Sharif University of Technology

Advance Neuroscience

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Evidence Accumulation

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1 Part 1

Aim: Simulation of evidence accumulation model and studying the relationship between accuracy and reaction time (RT) in decision making. We can represent a discrete version of diffusion drift model (DDM) for evidence accumulation as follows [**1**]:

$$dX = Bdt + \sigma dW$$

Here, dX represents the change in evidence (decision variable) during the time step dt B is a constant bias that directs the evidence total over time dW, represents a discretized Brownian motion term. Simply put, a Brownian motion is a random walk in which steps follow a Gaussian distribution. Formally, the Brownian motion term is characterized by three properties:

1. W(0) = 0
2. W(t) is continuous in t
3. For any two values of t, t1, and t2, the difference between W(t1) and W(t2) follows an independent normal distribution with mean 0 and variance equal to the difference t2-t1.

Consequently, one important aspect of the Brownian motion term is its scaling with respect to time. Since dW follows an N(0, dt) distribution, increasing or decreasing the time step size increases or decreases the variance of the random portion of the walk. The scalar scaling constant allows for adjustments to uncertainty. To complete the simple model, we need a start point x0. In most cases, we will choose a value of 0 for x0, so that a positive accumulation will indicate a positive choice.

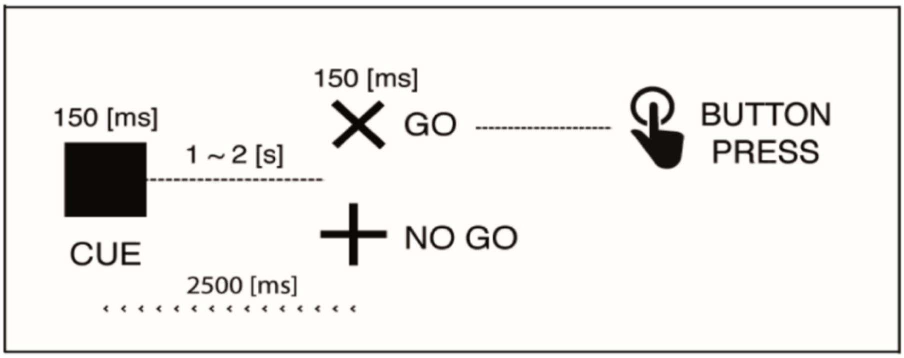


Figure 1: The general form of a Go/No Go task.

1. Here I am going to find the $X(t)$. We have:

$$dX = Bdt + \sigma dW$$

where

$$dW(t_i) \sim \mathcal{N}(0, dt)$$

$\overset{\int}{\Longrightarrow}$

$$\int_0^t dX = B\int_0^t dt + \ \sigma \int dW$$

If we assume $dW(t_i)$ are independent:

$$X(t) - X(0) = Bt + \sigma \sum dW$$

where

$$X(t) \sim \mathcal{N}(X(0) + Bt, \sigma t)$$

2. As we see on last section, $X(t)$ has a normal distribution with $\mu = X(0) + Bt$ and $\sigma^2 = dt$. Here we assume that if $X(t) > 0$, the decision is positive and vice versa, and with this point that we have two choices, so the probability of positive choice (1) comes from a Bernoulli distribution. To calculate the probability P we have:

$$P := p(X(t) > 0)$$

So P in Bernoulli distribution can be calculated from of $x(t)$ CDF.

$$p(X(t) > 0) = 1 - p(X(t) \leq 0)$$

$$p(X(t) > 0) = 1 - (0.5 \times (1 + erf(\frac{0 - \mu}{\sigma_1\sqrt{2}})))$$

where:

$$\mu = Bt + X(0) \ and \ \sigma_1 = \sqrt{\sigma t}$$

In Figure 3, we can see How $X(t)$ acts in different trials. Also we can see the probability P form 20 trials stimulation and P calculated from above formula in Figure ??. As we can see, the values from simulation and CDF are very close. The simulation parameters were: $B = 1$ $\sigma = 1$ $dt = 0.1$. The whole time of simulation was 1 second.

