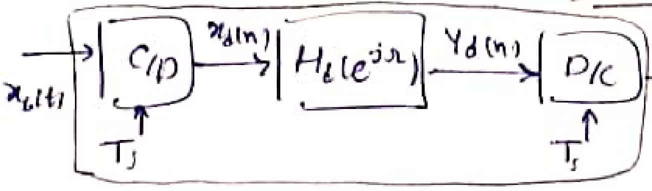
تمرین سری 4 DSP کیمیا پارسا رنی 400101204

سوال 1 : 27 :

$T_s = \frac{\pi}{\omega_M}$



$$H_d(e^{j\Omega}) = \begin{cases} \dfrac{e^{\frac{j\Omega}{2}} - e^{-\frac{j\Omega}{2}}}{T} & |\Omega|<\pi \\ \text{periodic} & \text{o.w.} \end{cases}$$

ω: فرکانس حالت پیوسته

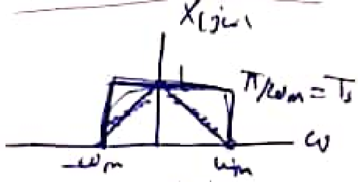
Ω: فرکانس حالت گسسته

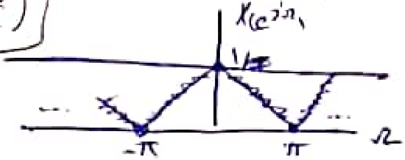
$H_c(j\omega)$ $\qquad \frac{\Omega}{T_s} = \omega$ داریم

(a)

$$H_c(j\omega) = H_d(e^{j\Omega})\Big|_{\Omega=\omega T_s} = \frac{e^{\frac{j\omega T_s}{2}} - e^{-\frac{j\omega T_s}{2}}}{T_s} = \frac{2j}{T_s}\sin\left(\frac{\omega T_s}{2}\right)$$

دقت کنید به شرط Nyquist برقرار است، تداخل فرکانسی پیش نمی‌آید.



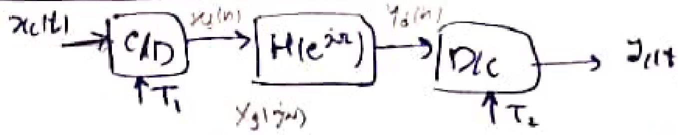


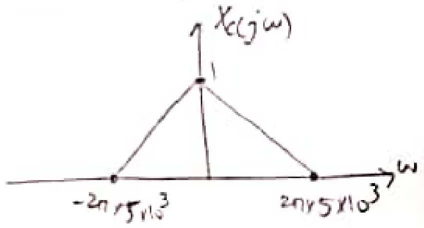
(b) $x_c(t) = \dfrac{\sin(\omega_m t)}{\omega_m t} \longrightarrow x_d(n) = x_c(nT_s) = \dfrac{\sin\left(\frac{\pi}{T_s}\times nT_s\right)}{\frac{\pi}{T_s}\times nT_s} = \dfrac{\sin(n\pi)}{n\pi} = 0 \qquad \Omega = \omega_m T_s = \pi$

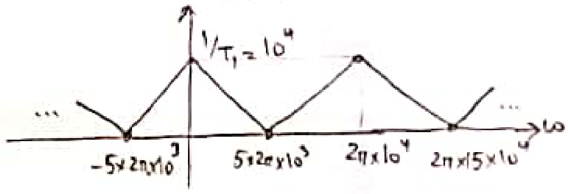
when $x_d(n) = 0 \longrightarrow Y_d(n) = 0$ $\longrightarrow Y_c(t) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} x(nT_s)\,\text{sinc}\left(\frac{t-nT_s}{\pi}\right) = 0$

سوال 2 : 30 :

