4001124 محمد پارسا امیری تمرین سری 6 کنترل

ⓐ $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \longrightarrow G(j\omega) = \dfrac{\omega_n^2}{(\omega_n^2 - \omega^2) + j\,2\omega\omega_n\zeta}$

$\longrightarrow |G(j\omega)|^2 = \dfrac{\omega_n^4}{(\omega^2-\omega_n^2)^2 + 4\zeta^2\omega_n^2\omega^2} \longrightarrow \dfrac{d}{d\omega}|G(j\omega)|^2 = \dfrac{4\omega_n^2\left[\omega_n^2(2\zeta^2-1)\omega + \omega^3\right]}{\left[(\omega_n^2-\omega^2)^2 + 4\zeta^2\omega_n^2\omega^2\right]} = 0$

$\Rightarrow$

$\omega = 0\,,\quad \boxed{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$ valid for $|\zeta| \le \frac{\sqrt{2}}{2}$ $\Bigg\}$ $M_{r_1} = |G(j\omega_r)| = \left|\dfrac{\omega_n^2}{-2\omega_n^2\zeta^2 + 2j\omega_n^2\zeta\sqrt{1-2\zeta^2}}\right| = \dfrac{1}{\sqrt{4\zeta^2 - 2\zeta^4}}$

$\longrightarrow \boxed{M_{r_1} = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}}$

ⓑ $G(s) = \dfrac{\omega_n(s+\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \longrightarrow G(j\omega) = \dfrac{j\omega\omega_n + \omega_n^2}{(\omega_n^2-\omega^2) + j\,2\omega\omega_n\zeta} \longrightarrow |G(\omega)|^2 = \dfrac{\omega_n^4 + \omega^2\omega_n^2}{\left[(\omega_n^2-\omega^2)^2 + 4\zeta^2\omega^2\omega_n^2\right]}$

$\dfrac{d}{d\omega}|G(\omega)|^2 = -\dfrac{2\omega_n^2\omega\left[\omega_n^4(4\zeta^2-3) + 2\omega_n^2\omega^2 + \omega^4\right]}{\left[\omega_n^4 + 2\omega_n^2\omega^2(2\zeta^2-1) + \omega^4\right]^2} \longrightarrow \omega \ne 0$

$\longrightarrow \omega^4 + 2\omega_n^2\omega^2 + \omega_n^4(4\zeta^2-3) = 0 \longrightarrow \omega^2 = \dfrac{-2\omega_n^2 \pm \sqrt{4\omega_n^4 - 16\zeta^2\omega_n^4 + 12\omega_n^4}}{2} \xrightarrow[\omega>0]{\omega\in\mathbb{R}}$

$\longrightarrow \omega_r = \omega_n\sqrt{2\sqrt{1-\zeta^2}-1} \longrightarrow M_{r_2} = |G(j\omega_r)|^2 = \dfrac{\omega_n^2\left[\omega_n^2 + \omega_n^2(-1+2\sqrt{1-\zeta^2})\right]}{\left(\cancel{\omega_n^2} - \cancel{\omega_n^2} + 2\omega_n^2\sqrt{1-\zeta^2}\right)^2 + 4\zeta^2\omega_n^2(-1+2\sqrt{1-\zeta^2})\omega_n^2}$

$\longrightarrow M_{r_2}^2 = \dfrac{2\omega_n^4\sqrt{1-\zeta^2}}{4\omega_n^4(1-\zeta^2) + 4\omega_n^4\zeta^2(-1+2\sqrt{1-\zeta^2})} = \dfrac{2\sqrt{1-\zeta^2}}{4-4\zeta^2 - 4\zeta^2 + 8\zeta^4\sqrt{1-\zeta^2}} = \dfrac{\sqrt{1-\zeta^2}}{2-4\zeta^2+4\zeta^4\sqrt{1-\zeta^2}}$

$\longrightarrow M_{r_2} = \dfrac{(1-\zeta^2)^{1/4}}{\sqrt{2}\left(1-2\zeta^2+2\zeta^4\sqrt{1-\zeta^2}\right)^{1/2}}$

ⓒ

supremum occurs when $\omega = \omega_r$

$\lim \dfrac{\sup|G_1(j\omega)|}{\sup|G_2(j\omega)|} = \dfrac{\sup\left|G_1\left(\lim\limits_{\zeta\to 0} j\omega\right)\right|}{\sup\left|G_2\left(\lim\limits_{\zeta\to 0} j\omega\right)\right|} = \dfrac{\left|G_1\left(\lim\limits_{\zeta\to 0} j\omega_{r_1}\right)\right|}{\left|G_2\left(\lim\limits_{\zeta\to 0} j\omega_{r_2}\right)\right|} = \dfrac{|G_1(j\omega_n)|}{|G_2(j\omega_n)|}$

since G_1 & G_2 are continuous function

$= \dfrac{|\omega_n^2|}{|j\omega_n^2 + \omega_n^2|} = \boxed{\dfrac{1}{\sqrt{2}}}$

$\lim\limits_{\zeta\to 0} \omega_{r_1} = \lim\limits_{\zeta\to 0} \omega_n\sqrt{1-2\zeta^2} = \omega_n$

$\lim\limits_{\zeta\to 0} \omega_{r_2} = \lim\limits_{\zeta\to 0} \omega_n\sqrt{2\sqrt{1-\zeta^2}-1} = \omega_n$

(a) $G(s) = \dfrac{k}{\tau s+1} \rightarrow G(j\omega) = \dfrac{k}{\tau j\omega+1} \rightarrow |G(j\omega)| = \dfrac{k}{\sqrt{1+\tau^2\omega^2}}$

DC gain $= k$

the system described above is indeed a low pass filter, hence:

$$|G(j\omega)| = \frac{k}{\sqrt{1+\tau^2\omega^2}} = \frac{G_{max}}{\sqrt{2}} = \frac{k}{\sqrt{2}} \rightarrow \tau^2\omega^2 = 1 \rightarrow \boxed{\omega_c = 1/\tau}$$

(b) $G(s) = \dfrac{k}{s^2+2\zeta\omega_n s+\omega_n^2} \rightarrow G(j\omega) = \dfrac{k/\omega_n^2}{\omega_n^2\left[(\omega_n^2-\omega^2)+j2\zeta\omega_n\omega\right]} \rightarrow |\tilde{G}(j\omega)|^2 = \dfrac{\omega_n^4}{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2}$

DC gain $= \dfrac{k}{\omega_n^2}$

$$20\log|\tilde{G}(j\omega)| = -10\log\left[(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2\right] + 10\log\left[\omega_n^4\right] = -3dB = -10\log(2)$$

$$\rightarrow \frac{\omega_n^4}{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2} = \frac{1}{2} \rightarrow 2\omega_n^4 = \omega_n^4+\omega^4-2\omega^2\omega_n^2+4\zeta^2\omega^2\omega_n^2$$

$$\rightarrow \omega^4-\omega^2(2\omega_n^2-4\zeta^2\omega_n^2)-\omega_n^4=0 \rightarrow \omega^2 = \frac{(2\omega_n^2-4\zeta^2\omega_n^2) \pm \sqrt{4\omega_n^4+16\zeta^4\omega_n^4-16\zeta^2\omega_n^4+4\omega_n^4}}{2}$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1-2\zeta^2 \pm \sqrt{2+4\zeta^4-4\zeta^2} \xrightarrow[\omega>0]{} \boxed{\omega_c = \omega_n \times \sqrt{1-2\zeta^2+\sqrt{2+4\zeta^4-4\zeta^2}}}$$

(c) $G(s) = \dfrac{2}{3} \times \dfrac{(s+200)(s+300000)}{(s+300)(s+200000)} \rightarrow |G(j\omega)| = \dfrac{2}{3} \times \dfrac{\sqrt{200^2+\omega^2} \times \sqrt{300000^2+\omega^2}}{\sqrt{300^2+\omega^2} \times \sqrt{200000^2+\omega^2}}$

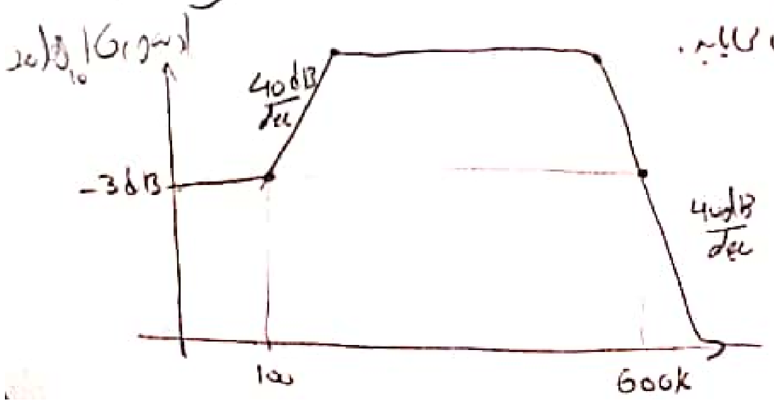
$$20\log|G(j\omega)| = 20\log\left(\frac{2}{3}\right) + 10\log(200^2+\omega^2) + 10\log(300000^2+\omega^2) - 10\log(300^2+\omega^2) - 10\log(200000^2+\omega^2) = -3dB = -10\log(2)$$

$$\rightarrow \log\left(\frac{(200^2+\omega^2)(300000^2+\omega^2)}{(300^2+\omega^2)(200000^2+\omega^2)}\right) = -\log(2)-\log\left(\frac{4}{9}\right) = \log\left(\frac{9}{8}\right) = 0.05115$$

$$\rightarrow \frac{(200^2+\omega^2)(300000^2+\omega^2)}{(300^2+\omega^2)(200000^2+\omega^2)} = 10^{0.0515} = 1.12499 \rightarrow \begin{array}{l}\omega \simeq 100, -100 \\ \omega \simeq \pm 600000\end{array}$$

(d) محدوده فرکانس صوت در بازه [0, 20kHz] است و این فیلتر میان‌گذر، فقط این محدوده را پوشش می‌دهد ولی چون این فیلتر فرکانس‌های از 20kHz تا 600kHz را نیز پوشش می‌دهد، سبب آن می‌شود که توان بیشتری از نویز نیز بر سیگنال سوار شده و در نتیجه SNR کاهش می‌یابد.

