0- Image Processing

0-1- Re-segmentation [1]

Re-segmentation updates the ROI mask R using the corresponding voxel intensities X_{gl}. It is applied after interpolation and is typically used to exclude voxels from an already segmented ROI; for example, removing air or bone voxels in CT. Two re-segmentation methods are described below. When methods are combined, the intersection of the intensity ranges defined by those methods is taken.

Intensity and Morphological Masks of an ROI

Although an ROI is conventionally represented by a single mask, re-segmentation can remove internal voxels or split the ROI into sub-volumes. To avoid repeatedly modifying the re-segmented ROI to enforce a plausible morphology, we define two masks:

- The morphological mask (G5KJ) is not re-segmented; it preserves the original morphology as determined by expert and/or (semi-)automatic segmentation algorithms.
- The intensity mask (SEFI) can be re-segmented and retains only the selected voxels. Many feature families use only this mask, whereas morphological and gray level distance zone matrix (GLDZM) features rely on both masks.

Range re-segmentation

Range-based re-segmentation removes from the intensity mask those voxels whose intensities lie outside a specified range. Examples include excluding HU corresponding to air or bone in CT tumour ROIs, or omitting low-activity regions in PET. Included-voxel ranges are typically given as a closed interval [ab] or a half-open interval [a). For arbitrary intensity units (e.g., raw MRI, uncalibrated microscopy images, and many spatial filters), no re-segmentation range can be specified.

When a user defines a re-segmentation range, it must be propagated and used wherever a specified intensity range is required (e.g., intensity-volume histogram features) and/or when employing fixed bin size discretisation. Recommendations for combining imaging intensity definitions, re-segmentation ranges, and discretisation algorithms are given in Table 0-1.

Intensity Outlier Filtering

Voxels in the ROI with outlier intensities may be removed from the intensity mask. According to IBSI [1], one approach computes the mean and standard deviation of gray levels over ROI voxels and then excludes voxels outside the range $[\mu - 3\sigma, \mu + 3\sigma]$.

Table 0-1: Recommendations for pairing imaging intensity definitions, re-segmentation ranges, and discretisation algorithms. A checkmark (✔) indicates a recommended combination of re-segmentation range and discretisation method; a cross (✗) indicates a combination that is not recommended. (1) PET and CT are modalities with calibrated intensity units (e.g., SUV and HU), whereas raw MRI has arbitrary intensity units. (2) Fixed bin number (FBN) discretisation uses the actual intensity range observed in the analysed ROI (whether re-segmented or not), not the user-defined re-segmentation range. (3) Fixed bin size (FBS) discretisation sets the minimum to the lower bound of the re-segmentation range; if no re-segmentation range is defined or cannot be specified, the minimum is taken from the observed ROI intensity minimum.

Image Intensity Units (1)	**Re-segmentation Range**	**FBN** (2)	**FBS** (3)
Calibrated	$[a, b]$	✔	✔
	$[a, \infty)$	✔	✔
	None	✔	✗
Arbitrary	None	✔	✗

0-2- Intensity Discretisation [1]

Inside the ROI, discretisation (or quantisation) of image intensities is often applied to make the calculation of texture features tractable, and it also has noise-suppressing effects.
Two schemes are commonly used: one discretises to a fixed number of bins, the other uses a fixed bin width. There is no intrinsic preference for either approach; each has particular characteristics (see below) that can make it more suitable for specific aims. Note that the lowest bin is always 1 (not 0), to keep texture-feature calculations consistent where gray level 0 is not allowed.

Fixed Bin Number

In the fixed bin number approach, intensities X_{gl} are mapped to a fixed N_g bins. It is defined as:

$$X_{d,k} = \begin{cases} \left\lfloor N_g \dfrac{X_{gl,k} - X_{gl,min}}{X_{gl,max} - X_{gl,min}} \right\rfloor + 1 & X_{gl,k} < X_{gl,max} \\ N_g & X_{gl,k} = X_{gl,max} \end{cases}$$

In short, the intensity $X_{gl,k}$ of voxel k is corrected by the lowest occurring intensity $X_{gl\,,min}$ in the ROI, divided by the bin width $\left(X_{gl\,,max} - X_{gl\,,min}\right)/N_g$, and then rounded down to the nearest integer.
This method breaks the relationship between image intensity and physiological meaning (if any). However, it introduces a normalising effect that can be beneficial when intensity units are arbitrary (e.g. raw MRI data and many spatial filters) and where contrast is important. In addition, because many feature values depend on the number of gray levels present in an ROI, using a fixed bin number enables direct comparison of feature values across multiple ROIs (e.g. across different samples).

Fixed Bin Size

Fixed bin size discretisation assigns a new bin to every intensity interval of width w_b, starting at a minimum $X_{gl\,,min}$. The minimum may be user-set (the lower bound of the re-segmentation range) or data-driven (the minimum intensity in the ROI, i.e. $X_{gl\,,min} = min(X_{gl}\,)$. In all cases, the method used and/or the set minimum value must be clearly reported. To maintain consistency between samples, it is strongly recommended to always set the same minimum value for all samples as defined by the lower bound of the re-segmentation range (e.g. HU of −500 for CT, SUV of 0 for PET, etc.). When no re-segmentation range can be defined due to arbitrary intensity units (e.g. raw MRI data and many spatial filters), use of the fixed bin size discretisation algorithm is not recommended.
A key advantage of the fixed bin size approach is that it maintains a direct relationship with the original intensity scale, which can be useful for functional imaging modalities such as PET. Discretised intensities are computed as:

$$X_{d,k} = floor\left(\frac{X_{gl,k} - X_{gl,min}}{w_b}\right) + 1$$

Other Methods

Other discretisation methods and variations exist but are not detailed here. Reference [2] described intensity histogram equalisation and Lloyd-Max algorithms for discretisation. Histogram equalisation redistributes intensities so that the resulting bins contain a similar number of voxels, i.e. contrast is increased by flattening the histogram as much as possible. Equalisation of the ROI intensities can be performed before any other discretisation algorithm (e.g. FBN, FBS, etc.), and it also requires defining the number of bins in the histogram to be equalised. The Lloyd-Max algorithm is an iterative clustering method that seeks to minimize mean-squared discretisation errors.