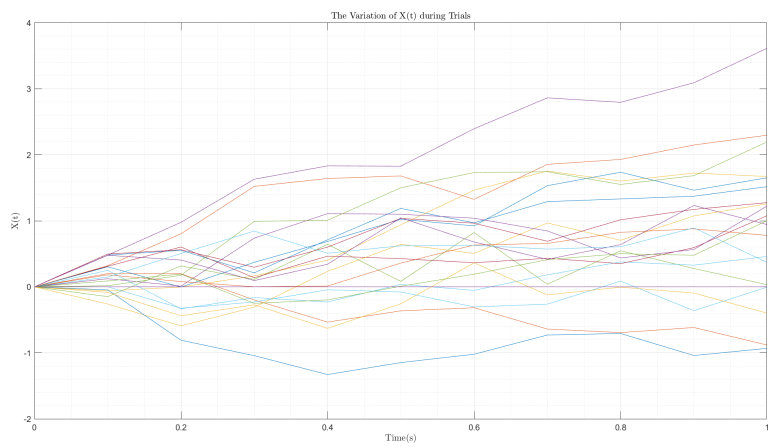


Figure 2: 20 Trials Simulation $X(t)$.

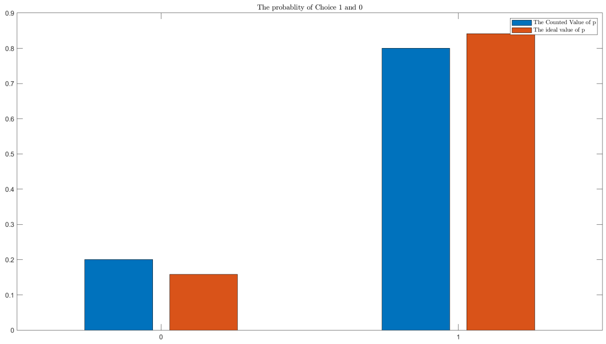
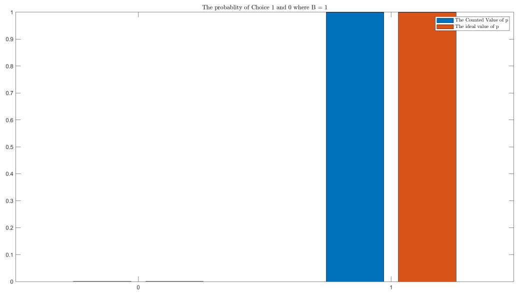
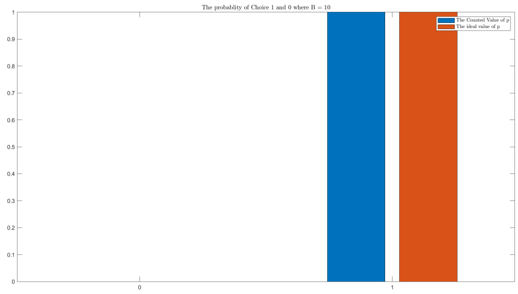


Figure 3: The Calculated Values of P from simulation and CDF.

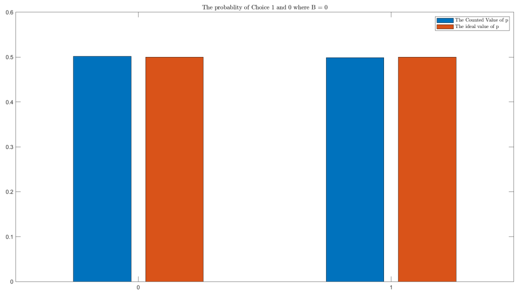
3. To see the effect of the model's parameters, I generate the last figure for different values for B. The values for B was = [1 , 10 , 0.1 , 0 , -1] and time of simulation was 10 seconds. As we can see in Figure 4, wit bigger B, we have correct decision with more probability (1 is correct and 0 is error). Also we can see with $B = 0$, we have same probability for correct and false decision; and for $B < 0$, the error rate is more than 0.5. We can define Error Rate as 1-P (the probability of 0). Another important point that we can see is the calculated probability from simulation is exact the same with calculated from CDF.



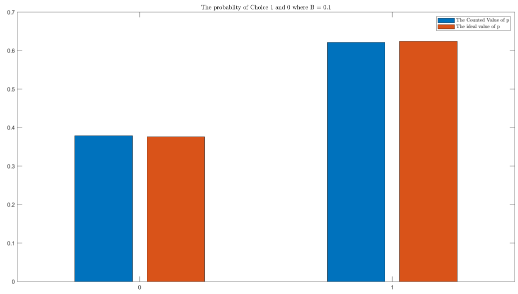
(a) $B = 1$



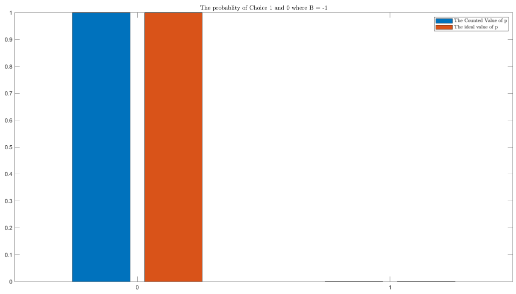
(b) $B = 10$



(c) $B = 0.1$



(d) $B = 0$



(e) $B = -1$

Figure 4: P Changes Over Different Value for B

4. As we said before, the error rate will be $1 - P$. We have:

$$P = 1 - (0.5 \times (1 + erf(\frac{0 - \mu}{\sigma_1 \sqrt{2}})))$$

where:

$$\mu = Bt + X(0) \ and \ \sigma_1 = \sqrt{\sigma t}$$

So:

$$1 - P = (0.5 \times (1 + erf(\frac{0 - (Bt + X(0))}{\sqrt{2\sigma t}})))$$

With that, we expect that with bigger time, we have smaller error rate. As we can see in Figure 7, we can see this change of error rate over time.

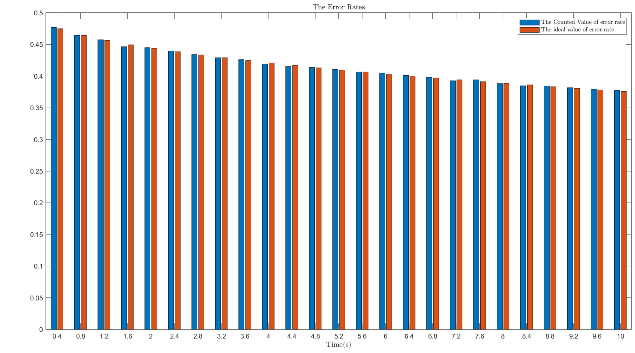
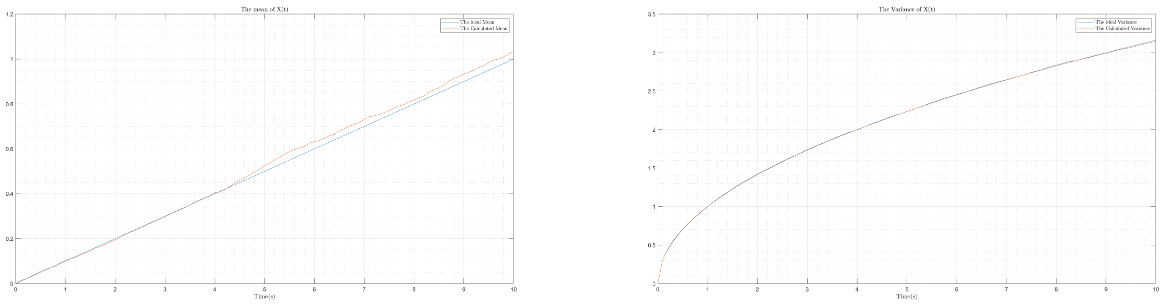


Figure 5: Simulation and Ideal Error Rates changes Over Time.

5. Here we want compare the simulation mean and standard deviation of $X(t)$ with calculated one from last section. As we see in section 1, the mean has a linear relation with time and standard deviation has a square root relation with time. As we can see in Figure 6, the difference between simulation and calculated mean and standard deviation is very small and they are really close.



(a) Mean

(b) Standard Deviation

Figure 6: Mean and Standard Deviation Comparison.

6. Here we want to see the effect of other parameters on P. As we can see in Figure **??**, the effect of other parameters, which includes : Starting Point, Time of trial, and Bias, are almost the same. Although there are some differences in amount of the effect, but all of them have positive correlation with P.

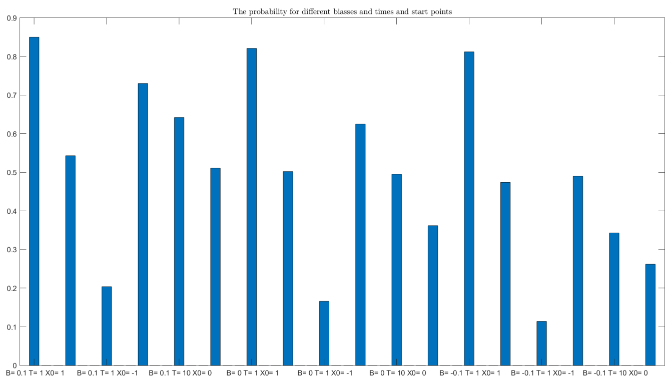
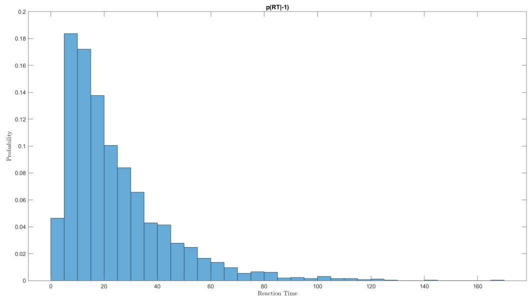


Figure 7: The Effect of Parameters on P.