و این به معنی افزایش تداخل است.



سوال 3:

ⓐ $G(s)=\dfrac{1}{Ts+1} \rightarrow |G(j\omega)|^2=\dfrac{1}{T^2\omega^2+1}=1 \longrightarrow \boxed{\omega_{cc}=0}$

ⓑ $G(s)=\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s} \rightarrow |G(j\omega)| = \dfrac{\omega_n^4}{\omega^4+4\zeta^2\omega_n^2\omega^2}=1 \longrightarrow \omega^4+4\zeta^2\omega_n^2\omega^2-\omega_n^4=0$

$\forall \zeta \in \mathbb{R}$

$\omega_{cc}^2=\dfrac{-4\zeta^2\omega_n^2 \pm\sqrt{16\zeta^4\omega_n^4+4\omega_n^4}}{2} \xrightarrow[\omega\in\mathbb{R}]{\omega_n>0} \omega_{cc}=\omega_n\sqrt{2\zeta^2\pm\sqrt{1+4\zeta^4}} \quad \xrightarrow{\text{since } \sqrt{1+4\zeta^4}\geq 2\zeta^2}$

$\longrightarrow \boxed{\omega_{cc}=\omega_n\sqrt{2\zeta^2+\sqrt{1+4\zeta^4}}}$

ⓒ $G(s)=\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \rightarrow |G(j\omega)|^2=\dfrac{\omega_n^4}{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2}=1$

$\longrightarrow \omega_n^4=\omega_n^4+\omega^4-2\omega_n^2\omega^2+4\zeta^2\omega_n^2\omega^2 \longrightarrow \omega^2\left(\omega^2+[4\zeta^2\omega_n^2-2\omega_n^2]\right)=0$

$\xrightarrow{\omega\neq 0} \boxed{\omega_{cc}=\omega_n\sqrt{2-4\zeta^2}}$

سوال 4:

ⓐ $y(t)=g(t)*u(t) \xrightarrow{\text{taking } \mathcal{F} \text{ transform}} \left.\begin{array}{l} Y(j\omega)=G(j\omega)U(j\omega) \\ Y(-j\omega)=G(-j\omega)U(-j\omega)\end{array}\right\} \longrightarrow$ let $\forall\omega\; G(j\omega)\leq M$

$\longrightarrow \dfrac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega)Y(-j\omega)d\omega=\|y\|_2^2=\dfrac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)G(-j\omega)U(j\omega)U(-j\omega)d\omega \leq \dfrac{M^2}{2\pi}\int_{-\infty}^{\infty}U(j\omega)U(-j\omega)d\omega$

$\Rightarrow \|y\|_2 \leq M\sqrt{\dfrac{1}{2\pi}\int_{-\infty}^{\infty}U(j\omega)U(-j\omega)d\omega} = \boxed{\max|G(j\omega)| \times \|u\|_2}$ ■ Q.E.D.

ⓑ we'll show that for the inequality above, the equality holds if $u(t)$ is an eigenfunction of the form: $u(t)=e^{j\omega_0 t} \longrightarrow y(t)=e^{j\omega_0 t}G(j\omega_0)$, $U(\omega)=\delta(\omega-\omega_0)$

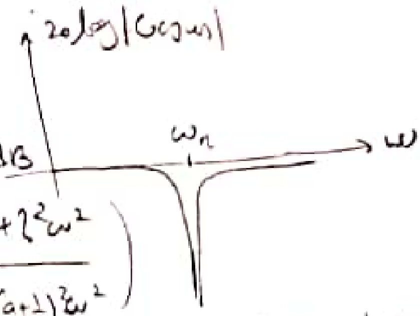
$\|y\|_2=\sqrt{\dfrac{1}{2\pi}\int_{-\infty}^{\infty}G(j\omega)G(-j\omega)\,\delta(\omega-\omega_0)\,\delta(\omega_0-\omega)\,d\omega} = \sqrt{\dfrac{1}{2\pi}G(j\omega_0)G(-j\omega_0)\int_{-\infty}^{\infty}\delta^2(\omega-\omega_0)d\omega}$

$=\sqrt{G(j\omega_0)G(-j\omega_0)}\sqrt{\dfrac{1}{2\pi}\int_{-\infty}^{\infty}e^{j\omega_0 t}e^{-j\omega_0 t}dt} = G(j\omega_0)\times\|1\| = G(j\omega_0)$

Notch filters characteristic

$$G(s) = \frac{s^2+2\zeta\omega_n s+\omega_n^2}{\omega_n^2} \times \frac{a\omega_n}{s+a\omega_n} \times \frac{\frac{\omega_n}{a}}{s+\frac{\omega_n}{a}} = \frac{s^2+2\zeta\omega_n s+\omega_n^2}{s^2+s\omega_n(a+\frac{1}{a})+\omega_n^2} \quad 50\text{Hz}$$

We wish to eliminate the frequency 50Hz while the cut-off frequencies are 49Hz, 51Hz. So we can tune "a" in a way that we get that.



$$20\log|G(j\omega)| = 10\log\left(\frac{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2}{(\omega_n^2-\omega^2)^2+(a+\frac{1}{a})^2\omega_n^2\omega^2}\right) = 20\log\left(\frac{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+4\zeta^2\omega^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+(a+\frac{1}{a})^2\omega^2}\right)$$

$\omega_n = 50\times 2\pi = 100\pi$

$$20\log|G(j\omega_n)| = 10\log\left(\frac{4\zeta^2}{(a+\frac{1}{a})^2}\right) = 20\log\left|\frac{2\zeta}{a+\frac{1}{a}}\right|$$

Now we must have

let $a>1$

$$\begin{cases} \frac{\omega_n}{a} = 2\pi\times 49 \\ a\omega_n = 2\pi\times 51 \end{cases} \quad a = \sqrt{\frac{51}{49}} = 1.02020$$

We can also check that $20\log|G(j\omega)|_{\omega=a\omega_n, \frac{\omega_n}{a}} = -10\log(2) = -3\text{dB}$ which is what we desired. Furthermore, we can check that $20\log|G(j\omega_n)|$ is also maximized since by AM-GM inequality we have $a+\frac{1}{a} \geq 2\sqrt{a\cdot\frac{1}{a}} = 2$ which since a is close to 1, $a+\frac{1}{a}$ will have a small deviation with respect to, thus: $a+\frac{1}{a} \approx 2$.

$$20\log|G(j\omega_n)| = 20\log\left(\frac{2\zeta}{a+\frac{1}{a}}\right) \approx \boxed{20\log(\zeta)}$$

Scanned by CamScanner

المسألة:

$$20\log|G(j2\pi\times 51)| = 10\log\left(\frac{0.0016+(4\pi^2)4\zeta^2\times 51^2}{0.0016+2(4\pi^2)\times 51^2}\right) = 10\log\left(\frac{1}{2}\right)$$

$\frac{0.1}{a} \simeq 1.0000377 \simeq 2$

$\omega_n = 50\times 2\pi$

$\omega = 51\times 2\pi$

$$\hookrightarrow 4\zeta^2-2\simeq 0 \rightarrow \zeta^2=\frac{1}{2} \rightarrow \zeta \approx \frac{1}{\sqrt{2}} = \boxed{0.7071}$$

eventually we get $\boxed{a=1.02020\ ,\ \zeta=0.7071\ ,\ \omega_n=100\pi}$

حل:

(a) Phase $=180° \longrightarrow GM = 80\,dB$

Gain $=0\,dB \longrightarrow PM \simeq 180-140 = 40°$

$\Big\}\ \zeta = \frac{\phi_m}{100} = 0.4$

$G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s}$

$$T(\omega) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \Rightarrow |T(\omega)| = \frac{\omega_n^2}{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2} \longrightarrow 20\log_{10}|T(\omega)| = 0\,dB$$

$$\rightarrow \omega_c = \omega_n\sqrt{2-2\zeta^2} = \omega_n\sqrt{2}\times\sqrt{1-(0.4)^2} = \omega_n\times 1.296148 \simeq \omega_n\times 1.3$$

Since cutoff frequency from the depicted graph can be estimated as $9\frac{rad}{s}$ we'll get: $\omega_n\times 1.3 = 9\frac{rad}{s} \rightarrow \omega_n = 6.92307 \simeq \boxed{6.92\ rad/s}$

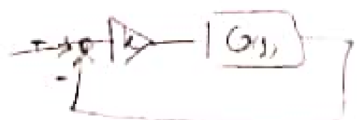
(b) $T(s) = \frac{kG(s)}{1+kG(s)} = \frac{k\omega_n^2}{s^2+2s\zeta\omega_n+k\omega_n^2}$ we depict the Routh-Hurwitz table for the denominator of $T(s)$

s^2	1	$k\omega_n^2$
s	$2\zeta\omega_n$	0
s^0	$+k\omega_n^2$	0

$2\zeta\omega_n > 0 \longrightarrow \zeta > 0$

$+k\omega_n^2 > 0 \longrightarrow k > 0$

$\longrightarrow k\in(0,\infty]$



the range of stability of closed loop is

$\Rightarrow$ the bigger the k is, the ~~more~~ stable the closed loop system becomes.

