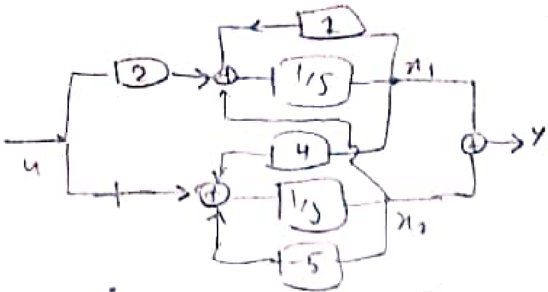
10%



let $Y = \begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix} \rightarrow \begin{cases} \frac{1}{s}\left(2U(s) + X_1(s) - 2X_2(s)\right) = X_1(s) \\ \frac{1}{s}\left(U(s) + 4X_1(s) - 5X_2(s)\right) = X_2(s) \end{cases}$

$$\mathcal{L}\{e^{-At}\} = (sI - A)^{-1}$$

$$\mathcal{L}\left\{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}\right\} = \begin{bmatrix} sX_1(s) \\ sX_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -2 \\ 4 & -5 \end{bmatrix}}_{A} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} U(s)$$

$$(sI - A)^{-1} = \begin{bmatrix} s-1 & 2 \\ -4 & s+5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{s+5}{s^2+4s+3} & \frac{-2}{(s+1)(s+3)} \\ \frac{4}{s^2+4s+3} & \frac{s-1}{(s+1)(s+3)} \end{bmatrix}$$

$$e^{-At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\} = \begin{bmatrix} 2e^{-t} - e^{-3t} & -e^{-3t}(e^{2t}-1) \\ 2e^{-3t}(e^{2t}-1) & -e^{-3t}(e^{2t}-2) \end{bmatrix} u(t)$$

20%

$$A \begin{cases} \dot{x}(t) = \underbrace{\begin{bmatrix} -3 & 7 \\ -6 & 4 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}_{B} u(t) \\ y(t) = \underbrace{\begin{bmatrix} 8 & 5 \end{bmatrix}}_{C} x(t) \end{cases} \qquad B \begin{cases} \dot{x}(t) = \underbrace{\begin{bmatrix} 2 & 1 \\ -1 & 5 \end{bmatrix}}_{A'} x(t) + \underbrace{\begin{bmatrix} 3 \\ 1 \end{bmatrix}}_{B'} u(t) \\ y(t) = \underbrace{\begin{bmatrix} 2 & -7 \end{bmatrix}}_{C'} x(t) \end{cases}$$

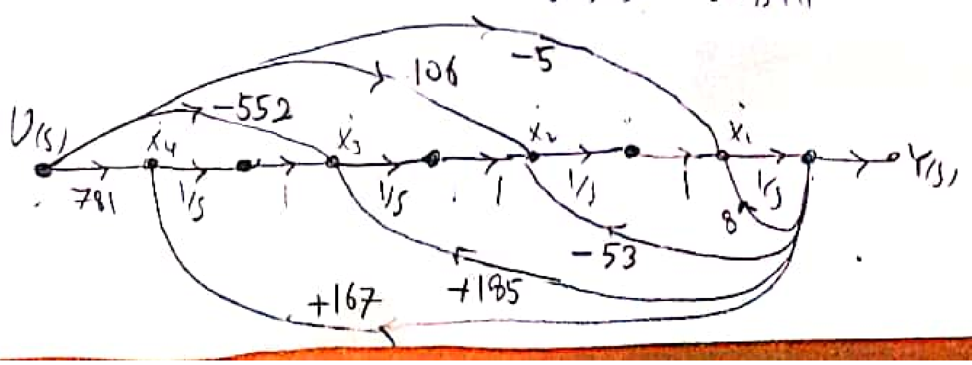
$$G_A(s) = C(sI-A)^{-1}B + \cancel{D} = \frac{\begin{bmatrix} 8 & 5 \end{bmatrix} \begin{bmatrix} s-4 & 7 \\ -6 & s+3 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix}}{s^2 - s + 30} = \frac{-5s - 71}{s^2 - s + 30}$$

$$\begin{bmatrix} s+3 & -7 \\ +6 & s-4 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{s-4}{s^2-s+30} & \frac{7}{s^2-s+30} \\ \frac{-6}{s^2-s+30} & \frac{s+3}{s^2-s+30} \end{bmatrix}$$

