تمرین سری 1 کنترل محمدیاسین دینی 400104204 سوال 1:

(I) $T(s) = \dfrac{s+6}{s(s+1)(s+3)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+3} \Rightarrow \boxed{\mathcal{L}^{-1}\{T(s)\} = 2u(t) - \frac{5}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)}$

$A = sT(s)\big|_{s=0} = 2 \;;\; B = (s+1)T(s)\big|_{s=-1} = -5/2 \;;\; C = (s+3)T(s)\big|_{s=-3} = 1/2$

(II) $T(s) = \dfrac{5}{s[(s+2)^2+1]} = \dfrac{A}{s} + \dfrac{B(s+2)}{(s+2)^2+1} + \dfrac{C}{(s+2)^2+1}$

$A = sT(s)\big|_{s=0} = 1 \;;\; B = -1 \;;\; C = -2$

$\boxed{\Rightarrow \mathcal{L}^{-1}\{T(s)\} = u(t) + e^{-2t}\cos(t)u(t) - e^{-2t}\sin(t)u(t)}$

(III) $T(s) = \dfrac{s^2+2s+3}{(s+2)^3} = \dfrac{A}{(s+2)^3} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+2} \;\boxed{\Rightarrow \mathcal{L}^{-1}\{T(s)\} = 3t^2e^{-2t}u(t) - 2te^{-2t}u(t) + e^{-2t}u(t)}$

$A = (s+2)^3T(s)\big|_{s=-2} = 3 \;;\; B = \frac{\partial}{\partial s}\left((s+2)^3T(s)\right)\Big|_{s=-2} = -2 \;;\; C = \frac{1}{2}\frac{\partial^2}{\partial s^2}\left((s+2)^3T(s)\right) = 1$

ok

سوال 2: با فرض علی بودن سیگنال داریم:

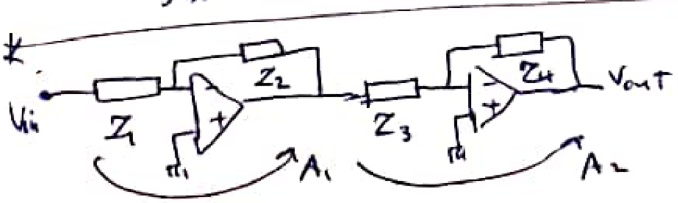
By initial value theorem we get: $F(s) = \dfrac{2s+1}{s(s+1)(s+2)}$

$f(0^+) = \lim\limits_{s\to\infty} sF(s) = \lim\limits_{s\to\infty} \dfrac{s(2s+1)}{s(s+1)(s+2)} = 0$

$f'(0^+) = \lim\limits_{s\to\infty} s\left(sF(s) - f(0^+)\right) = \lim\limits_{s\to\infty} s^2F(s) = \lim\limits_{s\to\infty} \dfrac{s^2(2s+1)}{s(s+1)(s+2)} = 2$

$f''(0^+) = \lim\limits_{s\to\infty} s\left(s^2F(s) - sf(0^+) - f'(0)\right) = \lim\limits_{s\to\infty} s^3F(s) - sf'(0) = \lim\limits_{s\to\infty} \dfrac{2s^3+s^2-2s(s^2+3s+2)}{(s+1)(s+2)} = \lim\limits_{s\to\infty} \dfrac{-5s^2-4s}{s^2+3s+2} = -5$

سوال 3:

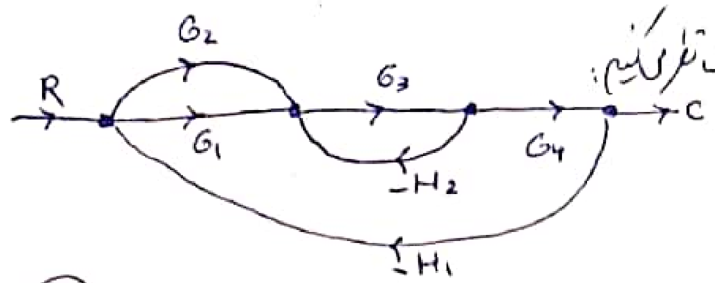


$\left(\frac{1}{Cs} \| R\right) = \dfrac{\frac{R}{Cs}}{\frac{1}{Cs}+R} = \dfrac{R}{1+RSC} = Z$

