Lecture 1: Introduction

Deep Generative Models

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Contents

Section 1

Supervised vs. Unsupervised Learning

Supervised Learning

- **Observations:** $\mathcal{D} = \{(\boldsymbol{x}_n, y_n)\}_{n=1}^{N}$

Supervised Learning

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- **Applications:**
  - Classification
  - Regression
  - ...

Supervised Learning

- **Observations:** $\mathcal{D} = \{(\boldsymbol{x}_n, y_n)\}_{n=1}^{N}$
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- **Model:** $p_\theta(y|\boldsymbol{x})$

Supervised Learning

- **Observations:** $\mathcal{D} = \{(\boldsymbol{x}_n, y_n)\}_{n=1}^{N}$
- **Applications:**
  - Classification
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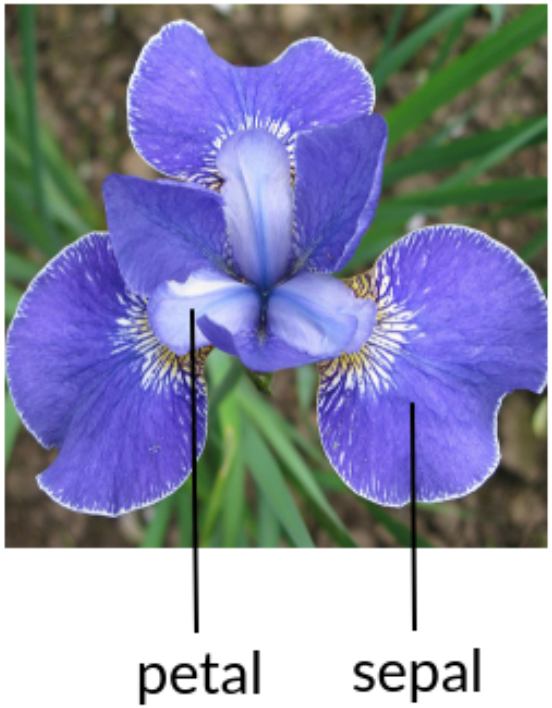
- **Model:** $p_\theta(y|\boldsymbol{x})$
- **Phases:**
  - *Training Phase:*

$$\boldsymbol{\theta}^\star = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{n=1}^{N} \text{Loss}\Big(y_n, p_\theta(y|\boldsymbol{x}_n)\Big)$$

  - *Application Phase:*

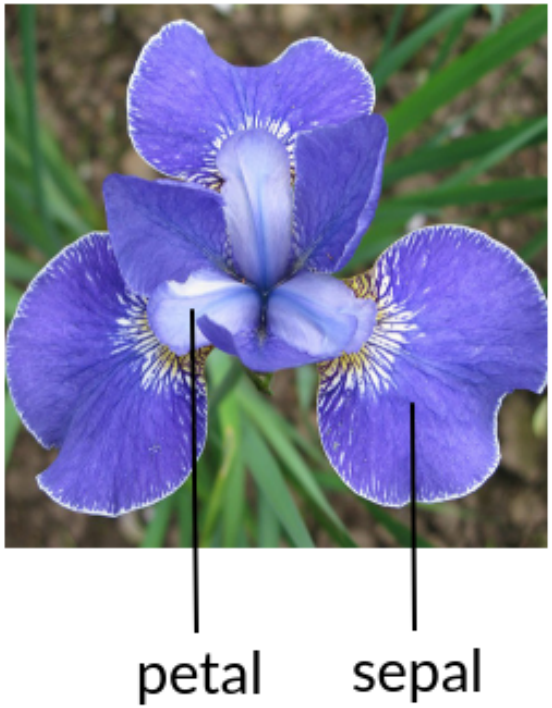
$$p_{\boldsymbol{\theta}^\star}(y|\boldsymbol{x}_{\text{new}})$$

Classification



(a) Setosa ($y = 1$)

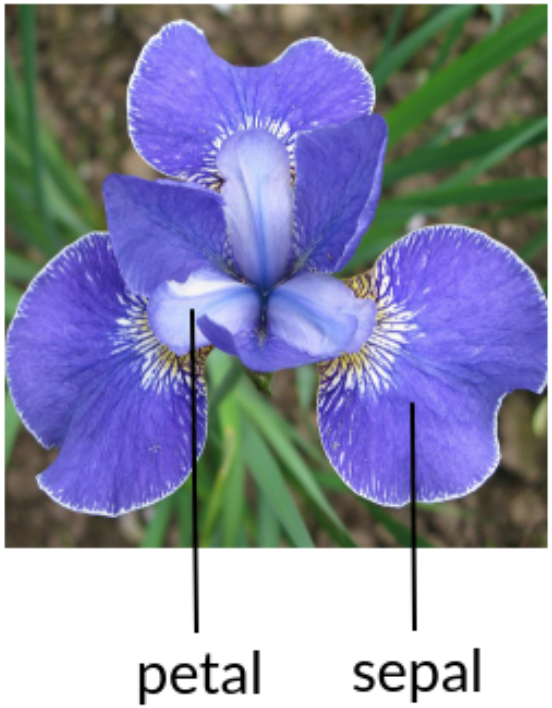
Classification



(a) Setosa ($y = 1$)

(b) Versicolor ($y = 2$)

Classification



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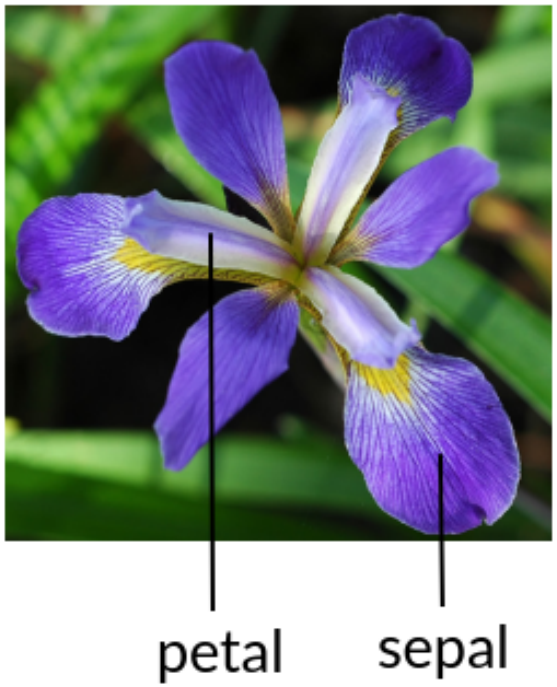

(c) Virginica ($y = 3$)

Figure: Iris flower dataset ($\boldsymbol{x} \in \mathbb{R}^{256 \times 256 \times 3}$ and $y \in \{1, 2, 3\}$)

