Two-Player Age of Information Game

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https://github.com/MohammadMahdiGhahramani7/TwoPlayerAoIGame_GameTheory

Abstract

This paper examines a scenario of periodical status updates sent from a transmitter to an intended receiver in the presence of an eavesdropper, modeling it as a static game between two active rational agents, with the transmitter being able to control the rate of data packet generation and consequently the load on the network, while the eavesdropper decides on its success rate of capturing these packets. A cost term is also considered, in association to both. We aim to provide an analytical solution to this model, as well as a software implementation supplying numerical results. These results are later analyzed to draw (sometimes counterintuitive) conclusions, and compared with other related contributions.

I. Introduction

Over the last 20 years, since the late 1990's [1-3], game theory has often served as a useful analytical tool for the study of wireless networks, growing in popularity with the rise of interest in decentralized networks and the increased complexity of the nodes making up these systems [4]. This is undoubtedly due to the capability of game theory to explore multi-agent interactions in increasingly complex environments where the autonomous decisions of each separate node have a direct effect on the whole network.

For the scope of this paper, we are considering a simple scenario of a network composed of just three nodes: a transmitter sending periodical status updates (T), the intended receiver of said updates (R), and a malicious eavesdropping node (E) whose intention is to intercept the transmitted packets and gain access to the information they contain. Needless to say, in such a setup, it is of critical importance for the information to be as recent as possible. A whole new research field has developed around the freshness of status updates and Age of Information (AoI) has emerged as the most widely used performance metric for quantifying it [5]. It is worth noting that ensuring the timeliness of an update is different from maximizing the throughput of the system, or minimizing the average transmission delay [6]. The AoI at the receiver is measured as:

$$\delta(t) = t - \sigma(t) \tag{1}$$

where t is the current time and $\sigma(t)$ the timestamp of the most recent packet. An update would be considered fresh if its timestamp was equal to the current time t, and therefore its age of information would be zero. In order to allow for equal computation of timestamps at both the transmitter and the receiver, we will not factor the propagation delay into the calculations. Such simplifications are commonly performed in literature [7-11]. However, the limited resources dictate that the updates must be periodical, which leads to a linearly increasing function of AoI, which gets reset to zero when the next update is received. The most common approach to modeling these systems is to represent the network as a queue to which the transmitter nodes supply status update packages in the manner of a stochastic process, with disregard for the state of the queue. These packages travel through the network as they get processed at the destination in first-come-first-served (FCFS) order. Kendall's notation is generally adopted to describe different types of queuing models, taking into consideration three main factors: A/B/c where A denotes the distribution of the inter-arrival times, B the distribution of the service time and c stands for the number of service channels.

The scenario we are analyzing consists of a single source and a single server, therefore c is 1. The simplest stochastic process that can be employed to model a queue is the M/M/1, where M stands for a Markovian process of exponentially distributed inter-arrival and service times. Denoting by λ the rate of packet generation (arrival at the back of the queue) and by μ the service rate at the receiver, the load/traffic intensity is $\rho = \lambda/\mu$. It has been proven that although the load approaching zero greatly reduces the delay while and the load approaching one increases the throughput none of these produce the optimal AoI, which is instead obtained through an intermediate value of $\rho = 0.531$ [6], updating frequently enough for the information to remain relatively fresh but not at such a high rate as to congest the queue [13-14].

The setup of this paper introduces a new twist through the presence of another actor, an eavesdropper which poses a threat to the confidentiality of the data. In this situation the goal of the transmitter is more complex, as besides ensuring the optimal AoI at the receiver, it also has to try and preserve the confidentiality of said information. Since the only thing within T's control is the rate at which it generates updates, this translates to maximizing the AoI at the eavesdropper so that the intercepted data is outdated and of little value. These are two clearly contrasting objectives, so we need to employ Bergson's theory of social welfare in order to define an appropriate function that would capture both objectives. Another factor to consider is that the higher the generation rate, the more resources are being consumed, and that is associated with a cost.

Moreover, E is an active actor on the system with its own strategy set and decision to make. The parameter within $E's$ control is the probability with which it can intercept a package, which we denote by β. Being a probability, β takes values in the range [0, 1]. Increasing the success rate also undoubtedly comes at a cost for E. For the sake of simplification and an indiscriminating analysis, we shall consider the costs to be the equivalent for both players.

All of the above is assumed to be common knowledge to the players, making this a game of Complete Information. We also assume that they both make their choice of parameters (their respective actions) simultaneously and independently of the other, resulting in what is known as a Static game.

