقیقی هستند

- خصوصیات فوق ما را قادر می‌سازند تا
  - عملگرهای حسابی روی اعداد فازی با کمک عملگرهای روی برش‌های آلفا تعریف شود
  - اگر * بیانگر هرکدام از چهار عملگر جمع (+)، تفریق (−)، ضرب (.) و تقسیم (/) باشد که بر روی بازه‌های بسته تعریف شده است، آنگاه

$$[a,b]*[d,e]=\{f*g \mid a\le f\le b, d\le g\le e\}$$

- رابطه $[a,b]/[d,e]$ روی $0\in[d,e]$ تعریف نشده است

اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- عملگرها

$$[a,b]*[d,e]=\{f*g \mid a\le f\le b, d\le g\le e\}$$

The four arithmetic operations on closed intervals are defined as follows:

$$[a,b]+[d,e] = [a+d, b+e], \tag{4.3}$$

$$[a,b]-[d,e] = [a-e, b-d], \tag{4.4}$$

$$[a,b]\cdot[d,e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)], \tag{4.5}$$

and, provided that $0 \notin [d, e]$,

$$\begin{aligned}[a,b]/[d,e] &= [a,b]\cdot[1/e, 1/d] \\ &= [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)].\end{aligned} \tag{4.6}$$

The following are a few examples illustrating the interval-valued arithmetic operations defined by (4.3)–(4.6):

$$[2,5]+[1,3] = [3,8] \qquad [0,1]+[-6,5] = [-6,6],$$

$$[2,5]-[1,3] = [-1,4] \qquad [0,1]-[-6,5] = [-5,7],$$

$$[-1,1]\cdot[-2,-0.5] = [-2,2] \qquad [3,4]\cdot[2,2] = [6,8],$$

$$[-1,1]/[-2,-0.5] = [-2,2] \qquad [4,10]/[1,2] = [2,10].$$

اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- فرض کنید $A = [a_1, a_2], B = [b_1, b_2], C = [c_1, c_2], \mathbf{0} = [0, 0], \mathbf{1} = [1, 1].$

The four arithmetic operations on closed intervals are defined as follows:

$$[a, b] + [d, e] = [a + d, b + e], \quad (4.3)$$

$$[a, b] - [d, e] = [a - e, b - d], \quad (4.4)$$

$$[a, b] \cdot [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)], \quad (4.5)$$

and, provided that $0 \notin [d, e]$,

$$[a, b]/[d, e] = [a, b] \cdot [1/e, 1/d] = [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)]. \quad (4.6)$$

1. $A + B = B + A$, $A \cdot B = B \cdot A$ (*commutativity*).
2. $(A + B) + C = A + (B + C)$ $(A \cdot B) \cdot C = A \cdot (B \cdot C)$ (*associativity*).
3. $A = \mathbf{0} + A = A + \mathbf{0}$ $A = \mathbf{1} \cdot A = A \cdot \mathbf{1}$ (*identity*).
4. $A \cdot (B + C) \subseteq A \cdot B + A \cdot C$ (*subdistributivity*).
5. If $b \cdot c \geq 0$ for every $b \in B$ and $c \in C$, then $A \cdot (B + C) = A \cdot B + A \cdot C$ (*distributivity*). Furthermore, if $A = [a, a]$, then $a \cdot (B + C) = a \cdot B + a \cdot C$.
6. $0 \in A - A$ and $1 \in A/A$.
7. If $A \subseteq E$ and $B \subseteq F$, then:

$$A + B \subseteq E + F,$$
$$A - B \subseteq E - F,$$
$$A \cdot B \subseteq E \cdot F,$$
$$A/B \subseteq E/F \text{ (inclusion monotonicity)}.$$

let $A = [0, 1], B = [1, 2], C = [-2, -1]$

Then, $A \cdot B = [0, 2], A \cdot C = [-2, 0], B + C = [-1, 1]$, and

$$A \cdot (B + C) = [-1, 1] \subset [-2, 2] = A \cdot B + A \cdot C.$$

اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- دو روش برای محاسبات فازی
  - محاسبات بازه‌ای (Interval arithmetic)
    - بر اساس تعاریف بیان شده برای بازه‌ها
    - کمک گرفتن از برش آلفا در محاسبات

The four arithmetic operations on closed intervals are defined as follows:

$$[a, b] + [d, e] = [a + d, b + e], \tag{4.3}$$

$$[a, b] - [d, e] = [a - e, b - d], \tag{4.4}$$

$$[a, b] \cdot [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)], \tag{4.5}$$

and, provided that $0 \notin [d, e]$,

$$[a, b]/[d, e] = [a, b] \cdot [1/e, 1/d] = [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)]. \tag{4.6}$$

  - اصل توسعه (Extension principle)
    - توسعه محاسبات حالت کریسپ برای حالت فازی

اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- محاسبات بازه‌ای . . .

Let A and B denote fuzzy numbers and let $*$ denote any of the four basic arithmetic operations. Then, we define a fuzzy set on $\mathbb{R}$, $A * B$, by defining its α-cut, ${}^{\alpha}(A * B)$, as

$$\boxed{{}^{\alpha}(A * B) = {}^{\alpha}A * {}^{\alpha}B} \tag{4.7}$$

for any $\alpha \in (0, 1]$. (When $\boxed{* = /}$, clearly, we have to require that $\boxed{0 \notin {}^{\alpha}B}$ for all $\alpha \in (0, 1]$.) Due to Theorem 2.5, $A * B$ can be expressed as

$$A * B = \bigcup_{\alpha \in [0,1]} {}_{\alpha}(A * B). \tag{4.8}$$

Since ${}^{\alpha}(A * B)$ is a closed interval for each $\alpha \in (0, 1]$ and A, B are fuzzy numbers, $A * B$ is also a fuzzy number.

1. محاسبه برش آلفای دو عدد
2. محاسبه روی بازه برش آلفا
3. محاسبه عدد حاصل از روی برش آلفا

Theorem 2.5 (First Decomposition Theorem). For every $A \in \mathcal{F}(X)$,

$$A = \bigcup_{\alpha \in [0,1]} {}_{\alpha}A, \tag{2.2}$$

where ${}_{\alpha}A$ is defined by (2.1) and $\cup$ denotes the standard fuzzy union.

$${}_{\alpha}A(x) = \alpha \cdot {}^{\alpha}A(x).$$

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اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- محاسبات بازه‌ای: مثال . . .
  - دو عدد فازی مثلثی

$$A(x) = \begin{cases} 0 & \text{for } x \le -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 < x \le 1 \\ (3-x)/2 & \text{for } 1 < x \le 3, \end{cases}$$

$$B(x) = \begin{cases} 0 & \text{for } x \le 1 \text{ and } x > 5 \\ (x-1)/2 & \text{for } 1 < x \le 3 \\ (5-x)/2 & \text{for } 3 < x \le 5. \end{cases}$$

