تمرین سری ۴ سیستم‌های کنترل خطی

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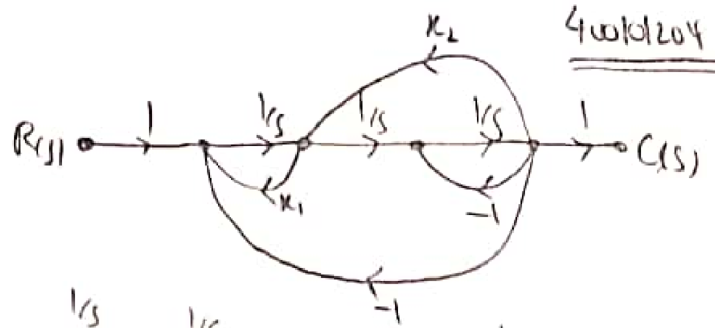
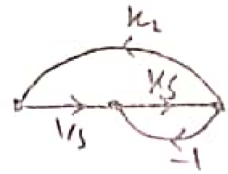
سوال ۱



ابتدا $H(s)=\frac{C(s)}{R(s)}$ را حساب می‌کنیم:

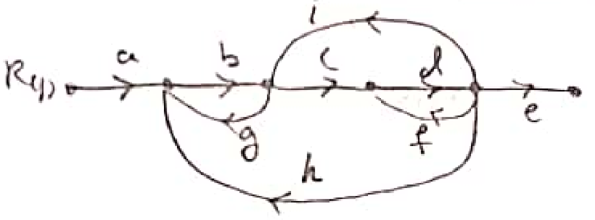
راه اول:

 $=\frac{1/s}{1/s+1}=\frac{1}{s+1}$ و  $=\frac{1}{s(s+1)}$ و  $=\frac{\frac{1}{s^2+s}}{1-k_2\left(\frac{1}{s^2+s}\right)}=\frac{1}{s^2+s-k_2}$

 $=\frac{1/s}{1-\frac{k_1}{s}}=\frac{1}{s-k_1} \Rightarrow H(s)=\frac{C(s)}{R(s)}=\frac{\frac{1}{s-k_1}\times\frac{1}{s^2+s-k_2}}{1+\frac{1}{s-k_1}\times\frac{1}{s^2+s-k_2}}=\frac{1}{1+(s-k_1)(s^2+s-k_2)}$

$$H(s)=\frac{1}{s^3+(1-k_1)s^2+(k_1+k_2)s+(1+k_1k_2)}$$

راه دوم: استفاده از Maison



forward paths $=\{abcde\} \Rightarrow P_1(s)=\frac{1}{s^3}$

loops $=\{bg,\ cdi,\ df,\ bcdh\}$

nontouchings $=\{(bg,df)\}$

$$\Delta(s)=1-\left(\frac{k_1}{s}+\frac{k_2}{s^2}-\frac{1}{s}-\frac{1}{s^3}\right)-\frac{k_1}{s^2}\quad,\quad \Delta_1(s)=0$$

$$T(s)=\frac{P_1(s)\Delta_1(s)}{\Delta(s)}=\frac{1/s^3}{1-\frac{k_1}{s}+\frac{k_2}{s^2}+\frac{1}{s}+\frac{1}{s^3}-\frac{k_1}{s^2}}=\frac{1}{s^3+s^2(1-k_1)+(k_1+k_2)s+(1+k_1k_2)}$$

$H(s)=\frac{1}{(s^2+\omega_0^2)(s+a)}$ برای اینکه سیستم ما، oscillatory، از فرم زیر باید داشته باشیم

$$(s^2+\omega_0^2)(s+a)=s^3+as^2+s\omega_0^2+a\omega_0^2\quad\begin{cases}1-k_1=a\\ k_1+k_2=-\omega_0^2\\ a\omega_0^2=1+k_1k_2\end{cases}$$

$$1+k_1+k_2+k_1k_2=\underbrace{(1+k_1)}_{a}(1+k_2)=\omega_0^2+a\omega_0^2=\omega_0^2(a+1)$$

$$1-k_2=\frac{\omega_0^2(a+1)}{a}\rightarrow k_2=\frac{-\omega_0^2(a+1)+a}{a}\rightarrow k_1=-\omega_0^2-k_2$$

$$\rightarrow k_1=\frac{+\omega_0^2(a+1)-a-a\omega_0^2}{a}=\frac{\omega_0^2-a}{a}\xRightarrow{a=1-k_1}\boxed{\omega_0=\sqrt{\frac{1+k_1k_2}{1-k_1}}}$$

(I)

سوال 2: راه اول: از آنجا که قطب $s=-30$ نسبت به قطب $s=-3$ بسیار دور است میتوان حذف کرد.

$$G_H(s) \simeq \frac{20/30}{(s+3)} = \frac{2}{3} \times \frac{1}{s+3}$$

راه دوم:

$$G_H(s) = \frac{20}{(s+3)(s+30)} \qquad G_L(s) = \frac{k}{s+P_1}$$

$$\frac{G_H(s)}{G_L(s)} = \frac{20(s+P_1)}{k(s+3)(s+30)} = \frac{M(s)}{D(s)}$$

$$\begin{cases} M^{(0)}_{(0)} = 20P_1 \\ M^{(1)}_{(0)} = 20 \\ M^{(k)} = 0 \quad \forall k \geq 2 \end{cases} \qquad \begin{cases} D^{(0)}_{(0)} = 90k \\ D^{(1)}_{(0)} = 33k \\ D^{(2)}_{(0)} = 2k \\ D^{(k)}_{(0)} = 0 \quad \forall k \geq 3 \end{cases}$$

$$q=0 \begin{cases} M_0 = \dfrac{M^{(0)}_{(0)}M^{(0)}_{(0)}}{1} = 400P_1^2 \\ D_0 = \dfrac{D^{(0)}_{(0)}D^{(0)}_{(0)}}{1} = 900k^2 \end{cases} \quad \left|\frac{P_1}{k}\right| = \frac{3}{2}$$

$$q=1 \begin{cases} M_2 = -\dfrac{M^{(0)}_{(0)}M^{(2)}_{(0)}}{2} + \dfrac{M^{(1)}_{(0)}M^{(1)}_{(0)}}{1} - \dfrac{M^{(2)}_{(0)}M^{(0)}_{(0)}}{2} = 400 \\ D_2 = -\dfrac{D^{(0)}_{(0)}D^{(2)}_{(0)}}{2} + \dfrac{D^{(1)}_{(0)}D^{(1)}_{(0)}}{1} - \dfrac{D^{(2)}_{(0)}D^{(0)}_{(0)}}{2} = -180k^2 + 33^2k^2 = 909k^2 \end{cases} \longrightarrow 909k^2 \stackrel{?}{=} 400 \longrightarrow k = \frac{20}{\sqrt{909}}$$

$$|P_1| = \frac{3}{2}|k| = \frac{10}{\sqrt{101}} \simeq 0.99 \simeq 1$$ قطب باید سمت چپ باشد: $P_1 \simeq 0.99$

$$G_L(0) > 0 \longrightarrow k > 0 \longrightarrow k = \frac{20}{\sqrt{909}} \simeq 0.66 \longrightarrow \boxed{k = \frac{2}{3},\ P_1 = 1} \Rightarrow G_L(s) = \frac{2/3}{s+1}$$