The rest of this paper is organized as follows. In Section II we provide a brief literature review of models concerning the AoI of queuing systems, AoI models which incorporate the security issues as well as applications of game theory to the objective of AoI minimization. Section III discusses our proposed extension of approaching this problem from a game-theoretical standpoint, modeling it as a classic Static Game of Complete Information between two rational, utility-maximizing agents, whose individual choices inevitably affect the other's utility and the final outcome. Employing the analytical framework of game theory we then try to define the Nash Equilibrium (NE) of such a system, the outcome where neither of the agents has a unilateral incentive to deviate and change its tuning of the respective parameter. Section IV illustrates our findings, presenting numerical results, as well as a discretized version of the originally continues game, solved by use of the Gambit software. Finally, Section V concludes the paper. The code implementation can be found on the Github repository of https://github.com/MohammadMahdiGhahramani7/TwoPlayerAoIGame_GameTheory.

II. Related Works

A. AoI in queuing systems

There have been countless studies assessing the AoI in queuing systems, and more specifically the single-server queues. For example Champati et al. have analyzed the distributional properties of the AoI for a D/G/1 type queue with respect to the frequency of generating updates [16]. More generalized versions of queuing systems have been explored in [17]. Other researchers have focused on multi-source queuing models and described how the same service channel can be shared by several independent sources [18-19].

Especially useful are the several works which derive expressions for the average AoI of the respective queues under investigation, such as M/M/1 [6], D/M/1 [19], and M/D/1 [20].

Other studies have examined further extensions such as applying preemption policies [21] and packet management strategies [22-23] which discard or replace packets in order to shorten the queue length. However, for the purpose of this paper, we will not be considering lossy queues but will instead focus on the simpler M/M/1 model.

B. AoI and Security

Interestingly, there is not a lot of research integrating AoI with the security aspect. Some papers examine its use in the context of a security indicator [24-25], effectively using it as a tool to enhance the security. [26] describes a very complex Internet of Everything system which makes use of distributed computing and some network coding to connect the packages together in order to encode the message in such a way that it can be decoded even if some of the packages get damaged or delayed. [27] and [14] study a very similar scenario to the one presented on this paper, with [14] being the direct inspiration for it. Both papers concern a source whose goal is to transmit timely status updates to the legitimate destination while deterring the eavesdropper from obtaining that information. Crosara et al. approaches the issue of contrasting objectives from Bergson's theory of social welfare, while Chen et al. have focused on one particular trade-off value. However, neither of them analyzes the scenario from a game-theoretic perspective (nor do they factor in the cost). The source is the only active agent in the models described in these papers, and our contribution consists in reframing the problem as an adversarial game setup, where the Eavesdropper is an active agent as well, competing against the Transmitter (Source) to decrease its own age of information.

C. AoI and Game Theory

While studies on AoI have been going on for more than a decade and game theoretical tools and concepts have been exploited to investigate distributed networking, especially in an adversarial setup (exploring various security issues such as jamming or denial-of-service attacks) for the past thirty years [28] (with effort continuing to this day [29]), the trend of publications relating to AoI minimization games has been very recent.

Oftentimes AoI is not the only parameter taken into consideration, but part of a joint optimization, like in [30] where the authors define a single utility function based on the average AoI as well as the Signal to Interference and Noise Ratio (SINR) metric for a wireless sensor network (WSN) channel under attack. Similarly, a year later, the same authors perform another joint optimization of AoI and SIRN (this time also considering Completeness and Energy) in defining a Coalition Formation Game for UAV-aided Sensed Data Collection Networks (SDCNs). Two algorithms are proposed for identifying the coalitions and positions of the mobile nodes, leading us to the NE of the game [31].

A lot of research in this field focuses on mobile edge computing (MEC) and in particular Unmanned Aerial Vehicles (UAV). In UAV-aided MEC networks the freshness of the data is a critical component of major influence on the real-time data-driven decisions on the server side and therefore AoI minimization is a crutial objective. Besides [31], discussed above, we could mention [32], which uses a game theoretic formulation and the NE found through means of a stochastic learning-based algorithm to determine an optimal AoI-based channel access strategy which achieves better efficiency and timeliness of data transmission compared to the baselines. Papers [33] and [34] incorporate the security aspect, analyzing the networks under Channel Access Attacks (CAAs) which attempt to deteriorate the availability of valuable channel resources and the overall performance of the channel. Similarly, [35] also considers an UAV-aided network under attack (this time in the context of a traffic monitoring network), however it applies a completely different formulation, modeling the AoI optimization problem as a Stackelberg adversarial game with the malicious attacking node being the leader and the legitimate UAVs the follower.