$A = A_1A_2 = -\dfrac{Z_2}{Z_1} \times -\dfrac{Z_4}{Z_3} = \left(\dfrac{R_2R_4}{R_1R_3}\right)\left(\dfrac{1+SR_1C_1}{1+SR_2C_2} \times \dfrac{1+SR_3C_3}{1+SR_4C_4}\right) = A(s) = \dfrac{V_{out}}{V_{in}}(s)$

سوال ۴: یک بار از ورودی D یک بار دیگر از ورودی R صرف نظر می‌کنیم:

$T(R(s)+D(s)) = T(R(s)) + T(D(s))$: by superposition



(*) هیچ دو حلقه‌ای نیست که non touching باشند.

(i) forward paths $= \{1 \to P_1 = G_1G_3G_4 ,\ 2 \to G_2G_3G_4 = P_2\}$

loops $= \{l_1 = -G_3H_2 ,\ l_2 = -G_1G_3G_4H_1 ,\ l_3 = -G_2G_3G_4H_1\}$

$$\overset{(*)}{\Rightarrow} \Delta(s) = 1 - \sum_{k=1}^{3} l_k = 1 + G_3H_2 + G_1G_3G_4H_1 + G_2G_3G_4H_1$$

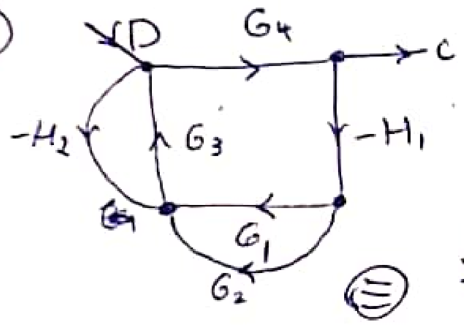
چون با حذف بال‌های شامل مسیرهای P_1 و P_2 حلقه‌ای باقی نمی‌ماند

$$\Delta_1 = 1 = \Delta_2$$

finalement

$$\Rightarrow T_r(s) = \frac{\sum_k P_k(s)\,\Delta_k(s)}{\Delta(s)} = \frac{G_1G_3G_4 + G_2G_3G_4}{1 + G_3H_2 + G_1G_3G_4H_1 + G_2G_3G_4H_1}$$

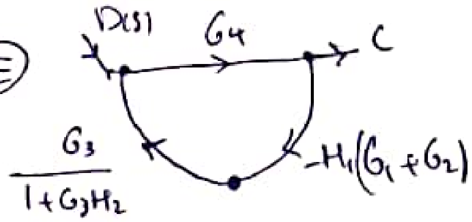
(ii)

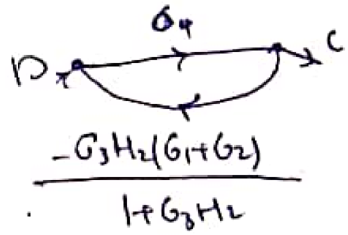


~~forward paths $= \{P_1 = G_4 ,\ P_2 = -H_2G_3G_4\}$~~

~~loops $= \{l_1 = -H_2G_3 ,\ l_2 = -H_1G_1G_3G_4 ,\ l_3 = -H_1G_2G_3G_4\}$~~

از راه ساده‌سازی مدار می‌رویم:





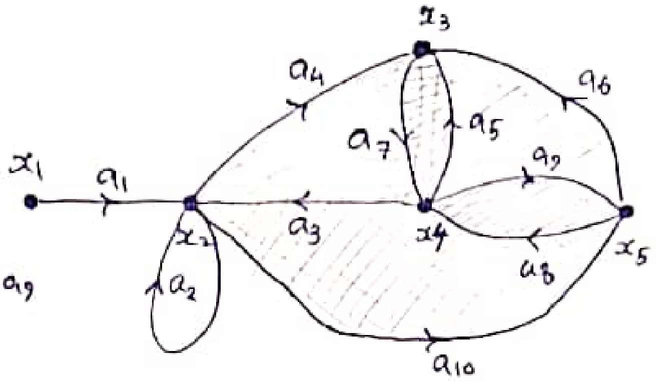
$$T_d(s) = \frac{G_4(s)}{1 + G_4(s)\left(\frac{G_3(s)H_2(s)(G_1(s)+G_2(s))}{1+G_3H_2}\right)} = \frac{G_4(1+G_3H_2)}{1+G_3H_2+(G_1+G_2)G_3G_4H_2}$$

$$\Rightarrow C(s) = \Bigg[\underbrace{\left(\frac{C(s)}{R(s)}\right)\Bigg|_{D(s)=0}}_{T_r(s)}\Bigg] R(s) + \Bigg[\underbrace{\left(\frac{C(s)}{D(s)}\right)\Bigg|_{R(s)=0}}_{T_d(s)}\Bigg] D(s)$$

$$\Rightarrow C(s) = \left[\frac{G_3(s)G_4(s)\left(G_1(s)+G_2(s)\right)}{1+G_3H_2+G_3(s)G_4(s)H_1(s)\left(G_1(s)+G_2(s)\right)}\right] R(s) + \left[\frac{G_4(s)\left(1+G_3(s)H_2(s)\right)}{1+G_3(s)H_2(s)+\left(G_1(s)+G_2(s)\right)H_2(s)G_3(s)G_4(s)}\right] D(s)$$

سوال 5:

$$\begin{cases} x_2 = a_1x_1 + a_2x_2 + a_3x_4 \\ x_3 = a_4x_2 + a_5x_4 + a_6x_5 \\ x_4 = a_7x_3 + a_8x_5 \\ x_5 = a_9x_4 + a_{10}x_2 \end{cases}$$



(i) forward paths: $\begin{cases} P_1 = a_1a_4a_7a_9 \\ P_2 = a_1a_{10} \end{cases}$

loops: $\begin{cases} l_1 = a_8a_9 \\ l_2 = a_7a_5 \\ l_3 = a_3a_4a_7 \\ l_4 = a_{10}a_6a_7a_3 \,? \\ l_5 = a_{10}a_8a_3 \\ l_6 = a_2 \\ l_7 = a_6a_7a_9 \end{cases}$

non-touching loops

$\begin{cases} l_6, l_2 \longrightarrow l_{2,6} = a_2a_7a_5 \\ l_6, l_1 \longrightarrow l_{1,6} = a_2a_8a_9 \\ l_7, l_6 \longrightarrow l_{6,7} = a_2a_6a_7a_9 \end{cases}$

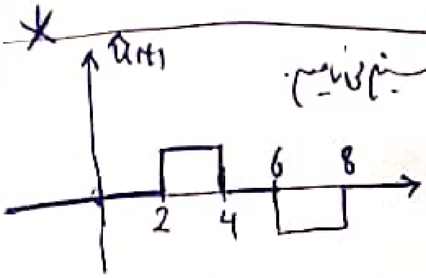
$$\Delta(s) = 1 - \sum_{k=1}^{7} l_k + \sum_{m,n} l_{mn} = 1 - \begin{pmatrix} a_2 + a_6a_7a_9 + a_{10}a_8a_3 + a_{10}a_6a_7a_3 \\ + a_3a_4a_7 + a_7a_5 + a_8a_9 \end{pmatrix} + \begin{pmatrix} a_2a_7a_5 \\ + a_2a_8a_9 \\ + a_2a_6a_7a_9 \end{pmatrix}$$

$\Delta_1 = 1$

$\Delta_2 = 1 - a_5a_7$

$$T(s) = \frac{\sum_i P_k(s)\Delta_k(s)}{\Delta(s)} = \frac{a_1a_4a_7a_9 + a_1a_{10}(1 - a_5a_7)}{1 - \begin{pmatrix} a_2 + a_6a_7a_9 + a_{10}a_8a_3 + a_{10}a_6a_7a_3 \\ + a_3a_4a_7 + a_7a_5 + a_8a_9 \end{pmatrix} + \begin{pmatrix} a_2a_7a_5 \\ + a_2a_8a_9 \\ + a_2a_6a_7a_9 \end{pmatrix}}$$