(I) $T_1 = T_2 = 10^{-4}$





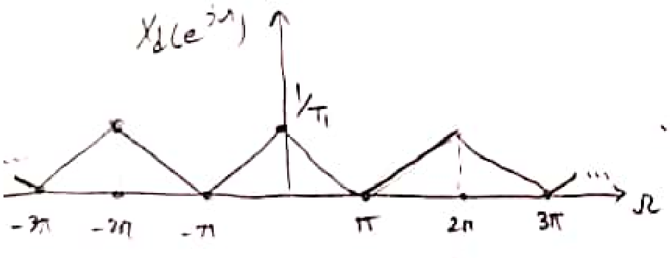


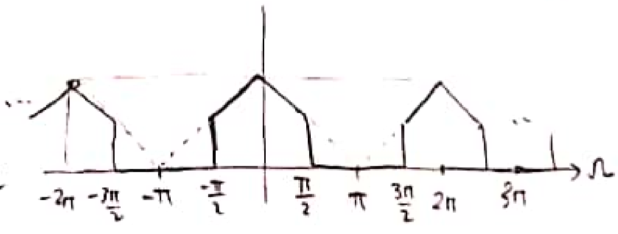
$\frac{\Omega}{T_s} = \omega_m$

$\Omega = 10^{-4}\times5\times2\pi\times10^3 = \pi$

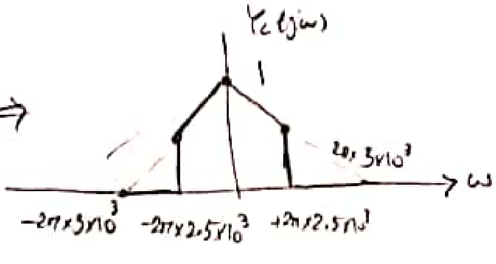
$\omega_1 = 2\pi\times10^4$

$Y_d(e^{j\Omega}) = H(e^{j\Omega})X_d(e^{j\Omega})$



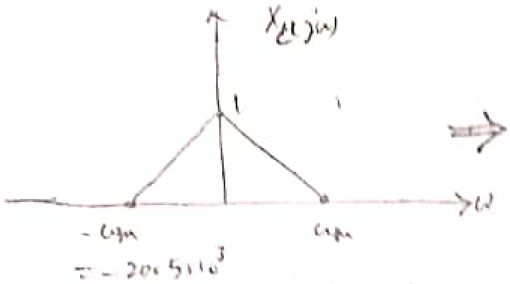


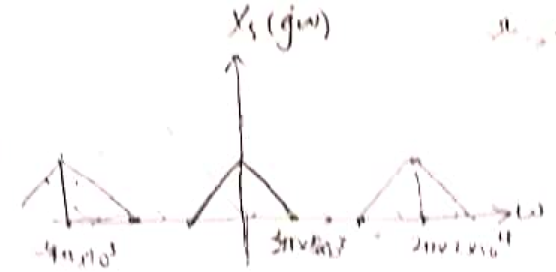
$\Rightarrow$



$\omega_M = \frac{\Omega}{T_s} = \frac{\pi/2}{10^{-4}} = 2\pi\times\frac{10^{+4}}{4} = 2500\times2\pi$