(c) $T(s) = \frac{e^{-Ts}G(s)}{1+e^{-Ts}G(s)} \longrightarrow D.M. = \frac{P.M}{\omega_c} = \frac{40}{9} = \boxed{4.444}$ ← Delay Margin

Stability Range for $\tau < 4.44 \Leftarrow \tau > 4.44 \rightarrow$ sign of phase changes $\rightarrow$ Not stable

(d)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1+\lim_{s\to 0}\frac{\omega_n^2}{s^2+2s\zeta\omega_n}} = \frac{1}{1+\infty} = \boxed{0}$$ ← Steady state error

(e) • $\zeta = 0.4 \rightarrow P.O. = 100 \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \boxed{25.38267\%}$ ← Percent of overshoot

• $T_{r_1} = \frac{0.6 + 2.16\zeta}{\omega_n} = \boxed{0.16766\ s}$ ← the Rising Time between 10% & 90% of the steady state value

• $T_s = \frac{4}{\zeta\omega_n} = \boxed{1.11\ s}$ ← the 95% setting time

$\begin{cases} \zeta = 0.4 \\ \omega_n = 1.36 \end{cases}$

the bandwidth of the closed loop system.

(f) $2\pi B = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 12.37065\ \frac{rad}{s} \longrightarrow B = \frac{12.37065}{2\pi} = \boxed{1.9688\,Hz}$

$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \boxed{1.363861}$ ← Peak amplitude of CLFR.

$\omega_r = \omega_n \sqrt{1 - 2\zeta^2} = \boxed{7.4215\ \frac{rad}{s}}$ ← Peak frequency of the closed loop freq. response

LCS HW6 Software Assignment

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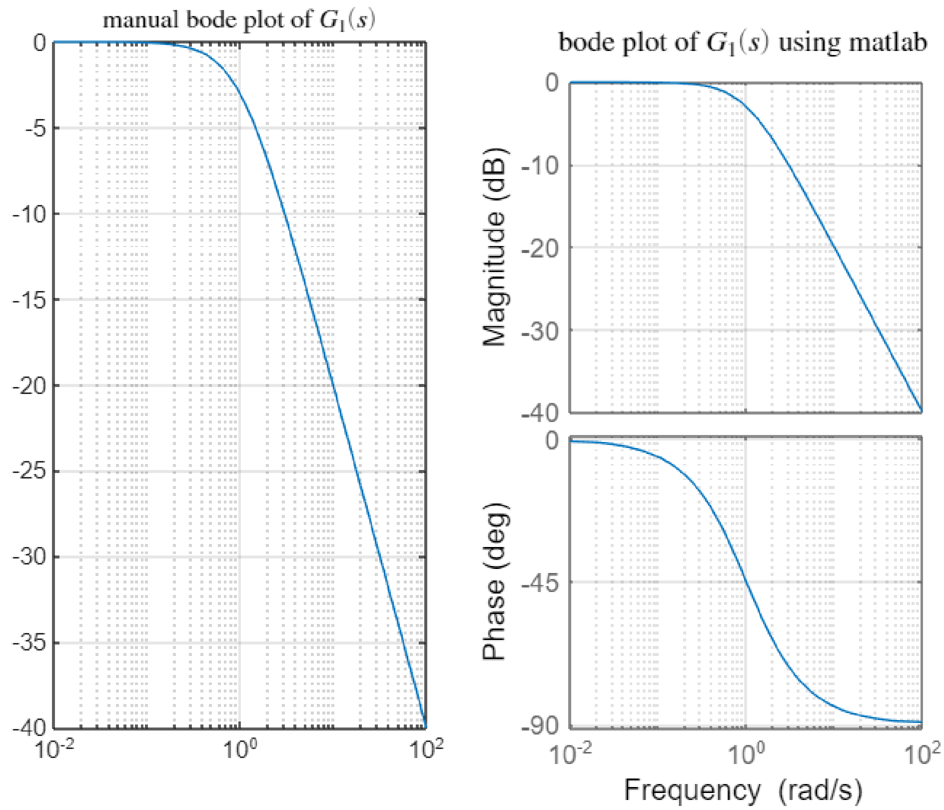
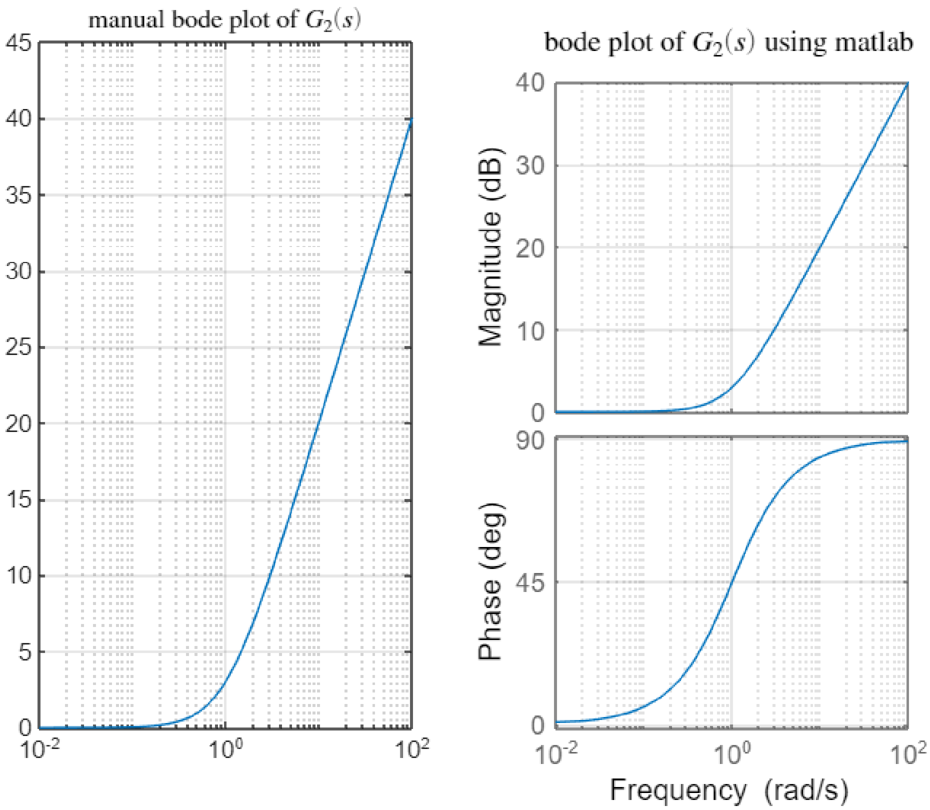
Problem 7:

Here the bode plots of these transfer functions are depicted:

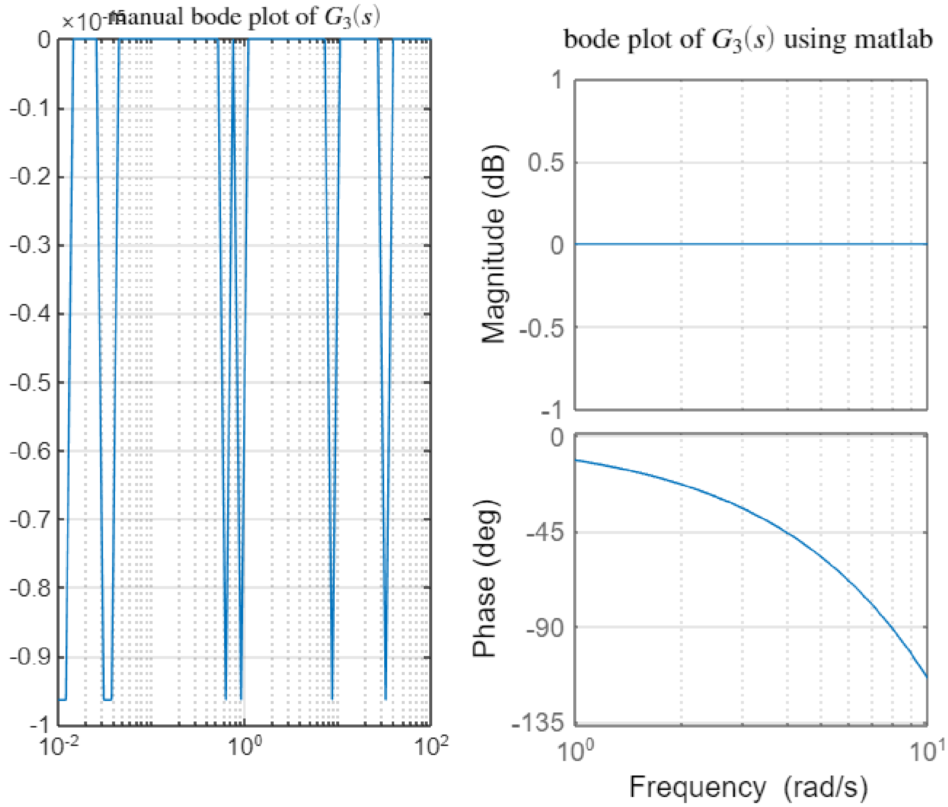
```
clear;
close all;
clc;

% G1
figure();
w = logspace(-2,2);
M1 = 1./(w*1i+1);
log_M1 = 20*log10(abs(M1));
subplot(1,2,1);
semilogx(w,log_M1);
grid on
title('manual bode plot of $G_1 (s)$',Interpreter='latex');
subplot(1,2,2);
G1 = tf(1 ,[1 1]);
bode(G1);
grid on
title('bode plot of $G_1 (s)$ using matlab',Interpreter='latex');
```