سوال 6: بکار اشتباه شدن u(t) را با step function , û(t) را ورودی بر سیستم داریم.



$$T(s) = \frac{50}{(s+5)(s+10)}, \quad \hat{U}(s) = \int_2^4 e^{-st}dt - \int_6^8 e^{-st}dt = \frac{-e^{-4s} + e^{-6s} + e^{-2s} + e^{-8s}}{s}$$

$$Y(s) = \hat{U}(s)T(s) = \left(\frac{1}{s} + \frac{1}{s+10} - \frac{2}{s+5}\right)\left(e^{-2s} + e^{-8s} - e^{-4s} - e^{-6s}\right) = X(s)\left(e^{-2s} + e^{-8s} - e^{-4s} - e^{-6s}\right)$$

$$\rightarrow x(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{1}{s+10} + \frac{2}{s+5}\right\} = u(t)\left(1 - 2e^{-5t} + e^{-10t}\right) \quad - u(t-4)\left(1 - e^{-5(t-4)} + e^{-10(t-4)}\right) - u(t-6)\left(1 - 2e^{-5(t-6)} + e^{-10(t-6)}\right)$$

$$\rightarrow y(t) = x(t-2) + x(t-8) - x(t-6) - x(t-4) = u(t-2)\left(1 - 2e^{-5(t-2)} + e^{-10(t-2)}\right) + u(t-8)\left(1 - 2e^{-5(t-8)} + e^{-10(t-8)}\right)$$

