Enhancing Economic Value: Finding Optimal Cut-Off Grades through Meta-Heuristic Algorithms in Stockpile Management Scenarios

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Abstract

The open pit mine production scheduling problem (OPMPSP) presents a substantial challenge in optimization. Its objective is to schedule material extraction within a mineral deposit across multiple time periods, maximizing operational profit while adhering to diverse physical and economic constraints. This research highlights the significant role of stockpiling in managing processing plant capacity and interplay of material flows—from mine to stockpile, mine to processing plant, and stockpile to plant. The incorporation of this approach greatly enhances the long-term profitability of a mine by providing optimum conditions for the goal function via the use of parameterized analysis.

The current research employed a meta-heuristic algorithm to ascertain optimal cut-off grades for diverse metal deposits while incorporating the stockpiling strategy led to a noteworthy 2.31% augmentation in net present value (NPV) and an extended mine life of six years compared to scenarios where the stockpiling option was neglected. Furthermore, diverse scenarios demonstrated that simultaneous implementation of stockpiling alongside mining operations, coupled with variations in the tonnage input rate from the stockpile to the processing plant, resulted in an impressive 4.95% increase in NPV compared to scenarios where stockpiling was only implemented at the end of the mine's operational lifespan.

Keywords: Net present value; Production optimization; Meta-heuristic algorithm; Open-pit mining; Stockpile management

1 Introduction

For decades, the mining industry has relied on the Lane model to determine metal mines' optimal cut-off grade strategy. The Lane model suggests a set of economic cut-off grades typically more extensive than the lowest grade at which processing is economically feasible, i.e., the breakeven or heuristic cut-off. According to the Lane model, if the grade of material in the deposit is higher than the cut-off grade, it is referred to as the ore and transported to a the processing plant. As shown in Figure 1, a part of the highest quality material (dark portion) is extracted and processed during the early production stages. Currently, if the grade of the material is lower than the cut-off grade, it is considered waste and is disposed of in a waste dump (light-colored part in Figure 1).

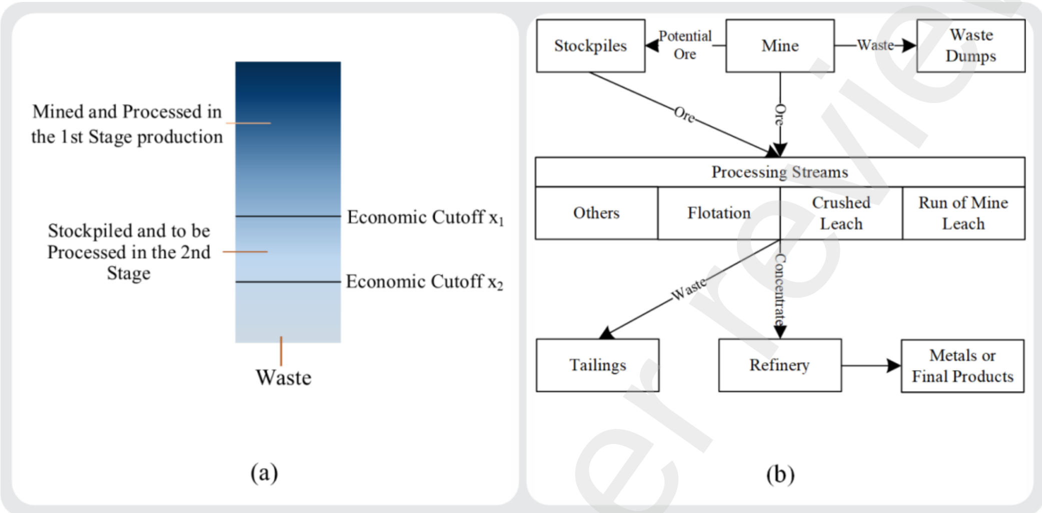

Fig. 1. (a) Determining the location of the material extracted from the mine by cut-off grade (Zhang and Kleit 2016), (b) the layout of an ideal open-pit mining system (Asad et al. 2016).

This mentioned strategy does not process the part of the ore because it may produce positive cash flows during or after mining. However, these materials are extracted during the generating process and are often stockpiled near the mine. Asad (2005) extended Lane's original theory to include stockpiling in production scheduling for deposits with two economic resources. When the active pit extraction is concluded, the stockpile acts as an additional pushback. Recent research indicates that Asad's efforts account for a significant portion. He initiated this by modifying the Lane algorithm to optimize the cut-off grade of two mineral deposits using the stockpile option. Then, he then proposed a model to determine the optimal cut-off grade by incorporating the stockpile option and technical and economic parameters (Asad and Topal 2011). Levitin et al. (2013) Solved the coal mixing and stockpile problem using a chance constraints and mixed integer programming model with the objective of maximizing the net present value. A mixed-integer linear programming (MILP) model considering stockpiles and the cost of uncertainty to generate long-term production schedules was presented by Koushavand et al. (2014). Moreno et al. (2017) proposed different models to investigate the stockpile option in mining planning and to compare the net present value of patterns. The long-term mine stockpiling model proposed a method to optimize mining operations by transporting ore from the stockpile to the processing plant (Dirkx and Dimitrakopoulos 2018). Rezakhah et al. (2020) adjusted a linear program to multiple mines, considering a variety of stockpiling and material flow strategies.

Some scholars presented models for large-scale production operations, considering economic criteria such as the cut-off grade of multi-metal deposits and proposed a new method for optimizing extraction sequencing with the aim of NPV maximization (Osanloo and Ataei 2003; Ramazan 2007; Reple et al. 2020). Cetin and Dowd

(2013) used the grid search method to improve the Lane model for mining projects, including multi-metal deposits. Implementation of the maximum parametric flow algorithm to optimize open pit mine planning and design by Asad et al. (2013). Lamghari et al. (2014) Applied an efficient algorithm to solve production planning problems in mines under metal uncertainty. Using scheduling optimization models to maximize the net present value of projects considering resource constraints (Zheng et al. 2018; Rostami et al. 2024). Tolouei et al. (2021) applied an enhanced Lagrangian relaxation method to solve production planning difficulties in open-pit mines with a grading policy. Nwaila et al. (2021) used an algorithmic technique to strategically optimize the cut-off grade and NPV while minimizing operational risks and maximizing the mine life of a gold mine. They demonstrated that their model increases NPV while satisfying functional restrictions.