Classification

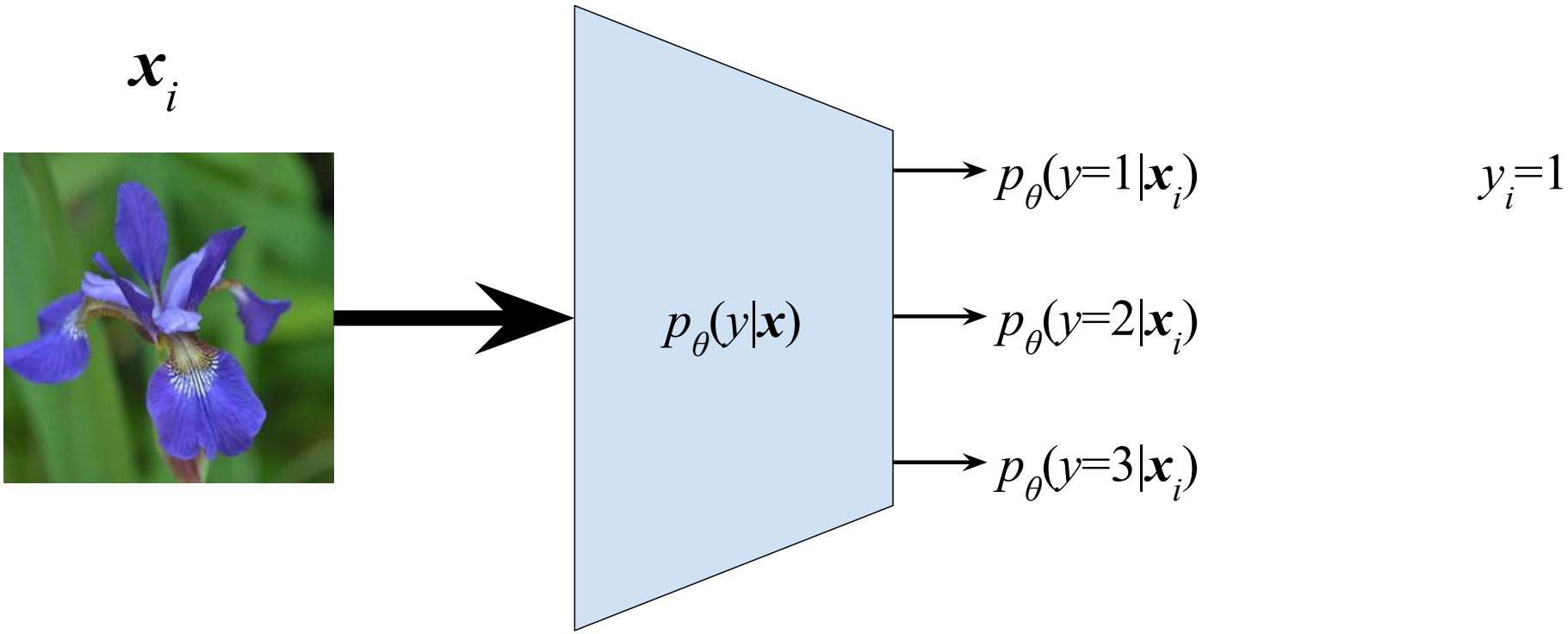

Figure: Training step 1: Evaluate $p_\theta(y|\boldsymbol{x}_i)$

Classification

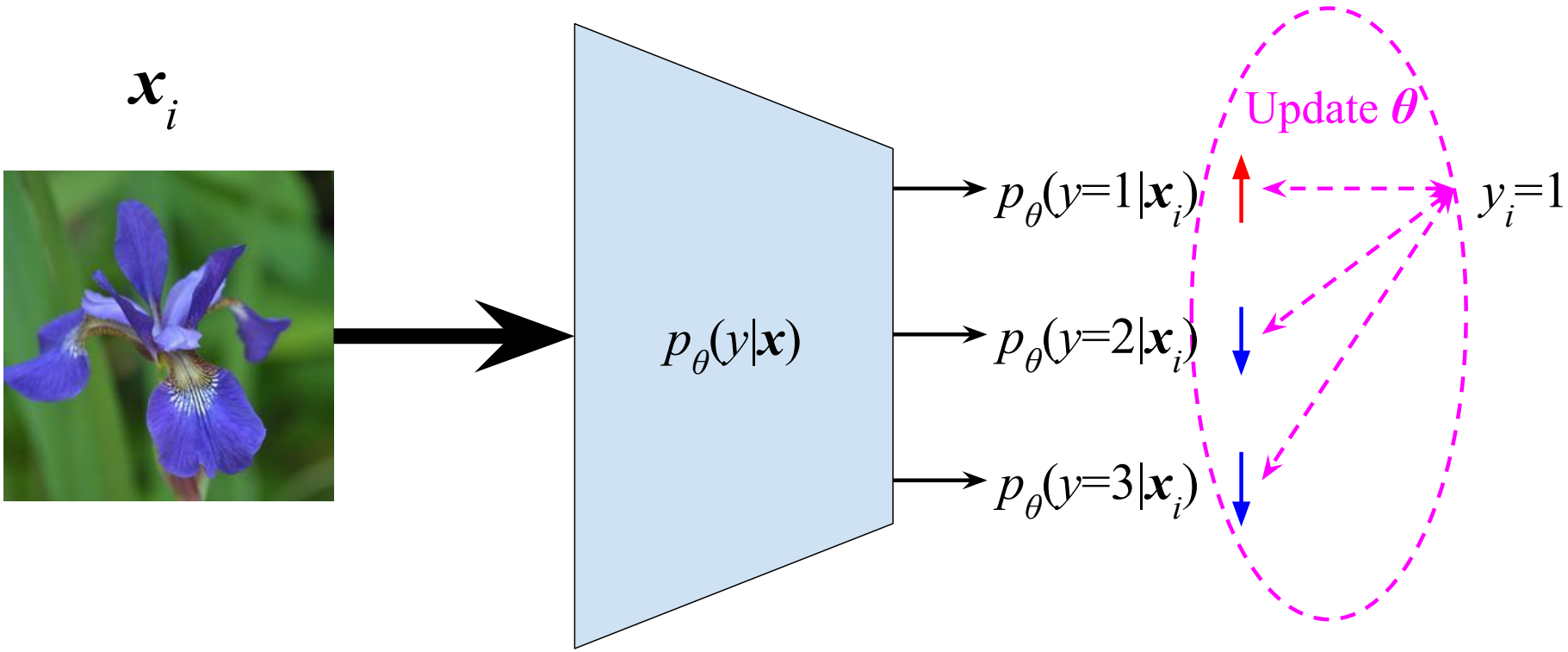

Figure: Training step 2: Update $\boldsymbol{\theta}$ to minimize cross entropy

Classification

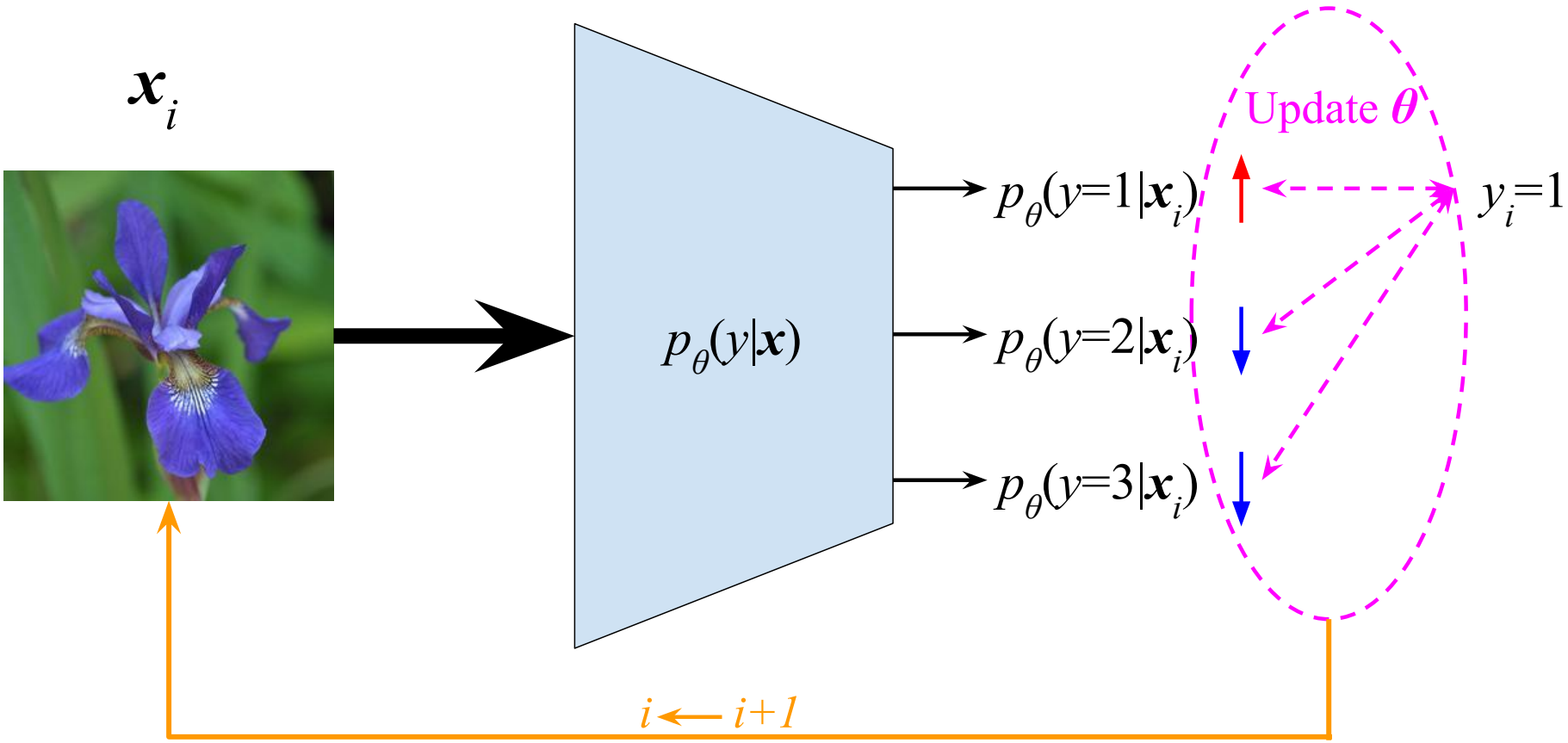

Figure: Training step 3: Go to the next training data

Classification

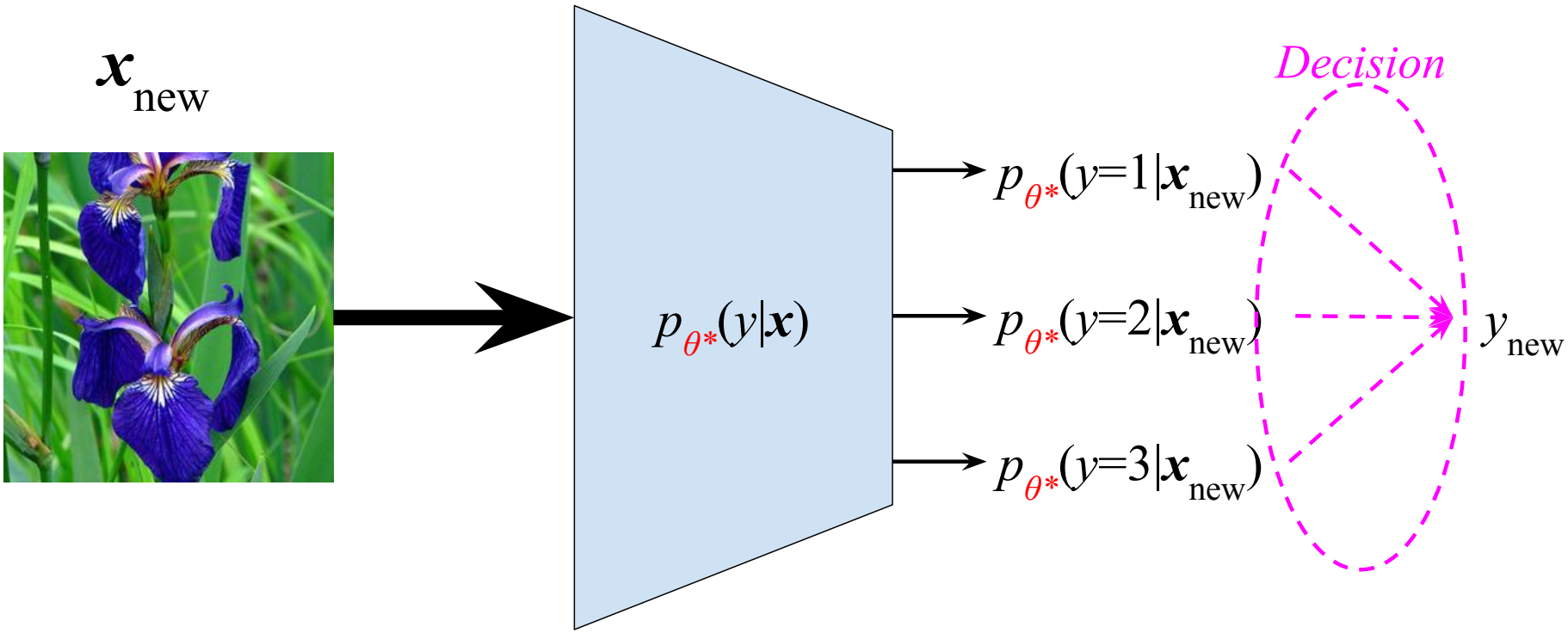

Figure: Inference: Evaluating the trained model $\boldsymbol{\theta}^\star$

Multinomial Logistic Regression Model

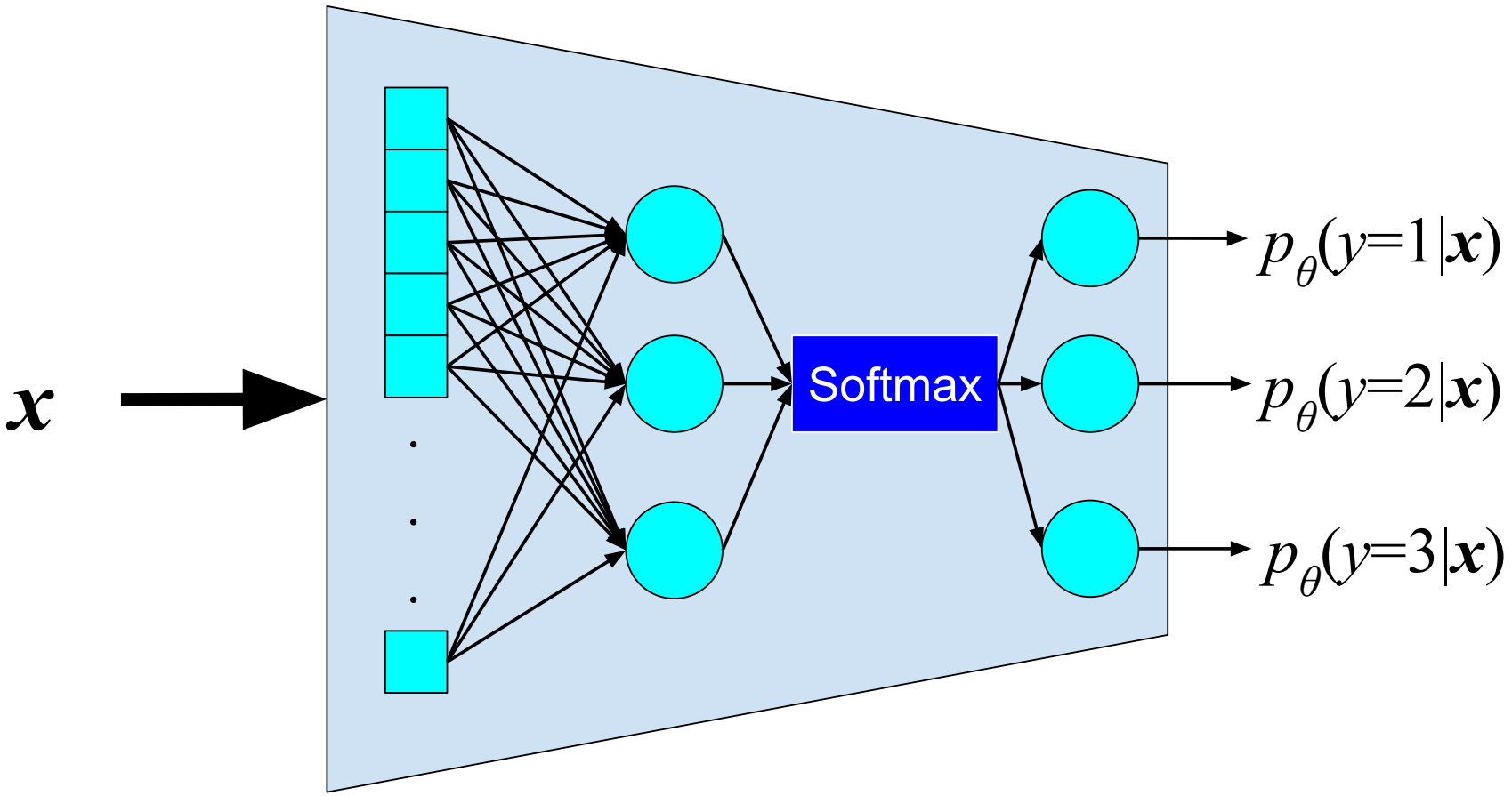

Figure: Multinomial logistic regression (MLR) model to implement $p_\theta(y|\boldsymbol{x})$ for Iris flower classification

Unsupervised Learning

- **Observations:** $\mathcal{D} = \{\boldsymbol{x}_n\}_{n=1}^{N}$

Unsupervised Learning

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- **Applications:**
  - Dimensionality reduction
  - Clustering
  - ...

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- **Applications:**
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- **Model:** *Dimensionality Reduction* $\Rightarrow \boldsymbol{y} = \boldsymbol{W}\boldsymbol{x}, \begin{cases} \boldsymbol{x} \in \mathbb{R}^D \\ \boldsymbol{W} \in \mathbb{R}^{L \times D} \\ \boldsymbol{y} \in \mathbb{R}^L \end{cases}$

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- **Phases:**
  - *Training Phase:*

$$\boldsymbol{W}^\star = \underset{\boldsymbol{W}}{\operatorname{argmin}} \sum_{n=1}^{N} \operatorname{Loss}\Big(\boldsymbol{x}_n, \boldsymbol{W}^T \boldsymbol{W} \boldsymbol{x}\Big)$$

  - *Application Phase:*

$$\boldsymbol{y}_{\text{new}} = \boldsymbol{W}^\star \boldsymbol{x}_{\text{new}}$$