The four arithmetic operations on closed intervals are defined as follows:

$$[a,b] + [d,e] = [a+d, b+e], \quad (4.3)$$

$$[a,b] - [d,e] = [a-e, b-d], \quad (4.4)$$

$$[a,b] \cdot [d,e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)], \quad (4.5)$$

and, provided that $0 \notin [d,e]$,

$$[a,b]/[d,e] = [a,b] \cdot [1/e, 1/d] = [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)]. \quad (4.6)$$

Their α-cuts are:

$$^{\alpha}A = [2\alpha - 1, 3 - 2\alpha],$$

$$^{\alpha}B = [2\alpha + 1, 5 - 2\alpha].$$

Using (4.3)–(4.7), we obtain

$$^{\alpha}(A+B) = [4\alpha, 8-4\alpha] \quad \text{for } \alpha \in (0,1],$$

$$^{\alpha}(A-B) = [4\alpha - 6, 2 - 4\alpha] \quad \text{for } \alpha \in (0,1],$$

$$^{\alpha}(A \cdot B) = \begin{cases} [-4\alpha^2 + 12\alpha - 5, 4\alpha^2 - 16\alpha + 15] & \text{for } \alpha \in (0, .5] \\ [4\alpha^2 - 1, 4\alpha^2 - 16\alpha + 15] & \text{for } \alpha \in (.5, 1], \end{cases}$$

$$^{\alpha}(A/B) = \begin{cases} [(2\alpha - 1)/(2\alpha + 1), (3 - 2\alpha)/(2\alpha + 1)] & \text{for } \alpha \in (0, .5] \\ [(2\alpha - 1)/(5 - 2\alpha), (3 - 2\alpha)/(2\alpha + 1)] & \text{for } \alpha \in (.5, 1] \end{cases}$$

اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- محاسبات بازه‌ای: مثال . . .

The resulting fuzzy numbers are then:

$$(A+B)(x) = \begin{cases} 0 & \text{for } x \le 0 \quad \text{and } x > 8 \\ x/4 & \text{for } 0 < x \le 4 \\ (8-x)/4 & \text{for } 4 < x \le 8, \end{cases}$$

$$(A-B)(x) = \begin{cases} 0 & \text{for } x \le -6 \quad \text{and } x > 2 \\ (x+6)/4 & \text{for } -6 < x \le -2 \\ (2-x)/4 & \text{for } -2 < x \le 2, \end{cases}$$

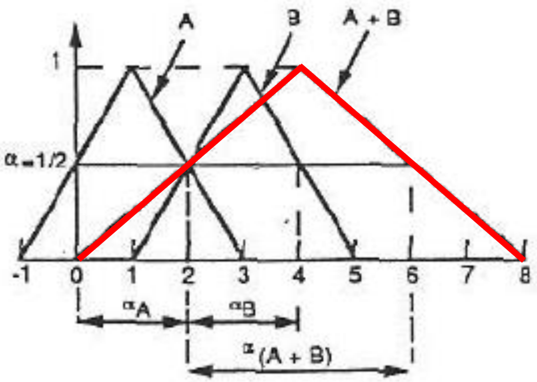
$${}^{\alpha}(A+B) = [4\alpha, 8-4\alpha]$$

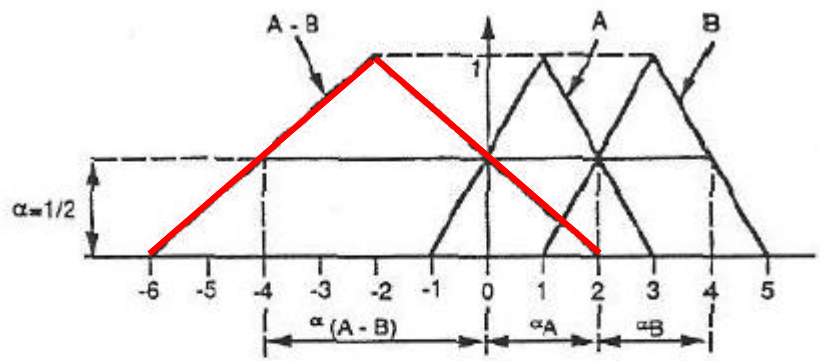
$$\text{Let } 4\alpha = x \Rightarrow f(x) = x/4$$

$$\alpha = 0 \Rightarrow x = 0; \quad \alpha = 1 \Rightarrow x = 4$$

$$\text{Let } 8-4\alpha = x$$

$$f(x) = \frac{(8-x)}{4}$$





$${}^{\alpha}(A+B) = [4\alpha, 8-4\alpha] \quad \text{for } \alpha \in (0,1],$$

$${}^{\alpha}(A-B) = [4\alpha-6, 2-4\alpha] \quad \text{for } \alpha \in (0,1],$$

اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- محاسبات بازه‌ای: مثال

The resulting fuzzy numbers are then:

$$(A/B)(x) = \begin{cases} 0 & \text{for } x < -1 \quad \text{and } x \geq 3 \\ (x+1)/(2-2x) & \text{for } -1 \leq x < 0 \\ (5x+1)/(2x+2) & \text{for } 0 \leq x < 1/3 \\ (3-x)/(2x+2) & \text{for } 1/3 \leq x < 3. \end{cases}$$

$$\boxed{{}^{\alpha}(A/B) = \begin{cases} [(2\alpha-1)/(2\alpha+1), (3-2\alpha)/(2\alpha+1)] & \text{for } \alpha \in (0, .5] \\ [(2\alpha-1)/(5-2\alpha), (3-2\alpha)/(2\alpha+1)] & \text{for } \alpha \in (.5, 1] \end{cases}}$$

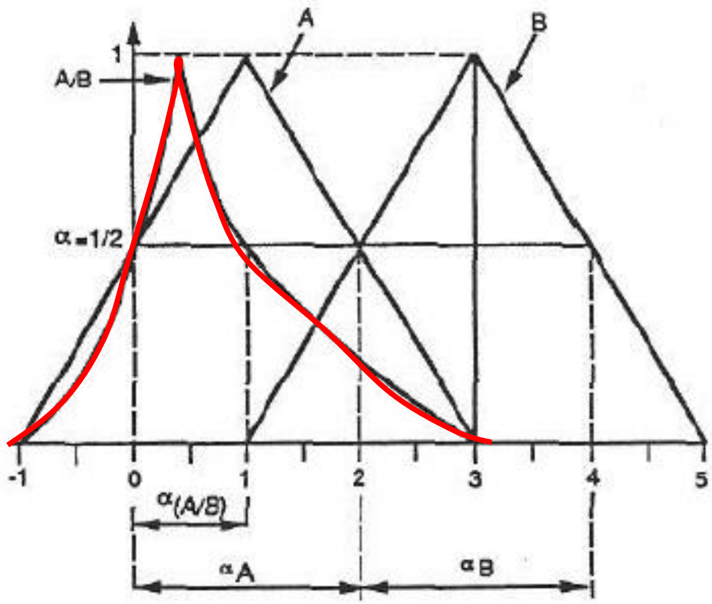
Let $x = \dfrac{2\alpha-1}{2\alpha+1} \Rightarrow \alpha = \dfrac{1+x}{2-2x}$