Control Systems

Software hw1

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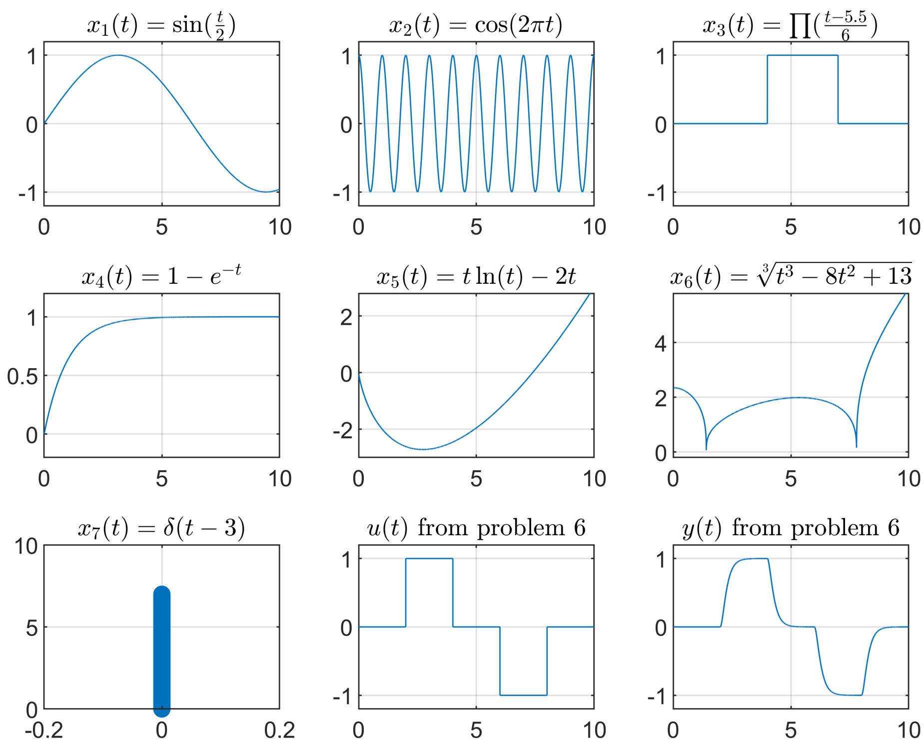
Problem 7: In this part we will plot some of the given signals versus time.

```
clear all;
close all;
% -------------------------
t_start = 0;
t_end = 10;
% ------ x1
subplot(3,3,1);
Ts1 = 0.001;
t1 = t_start:Ts1:t_end;
x1 = sin(t1/2);
plot(t1,x1);
title("$x_1(t) = \sin(\frac{t}{2})$",Interpreter="latex");
ylim([-1.2,1.2]);
xlim([0,10])
grid on
% ------ x2
subplot(3,3,2);
Ts2 = 0.001;
t_start = 0;
t_end = 10;
t2 = t_start:Ts2:t_end;
x2 = cos(2*pi*t2);
plot(t2,x2);
title("$x_2(t) = \cos(2\pi t)$",Interpreter="latex");
ylim([-1.2,1.2]);
xlim([0,10])
grid on
% ------ x3
subplot(3,3,3);
Ts3 = 0.001;
t3 = t_start:Ts3:t_end;
x3 = rectangularPulse(4,7,t3);
plot(t3,x3);
title("$x_3(t) = \prod(\frac{t-5.5}{6})$",Interpreter="latex");
ylim([-1.2,1.2]);
xlim([0,10])
grid on
% ------ x4
subplot(3,3,4);
Ts4 = 0.001;
t4 = t_start:Ts4:t_end;
```

```
x4 = 1 - exp(-t4);
plot(t4,x4);
title("$x_4(t) = 1 - e^{-t}$",Interpreter="latex");
ylim([-.2,1.2]);
xlim([0,10])
grid on
% ------ x5
subplot(3,3,5);
Ts5 = 0.001;
t5 = t_start+0.01:Ts5:t_end;
x5 = t5.*log(t5) - 2*t5;
plot(t5,x5);
title("$x_5(t) = t\ln(t) - 2t$",Interpreter="latex");
ylim([-3,2.8]);
xlim([0,10])
grid on
% ------ x6
subplot(3,3,6);
warning('off');
Ts6 = 0.001;
t6 = t_start+0.01:Ts6:t_end;
x6 = (t6.^3 - 8*t6.^2 + 13).^(1/3);
plot(t6,x6);
title("$x_6(t) = \sqrt[3]{t^3 - 8t^2 + 13}$",Interpreter="latex");
ylim([-.2,5.8]);
xlim([0,10])
grid on
% ------ x7
subplot(3,3,7);
Ts7 = 0.001;
t7 = t_start+0.01:Ts7:t_end;
x7 = dirac(t7-3);
stem(x7,t7-3);
title("$x_7(t) = \delta(t-3)$",Interpreter="latex");
xlim([-.2,.2]);
ylim([0,10])
grid on
% ------ x8
subplot(3,3,8);
% x(t) and y(t) from problem 6
Ts8 = 0.001;
t8 = t_start:Ts8:t_end;
x8 = (t8 >= 2) & (t8 <= 4);
x8 = x8 - ( (t8 >= 6) & (t8 <= 8));
y8 = (1 - 2*exp(-5*(t8-2))+ exp(-10*(t8-2))).*(t8>=2);
y8 = y8 - (1 - 2*exp(-5*(t8-4))+ exp(-10*(t8-4))).*(t8>=4) ;
y8 = y8 - (1 - 2*exp(-5*(t8-6))+ exp(-10*(t8-6))).*(t8>=6);
y8 = y8 + (1 - 2*exp(-5*(t8-8))+ exp(-10*(t8-8))).*(t8>=8) ;
plot(t8,x8);
ylim([-1.2,1.2]);
xlim([0,10])
grid on
title("$u(t)$ from problem 6",Interpreter="latex");
subplot(3,3,9);
```

```
plot(t8,y8);
xlim([0,10])
ylim([-1.2,1.2])
grid on
title("$y(t)$ from problem 6",Interpreter="latex");
```