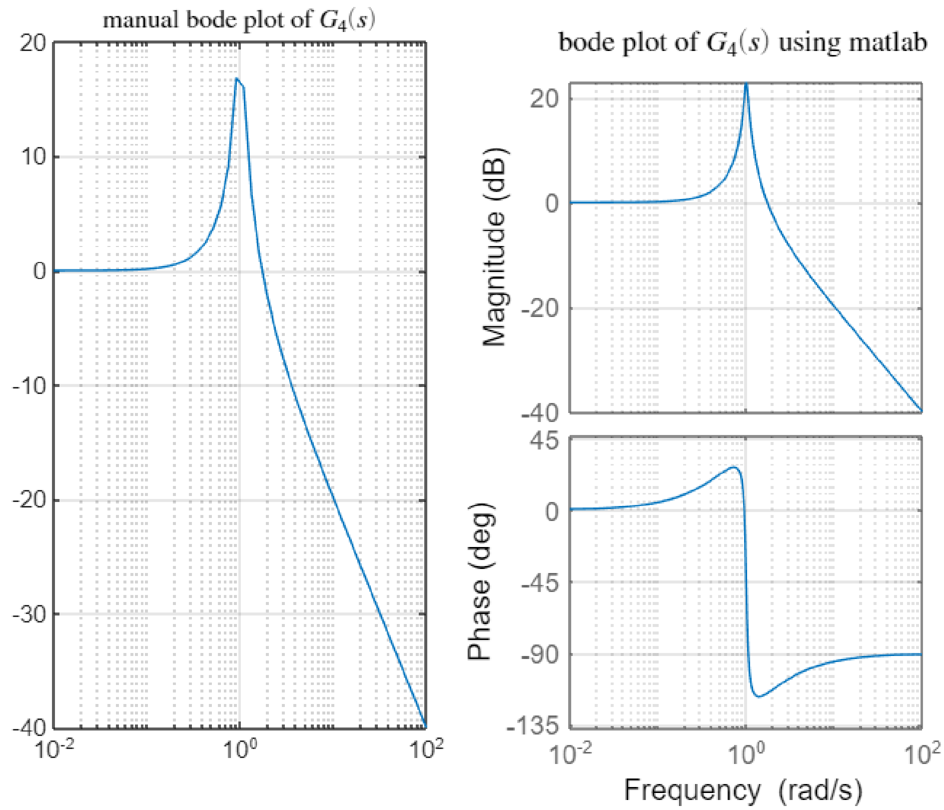

```
% G2 -------------------
clear;
figure();
w = logspace(-2,2);
M6 = (w*1i+1);
log_M2 = 20*log10(abs(M6));
subplot(1,2,1);
semilogx(w,log_M2);
grid on
title('manual bode plot of $G_2 (s)$',Interpreter='latex');
subplot(1,2,2);
G2 = tf([1 1],1);
bode(G2);
grid on
title('bode plot of $G_2 (s)$ using matlab',Interpreter='latex');
```



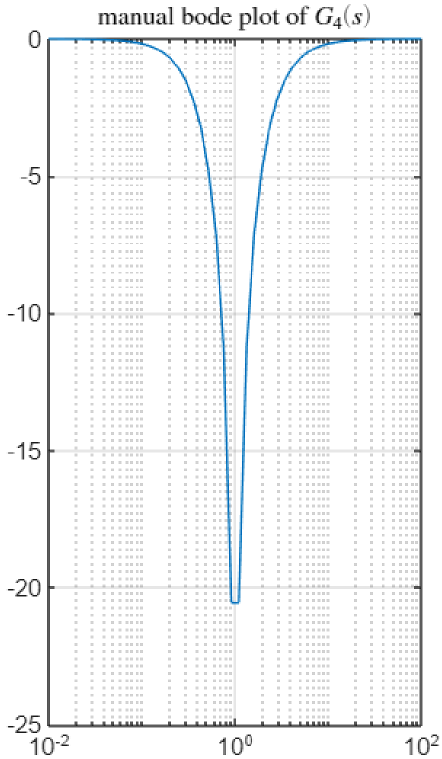

```
% G3 -------------------
clear;
figure();
w = logspace(-2,2);
M6 = exp(-0.2*w*1i);
log_M4 = 20*log10(abs(M6));
subplot(1,2,1);
semilogx(w,log_M4);
grid on
title('manual bode plot of $G_3 (s)$',Interpreter='latex');
subplot(1,2,2);
s = tf('s');
G6 = exp(-0.2*s);
bode(G6);
grid on
title('bode plot of $G_3 (s)$ using matlab',Interpreter='latex');
```

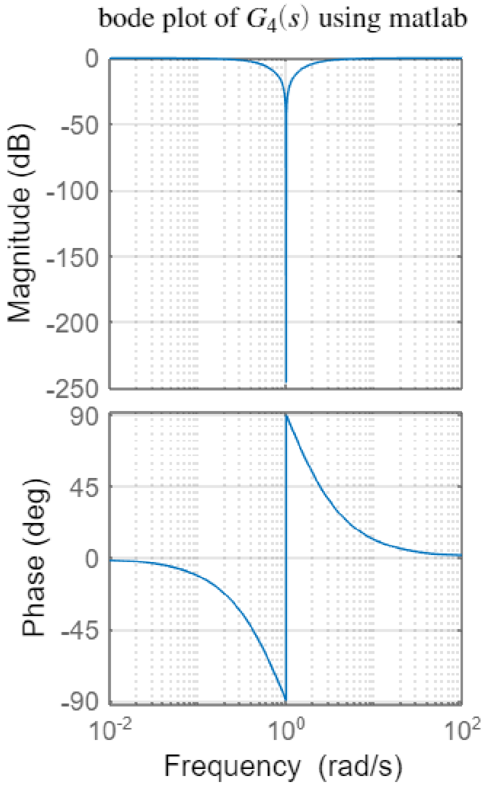



```
% G4 -------------------
clear;
figure();
w = logspace(-2,2);
M6 = (w*1i + 1)./(0.1*w*1i+1-w.*w);
log_M4 = 20*log10(abs(M6));
subplot(1,2,1);
semilogx(w,log_M4);
grid on
title('manual bode plot of $G_4 (s)$',Interpreter='latex');
subplot(1,2,2);
s = tf('s');
G6 = tf([1 1] ,[1 0.1 1]);
bode(G6);
grid on
title('bode plot of $G_4 (s)$ using matlab',Interpreter='latex');
```




```
% G5 -------------------
clear;
figure();
w = logspace(-2,2);
M6 = (-1*w.*w + 1)./(-1*w.*w + 2*w*1i + 1);
log_M = 20*log10(abs(M6));
subplot(1,2,1);
semilogx(w,log_M);
grid on
title('manual bode plot of $G_4 (s)$',Interpreter='latex');
subplot(1,2,2);
s = tf('s');
G6 = tf([1 0 1] ,[1 2 1]);
bode(G6);
grid on
title('bode plot of $G_4 (s)$ using matlab',Interpreter='latex');
```



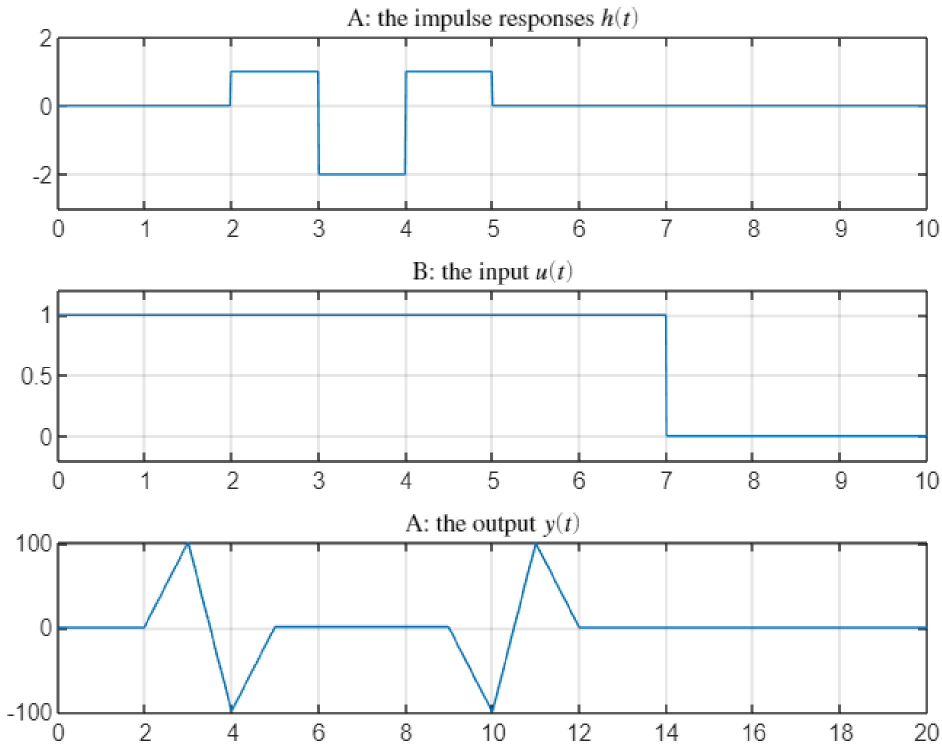


Both of these are resemblent, however we can see that matlab's plot is more accuarate around the singular point.