Also, [36] expands on the CCA scenario by exploring dynamic CCAs on multichannel Internet of Things (IoT) networks. They define a utility maximization problem for each of the sensor nodes, based on their expected AoI under DCAAs with probabilistic ACK feedback, and then proceed to model it as a Potential Game, and put forward a distributed learning algorithm to find the NE.

Amongst other game theoretic investigations of the AoI metric we may mention [37]. which considers a very dense IoT monitoring system with multiple devices competing for channel access in order to transmit timely updates to their respective receivers. The problem is formulated and analyzed as a mean-field game, which allows for the examination of each device's asymptotic performance. A related paper is [38] which also views the nodes as distributed players competing over a shared resource, the channel, modeled using a simple collision-based scheme. This model, similarly to ours, also considers a cost for each transmission. Each node is driven by minimizing its own AoI and the associated cost. An alternative view of this problem is given in [39], where the researchers consider just two pairs of transmitter-receiver, causing interference to each-other. The strategy sets of both players are composed of the power levels at which they may choose to transmit. This setup allows the authors to discuss the effect of interference on the timeliness of information.

A highly original approach is followed in [40], where the goal of AoI minimization is contrasted with that of throughput maximization (the latter being more important for WiFi networks) in a repeated two player game where the players are no longer the nodes but rather two networks representing these two different objectives.

Another important contribution in this field is [41], which investigates a scenario in which two independent sources both send status updates to the same receiver, and as in [38] pay an individual cost for each update. Therefore the sources separately and periodically decide on whether or not to update, evaluating the global benefit of reducing the AoI at the receiver against their own cost. This setup is completely unique in its type, because the nodes are not competing as in the previously mentioned papers, nor are they explicitly collaborating. Instead each node behaves in accordance with its own selfish interest, while having a shared global objective in common. The author identifies three NEs of varying efficiency and also defines the price of anarchy. A similar but more complex setup is presented in [13], where the different sources do not generate exactly the same content, but neither is the data entirely separate and independent either. Instead, It is assumed there exists some correlation between their measurements which results in the system displaying collaborative-like behaviors.

1. III. Analytical Model

A. The queue

As already stated in the previous sections, this paper models the communication between the transmitter (T) and the receiver (R) as an M/M/1 FCFS packet queue. New status updates are generated by T following a Poisson process with rate λ, while the service time on R's end follow an exponential distribution with rate μ, resulting in load $\rho = \lambda/\mu$. Without loss of generalization, we may normalize μ by setting it to 1, effectively making ρ depend only on λ, in a direct relation of $\rho = \lambda$. We can do this since in our model μ is just a constant, because R is a passive entity rather than a rational agent and cannot modulate its processing rate. Therefore we could always simply rescale back the obtained results by a factor μ to account for any value different from 1.

Moreover, we assume that every packet sent by T arrives at R, error-free. E is only able to "listen" to the conversation, without corrupting the data or preventing it from reaching its destination, and the queue does not employ any preemption or packet management strategies.

A portion β of the generated updates are also received by E, arriving at E's queue according to a Poisson process with rate $\beta\lambda$. Assuming E too has a service rate equal to one, the load factor of the eavesdropping channel is $\beta\rho$.

B. The Game

This setup is an adversarial static game of complete information $G = (S_T, S_E, u_T, u_E)$. It is adversarial because the two rational agents have competing objectives with T trying to maximize the AoI at E while E aims to minimize it. However, it is not a zero-sum game, because T's objective is more complex, made up of two parts. We could describe each of them as a separate utility function u_1 and u_2, with u_1 corresponding to the goal of providing fresh, timely updates to R and u_2 to that of leaving only stale information to E.

$$u_1(\rho) = \frac{1}{\Delta_R(\rho)}\ ,\ \ u_2(\rho) = \Delta_E(\rho) \tag{2}$$

where Δ_R and Δ_E denote the expected AoI at R and E respectively. These are clearly contrasting objectives because each time T generates a new packet, it risks resetting the AoI at E to zero with probability β. In other words, decreasing Δ_R inevitably leads to Δ_E decreasing as well. There exists no way in which to increase one of the utilities without decreasing the other, therefore we could say that all possible solutions are included in the Pareto frontier and are Pareto efficient. This certainly is not very helpful in reaching conclusions, so we turn to Bergson's approach, considering the weighted product of the two utility functions, which allows us to find a specific optimal point based on a trade-off factor between the two objectives. The objective function for T is then ultimately defined as:

$$U_T = [u_1(\rho)]^{a+1}u_2(\rho) - c\lambda = \frac{\Delta_E(\rho)}{\Delta_R(\rho)^{1+a}} - c\lambda \tag{3}$$

where a, ranging in (0, $+\infty$), is the trade-off parameter, and c is the unit cost for increasing the packet generation rate. We write a+1 because the objective of sending updates to R cannot be completely ignored as without it we would have a Transmitter than never updates and the very premise of this scenario would cease to exist.