Dimensionality Reduction

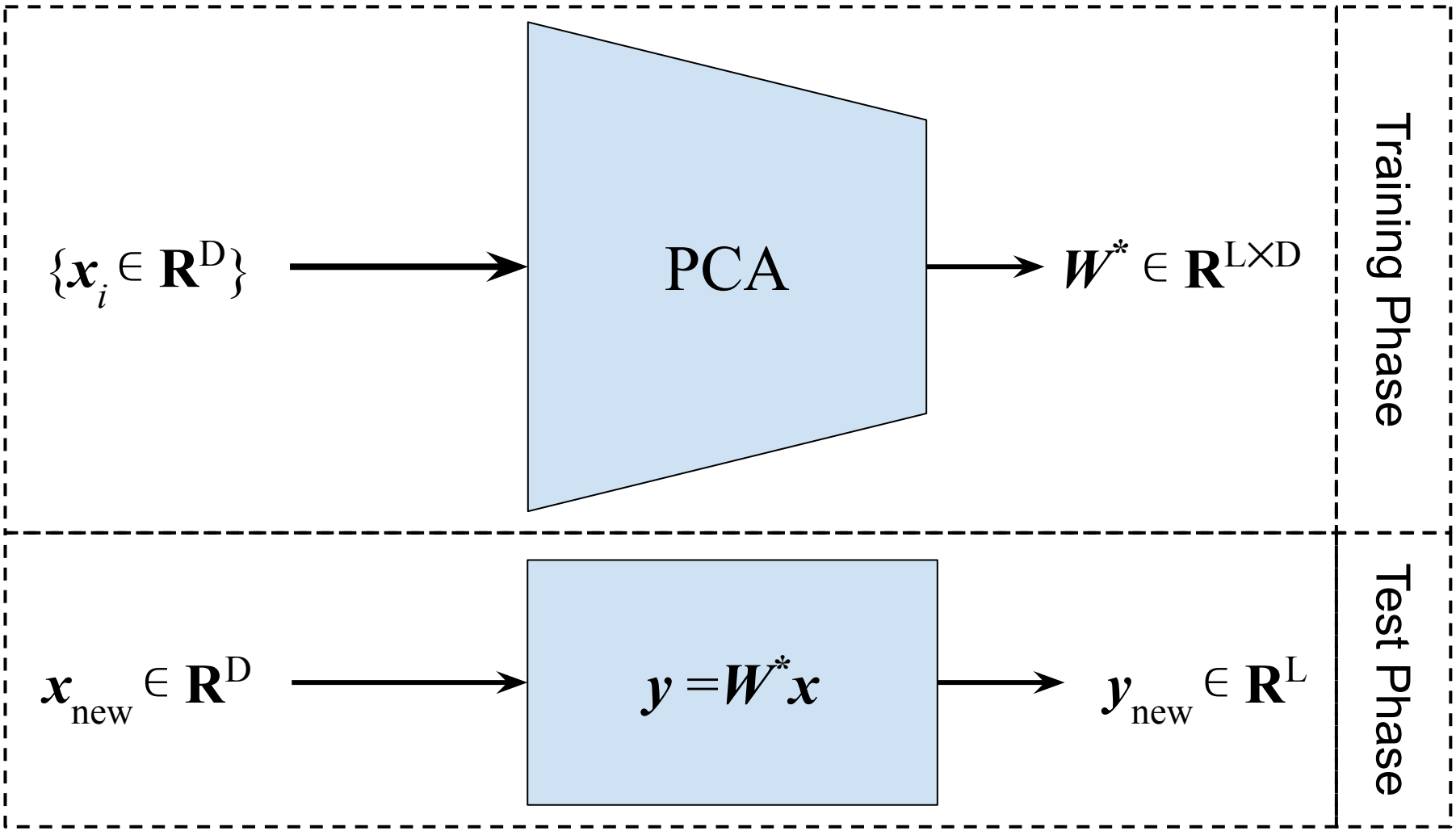

Figure: Training and Test for Dimensionality Reduction

Dimensionality Reduction

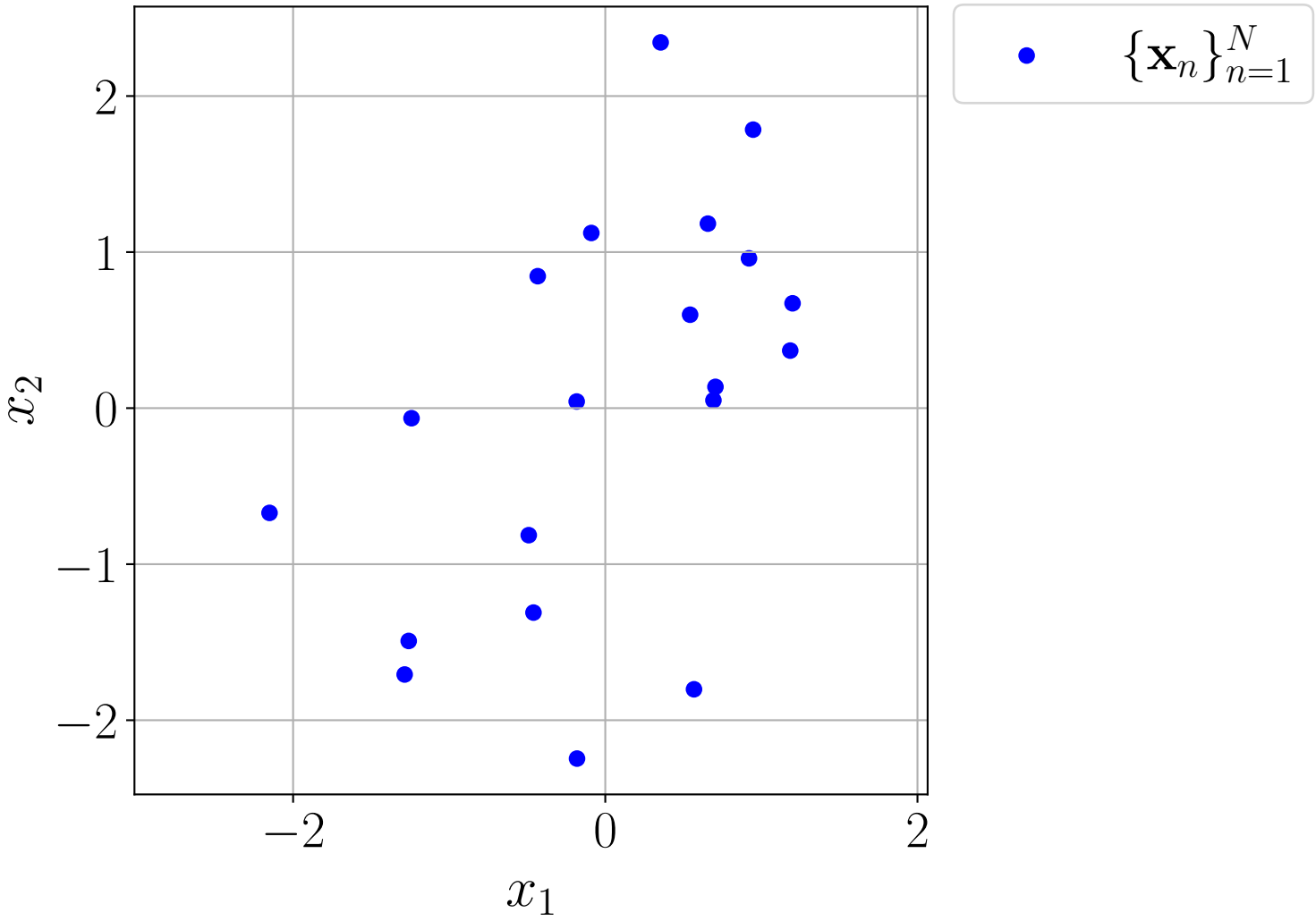

Figure: Training data for dimensionality reduction

Dimensionality Reduction

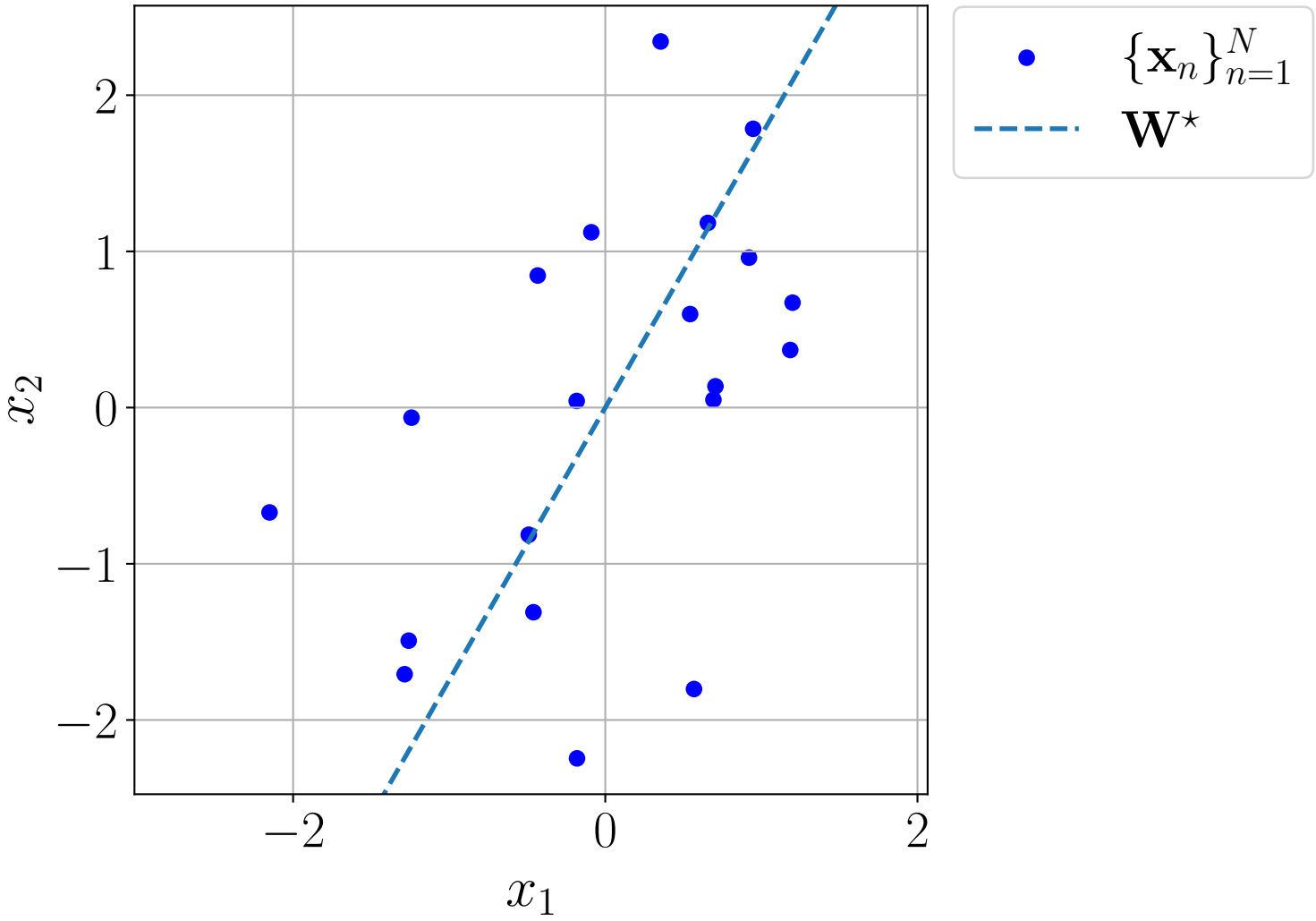

Figure: Learning principle component or projection matrix $\boldsymbol{W}^\star$

Dimensionality Reduction

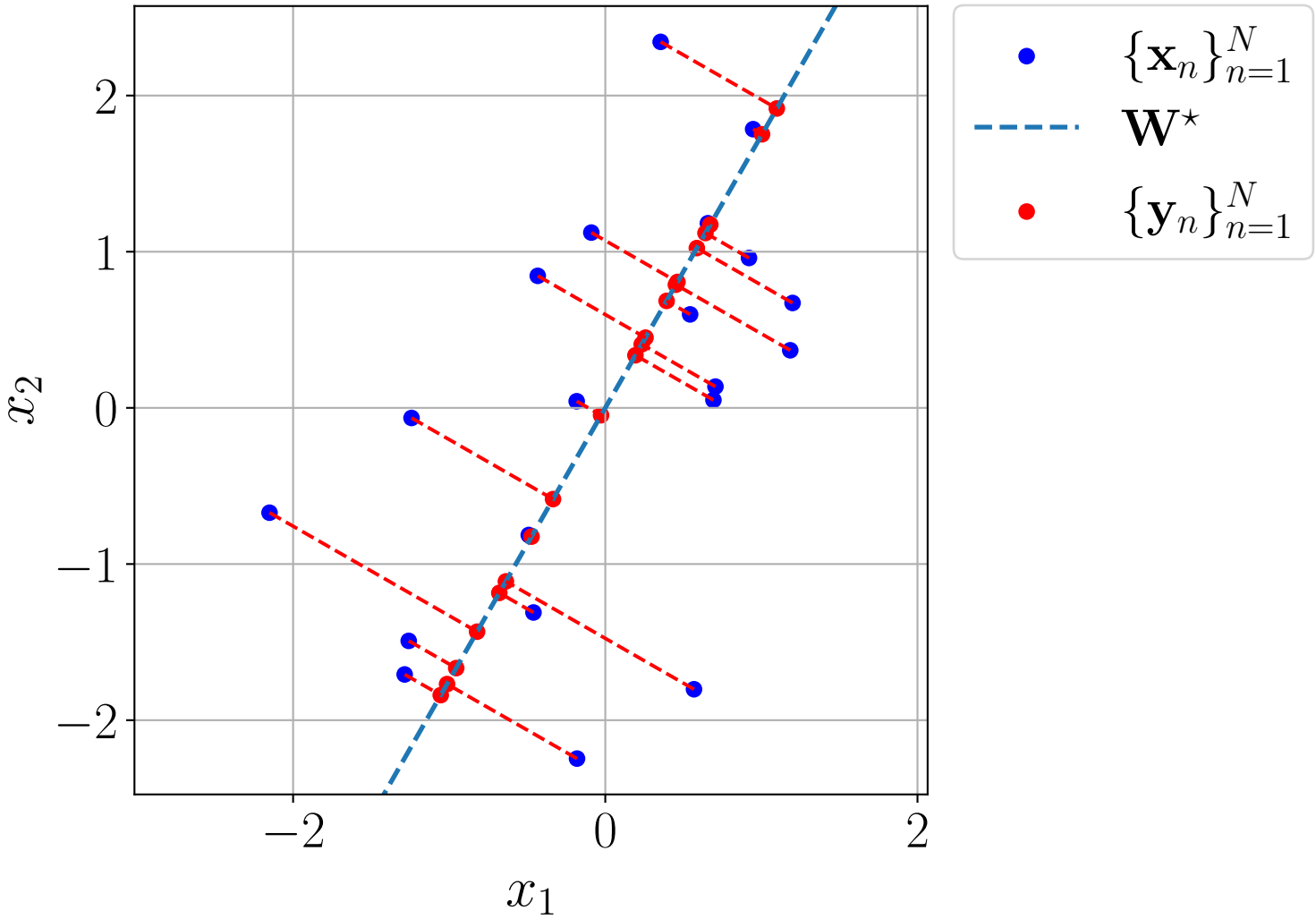

Figure: Reducing the dimensionality of training data

Altogether

Supervised Learning

- **Experience:**

$$\mathcal{D} = \{(\boldsymbol{x}_i, y_i)\}_{i=1}^{N}$$

- **Applications:**
  - Classification
  - Regression
  - Semantic segmentation
  - ...
- **Objective:**
  - Estimating $p_{\text{data}}(y|\boldsymbol{x})$

Altogether

Supervised Learning

- **Experience:**

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Unsupervised Learning

- **Experience:**

$$\mathcal{D} = \{\boldsymbol{x}_i\}_{i=1}^N$$

- **Applications:**
  - Clustering
  - Dimensionality Reduction
  - Density estimation
  - ...
- **Objective:**
  - Estimating $p_{\text{data}}(\boldsymbol{x})$

Section 2

Unconditional Generative Models as Unsupervised Learning Models

Unconditional Generative Model

Generative Model Learning

- **Observation:**

$$\mathcal{D} = \{\boldsymbol{x}_i\}_{i=1}^{N}$$

Unconditional Generative Model

Generative Model Learning

- **Observation:**

$$\mathcal{D} = \{\boldsymbol{x}_i\}_{i=1}^{N}$$

- **Model:** $p_\theta(\boldsymbol{x})$

Unconditional Generative Model

Generative Model Learning

- **Observation:**

$$\mathcal{D} = \{\boldsymbol{x}_i\}_{i=1}^{N}$$

- **Model:** $p_\theta(\boldsymbol{x})$
- **Tasks:**
  - Density estimation: $p_\theta(\boldsymbol{x}_{\text{new}})$

Unconditional Generative Model

Generative Model Learning

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- **Model:** $p_\theta(\boldsymbol{x})$
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Unconditional Generative Model

Generative Model Learning

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- **Tasks:**
  - Density estimation: $p_\theta(\boldsymbol{x}_{\text{new}})$
  - Generation: $\boldsymbol{x}_{\text{new}} \sim p_\theta(\boldsymbol{x})$
  - High level representation $\boldsymbol{z}$ (unsupervised representation learning):
    - Assume, $\boldsymbol{x}$ is IRIS flower, then ideally $z \in \{1, 2, 3\}$ corresponding to each type.