Problem 8:

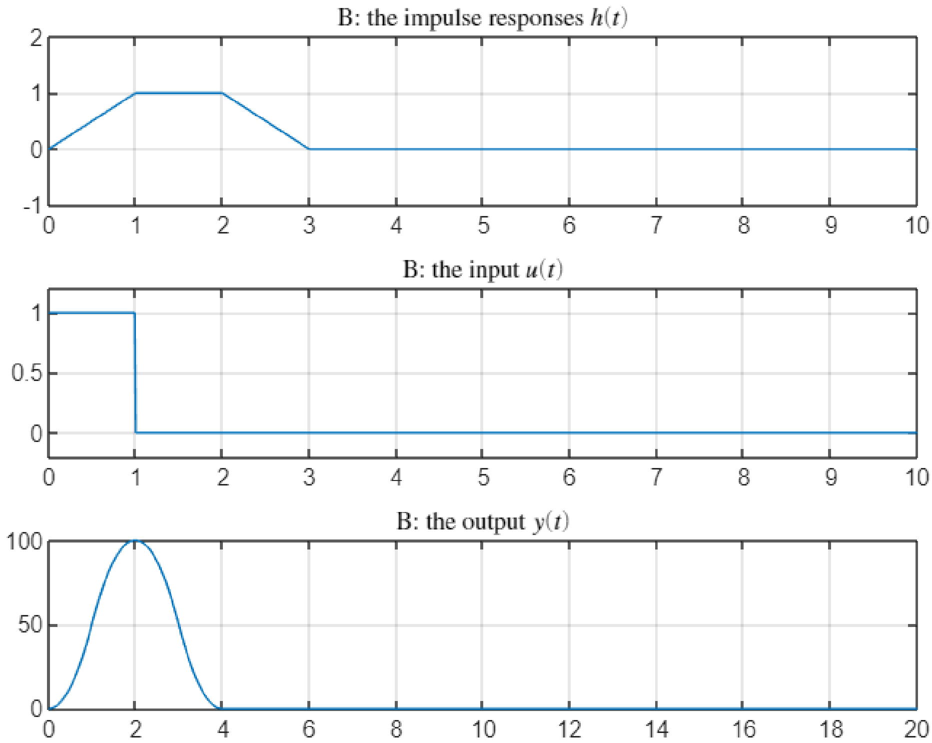
```
clear;
close all;
clc;

% A --------------------
Ts = 0.01;                  % sampling time step
t = 0 : Ts : 10;            % defining time
u = zeros(size(t));         % unit step
u(1:7/Ts) = 1;
figure();
h = zeros(size(t));         % defining the impulse response
h(2/Ts:3/Ts) = 1;
h(3/Ts +1:4/Ts) = -2;
h(4/Ts +1:5/Ts) = 1;
subplot(3,1,1);
plot(t,h);
title('A: the impulse responses $h(t)$',Interpreter='latex');
ylim([-3 ,2]);
grid on;
subplot(3,1,2);
plot(t,u);
title('B: the input $u(t)$',Interpreter='latex');
```

```
grid on;
ylim([-.2, 1.2]);
subplot(3,1,3);
t1 = 0:0.01:20;
plot(t1,conv(u,h));
title('A: the output $y(t)$',Interpreter='latex');
grid on;
```



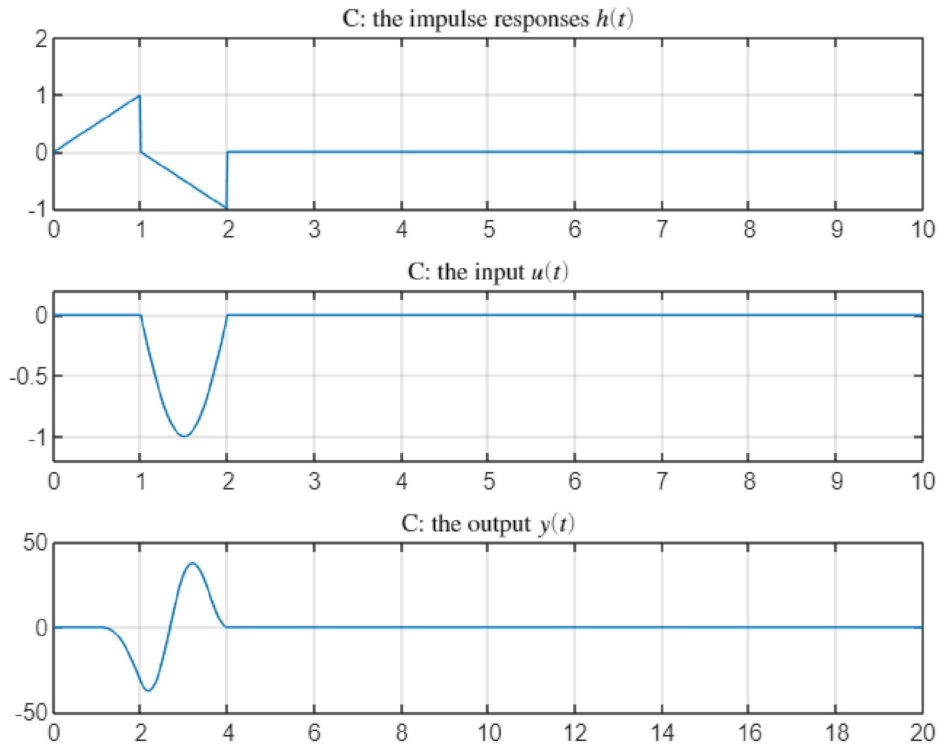

```
% B -------------------
Ts = 0.01;                  % sampling time step
t = 0 : Ts : 10;            % defining time
u = zeros(size(t));         % unit step
u(1:1/Ts) = 1;
figure();
h = zeros(size(t));         % defining the impulse response
h(1:1/Ts) = t(1:1/Ts);
h(1/Ts +1:2/Ts) = 1;
h(2/Ts +1:3/Ts) = 3 - t(2/Ts +1:3/Ts);
subplot(3,1,1);
plot(t,h);
title('B: the impulse responses $h(t)$',Interpreter='latex');
ylim([-1 2]);
grid on;
subplot(3,1,2);
plot(t,u);
title('B: the input $u(t)$',Interpreter='latex');
grid on;
ylim([-.2, 1.2]);
subplot(3,1,3);
```

```
t1 = 0:0.01:20;
plot(t1,conv(u,h));
title('B: the output $y(t)$',Interpreter='latex');
grid on;
```




```
% C -------------------
Ts = 0.01;                  % sampling time step
t = 0 : Ts : 10;            % defining time
u = zeros(size(t));         % unit step
%u(1+1/Ts,2/Ts) = sin(pi*t(1+1/Ts,2/Ts));
u(101:200) = sin(pi*t(101:200));
figure();
h = zeros(size(t));         % defining the impulse response
h(1:1/Ts) = t(1:1/Ts);
h(1+1/Ts:2/Ts) = 1 - t(1+1/Ts:2/Ts);
subplot(3,1,1);
plot(t,h);
title('C: the impulse responses $h(t)$',Interpreter='latex');
ylim([-1 2]);
grid on;
subplot(3,1,2);
plot(t,u);
title('C: the input $u(t)$',Interpreter='latex');
grid on;
ylim([-1.2, .2])
subplot(3,1,3);
t1 = 0:0.01:20;
```

```
plot(t1,conv(u,h));
title('C: the output $y(t)$',Interpreter='latex');
grid on;
```