(II)

$$G_H(s) = \frac{-8\left(\frac{s}{5}+1\right)}{(s+2)(s+4)(s+5)} \qquad G_L(s) = \frac{k}{(s+P_1)(s+P_2)}$$

$$\Rightarrow \frac{G_H(s)}{G_L(s)} = \frac{M(s)}{D(s)} = \frac{-8k(s-5)(s+P_1)(s+P_2)}{(s+2)(s+4)(s+5)}$$

$$\begin{cases} M^{(0)}_{(0)} = 40kP_1P_2 \\ M^{(1)}_{(0)} = -8k(P_1P_2 - 5P_1 - 5P_2) \\ M^{(2)}_{(0)} = -16k(P_1+P_2-5) \\ M^{(3)}_{(0)} = -48k \\ M^{(k)}_{(0)} = 0 \quad \forall k \geq 4 \end{cases} \qquad \begin{cases} D^{(0)}_{(0)} = 40 \\ D^{(1)}_{(0)} = 36 \\ D^{(2)}_{(0)} = 22 \\ D^{(3)}_{(0)} = 6 \\ D^{(k)}_{(0)} = 0 \quad \forall k \geq 4 \end{cases}$$

$$q=0 \begin{cases} D_0 = \dfrac{D^{(0)}_{(0)}D^{(0)}_{(0)}}{1} = \dfrac{40^2}{1} = 1600 \\ M_0 = \dfrac{M^{(0)}_{(0)}M^{(0)}_{(0)}}{1} = 40^2k^2P_1^2P_2^2 \end{cases} \longrightarrow \boxed{|kP_1P_2| = 1}$$

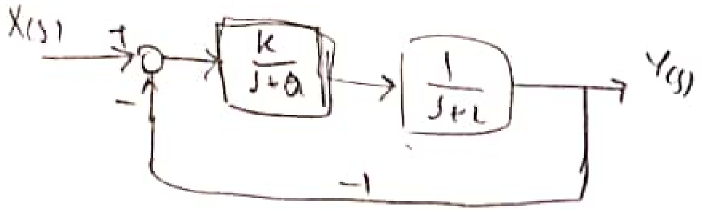
$$q=1 \begin{cases} -\dfrac{D^{(0)}_{(0)}D^{(2)}_{(0)}}{2} \times 2 + \dfrac{D^{(1)}_{(0)}D^{(1)}_{(0)}}{1} = D_2 = 36^2 - 40\times 22 = 416 \\ M_2 = -M^{(0)}_{(0)}M^{(2)}_{(0)} + M^{(1)}_{(0)}M^{(1)}_{(0)} = 64k^2(P_1P_2 - 5P_1 - 5P_2)^2 + 640P_1P_2(P_1+P_2-5)k^2 \\ \qquad = 1600k^2P_1^2 + 1600k^2P_2^2 + 64k^2P_1^2P_2^2 = 64(5k^2P_1^2 + 5k^2P_2^2 + k^2P_1^2P_2^2) \end{cases}$$

$$\longrightarrow 416 = 64\left(\frac{5}{P_1^2} + \frac{5}{P_2^2} + 1\right) \longrightarrow \boxed{\frac{1}{P_1^2} + \frac{1}{P_2^2} = \frac{416-64}{5\times 64} = \frac{6}{5}}$$

$$q=2 \begin{cases} D_4 = -\cancel{\dfrac{D^{(0)}_{(0)}D^{(4)}_{(0)}}{4!}}\times 2 + \dfrac{D^{(1)}_{(0)}D^{(3)}_{(0)}}{3!}\times 2 - \dfrac{D^{(2)}_{(0)}D^{(2)}_{(0)}}{2!\,2!} = \dfrac{36\times 6}{6}\times 2 - \dfrac{22\times 22}{2\times 2} \\ M_4 = -\cancel{\dfrac{M^{(0)}_{(0)}M^{(4)}_{(0)}}{4!}}\times 2 + \dfrac{M^{(1)}_{(0)}M^{(3)}_{(0)}}{6}\times 2 - \dfrac{M^{(2)}_{(0)}M^{(2)}_{(0)}}{4} = \end{cases}$$

$$= \frac{8\times 48k^2(P_1P_2 - 5P_1 - 5P_2)}{3} - \frac{16^2k^2(P_1+P_2-5)^2}{2}$$

سؤال 3



$$H(s)=\frac{Y(s)}{X(s)}=\frac{\frac{K}{(s+a)(s+2)}}{1+\frac{K}{(s+a)(s+2)}}$$

$$M(s)=\frac{K}{s^2+(a+2)s+(2a+K)}=\frac{K}{2a+K}\times\frac{2a+K}{s^2+(a+2)s+(2a+K)}=\frac{K}{2a+K}\,\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\left.\begin{aligned} f_v = \text{DC gain} &= \frac{K}{2a+K}\\ \omega_n &= \sqrt{2a+K}\\ 2\zeta\omega_n &= (a+2)\end{aligned}\right\}\quad \zeta=\frac{a+2}{2\sqrt{2a+K}}\qquad \omega_n=\sqrt{2a+K}$$

- Settling Time

$$T_s=\frac{4}{\omega_n\zeta}=\frac{4}{\sqrt{2a+K}\times\frac{a+2}{2\sqrt{2a+K}}}=\boxed{\frac{8}{a+2}\le 2}\quad (*)$$

- $M_{pt}=\left[1+\exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)\right]\times f_v$

- Overshoot Percentage $=P.O.=\frac{M_{pt}-f_v}{f_v}\times 100 < 5\%$

$$\frac{P.O.}{100}=\exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)\le\frac{1}{20}\longrightarrow\left(\frac{+\pi\zeta}{\sqrt{1-\zeta^2}}\right)\ge-\ln\left(\frac{1}{20}\right)=\ln(20)$$

$$\left.\begin{aligned}\sqrt{1-\zeta^2}&=\frac{\sqrt{-a^2+4a+4K-4}}{2\sqrt{2a+K}}\\ \zeta&=\frac{a+2}{2\sqrt{2a+K}}\end{aligned}\right\}\rightarrow\frac{\pi\zeta}{\sqrt{1-\zeta^2}}=\frac{(a+2)\pi}{\sqrt{-a^2+4a+4K-4}}\ge\ln(20)$$

let $a=4\longrightarrow\frac{6\pi}{\sqrt{4K-4}}\ge\ln(20)\quad,\quad K\ge1$

(*) هم برقرار است.

$$9.89=\left(\frac{3\pi}{\ln(20)}\right)^2\ge K$$

$$\Rightarrow\boxed{9.89\ge K\ge 1}$$

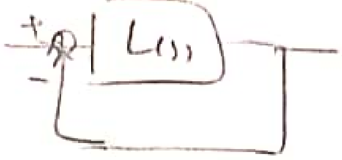
حال یک K از بازه، مطابق انتخاب می‌کنیم

مثلا

$$\boxed{\begin{aligned}K&=6\\ a&=2\end{aligned}}$$

Scanned by CamScanner

سوال ۴:



$$T(s) = \frac{L(s)}{L(s)+1}$$

(I) $L(s) = \dfrac{20}{s(s+1)(s+3)}$ $\xrightarrow{N=1}$ $e_{ss} = \begin{cases} 0 & \text{step } r_1(t) \\ \frac{3}{2} = \frac{10}{k_v} & \text{ramp } r_2(t) \\ \infty & \text{Parabola } r_3(t) \end{cases}$

$$k_v = \lim_{s\to 0} sL(s) = \lim_{s\to 0} \frac{20}{(s+1)(s+3)} = \frac{20}{3}$$

(II) $L(s) = \dfrac{100}{s(s+2)(s+4)(s+6)}$ $\xrightarrow{N=1}$ $e_{ss} = \begin{cases} 0 & \text{step} & r_1(t) \\ 24/5 = \frac{10}{k_v} & \text{ramp} & r_2(t) \\ \infty & \text{Parabola} & r_3(t) \end{cases}$

$$k_v = \lim_{s\to 0} sL(s) = \lim_{s\to 0} \frac{100}{(s+2)(s+4)(s+6)} = \frac{25}{12} \rightarrow \frac{10}{k_v} = \frac{\frac{10}{1}}{\frac{25}{12}} = \frac{120}{25} = \frac{24}{5}$$