$$G_B(s) = C'(sI-A')^{-1}B' + \cancel{D}' = \frac{\begin{bmatrix} 2 & -7 \end{bmatrix} \begin{bmatrix} s-5 & 1 \\ -1 & s-2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}}{s^2 - 7s + 11} = \frac{-s+7}{s^2 - 7s + 11}$$

$$\begin{bmatrix} s-2 & -1 \\ 1 & s-5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{s-5}{s^2-7s+11} & \frac{1}{s^2-7s+11} \\ \frac{-1}{s^2-7s+11} & \frac{s-2}{s^2-7s+11} \end{bmatrix}$$

$$H(s) = \frac{G_A(s)}{1 + G_A(s)G_B(s)} = \frac{-\frac{5s+71}{s^2-s+30}}{1 + \frac{(-5s-71)}{s^2-s+30} \cdot \frac{(-s+7)}{s^2-7s+11}} = \frac{-5s^3 + 106s^2 - 552s + 781}{s^4 - 8s^3 + 53s^2 - 185s + 167}$$



derived space model

سوال ۲: ادامه

$$\text{if } H(s)=\frac{b_{n-1}s^{n-1}+b_{n-2}s^{n-2}+\dots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+\dots+a_1 s+a_0}\qquad \begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)+Du(t)\end{cases}$$

$$H(s)=\frac{-5s^3+106s^2-552s+781}{s^4-8s^3+53s^2-185s-167}$$

(۱) فرم کانونی کنترل پذیر

$$A_{4\times4}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-167&-185&53&8\end{bmatrix},\ B=\begin{bmatrix}0\\0\\0\\1\end{bmatrix},\ C=\begin{bmatrix}781&-552&106&-5\end{bmatrix},\ D=0$$

(۲) فرم کانونی مشاهده پذیر

$$A_{4\times4}=\begin{bmatrix}-8&1&0&0\\-53&0&1&0\\185&0&0&1\\167&0&0&0\end{bmatrix},\ B=\begin{bmatrix}-5\\106\\-552\\781\end{bmatrix},\ C=\begin{bmatrix}1&0&0&0\end{bmatrix},\ D=0$$

سوال ۳:

$$\begin{cases}A:\ \dot{x}_1=A_1x_1+B_1u,\ y_1=C_1x_1+D_1u_1\\ B:\ \dot{x}_2=A_2x_2+B_2u,\ y_2=C_2x_2+D_2u_2\end{cases}\qquad\begin{cases}y=y_1+y_2\\ u_1=u_2=u\end{cases}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}A_1&0\\0&A_2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}B_1\\B_2\end{bmatrix},\quad y=\begin{bmatrix}C_1&C_2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\cancelto{0}{(D_1+D_2)}\,u$$

$$\hat{A}=\begin{bmatrix}A_1&0\\0&A_2\end{bmatrix}=\begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&-2&-3&0&0&0\\0&0&0&-3&1&0\\0&0&0&-2&0&1\\0&0&0&0&0&0\end{bmatrix},\ \hat{B}=\begin{bmatrix}B_1\\B_2\end{bmatrix}=\begin{bmatrix}0\\0\\1\\-2\\8\\1\end{bmatrix},\ \hat{C}=\begin{bmatrix}C_1&C_2\end{bmatrix}=\begin{bmatrix}1&-5&3&1&0&0\end{bmatrix}$$

$$G_{\hat{A}}(s)=\hat{C}(sI-\hat{A})^{-1}\hat{B}+\cancelto{0}{\hat{D}}=\begin{bmatrix}1&-5&3&1&0&0\end{bmatrix}\begin{bmatrix}s&-1&0&0&0&0\\0&s&-1&0&0&0\\0&2&s+3&0&0&0\\0&0&0&s+3&-1&0\\0&0&0&2&s&-1\\0&0&0&0&0&s\end{bmatrix}^{-1}\begin{bmatrix}0\\0\\1\\-2\\8\\1\end{bmatrix}=\frac{1}{s}$$

این بار فرض کنیم دو سیستم که بصورت interconnection بود را باهم جمع کردیم و بعد متغیر حالت ما 6 تا شد و دیدیم که $H(s)=\frac{1}{s}$. حالا این بار، سیستم‌های A و B را تا جایی که ممکن است را بصورت مجزا محاسبه کرده و در نهایت جمع می‌کنیم و خواهیم دید که باز هم به $H(s)=\frac{1}{s}$ می‌رسیم.