The full expressions for $\Delta_R(\rho)$ and $\Delta_E(\rho)$ are given in [14] as follows:

$$\Delta_R(\rho) = 1 + \frac{1}{\rho} + \frac{\rho^2}{1-\rho} \tag{4}$$

$$\Delta_E(\rho) = 1 + \frac{1}{\beta\rho} + \frac{\beta^2\rho^2}{1-\beta\rho} \tag{5}$$

The utility function for E is much simpler. Its aim is to decrease its AoI as much as the resources allow, sticking a balance between increasing its probability of successfully intercepting packets and the cost that entails. The function is defined as follows:

$$U_E = \frac{1}{\Delta_E(\rho)} - c\beta \tag{6}$$

The game is static because it is played only once (not repeatedly through different rounds), each player making a single decision, and that decision is taken independently, without knowledge of the other's choice. Therefore the whole game has a single information set.

Each of the players in this game has a continuous strategy set, made up of single action strategies of choosing the value of λ and β for T and E respectively. As discussed previously, the value 0.531 already serves as an upper bound to λ since the AoI starts to decline for values higher than that, therefore the strategy set S_T = (0, 0.531]. β represents a probability so the strategy set of player E, S_E = (0, 1].

The game is of Complete Information because all of the above, including the payoff functions for each of the players and their strategy sets, is common knowledge (all players know it and all players are aware that all other players know it).

C. Analytical Solution

In Game Theory the standard practice to analyze the outcome of the strategic interaction between several rational actors, especially in a non-cooperative context, is through the Nash Equilibrium [42]. A certain strategy profile is a Nash Equilibrium if each of the players is playing their best response to the choices of the other players. We evaluate a best response by maximizing the payoff function of player i while keeping the strategies of the other players (–i) fixed. If the strategy set had been discrete we could have simply picked the one giving the highest payoff from the normal form representation of the game, but since the game is continuous we need to get the derivative of the utility functions and equate them to zero in order to maximize them.

$$U_T = \frac{1 + \frac{1}{\beta\rho} + \frac{\beta^2\rho^2}{1-\beta\rho}}{(1 + \frac{1}{\rho} + \frac{\rho^2}{1-\rho})^{1+a}} - c\lambda, \tag{7}$$

$$U_T = \frac{1 + \frac{1}{\beta\lambda} + \frac{\beta^2\lambda^2}{1-\beta\lambda}}{(1 + \frac{1}{\lambda} + \frac{\lambda^2}{1-\lambda})^{1+a}} - c\lambda, \tag{8}$$

$$U_T = \frac{(\beta^3\lambda^3 - \beta^2\lambda^2 + 1)\lambda^a(1-\lambda)^{1+a}}{\beta(1-\beta\lambda)(\lambda^3 - \lambda^2 + 1)^{1+a}} - c\lambda \tag{9}$$

Then:

$$\lambda^\star = \arg\max_\lambda \; U_T. \tag{10}$$

The same calculation holds for E:

$$U_E = \frac{1}{1 + \frac{1}{\beta\rho} + \frac{\beta^2\rho^2}{1-\beta\rho}} - c\beta \tag{11}$$

$$U_E = \frac{1}{1 + \frac{1}{\beta\lambda} + \frac{\beta^2\lambda^2}{1-\beta\lambda}} - c\beta \tag{12}$$

$$U_E = \frac{\beta\lambda(1-\beta\lambda)}{\beta^3\lambda^3 - \beta^2\lambda^2 + 1} - c\beta \tag{13}$$