$\because \alpha \in (0,0.5]$

$\alpha = 0 \Rightarrow \alpha = \dfrac{1+x}{2-2x} = 0 \Rightarrow x = -1$

$\alpha = 0.5 \Rightarrow \alpha = \dfrac{1+x}{2-2x} = 0.5 \Rightarrow x = 0$

$\therefore -1 \leq x < 0$



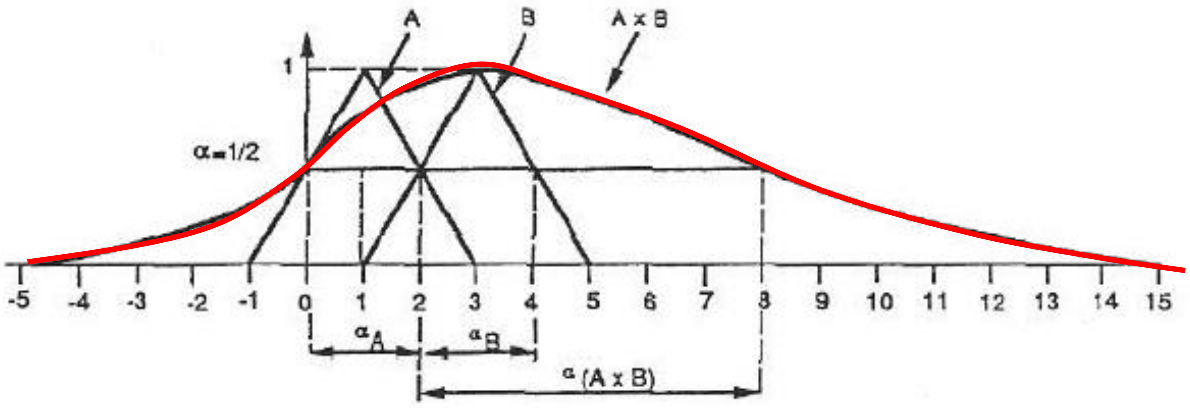
اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- محاسبات بازه‌ای: مثال . . .

The resulting fuzzy numbers are then:

$$(A \cdot B)(x) = \begin{cases} 0 & \text{for } x < -5 \quad \text{and } x \geq 15 \\ \left[3 - (4-x)^{1/2}\right]/2 & \text{for } -5 \leq x < 0 \\ (1+x)^{1/2}/2 & \text{for } 0 \leq x < 3 \\ \left[4 - (1+x)^{1/2}\right]/2 & \text{for } 3 \leq x < 15, \end{cases}$$

$$\boxed{{}^{\alpha}(A \cdot B) = \begin{cases} [-4\alpha^2 + 12\alpha - 5,\ 4\alpha^2 - 16\alpha + 15] & \text{for } \alpha \in (0, .5] \\ [4\alpha^2 - 1,\ 4\alpha^2 - 16\alpha + 15] & \text{for } \alpha \in (.5, 1], \end{cases}}$$



اعداد فازی: عملگرهای حسابی روی بازه‌ها . . .

- اصل توسعه

Let $*$ denote any of the four basic arithmetic operations and let A, B denote fuzzy numbers. Then, we define a fuzzy set on $\mathbb{R}$, $A * B$, by the equation

$$(A * B)(z) = \sup_{z=x*y} \min[A(x), B(y)] \tag{4.9}$$

for all $z \in \mathbb{R}$. More specifically, we define for all $z \in \mathbb{R}$:

$$(A + B)(z) = \sup_{z=x+y} \min[A(x), B(y)], \tag{4.10}$$

مثلاً اگر z=4، آنگاه (x,y) می‌تواند (0,4)، (1,3)، (2,2)، ... باشد

$$(A - B)(z) = \sup_{z=x-y} \min[A(x), B(y)], \tag{4.11}$$

$$(A \cdot B)(z) = \sup_{z=x\cdot y} \min[A(x), B(y)], \tag{4.12}$$

$$(A / B)(z) = \sup_{z=x/y} \min[A(x), B(y)]. \tag{4.13}$$

Theorem 4.2. Let $* \in \{+, -, \cdot, /\}$, and let A, B denote continuous fuzzy numbers. Then, the fuzzy set $A * B$ defined by (4.9) is a continuous fuzzy number.

$$(A * B)(z) = \sup_{z=x*y} \min[A(x), B(y)] \tag{4.9}$$

مبانی رایانش نرم

فازی: روابط

هادی ویسی

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دانشگاه تهران – دانشکده علوم و فنون نوین

روابط کلاسیک (کریسپ) . . .

- بیانگر وجود/عدم‌وجود نگاشت، تعامل یا پیوند بین عناصر دو/چند مجموعه

A *relation* among crisp sets $X_1, X_2, \ldots, X_n$ is a subset of the Cartesian product $\underset{i \in \mathbb{N}_n}{\times} X_i$. It is denoted either by $R(X_1, X_2, \ldots, X_n)$ or by the abbreviated form $R(X_i | i \in \mathbb{N}_n)$. Thus,

$$R(X_1, X_2, \ldots, X_n) \subseteq X_1 \times X_2 \times \ldots \times X_n,$$

so that for relations among sets $X_1, X_2, \ldots, X_n$, the Cartesian product $X_1 \times X_2 \times \ldots \times X_n$ represents the universal set.

- تابع مشخصه (**Characteristic function**)

Denoting a relation and its characteristic function by the same symbol R, we have

$$R(x_1, x_2, \ldots, x_n) = \begin{cases} 1 & \text{iff } \langle x_1, x_2, \ldots, x_n \rangle \in R, \\ 0 & \text{otherwise} \end{cases}$$

روابط کلاسیک (کریسپ) . . .

- نمایشی دیگر برای یک رابطه

A relation can be written as a set of ordered tuples. Another convenient way of representing a relation $R(X_1, X_2, \ldots, X_n)$ involves an *n*-dimensional membership array: $\mathbf{R} = [r_{i_1, i_2, \ldots, i_n}]$. Each element of the first dimension i_1 of this array corresponds to exactly one member of X_1 and each element of dimension i_2 to exactly one member of X_2, and so on. If the n-tuple $\langle x_1, x_2, \ldots, x_n \rangle \in X_1 \times X_2 \times \ldots \times X_n$ corresponds to the element $r_{i_1, i_2, \ldots, i_n}$ of $\mathbf{R}$, then

$$r_{i_1, i_2, \ldots, i_n} = \begin{cases} 1 & \text{if and only if } \langle x_1, x_2, \ldots, x_n \rangle \in R, \\ 0 & \text{otherwise.} \end{cases}$$

- مثال

Example 5.1

Let R be a relation among the three sets X = {English, French}, Y = {dollar, pound, franc, mark} and Z = {US, France, Canada, Britain, Germany}, which associates a country with a currency and language as follows:

$$R(X, Y, Z) = \{\langle \text{English, dollar, US} \rangle, \langle \text{French, Franc, France} \rangle, \langle \text{English, dollar, Canada} \rangle, \langle \text{French, dollar, Canada} \rangle, \langle \text{English, pound, Britain} \rangle\}.$$

This relation can also be represented with the following three-dimensional membership array:

English

	US	*Fra*	*Can*	*Brit*	*Ger*
dollar	1	0	1	0	0
pound	0	0	0	1	0
franc	0	0	0	0	0
mark	0	0	0	0	0

French

	US	*Fra*	*Can*	*Brit*	*Ger*
dollar	0	0	1	0	0
pound	0	0	0	0	0
franc	0	1	0	0	0
mark	0	0	0	0	0

خواص رابطه کریسپ . . .