Scanned by CamScanner

سوال 3

$$A\begin{cases}\dot{x}(t)=\begin{bmatrix}0&1&0\\0&0&1\\0&-2&-3\end{bmatrix}x(t)+\begin{bmatrix}0\\0\\1\end{bmatrix}u(t)\\ y(t)=\begin{bmatrix}1&-5&3\end{bmatrix}x(t)\end{cases}\qquad B\begin{cases}\dot{x}(t)=\begin{bmatrix}-3&1&0\\-2&0&1\\0&0&0\end{bmatrix}x(t)+\begin{bmatrix}-2\\8\\1\end{bmatrix}u(t)\\ y(t)=\begin{bmatrix}1&0&0\end{bmatrix}x(t)\end{cases}$$

$$G_A(s)=C(sI-A)^{-1}B+\cancel{D}^{0}=\begin{bmatrix}1&-5&3\end{bmatrix}\begin{bmatrix}s&-1&0\\0&s&-1\\0&2&s+3\end{bmatrix}^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix}=\frac{-3s^2-5s+1}{s(s+1)(s+2)}$$

$$G_B(s)=C'(sI-A')^{-1}B'+\cancel{D'}=\begin{bmatrix}1&0&0\end{bmatrix}\begin{bmatrix}s+3&-1&0\\2&s&-1\\0&0&s\end{bmatrix}^{-1}\begin{bmatrix}-2\\8\\1\end{bmatrix}=\frac{-2s^2+8s+1}{s(s+1)(s+2)}$$

$$H(s)=G_A(s)+G_B(s)=\frac{s^2+3s+2}{s(s+1)(s+2)}=\frac{(s+1)(s+2)}{s(s+1)(s+2)}=\frac{1}{s}$$

حال به صورت عملی می‌توان دید که تابع تبدیل در روش فوق دقیقاً حاصل شد. به جای انجام ضرب ماتریس‌ها در
خود به دست آوردن تابع شبکه‌ها و جمع آن‌ها برای رسیدن به تابع شبکه کل.

سوال 4:

$$S\begin{cases}\dot{x}(t)=\begin{bmatrix}-1&-2\\1&-4\end{bmatrix}x(t)+\begin{bmatrix}0\\1\end{bmatrix}u(t)\\ y(t)=\begin{bmatrix}0&1\end{bmatrix}x(t),\quad x(0)=\begin{bmatrix}1\\0\end{bmatrix}\end{cases}\qquad u(t)=\mathbb{1}(t\geq 0)$$

$$H(s)=C(sI-A)^{-1}B=\begin{bmatrix}0&1\end{bmatrix}\begin{bmatrix}s+1&2\\-1&s+4\end{bmatrix}^{-1}\begin{bmatrix}0\\1\end{bmatrix}=\frac{s+1}{s^2+5s+6}=\frac{s+1}{(s+2)(s+3)}$$

$$\dot{x}=Ax+BU\rightarrow sX(s)-x(0)=AX(s)+BU(s)\rightarrow X(s)=x(0)(sI-A)^{-1}+B(sI+A)^{-1}U(s)$$

$$\rightarrow X(s)=(sI-A)^{-1}(x(0)+BU(s))$$

$$X(s)=\begin{bmatrix}\frac{s+4}{(s+4)(s+2)}&\frac{-2}{(s+2)(s+3)}\\\frac{1}{(s+2)(s+3)}&\frac{s+1}{(s+2)(s+3)}\end{bmatrix}\begin{bmatrix}1\\1/s\end{bmatrix}=\begin{bmatrix}\frac{s^2+4s-2}{s(s+2)(s+3)}\\\frac{2s+1}{s(s+2)(s+3)}\end{bmatrix}\Rightarrow x(t)=\begin{bmatrix}-\frac{1}{3}u(t)+3e^{-2t}u(t)-5/3e^{-3t}u(t)\\\frac{1}{6}u(t)+\frac{3}{2}e^{-2t}u(t)-5/3e^{-3t}u(t)\end{bmatrix}$$

$$Y(s)=\begin{bmatrix}0&1\end{bmatrix}X(s)=\frac{2s+1}{s(s+2)(s+3)}=\frac{1}{6s}+\frac{3}{2}\frac{1}{s+2}+\frac{5}{3}\frac{1}{s+3}$$

$$\rightarrow y(t)=\frac{1}{6}u(t)+\frac{3}{2}e^{-2t}u(t)-\frac{5}{3}e^{-3t}u(t)$$