One of the significant hardships in cut-off grade optimization is the presence of optimal local values broadly detached from the optimum global point and each other. The actual challenge in dealing with such issues is finding solutions close to the global ideal for a limited time. The genetic algorithm (GA) is more robust in this regard than many other existing search methods. This algorithm is highly accurate and well-suited for solving large-scale mine optimization problems. Additionally, it generates a population of possible solutions. Numerous researchers have used the GA to solve problems involving cut-off grade optimization. Ataei and Osanloo used a hybrid GA in conjunction with the grid search method to determine the optimal cut-off grades using Lane's formulation (2004). Other researchers have employed the GA to derive the cut-off grade for NPV maximization (Azimi and Osanloo 2011; Akbari and Rahimi 2016; Ahmadi and Shahabi 2018). Notably, mining operations planning is a necessary production process that Zhang & Wong used a genetic algorithm to optimize this process (2015). The GA has been devised as a strategy for optimizing the cut-off grade of multi-metal deposits and has been compared to another pattern (Cetin and Dowd 2016). Implementing of Genetic Algorithm in solving operation scheduling problems with multi-mode stock constraints by Alcaraz et al. (2003). Besides, mutation assists in overcoming the problem of optimal local trapping, and the population will be evaluated based on the offspring produced by the GA procedure (Ahmadi and Golshadi 2012; Ahmadi et al. 2015; Samareh et al. 2017; Matamoros and Kumral 2019). According to the literature review available on cut-off grade optimization, researchers rarely used a combination of multi-metal deposit cut-off grade optimization and the stockpile option.

The present study aims primarily to model the flow of mining performance while considering technical and economic constraints for mining, processing, and marketing/refining capacities during operations using a new approach that incorporates the stockpile implementation into two different plans. These plans include parallel mining operations with varying percentages of tonnage input (constant, descending, and rising) from the stockpile to the processing plant and the end of the life of the mine to maximize NPV. A genetic algorithm is employed as a meta-heuristic and intelligent strategy to optimize the objective function. This program initiates a recurrent optimization process among different design constituents and leads them toward the most economically viable mine settings. Thus, a solution using this method is accomplished efficiently and rapidly. The following sections discuss the objective function. In the following sections, the different sections continue the discussions as follows: the Lane model is introduced in Section 3. Then discusses a developed approach for multi-metal cut-off grade optimization. Section 4 illustrates the model's development through a case study.

Section 5 analyzes with and without the stockpile option and plans with a range of stockpile managing options. Section 6 summarizes this new technique.

2 Objective function

In the realm of 'open pit mine production scheduling' studies utilizing mathematical modeling, it is typically assumed that the cut-off grade is predetermined—a realistic approach for long-term planning. Alternatively, it may be treated as one of the variables within the problem. This research examines the cut-off grade as a problem variable, which is further analyzed using mathematical models. This research divides the objective function into four components to maximize the NPV. The first section demonstrates the NPV achieved by extracting mineral blocks in various cycles. The goal function's value must be more significant than the costs associated with NPV and block extraction (second part), the usage of minerals in succeeding periods, and the profits earned from them (third part). As a result, the stockpile's material is utilized in two distinct ways: in parallel with mining activities, with varying percentages of tonnage input (constant, decreasing, and increasing) from the stockpile to the processing facility, and in the process of maximizing NPV at the mine's end of life. Restrictions on both the quantity and quality attributes of the mineral are included in the suggested model's objective function, ensuring the formula's practicality (fourth part). The mining method is depicted in Figure 2.

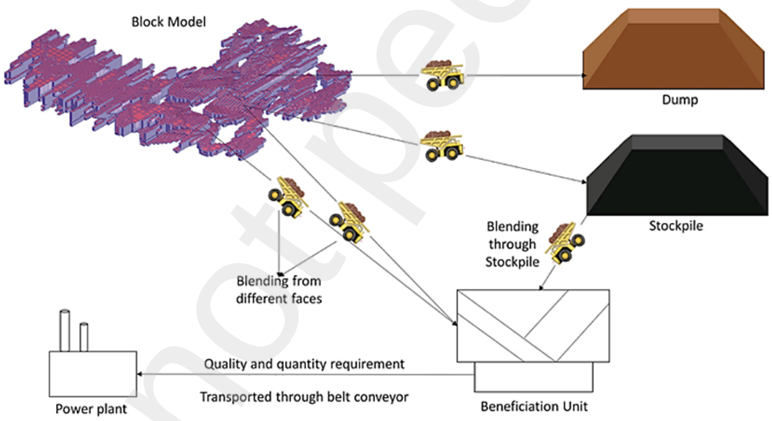

Fig. 2. Schematic diagram of mining operations from extraction to processing considering the stockpile option (Kumar and Chatterjee 2017).

3 Lane's model

Lane's model is a technique to establish the cut-off grade by considering the capacities of mining, processing, and refining stages, and operational costs. The following mathematical calculations demonstrate that the model aims to maximize the project's NPV and profit. The model's parameters and symbols are listed in Table 1.

$$Max\ NPV = \sum_{i=1}^{N} \frac{P_i}{(1+d)^i} \tag{1}$$

Subject to:

$$Q_m \leq M \tag{2}$$

$$Q_c \leq C \tag{3}$$

$$Q_{r1} \leq R_1 \tag{4}$$

$$Q_{r2} \leq R_2 \tag{5}$$

here,

$$P = (s_1 - r_1)Q_{r_1} + (s_2 - r_2)Q_{r_2} - cQ_c - mQ_m - fT \tag{6}$$

Infinite points are considered for the optimum cut-off grade to establish ideal cut-off grades in multi-metal deposits. In contrast, only six locations are chosen in single-metal deposits. Therefore, unlike single-metal deposits, multiple-metal deposits have objective functions that are multivariate and nonlinear. These functions conceal the cut-off grade variables for each metal and the grade-tonnage distribution function, which are not derivative or continuous. Thus, analytical methods such as the Lane model are incapable of determining the optimal sites, and meta-heuristic algorithms are required.

The strategy discussed in this research introduces a new approach that incorporates stockpiling into two different plans concurrently, optimizing NPV both during parallel mining operations and at the end of the mine's life. This method employs a genetic algorithm to achieve the stated goals. The evolutionary algorithm is a meta-heuristics and intelligent strategy known for high accuracy and speed in efficiently addressing mining challenges. This approach involves a recurring optimization process among various design components, guiding them toward the highest economic parameters for the mine without getting trapped in local extremums.

Table 1. Effective parameters in the Lane method.

