Cairo University
Faculty of Engineering
Aerospace Engineering Dept.

Report (2)
AER 3110 - Aerodynamics
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Assignment 2: Q11, 12

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Contents

Part I

Question 11:

The velocity potential of a two-dimensional motion is given by;

$\phi = kxy$; k = const.

Determine the stream function equation and sketch the flow pattern .

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}, \frac{\partial\phi}{\partial y} = \frac{-\partial\psi}{\partial x}$$

$$\frac{\partial\phi}{\partial x} = ky = \frac{\partial\psi}{\partial y}$$

$$\psi = \frac{k}{2}y^2 + f(x)$$

$$\frac{\partial\phi}{\partial y} = kx = \frac{-\partial\psi}{\partial x}$$

$$\psi = \frac{-k}{2}x^2 + g(y)$$

By comparison

$$f(x) = \tfrac{-k}{2}x^2,\ g(y) = \tfrac{k}{2}y^2$$

$$\psi = \frac{k}{2}(y^2 - x^2)$$

or

$$\frac{\partial\phi}{\partial x} = u = ky,\ \frac{\partial\phi}{\partial y} = v = kx$$

$\psi = \int kydy = \frac{k}{2}y^2 + f(x)$
$\frac{\partial\psi}{\partial x} = f`(x) = v = -kx$
$\int f`(x) = \frac{-k}{2}x^2 + c$
$\psi = \frac{k}{2}y^2 + \frac{-k}{2}x^2 + c$

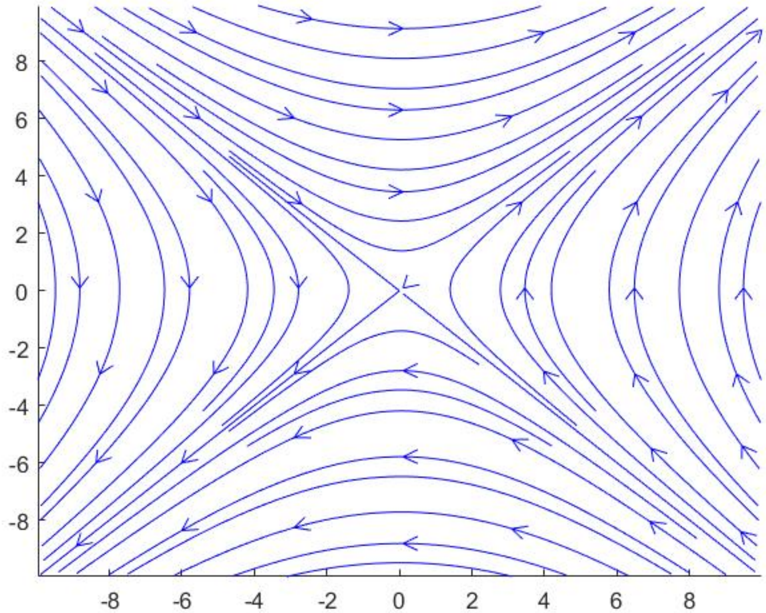

Figure 1: Stream function

Part II

Question 12:

a) Show that the two-dimensional flow described by the equation $\psi = x + 2x^2 - 2y^2$, is irrotational.

b) What is the velocity potential function of that flow?

c) If the density of the fluid is $1.12 \mathrm{Kg/m^3}$ and the pressure at the point $(1, -2)$ is 4.8kPa , what is the pressure at the point $(9, 6)$?

a) irrotational flow means:

$$\nabla^2 \psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} = 4,\ \frac{\partial^2 \psi}{\partial y^2} = -4 \rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

b)

$$\frac{\partial \psi}{\partial x} = 1 + 4x = -\frac{\partial \phi}{\partial y} = v,\ \frac{\partial \psi}{\partial y} = -4y = \frac{\partial \phi}{\partial x} = u$$

$$\phi = -4xy + f(x)$$

$$\frac{\partial \phi}{\partial y} = -4x + f`(x) = v = -(1+4x) \to f`(x) = -1 \to f(x) = -y + c_1$$

$$\phi = -4xy - y + c_1$$

or

$$\frac{\partial \phi}{\partial y} = -1 - 4x,\ \frac{\partial \phi}{\partial x} = -4y$$

integrate both equations

$$\phi|_y = -y - 4xy + f(x),\ \phi|_x = -4xy + g(y)$$

by comparison:

$$f(x) = 0,\ g(y) = -y \to \phi = -4xy - y$$

c)

$$-1 - 4x = v,\ -4y = u$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

for the X-direction:

$$-4y * 0 + (-1 - 4x)(-4) = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}(0)$$

$$\frac{-1}{\rho}\frac{\partial p}{\partial x} = 4 + 16x \to p = 4x + 8x^2 + g(y)$$

$$\frac{\partial p}{\partial y} = g`(y)$$

for the Y-direction:

$$16y + (-1 - 4x) * 0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}(0) \to g`(y) = 16y$$

$$g(y) = 8y^2 + c_2$$

$$p = -\rho(4x + 8x^2 + 8y^2 + c_2)$$

at the point $(1, -2)$ p is 4.8kPa $\longrightarrow c_2 = -4329.71$

$$p = -\rho(4x + 8x^2 + 8y^2 - 4329.71)$$

at $(9, 6)$ p is:

$$p = -1.12(36 + 648 + 288 - 4329.71) = 3760.6352pa$$