



Amirkabir University of Technology
(Tehran Polytechnic)

Mellin Transform

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Introduction

- Brief introduction to Mellin Transform.
- Applications in mathematical analysis.
- Why mellin transform?

Mathematical Background

- Key mathematical definitions.
- Important integral formulas.

$$\mathcal{M}\{f(x)\}(s) = \int_0^{\infty} x^{s-1} f(x) dx \quad (1)$$

$$\Gamma(s) = \int_0^{\infty} x^{s-1} e^{-x} dx \quad (2)$$

Property Proof

- Linearity of Mellin Transform.
- Scaling properties.

$$\mathcal{M}\{x^a f(x)\}(s) = \mathcal{M}\{f(x)\}(s + a). \quad (3)$$

$$\mathcal{M}\{f(ax)\}(s) = a^{-s} \mathcal{M}\{f(x)\}(s). \quad (4)$$

Examples

- Examples.

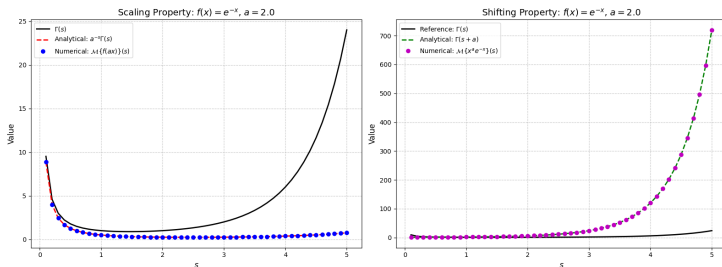


Figure 1: Shifting and Scaling Plots

Gamma and Mellin Transform Relations

- Connection between Gamma functions and Mellin Transform.
- Relation of arithmetic number factorial and gamma function.

if $f(x) = e^{-x}$:

$$\mathcal{M}\{e^{-x}\}(s) = \int_0^{\infty} x^{s-1} e^{-x} dx = \Gamma(s). \quad (5)$$

$$n! = \Gamma(n+1) \quad (6)$$

Case Study Result

- Description of the case study.

$$r^2 \phi_{rr} + r \phi_r + \phi_{\theta\theta} = 0 \quad (7)$$

$$\phi(r, \alpha) = f(r), \quad \phi(r, -\alpha) = g(r), \quad 0 \leq r < \infty, \quad (8)$$

$$\phi(r, \theta) \rightarrow 0 \quad \text{as} \quad r \rightarrow \infty \quad \text{for all} \quad \theta \in (-\alpha, \alpha). \quad (9)$$

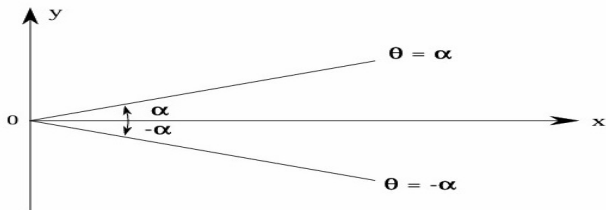


Figure 2:Infinite Wedge

- Plot for phi in designated variables.

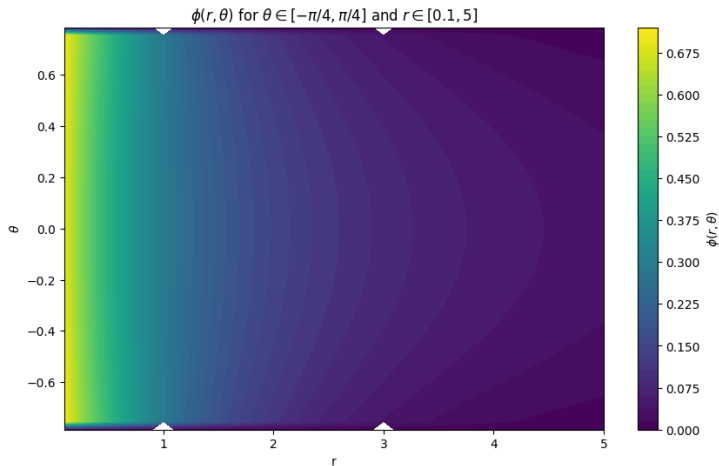


Figure 3:Phi plot

Conclusion

- Summary of key takeaways.

- Debnath, Lokenath, and Dambaru Bhatta. *I Transforms and Their Applications*. 2nd ed., CRC Press, 1995.
- Github link, *Mellin-transform* at <https://github.com/MohammadMahdiElyasi/Mellin-transform>.

Thank you for your attention!



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