Unconditional Generative Model

Generative Model Learning

- **Observation:**

$$\mathcal{D} = \{\boldsymbol{x}_i\}_{i=1}^{N}$$

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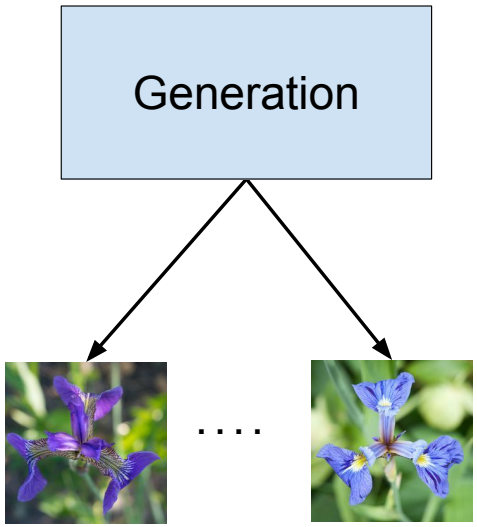
- **Tasks:**
  - Density estimation: $p_\theta(\boldsymbol{x}_{\text{new}})$
  - Generation: $\boldsymbol{x}_{\text{new}} \sim p_\theta(\boldsymbol{x})$
  - High level representation $\boldsymbol{z}$ (unsupervised representation learning):
    - Assume, $\boldsymbol{x}$ is IRIS flower, then ideally $z \in \{1, 2, 3\}$ corresponding to each type.
- **Objective:**
  - Estimating distribution $p_{\text{data}}(\boldsymbol{x})$

$$\boldsymbol{\theta}^\star = \underset{\boldsymbol{\theta}}{\text{argmin}}\, \text{dist}(p_{\text{data}}, p_\theta)$$

Unconditional Generative Model



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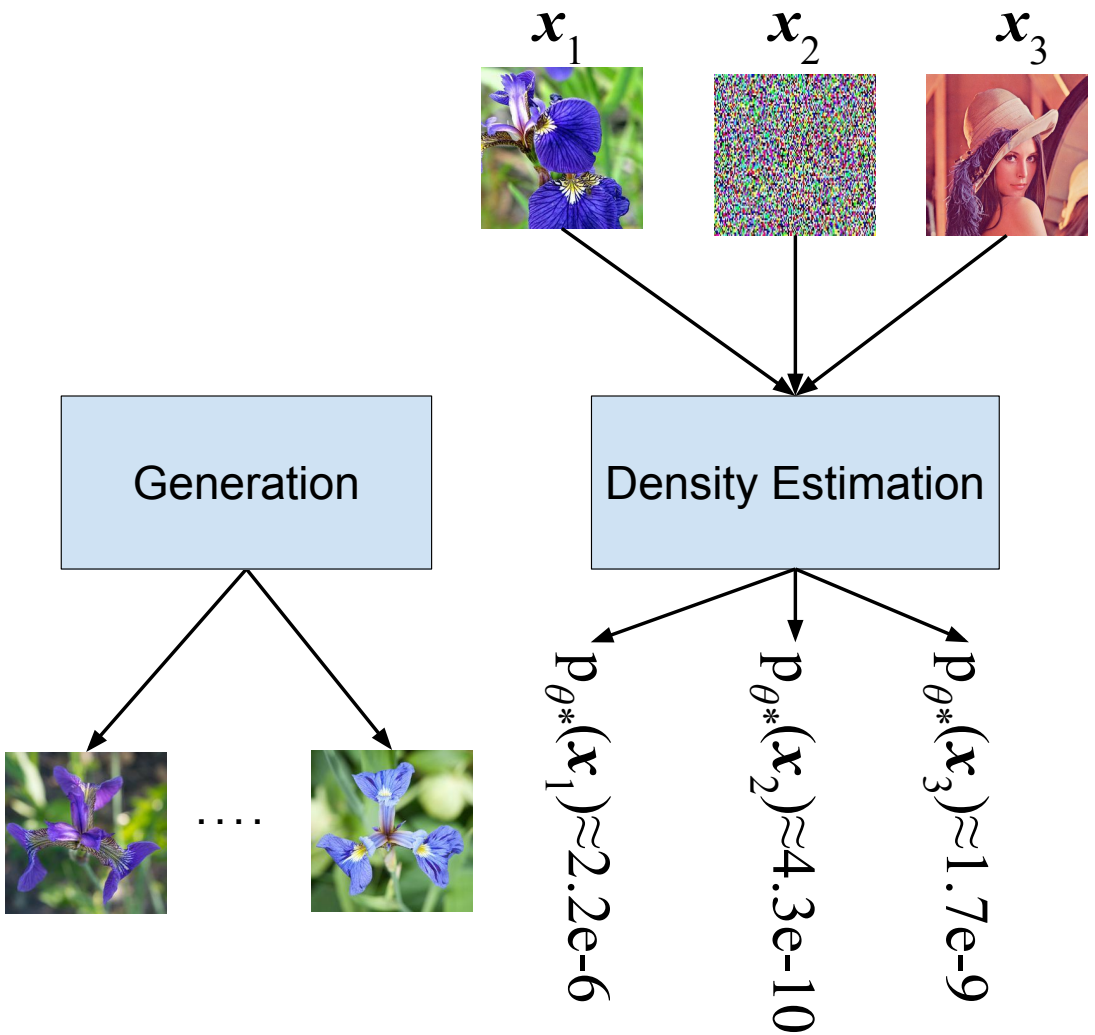
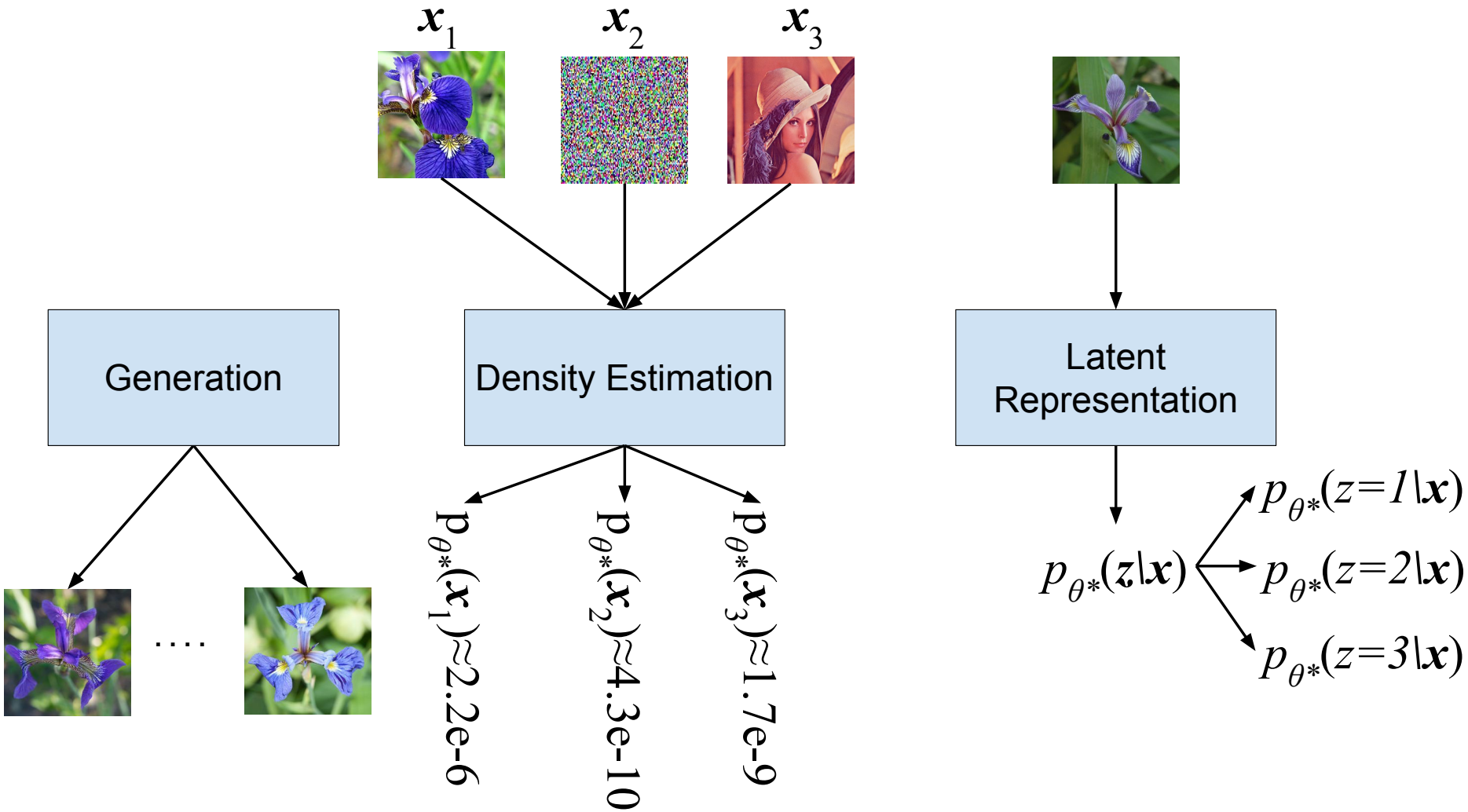


Unconditional Generative Model



Section 3

Conditional Generative Models as Extention of Supervised Learning Models

Conditional Generative Model

Conditional Generative Model Learning

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$$\mathcal{D} = \{(\boldsymbol{x}_i, \boldsymbol{y}_i)\}_{i=1}^{N}$$

Conditional Generative Model

Conditional Generative Model Learning

- **Observation:**

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- **Model:** $p_\theta(\boldsymbol{y}|\boldsymbol{x})$

Conditional Generative Model

Conditional Generative Model Learning

- **Observation:**

$$\mathcal{D} = \{(\boldsymbol{x}_i, \boldsymbol{y}_i)\}_{i=1}^{N}$$

- **Model:** $p_\theta(\boldsymbol{y}|\boldsymbol{x})$
- **Tasks:**
  - Conditional density estimation: $p_\theta(\boldsymbol{y}|\boldsymbol{x}_{\text{new}})$

Conditional Generative Model

Conditional Generative Model Learning

- **Observation:**

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Conditional Generative Model

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- **Objective:**
  - Estimating distribution $p_{\text{data}}(\boldsymbol{y}|\boldsymbol{x})$

$$\boldsymbol{\theta}^\star = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \text{dist}(p_{\text{data}}, p_\theta)$$

Conditional Generative Model - Density Estimation

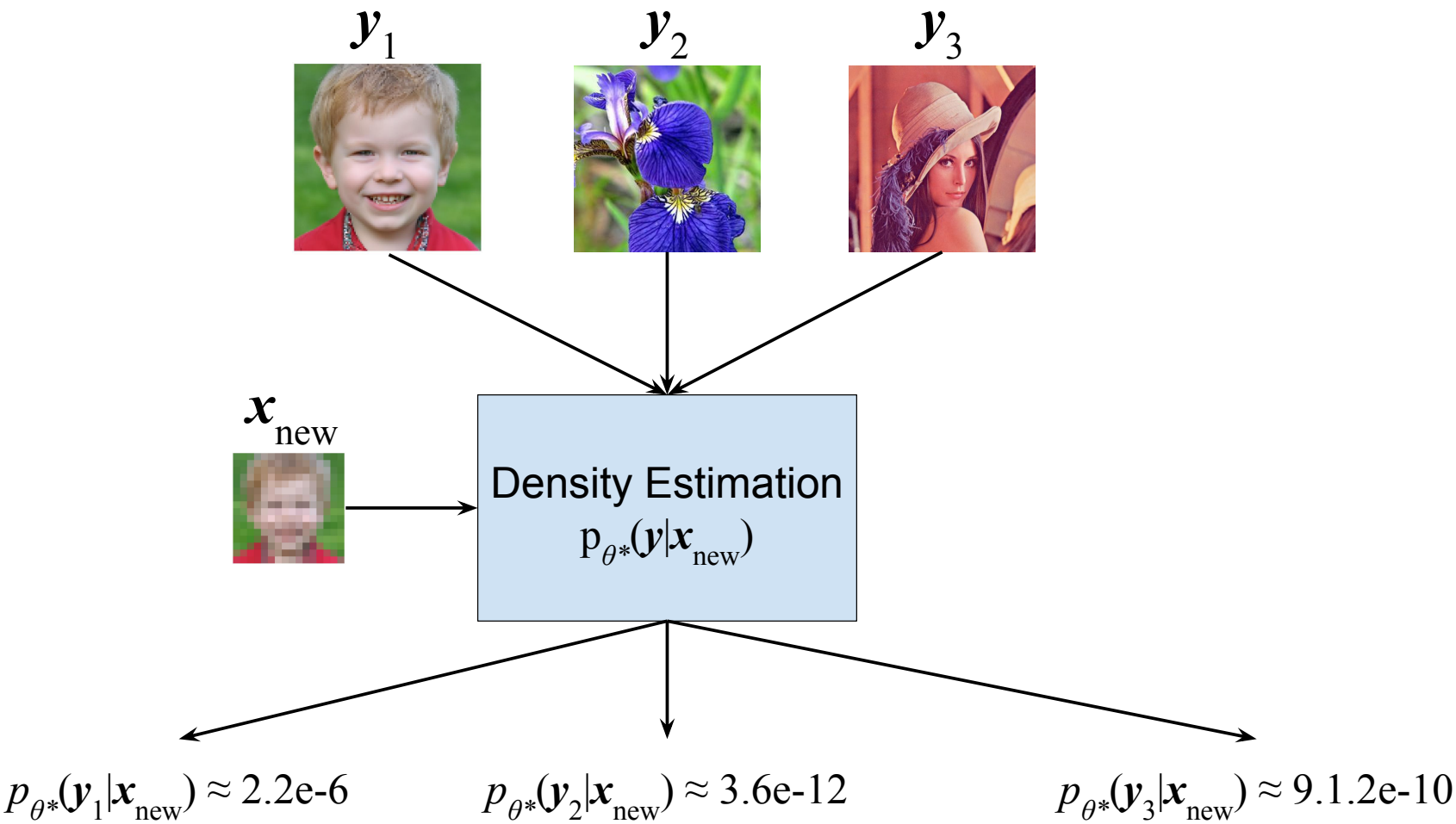

Figure: Assume we trained a conditional generative model and estimate $\boldsymbol{\theta}^\star$. Then we can use it for density estimation of different $\boldsymbol{y}$ vector given $\boldsymbol{x}_{\text{new}}$ (image source: [1])