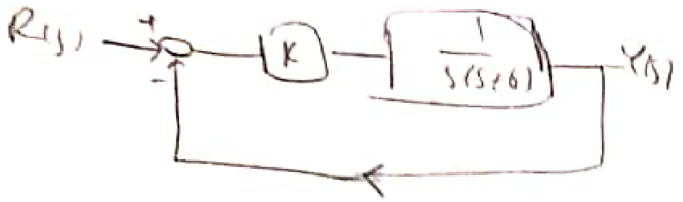
(III) $L(s) = \dfrac{10(s+7)}{(s+1)(s+2)}$ $\xrightarrow{N=0}$ $e_{ss} = \begin{cases} k_p = 35 & \text{step} & r_1(t) \\ \infty & \text{ramp} & r_2(t) \\ \infty & \text{Parabola} & r_3(t) \end{cases}$

$$k_p = \lim_{s\to 0} L(s) = \lim_{s\to 0} \frac{10(s+7)}{(s+1)(s+2)} = 35$$

(IV) $L(s) = \dfrac{5(s+1)}{s^2(s+3)}$ $\xrightarrow{N=2}$ $e_{ss} = \begin{cases} 0 & \text{Step} & r_1(t) \\ 0 & \text{ramp} & r_2(t) \\ k_a = \frac{5}{3} & \text{Parabola} & r_3(t) \end{cases}$

$$k_a = \lim_{s\to 0} s^2 L(s) = \lim_{s\to 0} \frac{5(s+1)}{s+3} = \frac{5}{3}$$

سوال ۵:



$r(t) = u(t)$

$$T(s) = \frac{\frac{k}{s^2+6s}}{1+\frac{k}{s^2+6s}} = \frac{k}{s^2+6s+k} = \frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$$

(I) $\begin{cases} \omega_0 = \sqrt{k} \\ \zeta = \frac{3}{\sqrt{k}} \end{cases} \longrightarrow$ 2% setting time $= T_s = \frac{4}{\zeta\omega_0} = \frac{4}{3}$

همان‌طور که دیده می‌شود، T_s اساساً به k ندارد.

هوالرسیم undamped باشد و از میان overdamped و critically damped و overdamped ها ζ کمتر باشد چون که در حالت میرای زیاد (?) te^{-at} داریم. همچنین از آنجا میرای معادلات underdamped و underdamped سریعتر به تعادل می‌رسد ← setting time کمتری دارد.

شرط underdamped $\longrightarrow \zeta = 1 = 3/\sqrt{k} \rightarrow \boxed{k = 9}$

شرط بین overdamped $\Rightarrow 0 < \zeta = \frac{3}{\sqrt{k}} < 1 \longrightarrow \boxed{k > 9} \Rightarrow \arg\min_k T_s(k) = \{9\}$

(II) $\arg\min_k T_s(k) = \{k \mid k > 9\}$

$y(t) \leq u(t)$

اینکه $y(t)$ همواره یک مقدار کمتر از ورودی $u(t)$ باشد، صرفاً در critically damped و overdamped رخ می‌دهد.

ولی شرایط استدلال فوق overdamped بیشتر از critically damped است.

شرط overdamped $\rightarrow 0 < \zeta = \frac{3}{\sqrt{k}} < 1 \longrightarrow k > 9$

(III) $\arg\min_{k\,\infty} T_s(k) = ?$

s.t. $\min\{\int e(t)^2 dt\}$

برای حالت میرای کمتر که overdamped باشد، چون که T_s کمتری دارد نسبت به بقیه حالات، داریم:

$$\forall t \geq 0: e(t) = 1 - y(t) = \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin(\omega_0\sqrt{1-\zeta^2}\,t + \cos^{-1}(\zeta)) \quad , \zeta = \frac{3}{\sqrt{k}}$$

$$\longrightarrow e(t) = \sqrt{\frac{k}{k-9}}\, e^{-3t} \sin\left(\sqrt{k-9}\,t + \cos^{-1}\left(\frac{3}{\sqrt{k}}\right)\right)$$

$$|I(k)| = \left|\int_0^\infty e_k(t)^2 dt\right| \leq \left(\frac{k}{k-9}\right)\int_0^\infty e^{-2\zeta\omega_0 t} dt = \frac{k}{k-9} \times \frac{1}{2\omega_0\zeta} = \frac{k}{6(k-9)}$$

حال مسأله به این تبدیل می‌شود که

$$\arg\min_k \frac{k}{6(k-9)} \text{ s.t. } k > 9 \quad = \quad \arg\min_{k\,\infty} T_s(k) \text{ s.t. } \min\{\int e(t)^2 dt\}$$

و تابع $f(k) = \frac{k}{6(k-9)}$ تنها زمانی min می‌شود که $k \to \infty$ و در این صورت کران بالا برای $|I(k)|$ پیدا می‌شود

$$\arg\min_{\text{s.t. } k>9} \frac{k}{6(k-9)} = \infty \longrightarrow |I(k)| = \lim_{k\to\infty} \frac{k}{6(k-9)} = \frac{1}{6}$$

LCS HW3 Software Assignment

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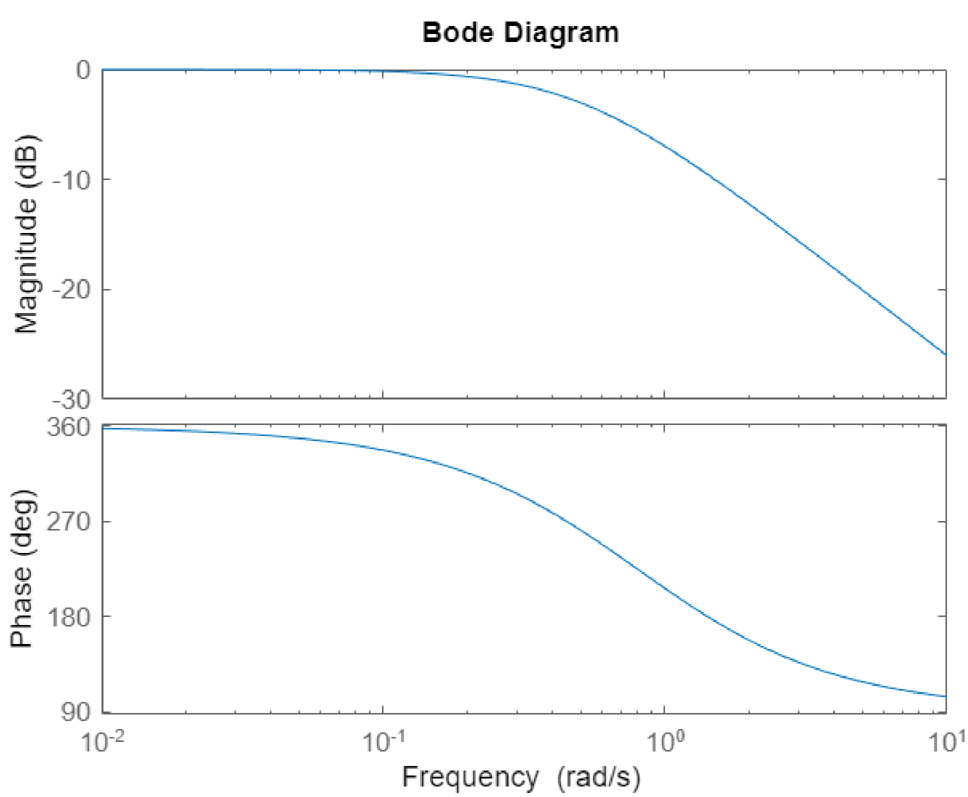
Part 6:

6.1

We are given a transfer function $H(s) = \dfrac{1-s}{(s+1)(2s+1)}$, we wish to show its frequency response first:

```
clear;
clc;
figure();
s = tf('s');
G = (1-s)/((s+1)*(2*s+1));
bode(G);
```



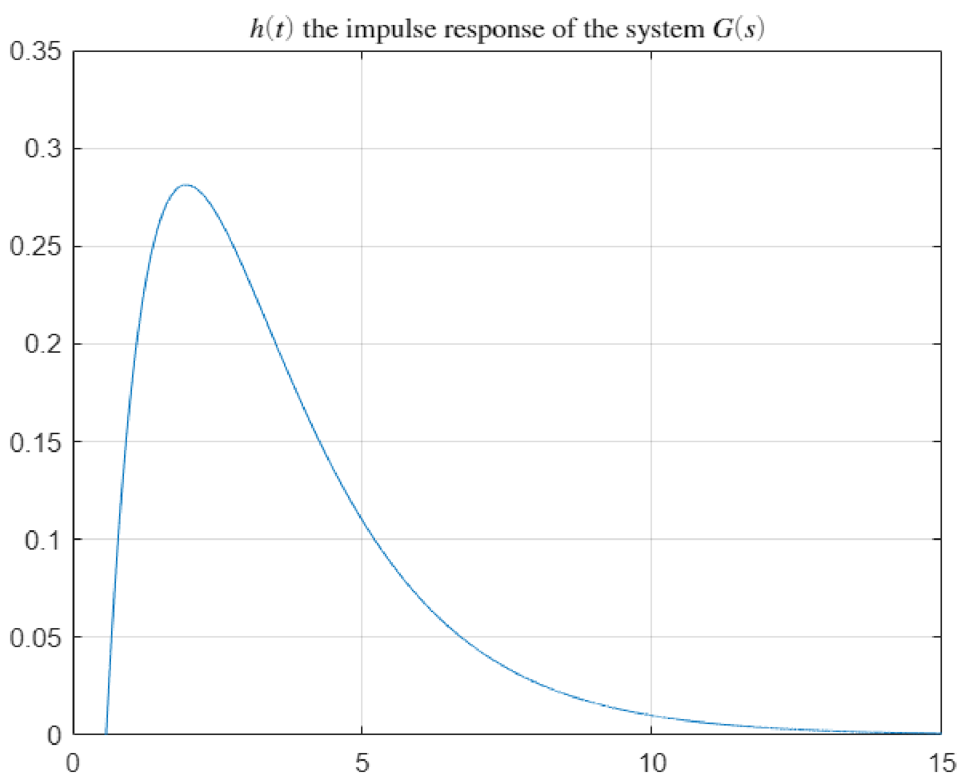
As we can see $G(s)$ is a low pass filter.

Now we want to generate the impulse signal to find the impulse response of the system.

we will define an approximation of Dirac delta function: $f_\epsilon = 1(-\frac{\epsilon}{2} \leq t \leq \frac{\epsilon}{2})$ and we will approximate it as below:

```
% defining the time sample
```

```
dt = 0.001;
epsilon = dt;
t = 0:dt:15;
% defining the input
delta_dirac = zeros(size(t));
delta_dirac(1) = 1/(3*epsilon);
delta_dirac(2) = 1/(3*epsilon);
delta_dirac(3) = 1/(3*epsilon);
% plotting
h_impulse = lsim(G,delta_dirac,t);
figure();
plot(t,h_impulse);
grid on;
title('$h(t)$ the impulse response of the system $G(s)$',Interpreter='latex');
xlim([0,15]);
ylim([0 0.35]);
```

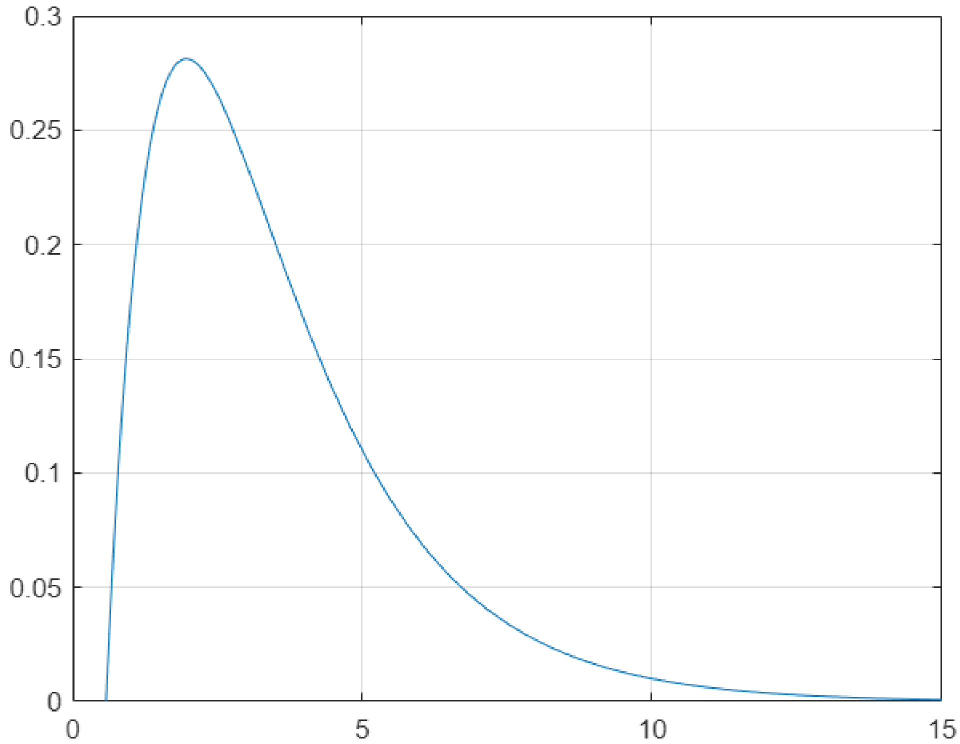


6.2

now we will get the impulse resonse using the ***ilaplace()*** of matlab we will get the inverse Laplace tranform of the transfer function which infact is its time domain impulse response and we will see that the results match!

```
clear;
syms s;
figure();
dt = 0.001;
G = (1-s)/((s+1)*(2*s+1));
fplot(ilaplace(G),[0 15]);
```

```
ylim([0 0.3]);
grid on;
```



6.3

Here we will get the space model representation of the system

```
num = [-1 1];
den = [2 3 1];
syms t;
[A, B, C, D] = tf2ss(num, den)
```

```
A = 2×2
   -1.5000   -0.5000
    1.0000         0
B = 2×1
     1
     0
C = 1×2
   -0.5000    0.5000
D = 0
```