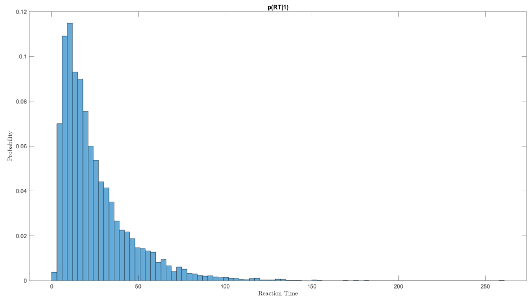
7. In this section, we are going to see the relation between Reaction Time and Error Rate. First we assume two Thresholds for each correct and incorrect choices. Then when we cross one of the thresholds, the decision is taken and Reaction Time can be calculate. As we can see in Figure 8, for Choice 1, we have a longer tale in Reaction Time distribution and the shape of it is narrower than Reaction Time in Incorrect Choice. This result is compatible with [**2**], where incorrect choices have smaller Reaction Time.

It can be meaningful in this way that when we have error, we choose very fast which we don't have enough time to accumulate enough evidence, or the trial has ambiguity in it.

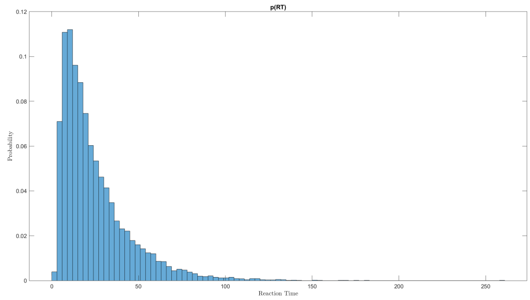
With the assumption that we don't have ambiguity here, the results also compatible with our background. Another important point is the Reaction Times distribution is like Inverse Gaussian distribution, which is also noted in [**2**]. The reason is that we have a Drift Diffusion, where we accumulate evidences like a Drift Diffusion, and when we cross one threshold, we make our decision.



(a) Incorrect Choice Reaction Time Histogram



(b) Correct Choice Reaction Time Histogram



(c) Reaction Time Histogram

Figure 8: Different Conditions Reaction Times.

8. Here we want to create a race model where two racer X_1 and X_2, race against each other to see which one cross it's threshold first. We can see one race in Figure 9, where the racer X_2 is winner.

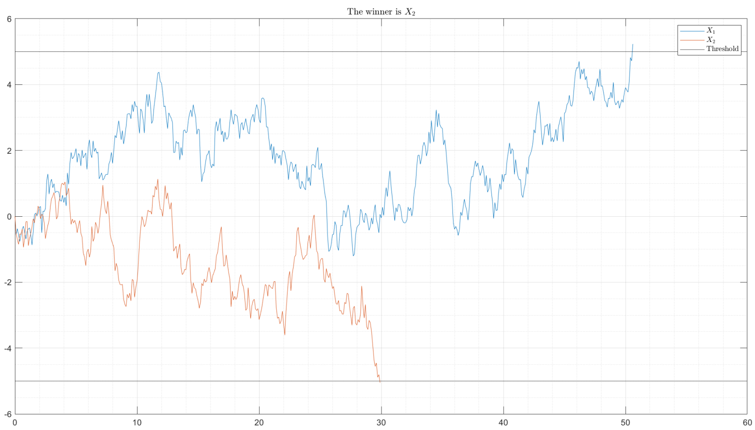
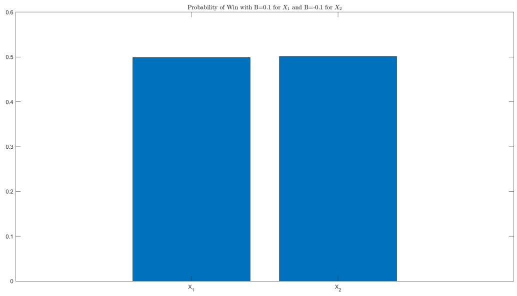


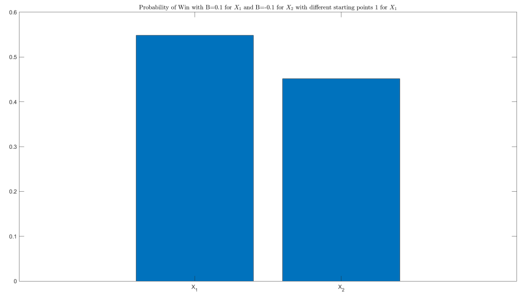
Figure 9: The Race Between X_1 and X_2.