Symbol	Definition	Unit
P	Profit	dollar per year
Q_m	Tons of material mined	tons per year
Q_c	Tons of ore processed	tons per year
Q_{r1}	Tons of mineral 1 refined	tons per year
Q_{r2}	Tons of mineral 2 refined	tons per year
M	Mining capacity	tons per year
C	Milling capacity	tons per year
R_1	Refining capacity of mineral 1	tons per year
R_2	Refining capacity of mineral 2	tons per year
S	Stockpile handling cost	dollar per tons
m	Mining cost	dollar per tons
c	Milling cost	dollar per tons
r_1	Refining cost	dollar per tons
r_2	Refining cost	dollar per tons
f	Fixed costs	dollar per year
s_1	Selling price	dollar per tons
s_2	Selling price	dollar per tons
y_1	Recovery	%
y_2	Recovery	%
g_1	Average grade of metal 1	%
g_2	Average grade of metal 2	%
T	Life span	N
d	Discount rate	%

1. The multi-metal cut-off grade optimization model

Typically, open-pit mining processes involve three stages: (i) the mining stage, (ii) the concentration phase, where the ore is processed and concentrated, and (iii) the refinement stage. For simplicity, two metal deposit mines were assumed. The ore is dispatched to a concentrator in these deposits, and the concentrator will yield two concentrates. Each concentration is sent to a processing unit for smelting and refining. Each phase has its costs and restricting capacities. As Figure 3 shows a representation of the parameters and symbols used in the model.

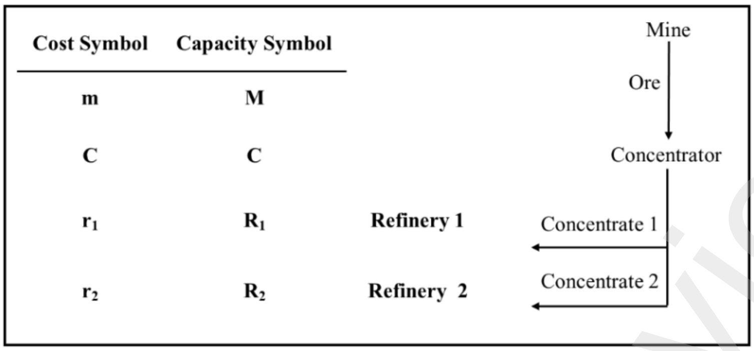

Fig. 3. The flowchart of mining operation in two metal deposits.

This study employed a genetic algorithm to optimize the cut-off grades and maximize NPV. In the algorithm, the first step involves calling the objective function and defining all inputs and variables. The number of variables (nvar) includes the cut-off grades, mining rate, the number of processing units, and the annual production from refinery units. Additionally, the algorithm incorporated constraints for extraction, processing, and smelting. Equations were coded to optimize the objective function, with the maximum number of iterations serving as the termination condition.

Mutation and crossover operators are defined and encoded as separate functions from the input data required in the algorithm. In each iteration, the best value of the answer from Equation (1-6), along with other parameters, was determined. When the termination condition was met, the optimal solution, adhering to all constraints, was chosen. The grade-tonnage distribution assumed K individual grade categories for mineral 1 and M grade categories for mineral 2, represented as:

$$[g_1(1), g_1(2)], [g_1(2), g_1(3)], \ldots, [g_1(K-1), g_1(K)]$$

$$[g_2(1), g_2(2)], [g_2(2), g_2(3)], \ldots, [g_2(M-1), g_2(M)]$$

For each grade category of mineral 1 (k*) and M grade categories of mineral 2 (m*), there are t tons of material. Similarly, the stockpile's grade categories align with the deposit's original grade categories between the breakeven and economic cut-off grades for minerals 1 and 2. Material from the stockpile is processed in the same manner as material from the mine. If k* represents the grade category [g_1(k), g_1(k +1)] for mineral 1, where g_1(k) is the cut-off grade, and m* corresponds to grade category [g_2(m), g_2(m +1)] for mineral 2, where g_2 (m) is the cut-off grade, the quantities of ore T_o and waste T_w, and the average grade of mineral 1($\bar{g}_1$), and the average grade of mineral 2 ($\bar{g}_2$) are the given in Equation (7-10):

$$T_o(k^*, m^*) = \sum_{k=k^*}^{K} \sum_{m=m^*}^{M} t_{(k,m)} \qquad (7)$$

$$T_w(k^*,m^*) = \sum_{k=1}^{k^*-1} \sum_{m=1}^{m^*-1} t_{(k,m)} \tag{8}$$

$$\bar{g}_1(k^*) = \frac{\sum_{k=k^*}^{K} \left[\left(\sum_{m=m^*}^{M} t_{(k,m)} \right) \left(\frac{g_1(k) + \ g_1(k+1)}{2} \right) \right]}{T_o(k^*,m^*)} \tag{9}$$

$$\bar{g}_2(m^*) = \frac{\sum_{k=k^*}^{K} \left[\left(\sum_{m=m^*}^{M} t_{(k,m)} \right) \left(\frac{g_2(m) + \ g_2(m+1)}{2} \right) \right]}{T_o(k^*,m^*)} \tag{10}$$

now,

$$\begin{cases} Q_c = C, & if\ T_o > C \\ Q_c = T_o, & otherwise \end{cases} \tag{11}$$

$$Q_m = Q_c \left(1 + \frac{T_w}{T_o} \right) \tag{12}$$

$$Q_{r_1} = \bar{g}_1.y_1.Q_c \tag{13}$$

$$Q_{r_2} = \bar{g}_2.y_2.Q_c \tag{14}$$

To calculate the amount of material Q_m over time (T), Equation (13-14) with Equation (6) have been substitute to determine the profit generated by Q_m at the end of time T:

$$P = [(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c]Q_c - mQ_m - fT \tag{15}$$

Assuming V is the maximum NPV of future profits at time zero, and W is the most significant attainable NPV of future profits (p_{T+1} to p_N) collected from reserves after mining the next Q_m of material, i.e., in time T, obtaining knowledge about the discount rate d:

$$W = \frac{p_{T+1}}{(1+d)^{T+1}} + \ \dots \ + \frac{p_N}{(1+d)^N} \tag{16}$$

$$V = \frac{P + W}{(1+d)^T} \tag{17}$$

Rise in present value *v* occurs due to subsequent mining of Q_m material, and the difference between V and W reflect this rise. Equation (18) is simplified due to the brief time interval T

$$v = (V - W) = P - dVT \tag{18}$$

By substituting Equation (15) into Equation (18), the fundamental present value expression used to calculate the optimal cut-off grades is obtained:

$$v = [(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c]Q_c - mQ_m - (f + Vd)T \tag{19}$$

Mining, processing, and refinery capacities for mineral 1 or mineral 2 determine time T, resulting in four values depending upon the actual constraining capacity.