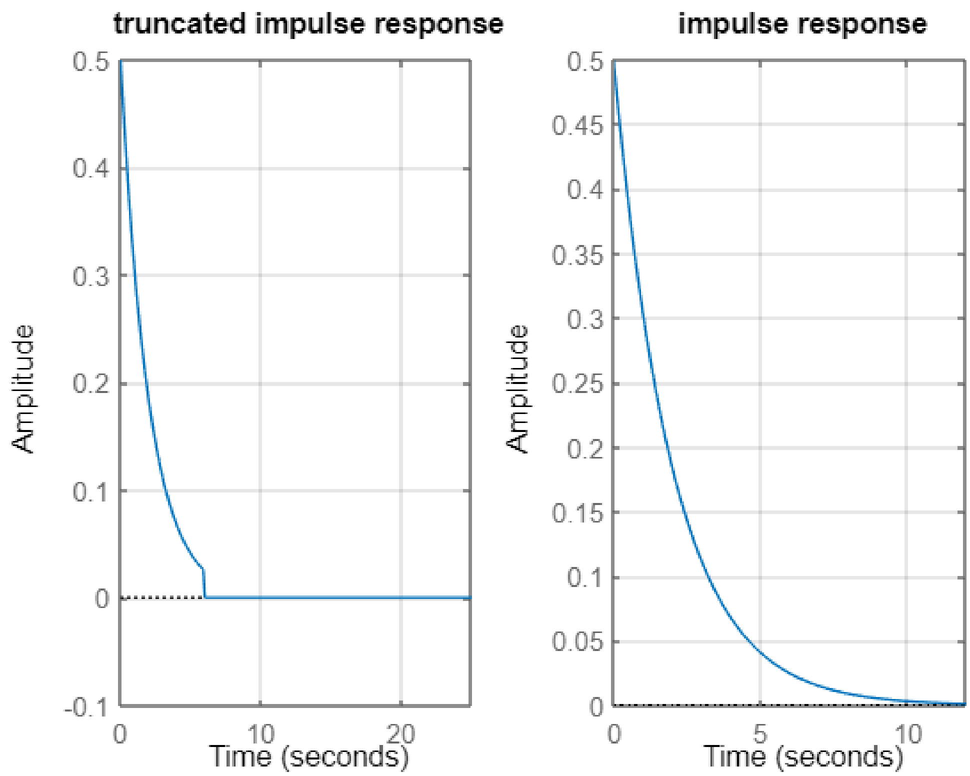
Problem 9: Part A

In this case, $T_1(s) = \dfrac{1}{2s+1} \rightarrow T_1(t) = exp(-t/2)$ and we wish to truncate it from $t = T_s$, so the truncated signal becomes $M_1(t) = T_1(t)(u(t) - \exp(-T_s/2)u(t - T_s))$ which implies:

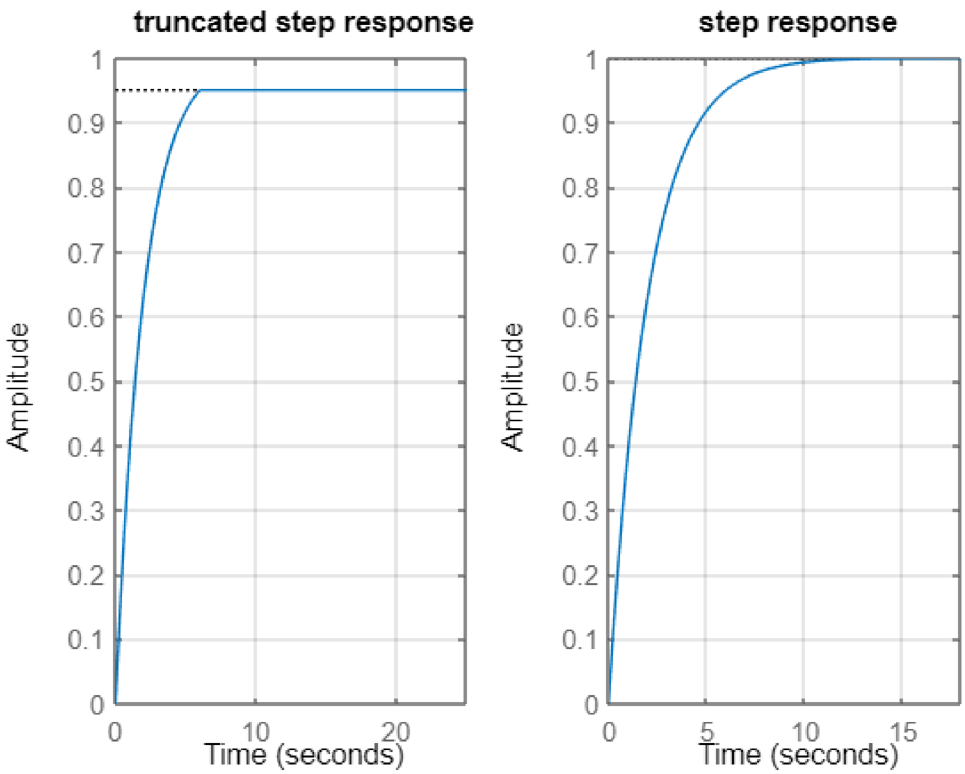
$M_1(s) = T_1(s)(1 - \exp(-T_s/2)\exp(-T_s s))$. Since we wanted to truncate the signal in time domain, we must scale the shifted version of $\exp(-t/2)$ in order to let them cancel each other for all $t \geq T_s$.

```
c = 3;
s = tf('s');
T = 2*c;
M1 = (1 - exp(-0.5*T)*exp(-T*s) )/(2*s+ 1 );
T1 = 1/(2*s+1);

figure();
subplot(1,2,1);
impulse(M1);
title('truncated impulse response');
grid on;
subplot(1,2,2);
impulse(T1);
title('impulse response');
```

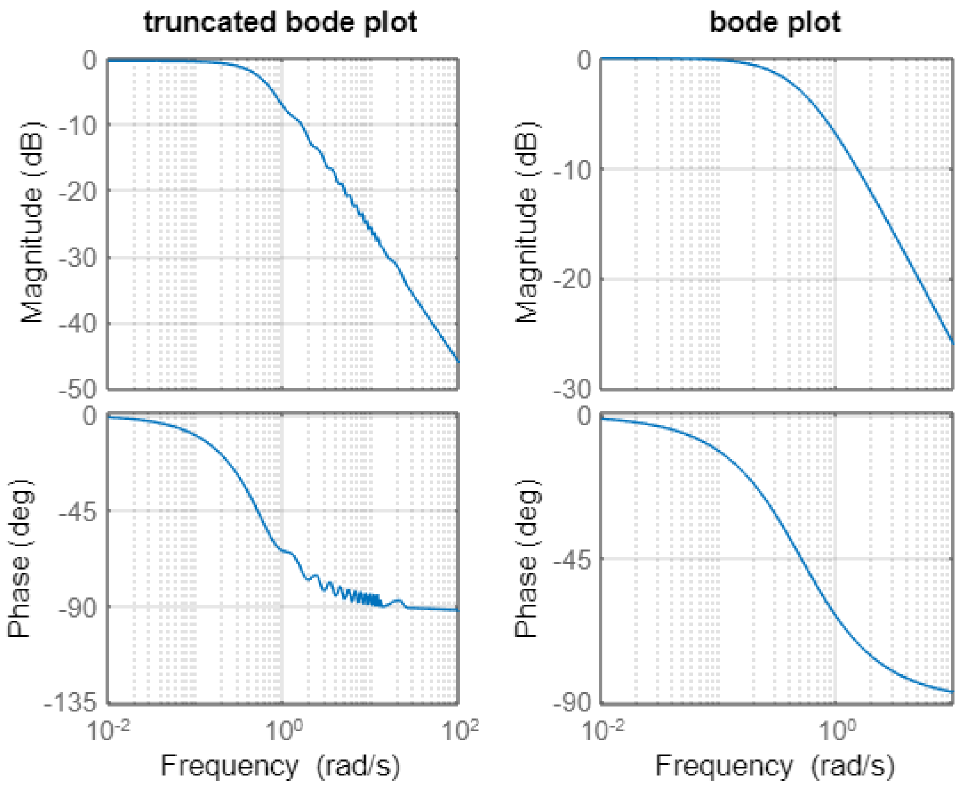
```
grid on;
```




```
figure();
subplot(1,2,1);
step(M1);
title('truncated step response');
grid on;
subplot(1,2,2);
step(T1);
title('step response');
grid on;
```




```
figure();
subplot(1,2,1);
bode(M1);
title('truncated bode plot');
grid on;
subplot(1,2,2);
bode(T1);
title('bode plot');
grid on;
```



It is obvious that when the signal gets truncated, it has a noisy behaviour with respect to the original version of the signal.

Problem 9: Part B

We have $T_2(s) = \dfrac{1}{s^2+s+1} \rightarrow T_2(t) = \dfrac{2\exp(-t/2)\sin(\sqrt{3}\,t/2)}{\sqrt{3}}$. we have $\omega_n = 1, \zeta = 0.5$ and $T_s = \dfrac{4}{\zeta\omega_n} = 8$. In order to truncate the main signal, we can use a pulse signal as $h(t) = u(t) - u(t - T_s)$ which is laplace transform is $H(s) = \dfrac{1-\exp(-T_s s)}{s}$, therefore we need to convolve $H(s)$ and $T_2(s)$ in s domain, in order to do so, I've used a technique I;ve learnt in DSP course, we can map this continuous space into discrete Z space, and then using bilinar transform, convolve the coefficients and finally using conversion into s domain we get our truncated signal.