- بازتابی-انعکاسی (Reflexivity)

  - انعکاسی: هر عنصر در رابطه با خودش مرتبط است

A crisp relation $R(X, X)$ is *reflexive* iff $\langle x, x\rangle \in R$ for each $x \in X$

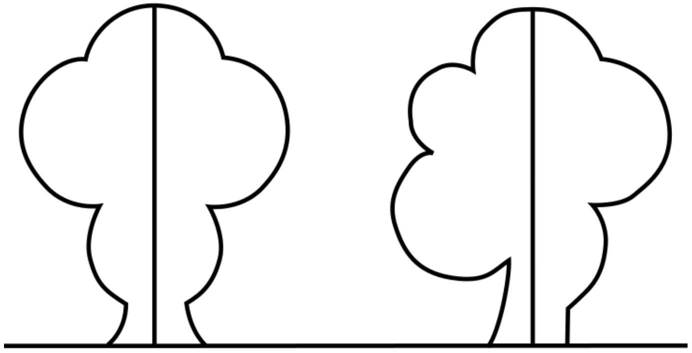
  - غیر انعکاسی (Irreflexive)

Otherwise, $R(X, X)$ is called *irreflexive*.

  - ضد انعکاسی (Antireflexive)

If $\langle x, x\rangle \notin R$ for every $x \in X$, the relation is called *antireflexive*.

خواص رابطه کریسپ . . .



تقارن (Symmetry)

- متقارن

A crisp relation $R(X, X)$ is *symmetric* iff for every $\langle x, y\rangle \in R$, it is also the case that $\langle y, x\rangle \in R$, where $x, y, \in X$.

- نامتقارن (Asymmetric)

If this is not the case for some x, y, then the relation is called *asymmetric*.

- پادمتقارن (Antisymmetric)

If both $\langle x, y\rangle \in R$ and $\langle y, x\rangle \in R$ implies $x = y$, then the relation is called *antisymmetric*.

هیچ جفت <x,y> و <y,x>ی وجود ندارد که x≠y باشد

- اکیداً پادمتقارن (Strictly antisymmetric)

If either $\langle x, y\rangle \in R$ or $\langle y, x\rangle \in R$, whenever $x \neq y$, then the relation is called *strictly antisymmetric*.

اگر یکی از جفت‌های <x,y> یا <y,x> وجود دارد که x≠y باشد

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خواص رابطه کریسپ . . .

انتقالی-ترایا (Transitivity)

- انتقالی

A crisp relation $R(X, X)$ is called *transitive* iff $\langle x, z\rangle \in R$ whenever both $\langle x, y\rangle \in R$ and $\langle y, z\rangle \in R$ for at least one $y \in X$.

- غیرانتقالی (Nontransitive)

A relation that does not satisfy this property is called *nontransitive*.

- پادانتقالی (Antitransitive)

If $\langle x, z\rangle \notin R$ whenever both $\langle x, y\rangle \in R$ and $\langle y, z\rangle \in R$, then the relation is called *antitransitive*.

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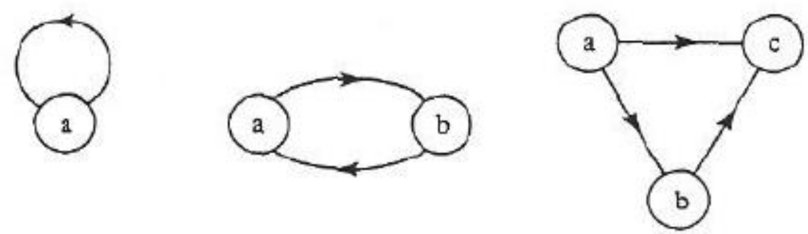
خواص رابطه کریسپ

- مثال



Reflexivity Symmetry Transitivity

Example 5.6

Let R be a crisp relation defined on $X \times X$, where X is the set of all university courses and R represents the relation "is a prerequisite of." R is antireflexive because a course is never a prerequisite of itself. Further, if one course is a prerequisite of another, the reverse will never be true. Therefore, R is antisymmetric. Finally, if a course is a prerequisite for a second course that is itself a prerequisite for a third, then the first course is also a prerequisite for the third course. Thus, the relation R is transitive.

روابط فازی . . .

- تعریف میزان عضویت برای ارتباط‌ها در رابطه کلاسیک
  - تعریف روی ضرب مجموعه‌های کلاسیک $X_1, X_2, \ldots, X_n$
  - تعریف عضویت برای چندتایی $\langle x_1, x_2, \ldots, x_n \rangle$

- مثال

Example 5.2

Let R be a fuzzy relation between the two sets X = {New York City, Paris} and Y = {Beijing, New York City, London}, which represents the relational concept "very far." This relation can be written in list notation as

$$R(X, Y) = 1/\text{NYC, Beijing} + 0/\text{NYC, NYC} + .6/\text{NYC, London} + .9/\text{Paris, Beijing} + .7/\text{Paris, NYC} + .3/\text{Paris, London}.$$

This relation can also be represented by the following two-dimensional membership array (matrix):

	NYC	*Paris*
Beijing	1	.9
NYC	0	.7
London	.6	.3

روابط فازی: تصویر (Projection) . . .

زیردنباله(Subsequence)

- y را زیردنباله x گویند ($\mathbf{y} \prec \mathbf{x}$)

Consider the Cartesian product of all sets in the family $\mathcal{X} = \{X_i | i \in \mathbb{N}_n\}$. For each sequence ($n$-tuple)

$$\mathbf{x} = \langle x_i | i \in \mathbb{N}_n \rangle \in \bigtimes_{i \in \mathbb{N}_n} X_i$$

and each sequence (r-tuple, $r \leq n$)

$$\mathbf{y} = \langle y_j | j \in J \rangle \in \bigtimes_{j \in J} X_j,$$

where $J \subseteq \mathbb{N}_n$ and $|J| = r$, let $\mathbf{y}$ be called a *subsequence* of $\mathbf{x}$ iff $y_j = x_j$ for all $j \in J$.

حالت خاص: رشته

x=<This is a string>

y= <s a str>

تصویر (Projection)

Given a relation $R(X_1, X_2, \ldots, X_n)$, let $[R \downarrow \mathcal{Y}]$ denote the *projection* of R on $\mathcal{Y}$ that disregards all sets in X except those in the family

$$\mathcal{Y} = \{X_j | j \in J \subseteq \mathbb{N}_n\}.$$

Then, $[R \downarrow \mathcal{Y}]$ is a fuzzy relation whose membership function is defined on the Cartesian product of sets in $\mathcal{Y}$ by the equation

$$\boxed{[R \downarrow \mathcal{Y}](\mathbf{y}) = \max_{\mathbf{x} \succ \mathbf{y}} R(\mathbf{x}).} \qquad (5.1)$$

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روابط فازی: تصویر (Projection) . . .