$$T_m = {Q_m}/{M} \tag{20}$$

$$T_c = {Q_c}/{C} \tag{21}$$

$$T_r = {Q_{r_1}}/{R_1} \tag{22}$$

$$T_r = {Q_{r_2}}/{R_2} \tag{23}$$

Substituting Equation (20-23) into Equation (19) yields:

$$v_m = [(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c]Q_c - (m + {f + Vd}/{M})Q_m \tag{24}$$

$$v_c = [(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c + {f + Vd}/{C}]Q_c - mQ_m \tag{25}$$

$$v_{r_1} = [\left(s_1 - r_1 - {f + Vd}/{R_1}\right)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c]Q_c - mQ_m \tag{26}$$

$$v_{r_2} = [(s_1 - r_1)\bar{g}_1 y_1 + \left(s_2 - r_2 - {f + Vd}/{R_2}\right)\bar{g}_2 y_2 - c]Q_c - mQ_m \tag{27}$$

For any given values of g_1 and g_2, v_m,v_c,v_{r1} and v_{r2} can be calculated. acts as the capacity-limiting factor, maximizing the objective function according to Equation (28).

$$v_{max}(\gamma_1,\gamma_2) = \max\left[\min(v_m, v_c, v_{r_1}, v_{r_2})\right] \tag{28}$$

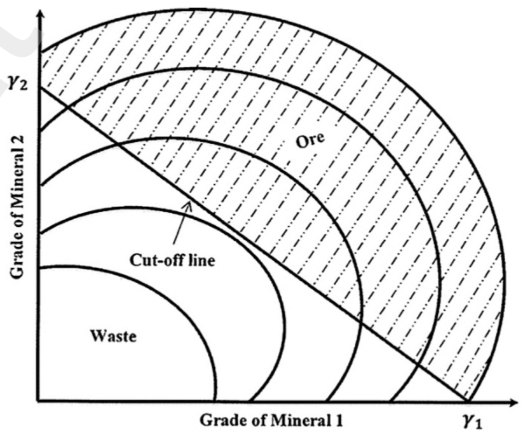

Fig. 4. Grade-tonnage distribution of the deposit in two-minerals cases (Lane 1988).

Intercept is depicted in Figure 4 on a surface representing the grade-tonnage distribution (Lane 1988). The value of v_{max} corresponds to each of v_m,v_c,$v_{r1}$and v_{r2}. Moreover, the capacity of that unit will be the determinant constraint. The mine life and the annualized production values of various units are estimated according to their capacity.

Annual producing different units, mine life, discount rate, annual profit amounts, and NPV can be calculated. Stockpile grade categories are created and are accumulated the reserves available in each category, as was discussed in this section and this process continues until the end of the deposit. The grade-tonnage distribution table can then be adjusted, and the calculations repeated for the remainder of the mine's life based on the amount of mineral and waste extracted in the first year and the intermediate grade material stockpiled during mine life. After the mine deposit is exhausted, the stockpile materials are processed like a small mine. Equation (15) is modified as:

$$P = [(s_1 - r_1)\bar{g}_1 y_1 + (s_2 - r_2)\bar{g}_2 y_2 - c]Q_c - SQ_m - fT \qquad (29$$

Here, material handling cost constitutes 45% of the mining cost, comprising 40% for material handling and 5% for supervision (Schellman 1989). These steps are repeated until the completion of this small-scale mine.

2. Background if the method used

The current research explores the application of metaheuristic algorithms to the long-term open-pit mining scheduling problem, aiming to establish optimal cut-off grades for multiple metal resources. In this endeavor, a GA was employed as a strategic tool. The GA, inspired by nature's optimization strategies, is a randomized optimization algorithm, well-suited for tackling complex problems with uncertain search spaces. While GAs have demonstrated effectiveness in handling linear and convex challenges, their true strength lies in resolving nonlinear and discrete problems. One distinctive feature of GAs is their ability to simultaneously explore multiple potential solutions and assess various locations within the intended search space.

The problem parameters must first be encoded as binary strings to initiate the GA. Each response is assigned a fitness value that shows the response's quality compared to other responses within the population. Consequently, responses that are more suitable have a higher chance of survival, reproduction, and being passed on to subsequent generations. The algorithm involves three fundamental operators: selection, crossover, and mutation.

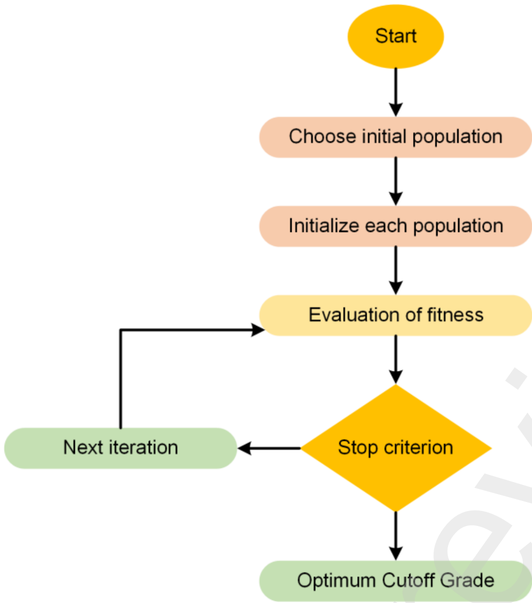

Fig. 5. Graphical illustration of GA.

A. **Selection:** This step involves choosing a process in which some strings are matched to their fitness function. The easiest way is to use a roulette wheel, in which each strand of the population, in proportion to the amount of self-discipline, comes from the wheel. With each wheel rotation you want, a candidate is selected.

B. **Crossover:** Intersection is performed in two steps; First, members are randomly selected for mating, and then each pair of strings is randomly cut, and pieces are cut off after cutting.

C. **Mutation:** After the crossover operator, the strings are exposed to the mutation operator. The operation mutation is random, that is, the locale of the string is randomly selected, and the mutation applied to it.

In this study, a coded genetic algorithm was used to optimize the objective functions. To implement genetic algorithm optimization, limitations of the parameters was defined. Therefore, three restrictions including limitations of extraction, processing, and smelter and were coded.

4 Case Study

The method considered in this research involves utilizing stockpiling in two plans: during and after the mining operation. This method was implemented using a genetic algorithm. In this section, some of the production planning activities involved in an open-pit lead-zinc mine will be discussed. Specifically, attention will be devoted to mine life and production rate determinations. The hypothetical lead-zinc deposit studied in this research contains 12 million tons of ore and waste products inside an open-pit mine. The economic characteristics and operational capabilities of this hypothetical mine are shown in Table 2. The deposit's grade-tonnage distribution is depicted in Table 3 and Figure 6.