II $\frac{1}{T_1} = \frac{1}{T_2} = 2\times10^{+4}$

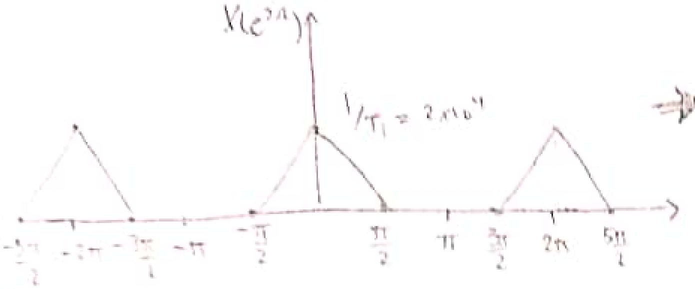


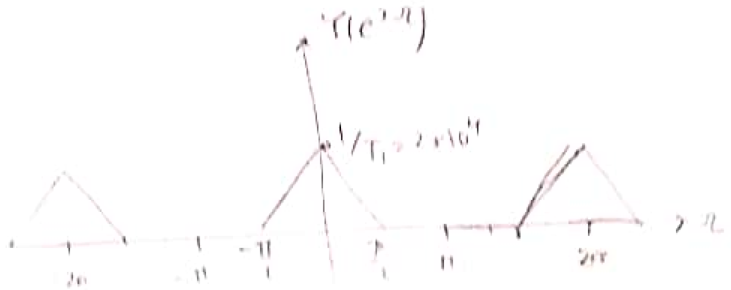


$\Omega = \omega T_s = \frac{1}{2}\times10^{-4}\times2\pi\times5\times10^3$

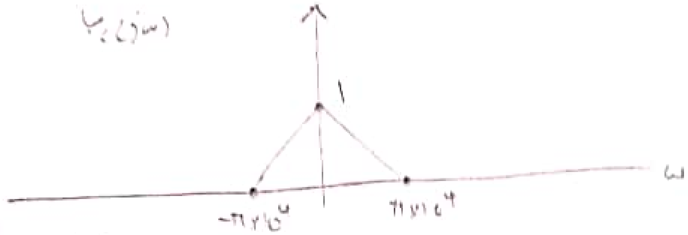
$= \frac{\pi}{2}$

$\omega_1 = \frac{2\pi}{T_1} = 2\pi\times10^4\times2$





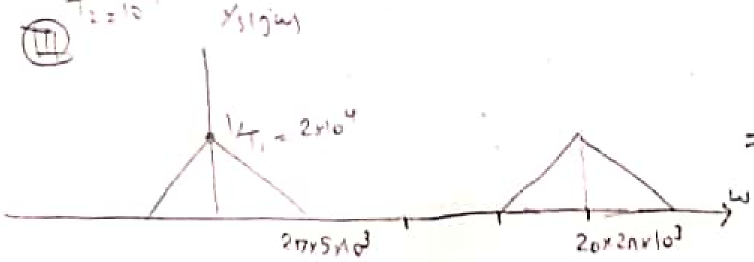
$\omega_M = \frac{\Omega}{T_s} = 2\times10^4\times\frac{\pi}{2} = 2\pi\times5\times10^3$

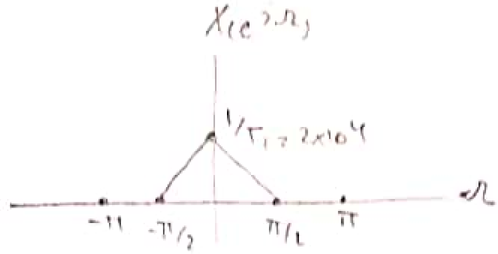


$\frac{1}{T_1} = 10^4\times2$

$T_2 = 10^{-4}$

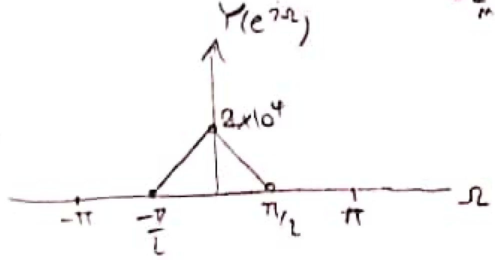
III

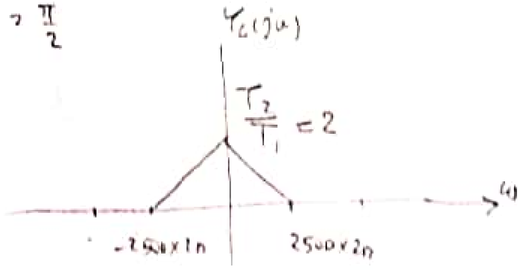




$\omega_1 = 4\pi\times10^4$

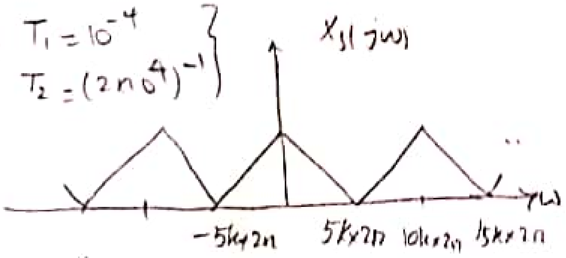
$\Omega_m = \omega_m T_s = \frac{2\pi\times5\times10^3}{2\times10^4} = \frac{\pi}{2}$