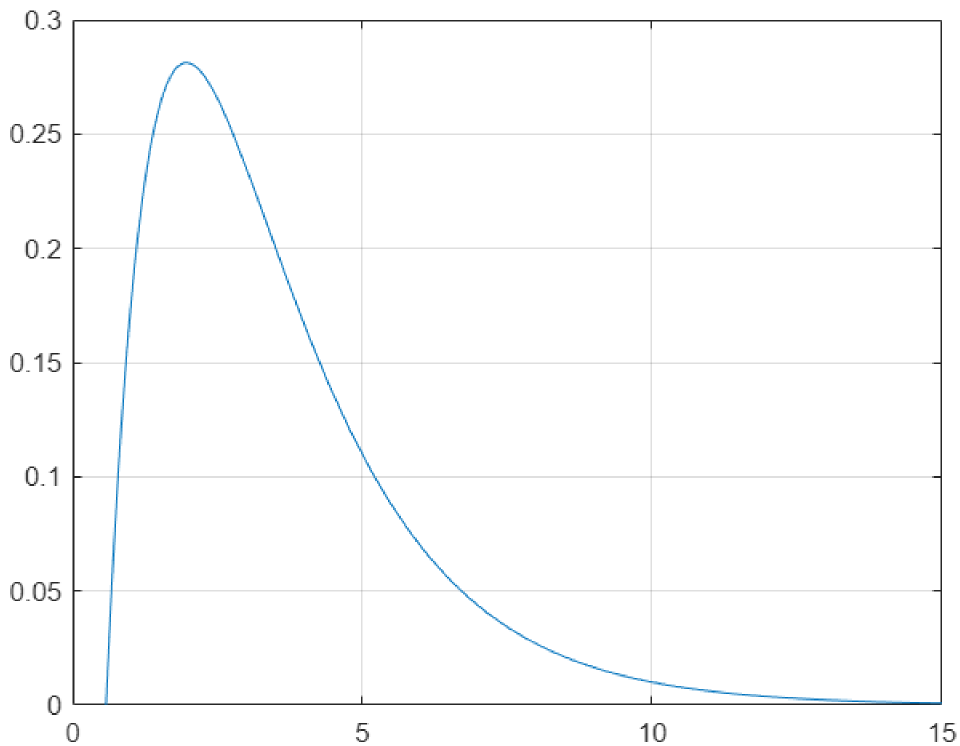
So using these equations we will reach the space model representation of $G(s)$

$\dot{x} = A\,x + B\,u$

$y = C\,x + D\,u$

We can also use $G(s) = C(sI - A)^{-1}B + D$ to plot the impulse response:

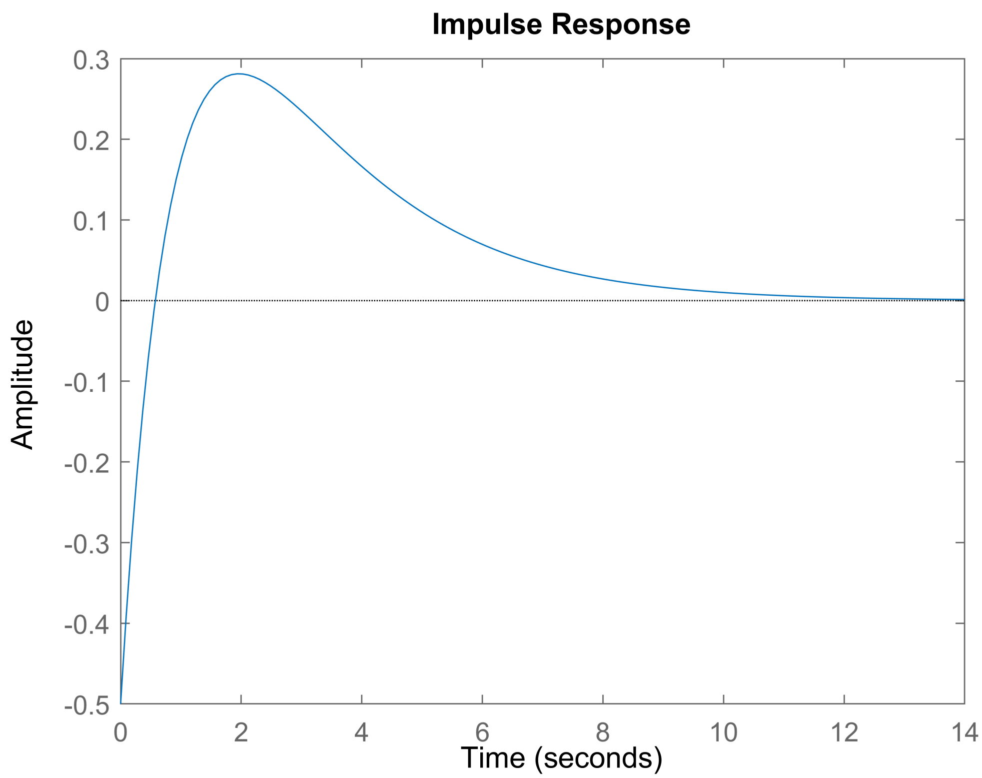
```
figure();
h_ss = ss(tf(num, den));
fplot(h_ss.C * expm((h_ss.A) * t) * h_ss.B + h_ss.D, [0 15] );
ylim([0 0.3]);
grid on;
```



6.4

This is the result of getting the impulse response with ***impulse()*** function.

```
clear;
figure();
s = tf('s');
G = (1-s)/((s+1)*(2*s+1));
impulse(G)
```



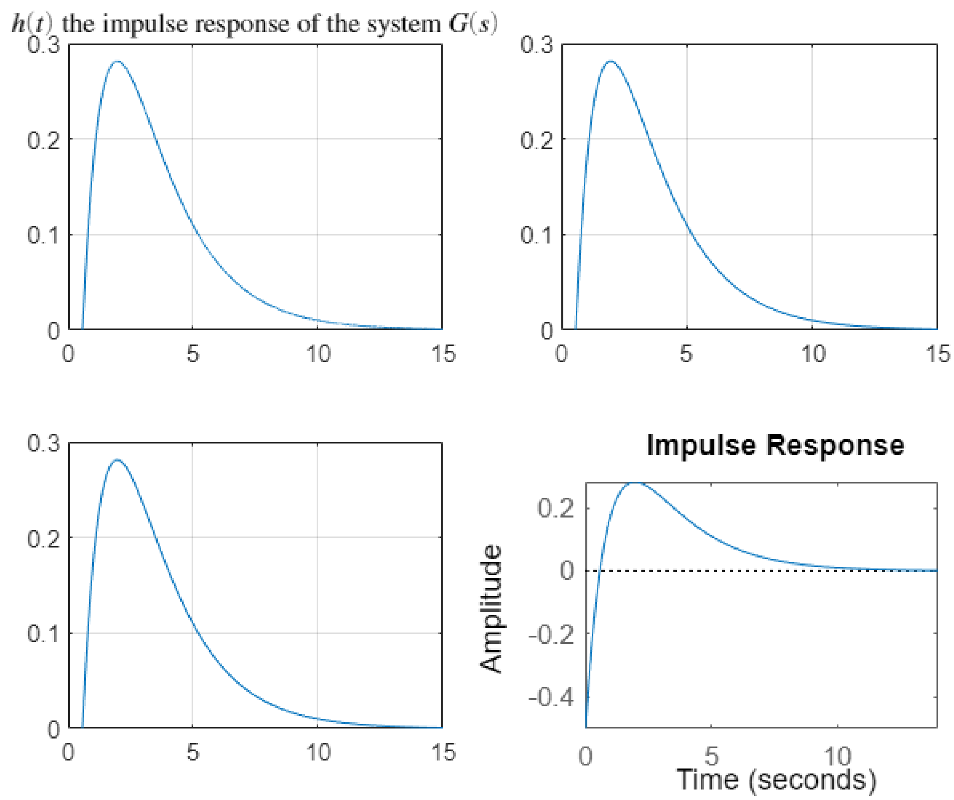
6.5

```
figure();
subplot(2,2,1);
s = tf('s');
G = (1-s)/((s+1)*(2*s+1));
dt = 0.001;
epsilon = dt;
t = 0:dt:15;
delta_dirac = zeros(size(t));
delta_dirac(1) = 1/(3*epsilon);
delta_dirac(2) = 1/(3*epsilon);
delta_dirac(3) = 1/(3*epsilon);
h_impulse = lsim(G,delta_dirac,t);
plot(t,h_impulse);
grid on;
title('$h(t)$ the impulse response of the system $G(s)$',Interpreter='latex');
xlim([0,15]);
ylim([0 0.3]);

subplot(2,2,2);
syms s;
dt = 0.001;
G = (1-s)/((s+1)*(2*s+1));
fplot(ilaplace(G),[0 15]);
ylim([0 0.3]);
grid on;
```

```
subplot(2,2,3);
syms t;
num = [-1 1];
den = [2 3 1];
h_ss = ss(tf(num, den));
fplot(h_ss.C * expm((h_ss.A) * t) * h_ss.B + h_ss.D, [0 15] );
ylim([0 0.3]);
grid on;

subplot(2,2,4);
s = tf('s');
G = (1-s)/((s+1)*(2*s+1));
impulse(G);
```