Table 2. Economic parameters for a manual example.

Parameter	Unit	Quantity
Q_m	tons per year	1000000
Q_c	tons per year	500000
Q_{r1}	tons per year	150000
Q_{r2}	tons per year	60000
M	dollar per tons	1
C	dollar per tons	16.25
r_1	dollar per tons	525
r_2	dollar per tons	220
F	dollar per year	10000000
s_1	dollar per tons	775
s_2	dollar per tons	435
y_1	%	80
y_2	%	60
D	%	20

Table 3. Grade-tonnage distribution of lead and zinc.

Zinc (%)	Lead (%)				
	0-4	4-8	8-12	12-16	>16
0-4	250000	110000	5000	5000	2000
4-7	900000	100000	10000	70000	30000
7-10	700000	100000	50000	60000	150000
10-14	185000	170000	110000	200000	370000
14-17	500000	170000	170000	90000	345000
17-20	570000	390000	150000	150000	280000
20-24	900000	770000	270000	190000	365000
24-27	420000	200000	250000	370000	225000
27-30	350000	170000	540000	230000	25000
>30	300000	115000	60000	35000	3000

Each grade category of zinc has five lead grade categories, as shown in Table 3. Due to the tonnage-grade distribution, the overall cut-off grade for the lead was believed to be between 0 and 16, and for zinc, it was regarded to be between 0 and 30. In a genetic algorithm, the initial population equals 200, gained by a try and error method. Also, the number of variables needed to solve equals 27, and the maximum iteration number in the genetic algorithm equals 600. The obtained results are presented by considering this algorithm to maximize the mine's economic parameters without being trapped in the local extremum.

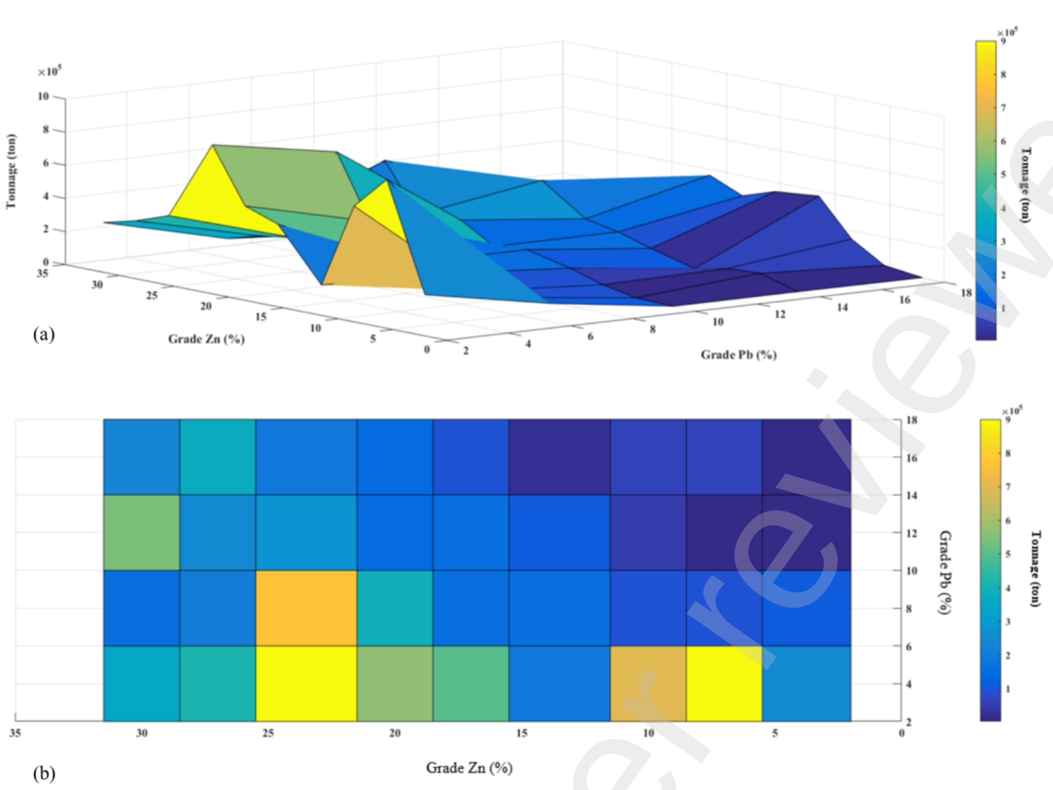

Fig. 6. Grade-tonnage distribution for the lead-zinc deposit. (a) 3-D, (b) 2-D

Table 4. Optimum cut-off policy without stockpiling option.

Year	Cut-off Grade		Profit ($)	NPV ($)
	Zinc (%)	Lead (%)		
1	16.502	2.338	11,031,348	48,481,667
2	15.104	2.202	11,031,117	47,146,652
3	16.718	0.868	11,030,588	45,544,866
4	14.827	1.035	11,030,095	43,623,251
5	14.767	0.394	10,925,841	41,317,806
6	13.77	0.094	10,923,842	38,655,526
7	10.065	0.036	10,721,311	35,462,790
8	8.835	0.231	10,548,317	31,834,037
9	6.89	0.037	10,364,825	27,652,527
10	6.151	0.017	9,564,562	22,818,208
11	2.265	0.049	8,904,023	17,817,287
12	6.957	0.021	8,164,501	12,476,722
13	6.332	0.18	5,880,110	6,807,565
14	5.799	0.301	2,746,763	2,288,969

Table 4 illustrates the optimal cut-off policy in the absence of stockpiling. As shown in Table 4, the optimal cut-off grades for the lead and zinc in year 1 obtained 2.338% and 16.502%, respectively. The optimal cut-off grades for lead and zinc decreased to 0.301% and 5.799%, respectively, in year 14, due to the declining effect of NPV from year 1 ($48,481,667) to year 14 ($2,288,969). Comparing Table 4 to the findings of Ataei and Osanloo confirms the results' similarity and the method's correctness (2004). The optimal number of generations, crossover rate, and mutation rate for obtaining the optimal cut-off grade are 100, 90%, and 45%, respectively.

Because of the of higher-grade minerals extraction in the early years of mine life, the NPV of the deposit is high and falls with mine lifespan. After the mine completing, the minerals are stored in the stockpile during the mine life and sent to the processing plant.