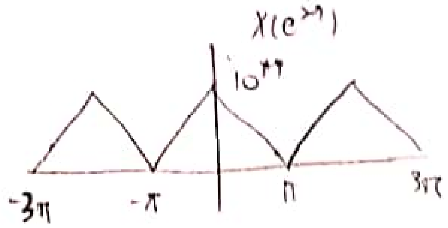




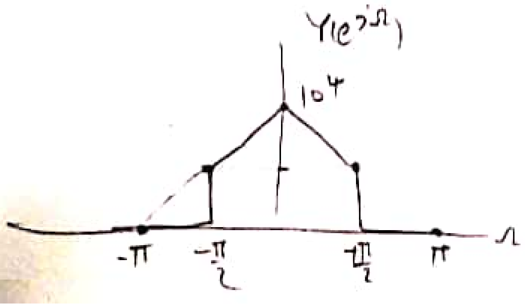
$\omega_M = \frac{\pi}{2}\times10^4 = 2500\times2\pi$

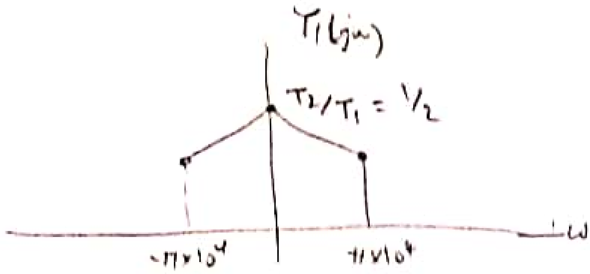
IV $\left.\begin{array}{l} T_1 = 10^{-4} \\ T_2 = (2\times10^4)^{-1} \end{array}\right\}$





$\omega_1 = 2\pi\times10^4$





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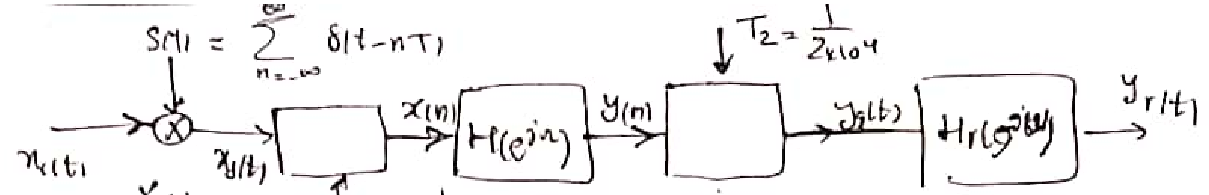
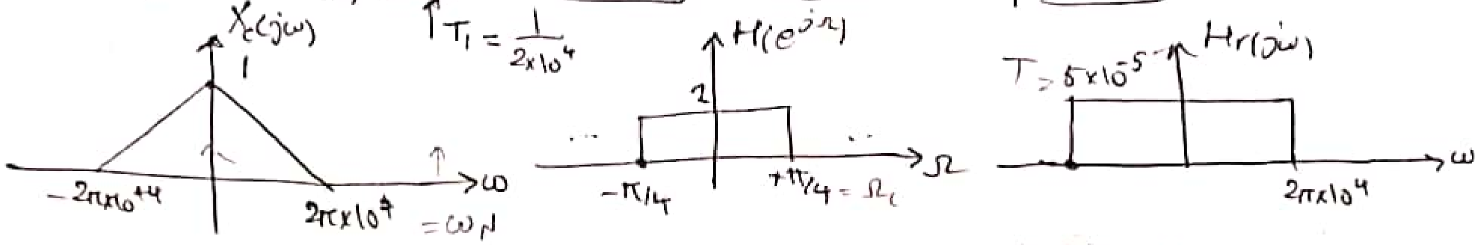
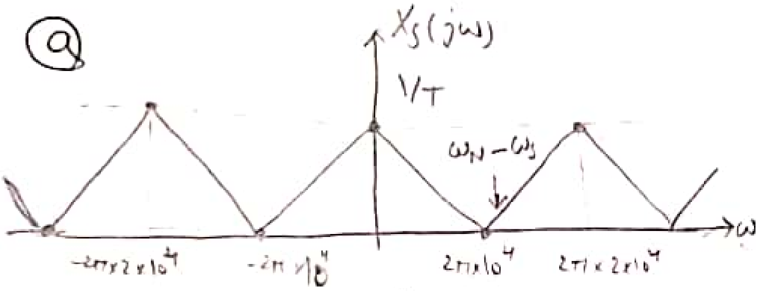
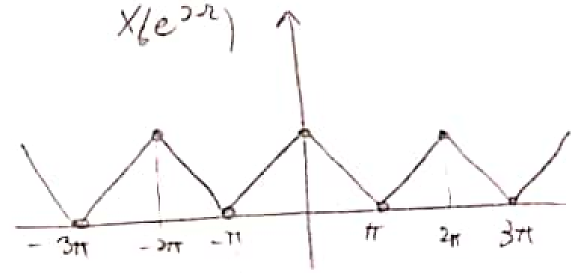