Recommendations

The discretisation approach that yields optimal inter- and intra-sample reproducibility is modality-dependent. Usage recommendations for combinations of imaging intensity definitions, re-segmentation ranges, and discretisation algorithms are provided in Table 0.1. Overall, the choice of discretisation has a substantial impact on intensity distributions, feature values, and reproducibility.

1- Morphological Features

1-1- Introduction [1]

Morphological features quantify the geometric properties of a region of interest (ROI), such as volume, surface area, or shape descriptors. They are derived from voxel representations of the ROI and provide information about both size and shape.

ROI Representations

Three voxel representations of the ROI volume are possible:

1. **Voxel volume representation:** the ROI is treated as a collection of voxels, each with its defined volume. This representation is not commonly used, as it poorly handles partial volume effects at the ROI boundaries and may cause inconsistencies when mixing representations.
2. **Voxel point set representation:** the ROI is represented by the coordinates of voxel centers. This representation is used when the internal structure of the ROI is important.
3. **Surface mesh representation:** the ROI boundary is represented as a surface mesh. This representation is used when the outer surface geometry is of interest.

Mesh-Based Representation

A mesh-based surface representation provides consistent evaluation of surface area and volume. Voxel-based methods tend to overestimate surface area due to partial volume effects. The surface is constructed into a triangle mesh using a meshing algorithm.
The Marching Cubes algorithm is recommended due to its wide availability, efficiency, and reasonable accuracy. Different algorithms may produce small differences in feature values.
A triangle mesh consists of faces (triangles) and vertices. For a triangle face spanned by vertices a, b, and c, the edges are defined as $ab = b - a$, $bc = c - b$, and $ca = a - c$. The face normal n is given by the cross product:

$$n = \frac{(ab \times bc)}{\left|\left|ab \times bc\right|\right|}$$

Meshes are constructed from the ROI voxel point set X_c. For Marching Cubes, the default iso-level is 0.5, corresponding to the midpoint between voxel centers. Correct padding of the ROI mask may be required to ensure proper meshing at the boundaries.
The mesh contains N_{fc} triangular faces and N_{vx} vertices, forming the vertex set X_{vx}. Consistency of face normal orientation is necessary for accurate volume calculation. In a closed mesh, each edge is shared by exactly two faces. Fig. 1-1 shows an example triangle face.

ROI Morphological and Intensity Masks

The ROI consists of two primary masks:

- The morphological mask defines the structure of the ROI and is used to generate X_c for morphological features. Holes inside the mask are considered intentional (results of segmentation).
- The intensity mask defines the voxel intensities within the ROI and is used to generate X_{gl} and the corresponding point set $X_{c,gl}$.

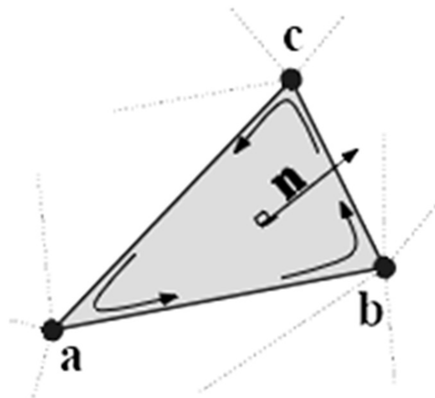

Fig. 1-1: An example triangle face. [1]

Aggregation of Morphological Features

Morphological features are always computed in 3D. They are not aggregated slice-by-slice.

Units of Measurement

Morphological features are expressed in physical units of length as defined in the DICOM standard (millimeters for most imaging modalities).

- If another unit of length is explicitly defined in the metadata, that unit should be used, but consistency across the dataset must be ensured.
- Conversion to other units (e.g. cm) should be performed only after the feature calculation.

1-2- Explanation of Features [1]

1-2-1- Volume (Mesh) (morph_volume)

Compute the mesh-based volume V from the ROI surface mesh. For each triangular face k, form a tetrahedron with the origin $(0, 0, 0)$; its signed volume is:

$$V_k = \frac{a.(b \times c)}{6}$$

Where a, b and c are the coordinates of the face vertices. The sign depends on the orientation of the face normal; therefore, all face normals must have a consistent orientation (all outward or all inward). The final ROI volume is:

$$F_{mor\quad.vol} = V \;= \left| \sum_{k=1}^{N_{f\,c}} \quad V_k \right|$$

Where N_{fc} is the total number of faces in the mesh.

Note: In positron emission tomography, the ROI volume is often named according to the tracer (e.g., metabolically active tumour volume (MATV) for 18F-FDG).

Table 1-1: Reference values for the volume (mesh) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	556	4	Very strong
Config. A	3.58×10^5	5×10^3	Very strong
Config. B	3.58×10^5	5×10^3	Strong
Config. C	3.67×10^5	6×10^3	Strong
Config. D	3.67×10^5	6×10^3	Strong
Config. E	3.67×10^5	6×10^3	Strong

1-2-2- Volume (Voxel Counting) (morph_vol_approx)

Voxel counting volume is defined as:

$$F_{morph.approx.vol} \;= \sum_{K=1}^{N_v} \quad V_k$$

Where N_v is the number of voxels in the morphological mask and V_k the physical volume of voxel k.