Table 5. An optimum cut-off policy with stockpiling option

Year	Cut-off Grade			
	Zinc (%)	Lead (%)	Profit ($)	NPV ($)
1	16.502	2.338	11,031,348	49,602,803
2	15.104	2.202	11,031,117	48,492,016
⋮	⋮	⋮	⋮	⋮
13	6.332	0.18	5,880,110	16,803,734
14	5.799	0.301	2,746,763	14,284,371
15	0.305	2	6,021,878	14,394,482
16	0.103	1.239	5,243,000	11,251,501
17	1.262	1.158	4,579,891	8,258,801
18	1.355	0.597	3,073,066	5,330,670
19	0.61	0.344	2,633,432	3,323,739
20	0.225	0.17	1,626,066	1,355,055

The optimal cut-off grade policy with the stockpiling option is shown in Table 5, along with the NPV growth from $48,481,667 to $49,602,803, representing a 2.31% increase. Additionally, the mine's life is extended from 14 to 20 years, with material supplied from the stockpile in years 15 to 20. Figure 7 illustrates these findings schematically. Besides, Figure 7 illustrates the negative trend in the NPV over the mine's life. This declining tendency is visible in Figure 8 when the NPV contour is plotted. The higher the level, the greater the NPV, and the lower the level, the lesser the NPV during mining operations.

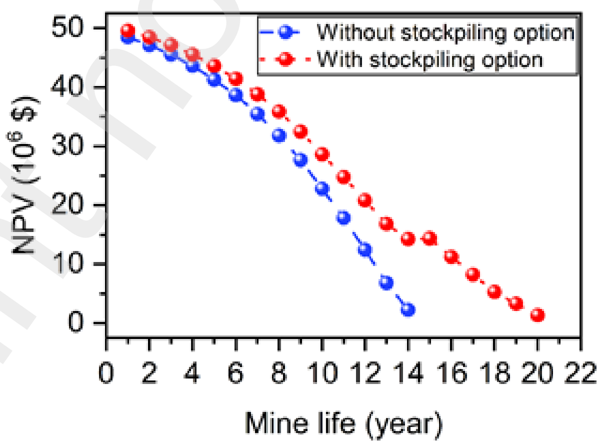

Fig. 7. Comparison NPV without and with stockpiling option during mine life.

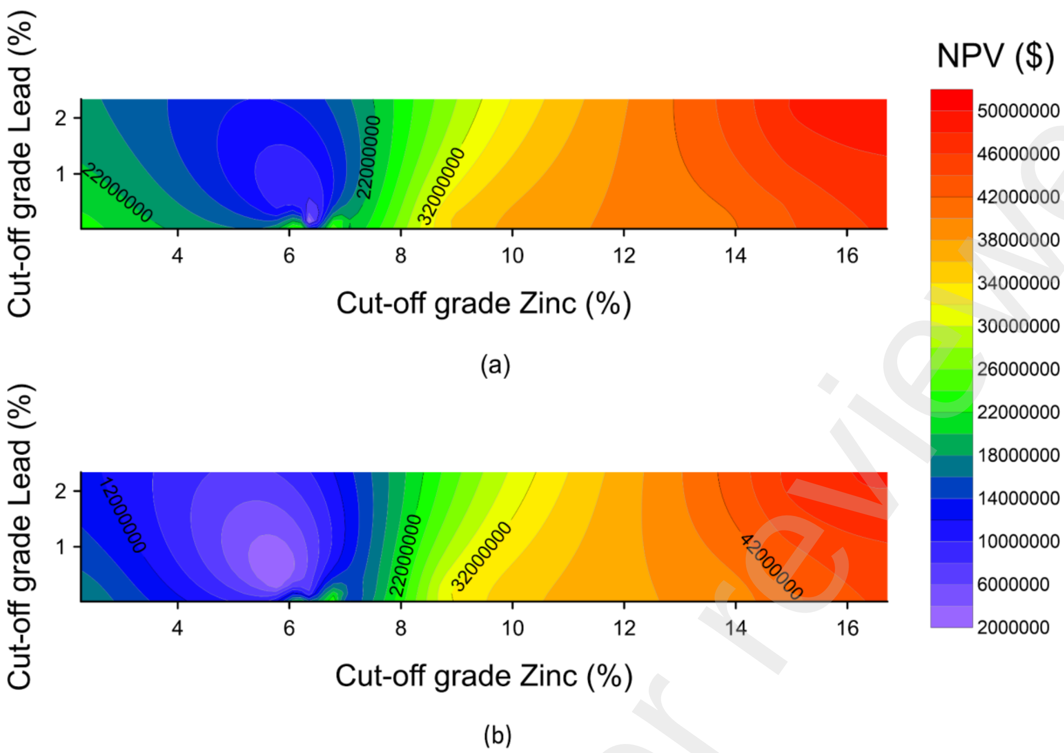

Fig. 8. Contour NPV during the mine life. (a) with stockpiling, (b) without stockpiling

5 Discussion

In the present study, using the findings of the first scenario with the subject without and with the stockpile, this section discusses various techniques to implement the stockpile option to compare NPV and select the optimum scenario. Mining operations are carried out without a stockpile from the first year to the end of the sixth year of the mine's life. Nonetheless, the feed to the processing plant will notably be different as the seventh year begins. The feed provided to the processing plant is a combination of mine feed and stockpile feed, with no restrictions on extraction capacity or processing plant capacity. Whereas the tonnage transmitted from the stockpile is assumed to be a 60% constant rate, a descending tendency from 90% to 10% stepwise, and an ascending trend from 10% to 90% gradually in the latter scenario is assumed in each mine life. The GA was implemented according to different scenarios. Despite existing limitations, the optimal cut-off grade value and maximum NPV were estimated and the NPV for three scenarios during mine life were compared (Figure 9).

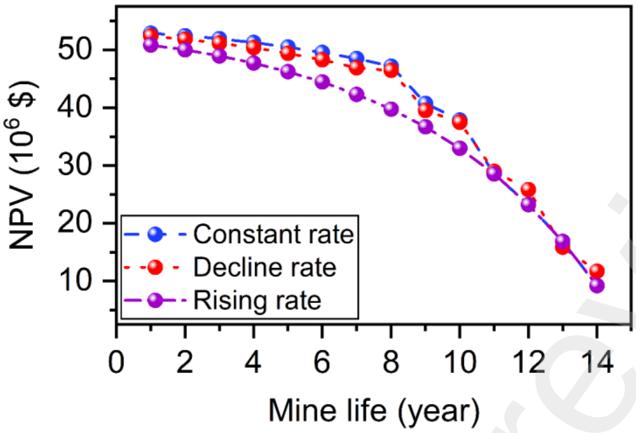

Fig. 9. Comparison NPV for three scenarios during the mine life.