So we create our model, we can see the effect of parameters in probability of winning each racer. As we can see in Figure 10, each racer parameter has a positive correlation with the probability of winning that racer.

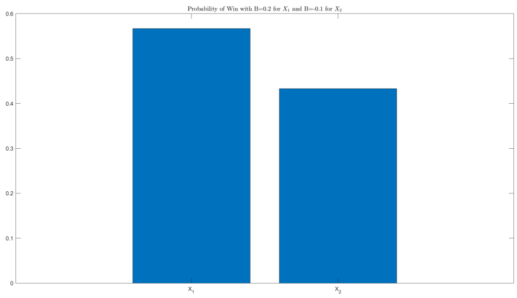
9. In this section, I try to my model for the time we have constant Time interval. As we can see in Figure



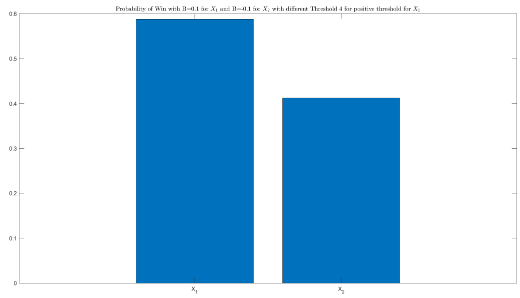
(a) B for X_1 = 0.1 and B for X_1 = -0.1



(b) B for X_1 = 0.1 and B for X_1 = -0.1 with Different Starting Point 1 for X_1



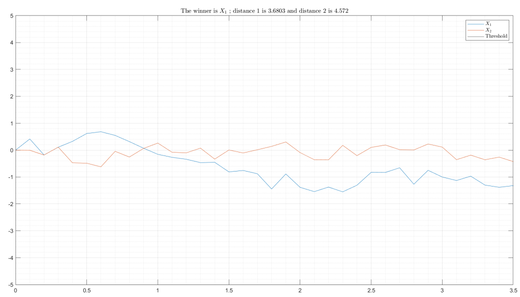
(c) B for X_1 = 0.2 and B for X_1 = -0.1



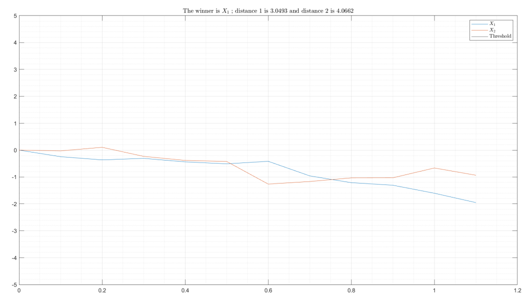
(d) B for X_1 = 0.1 and B for X_1 = -0.1 with Different Thresholds +4 for X_1

Figure 10: Effect of Different Parameters on Winner.

13, at the end of the time interval, the distance between each racer and it's threshold is computed and the smaller distance is the winner.



(a) Time of Trial = 1 Second



(b) Time of Trial = 3.5 Second

Figure 11: Two Different Races With Different Time Interval.

Also, we can see the probability of winning for each racer over different time intervals in Figure 12. It seems that there is no correlation between different time intervals and probability of winning each racer.