Then:

$$\beta^\star = \arg\max_\beta U_E \tag{14}$$

We can rewrite (10) and (14) as following:

$$\lambda^\star = \arg\max_\lambda \; \frac{f_1}{g_1} \tag{15}$$

$$\beta^\star = \arg\max_\beta \; \frac{f_2}{g_2} \tag{16}$$

Using derivation to solve these optimizations, we will have:

$$\lambda^\star = \{\lambda \mid \frac{\partial f_1}{\partial \lambda} g_1 = f_1 \frac{\partial g_1}{\partial \lambda}\} \tag{17}$$

$$\beta^\star = \{\beta \mid \frac{\partial f_2}{\partial \beta} g_2 = f_2 \frac{\partial g_2}{\partial \beta}\}. \tag{18}$$

By rewriting (17) and (18) we will have that λ^*s and β^*s should firstly make the the equation corresponding to (17) true:

$$\begin{gathered}
[((a+3)\beta^3\lambda^{a+2} - (a+2)\beta^2\lambda^{1+a} + a\lambda^{a-1})(1-\lambda)^{1+a} + \\
(\beta^3\lambda^{a+3} - \beta^2\lambda^{a+2} + \lambda^a)(-1-a)(1-\lambda)^a - (c\beta - 2c\beta^2\lambda) \\
(\lambda^3 - \lambda^2 + 1)^{1+a} - (c\lambda\beta - v\lambda^2\beta^2)(1+a)(\lambda^3 - \lambda^2 + 1)^a \\
(3\lambda^2 - 2\lambda)] * [\beta(1-\beta\lambda)(\lambda^3 - \lambda^2 + 1)^{1+a}] \\
= \\
[-\beta^2(\lambda^3 - \lambda^2 + 1) + (\beta - \beta^2\lambda)(1+a)(\lambda^3 - \lambda^2 + 1)^a \\
(3\lambda^2 - 2\lambda)] * [(\beta^3\lambda^{a+3} - \beta^2\lambda^{a+2} + \lambda^a)(1-\lambda)^{1+a} \\
- (c\beta\lambda - c\beta^2\lambda^2)(\lambda^3 - \lambda^2 + 1)^{1+a}],
\end{gathered}$$

and then they should make the equation corresponding to (18) true:

$$\begin{gathered}
[\beta^3\lambda^3 - \beta^2\lambda^2 + 1] * [\lambda - 2\beta\lambda^2 - 4c\beta^3\lambda^3 + 3c\beta^2\lambda^2 - c] \\
= \\
[\beta\lambda - \beta^2\lambda^2 - c\beta^4\lambda^3 + c\beta^3\lambda^2 - c\beta] * [3\beta^2\lambda^3 - 2\beta\lambda^2].
\end{gathered}$$

Since, it is impractical to solve the aforementioned system of equations analytically, we compute NE s through simulation carried out by a software and verify that the equations hold.

IV. Numerical Results

In this section we present quantitative evaluations of the game model described above, taking into consideration different values of the cost and of the trade-off parameter a. The Nash Equilibriums of the games are computed using Gambit [43], an open-source software designed for numerically solving non-cooperative games. Unfortunately, one of its limitations is that Gambit does not support non finite games, such as ours where the players have a continuum of actions to choose from. To the best of our knowledge, nor does any other of the available open-source tools and software. Therefore we have explored a discretized version of the game, by partitioning the continuous variables into intervals of size 0.01, providing precision up to the second decimal point, without blowing up the number of outcomes such as to make the computation infeasible. Before presenting the actual results, let us first comment on some intuition and the trivial solutions. If the cost is 0 the only rational choice for E is to maximize β. This would mean that each packet sent by T would be intercepted with absolute certainty and the AoI at both R and E will be the same at all times. There are two cases:

- $a = 0$
  This would mean that T places the same importance on the goal of evading the eavesdropper as it does on sending timely updates. In this case there is no optimal solution for T. We cannot identify a single best response as all its choices would lead to exactly the same payoff. In short, we have infinitely many NEs, where E plays 1 and T plays any of its possible choices or any combination of them.
- $a > 0$
  In this case, updating the receiver is a priority for T. Since it cannot avoid E eavesdropping on the information, T has no choice but to ignore it, transmitting at the rate which minimizes the AoI (the highest rate in the strategy set ST).