Execution of the stockpile option was considered in three curves from the seventh year of the mine life at different tonnage amounts sent from the stockpile to the processing plant. Total NPV with a rising rate from \$48481667 to \$50860869 with a growth of 4.90% and total NPV with a discount rate from \$48481667 to \$52391197 increasing by 8.06% and total NPV with a constant rate from \$48481667 to \$52919094 which was with a growth of 9.15%. Figure 10 shows the evaluation of NPV for three scenarios during the mine life.

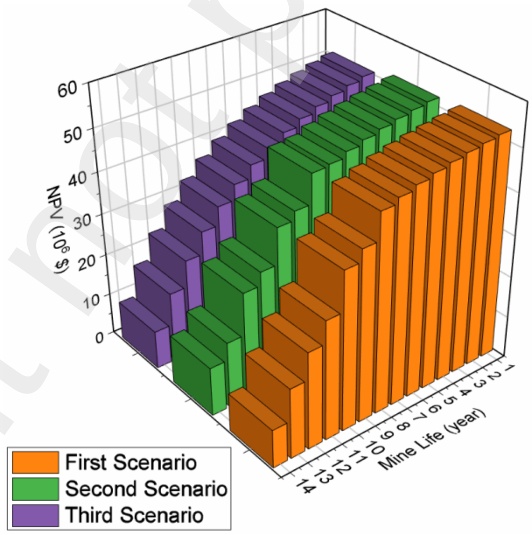

Fig. 10. Evaluation of NPV for three scenarios during mine life.

Based on the results obtained in three scenarios, NPV has a downward trend during the mine life. This downward trend is evident in Figure 11. Higher levels are the maximum NPV, and lower levels are the minimum NPV during mining operations.

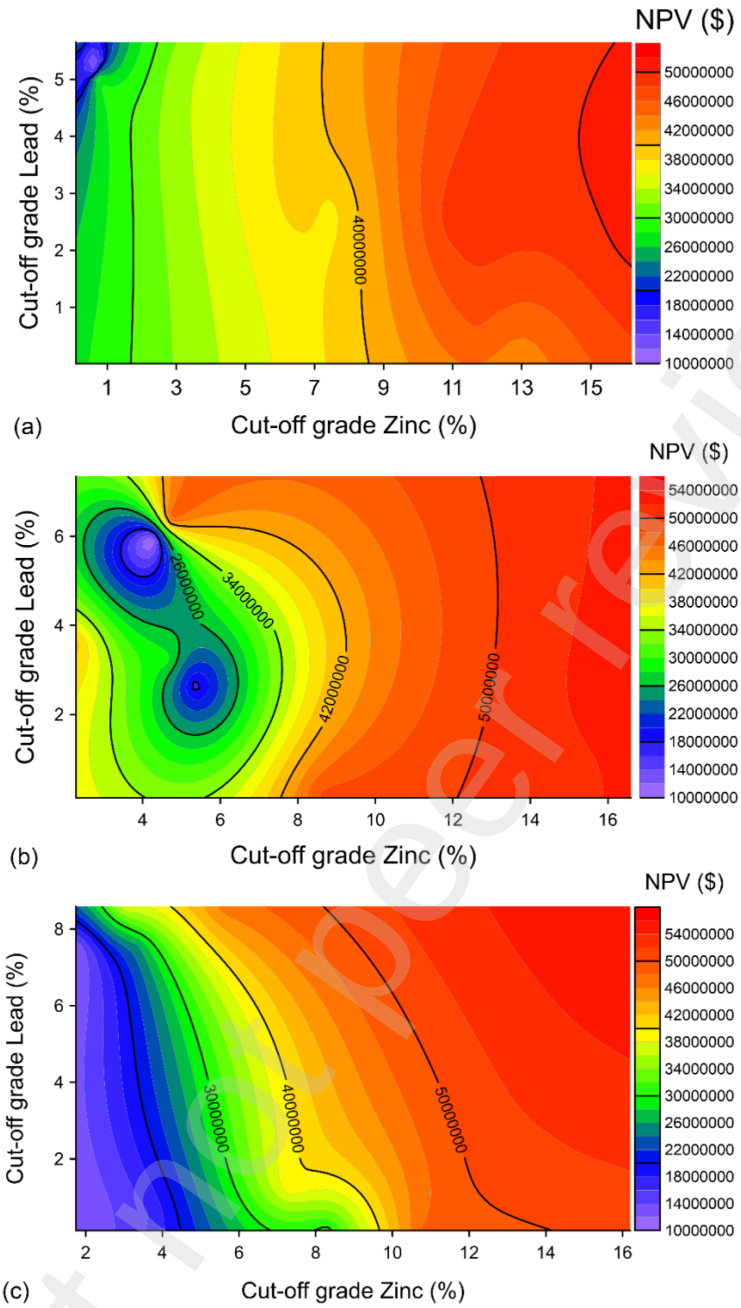

Fig. 11. Contour NPV with stockpiling during the mine life. (a) rising rate, (b) constant rate, (c) decline rate.

In the first year, the mine life of the optimum cut-off grade for lead and zinc is 2.487% and 15.842%, respectively. In the 14th year, the optimum cut-off grades were reduced to 5.919% and 4.237% for lead and zinc, respectively. In the second scenario, the optimum cut-off grade for lead and zinc is 2.28% and 16.209%. In year 14th, it declined to 7.488% and 1.742%, but in the latter scenario, the optimum cut-off grade for lead and zinc is 2.184% and 16.187%, dropping to 5.216% and 0.647% in year 14th, respectively. However, due to the two ore deposits, this downward trend in each year of the mine life is more evident in one of the ores than in the previous year. In other words, the downward trend in the mine life is not consecutive for a cut-off grade of each ore deposit (Figs. 12-14).