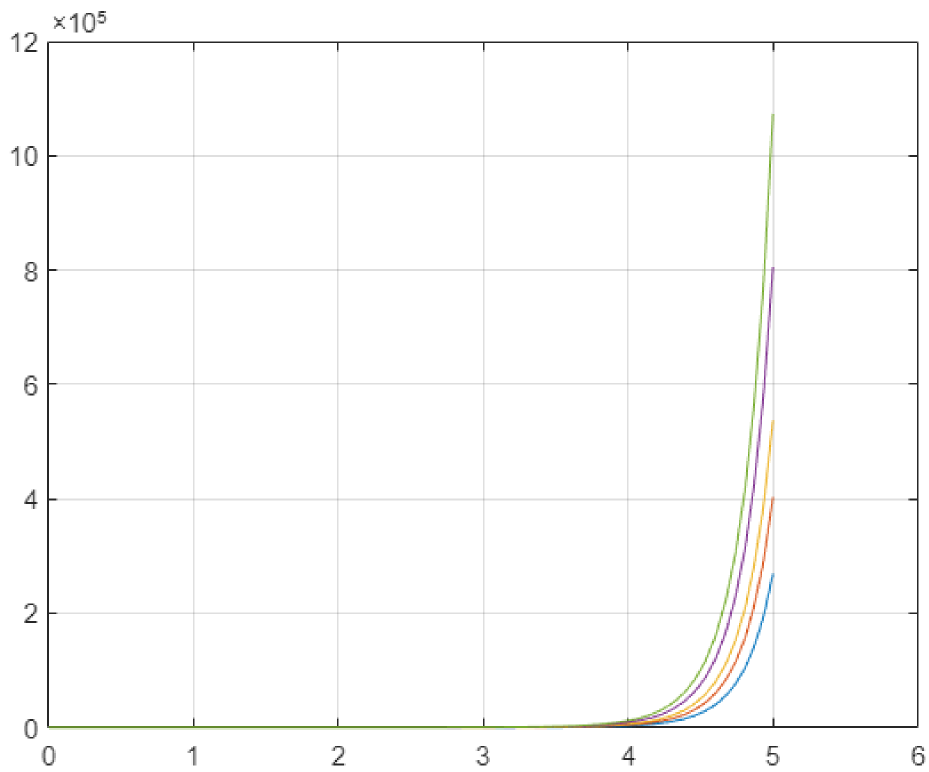
Problem 8: In this part we will change the initial condition of the differentaial equation and we observe the solution to it. The output is depicted in this figure dwn here:

```
%initial conditions
close all;

y0_1 = 1;
y0_2 = 1.5;
y0_3 = 2;
y0_4 = 3;
y0_5 = 4;

% y0 = 1
tspan = [0 5];
[t_1, y_1] = ode45(@myode1, tspan, y0_1);
plot(t_1,y_1)
grid on
hold on
% y0 = 1.5
tspan = [0 5];
[t_2, y_2] = ode45(@myode1, tspan, y0_2);
```

```
plot(t_2,y_2)
grid on
hold on
% y0 = 2
tspan = [0 5];
[t_3, y_3] = ode45(@myode1, tspan, y0_3);
plot(t_3,y_3)
grid on
hold on
% y0 = 3
tspan = [0 5];
[t_4, y_4] = ode45(@myode1, tspan, y0_4);
plot(t_4,y_4)
grid on
hold on
% y0 = 4
tspan = [0 5];
[t_5, y_5] = ode45(@myode1, tspan, y0_5);
plot(t_5,y_5)
grid on
xlim([0,6]);
```

