Amirkabir University of Technology
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Application of Wavelet on Electromagnetic Integral Equation

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Brief Overview

- Basics of wavelets.
- Overview of electromagnetic integral equations.
- Applications and results.

Importance of Integral Equations

- Provide a mathematical framework for solving electromagnetic problems.
- Convert differential equations into a solvable integral form.
- Enable numerical solutions for complex geometries and boundary conditions.

$$f(x) = \lambda \int_a^b K(x,t)\phi(t)\,dt + g(x)$$

Motivation of Using Wavelet

- Efficient representation of functions and signals.
- Localized analysis in both time and frequency domains.
- Reduction in computational complexity for solving integral equations.

Wavelet Basis

- Wavelet basis functions provide localized representations of signals.
- Constructed through dilation and translation of a mother wavelet.
- Enable sparse representation of complex signals.

$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k),\ k \in \mathbb{Z}$$

Types of Wavelets

- Haar wavelets: Simplest wavelets with step-like functions.
- Daubechies wavelets: Compactly supported wavelets with various smoothness.
- Morlet wavelets: Used for time-frequency analysis.

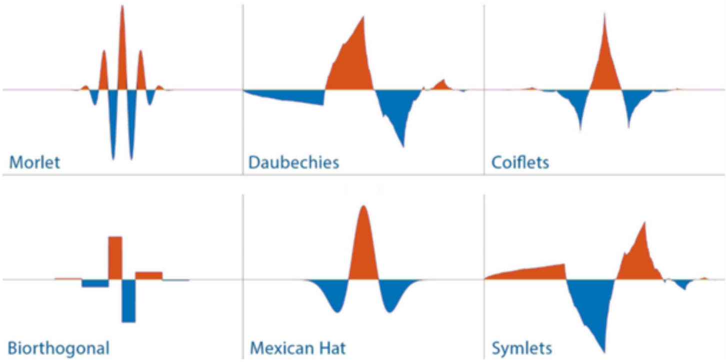

Figure 1: Different Wavelets.

Advantages of Wavelet

- Multi-resolution analysis for efficient signal representation.
- Localized analysis in time and frequency domains.
- Reduced computational complexity for large-scale problems.

Types of Integral Equations

- Fredholm integral equations: Defined over a fixed range.
- Volterra integral equations: Defined over a variable range.
  - Boundary integral equations: Commonly used in electromagnetic problems.

$$f(x) = \int_a^x K(x,t)\phi(t)\,dt + g(x)$$

Volterra IE

$$f(x) = \lambda \int_a^b K(x,t)\phi(t)\,dt + g(x)$$

Fredholm IE

Challenges with Traditional Methods

- High computational cost for large-scale problems.
- Poor convergence for complex geometries.
- Difficulty in handling multi-scale behavior.

Why Wavelet Help

- Provide sparse representations of integral operators.
- Reduce computational complexity with efficient algorithms.
- Handle multi-scale behavior effectively.

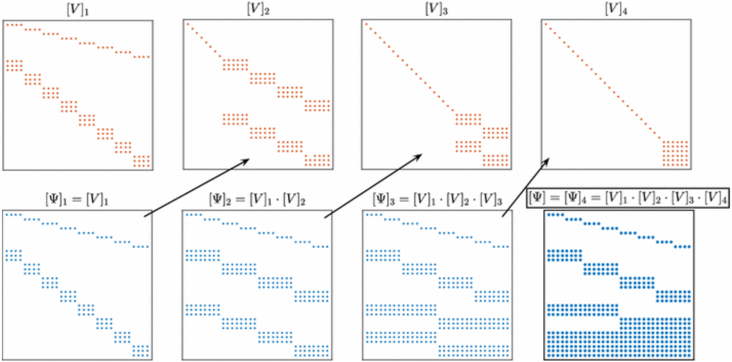

Figure 2: Example Sparse Matrix

Mathematical Foundation

- Underlying mathematical principles of wavelet theory.
- Connection between wavelet transforms and integral equations.
- Basis for developing efficient computational algorithms.

$$\sum_{j=0}^{n-1} k(\theta_i - \theta_j)\rho(\theta_j)w_j = f(\theta_i), \quad i = 0, 1, \ldots, n-1.$$

$$\mathbf{K}\boldsymbol{\rho} = \mathbf{f},$$

Solved Case

- Case study of wavelet application in a real-world problem.
- Comparison with traditional methods.
- Demonstration of improved efficiency and accuracy.