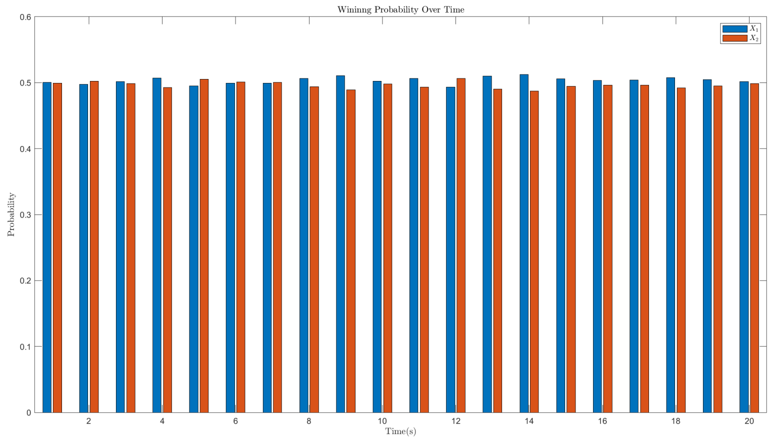
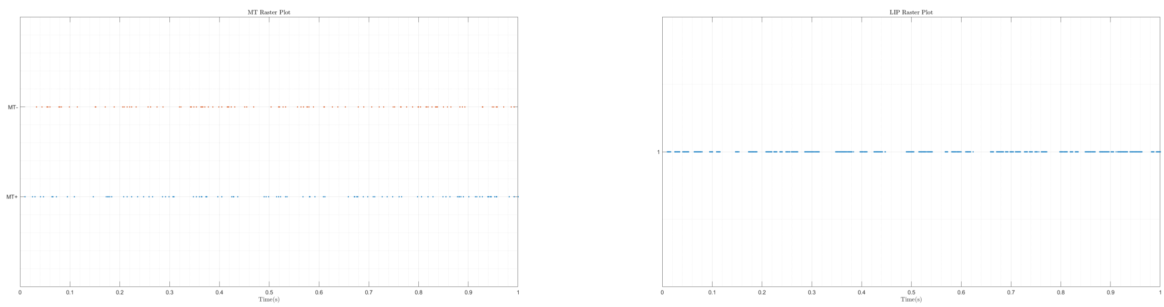


Figure 12: Probability of Winning Over Different Time Intervals.

2 Part 2

Aim: Implementing the model proposed by Shadlen and Newsome (2001)[**3**] for the interaction between area MT and LIP

1. We can simulate two MT Neurons with different weights and Different signs in synapses with LIP Neuron wit the given Model. We can see the raster plot of two MT Neurons and LIP Neuron in Figure **??**. As it can be seen, the weight of positive synapse is more than negative synapse.



(a) Time of Trial = 1 Second

(b) Time of Trial = 3.5 Second

Figure 13: Two Different Races With Different Time Interval.

Also, we can see the rate of LIP Neuron. As we know from [**3**], LIP Neurons get inputs from MT Neurons and in fact, they accumulate the evidences from MT Neurons and when the rate cross a threshold, the decision is made. We can see the evidence accumulation process in Figure 14, which means our LIP Neuron is accumulating the evidences.

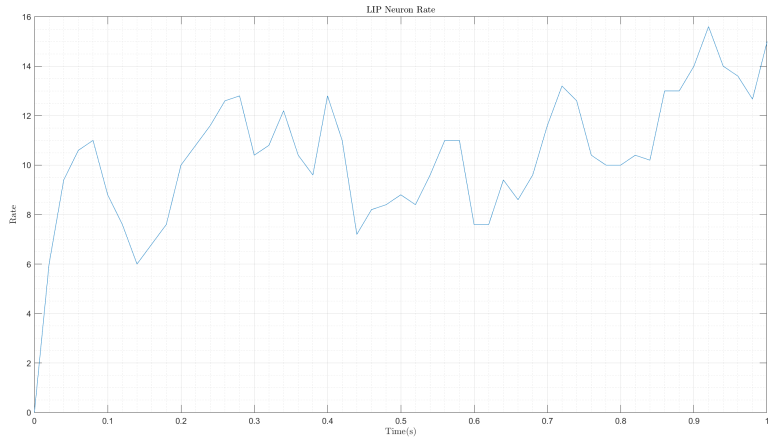


Figure 14: LIP Neuron Rate.

2. Assume the neural network shown in Figure 15. Here I am trying to model a simple neural network which contains two MT Neurons and two LIP Neurons.