Note: In clinical workflows, volume is often estimated by counting voxels. For ROIs with a large number of voxels (thousands), voxel counting and mesh-based volume estimates typically agree closely. For small ROIs (tens to hundreds of voxels), voxel counting tends to overestimate volume relative to the mesh method. Consequently, it is used only as a reference feature and not to derive other morphological features.

Table 1-2: Reference values for the volume (voxel counting) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	592	4	Very strong
Config. A	3.59×10^5	5×10^3	Strong
Config. B	3.58×10^5	5×10^3	Strong
Config. C	3.68×10^5	6×10^3	Strong
Config. D	3.68×10^5	6×10^3	Strong
Config. E	3.68×10^5	6×10^3	Strong

1-2-3- Surface Area (Mesh) (morph_area_mesh)

The mesh surface area A is computed by summing triangular face areas. For a face k:
The surface area A is also calculated from the ROI mesh by summing over the triangular face surface areas [46]. By definition, the area of face k is:

$$A_k = \frac{|ab \times ac|}{2}$$

Where $ab = b - a$, and $ac = c - a$ (see Fig 1-1). The total surface area A is then:

$$F_{morph.area} = A = \sum_{k=1}^{N_{fc}} A_k$$

Table 1-3: Reference values for the surface area (mesh) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	592	3	Very strong
Config. A	3.57×10^4	300	Strong
Config. B	3.37×10^4	300	Strong
Config. C	3.43×10^4	400	Strong
Config. D	3.43×10^4	400	Strong
Config. E	3.43×10^4	400	Strong

1-2-4- Surface To Volume Ratio (morph_av)

This feature is calculated by:

$$F_{morph.av} = \frac{A}{V}$$

Note: This feature is not dimensionless.

Table 1-4: Referencevaluesforthesurfacetovolumeratio feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.698	0.004	Very strong
Config. A	0.0996	0.0005	Strong
Config. B	0.0944	0.0005	Strong
Config. C	0.0934	0.0007	Strong
Config. D	0.0934	0.0007	Strong
Config. E	0.0934	0.0007	Strong

1-2-5- Compactness 1 (morph_comp_1)

A group of related measures—Compactness 1 and 2, Spherical disproportion, Sphericity, and Asphericity—quantify how closely the ROI resembles a spheroid. These measures can be derived from one another and may therefore be highly correlated (potential redundancy). Compactness 1 is defined as:

$$F_{morph.comp.1} = \frac{V}{\pi^{\frac{1}{2}} A^{\frac{3}{2}}}$$

Table 1-5: Reference values for the compactness1 feature. An unset value(__) indicates the lack of a reference value. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.0411	0.0003	Strong
Config. A	0.03	0.0001	Strong
Config. B	0.0326	0.0001	Strong
Config. C	__	__	Moderate
Config. D	0.0326	0.0002	Strong
Config. E	0.0326	0.0002	Strong

1-2-6- Compactness 2 (morph_comp_2)

This feature is defined as:

$$F_{morph.comp.2} = 36\pi \frac{V^2}{A^3}$$

Note: By definition $F_{morph.comp.1} = \frac{1}{6\pi(F_{morph.comp.2})^{\frac{1}{2}}}$.

Table 1-6: Reference values for the compactness 2 feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.599	0.004	Strong
Config. A	0.319	0.001	Strong
Config. B	0.377	0.001	Strong
Config. C	0.378	0.004	Strong
Config. D	0.378	0.004	Strong
Config. E	0.378	0.004	Strong

1-2-7- Spherical Disproportion (morph_sph_dispr)

This feature is defined as:

$$F_{morph.sph.dispr} = \frac{A}{4\pi R^2} = \frac{A}{(36\pi V^2)^{\frac{1}{3}}}$$

Note: By definition $F_{morph.sph.dispr} = (F_{morph.comp.2})^{-\frac{1}{3}}$.

Table 1-7: Reference values for the spherical disproportion feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.19	0.01	Strong
Config. A	1.46	0.01	Strong
Config. B	1.38	0.01	Strong
Config. C	1.38	0.01	Strong
Config. D	1.38	0.01	Strong
Config. E	1.38	0.01	Strong

1-2-8- Sphericity (morph_sphericity)

This feature is defined as:

$$F_{morph.sphericit} = \frac{(36\pi V^2)^{\frac{1}{3}}}{A}$$

Note: By definition $F_{morp\ \ .sphericit} = (F_{morph.comp.2})^{\frac{1}{3}}$.

Table 1-8: Reference values for the sphericity feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.843	0.005	Very strong
Config. A	0.683	0.001	Strong
Config. B	0.722	0.001	Strong
Config. C	0.723	0.003	Strong
Config. D	0.723	0.003	Strong
Config. E	0.723	0.003	Strong

1-2-9- Asphericity (morph_asphericity)

Asphericity is defined as:

$$F_{morp\ \ .asphericity} = \left(\frac{1}{36\pi}\frac{A^3}{V^2}\right)^{\frac{1}{3}} - 1$$

This measure is zero for a perfect sphere.

Note: By definition $F_{morp\ \ .asphericit} = (F_{morph.comp.2})^{-\frac{1}{3}} - 1$.

Table 1-9: Reference values for the asphericity feature. [1]

Data	Value	Tol.	Consensus

Dig. phantom	0.186	0.001	Strong
Config. A	0.463	0.002	Strong
Config. B	0.385	0.001	Moderate
Config. C	0.383	0.004	Strong
Config. D	0.383	0.004	Strong
Config. E	0.383	0.004	Strong

1-2-10- Centre of Mass Shift (morph_com)

This feature expresses the distance between the geometric centroid of the ROI volume and the intensity-weighted centroid, reflecting the spatial distribution of lower and higher intensities within the ROI.