Conditional Generative Model - Generation

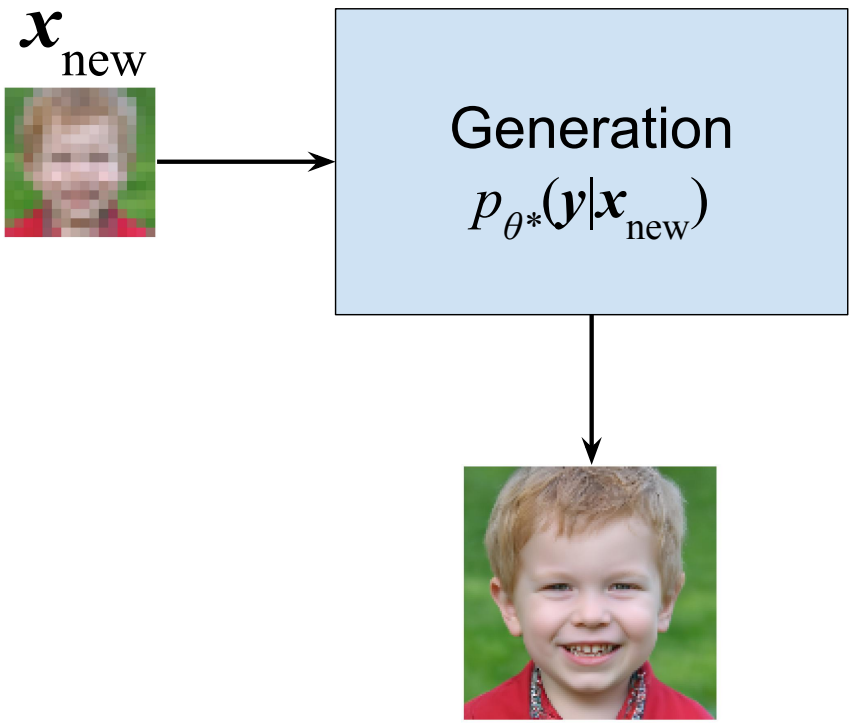

Figure: Assume we trained a conditional generative model and estimate $\boldsymbol{\theta}^\star$. Then we can use it for sampling $\boldsymbol{y}$ given $\boldsymbol{x}_{\text{new}}$ (image source: [1])

Conditional Generative Model - Generation

MLR for Conditional Generative Model

Assume a simple super-resolution problem:

- $\boldsymbol{x} \in \{0,1\}^{r \times c}$
- $\boldsymbol{y} \in \{0,1\}^{h \times w}$

Conditional Generative Model - Generation

MLR for Conditional Generative Model

Assume a simple super-resolution problem:

- $\boldsymbol{x} \in \{0,1\}^{r \times c}$
- $\boldsymbol{y} \in \{0,1\}^{h \times w}$

Then MLR must assign a probability to each possible output which is:

$$\text{Number of possible } \boldsymbol{y} \text{ vectors } \Rightarrow 2^{h \times w}$$

$$\text{Number of MLR parameters} \Rightarrow r \times c \times 2^{h \times w}$$

Conditional Generative Model - Generation

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So using MLR and more complex models such as supervised deep models is not practical.

Conditional vs Unconditional Deep Generative Models

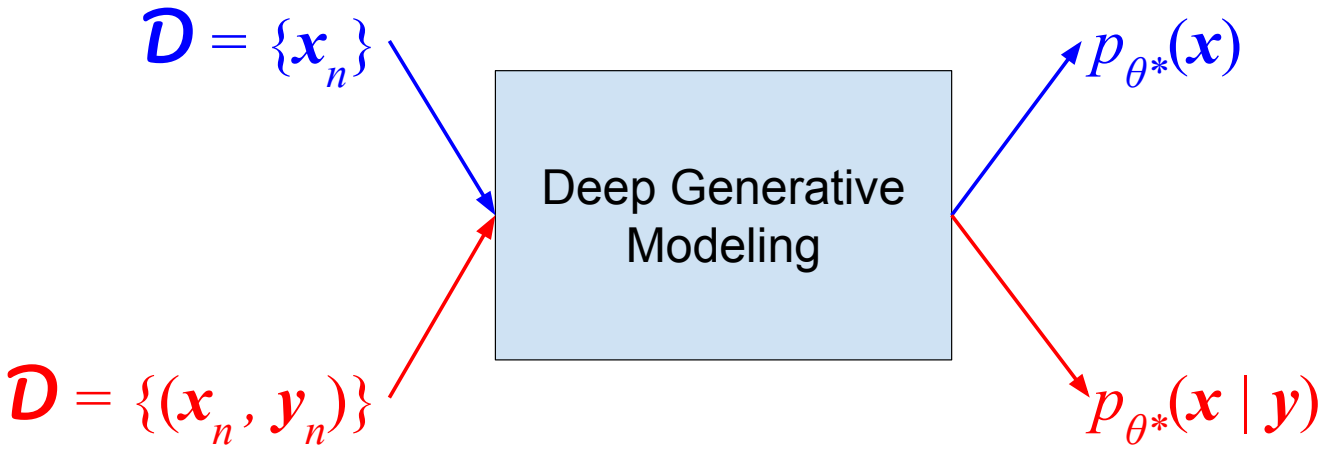

Figure: The methods to train conditional and unconditional generative modeling are almost shared in different approaches

Conditional vs Unconditional Deep Generative Models

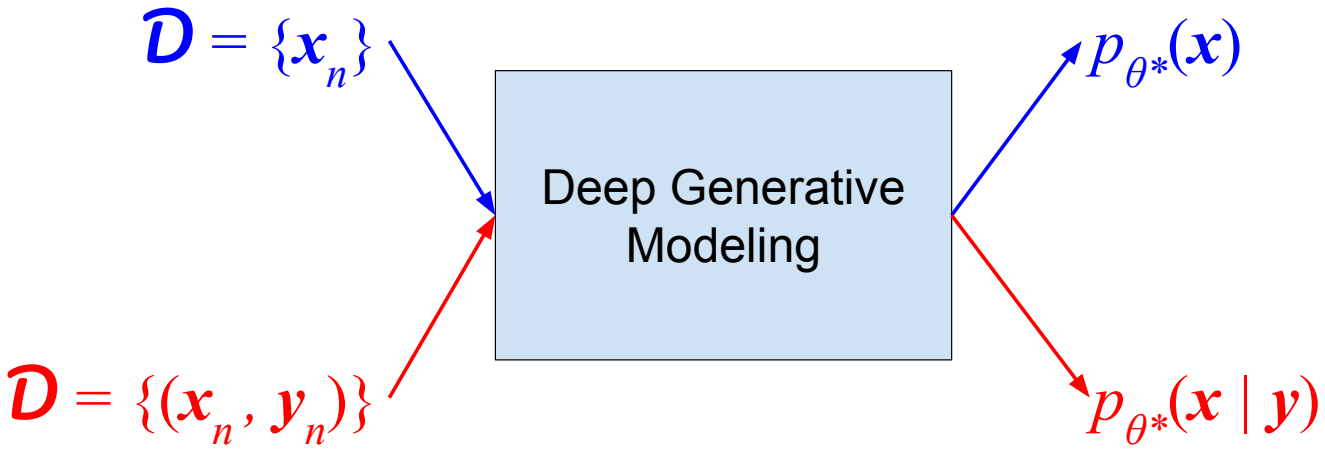

Figure: The methods to train conditional and unconditional generative modeling are almost shared in different approaches

Notation Update

To better share the notation across conditional and unconditional DGM, we use $p_\theta(\boldsymbol{x}|\boldsymbol{y})$ instead of $p_\theta(\boldsymbol{y}|\boldsymbol{x})$ for the conditional case. So we are always working on a probabilistic model over $\boldsymbol{x}$.

Text-to-Speech Models

Text-to-Speech Models

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{An audio file} \\ \boldsymbol{y} : \text{A text} \end{cases}$$

Text-to-Speech Models

Text-to-Speech Models

$$p(\boldsymbol{x}|\boldsymbol{y}): \begin{cases} \boldsymbol{x}: \text{An audio file} \\ \boldsymbol{y}: \text{A text} \end{cases}$$

Real-World Sample

Listen to the following speech synthesis (source: [2])

$$\boldsymbol{y} = \text{“A single Wavenet can capture the characteristics of many different speakers with equal fidelity, not it's fast.”} \xrightarrow{\text{Sampling} p(\boldsymbol{x}|\boldsymbol{y})} \boldsymbol{x} = \boxed{\text{Play}}$$

Text-to-Image Models

Text-to-Image Models

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{An image} \\ \boldsymbol{y} : \text{A text} \end{cases}$$

Text-to-Image Models

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{An image} \\ \boldsymbol{y} : \text{A text} \end{cases}$$

Figure: $\boldsymbol{x}$ for $\boldsymbol{y}$ = "Teddy bears swimming at the Olympics 400m Butterfly event." (source: [3])

Image-to-Image Translation

Image Colorization

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{A Colored image} \\ \boldsymbol{y} : \text{A Gray} - \text{scale image} \end{cases}$$

Image-to-Image Translation

Image Colorization

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{A Colored image} \\ \boldsymbol{y} : \text{A Gray} - \text{scale image} \end{cases}$$

(a) $\boldsymbol{y}$

(b) $\boldsymbol{x}$

(c) Ground truth

Figure: Image colorization (source: [4])

Image-to-Image Translation

Image Inpainting

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{A clean image} \\ \boldsymbol{y} : \text{A corrupted image} \end{cases}$$

Image-to-Image Translation

Image Inpainting

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{A clean image} \\ \boldsymbol{y} : \text{A corrupted image} \end{cases}$$

(a) $\boldsymbol{y}$

(b) $\boldsymbol{x}$

(c) Ground truth

Figure: Image inpainting (source: [4])

Image-to-Image Translation

Image Uncropping

$$p(\boldsymbol{x}|\boldsymbol{y}) : \begin{cases} \boldsymbol{x} : \text{A clean image} \\ \boldsymbol{y} : \text{A cropped image} \end{cases}$$

Image-to-Image Translation

Image Uncropping

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Figure: Image uncropping (source: [4])

Image-to-Image Translation

Image Restoration

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Image Restoration

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(c) Ground truth

Figure: Image restoration (source: [4])

Section 4

Discriminative Vs. Generative Classification

Discriminative Approach for Classification

Multinomial Logistic Regression

- Model

$$p_\theta(y|\boldsymbol{x}) = \text{Cat}(y|\mathcal{S}(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b}))$$

Discriminative Approach for Classification

Multinomial Logistic Regression

- Model

$$p_\theta(y|\boldsymbol{x}) = \text{Cat}(y|\mathcal{S}(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b}))$$

where:

$y \in \{1, 2, 3, \ldots, C\}$	Label
$\boldsymbol{x} \in \mathbb{R}^D$	Input pattern
$\mathcal{S}(\cdot)$	Softmax function
$\boldsymbol{W} \in \mathbb{R}^{C \times D}$	Weight matrix
$\boldsymbol{b} \in \mathbb{R}^C$	Bias vector
$\boldsymbol{\theta} = (\boldsymbol{W}, \boldsymbol{b})$	Model parameters

- Number of parameters:

$$C \times D + C$$

Discriminative Approach for Classification

Multinomial Logistic Regression

Assume:

$$\boldsymbol{x} = \text{vec}\left(\text{  } \in \mathbb{R}^{600 \times 800} \right)$$

$$y \in \{1 \text{ (LivingRoom)}, 2 \text{ (BedRoom)}, 3 \text{ (DiningRoom)}\}$$

Discriminative Approach for Classification

Multinomial Logistic Regression

Assume:

$$\boldsymbol{x} = \text{vec}\left(\text{  } \in \mathbb{R}^{600 \times 800} \right)$$

$$y \in \{1 \text{ (LivingRoom)}, 2 \text{ (BedRoom)}, 3 \text{ (DiningRoom)}\}$$

then:

$$p_\theta\left(y \middle| \text{  } \right) = \text{Cat}(y|[0.1, 0.7, 0.2]^T) \xrightarrow{\text{Sampling}} \begin{cases} 2 \\ 2 \\ 2 \\ 3 \\ 2 \end{cases}$$

Generative Approach for Classification

Discriminant Analysis

- Model

$$p_\theta(y = c|\boldsymbol{x}) = \frac{p_\theta(\boldsymbol{x}|y = c)p_\theta(y = c)}{\sum_{c'} p_\theta(\boldsymbol{x}|y = c')p_\theta(y = c')}$$

Generative Approach for Classification

Discriminant Analysis

- Model

$$p_\theta(y = c|\boldsymbol{x}) = \frac{p_\theta(\boldsymbol{x}|y = c)p_\theta(y = c)}{\sum_{c'} p_\theta(\boldsymbol{x}|y = c')p_\theta(y = c')}$$

where:

$y \in \{1, 2, 3, \ldots, C\}$	Label
$\boldsymbol{x} \in \mathbb{R}^D$	Input pattern
$p_\theta(y = c)$	Prior probability over labels
$p_\theta(\boldsymbol{x}|y = c)$	Class conditional density
$\boldsymbol{\theta}$	Model parameters

Generative Approach for Classification

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$p_\theta(\boldsymbol{x}|y = c)$	Class conditional density
$\boldsymbol{\theta}$	Model parameters

- Number of Parameters (Assuming $p_\theta(\boldsymbol{x}|y = c)$ to be Multivariate Normal):

$$(C - 1) + \overbrace{C \times D}^{\text{Mean Vector}} + \overbrace{C \times \frac{D(D + 1)}{2}}^{\text{Covariance matrix}}$$

Generative Approach for Classification

Discriminant Analysis

Assume:

$$\boldsymbol{x} = \text{vec}\left(\text{  } \in \mathbb{R}^{600 \times 800} \right)$$

$$y \in \{1 \text{ (LivingRoom)}, 2 \text{ (BedRoom)}, 3 \text{ (DiningRoom)}\}$$

then:

$$p_\theta(\boldsymbol{x}|y=2) \xrightarrow{\text{Sampling}}$$




Comparing Discriminative and Generative Approaches

Training

- Number of parameters (Assuming $p_\theta(\boldsymbol{x}|y=c)$ to be Multivariate Normal):

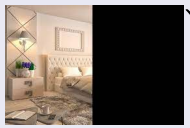
$$\overbrace{C \times D + C}^{\text{Discriminative}} \ll \overbrace{(C-1) + C \times D + C \times \frac{D(D+1)}{2}}^{\text{Generative}}$$

- Training a Discriminative model is much simpler

Comparing Discriminative and Generative Approaches

Challenging Situations

- Discriminative Model:

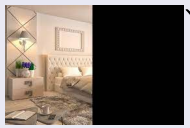
$$p_\theta\left(y \middle| \text{  }\right) = \text{Cat}(y|[0.7, 0.1, 0.2]^T)$$

Comparing Discriminative and Generative Approaches

Challenging Situations

- Discriminative Model:

$$p_\theta\left(y \middle| \right) = \text{Cat}(y|[0.7, 0.1, 0.2]^T)$$



- Generative Model: Assume $\boldsymbol{x}^v$ and $\boldsymbol{x}^m$ to be the visible and masked parts, respectively, we can calculate $p(y|\boldsymbol{x}^v)$ and thus:

$$\max_y p(y|\boldsymbol{x}^v)$$

and decide based on the visible part which is much more *robust*.