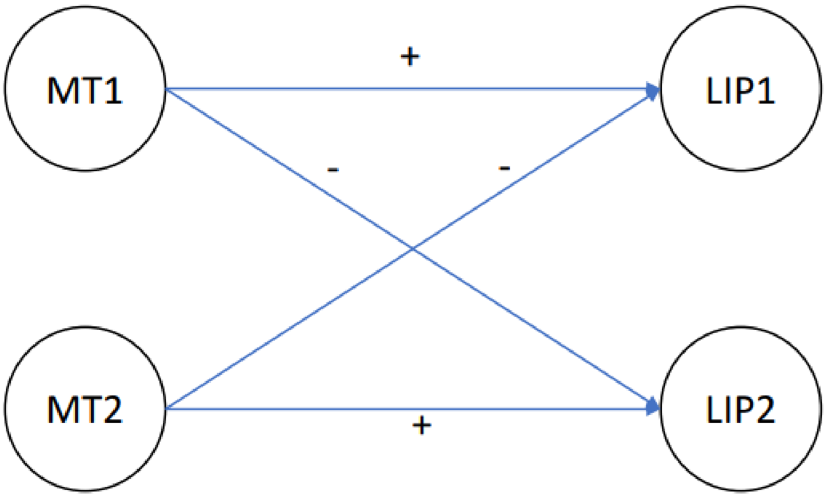
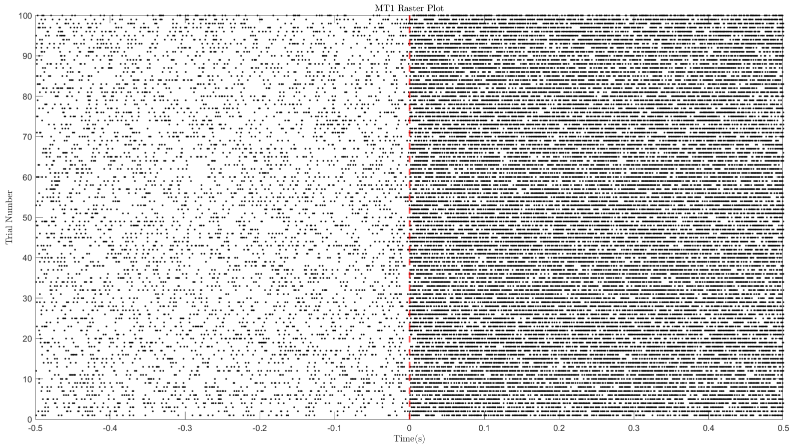
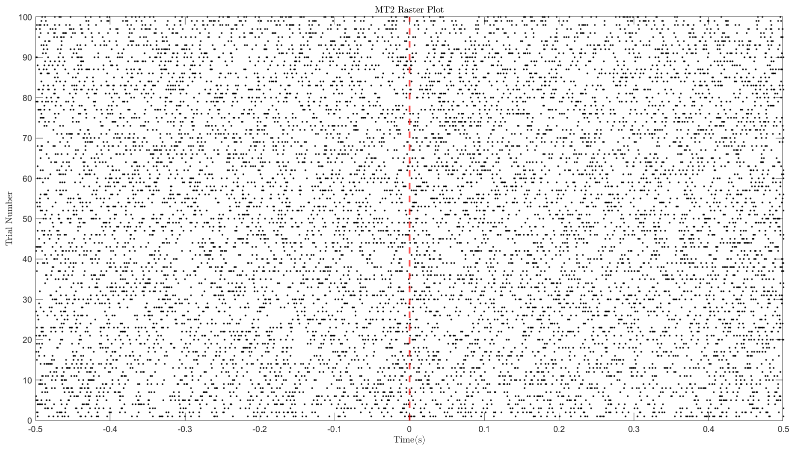


Figure 15: The Desired Neural Network.

Assume we have a trial, where before onset, we are showing an stimulus which is not in Receptive Field of MT1 neither MT2. After onset, we change the stimulus and put it on Receptive Field of MT1. The MT1 firing rate will increase while the MT2 firing rate doesn't change and continue firing with it's baseline rate. We can see this change in Figure 16.