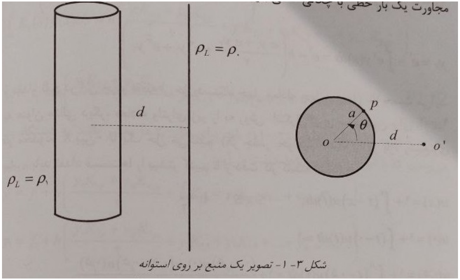

Figure 3: Case Study

Results

- Solved Case for two diferent compilers,"Matlab and Python".

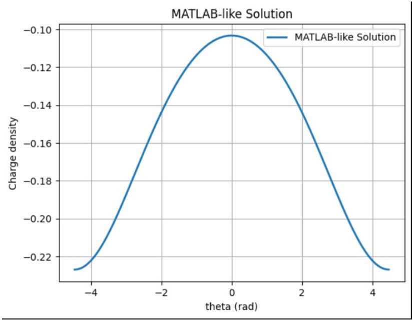

Figure 4:MATLAB-like Solution

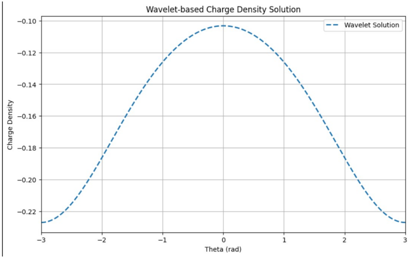

Figure 5:Wavelet-based Charge Density Solution

Plots

- Extraction of Coefficient with Multi-Resolution method.

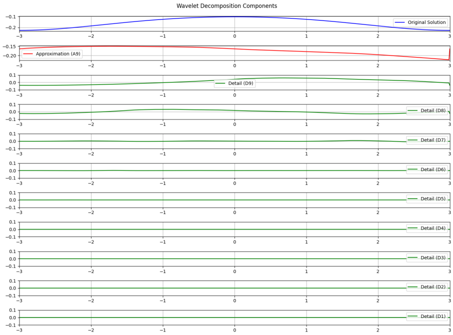

Figure 6:Detailed and Approximation

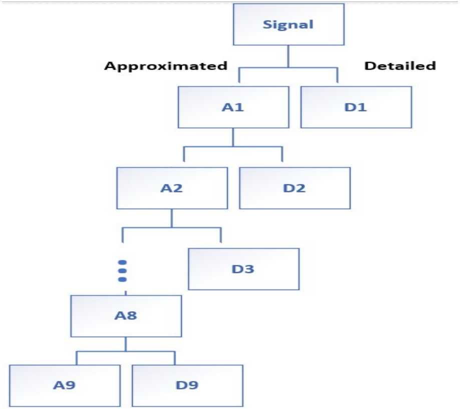

Figure 7:Tree Graph

Conclusion

- Benefits of wavelets for electromagnetic integral equations.
- Wavelet or other methods.

References

1 G.Moradi, Advanced Engineering Mathematics,AmirKabir University Of Technology, 2012 .

2 I. Daubechies, Ten Lectures on Wavelets, Society for Industrial and Applied Math ematics, 1992.

3 Github link,Application of Wavelet on EFIE at https://github.com/MohammadMahdiElyasi/Application-of-Wavelet-in-Elctromagnetic-Integral-Equation.

Thank you for your attention!

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