Section 5

Big Picture

Course Big Picture

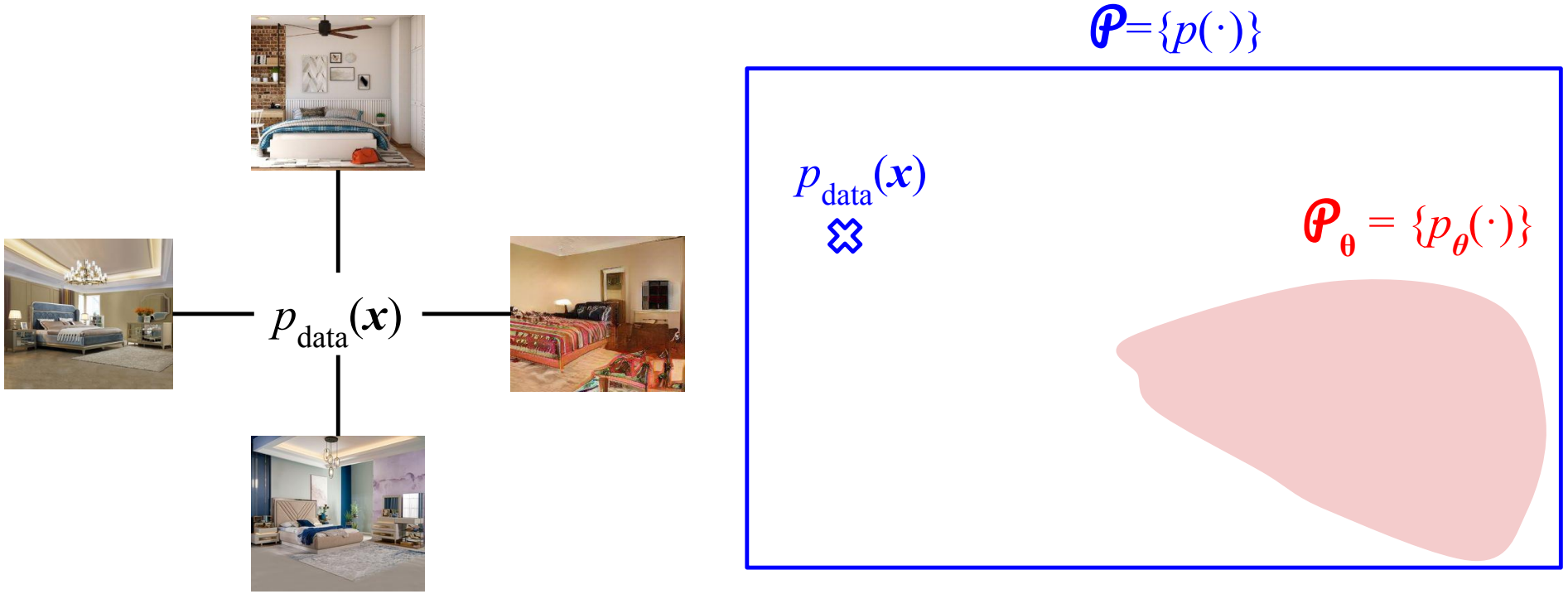
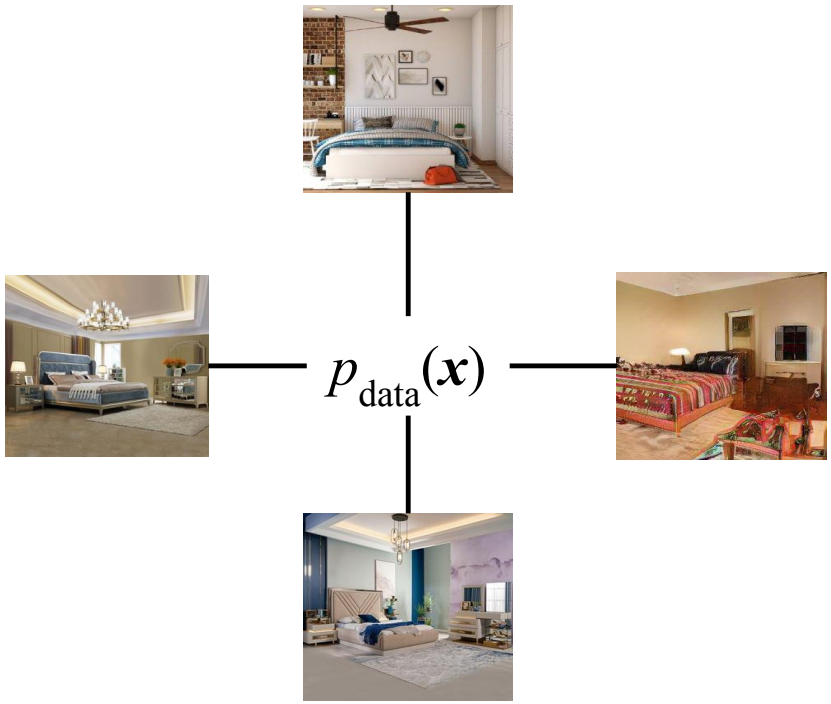


Figure: Selecting a family [5]

Course Big Picture



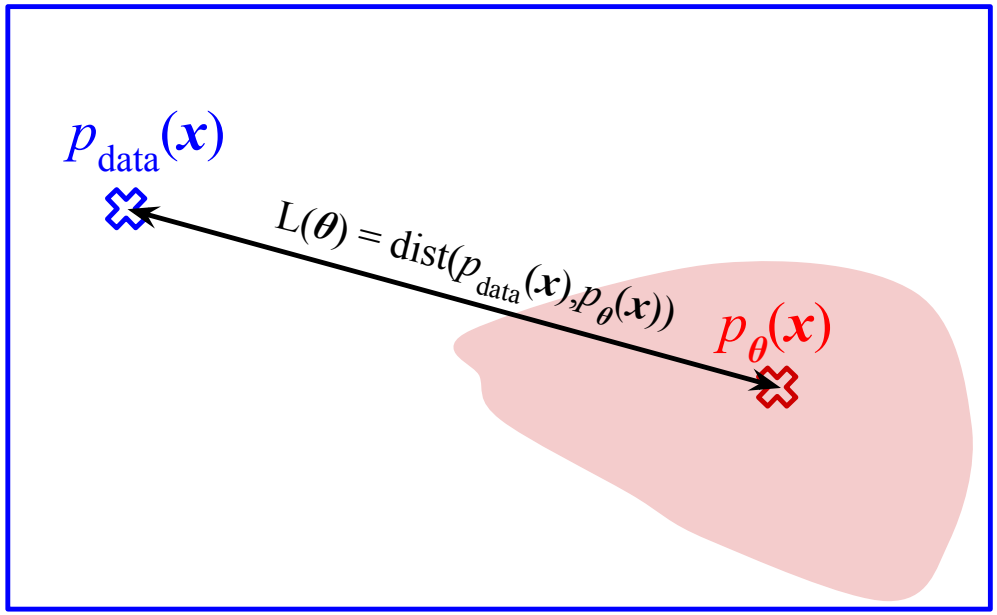
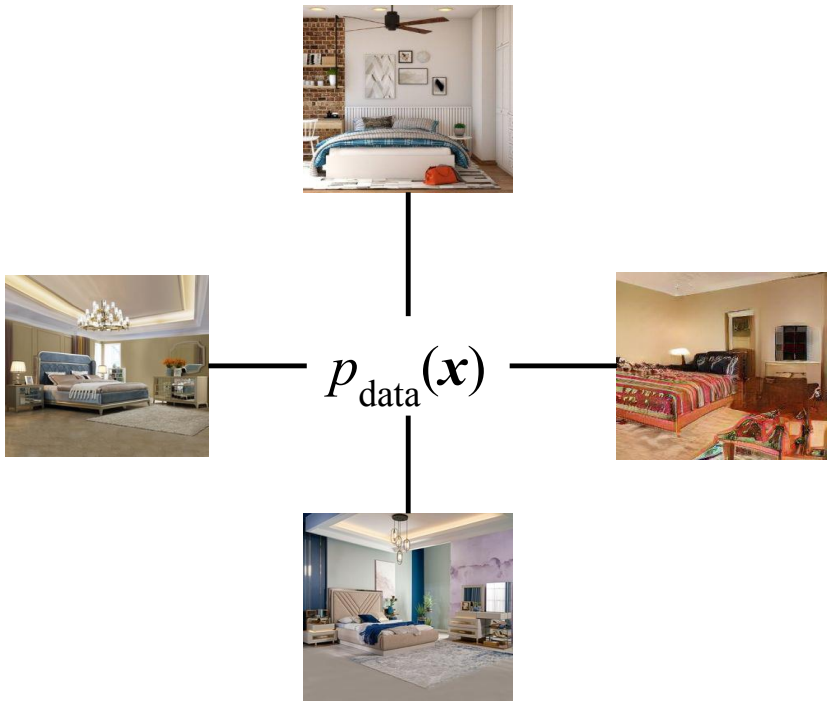


Figure: Developing a distance metric [5]

Course Big Picture



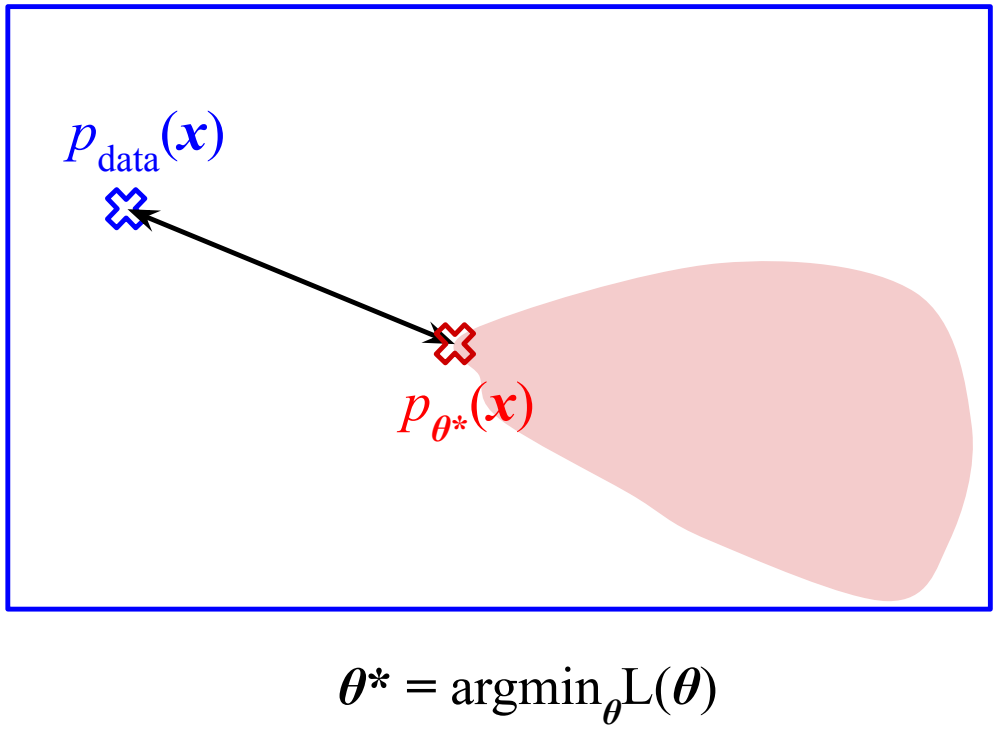
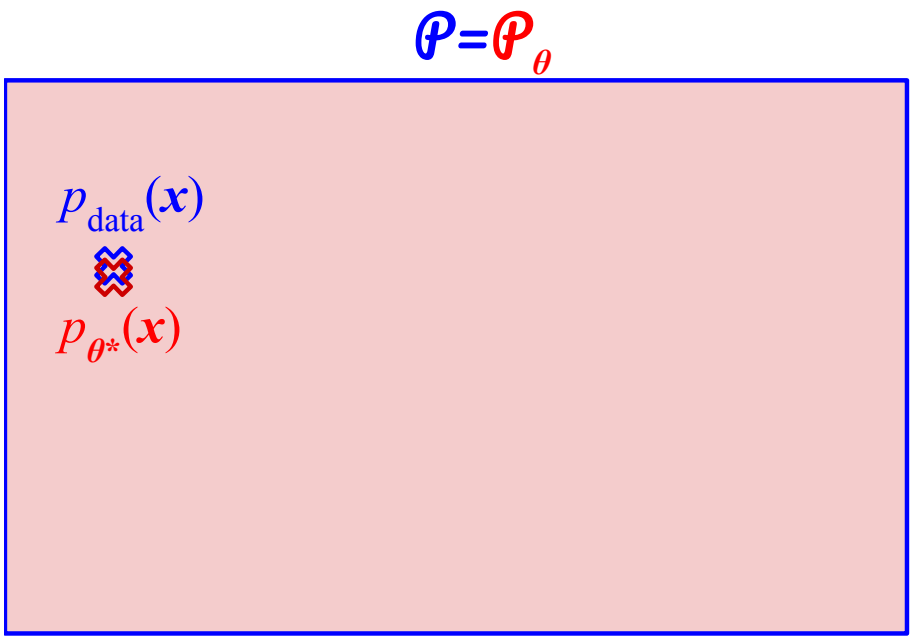
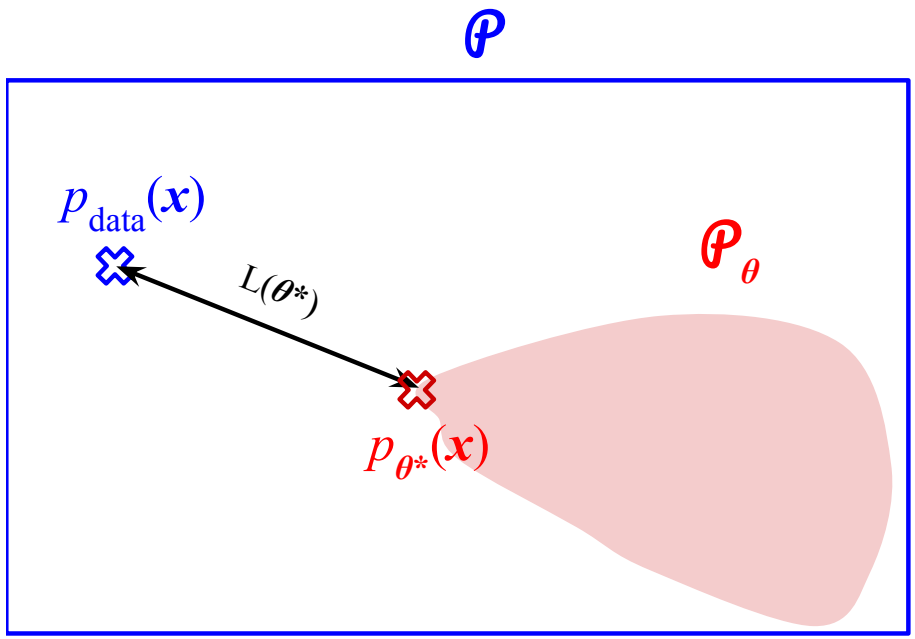


