National University of Computer and Emerging Sciences, Lahore Campus

THE PROPERTY OF THE PARTY OF TH	Course Name:	Design and Analysis of Algorithms	Course Code:	CS2009
	Program:	BS(Computer Science)	Semester:	Fall-2025
			Total Marks:	60
			Section:	5G, 5H, 5J
	Exam Type:	Assignment 1	Page(s):	2

Q1. In the Republic of Chatter, each of the n citizens sends a message to every other citizen.

[10 points]

- 1. Find the total number of messages exchanged.
- 2. Derive the time complexity of this process.

Q2. A startup company measures the performance of their recursive server load-balancing algorithm and models it as [15 points]:

1.
$$T(n) = 2T(n/3) + n$$

2.
$$T(n) = T(n/2) + \sqrt{n}$$

3.
$$T(n) = 3T(n/2) + n$$

For each recurrence:

- Solve it using recursion tree.
- Give the tightest asymptotic bound.

Q3. In the city of Festival Town, an array of n flags is arranged in random order. Each flag is either red, orange, or blue. For the parade, all red flags must appear first, followed by all orange flags, and finally all blue flags.

You are asked to design an algorithm to rearrange the flags into the correct order. You are not allowed to count occurrences - only direct comparisons with "red", "orange", or "blue" are permitted. Write the algorithm and derive its time complexity. [10 points]

Q4. Derive the recurrence of the following algorithm [10 points]:

```
SplitData(Arr[], left, right) {
    if (left < right) {
       SplitData(Arr, left, right - 1);
      SplitData(Arr, left + 1, right);
       ViewData(Arr, left, right);
    }
 }
 ViewData(Arr[], left, right) {
    i = left;
    N_Arr[left + right];
    while (i < right) {
      j = left;
      k = 1;
       while (j < right) {
         N_Arr[k] = Arr[j];
         print Arr[i];
         k = k + 1;
         j = j * 2;
       }
     i++
   }
}
```

Q5. Consider the following variation on Merge Sort, that instead of dividing input in half at each step of Merge Sort, you divide into three part, sort each part, and finally combine all of them using a three-way merge subroutine. What is the overall asymptotic running time of this algorithm? [10 points]

Q6. Prove $n^3 - 2n + 1 = O(n^3)$. Determine the values of constant c and n0. [5 points]