Introduction to Compressed Sensing

- Detecting the fake coin using a digital scale
- Detecting the fake coin using a two-sided balance
- General CS problem

Digital Scale example

- 8 coins, one is fake

- Find the fake with a digital balance

 $= 10^{\mathrm{gr}}$

Digital Scale example

- 8 coins, one is fake

- None of the coins in experiment 1 is fake.
- All coins not used in experiment 2 are authentic.
- All coins not used in experiment 3 are authentic.

F

$= 8.3^{\text{gr}}$

Digital Scale example

- Mathematical Formulation

$$\begin{bmatrix} 40 \\ 38.3 \\ 38.3 \end{bmatrix} = \underbrace{\begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}}_{\mathbf{\Phi}_{3\times 8}} \begin{bmatrix} w_A \\ w_B \\ w_C \\ w_D \\ w_E \\ w_F \\ w_G \\ w_H \end{bmatrix}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 0 \\ -1.7 \\ -1.7 \end{bmatrix}}_{\mathbf{y}_{3\times 1}} = \mathbf{\Phi}_{3\times 8} \underbrace{\begin{bmatrix} w_A - 10 \\ w_B - 10 \\ w_C - 10 \\ w_D - 10 \\ w_E - 10 \\ w_F - 10 \\ w_G - 10 \\ w_H - 10 \end{bmatrix}}_{\mathbf{x}_{8\times 1}} \quad \Rightarrow \quad \mathbf{y}_{3\times 1} = \mathbf{\Phi}_{3\times 8} \cdot \mathbf{x}_{8\times 1}$$

- Underdetermined LSoE!
- $\mathbf{x}_{8\times 1} =$ 1-sparse !

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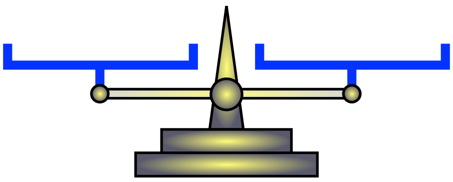
Two-sided Balance example

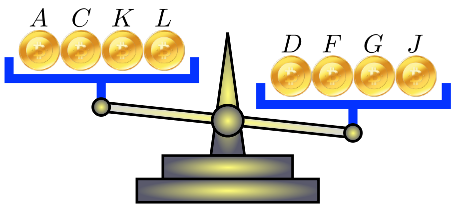
- 13 coins, one is fake

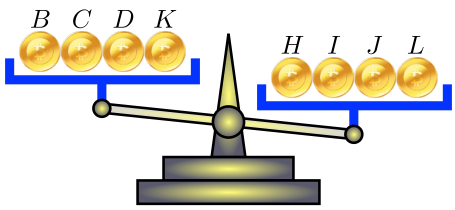
- Find the fake with an analog balance

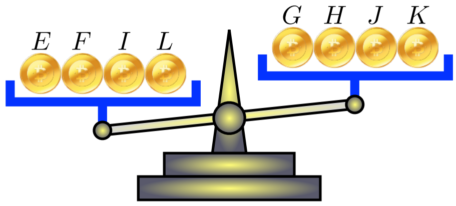


Two-sided Balance example

- 13 coins, one is fake







- Fake coin is present in all experiments.
- Fake coin is in the same side in experiments 1 and 2.
- Fake coin is in different sides in experiments 2 and 3.

Two-sided Balance example

- Mathematical Formulation

$$\begin{bmatrix}-1\\-1\\1\end{bmatrix} = \begin{bmatrix}\mathrm{sign}(d_1)\\\mathrm{sign}(d_2)\\\mathrm{sign}(d_3)\end{bmatrix} \qquad \begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix} = \underbrace{\begin{array}{c}\begin{array}{ccccccccccccc}A & B & C & D & E & F & G & H & I & J & K & L & M\end{array}\\ \begin{bmatrix}1 & 0 & 1 & -1 & 0 & -1 & -1 & 0 & 0 & -1 & 1 & 1 & 0\\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & -1 & 0\\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 0\end{bmatrix}\end{array}}_{\mathbf{\Phi}_{3\times 13}} \begin{bmatrix}w_A\\w_B\\\vdots\\w_L\\w_M\end{bmatrix}$$