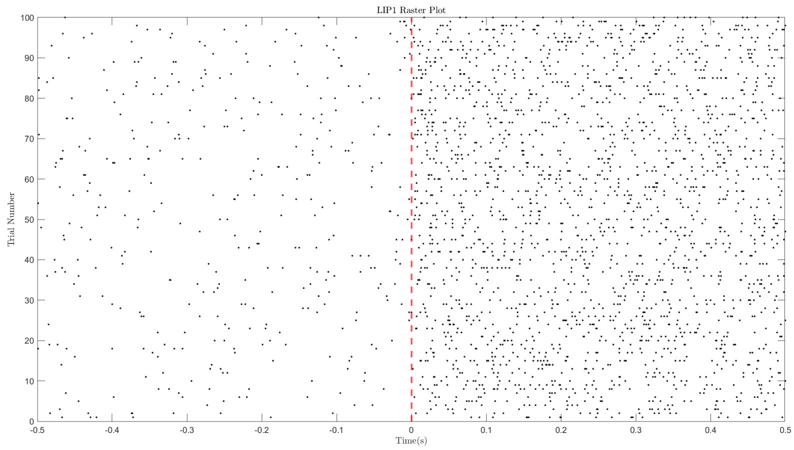
(a) MT1 Raster Plot



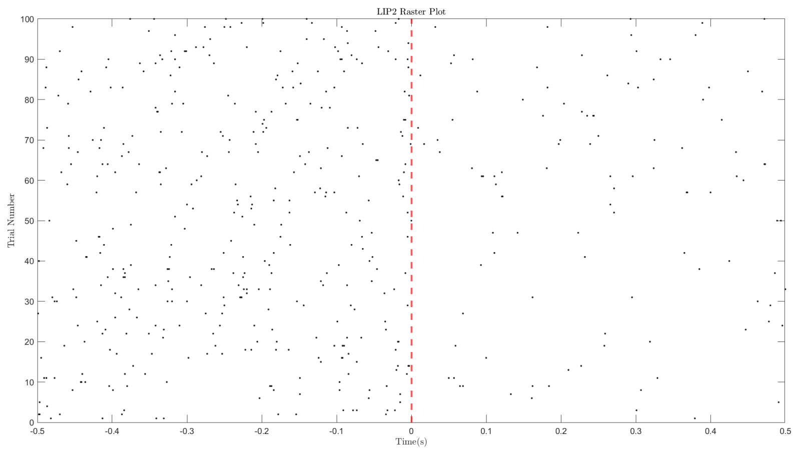
(b) MT2 Raster Plot

Figure 16: Two MT Neurons Raster Plot.

With considering the desired neural network, we can find the spikes of two LIP Neurons we have in Figure 15. We can see the raster plot in Figure 17. As we can see, the neuron with positive synapse with MT1, has an increase in it's rate (LIP1), and the neuron with negative synapse has a decrease in it's rate(LIP2). We can see this also in Figure 18. This was a good example of a simple neural network contains MT and LIP Neurons.



(a) LIP1 Raster Plot



(b) LIP2 Raster Plot

Figure 17: Two LIP Neurons Raster Plot.

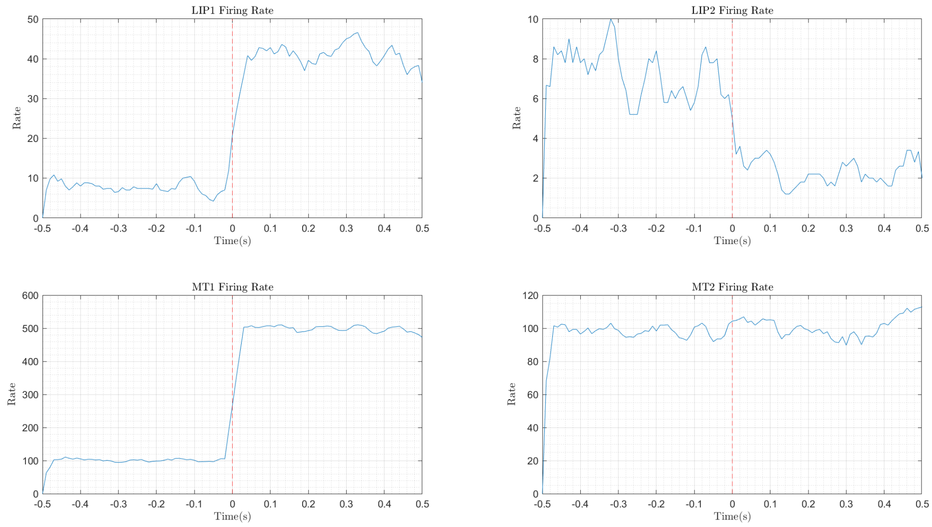


Figure 18: The Neural Network's Neurons Firing Rates.

References

[1] Ratcliff, R. (1978). A theory of memory retrieval. Psychological Review, 85(2), 59–108. https://doi.org/10.1037/0033-295X.85.2.59

[2] Ratcliff R, McKoon G. The diffusion decision model: theory and data for two-choice decision tasks. Neural Comput. 2008 Apr;20(4):873-922. doi: 10.1162/neco.2008.12-06-420. PMID: 18085991; PMCID: PMC2474742.

[3] Shadlen MN, Newsome WT. Neural basis of a perceptual decision in the parietal cortex (area LIP) of the rhesus monkey. J Neurophysiol. 2001 Oct;86(4):1916-36. doi: 10.1152/jn.2001.86.4.1916. PMID: 11600651.