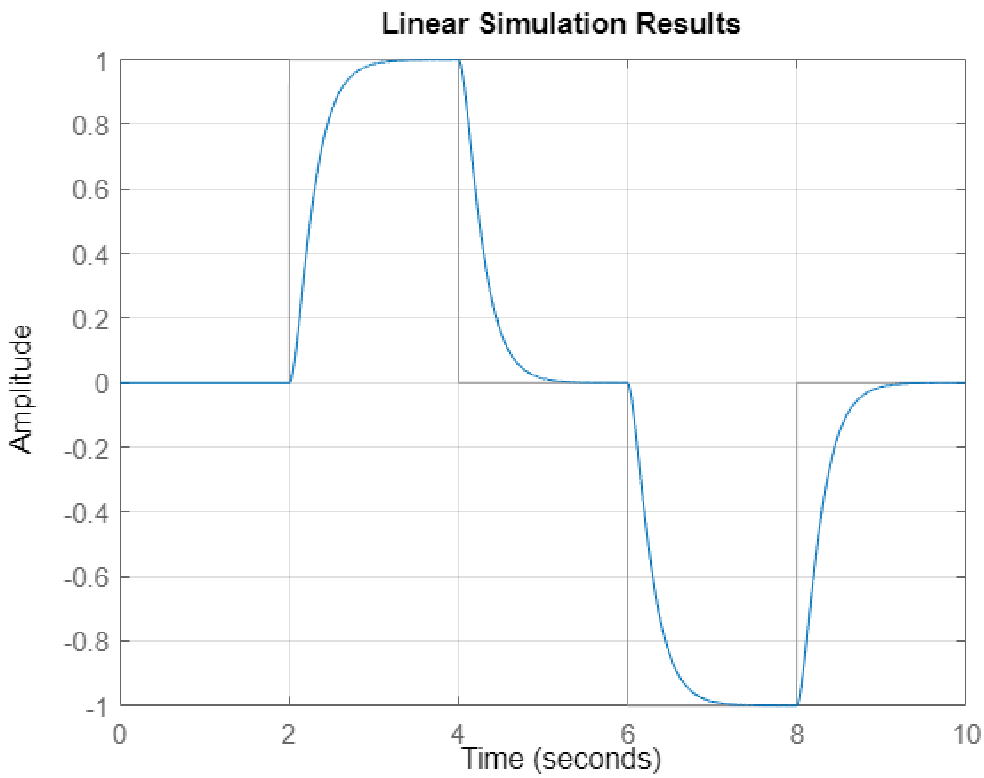


Problem 9:

$H(s) = \frac{Y(s)}{X(s)} = \frac{50}{(s+5)(s+10)} = \frac{10}{s+5} + \frac{-10}{s+10}$ which suggests $50X(s) = (s^2 + 15s + 50)Y(s)$. Now that we know $H(s)$, we can get the simulation done with the help of matlab's $lsim$ commmand.

As you can see is in synch with our result in the previous chapter.

```
close all;
clear all;
s = tf('s');
H_sys = 50/((s+5)*(s+10));
Ts = 0.001;
tt = 0:Ts:10;
uu = ((tt>=2)&(tt<=4));
uu = uu - ((tt<=8)&(tt>=6));
lsim(H_sys,uu,tt)
grid on
```



$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{50}{(s+5)(s+10)} = \dfrac{10}{s+5} + \dfrac{-10}{s+10}$ which suggests $50X(s) = (s^2 + 15s + 50)Y(s)$ which implies $50x(t) = y''(t) + 15y'(t) + 50y(t)$. Now if we let $y_d[n] = y(t = nh)$ then we get:

$y'(t) \equiv \dfrac{y_d[n] - y_d[n-1]}{h}$ and $y''(t) \equiv \dfrac{y_d[n] - 2y_d[n-1] + y_d[n-2]}{h^2}$.

Which will result in:

$$50x(t) = 50x_d[n] = \frac{1}{h^2}(y_d[n] - 2y_d[n-1] + y_d[n-2]) + \frac{15}{h}(y_d[n] - y_d[n-1]) + 50y_d[n]$$

which follows:

$$y_d[n](\frac{1}{h^2}+\frac{15}{h}+50) = (\frac{2}{h^2}-\frac{15}{h})y_d[n-1]-\frac{1}{h^2}y_d[n-2]+50x[n]$$