(a)



$\Rightarrow$



(b) از آنجا که $H(e^{j\Omega})$ یک فیلتر ایده‌آل با پهنای باند $\Omega_c = \frac{\pi}{4}$ است، از این رو T باید به گونه‌ای باشد که اطلاعات در خارج از بازه $[-\Omega_c, +\Omega_c] = [-\frac{\pi}{4}, \frac{\pi}{4}]$ نتوانسته باشد. (همچنین شرط نایکوئیست باید برقرار باشد)

$\frac{\Omega}{T_s} = \omega$

شرط نایکوئیست $\rightarrow$ $\omega_N - \omega_s \geq \omega_s \longrightarrow \omega_s = \frac{2\pi}{T_s} \leq \frac{\omega_N}{2} \longrightarrow \frac{1}{T_s} \leq \frac{10^4}{2} = 5KHz$

شرط عدم تداخل فرکانس در داخل فیلتر $\rightarrow T_s(\omega_s - \omega_N) \geq \pi/4 = \Omega_c \longrightarrow T_s(\frac{2\pi}{T_s}) - T_s \times 2\pi\times10^4 \geq \frac{\pi}{4} \longrightarrow \frac{7}{16}\times10^{-4} \geq T_s$

شرط اینکه تمام فرکانس‌ها در داخل فیلتر قرار بگیرد و محتوا تغییر نکند. $\rightarrow T_s\omega_N \leq \Omega_c = \pi/4 \longrightarrow T_s \times 2\pi\times10^4 \leq \frac{\pi}{4} \longrightarrow T_s \leq \frac{10^{-4}}{8}$

سوال 5: 45

$\mathbb{E}(x(n)^2) = 1 = \sigma_x^2$

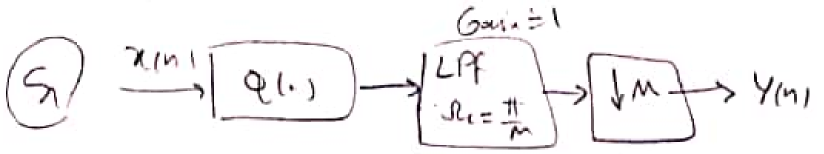
$m_x = 0 = \mathbb{E}\, x(n)$ WSS

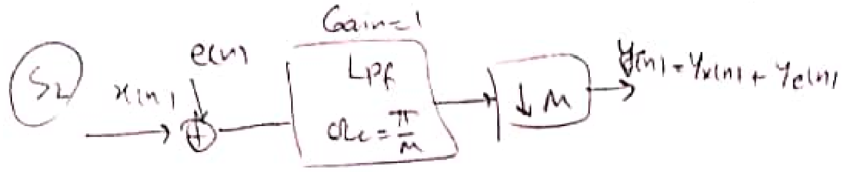
Spectral density $\Phi_{xx}(e^{j\Omega})$

band limited to $\frac{\pi}{M}$

$\mathbb{E}(e(n)) = m_e = 0, \quad \sigma_e^2 = \frac{\Delta^2}{12}$

$\mathbb{E}(x(n)\,e(n)) = 0$





a) $SNR = 10\log\left(\frac{\mathbb{E}(Y_x^2(n))}{\mathbb{E}(Y_e^2(n))}\right) =$

s(n) قطار ضربه M تایی است

ابتدا توجه می کنیم که سیگنال ها در حوزه فرکانس Band limited هستند $\sim \left(\frac{\pi}{M}, -\frac{\pi}{M}\right)$ در بازه $\left(\frac{M\pi}{M}\right)$ است و مشکلی از نظر تداخل ایجاد نمی کند $\Omega \to \frac{\Omega}{M}$

$$y_e(n) = e(Mn) \longrightarrow \mathbb{E}(y_e(n)^2) = \mathbb{E}\left[(e(n) * s(n))^2\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{ee}(e^{\frac{j\Omega}{M}})\, d\Omega$$

