National University of Computer and Emerging Sciences
Lahore Campus

D & A of Algorithms (CS2009)

Sessional-I Exam

Date: Sep 22 2025

Course Instructor(s)

Dr. MB, Dr. SK, Dr. MB, Dr. MAQ, AA, UH, SK

Total Time (Hrs): 1
Total Marks: 15
Total Questions: 3

_______________ Roll No   _______________ Section   _______________ Student Signature

Instructions: Answer in the space provided. Do not attach rough sheets with this exam.

CLO 2: **Analyze** the time and space complexity of different algorithms by using standard asymptotic notations for recursive and non-recursive algorithms.

Question 1: [2+ 2 + 3 = 7 Marks]

(a) $f(n) = 30*2^n + 15*4^n + 3*16^n$

Which of the following statements are true about f(N)

i. $f(n) = O(4^{2n})$

ii. $f(n) = O(8^{n+2})$

iii. $f(n) = O(4^{n+4})$

iv. $f(n) = O(2^{4n})$

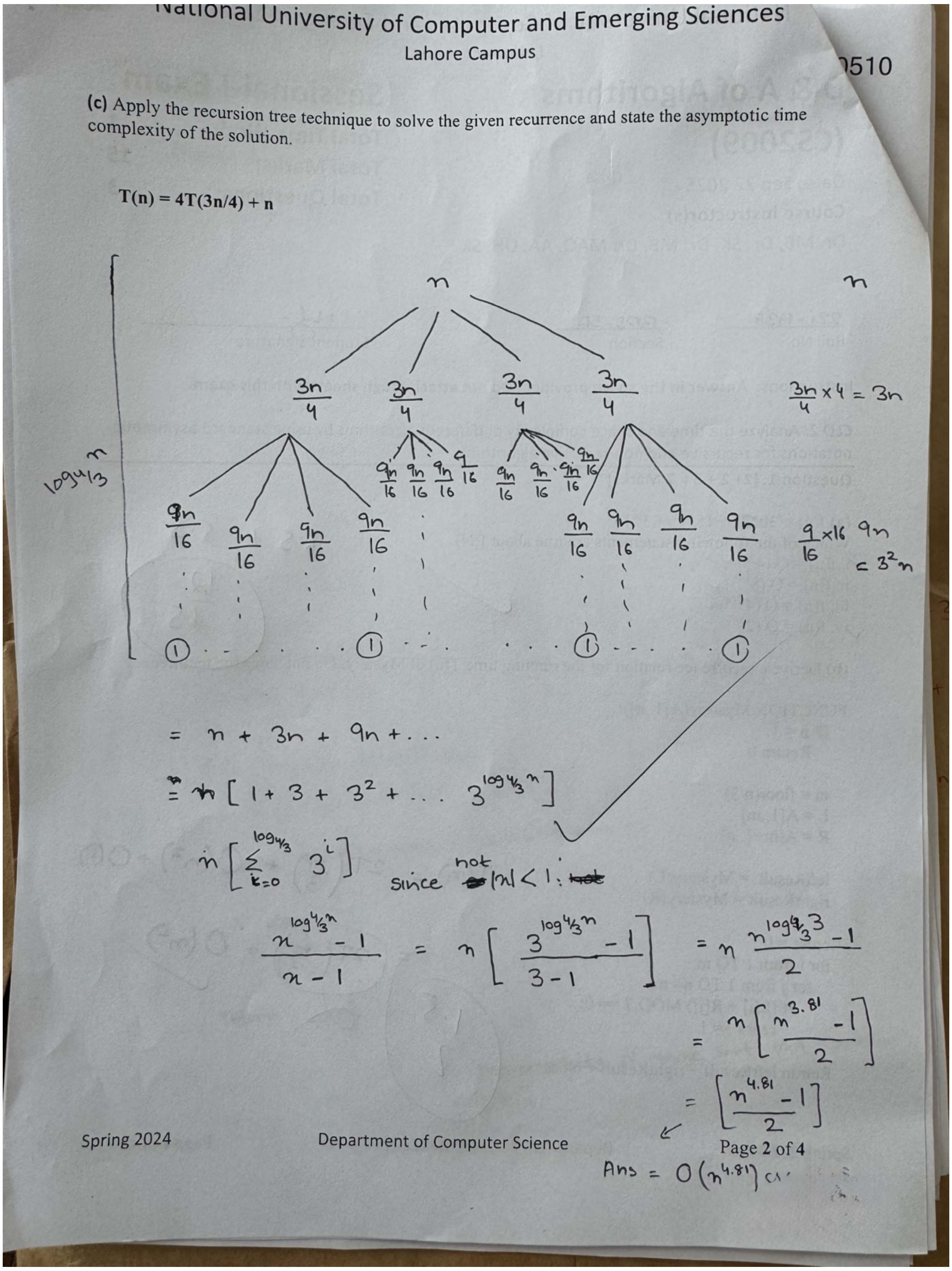
(b) Derive a recurrence relation for the running time T(n) of Mystery. Do not solve the recurrence.

```
FUNCTION Mystery(A[1..n]):
   IF n ≤ 1:
      Return 0

   m = floor(n/3)
   L = A[1..m]
   R = A[m+1..n].

   leftResult  = Mystery(L)
   rightResult = Mystery(R)

   cross = 0
   for i from 1 TO m:
      for j from 1 TO n - m:
         If (L[i] + R[j]) MOD 7 == 0:
            cross += 1

   Retrun leftResult + rightResult + cross
```