Let $N_{v,m}$ be the number of voxels in the morphological mask. The geometric centre of mass is:

$$CoM_{geom} = \frac{1}{N_{v,m}} \sum_{k=1}^{N_{v.m}} X_{c,k}$$

Using the intensity mask, where $X_{c,gl}$ are voxel-centre coordinates and X_{gl} the corresponding intensities, and $N_{v,gl}$ is the number of intensity-mask voxels:

$$CoM_{gl} = \frac{\sum_{k=1}^{N_{v,gl}} X_{gl,k} X_{c,gl,k}}{\sum_{k=1}^{N_{v,gl}} X_{gl,k}}$$

The centre of mass shift is then:

$$F_{morp.com} = \left|\left|CoM_{geom} - CoM_{gl}\right|\right|_2$$

Table 1-10: Reference values for the centre of mass shift feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.672	0.004	Very trong

Config. A	52.9	28.7	Strong
Config. B	63.1	29.6	Strong
Config. C	45.6	2.8	Strong
Config. D	64.9	2.8	Strong
Config. E	68.5	2.1	Moderate

1-2-11- Maximum 3D Diameter (morph_diam)

The maximum 3D diameter is the largest Euclidean distance between any two vertices of the ROI mesh X_{vx}:

$$F_{mor\quad .diam} = max\left(\left\|X_{vx,k_1} - X_{vx,k_2}\right\|_2\right), \quad k_1 = 1,\dots,N \quad k_2 = 1,\dots,N$$

A practical approach is to compute the convex hull first. Its vertex set $X_{vx,convex}$ is guaranteed to contain the farthest pair, which reduces the computational burden. Although pairwise distances on the hull are still $O(n^2)$, $X_{vx,convex}$ is typically much smaller than X_{vx}. The convex hull is also used in later morphology features (see 1-2-25 and 1-2-26)

Table 1-11: Reference values for the maximum 3D diameter feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	13.1	0.1	Strong
Config. A	125	1	Strong
Config. B	125	1	Strong
Config. C	125	1	Strong
Config. D	125	1	Strong
Config. E	125	1	Strong

1-2-12- Major Axis Length (morph_pca_maj_axis)

Principal component analysis (PCA) determines the main orientation of a 3D ROI and yields three orthogonal eigenvectors $\{e_1, e_2, e_3\}$ with corresponding eigenvalues $\{\lambda_1, \lambda_2, \lambda_3\}$. These describe a triaxial ellipsoid: the eigenvectors set its orientation, and the eigenvalues quantify its extent along each axis. Several features use PCA results: major/minor/least axis length, elongation, flatness, and the approximate enclosing ellipsoid (volume and area densities).

Order eigenvalues so that $\lambda_{major} \geq \lambda_{min} \quad \geq \lambda_{least}$, corresponding to the major, minor, and least axes of the ellipsoid. The semi-axis lengths are $a = 2\sqrt{\lambda_{major}}$, $b = 2\sqrt{\lambda_{minor}}$, and $c = 2\sqrt{\lambda_{least}}$. The major axis length is twice the semi-axis a:

$$F_{morp \quad .pca.major} = 2a = 4\sqrt{\lambda_{major}}$$

Table 1-12: Reference values for the major axis length feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	11.4	0.1	Very Strong
Config. A	92.7	0.4	Very Strong
Config. B	92.6	0.4	Strong
Config. C	93.3	0.5	Strong
Config. D	93.3	0.5	Strong
Config. E	93.3	0.5	Strong

1-2-13- Minor Axis Length (morph_pca_min_axis)

The minor axis length reflects the extent along the second-largest axis (see 1-2-12). It is twice the semi-axis b:

$$F_{morph.pca.minor} = 2b = 4\sqrt{\lambda_{minor}}$$

Table 1-13: Reference values for the minor axis length feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	9.31	0.06	Very Strong
Config. A	81.5	0.4	Very Strong

Config. B	81.3	0.4	Strong
Config. C	82	0.5	Strong
Config. D	82	0.5	Strong
Config. E	82	0.5	Strong

1-2-14- Least Axis Length (morph_pca_least_axis)

The least axis is the direction along which the object is least extended (see 1-2-12). Its length is twice the semi-axis c:

$$F_{morph.pca.least} = 2c = 4\sqrt{\lambda_{least}}$$

Table 1-14: Reference values for the least axis length feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	8.54	0.05	Very Strong
Config. A	70.1	0.3	Strong
Config. B	70.2	0.3	Strong
Config. C	70.9	0.4	Strong
Config. D	70.9	0.4	Strong
Config. E	70.9	0.4	Strong

1-2-15- Elongation (morph_pca_elongation)

The ratio of major to minor axis lengths indicates how much longer the object is than it is wide (eccentricity). For numerical stability, elongation is expressed as an inverse ratio. A value of 1 indicates no elongation (e.g., a sphere); smaller values indicate greater elongation:

$$F_{morph.pca.elongation} = \sqrt{\frac{\lambda_{minor}}{\lambda_{major}}}$$

Table 1-15: Reference values for the elongation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.816	0.005	Very Strong
Config. A	0.879	0.001	Strong

Config. B	0.878	0.001	Strong
Config. C	0.879	0.001	Strong
Config. D	0.879	0.001	Strong
Config. E	0.879	0.001	Strong

1-2-16- Flatness (morph_pca_flatness)