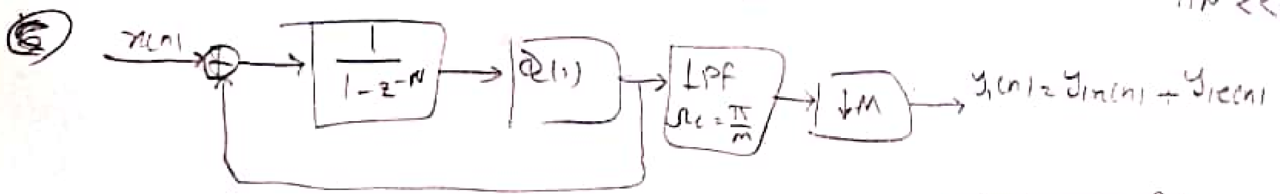
$$= \frac{M}{2\pi}\int_{-\pi}^{\pi} \Phi_{ee}(e^{j\Omega})\frac{d\Omega}{M} = M\frac{\mathbb{E}(e(n)^2)}{M} = \frac{M\sigma_e^2}{M} = \sigma_e^2$$

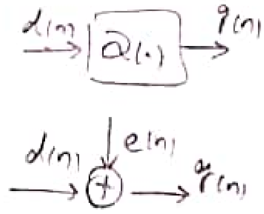
به طریق مشابه برای x(n) هم ثابت می شود که

$$y_x(n) = x(nM) \longrightarrow \mathbb{E}(y_x(n)^2) = M\,\mathbb{E}(x(n)^2) = M\sigma_x^2 = M$$

$$SNR = 10\log\left(\frac{M\sigma_x^2}{\sigma_e^2}\right) = 10\log\left(\frac{M}{\sigma_e^2}\right)$$

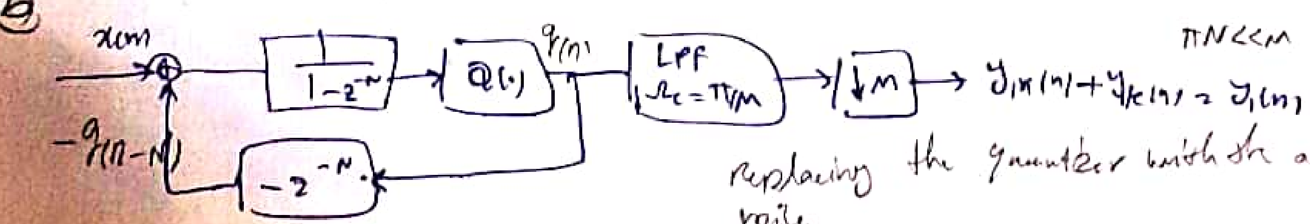
b) $\pi N \ll M$



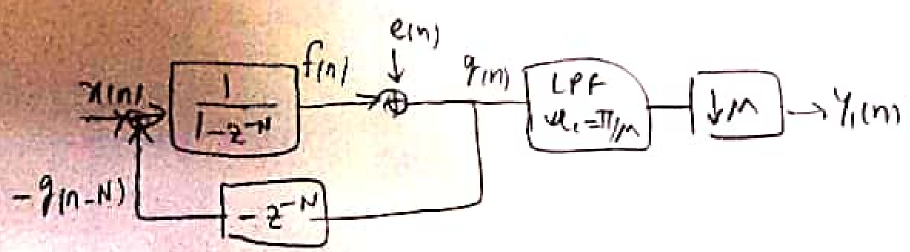


Shift and scaling has no effect on being a WSS stochastic process therefore both outputs will be a WSS stochastic process.