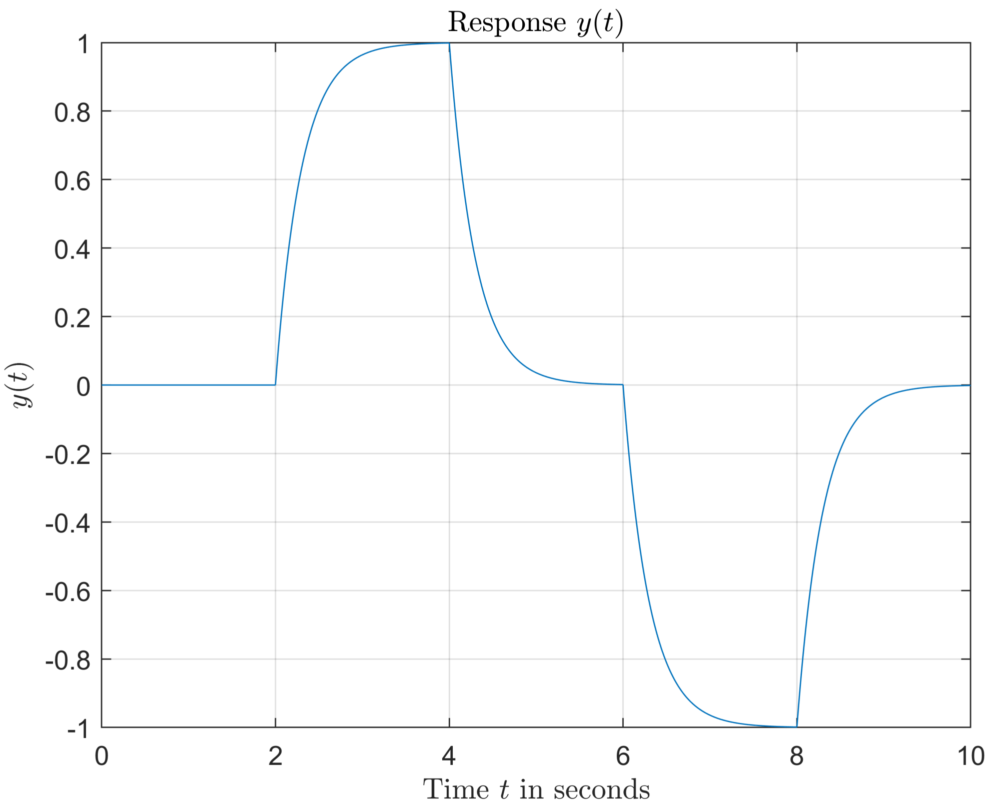
and by approximation we get:

$$y_d[n](16+50h) = 50hx[n]+17y_d[n-1]-y_d[n-2]\ .$$

Finally, this is the implementation of the Euler's method for the differential equation:

As you can see, the output is also in sunvh with the results above and the result from part 6.

```
clc;
clear all;
close all;
% defining the sampling rate
Ts = 0.001;
h = Ts;
% the interval's boundaries
t_start =0;
t_end = 10;
% Create a time vector 't'
t = 0:Ts:10;
% Define the input signal 'X' and the output signal 'Y'
U = heaviside(t-2) - heaviside(t-4) - heaviside(t-6) + heaviside(t-8);
X = ((t >= 2) & (t <= 4)) - ((t >= 6) & (t <= 8));
Y = zeros(size(t));
% Set the initial conditions for Y
Y(1) = 0;
Y(2) = 0;
% Loop to solve the difference equation
for i = 3:length(t)
    Y(i) = g(i, Y(i-1), Y(i-2), X(i), h);
end
% Plotting the result
plot(t,Y);
title('Response $y(t)$',Interpreter='latex');
xlabel('Time $t$ in seconds', Interpreter='latex');
ylabel('$y(t)$',Interpreter='latex');
grid on;
```




```
function [dydt1] = myode1(t, y)
dydt1 = t*y;
end


function [result] = f(i, y_i1, y_i2, u , h)
result =  -(-50*(h^2)*u + (y_i1*(15*h -2)/(h^2)) + (y_i2/(h^2)))/(1 + 15*h + h^2);
end

function [result] = g(i, y_i1, y_i2, u , h)
result =  (50*h*u- y_i2 + 17*y_i1)/(16 + 50*h);
end
```