However, there is a better approach: we can actually, find the laplace transform of that signal:

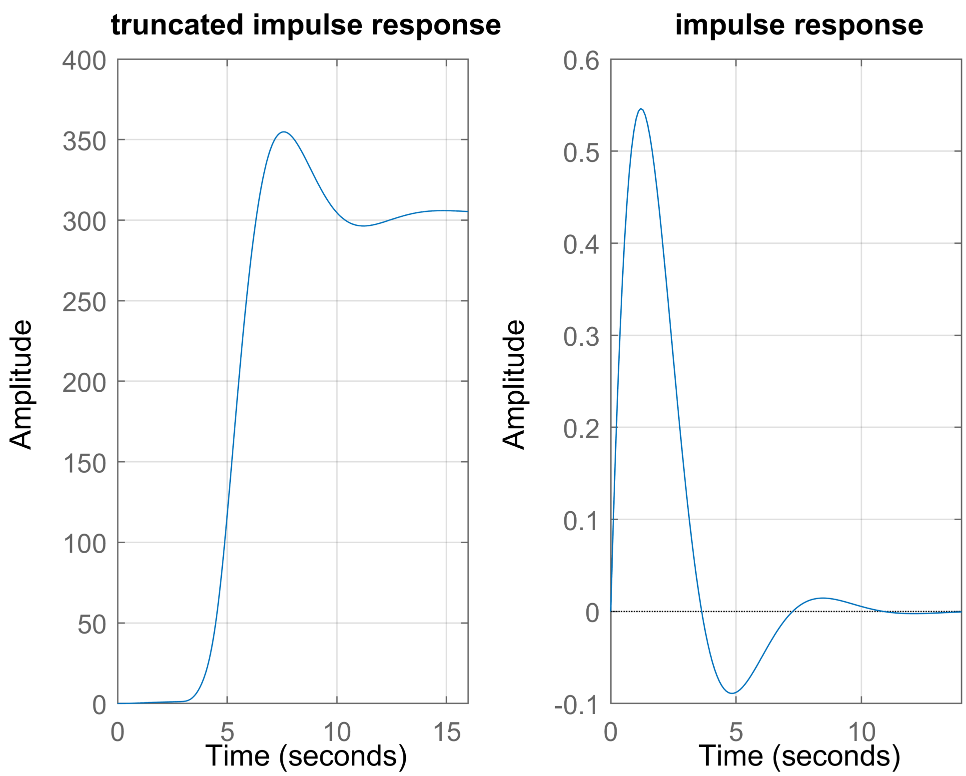
$M_2(t) = T_2(t)(u(t) - u(t - T_s))$ which leads to

$$M_2(s) = \frac{1}{s^2+s+1} + \frac{-\exp(-T_s/2)}{j\sqrt{3}}\left(\frac{\exp(-T_s(s - j\sqrt{3}/2))}{s+1/2-j\sqrt{3}/2} + \frac{\exp(-T_s(s + j\sqrt{3}/2))}{s+1/2+j\sqrt{3}/2}\right)$$

Both will lead to same results!

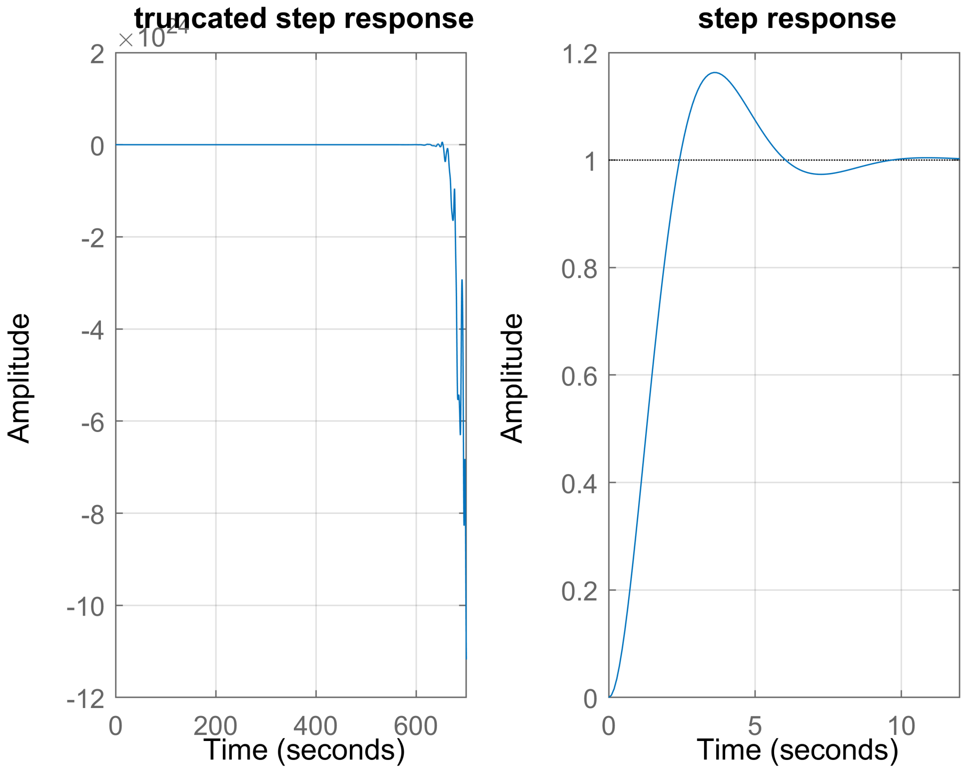
```
s = tf('s');
Ts = 8;
```

```
T2 = 1/(s^2 + s+1);
H = (1 -exp(-Ts*s))/s;
Td = 0.1; % Sampling period
z = tf('z',Td);
H1z = c2d(H,Td,'tustin');
H2z = c2d(T2,Td,'tustin');
[num1,den1] = tfdata(H1z,'v');
[num2,den2] = tfdata(H2z,'v');
num = conv(num1,num2);
den = conv(den1,den2);
Hz = tf(num,den,Td);
M2 = d2c(Hz,'tustin');

figure();
subplot(1,2,1);
impulse(M2);
title('truncated impulse response');
grid on;
subplot(1,2,2);
impulse(T2);
title('impulse response');
grid on;
```

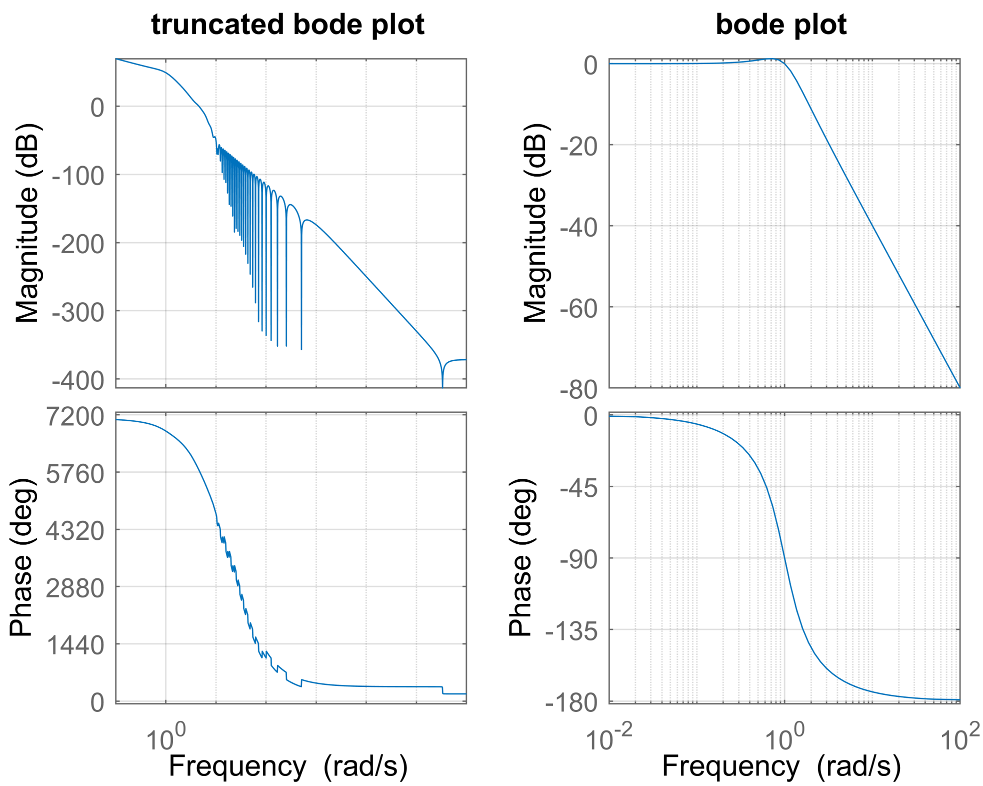



```
figure();
subplot(1,2,1);
step(M2);
title('truncated step response');
grid on;
subplot(1,2,2);
```

```
step(T2);
title('step response');
grid on;
```




```
figure();
subplot(1,2,1);
bode(M2);
title('truncated bode plot');
grid on;
subplot(1,2,2);
bode(T2);
title('bode plot');
grid on;
```



As we can see, the signal is very much distorted this way...