It's crystal clear that all these plots are showing the impulse response of $G(s)$

As expected, e^{At} matrices calculated from those ways are identical!

```
clear;
num = [-1 1];
den = [2 3 1];
syms s t;
h_ss = ss(tf(num, den));
G = inv((s*eye(2,2) - h_ss.A));
exp_At = ilaplace(G);
exp_At_ = expm(h_ss.A * t);
```

```
exp_At
```

exp_At =

$$\begin{pmatrix} 2\,e^{-t} - e^{-\frac{t}{2}} & e^{-t} - e^{-\frac{t}{2}} \\ 2\,e^{-\frac{t}{2}} - 2\,e^{-t} & 2\,e^{-\frac{t}{2}} - e^{-t} \end{pmatrix}$$

```
exp_At_
```

exp_At_ =

$$\begin{pmatrix} 2\,e^{-t} - e^{-\frac{t}{2}} & e^{-t} - e^{-\frac{t}{2}} \\ 2\,e^{-\frac{t}{2}} - 2\,e^{-t} & 2\,e^{-\frac{t}{2}} - e^{-t} \end{pmatrix}$$

```
G
```

G =

$$\begin{pmatrix} \frac{2\,s}{2\,s^2 + 3\,s + 1} & -\frac{1}{2\,s^2 + 3\,s + 1} \\ \frac{2}{2\,s^2 + 3\,s + 1} & \frac{2\,s + 3}{2\,s^2 + 3\,s + 1} \end{pmatrix}$$

LCS Software Assignment HW 3

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Part 7:

7.1, 7.2:

Yes, the system is LTI since we can simplify the given equations as:

$$\begin{pmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{1}{m} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} u(t) = AX(t) + Bu(t)$$

$$y(t) = (1 \quad 0) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} u(t) = CX(t) + Du(t)$$

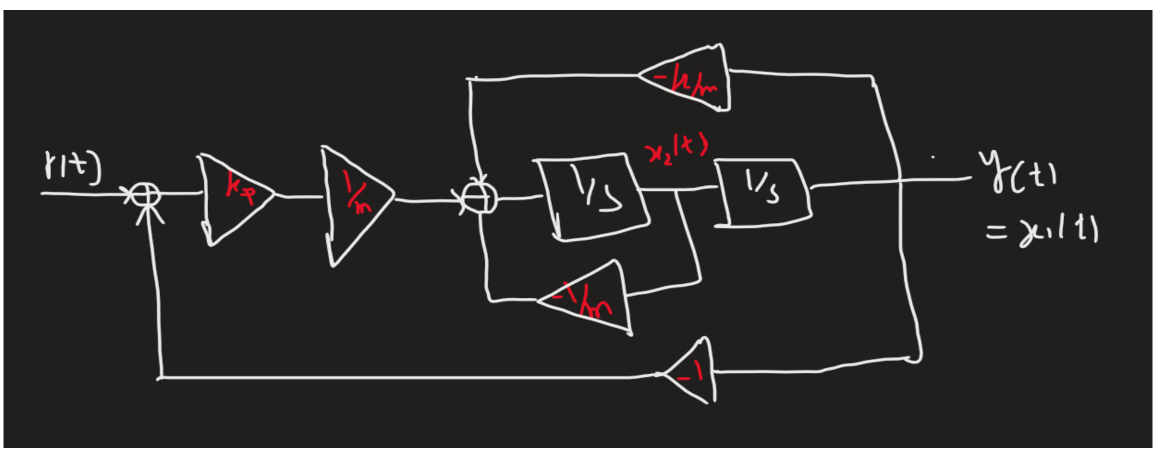
7.3:

$$(sI - A)^{-1} = \begin{pmatrix} s & -1 \\ \frac{k}{m} & s + \frac{1}{m} \end{pmatrix}^{-1} = \frac{1}{s^2 + \frac{s}{m} + \frac{k}{m}} \begin{pmatrix} s + \frac{1}{m} & 1 \\ -\frac{k}{m} & s \end{pmatrix}$$

$$G(s) = C\,(sI - A)^{-1}B + D = \frac{1}{ms^2 + s + k} = \frac{\frac{1}{m}}{s^2 + \frac{1}{m}s + \frac{k}{m}}$$

7.4:

Here is the diagram of the closed-loop system



When r(t) and x_1 (t) add up together, then amplified by k_p, then We get u(t) :

$$u(t) = k_p(r(t) - x_1(t))$$

7.5:

Here are the state-space representation of the closed-loop system:

$$\dot{x_1}(t) = x_2(t)$$

$$\dot{x}_2(t) = \frac{k_p}{m}u(t) - \left(\frac{k_p}{m} - \frac{k}{m}\right)x_1(t) - \frac{1}{m}x_2(t)$$

$$y(t) = x_1(t)$$

7.6 , 7.7:

Here is the closed-loop transfer function:

$$H(s) = \frac{Y(s)}{R(s)} = \frac{k_p G(s)}{k_p G(s) + 1} = \frac{k_p}{ms^2 + s + k + k_p} = \frac{4k_p}{s^2 + 4s + (1 + 4k_p)}$$

$$s_{1,2} = \frac{-4 \pm \sqrt{16 - 4 - 16k_p}}{2} = -2 \pm \surd(3 - 4k_p)$$

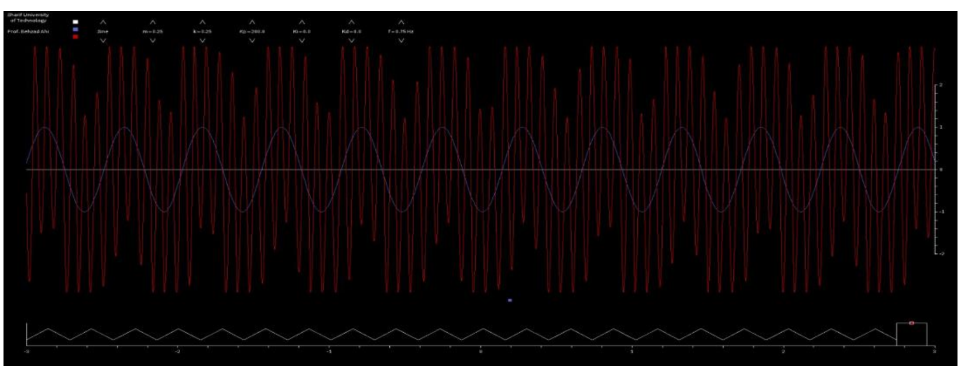
The Roots must be on the left side of jw axis, therefore:

If $3 - 4k_p < 0$, then the real part of s_1,2 will be negative which is sufficient. Now we assume the otherwise is true $(3 - 4k_p > 0)$. Then, we get $k_p > \frac{3}{4}$ && $k_p > -\frac{1}{4}$. So, eventually we get :

$$-\frac{1}{4} \leq k_p$$

7.8

Letting m = 0.25, k = 0.25 and Kp = 200

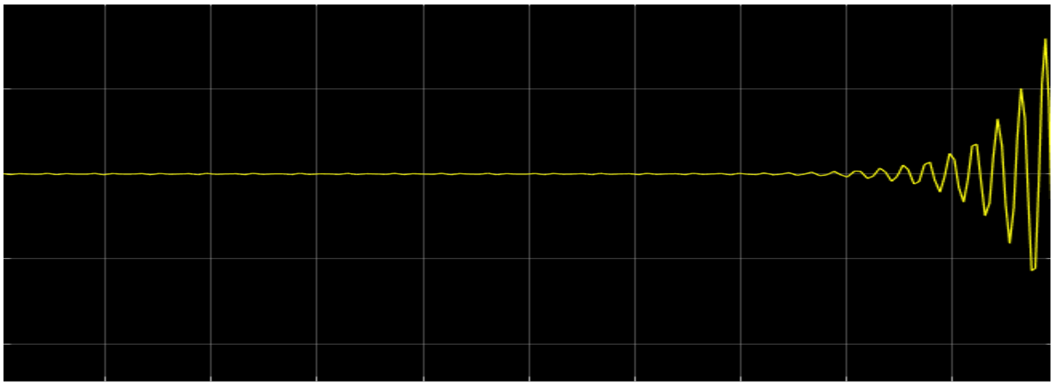


As we can see, the output is stable. This is due to the fact that the poles of H(s) have a negative real part.

7.9

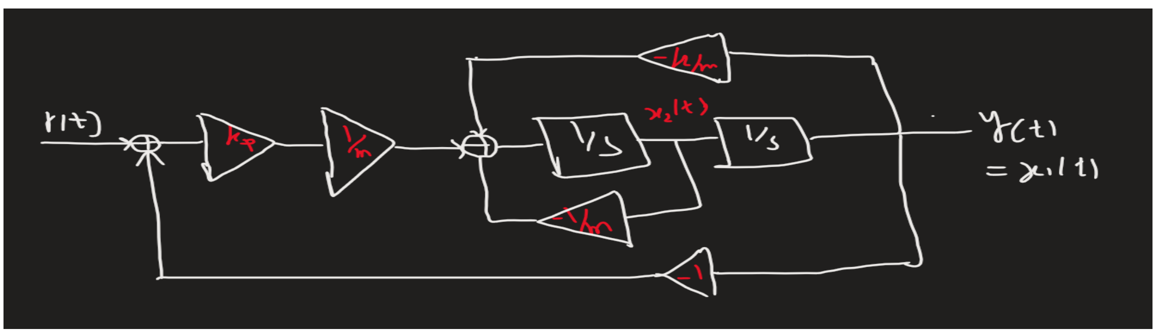
The system per se is unstable as we can see in the figure the output starts to rise monotonically. But using the feedback in next part we will stabilize the system.

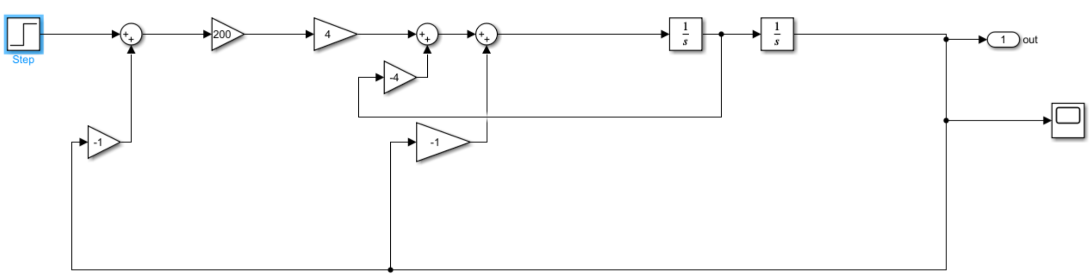
Ts = 0.01 / m = 0.25 / k =0.25 / k_ p = 200



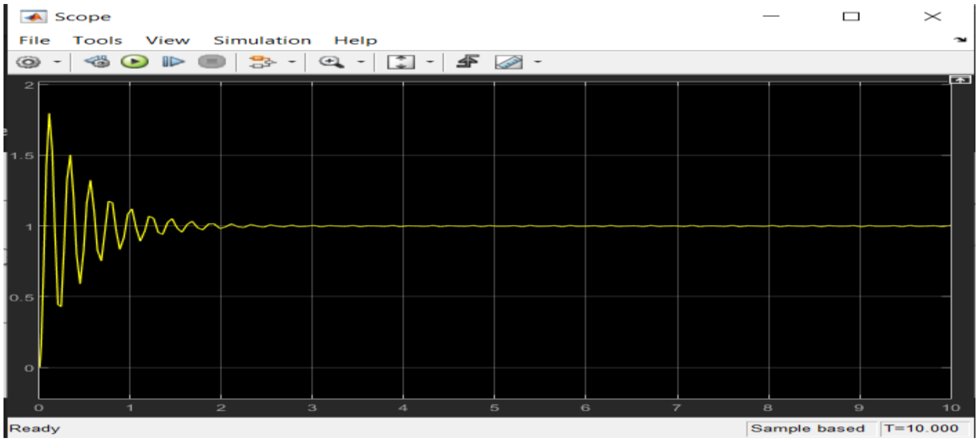
7.10

Here we implemented the system and simulated it:





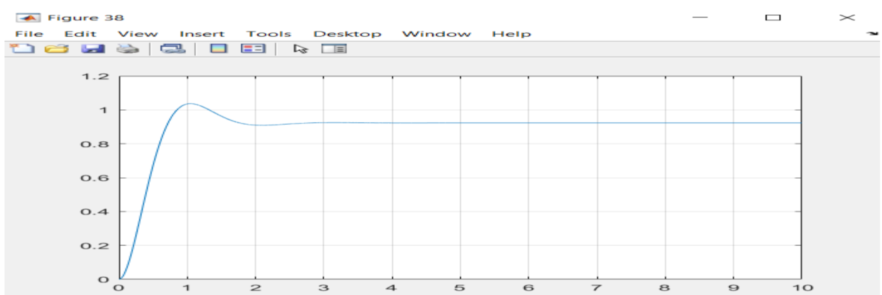
The output to step input will look like this :



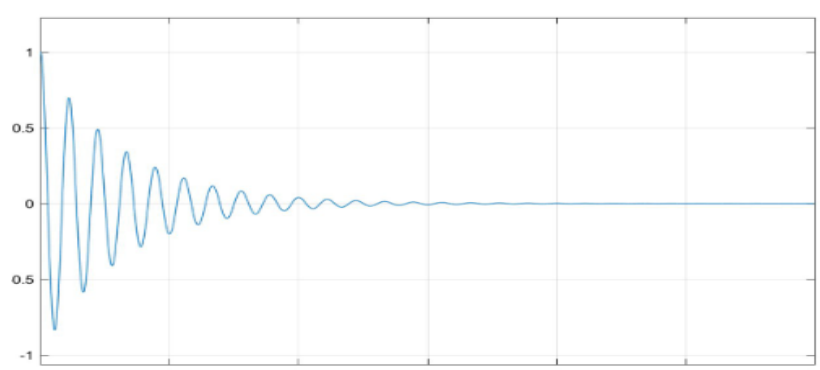
Using the given time step and the solver in the instruction and the results, we can deduce that the system is stable, since by getting bounded input we got bunded output and furthermore the output converges to zero as time passes. The feedback indeed stabilized the system!