We also expect the cost to have a negative impact on both variables, and not just because it is associated with a negative sign. We mentioned above that a negligible cost would make E maximize its β. This is not simply so it has a better chance of obtaining the information, but also because based on intuition as well as the results discussed in [14], a higher β makes T transmit more often. A higher cost lowers β and this in turn results in a lower λ, even if there was not a cost factor associated with T itself. We solved different instances of the game, for several combinations of different values of the parameters a and c, and presented the NEs on the following plots. It is worth mentioning that the majority of the games had only one unique pure Nash Equilibrium. There were a few instances where two pure equilibriums were present (more concretely for (a=0.1, c=0.1) and (a=0.5, c=0.2)). In the first case the two equilibriums correspond to the two extreme choices for β, while in the second case the two equilibriums display consecutive values, so it is more probably a continuous set of NEs along the interval between these two values. There are also instances where the algorithm failed to find a pure NE. However, if we are to look at their mixed NE (which is always a unique one) it becomes clear that the issue arises due to rounding errors and the discretization of the strategy sets, because the strategy is a mix between two consecutive values. This tells us that the actual best response is an intermediate value between the two, closer to the one which is played with higher probability. Therefore, these can be considered as pure strategy NEs as well. None of the other games (the ones with clearly defined, pure NEs) have mixed strategy NEs.

Figure 1 shows the plots of pairs ($\lambda^\star$, $\beta^\star$) for various values of a, on different plots depending on the cost c. Figure 1.a illustrates the trivial solution mentioned above. For a equal to zero we have what appears to be a straight line, going through the full interval of ST. All the other values of a lead to the same NE: (0.53, 1.0), as predicted. This may not be very visible in the graph, because the points overlap and you can only see the last one, corresponding to a=25. In all subsequent graphs of Figure 1, both λ and β start at their minimum for a=0. A slight increase in a (from 0 to 0.1) leads to a noticeable increase in the transmission rate, but it has no effect on beta which continues to stay at its minimum, except for the case when the cost is negligible (c=0.1) and another NE presents itself, where E essentially ignores the cost and proceeds to increase β to the max. In accordance with the conclusions drawn in [14] and our expectations, the increase in β leads to a massive increase of λ, in this case by a factor of 4 (from 0.1 for minimal β to 0.4 for maximal β). λ keeps increasing with a up to a certain value of a before it starts decreasing again and dropping to the minimum. β appears to mimic the same behavior. The threshold of a for which this switch occurs increases based on the cost. For cost 0.1 for example, the rates keep growing till a=2, while for cost 0.5 the increase continuous till a=4. The maximum λ reached is somewhere between 0.39 and 0.47, depending on the cost. The cost factor impacts E much more than T. For any $c > 0.4$ the best response for E is to always play the minimum success rate, because no matter how much it's age of information might increase, it simply does not justify the costs. The impact on the transmission rate, while still present, is significantly more mild. Increasing the cost by a factor of 100 (from 0.1 to 10) has only decreased λ by approximately 25%.

Figure 2 presents another view of the same phenomena. This time the pairs ($\lambda^\star$, $\beta^\star$) are plotted for varying costs, while keeping parameter a fixed. Once again, in Figure 2.a we see the trivial solution for (a=0, c=0). We also observe what happens for a=0 and c different from 0, both rates being set at their minimum. In these graphs it becomes even more clear how disproportionately the effect of the cost is felt on both T and E, especially when looking at the graphs for a = 0.5, 1 and 2. When a gets too high however, the cost does not matter any longer, unless it is completely nonexistent. For any cost other than zero and $a > 4$ the system appears to have only one NE: (0.01, 0.01). This seems a little counterintuitive, because a large a would mean that the goal sending fresh updates to R is significantly more important than evading E, therefore for a very large a we would expect T to simply ignore E's existence. However, E can anticipate that, and knows that it will have a good chance of obtaining plenty of up-to-date information without having to spend on increasing its success probability, but simply because T will be transmitting more often. T on the other hand can anticipate that E will choose a low β and responds by lowering its λ.

Figures 3 and 4 separates λs from βs to present them in relation to the parameters a and c, with Figure 3 showing λ as a function of a for various c's. The same approach if followed for β on Figures 4. These plots once again illustrate the same important behavior: the two rates increase along with a, reach a peak, and then fall sharply. Also, the values of λ for different costs are very close to each-other, and in general part of the same interval. The values for β are much more spaced out, ranging anywhere from 0 to 1 depending on the cost, which even when relatively small in value still manages to overshadow the goal of AoI minimization.

V. Conclusions

We have presented the problem of status updates in the presence of an eavesdropper from a game theory standpoint, employing several of the tools, concepts and major contributions in the field such as Bergson's theory of social welfare and the Nash Equilibrium. On building our mode, we have also utilized previous analytical findings on queuing systems, expressing the AoI as a function of the load factor.