$$\begin{array}{lcl}\widetilde{w}_A & \triangleq & \frac{w_A - w_{\text{normal}}}{|w_{\text{fake}} - w_{\text{normal}}|}\\ & \vdots & \\ \widetilde{w}_M & \triangleq & \frac{w_M - w_{\text{normal}}}{|w_{\text{fake}} - w_{\text{normal}}|}\end{array} \Rightarrow \underbrace{\begin{bmatrix}-1\\-1\\1\end{bmatrix}}_{\mathbf{y}_{3\times 1}} = \mathbf{\Phi}_{3\times 13} \underbrace{\begin{bmatrix}\widetilde{w}_A\\\widetilde{w}_B\\\vdots\\\widetilde{w}_L\\\widetilde{w}_M\end{bmatrix}}_{\mathbf{x}_{13\times 1}} \Rightarrow \mathbf{y}_{3\times 1} = \mathbf{\Phi}_{3\times 13}\cdot \mathbf{x}_{13\times 1}$$

➢ $\mathbf{x}_{13\times 1} =$ 1-sparse !

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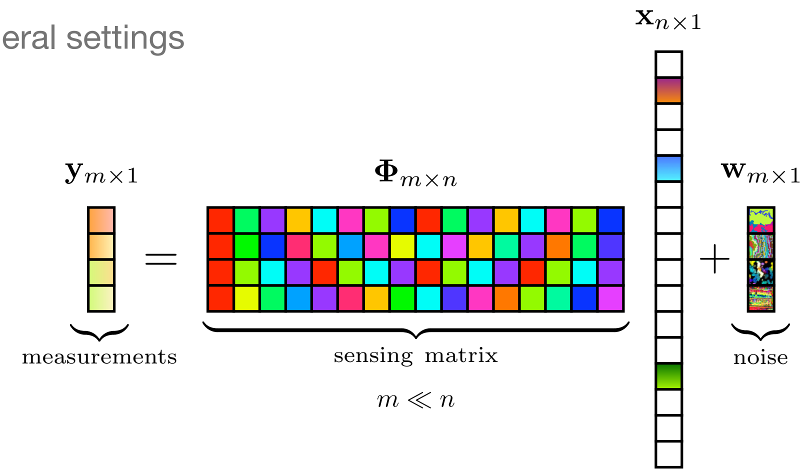
CS problem statement

- More general settings



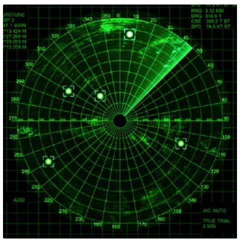
$$\left\{\begin{array}{lcl} \mathbf{y}_{m\times 1} & = & \mathbf{\Phi}_{m\times n}\cdot \mathbf{x}_{n\times 1} + \mathbf{w}_{m\times 1} \\ \mathbf{x}_{n\times 1} & = & k\text{-sparse} \end{array}\right. \quad \Rightarrow \quad \mathbf{x}_{n\times 1} = ?$$

$$m < n \quad \Rightarrow \quad \text{infinitely many solutions}$$

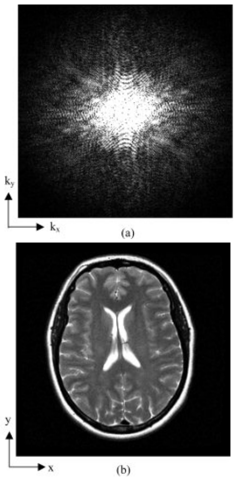
CS problem statement

- Similar problems

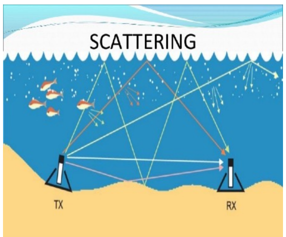
DOA estimation



MRI scanning



Channel estimation



Group testing