c) $\pi N \ll M$

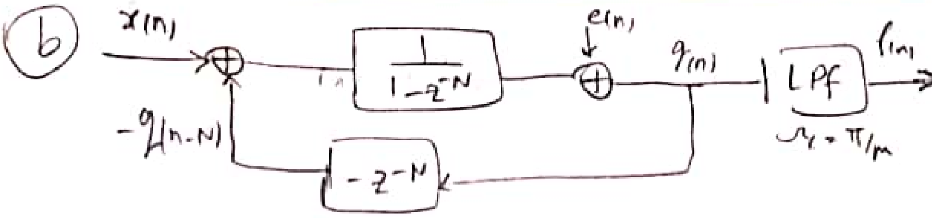


replacing the quantizer with the additive noise



ادامه سوال 5:

(b)



$q_x(n)$ حاصل از $x(n)$
$q_e(n)$ حاصل از $e(n)$

$$\text{let}\begin{cases} y(n) = f(Mn), \quad f_x(n) = q_x(n) * \dfrac{\sin(\Omega_c n)}{\pi n}, \quad f_e(n) = q_e(n) * \dfrac{\sin(\Omega_c n)}{\pi n} \\ q(n) = q_x(n) + q_e(n) \ \& \ y(n) = y_e(n) + y_x(n) \end{cases} \text{ by Superpositioning}$$

$$\frac{X(z) - z^{-N} Q(z)}{1 - z^{-N}} + E(z) = Q(z) \begin{cases} \xrightarrow[\text{using Superposition}]{\text{if } E(z)=0} & X(z) = Q(z) \Rightarrow x_q(n) = x(n) \\ \xrightarrow{\text{if } X(z)=0} & E(z)(1 - z^{-N}) = Q(z) \Rightarrow x_e(n) = e(n) - e(n-N) \end{cases}$$

$$\begin{cases} y_x(n) = f_x(Mn) = q_x(Mn) * \dfrac{\sin(\Omega_c Mn)}{\pi Mn} = x(Mn) * \dfrac{\sin(\Omega_c Mn)}{\pi Mn} \\ y_e(n) = f_q(Mn) = q_e(Mn) * \dfrac{\sin(\Omega_c Mn)}{\pi Mn} = \left(e(Mn) - e(Mn-N)\right) * \dfrac{\sin(\Omega_c Mn)}{\pi Mn} \end{cases}$$

$$y(n) = y_x(n) + y_e(n) = \left[x(Mn) + e(Mn) - e(Mn-N)\right] * \frac{\sin(\Omega_c Mn)}{\pi Mn}$$

(d) برای اینکه اثر نویز e(n) را بهتر درک کنیم، عامل کوانتیزه شدن e(n) یعنی $q_e(n)$ را بررسی کرده و مقدار چگالی توان آن را بررسی می‌کنیم.

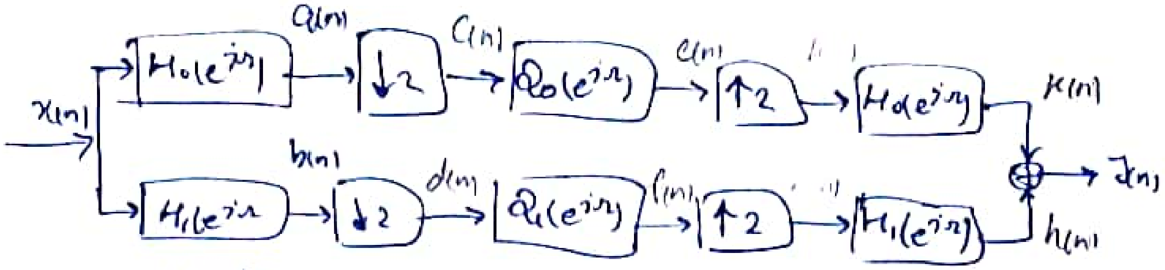
$$\Phi_{q_e q_e}(e^{j\Omega}) = \mathcal{F}\{\sigma_e^2 \delta(m)\}\, \mathcal{F}\{\delta(m) - \delta(m-N)\}\, \mathcal{F}\{\delta(m) - \delta(m-N)\}^*$$

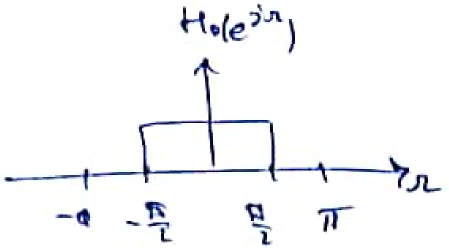
$$= \sigma_e^2 \left(1 - e^{-j\Omega N}\right)\left(1 - e^{j\Omega N}\right) = \sigma_e^2\left(2 - e^{-j\Omega N} - e^{j\Omega N}\right)$$