The ratio of major to least axis lengths reflects how flat the object is relative to its length. For numerical stability, flatness is expressed as an inverse ratio. A value of 1 indicates a non-flat object (e.g., a sphere); smaller values indicate increasing flatness:

$$F_{morph.pca.flatness} = \sqrt{\frac{\lambda_{least}}{\lambda_{major}}}$$

Table 1-16: Reference values for the flatness feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.749	0.005	Very Strong
Config. A	0.756	0.001	Strong
Config. B	0.758	0.001	Strong
Config. C	0.76	0.001	Strong
Config. D	0.76	0.001	Strong
Config. E	0.76	0.001	Strong

1-2-17- Volume Density (Axis-aligned Bounding Box) (morph_vol_dens_aabb)

Volume density is the fraction between the ROI volume and that of a comparison volume. Here the comparison is the axis-aligned bounding box (AABB) enclosing X_{vx} (or $X_{vx,convex}$); both yield the same box, defined as the smallest axis-aligned box enclosing the vertex set:

$$F_{morph.v.dens.aabb} = \frac{V}{V_{aabb}}$$

This feature is also known as extent.

Table 1-17: Reference values for the volume density (AABB) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.869	0.005	Strong
Config. A	0.486	0.003	Strong
Config. B	0.477	0.003	Strong
Config. C	0.478	0.003	Strong
Config. D	0.478	0.003	Strong
Config. E	0.478	0.003	Strong

1-2-18- Area Density (Axis-aligned Bounding Box) (morph_area_dens_aabb)

Analogous to the volume-density (AABB), area density is the ratio between the ROI surface area and the surface area A_{aabb} of the AABB enclosing X_{vx}. The bounding box is identical to that used for the volume-density (AABB) feature:

$$F_{mor\quad .a.dens.aabb} = \frac{A}{A_{aabb}}$$

Table 1-18: Reference values for the area density (AABB) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.866	0.005	Strong
Config. A	0.725	0.003	Strong
Config. B	0.678	0.003	Strong
Config. C	0.678	0.003	Strong
Config. D	0.678	0.003	Strong

Config. E	0.678	0.003	Strong

1-2-19- Volume Density (Oriented Minimum Bounding Box) (morph_vol_dens_ombb)

Note: This feature currently has no reference values and should not be used.
The AABB volume is not necessarily minimal. By orienting the box along different axes, a smaller enclosing volume can be obtained. The oriented minimum bounding box (OMBB) of X_{vx} (or $X_{vx,convex}$) encloses the vertices with the smallest possible volume. Due to the computational cost of exact 3D rotating callipers, lower-complexity approximations are commonly employed. This feature is calculated as follows:

$$F_{morph.v.dens.ombb} = \frac{V}{V_{ombb}}$$

Here Vombb is the volume of the OMBB.

1-2-20- Area Density (Oriented Minimum Bounding Box) (morph_area_dens_ombb)

Note: This feature currently has no reference values and should not be used.
Using the same OMBB, define:

$$F_{morp\ \ .a.dens.ombb} = \frac{A}{A_{ombb}}$$

Where A_{ombb} is the OMBB surface area.

1-2-21- Volume Density (Approximate Enclosing Ellipsoid) (morph_vol_dens_aee)

The PCA eigenvectors and eigenvalues of X_c can be used to describe an ellipsoid approximating the point cloud—the approximate enclosing ellipsoid (AEE). With semi-axes a, b, c (see 1-2-12), the ellipsoid volume is $V_{aee} = 4\pi\ a\ b\ c/3$. Then:

$$F_{morph.v.dens.aee} = \frac{3V}{4\pi abc}$$

Table 1-19: Reference values for the volume density (AEE) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.17	0.01	Moderate
Config. A	1.29	0.01	Strong
Config. B	1.29	0.01	Strong
Config. C	1.29	0.01	Moderate
Config. D	1.29	0.01	Moderate

Config. E	1.29	0.01	Strong

1-2-22- Area Density (Approximate Enclosing Ellipsoid) (morph_area_dens_aee)

The surface area of an ellipsoid generally lacks a closed-form expression. It can be approximated using an infinite Legendre-polynomial series P_v. Using the same semi-axes a, b, c as in the AEE volume-density feature, define eccentricities $\alpha = \sqrt{1 - \frac{b^2}{a^2}}$ and $\beta = \sqrt{1 - \frac{c^2}{a^2}}$, and approximate:

$$A_{aee}(a,b,c) \ = 4\pi ab \sum_{v=0}^{\infty} \frac{(\alpha\beta)^v}{1-4v^2} P_v\left(\frac{\alpha^2+\beta^2}{2\alpha\beta}\right)$$

Where the series converges and can be truncated once further terms yield negligible improvement (default $L = 20$). The area density is then:

$$F_{morph.a.dens.aee} \ = \frac{A}{A_{aee}}$$

Table 1-20: Reference values for the area density (AEE) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.36	0.01	Moderate
Config. A	1.71	0.01	Moderate
Config. B	1.62	0.01	Moderate
Config. C	1.62	0.01	Moderate
Config. D	1.62	0.01	Moderate
Config. E	1.62	0.01	Strong

1-2-23- Volume Density (Minimum Volume Enclosing Ellipsoid) (morph_vol_dens_mvee)

Note: This feature currently has no reference values and should not be used.

The minimum-volume enclosing ellipsoid (MVEE) is the smallest ellipsoid enclosing the ROI, in contrast to the AEE. Direct computation is generally infeasible and is therefore approximated using iterative methods (e.g., barycentric coordinate descent with a default stopping tolerance of $\tau = 0.001$).

It is recommended to use the convex mesh set $X_{vx,convex}$ due to its sparsity compared to the full vertex set.