Figure: Minimizing the distance metric [5]

Limited Vs. Unlimited Selection



(a) Unlimited distribution selection



(b) Limited distribution selection

Generative Model for Simple Random Variables

Bernoulli Random Variable (Tossing a coin)

Assume we have a Bernoulli random variable:

$$X \sim \text{Ber}(\beta) \Rightarrow \begin{cases} p(X=1) = \beta \\ p(X=0) = 1-\beta \end{cases}$$

Generative Model for Simple Random Variables

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Then:

- The description for all possible distributions is:

$$\mathcal{P} = \{\text{Ber}(\alpha);\ 0 \leq \alpha \leq 1\} \Rightarrow \#\text{Parameters} = 1$$

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$$\mathcal{P} = \{\text{Ber}(\alpha);\ 0 \leq \alpha \leq 1\} \Rightarrow \#\text{Parameters} = 1$$

- The description for a subset of the whole family is:

$$\mathcal{P}_\theta = \{\text{Ber}(\theta);\ 0.2 \leq \theta \leq 0.3\} \Rightarrow \#\text{Parameters} = 1$$

Generation and Density Estimation from a Trained Model

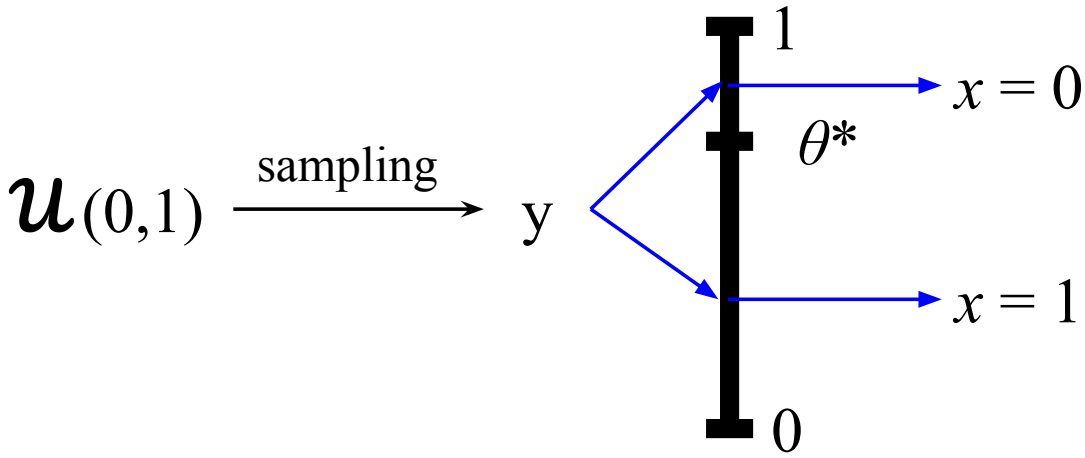

Figure: Sample generation using a trained model for Bernoulli distribution

Generation and Density Estimation from a Trained Model

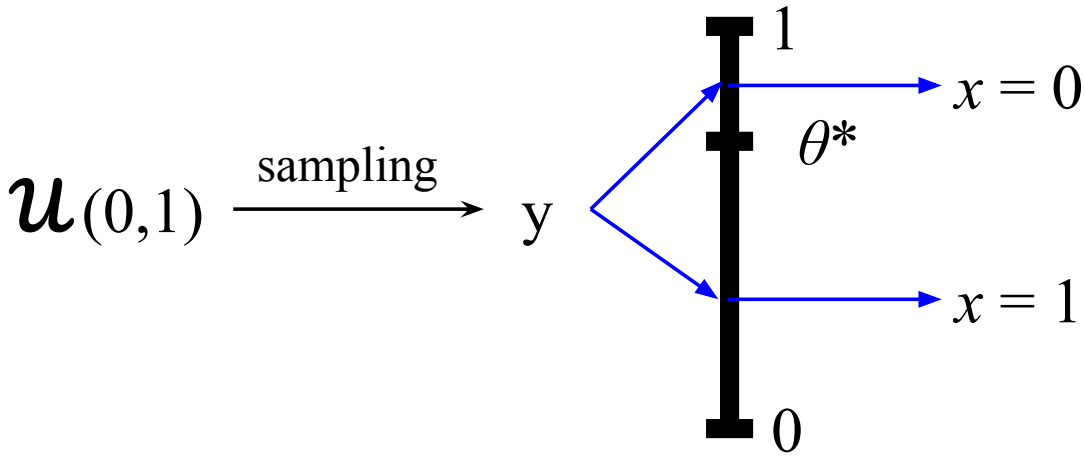

Figure: Sample generation using a trained model for Bernoulli distribution

Density Estimation

You have two states in your probabilistic world $X \in \{0, 1\}$, thus:

$$\begin{cases} p(X = 1) = \theta^\star \\ p(X = 0) = 1 - \theta^\star \end{cases}$$

Generative Model for Simple Random Variables

Categorical Random Variable (Rolling a die)

Assume we have a Categorical random variable with six states:

$$X \sim \text{Cat}([\beta_1, \ldots, \beta_6]) \Rightarrow p(X = i) = \beta_i$$

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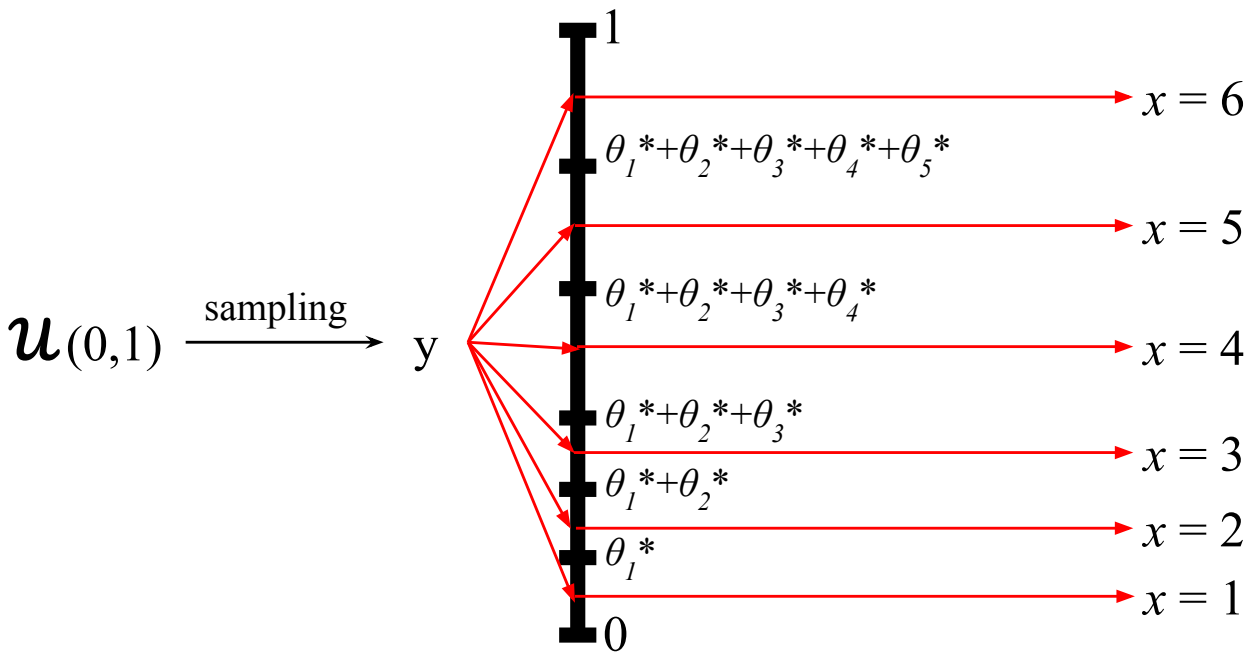
- The description for a subset of the whole family is:

$$\mathcal{P}_{\boldsymbol{\theta}} = \left\{ \text{Cat}(\boldsymbol{\theta}); 0 \leq \theta_i \leq 1, \sum_i \theta_i = 1, \begin{cases} \theta_1 = \theta_2 = \theta_3 \\ \theta_4 = \theta_5 = \theta_6 \end{cases} \right\}$$

$$\Rightarrow \#\text{Parameters} = 2 - 1$$

Generation and Density Estimation from a Trained Model



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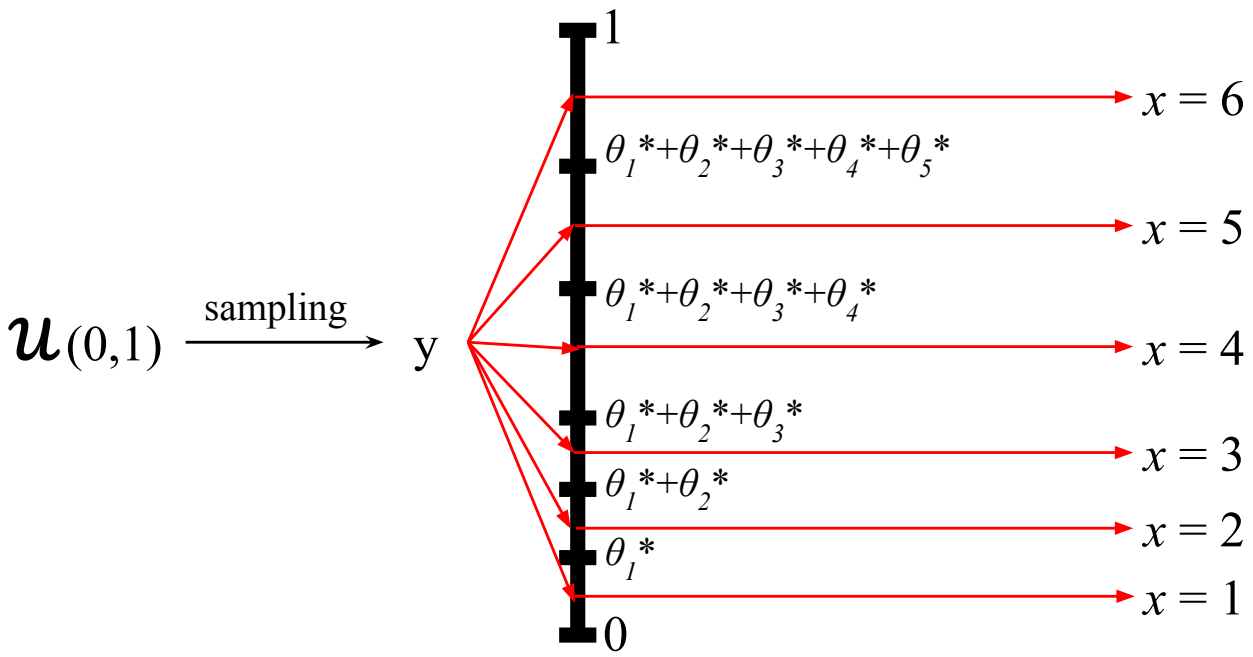

Figure: Sample generation using a trained model for Bernoulli distribution

Density Estimation

You have six states in your probabilistic world $X \in \{1, 2, 3, 4, 5, 6\}$, thus:

$$p(X = i) = \theta_i^\star$$

Real-World Categorical Distribution

RGB pixel

Assume $(r, g, b) \sim p(R, G, B)$, $\begin{cases} \text{SampleSpace}(R) = \{0, \dots, 255\} \\ \text{SampleSpace}(G) = \{0, \dots, 255\} \\ \text{SampleSpace}(B) = \{0, \dots, 255\} \end{cases}$ to be an outcome of distributions over RGB pixel, then:

- Number of Categories: 256^3

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- Number of Categories: 256^3
- #Parameters for $\mathcal{P}$: $256^3 - 1$

Real-World Binary Image Distribution

Binary image with n pixels

Assume $\boldsymbol{x} \sim p(\mathbb{X})$ to be a binary image of digits where $\mathbb{X} = [X_1, \ldots, X_n]$, Then:

- Number of Categories: 2^n
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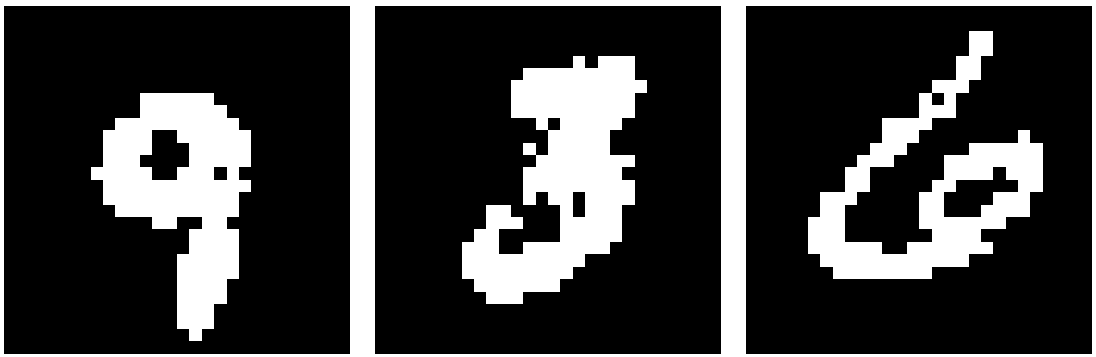

(a) Generated samples from $p_{\theta^\star}$ assuming $\mathcal{P} = \mathcal{P}_\theta$ we can find $p_{\theta^\star}$ efficiently (source: [5])

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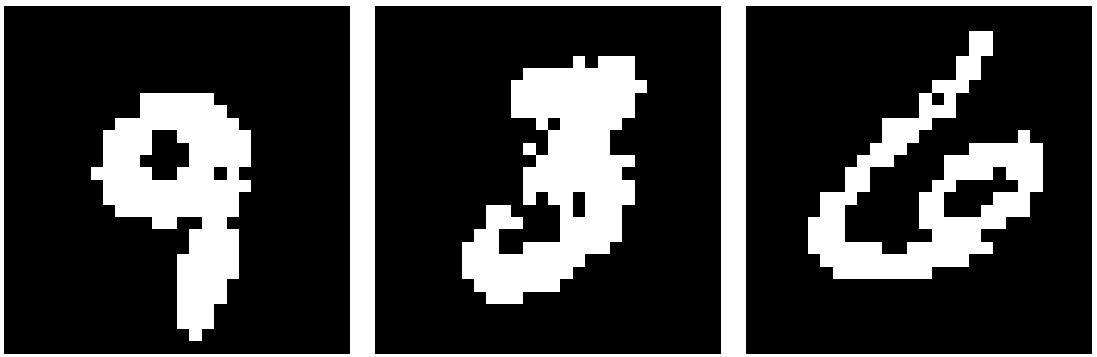

(a) Generated samples from $p_{\theta^\star}$ assuming $\mathcal{P} = \mathcal{P}_\theta$ we can find $p_{\theta^\star}$ efficiently (source: [5])

(b) Typical $\mathcal{P}$ and $\mathcal{P}_\theta$

Independence as Simplifying Tool

Binary image with n independent pixels

Assume $\boldsymbol{x} \sim p(\mathbb{X})$ to be a binary image of digits where the pixels are independent random variables, in other words, $p(\boldsymbol{x}) = p(x_1) \times p(x_2) \dots p(x_n)$, Then:

- Number of Categories: 2^n
- #Parameters for $\mathcal{P}$: $2^n - 1$
- #Parameters for $\mathcal{P}_\theta$: n

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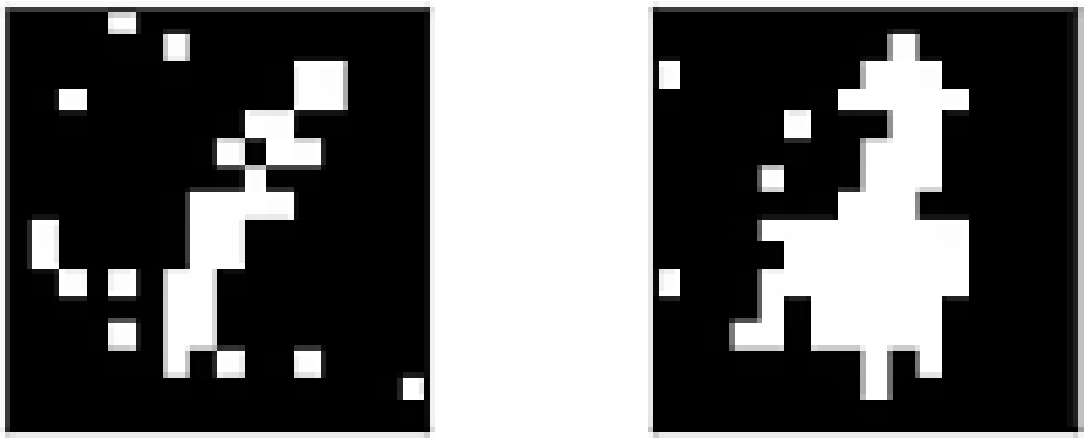

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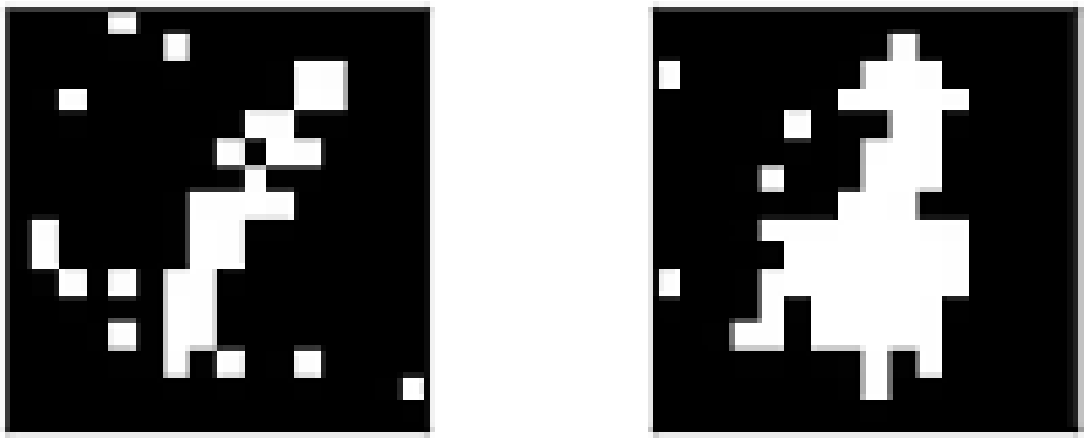

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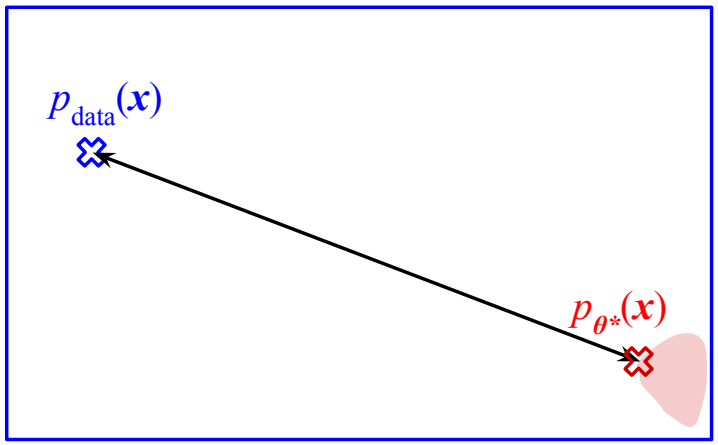

(b) Typical $\mathcal{P}$ and $\mathcal{P}_\theta$

Chain Rule

Binary image with n pixels

Using chain rule, we can write:

$$p(\boldsymbol{x}) = \overbrace{p(x_1)}^{\#1}\overbrace{p(x_2|x_1)}^{\#2}\ldots\overbrace{p(x_n|x_{n-1},\ldots,x_1)}^{\#2^{n-1}}$$

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Then:

- Number of Categories: 2^n
- #Parameters for $\mathcal{P}$: $2^0 + 2^1 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$

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Binary image with n pixels

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Then:

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- #Parameters for $\mathcal{P}$: $2^0 + 2^1 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$

Chair rule

Note that the chain rule does not impose any restriction on the distribution. Thus again we are parameterizing the $\mathcal{P}$ and thus the number of needed parameters is unchanged.

Chain Rule + Conditional Indepence

Binary Image with Conditional Independence Property

Now assum $X_{i+1} \perp X_{i-1}, \ldots, X_1 | X_i$, then we can simplify the distribution as:

$$\begin{aligned} p(\boldsymbol{x}) &= p(x_1)p(x_2|x_1)p(x_3|x_2,x_1)\ldots p(x_n|x_{n-1},\ldots,x_1) \\ &= \underbrace{p(x_1)}_{\#1}\underbrace{p(x_2|x_1)}_{\#2}\underbrace{p(x_3|x_2))}_{\#2}\ldots\underbrace{p(x_n|x_{n-1})}_{\#2} \end{aligned}$$

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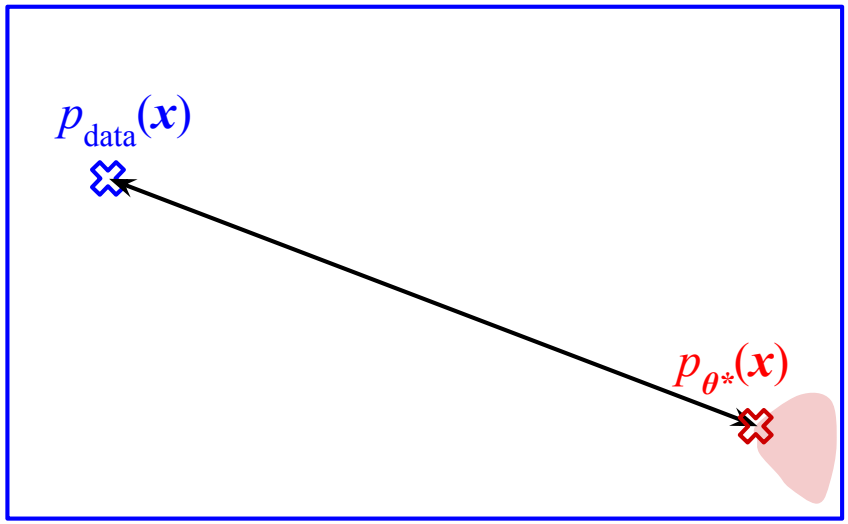
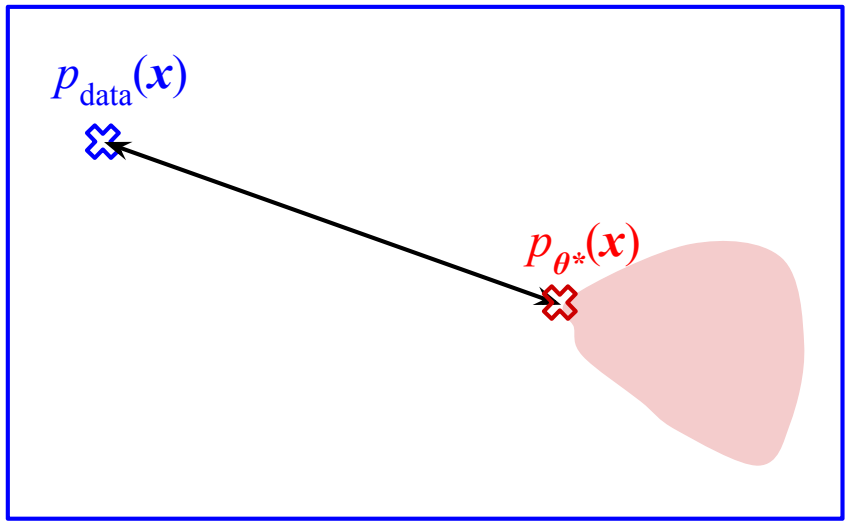
Then:

- Number of Categories: 2^n
- #Parameters for $\mathcal{P}$: $2n - 1$

Chain Rule + Conditional Indepence



(a) Typical $\mathcal{P}$ and $\mathcal{P}_\theta$ for independent case



(b) Typical $\mathcal{P}$ and $\mathcal{P}_\theta$ for first-order Markov chain

Figure: With first-order Markov chain, while controlling the tractability for the number of parameters, the chance to find a better fit to p_{data} increases

Section 6

Conclusions

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Our Objective in This Course

- Introduce suitable subset of probability distribution functions $\mathcal{P}_\theta$:
  - Not too large (proposing $\mathcal{P}$ as $\mathcal{P}_\theta$)

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  - Aware of input type (such as image, text, audio, video, etc.)

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- Introduce suitable distance metrics between two distributions

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- Introduce suitable subset of probability distribution functions $\mathcal{P}_\theta$:
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  - Aware of input type (such as image, text, audio, video, etc.)
- Introduce suitable distance metrics between two distributions
- Introduce the optimization methods to reduce the distance metric between model distribution and true distribution

Conclusions

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- Introduce suitable subset of probability distribution functions $\mathcal{P}_\theta$:
  - Not too large (proposing $\mathcal{P}$ as $\mathcal{P}_\theta$)
  - Not too small (image with independent pixels)
  - Aware of input type (such as image, text, audio, video, etc.)
- Introduce suitable distance metrics between two distributions
- Introduce the optimization methods to reduce the distance metric between model distribution and true distribution
- Evaluate your modeling performance

List of Abbreviations

Complete	Abbreviation
Deep Generative Model	DGM
Multinomial Logistic Regression	MLR

References I


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