Solution: $T(n) = T(n/3) + T(2n/3) + O(n^2)$

(c) Apply the recursion tree technique to solve the given recurrence and state the asymptotic time complexity of the solution.

T(n) = 4T(3n/4) + n



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(c) Apply the recursion tree technique to solve the given recurrence and state the asymptotic time complexity of the solution.

T(n) = 4T(3n/4) + n

n — n

$\frac{3n}{4}$ $\frac{3n}{4}$ $\frac{3n}{4}$ $\frac{3n}{4}$ — $\frac{3n}{4} \times 4 = 3n$

$\frac{9n}{16}$ $\frac{9n}{16}$ $\frac{9n}{16}$ $\frac{9n}{16}$ … — $\frac{9}{16} \times 16 \; 9n = 3^2 n$

$\log_{4/3} n$

1 … 1 … 1 … 1

$= n + 3n + 9n + \dots$

$= n \left[1 + 3 + 3^2 + \dots 3^{\log_{4/3} n} \right]$

$n \left[\sum_{i=0}^{\log_{4/3}} 3^i \right]$ since not $|x| < 1$ ~~not~~

$\frac{x^{\log_{4/3} n} - 1}{x - 1} = n \left[\frac{3^{\log_{4/3} n} - 1}{3 - 1} \right] = n \, \frac{n^{\log_{4/3} 3} - 1}{2}$

$= n \left[\frac{n^{3.81} - 1}{2} \right]$

$= \left[\frac{n^{4.81} - 1}{2} \right]$

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Ans $= O(n^{4.81})$

CLO #1: Design algorithms using different algorithms design techniques i.e. Brute Force, Divide and Conquer, Dynamic Programming, Greedy Algorithms and apply them to solve problems in the domain of the program

Question 2: [2 Marks] Quick Sort's efficiency depends heavily on how the pivot is chosen. Describe a specific input and pivot-selection strategy that causes Quick Sort to run in $\Theta(n^2)$ time.

When the array is sorted, you always select the first or the last element of the array as the pivot.

Question 3: [2+4 = 6 Marks] Recall the problem of finding the number of inversions. We are given a sequence of n numbers $a_1, \ldots, a_n$, which we assume are all distinct, and we define an inversion to be a pair $i < j$ such that $a_i > a_j$. We motivated the problem of counting inversions as a good measure of the sortedness of an array. However, one might feel that this measure is too sensitive. Let's call a pair a significant inversion **if** $i < j$ **and** $a_i > 2a_j$.

(a) Give an $O(n^2)$ algorithm to count the number of significant inversions between two orderings.[2 Marks]

```
function count_significant_inversions_bruteforce(A[0..n-1]):

   count ← 0

   for i ← 0 to n-2:

      for j ← i+1 to n-1:

         if A[i] > 2 * A[j]:

            count ← count + 1

   return count
```

(b) Give an O(n log n) algorithm to count the number of significant inversions between two orderings.[4 Marks]

```
function countSignificantInversions(A, left, right):
    if left >= right:
        return 0

    mid = (left + right) // 2
    count = 0
    count += countSignificantInversions(A, left, mid)
    count += countSignificantInversions(A, mid+1, right)

    # Count cross inversions
    j = mid + 1
    for i from left to mid:
        while j <= right and A[i] > 2 * A[j]:
            count += (mid – i + 1))
            j += 1


    # Merge step (to keep array sorted)
    merge(A, left, mid, right)

    return count


def merge(A, left, mid, right):
    # Copy halves
    L = A[left:mid + 1]
    R = A[mid + 1:right + 1]

    i = 0
    j = 0
    k = left

    # Merge sorted halves back into A
    while i < len(L) and j < len(R):
        if L[i] <= R[j]:
            A[k] = L[i]
            i += 1
        else:
            A[k] = R[j]
            j += 1
        k += 1

    # Copy remaining elements (if any)
    while i < len(L):
        A[k] = L[i]
```

```
    i += 1
    k += 1

while j < len(R):
    A[k] = R[j]
    j += 1
    k += 1
```