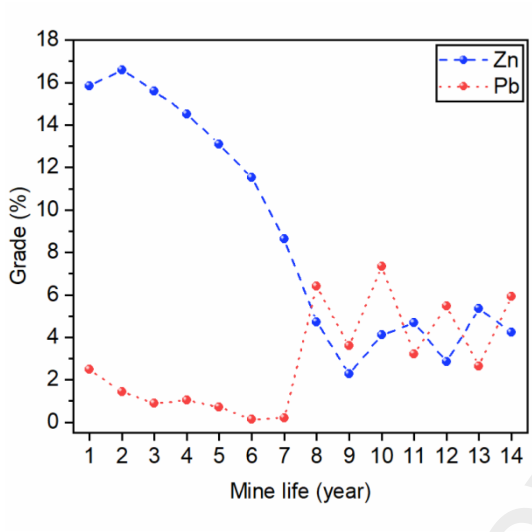

Fig. 12. Evaluation of cut-off grade for lead and zinc in the first scenario during the mine life

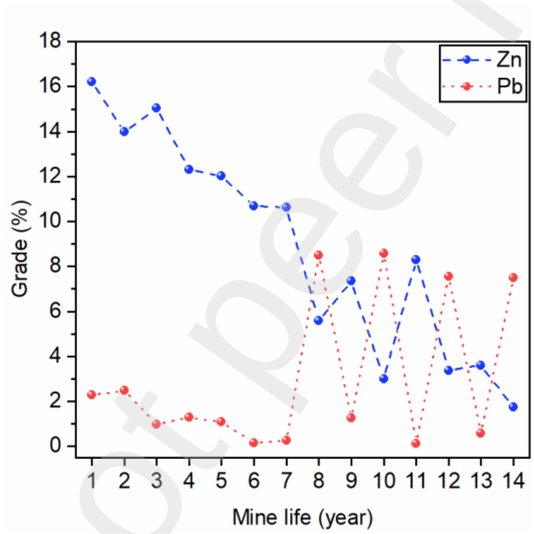

Fig. 13. Evaluation of cut-off grade for lead and zinc in the second scenario during the mine life.

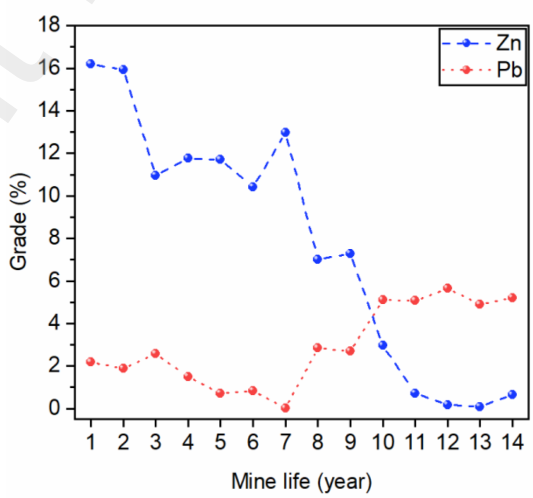

Fig. 14. Evaluation of cut-off grade for lead and zinc in the third scenario during the mine life.

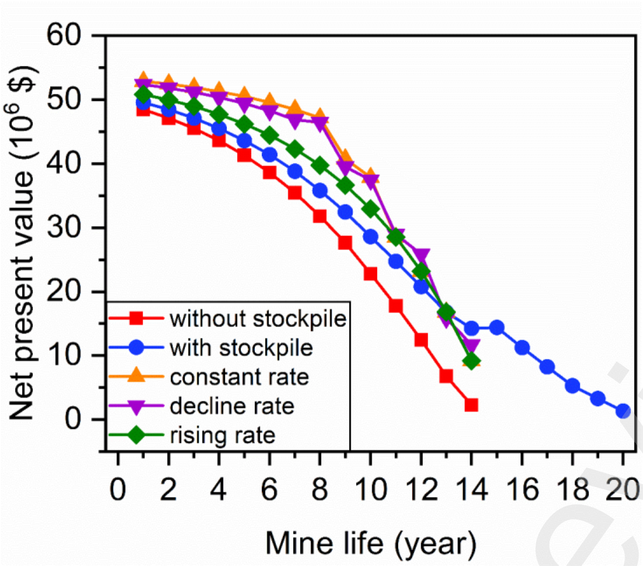

Fig. 15. Comparison of NPV with considering different scenarios

According to Figure 15, implementing the stockpile option from the 7th year with a constant rate provides the total NPV. However, the executing the stockpile option from the seventh year with a discount rate has a relatively good overlap with the stockpile option from the seventh year at a constant rate.

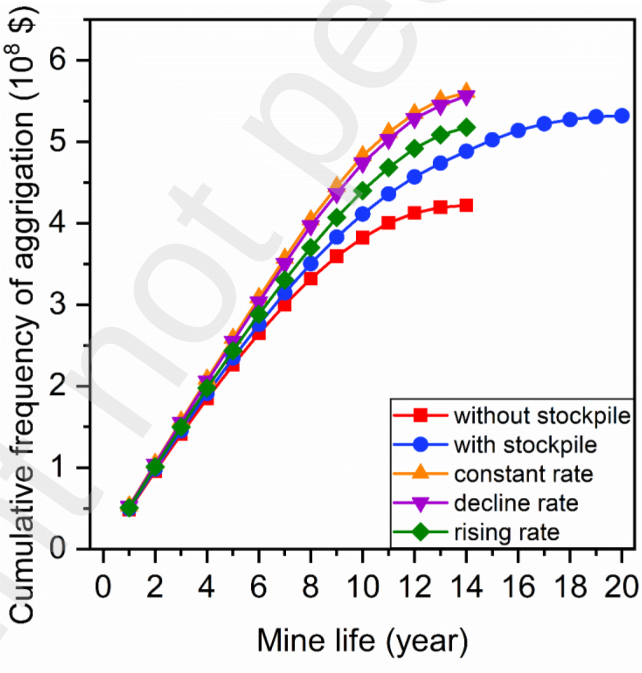

Fig. 16. Comparison of the cumulative frequency of the different scenarios.

Also, Figure 16 shows the cumulative frequency of different scenarios, suggesting that the stockpile option from the seventh year of the mine life with a constant rate is the best option available.

6 Conclusion

Stockpiling is a critical aspect of production planning since it directly impacts the ideal mining rate, cut-off grade strategy, and mine profit, accounting for up to 15% of total mine profit. The proposed model, a mix of stockpile and optimization cut-off grades with consideration for restrictions, was studied in this research. The data and problem variables, and the problem's objective function, were determined, and a numerical example was computed. This research’s financial results showed that the optimum cut-off grade at the beginning of the mine life is 2.338% and 16.502% for lead and zinc, respectively. In the final years, it reaches 0.301% and 5.799%, respectively. NPV during the mine life is $48481667.

On the other hand, the stockpiling option was evaluated in the following scenarios. Since the materials in the deposit are computed after mining and in parallel with the mining operation, the NPV in these scenarios is more significant than in the first scenario. This study reveals that a genetic meta-heuristic algorithm is an effective tool for tackling polymetallic ore deposits, more precise and efficient than existing methods. while analytical methods such as the Lane model are incapable of determining the optimal cut-off grade sites.

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8 Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

9 Data availability

The datasets supporting the case study are available by contacting the corresponding author. (10.6084/m9.figshare.24516208)

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