- مثال . . .

Example 5.3

Consider the sets $X_1 = \{0, 1\}, X_2 = \{0, 1\}, X_3 = \{0, 1, 2\}$ and the ternary fuzzy relation on $X_1 \times X_2 \times X_3$ defined in Table 5.1. Let $R_{ij} = [R \downarrow \{X_i, X_j\}]$ and $R_i = [R \downarrow \{X_i\}]$ for all $i, j \in \{1, 2, 3\}$. Using this notation, all possible projections of R are given in Table 5.1. A detailed calculation of one of these projections, R_{12}, is shown in Table 5.2.

حذف x_3

TABLE 5.1 TERNARY FUZZY RELATION AND ITS PROJECTIONS

$\langle x_1,$	$x_2,$	$x_3\rangle$	$R(x_1, x_2, x_3)$	$R_{12}(x_1, x_2)$	$R_{13}(x_1, x_3)$	$R_{23}(x_2, x_3)$	$R_1(x_1)$	$R_2(x_2)$	$R_3(x_3)$
0	0	0	0.4	0.9	1.0	0.5	1.0	0.9	1.0
0	0	1	0.9	0.9	0.9	0.9	1.0	0.9	0.9
0	0	2	0.2	0.9	0.8	0.2	1.0	0.9	1.0
0	1	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0	1	1	0.0	1.0	0.9	0.5	1.0	1.0	0.9
0	1	2	0.8	1.0	0.8	1.0	1.0	1.0	1.0
1	0	0	0.5	0.5	0.5	0.5	1.0	0.9	1.0
1	0	1	0.3	0.5	0.5	0.9	1.0	0.9	0.9
1	0	2	0.1	0.5	1.0	0.2	1.0	0.9	1.0
1	1	0	0.0	1.0	0.5	1.0	1.0	1.0	1.0
1	1	1	0.5	1.0	0.5	0.5	1.0	1.0	0.9
1	1	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0

روابط فازی: تصویر (Projection)

- مثال

TABLE 5.1 TERNARY FUZZY RELATION AND ITS PROJECTIONS

$(x_1,$	$x_2,$	$x_3)$	$R(x_1, x_2, x_3)$	$R_{12}(x_1, x_2)$	$R_{13}(x_1, x_3)$	$R_{23}(x_2, x_3)$	$R_1(x_1)$	$R_2(x_2)$	$R_3(x_3)$
0	0	0	0.4	0.9	1.0	0.5	1.0	0.9	1.0
0	0	1	0.9	0.9	0.9	0.9	1.0	0.9	0.9
0	0	2	0.2	0.9	0.8	0.2	1.0	0.9	1.0
0	1	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0	1	1	0.0	1.0	0.9	0.5	1.0	1.0	0.9
0	1	2	0.8	1.0	0.8	1.0	1.0	1.0	1.0
1	0	0	0.5	0.5	0.5	0.5	1.0	0.9	1.0
1	0	1	0.3	0.5	0.5	0.9	1.0	0.9	0.9
1	0	2	0.1	0.5	1.0	0.2	1.0	0.9	1.0
1	1	0	0.0	1.0	0.5	1.0	1.0	1.0	1.0
1	1	1	0.5	1.0	0.5	0.5	1.0	1.0	0.9
1	1	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0

حذف x_3

TABLE 5.2 CALCULATION OF THE PROJECTION R_{12} IN EXAMPLE 5.3

$(x_1,$	$x_2,$	$x_3)$	$R(x_1, x_2, x_3)$	$R_{12}(x_1, x_2)$
0	0	0	0.4	
0	0	1	0.9	$\max[R(0,0,0), R(0,0,1), R(0,0,2)] = 0.9$
0	0	2	0.2	
0	1	0	1.0	
0	1	1	0.0	$\max[R(0,1,0), R(0,1,1), R(0,1,2)] = 1.0$
0	1	2	0.8	
1	0	0	0.5	
1	0	1	0.3	$\max[R(1,0,0), R(1,0,1), R(1,0,2)] = 0.5$
1	0	2	0.1	
1	1	0	0.0	
1	1	1	0.5	$\max[R(1,1,0), R(1,1,1), R(1,1,2)] = 1.0$
1	1	2	1.0	

روابط فازی: گسترش (Extension) . . .

- گسترش استوانه‌ای (Cylindric extension)
  - معکوس عمل تصویر (projection)

Let $\mathcal{X}$ and $\mathcal{Y}$ denote the same families of sets as employed in the definition of projection. Let R be a relation defined on the Cartesian product of sets in the family $\mathcal{Y}$, and let $[R \uparrow \mathcal{X} - \mathcal{Y}]$ denote the cylindric extension of R into sets $X_i (i \in \mathbb{N}_n)$ that are in $\mathcal{X}$ but are not in $\mathcal{Y}$. Then,

$$[R \uparrow \mathcal{X} - \mathcal{Y}](\mathbf{x}) = R(\mathbf{y}) \tag{5.2}$$

for each $\mathbf{x}$ such that $\mathbf{x} \succ \mathbf{y}$.

روابط فازی: گسترش (Extension)

- مثال

Example 5.4

Membership functions of cylindric extensions of all the projections in Example 5.3 are actually those shown in Table 5.1 under the assumption that their arguments are extended to $\langle x_1, x_2, x_3 \rangle$. For instance:

$$[R_{23} \uparrow \{X_1\}](0,0,2) = [R_{23} \uparrow \{X_1\}](1,0,2) = R_{23}(0,2) = 0.2.$$

We can see that none of the cylindric extensions (identical with the respective projections in Table 5.1) are equal to the original fuzzy relation from which the projections involved in the cylindric extensions were determined. This means that some information was lost when the given relation was replaced by any one of its projections in this example.

TABLE 5.1 TERNARY FUZZY RELATION AND ITS PROJECTIONS

$\langle x_1,$	$x_2,$	$x_3\rangle$	$R(x_1,x_2,x_3)$	$R_{12}(x_1,x_2)$	$R_{13}(x_1,x_3)$	$R_{23}(x_2,x_3)$	$R_1(x_1)$	$R_2(x_2)$	$R_3(x_3)$
0	0	0	0.4	0.9	1.0	0.5	1.0	0.9	1.0
0	0	1	0.9	0.9	0.9	0.9	1.0	0.9	0.9
0	0	2	0.2	0.9	0.8	0.2	1.0	0.9	1.0
0	1	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0	1	1	0.0	1.0	0.9	0.5	1.0	1.0	0.9
0	1	2	0.8	1.0	0.8	1.0	1.0	1.0	1.0
1	0	0	0.5	0.5	0.5	0.5	1.0	0.9	1.0
1	0	1	0.3	0.5	0.5	0.9	1.0	0.9	0.9
1	0	2	0.1	0.5	1.0	0.2	1.0	0.9	1.0
1	1	0	0.0	1.0	0.5	1.0	1.0	1.0	1.0
1	1	1	0.5	1.0	0.5	0.5	1.0	1.0	0.9
1	1	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0

روابط فازی: بستار (closure) . . .