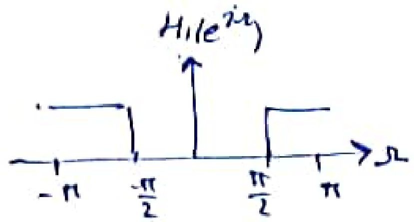
$$= 2\sigma_e^2 \left(1 - \cos(\Omega N)\right) =$$

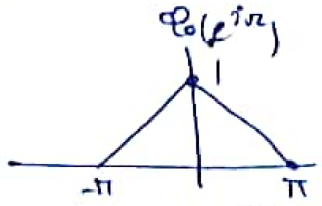
$$E\left(f_e(n)^2\right) = \phi_{f_e f_e}(m)\Big|_{m=0} = \frac{1}{2\pi}\int_{-\pi/M}^{\pi/M} \Phi_{q_e q_e}(e^{j\Omega})\, d\Omega \approx \frac{2}{2\pi}\int_0^{\pi/M} \frac{4\sigma_e^2 N^2 \Omega^2}{4}\, d\Omega = \boxed{\frac{\sigma_e^2 N^2 \pi^2}{3M^3}}$$

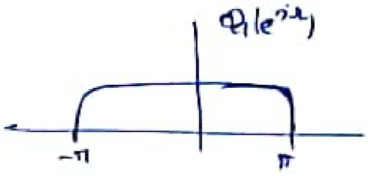
: 49 : 6 سوال











$$G(e^{j\Omega}) = \frac{X(e^{j\Omega})}{X(e^{j\Omega})}$$

let $\mathcal{F}\{x(n)\} = X(e^{j\Omega})$

$$A(e^{j\Omega}) = \begin{cases} X(e^{j\Omega}) & |\Omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\Omega| < \pi \end{cases} \; ; \; B(e^{j\Omega}) = \begin{cases} 0 & |\Omega| < \pi/2 \\ X(e^{j\Omega}) & \frac{\pi}{2} < |\Omega| < \pi \end{cases}$$

$$C(e^{j\Omega}) = \begin{cases} \frac{1}{2} X(e^{j\frac{\Omega}{2}}) & |\Omega| < \pi \\ \text{periodic} & \text{o.w.} \end{cases} \; ; \; D(e^{j\Omega}) = \begin{cases} \frac{1}{2} X(e^{j\frac{(\Omega-2\pi)}{2}}) & |\Omega| < \pi \\ \text{Periodic} & \text{o.w.} \end{cases}$$

$$E(e^{j\Omega}) = \begin{cases} \frac{1}{2} X(e^{j\frac{\Omega}{2}}) Q_0(e^{j\Omega}) & |\Omega| < \pi \\ \text{Periodic} & \text{o.w.} \end{cases} \; ; \; F(e^{j\Omega}) = \begin{cases} \frac{1}{2} X(e^{\frac{j(\Omega-2\pi)}{2}}) Q_1(e^{j\Omega}) & |\Omega| < \pi \\ \text{Periodic} & \text{o.w.} \end{cases}$$

$$K(e^{j\Omega}) = \begin{cases} X(e^{j\Omega}) Q_0(e^{2j\Omega}) & |\Omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\Omega| < \pi \end{cases} \; ; \; H(e^{j\Omega}) = \begin{cases} X(e^{\frac{j(\Omega-2\pi)}{2}}) Q_1(e^{j2\Omega}) & \frac{\pi}{2} < |\Omega| < \pi \\ 0 & \frac{\pi}{2} > |\Omega| \end{cases}$$

$$Y(e^{j\Omega}) = K(e^{j\Omega}) + H(e^{j\Omega}) = \begin{cases} X(e^{j\Omega}) Q_1(e^{j\Omega}) & \frac{\pi}{2} < |\Omega| < \pi \\ X(e^{j\Omega}) Q_0(e^{2j\Omega}) & \frac{\pi}{2} > |\Omega| \end{cases}$$