Neuroscience of Learning, Memory, Cognition
HW 2 (Solution) Part I

Mohammad Parsa Dini – Student ID: 400101204

1. Linear Filter Model of a Synapse

We assume that $\rho_b(t) = \sum_i \delta(t - t_i)$ is the spike-train signal, $K(t) = e^{-\tau/\tau_s}$ is the exponential decay signals and $P_s(t) = \sum_{t>t_i} K(t - t_i)$ is the probability of that postsynaptic ion channels are open at time t.

1.1 Background and Introduction

Given the spike train times $\mathcal{T} = \{1ms, 3ms, 7ms, 9ms, 18ms\}$, we can model as the firing-rate as

$$\rho_b(t) = \sum_{\tau \in \mathcal{T}} \delta(t - \tau) = \delta(t - 0.001) + \delta(t - 0.003) + \delta(t - 0.007) + \delta(t - 0.009) + \delta(t - 0.018)$$

Now, given $\tau_s = 5ms$, we can compute $P_S(10^{-2})$ as:

$$P_s(10ms) = K(1ms) + K(3ms) + K(7ms) + K(9ms) = e^{-\frac{1}{5}} + e^{-\frac{3}{5}} + e^{-\frac{5}{5}} + e^{-\frac{9}{5}} \approx 1.7794$$

We can easily see that **higher** τ_s means slower decay of postsynaptic response and therefore, longer-lasting influence of each spike, temporal summation is more significant. However, **lower** τ_s means faster decay and thus each spike's effect is brief, less summation.

1.2 Shifting property and convolution interpretation

Assuming that the spiking train $\rho_b(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$ passes through the kernel $h(t) = e^{-t}u(t)$, the filtered output will be:

$$y(t) = h(t) * \rho_b(t) = h(t-1) + h(t-2) + h(t-3) = e^{-t+1}u(t-1) + e^{-t+2}u(t-2) + e^{-t+3}u(t-3)$$

1.3 Calculating the system's response to a spike train

We have the total synaptic input current as $I_s(t) = \sum_{b=1}^{n} w_b \int_{-\infty}^{t} K(t-\tau)\rho_b(t)$, since $\rho_b(t) = \sum_i \delta(t - t_i^{(b)})$, we obtain that:

$$I_s(t) = \sum_{b=1}^{n} w_b \sum_{t_i^{(b)}<t} K(t - t_i^{(b)})$$

where $K(t) = e^{-t/}$ and $= 5ms$. We have two synapses synapse1 and synapse2 who spike at times $\mathcal{T}_1 = \{1ms, 6ms\}$ and $\mathcal{T}_2 = \{2ms, 8ms, 11ms\}$ with synaptic weights $w_1 = 1$ and $w_2 = 0.5$, respectively.

Now we will compute total input current at $t = 15ms$. Let I_1, I_2 be the Synapse 1 and 2 contributions at $t = 15ms$:

$$I_1 = w_1(K(14\text{ms}) + K(9\text{ms})) = 1 \times (e^{-1.8} + e^{-2.8}) = 0.2261095$$
$$I_2 = w_2(K(13\text{ms}) + K(7\text{ms}) + K(4\text{ms})) = 0.2 \times (e^{-2.6} + e^{-1.4} + e^{-0.8}) = 0.3850998$$
$$I_s = I_1 + I_2 = 0.2261095 + 0.3850998 = 0.6112093 \approx 0.611$$

Here is the code of the simulation of the spikes:

```
% --------------- Simulation of Spikes -----------------
tau_s = 5.0;
t_vals = linspace(0, 25, 1000);

spikes1 = [1, 6];
spikes2 = [2, 8, 11];

% Define the P_b function
function result = P_b(t, spikes, tau_s)
    result = zeros(size(t));
    for i = 1:length(spikes)
        spike_time = spikes(i);
        result = result + (t > spike_time) .* exp(-(t - spike_time) /
            tau_s);
    end
end

% Calculate postsynaptic responses
P1 = P_b(t_vals, spikes1, tau_s);
P2 = P_b(t_vals, spikes2, tau_s);
Is = P1 + 0.5 * P2;
```

In the next page, you can see the result of the simulation where we can clearly see that after each neuron spikes, the $I_s(t)$ jumps a little bit with respect to the weights and then it would continue its decay. Here the graphs of the current $I_s(t)$ and post-synaptic responses are depicted:

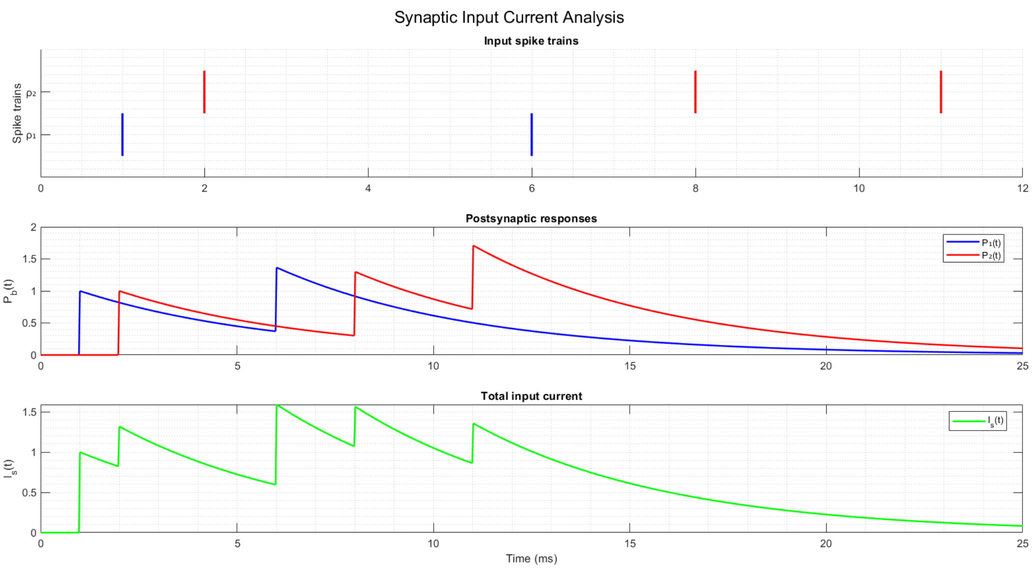

Figure 1: total synaptic input current $I_s(t)$ over time.