- بستار استوانه‌ای (Cylindric closure)
  - بدست آوردن یک رابطه از روی چند تصویر آن رابطه با کمک اشتراک گرفتن از گسترش‌های آن

Hence, given a set of projections $\{P_i | i \in I\}$ of a relation on $\mathcal{X}$, the cylindric closure, cyl $\{P_i\}$, based on these projections is defined by the equation

$$\text{cyl}\{P_i\}(\mathbf{x}) = \min_{i \in I}[P_i \uparrow \mathcal{X} - \mathcal{Y}_i](\mathbf{x})$$

for each $\mathbf{x} \in \mathcal{X}$ where $\mathcal{Y}_i$ denotes the family of sets on which P_i is defined.

- رابطه اولیه را به طور کامل بازیابی نمی‌کند
  - از دست دادن اطلاعات با تصویر کردن

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روابط فازی: بستار (closure)

- مثال

TABLE 5.3 CYLINDRIC CLOSURES OF THREE FAMILIES OF PROJECTIONS CALCULATED IN EXAMPLE 5.3

$\langle x_1,$	$x_2,$	$x_3\rangle$	$\text{cyl}\{R_{12}, R_{13}, R_{23}\}$	$\text{cyl}\{R_1, R_2, R_3\}$	$\text{cyl}\{R_{12}, R_3\}$
0	0	0	0.5	0.9	0.9
0	0	1	0.9	0.9	0.9
0	0	2	0.2	0.9	0.9
0	1	0	1.0	1.0	1.0
0	1	1	0.5	0.9	0.9
0	1	2	0.8	1.0	1.0
1	0	0	0.5	0.9	0.5
1	0	1	0.5	0.9	0.5
1	0	2	0.2	0.9	0.5
1	1	0	0.5	1.0	1.0
1	1	1	0.5	0.9	0.9
1	1	2	1.0	1.0	1.0

$$\boxed{\text{cyl}\{P_i\}(\mathbf{x}) = \min_{i \in I}[P_i \uparrow \mathcal{X} - \mathcal{Y}_i](\mathbf{x})}$$

TABLE 5.1 TERNARY FUZZY RELATION AND ITS PROJECTIONS

$\langle x_1,$	$x_2,$	$x_3\rangle$	$R(x_1, x_2, x_3)$	$R_{12}(x_1, x_2)$	$R_{13}(x_1, x_3)$	$R_{23}(x_2, x_3)$	$R_1(x_1)$	$R_2(x_2)$	$R_3(x_3)$
0	0	0	0.4	0.9	1.0	0.5	1.0	0.9	1.0
0	0	1	0.9	0.9	0.9	0.9	1.0	0.9	0.9
0	0	2	0.2	0.9	0.8	0.2	1.0	0.9	1.0
0	1	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0	1	1	0.0	1.0	0.9	0.5	1.0	1.0	0.9
0	1	2	0.8	1.0	0.8	1.0	1.0	1.0	1.0
1	0	0	0.5	0.5	0.5	0.5	1.0	0.9	1.0
1	0	1	0.3	0.5	0.5	0.9	1.0	0.9	0.9
1	0	2	0.1	0.5	1.0	0.2	1.0	0.9	1.0
1	1	0	0.0	1.0	0.5	1.0	1.0	1.0	1.0
1	1	1	0.5	1.0	0.5	0.5	1.0	1.0	0.9
1	1	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0

رابطه فازی: دودویی . . .

- رابطه دودویی: رابطه بین دو مجموعه

Given a fuzzy relation $R(X, Y)$, its *domain* is a fuzzy set on X, dom R, whose membership function is defined by

$$\operatorname{dom} R(x) = \max_{y \in Y} R(x, y) \quad \text{for each } x \in X. \tag{5.3}$$

The *range* of $R(X, Y)$ is a fuzzy relation on Y, ran R, whose membership function is defined by

$$\operatorname{ran} R(y) = \max_{x \in X} R(x, y) \quad \text{for each } y \in Y. \tag{5.4}$$

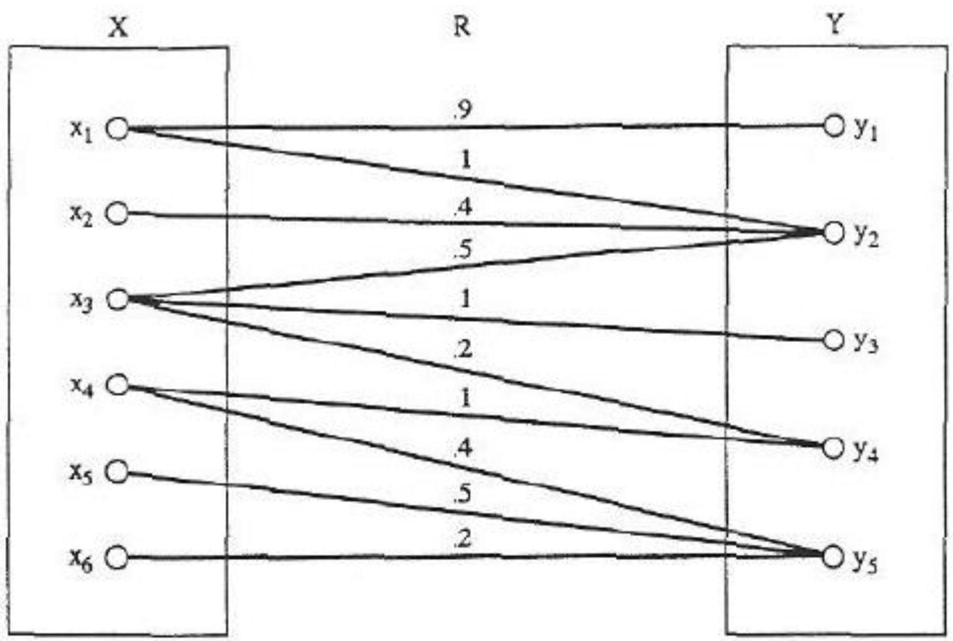
The *height* of a fuzzy relation $R(X, Y)$ is a number, $h(R)$, defined by

$$h(R) = \max_{y \in Y} \max_{x \in X} R(x, y). \tag{5.5}$$

That is, $h(R)$ is the largest membership grade attained by any pair $\langle x, y \rangle$ in R.

رابطه فازی: دودویی . . .

- نمایش . . .
  - ماتریس عضویت
  - نموداری (Sagittal diagram)



(a)

$$
R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array}
\begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\
\begin{bmatrix} .9 & 1 & 0 & 0 & 0 \\ 0 & .4 & 0 & 0 & 0 \\ 0 & .5 & 1 & .2 & 0 \\ 0 & 0 & 0 & 1 & .4 \\ 0 & 0 & 0 & 0 & .5 \\ 0 & 0 & 0 & 0 & .2 \end{bmatrix} \end{array}
$$

(b)

Figure 5.2 Examples of two convenient representations of a fuzzy binary relation: (a) sagittal diagram; (b) membership matrix.