With semi-axes a, b, c (see 1-2-12), and $V_{mvee} = 4\pi \frac{a\,b\,c}{3}$:

$$F_{morph.v.dens.mvee} = \frac{V}{V_{mvee}}$$

1-2-24- Area Density (Minimum Volume Enclosing Ellipsoid) (morph_area_dens_mvee)

Note: This feature currently has no reference values and should not be used.
Approximate the MVEE surface area A_{mvee} as in Section 1-2-22 and compute:

$$F_{mor\quad .a.dens.mvee} = \frac{A}{A_{mvee}}$$

1-2-25- Morph_vol_dens_conv_hull (Volume density (convex hull))

The convex hull encloses X_{vx} and consists of the vertex set $X_{vx,convex}$ with corresponding faces (see 1-2-11). Compute the hull volume V_{convex} as in the mesh-volume feature (1-2-1). Then:

$$F_{mor\quad .v.dens.conv.hull} = \frac{V}{V_{convex}}$$

This feature is also known as solidity.

Table 1-21: Reference values for the area density (AEE) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.961	0.006	Strong
Config. A	0.827	0.001	Moderate
Config. B	0.829	0.001	Moderate
Config. C	0.834	0.002	Moderate
Config. D	0.834	0.002	Moderate
Config. E	0.834	0.002	Moderate

1-2-26- Area Density (Convex Hull) (morph_area_dens_conv_hull)

Let A_{convex} be the convex hull surface, defined as the sum of areas of hull faces (computed as in 1-2-3). Using the same hull as in 1-2-25, this feature is defined as:

$$F_{morph.a.dens.conv.hull} = \frac{A}{A_{convex}}$$

Table 1-22: Reference values for the area density (convexhull) feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.03	0.01	Strong

Config. A	1.18	0.01	Moderate
Config. B	1.12	0.01	Moderate
Config. C	1.13	0.01	Moderate
Config. D	1.13	0.01	Moderate
Config. E	1.13	0.01	Moderate

1-2-27- Integrated Intensity (morph_integ_int)

Integrated intensity is the product of the ROI volume and its mean intensity. In 18F-FDG-PET, this is often referred to as total lesion glycolysis. Thus:

$$F_{morph.integ.int} = V \frac{1}{N_{v,gl}} \sum_{k=1}^{N_{v,gl}} X_{gl,k}$$

Where $N_{v,gl}$ is the number of voxels in the intensity mask.

Table 1-23: Reference values for the integrated intensity feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.2×10^{3}	10	Moderate
Config. A	4.81×10^{6}	3.2×10^{5}	Strong
Config. B	4.12×10^{6}	3.2×10^{5}	Strong
Config. C	-1.8×10^{7}	1.4×10^{6}	Strong
Config. D	-8.64×10^{6}	1.56×10^{6}	Strong
Config. E	-8.31×10^{6}	1.6×10^{6}	Strong

1-2-28- Moran's I Index (morph_moran_i)

Moran's I measures spatial autocorrelation:

$$F_{morph.moran.i} = \frac{N_{v,gl}}{\sum_{k_1=1}^{N_{v,gl}} \sum_{k_2=1}^{N_{v,gl}} w_{k_1 k_2}} \frac{\sum_{k_1=1}^{N_{v,gl}} \sum_{k_2=1}^{N_{v,gl}} w_{k_1 k_2} \left(X_{gl,k_1} - \mu\right)\left(X_{gl,k_2} - \mu\right)}{\sum_{k=1}^{N_{v,gl}} \left(X_{gl,k} - \mu\right)^2}, \quad k_1 \neq k_2$$

Here, $N_{v,gl}$ is the number of voxels in the intensity mask, μ the mean intensity, and $w_{k_1 k_2}$ a weight equal to the inverse Euclidean distance between voxels k_1 and k_2 in $X_{c,gl}$. Values near 1.0, 0.0

and -1.0 indicate high spatial autocorrelation, no spatial autocorrelation and high spatial anti-autocorrelation, respectively.

Note: The computational cost is $O(n^2)$; approximation via repeated subsampling may be required to make computation feasible for large ROIs (with a trade-off in accuracy).

Table 1-24: Reference values for the Moran's I index feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.0397	0.0003	Strong
Config. A	0.0322	0.0002	Moderate
Config. B	0.0329	0.0001	Moderate
Config. C	0.0824	0.0003	Moderate
Config. D	0.0622	0.0013	Moderate
Config. E	0.0596	0.0014	Moderate

1-2-29- Geary's C Measure (morph_geary_c)

Geary's C also quantifies spatial autocorrelation, but directly evaluates intensity differences between voxels and is more sensitive to local structure:

$$F_{morph.geary.c} = \frac{N_{v,gl}-1}{2\sum_{k_1=1}^{N_{v,gl}}\sum_{k_2=1}^{N_{v,gl}} w_{k_1k_2}} \frac{\sum_{k_1=1}^{N_{v,gl}}\sum_{k_2=1}^{N_{v,gl}} w_{k_1k_2}\left(X_{gl,k_1}-X_{gl,k_2}\right)^2}{\sum_{k=1}^{N_{v,gl}}\left(X_{gl,k}-\mu\right)^2}, \quad k_1 \neq k_2$$

As above, $N_{v,gl}$ is the number of voxels in the intensity mask, μ is the mean of X_{gl} and $w_{k_1k_2}$ is a weight equal to the inverse Euclidean distance between voxels k_1 and k_2 in $X_{c,gl}$. Like 1-2-28, the computation is $O(n^2)$; approximation strategies may be required for large ROIs.