After deriving the system of equations which analytically solves the proposed setup, we considered a discrete version of the game, constructed its normal form representation and reached the solution by way of software, for var-

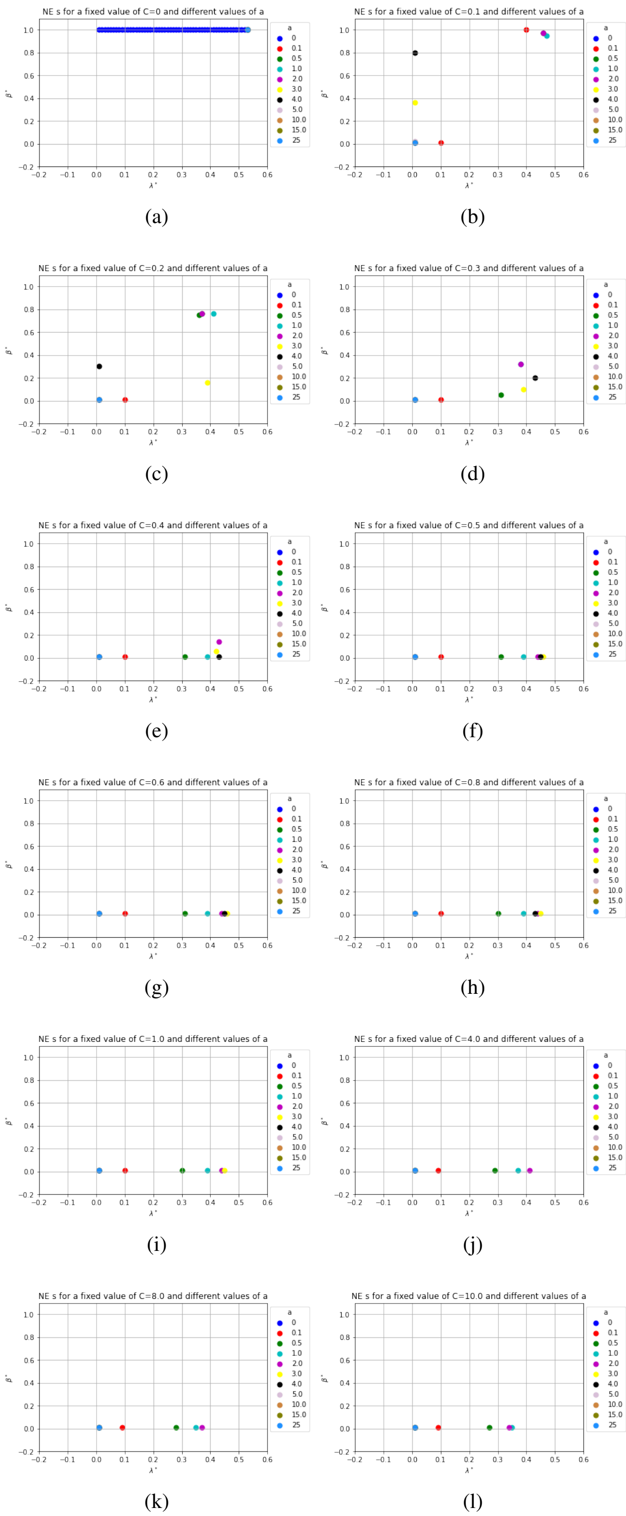

Figure 1: **NEs for a fixed *c* and different *a*s.**

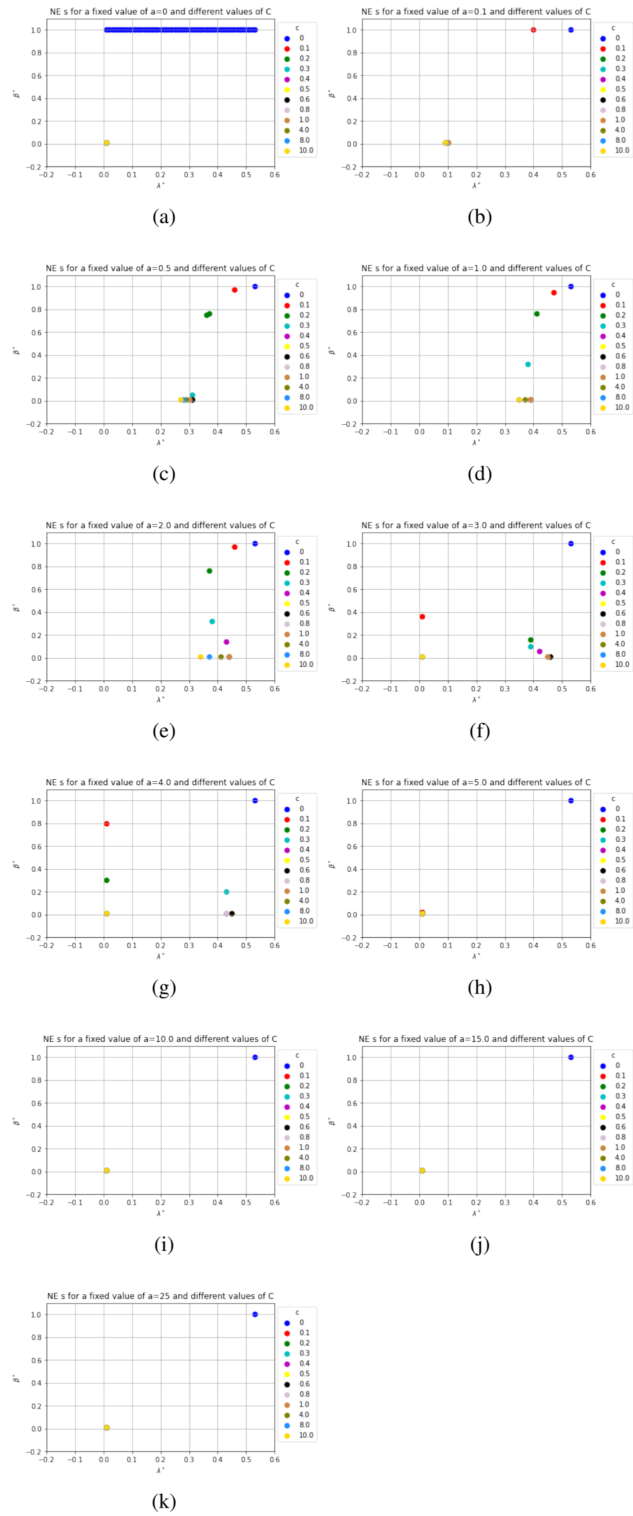

Figure 2: **NEs for a fixed *a* and different *c*s.**

ious parameter values. The results confirmed prior findings relating to λ and β which show that as β decreases

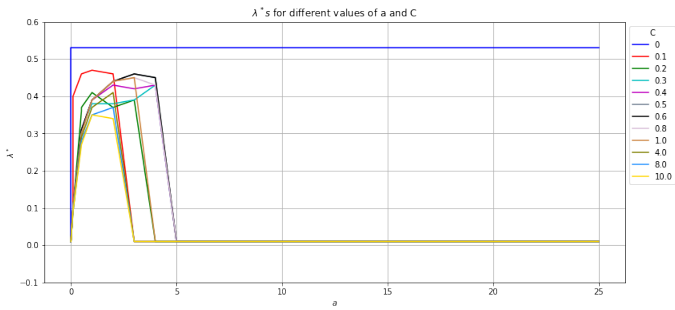

Figure 3: $\lambda^{\star}$**s for different** a**s and** c**s.**

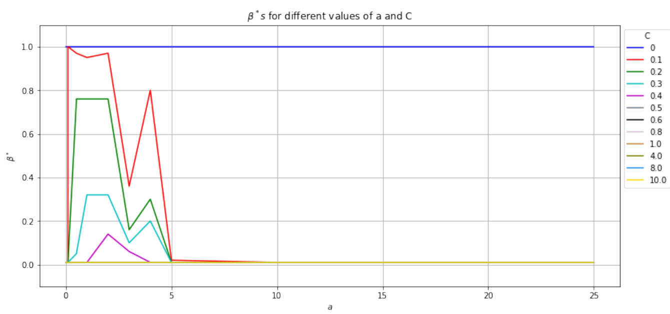

Figure 4: $\beta^{\star}$**s for different** a**s and** c**s.**

the transmitter must reduce the load factor in order to comply with the goal of only leaving the eavesdropper outdated information. We also observed that the cost factor disproportionally impacts and serves as a very efficient deterrent to the malicious activity of the eavesdropper. The results also demonstrated a seemingly counterintuitive conclusion: for higher values of a (greater than 4), in other words cases where minimizing the AoI at the receiver is the significantly predominating goal, the best strategy for the Transmitter is to update at the very minimal rate, as the eavesdropper will also not invest on increasing its probability of successfully capturing packets. The plots of the NEs for various numerical instances of a and c clearly illustrate the behavior of both λ and β as functions of a, swiftly rising for smaller a's, reaching a peak value and then plummeting once again.

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