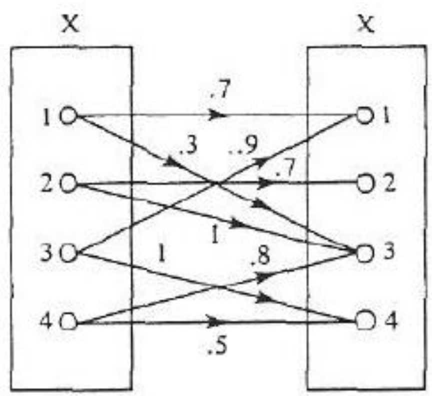
رابطه فازی: دودویی . . .

- نمایش

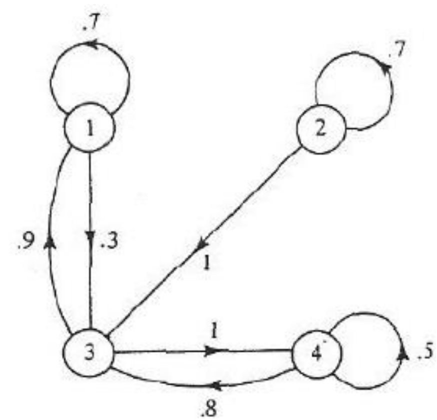
X

X	1	2	3	4
1	.7	0	.3	0
2	0	.7	1	0
3	.9	0	0	1
4	0	0	.8	.5

Membership matrix



Sagittal diagram



Simple diagram

x	y	R(x, y)
1	1	.7
1	3	.3
2	2	.7
2	3	1
3	1	.9
3	4	1
4	3	.8
4	4	.5

Table

رابطه فازی: دودویی . . .

- معکوس رابطه دودویی

$R^{-1}(y,x) = R(x,y)$ for all $x \in X$ and all $y \in Y$.

➡ $\mathbf{R}^{-1} = [r_{yx}^{-1}]$

➡ $(\mathbf{R}^{-1})^{-1} = \mathbf{R}$

- مثال

$$R = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 1 \\ 0.6 & 0.4 \end{bmatrix} \Rightarrow R^{-1} = R^{T} = \begin{bmatrix} 0.3 & 0 & 0.6 \\ 0.2 & 1 & 0.4 \end{bmatrix}$$

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رابطه فازی: دودویی . . .

- ترکیب استاندارد (max-min composition) روابط

$P(X, Y)$ and $Q(Y, Z)$

$$R(x, z) = P(X, Y) \circ Q(Y, Z)$$

$$\boxed{R(x, z) = [P \circ Q](x, z) = \max_{y \in Y} \min[P(x, y), Q(y, z)]} \qquad (5.7)$$

for all $x \in X$ and all $z \in Z$.

- ترکیب استاندارد دارای خاصیت انجمنی (associative) است و معکوس آن برابر است با ترکیب برعکس روابط معکوس

$$[P(X, Y) \circ Q(Y, Z)]^{-1} = Q^{-1}(Z, Y) \circ P^{-1}(Y, X),$$

$$[P(X, Y) \circ Q(Y, Z)] \circ R(Z, W) = P(X, Y) \circ [Q(Y, Z) \circ R(Z, W)].$$

- ترکیب استاندارد جابجایی‌پذیر (commutative) نیست

Even if $X = Z$ $\Rightarrow$ $P(X, Y) \circ Q(Y, Z) \neq Q(Y, Z) \circ P(X, Y)$.

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رابطه فازی: دودویی . . .

- ترکیب: نمایش ماتریسی

Let $\mathbf{P} = [p_{ik}]$, $\mathbf{Q} = [q_{kj}]$, and $\mathbf{R} = [r_{ij}]$ be membership matrices of binary relations such that $R = P \circ Q$.

$$[r_{ij}] = [p_{ik}] \circ [q_{kj}],$$
$$\text{where} \quad r_{ij} = \max_k \min(p_{ik}, q_{kj}). \tag{5.8}$$

- مثال
  - مشابه ضرب ماتریسی!
    - با عملگرهای فازی

$$\begin{bmatrix} \boxed{.3 \quad .5 \quad .8} \\ 0 \quad .7 \quad 1 \\ \boxed{.4 \quad .6 \quad .5} \end{bmatrix} \circ \begin{bmatrix} .9 & .5 & .7 & .7 \\ .3 & .2 & 0 & .9 \\ 1 & 0 & .5 & .5 \end{bmatrix} = \begin{bmatrix} \boxed{.8} & .3 & .5 & .5 \\ 1 & .2 & .5 & .7 \\ .5 & \boxed{.4} & .5 & .6 \end{bmatrix}.$$

$$\begin{aligned} \boxed{.8}(= r_{11}) &= \max[\min(.3, .9), \min(.5, .3), \min(.8, 1)] \\ &= \max[\min(p_{11}, q_{11}), \min(p_{12}, q_{21}), \min(p_{13}, q_{31})], \\ \boxed{.4}(= r_{32}) &= \max[\min(.4, .5), \min(.6, .2), \min(.5, 0)] \\ &= \max[\min(p_{31}, q_{12}), \min(p_{32}, q_{22}), \min(p_{33}, q_{32})]. \end{aligned}$$

رابطه فازی: دودویی . . .

- اتصال (join) رابطه‌ای $P * Q$ برای دو رابطه فازی $P(X,Y)$ and $Q(Y,Z)$

$$R(x,y,z) = [P * Q](x,y,z) = \min[P(x,y), Q(y,z)] \qquad (5.9)$$

for each $x \in X, y \in Y$, and $z \in Z$.

- ترکیب استاندارد را می‌توان بر حسب اتصال فوق نوشت

$$[P \circ Q](x,z) = \max_{y \in Y} [P * Q](x,y,z) \qquad (5.10)$$

for each $x \in X$ and $z \in Z$.

$$\boxed{R(x,z) = [P \circ Q](x,z) = \max_{y \in Y} \min[P(x,y), Q(y,z)] \qquad (5.7)}$$

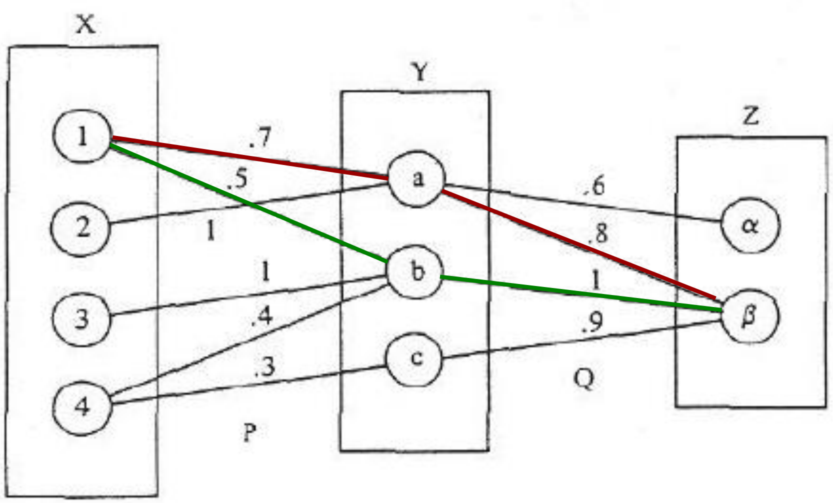
رابطه فازی: دودویی . . .