7.11:

Letting m=0.25, k=0.25 and k_p = 3: => steady state error = 0.89

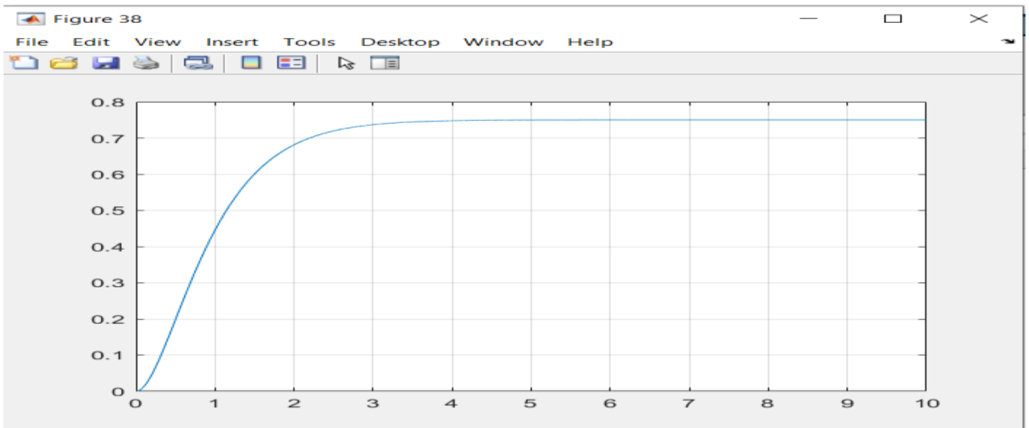


Letting m=0.25, k=0.25 and k_p = 200: => steady state error = 0.99



7.12:

We know that $H(s) = \frac{4k_p}{s^2+4s+(1+4k_p)} = 4k_p(\frac{1}{s^2+2\zeta\omega_0 s+\omega_0^2})$. If we want the system to show critically damped behavior, then we must have $\zeta = 1$which results in$\zeta = \frac{2}{\sqrt{1+4k_p}} = 1 \rightarrow k_p = \frac{3}{4}$.

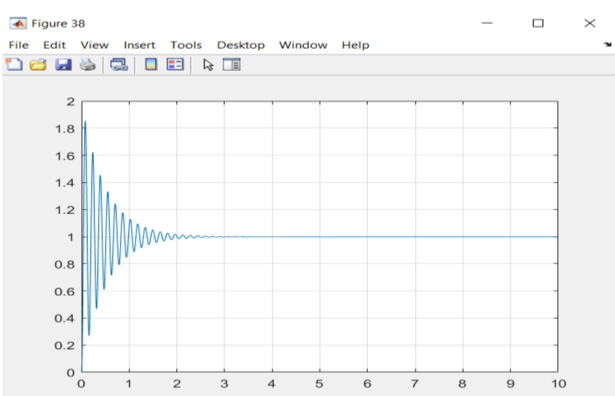


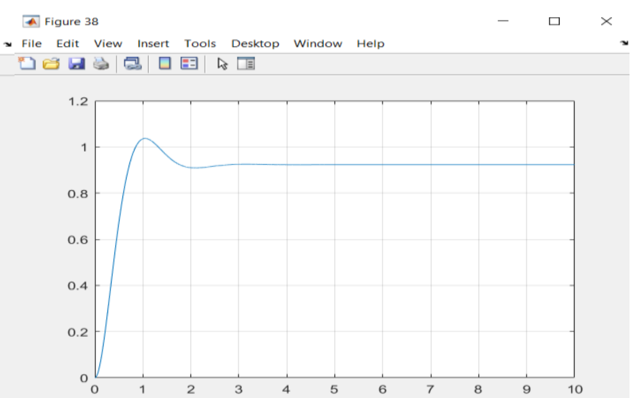
In this case, the step response is depicted above. Our output is $Y(s) = \frac{1}{s(s+2)\char94 2}$which translates to

$y(t) = \frac{3}{4} - \frac{3}{2}te^{-2t} - \frac{3}{4}e^{-2t}$ in time domain.

7.13:

Among all systems, underdamped is said to have the least setting time, as a result we need $1 > \zeta > 0$. Thus, $0 < \frac{2}{\sqrt{1+4k_p}} < 1 \rightarrow k_p > \frac{3}{4}$. In addition to this the setting time will be $T_s = \frac{4}{\zeta\omega_0} = 2$s. for each k_p in interval [3/4 , infinity] this will work! The results are depicted for k_p = 400 and k_p = 1:





So we will chose smaller k's in order to avoid high frequency oscillations and obtain less overshoot percentage.

7.14:

Since $H(s) = \frac{G(s)}{G(s)+1} = \frac{1}{ms^2+s+(k+1)}$, in order to achieve a pure oscillation, we should have $H(s) = \frac{f_v \omega_0^2}{s^2+\omega_0^2}$ which clearly can not happen!

Part 8:

8.1:

Here are the state-space representation of the open-loop system:

$$\begin{pmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \\ \dot{x_4}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{2k}{m} & -\frac{1}{m} & \frac{2k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2k}{M} & 0 & -\frac{2k}{M} & -\frac{1}{M} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M} \end{pmatrix} u(t) = AX\,(t) + Bu(t)$$

$$y_1(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix} + (0)u(t) = C_1X(t) + D_1u(t)$$

$$y_2(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix} + (0)u(t) = C_2X(t) + D_2u(t)$$

8.2:

Here are the open-loop transfer functions:

$$G_1(s) = C_1(sI - A)^{-1}B_1 + D_1 = \frac{2k}{Mms^4 + (m+M)s^3 + 2k(m+M+1)s^2 + 4ks}$$

$$G_2(s) = C_2(sI - A)^{-1}B_2 + D_2 = \frac{2k}{Mms^3 + (m+M)s^2 + 2k(m+M+1)s^1 + 4ks}$$

8.3:

Here are the closed-loop transfer functions:

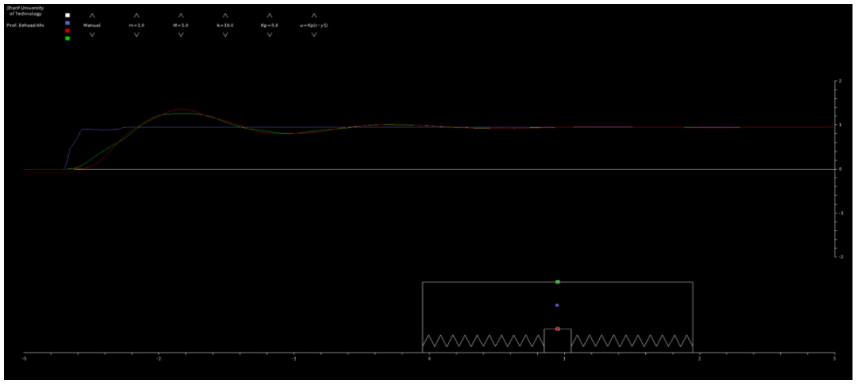
$$T_1(s) = \frac{k_pG_1(s)}{1 + k_pG_1(s)} = \frac{2k\ k\_p}{Mms^4 + (m+M)s^3 + 2k(m+M+1)s^2 + 4ks + 2k\ k_p}$$

$$T_2(s) = \frac{k_pG_2(s)}{1 + k_pG_2(s)} = \frac{2k\ k\_p}{Mms^3 + (m+M)s^2 + 2k(m+M+1)s + (4k + 2k\ k_p)}$$

8.4:

M = 1 / m = 1 / k=10 / k_p = 5

Steady state error = 0



M = 2 / m = 1 / k= 5 / k_p = 1

Steady state error = 0