Table 1-25: Reference values for the Geary's C measure feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.974	0.006	Strong
Config. A	0.863	0.001	Moderate
Config. B	0.862	0.001	Moderate
Config. C	0.846	0.001	Moderate

Config. D	0.851	0.001	Moderate
Config. E	0.853	0.001	Moderate

2- Local Intensity Features (SUVpeak)

Table of Content

2-1- Introduction [1]

Local intensity features use voxel intensities from a defined neighbourhood around a centre voxel. Unlike many other feature sets, these features are not restricted to intensities inside the ROI alone: the centre voxel must lie within the ROI intensity mask, but the local neighbourhood includes all voxels regardless of ROI membership.

Aggregating features

By definition, local intensity features are computed in 3D and not per slice.

2-2- Explanation of Features [1]

2-2-1- Local Intensity Peak (loc_peak_loc)

Originally introduced to reduce variance in determining standardised uptake values, the local intensity peak is defined as the mean intensity within a 1 cm³ spherical region (world coordinates) centred on the voxel with the maximum intensity inside the ROI intensity mask.

To compute $F_{loc.peak.local}$, select all voxels whose centres lie within radius $r = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \approx 0.62\ cm$ of the maximum-intensity voxel, then calculate the mean intensity of the selected voxels (including the centre voxel).
If multiple voxels share the maximum intensity inside the ROI, evaluate the local intensity peak for each such voxel and retain the highest result.

Table 2-1: Reference values for the local intensity peak feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	2.6	—	Strong
Config. A	-277	10	Moderate
Config. B	178	10	Moderate
Config. C	169	10	Moderate
Config. D	201	10	Strong
Config. E	181	13	Moderate

2-2-2- Global Intensity Peak (loc_peak_glob)

The global intensity peak $F_{loc.peak.global}$ mirrors the local definition, but instead of limiting to the voxel(s) at the maximum intensity, it computes the mean intensity within a 1 cm³ neighbourhood for every voxel in the ROI intensity mask, and then selects the highest peak value.
Computation can be accelerated by constructing and applying an appropriate spatial spherical mean convolution filter (convolution theorem). Build an empty 3D filter that fits a 1 cm³ sphere. Represent filter voxels as a point set (akin to X_c in Section 1-1). Compute Euclidean distances (world spacing) between the filter's central voxel and every other filter voxel. If the distance is within radius $r = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \approx 0.62\ cm$, assign label 1; otherwise 0. Summing labels yields N_s, the number of voxels inside the 1 cm³ sphere. Convert to a spherical mean filter by dividing labels by N_s.

Table 2-2: Reference values for the global intensity peak feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	3.1	—	Strong
Config. A	189	5	Moderate
Config. B	178	5	Moderate
Config. C	180	5	Moderate
Config. D	201	5	Moderate
Config. E	181	5	Moderate

3- Intensity-based Statistical Features

3-1- Introduction [1]

These features characterise how intensities inside the region of interest (ROI) are distributed. They do not require discretisation and can be applied to a continuous intensity distribution. Intensity-based statistical features are not meaningful when the intensity scale is arbitrary.

Aggregating features

We recommend computing intensity-based statistical features on the 3D volume. The per-slice approach with subsequent averaging is not recommended.

3-2- Explanation of Features [1]

Let the set of intensities from the N_v voxels in the ROI intensity mask be:

$$X_{gl} = \{X_{gl,1}, X_{gl,2}, \ldots, X_{gl,N_v}\}$$

3-2-1- Mean Intensity (stat_mean)

The mean intensity of X_{gl} is:

$$F_{stat.mean} = \frac{1}{N_v}\sum_{k=1}^{N_v} X_{gl,k}$$

Table 3-1: Reference values for the mean feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	2.15	—	Very Strong
Config. A	13.4	1.1	Very Strong
Config. B	11.5	1.1	Strong
Config. C	−49	2.9	Very Strong
Config. D	−23.5	3.9	Strong
Config. E	−22.6	4.1	Strong

3-2-2- Intensity Variance (stat_var)

The intensity variance of X_{gl} is:

$$F_{stat.var} = \frac{1}{N_v}\sum_{k=1}^{N_v} \left(X_{gl,k} - \mu\right)^2$$

Note: do not apply a bias correction when computing the variance.

Table 3-2: Reference values for the variance feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	3.05	—	Very Strong
Config. A	1.42×10^{4}	400	Very Strong
Config. B	1.44×10^{4}	400	Strong
Config. C	5.06×10^{4}	1.4×10^{3}	Very Strong
Config. D	3.28×10^{4}	2.1×10^{3}	Strong
Config. E	3.51×10^{4}	2.2×10^{3}	Strong

3-2-3- Intensity Skewness (stat_skew)

Skewness of the distribution of X_{gl}:

$$F_{stat.skew} = \frac{\frac{1}{N_v}\sum_{k=1}^{N_v} \left(X_{gl,k} - \mu\right)^3}{\left(\frac{1}{N_v}\sum_{k=1}^{N_v} \left(X_{gl,k} - \mu\right)^2\right)^{\frac{3}{2}}}$$

Here μ corresponds to the mean intensity $F_{stat.mean}$. If $F_{stat.var} = 0$, then $F_{stat.skew} = 0$.