- مثال: ترکیب و اتصال

Example 5.5

The join $S = P * Q$ of relations P and Q given in Fig. 5.3a has the membership function given in Fig. 5.3b. To convert this join into the corresponding composition $R = P \circ Q$ by (5.10), the two indicated pairs of values of $S(x, y, z)$ in Fig. 5.3b are aggregated by the max operator. For instance,

$$R(1, \beta) = \max[S(1, a, \beta), S(1, b, \beta)]$$
$$= \max[.7, .5] = .7.$$



(a)

Join: S = P * Q

x	y	z	$\mu_S(x, y, z)$
1	a	α	.6
1	a	β	.7
1	b	β	.5
2	a	α	.6
2	a	β	.8
3	b	β	1
4	b	β	.4
4	c	β	.3

(b)

Composition: R = P ∘ Q

x	z	$\mu_R(x, z)$
1	α	.6
1	β	.7
2	α	.6
2	β	.8
3	β	1
4	β	.4

(c)

رابطه فازی: خواص رابطه دودویی . . .

- انعکاس

$R(X, X)$ is *reflexive* iff $R(x, x) = 1$ for all $x \in X$.
If this is not the case for some $x \in X$, the relation is called *irreflexive*;
if it is not satisfied for all $x \in X$, the relation is called *antireflexive*.

A weaker form of reflexivity, referred to as *ε-reflexivity*, $R(x, x) \geq \varepsilon$, where $0 < \varepsilon < 1$.

- تقارن

A fuzzy relation is *symmetric* iff $R(x, y) = R(y, x)$ for all $x, y \in X$.
Whenever this equality is not satisfied for some $x, y \in X$, the relation is called *asymmetric*.
Furthermore, when $R(x, y) > 0$ and $R(y, x) > 0$ implies that $x = y$ for all $x, y \in X$, the relation R is called *antisymmetric*.

- انتقال

A fuzzy relation $R(X, X)$ is *transitive* (or, more specifically, *max-min transitive*) if

$$R(x, z) \geq \max_{y \in Y} \min[R(x, y), R(y, z)] \tag{5.11}$$

is satisfied for each pair $\langle x, z \rangle \in X^2$.

A relation failing to satisfy this inequality for some members of X is called *nontransitive*, and if

$$R(x, z) < \max_{y \in Y} \min[R(x, y), R(y, z)],$$

for all $\langle x, z \rangle \in X^2$, then the relation is called *antitransitive*.

رابطه فازی: خواص رابطه دودویی . . .

- مثال

Example 5.7

Let R be the fuzzy relation defined on the set of cities and representing the concept *very near*. We may assume that a city is certainly (i.e., to a degree of 1) very near to itself. The relation is therefore reflexive. Furthermore, if city A is very near to city B, then B is certainly very near to A to the same degree. Therefore, the relation is also symmetric. Finally, if city A is very near to city B to some degree, say .7, and city B is very near to city C to some degree, say .8, it is possible (although not necessary) that city A is very near to city C to a smaller degree, say 0.5. Therefore, the relation is nontransitive.

رابطه فازی: خواص رابطه دودویی (مباحث دیگر) . . .

- همارزی (equivalence)
  - رابطه‌ای که انعکاسی، متقارن و انتقالی باشد
- سازگاری (compatibility)
  - رابطه‌ای که انعکاسی و متقارن باشد
- ترتیبی (ordering)
  - رابطه‌ای که انعکاسی، پادمتقارن و انتقالی باشد

رابطه فازی: خواص رابطه دودویی

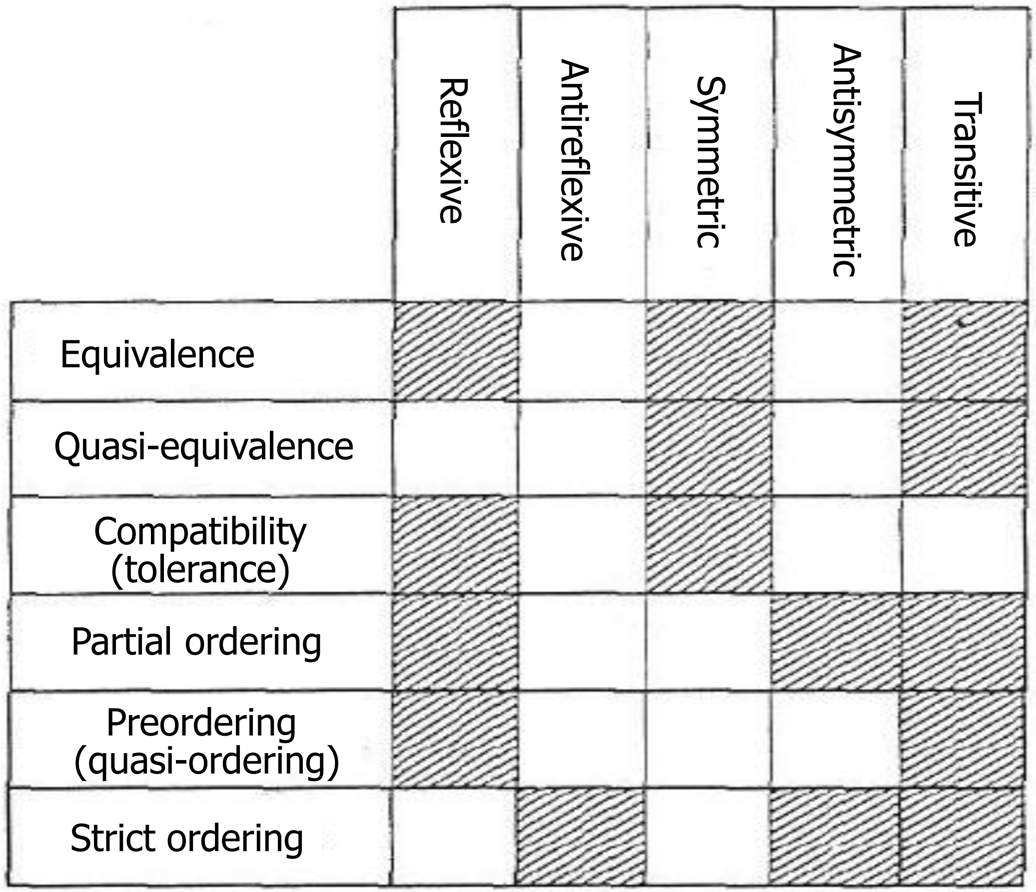


Figure 5.6 Some important types of binary relations *R* (*X*, *X*)