سوال ۳:

by getting Laplace transform from equations, we get:

$$\left.\begin{array}{l} \ddot{S}_1(t) + 3\dot{S}_2(t) + 2S_1(t) = 3\dot{u}(t) - u(t) \\ \dot{S}_2(t) - 2\dot{S}_1(t) + S_2(t) = 0 \\ y(t) = \dot{S}_1(t) + \dot{S}_2(t) \end{array}\right\} \quad \begin{array}{ll} (s^2+2)S_1(s) + 3sS_2(s) = U(s)(3s-1) & (a) \\ S_2(s)(s+1) - 2sS_1(s) = 0 & (b) \\ Y(s) = s\left(S_1(s) + S_2(s)\right) & (c) \end{array}$$

$$(b) \rightarrow \frac{S_2(s)}{S_1(s)} = \frac{2s}{s+1} \quad (d)$$

$$\xrightarrow{(a),(d)} S_1(s)\left(2s^3+6s^2+2s+2\right) = U(s)(3s-1)(s+1) \rightarrow \frac{S_1(s)}{U(s)} = \frac{3s^2+2s-1}{2s^3+6s^2+2s+2} \quad (e)$$

$$\xrightarrow{(e),(c),(d)} \frac{Y(s)}{U(s)} = s\left(\frac{3s^2+2s-1}{2s^3+6s^2+2s+2}\right)\left(1+\frac{2s}{s+1}\right) = \frac{s(3s-1)(3s+1)}{2s^3+6s^2+2s+2} = \frac{9s^3-s}{2s^3+6s^2+2s+2} \quad (g)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\frac{9}{2}s^3 - \frac{1}{2}s}{s^3+3s^2+s+1} = \underbrace{\frac{9}{2}}_{D} + \frac{-\frac{27}{2}s^2 - 5s - \frac{9}{2}}{s^3+3s^2+s+1}$$

حال که تابع تبدیل را داریم، می‌توانیم ماتریس‌های A، B، C را بیابیم

$$\begin{cases} \dot{\underline{X}} = A\underline{X} + B\underline{U} \\ Y = CX + DU \end{cases}$$

$$\begin{cases} D = \frac{9}{2} \\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} -\frac{9}{2} & -5 & -\frac{27}{2} \end{bmatrix} \end{cases}$$

سوال ۲ج: از آنجا که در سوال ربط دلیل بود $\dot{u}(t)$ مشتق است که ما باید یک State کمکی نیز داشته باشیم

let $S_3(t) = \dot{S}_1(t)$ → then we get

$$\left.\begin{array}{l} \ddot{S}_1 + 3\dot{S}_2 + 2S_1 = 3\dot{u} - u \\ \dot{S}_2 - 2\dot{S}_1 + S_2 = 0 \\ y = \dot{S}_1 + \dot{S}_2 \end{array}\right\} \quad \begin{array}{l} \dot{S}_3 + 3\dot{S}_2 + 2S_1 = 3\dot{u} - u \\ \dot{S}_2 - 2S_3 + S_2 = 0 \\ S_3 = \dot{S}_1 \end{array}$$

$$\dot{Y} = \begin{bmatrix} \dot{S}_1(t) \\ \dot{S}_2(t) \\ \dot{S}_3(t) \end{bmatrix} \to \begin{bmatrix} \dot{S}_1(t) \\ \dot{S}_2(t) \\ \ddot{S}_1(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ -2 & 3 & -6 \end{bmatrix} \begin{bmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix}$$

(**) $Y = \dot{S}_1 + \dot{S}_2 = S_3 - S_2 + 2S_3 = 3S_3 - S_2$

(*) $\dot{S}_3 = -3\dot{S}_2 - 2S_1 + 3\dot{u} - u = -3(2S_3 - S_2) - 2S_1 + 3\dot{u} - u$

$$= -6S_3 + 3S_2 - 2S_1 + 3\dot{u} - u$$

$$\begin{cases} \dot{Y} = \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ -2 & 3 & 6 \end{bmatrix}}^{A} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 3 \end{bmatrix}}_{B} \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix} \\ Y = \underbrace{\begin{bmatrix} 0 & -1 & 3 \end{bmatrix}}_{C} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \end{cases}$$