Therfore, we don't really expect to get our time domain signals having resemblance

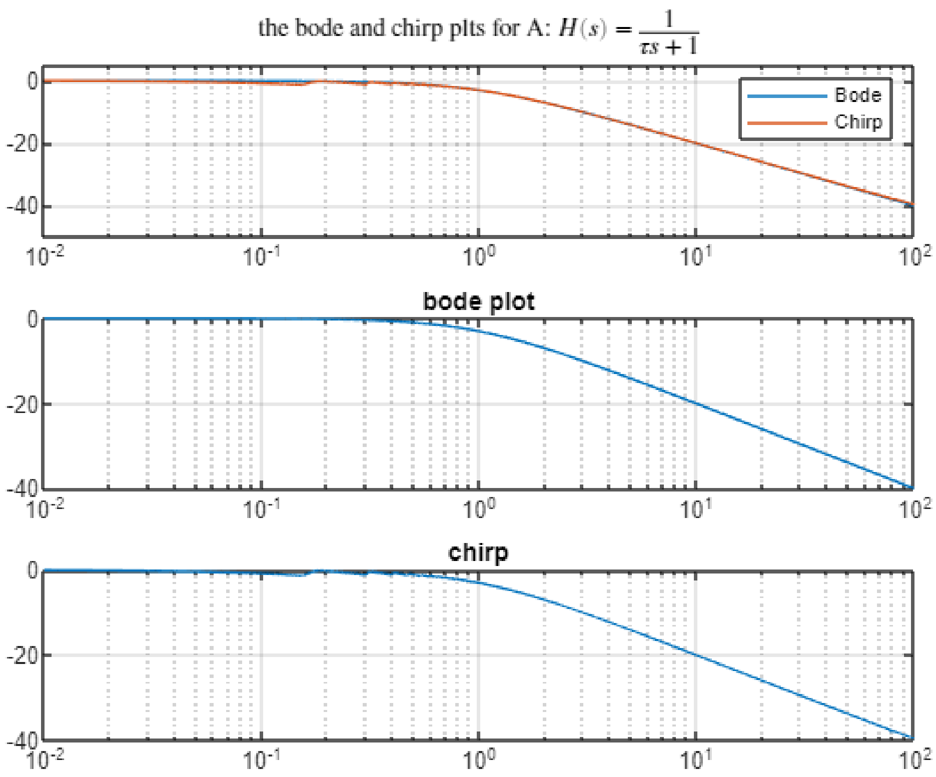
Problem 10:

We have

Note that before running thos peace of code we need to simulate the other attatched file, so that it works properly.

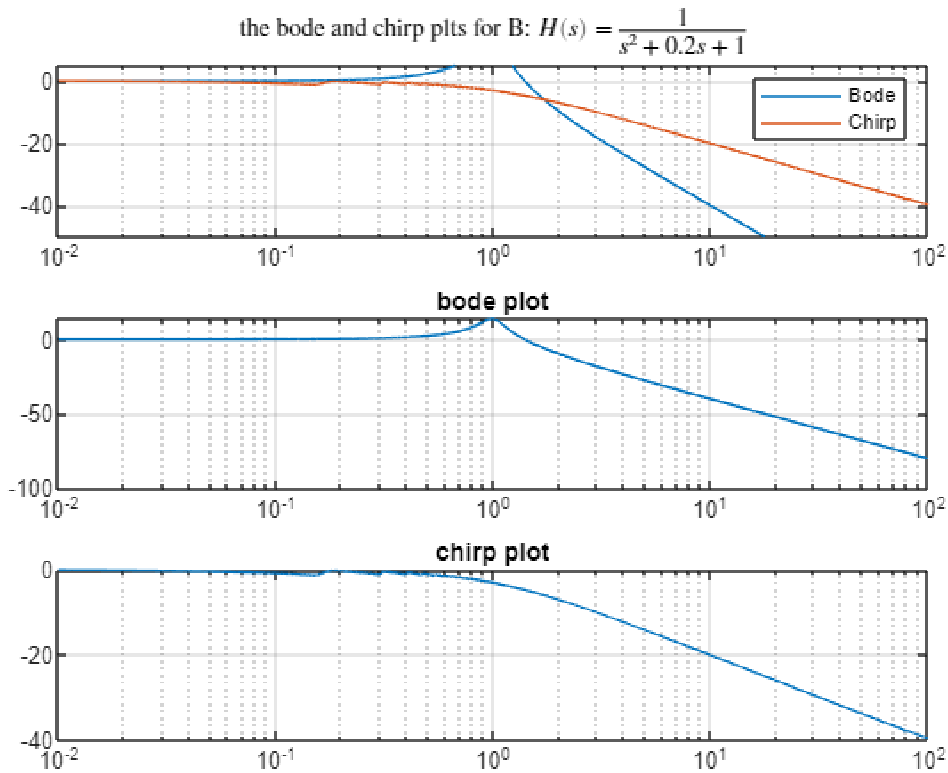
```
% 10 ----------------------------- A
H_s = tf(1,[1 1]);
simul1 = out.simout.Data;
w = linspace(0.01 , 100 , length(simul1));
h = reshape(bode(H_s,w),size(w));
figure();
subplot(3,1,1);
semilogx(w,20*log10(h));
hold on;
semilogx(w,20*log10(simul1));
grid on;
title('the bode and chirp plts for A: $H(s) = \frac{1}{\tau s +1}$',Interpreter='latex');
ylim([-50 ,5]);
legend({'Bode' , 'Chirp'});
subplot(3,1,2);
semilogx(w,20*log10(h));
```

```
title('bode plot');
grid on;
subplot(3,1,3);
semilogx(w,20*log10(simul1));
title('chirp');
grid on;
```

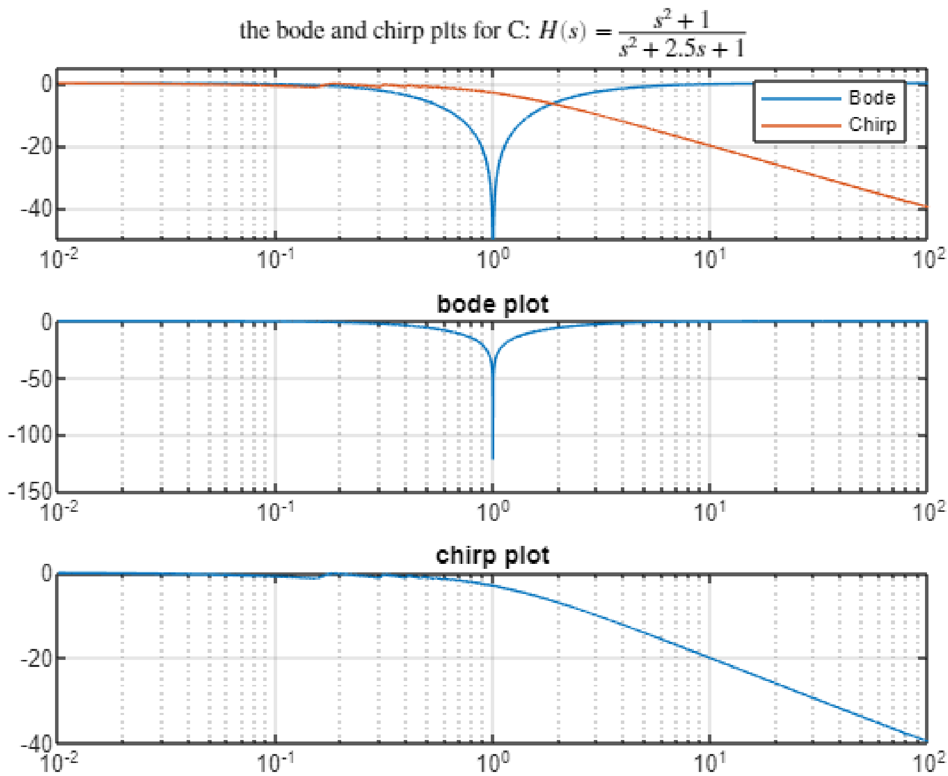



```
% 10 ----------------------------- B
H_s = tf(1,[1 0.2 1]);
simul1 = out.simout.Data;
w = linspace(0.01 , 100 , length(simul1));
h = reshape(bode(H_s,w),size(w));
figure();
subplot(3,1,1);
semilogx(w,20*log10(h));
hold on;
semilogx(w,20*log10(simul1));
grid on;
title(['the bode and chirp plts for B: $H(s) = \frac{1}{s^2 + 0.2s +1}$'],Interpreter='latex');
ylim([-50 ,5]);
legend({'Bode' , 'Chirp'});
subplot(3,1,2);
semilogx(w,20*log10(h));
title('bode plot');
grid on;
subplot(3,1,3);
semilogx(w,20*log10(simul1));
```

```
title('chirp plot');
grid on;
```




```
% 10 ---------------------------- C
H_s =  tf([1 0 1],[1 2.5 1]);
simul1 = out.simout.Data;
w = linspace(0.01 , 100 , length(simul1));
h = reshape(bode(H_s,w),size(w));
figure();
subplot(3,1,1);
semilogx(w,20*log10(h));
hold on;
semilogx(w,20*log10(simul1));
grid on;
title(['the bode and chirp plts for C: $H(s) = \frac{s^2 +1}{s^2 + 2.5s +1}$'],Interpreter='lat
ylim([-50 ,5]);
legend({'Bode' , 'Chirp'});
subplot(3,1,2);
semilogx(w,20*log10(h));
title('bode plot');
grid on;
subplot(3,1,3);
semilogx(w,20*log10(simul1));
title('chirp plot');
grid on;
```



the bode and chirp plts for C: $H(s) = \frac{s^2+1}{s^2+2.5s+1}$
Bode
Chirp
bode plot
chirp plot