Table 3-3: Reference values for the skewness feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.08	—	Very Strong
Config. A	-2.47	0.05	Very Strong
Config. B	-2.49	0.05	Strong
Config. C	-2.14	0.05	Very Strong
Config. D	-2.28	0.06	Strong
Config. E	-2.3	0.07	Strong

3-2-4- Intensity Kurtosis (stat_kurt)

Kurtosis (excess kurtosis) as a measure of peakedness of X_{gl} is defined as:

$$F_{stat.kurt} = \frac{\frac{1}{N_v}\sum_{k=1}^{N_v} \left(X_{gl,k} - \mu\right)^4}{\left(\frac{1}{N_v}\sum_{k=1}^{N_v} \left(X_{gl,k} - \mu\right)^2\right)^2} - 3$$

Here $\mu = F_{stat.mean}$. If $F_{stat.var} = 0$, then $F_{stat.kurt} = 0$.
Kurtosis includes a Fisher correction of −3 to centre it at 0 for normal distributions.

Table 3-4: Reference values for the (excess) kurtosis feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	-0.355	—	Very Strong
Config. A	5.96	0.24	Very Strong
Config. B	5.93	0.24	Strong
Config. C	3.53	0.23	Very Strong
Config. D	4.35	0.32	Strong
Config. E	4.44	0.33	Strong

3-2-5- Median Intensity (stat_median)

The median intensity $F_{stat.median}$ is the sample median of X_{gl}, i.e.:

$$F_{stat.median} = median\left(X_{gl}\right)$$

Table 3-5: Reference values for the median feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	__	Very Strong
Config. A	46	0.3	Very Strong
Config. B	45	0.3	Strong
Config. C	40	0.4	Very Strong
Config. D	42	0.4	Strong
Config. E	43	0.5	Strong

3-2-6- Minimum Intensity (stat_min)

The minimum intensity equals the lowest value in X_{gl}, i.e.:

$$F_{stat.min} = min\left(X_{gl}\right)$$

Table 3-6: Reference values for the minimum feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	__	Very Strong
Config. A	-500	__	Very Strong
Config. B	-500	__	Strong
Config. C	-939	4	Very Strong
Config. D	-724	12	Strong
Config. E	-743	13	Strong

3-2-7- 10th Intensity Percentile (stat_p10)

P_{10} is the 10th percentile of X_{gl} and serves as a more robust alternative to the minimum intensity.

Table 3-7: Reference values for the 10th percentile feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	__	Very Strong
Config. A	-129	8	Very Strong

Config. B	-136	8	Strong
Config. C	-424	14	Very Strong
Config. D	-304	20	Strong
Config. E	-310	21	Strong

3-2-8- 90th Intensity Percentile (stat_p90)

P_{90} is the 90th percentile of X_{gl} and is a more robust alternative to the maximum intensity. Note. For the digital phantom, the 90th percentile may differ from the listed reference depending on the software implementation; some implementations yield 4.2 instead of 4.

Table 3-8: Reference values for the 90th percentile feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	4	—	Very Strong
Config. A	95	—	Strong
Config. B	91	—	Strong
Config. C	86	0.1	Strong
Config. D	86	0.1	Strong
Config. E	93	0.2	Strong

3-2-9- Maximum Intensity (stat_max)

The maximum intensity equals the highest value in X_{gl}, i.e.:

$$F_{stat.max} = max(X_{gl})$$

Table 3-9: Reference values for the maximum feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	6	—	Very Strong
Config. A	377	9	Very Strong

Config. B	391	9	Strong
Config. C	393	10	Very Strong
Config. D	521	22	Strong
Config. E	345	9	Strong

3-2-10- Intensity Interquartile Range (stat_iqr)

The interquartile range (IQR) of X_{gl} is defined as:

$$F_{stat.iqr} = P_{75} - P_{25}$$

Where P_{25} and P_{75} are the 25th and 75th percentiles of X_{gl}, respectively.

Table 3-10: Reference values for the interquartile range feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	3	—	Very Strong
Config. A	56	0.5	Very Strong
Config. B	52	0.5	Strong
Config. C	67	4.9	Very Strong
Config. D	57	4.1	Strong
Config. E	62	3.5	Strong

3-2-11- Intensity Range (stat_range)

The intensity range is defined as:

$$F_{stat.range} = max(X_{gl}) - min(X_{gl})$$

Table 3-11: Reference values for the range feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	5	—	Very Strong
Config. A	877	9	Very Strong

Config. B	891	9	Strong
Config. C	1.33×10^{3}	20	Very Strong
Config. D	1.24×10^{3}	40	Strong
Config. E	1.09×10^{3}	30	Strong

3-2-12- Mean Absolute Deviation (stat_mad)

Mean absolute deviation as a dispersion measure from the mean of X_{gl}:

$$F_{stat.mad} = \frac{1}{N_v}\sum_{k=1}^{N_v} \left|X_{gl,k} - \mu\right|$$

Here $\mu = F_{stat.mean}$.

Table 3-12: Reference values for the mean absolute deviation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.55	—	Very Strong
Config. A	73.6	1.4	Very Strong
Config. B	74.4	1.4	Strong
Config. C	158	4	Very Strong
Config. D	123	6	Strong
Config. E	125	6	Strong

3-2-13- Robust Mean Absolute Deviation (stat_rmad)

Outliers may influence the mean absolute deviation. To increase robustness, restrict to intensities closer to the centre of the distribution. Let:

$$X_{gl,10-90} = \left\{x \in X_{gl} \middle| P_{10}\left(X_{gl}\right) \leq x \leq P_{90}\left(X_{gl}\right)\right\}$$

Then $X_{gl,10-90}$ contains $N_{v,10-90} \leq N_v$ voxels. The robust mean absolute deviation is:

$$F_{stat.rmad} = \frac{1}{N_{v,10-9}}\sum_{k=1}^{N_{v,10-9}} \left|X_{gl,10-\ \ ,k} - \underline{X}_{gl,10-9}\ \right|$$

Where $\underline{X}_{gl,10-9}$ is the sample mean of $X_{gl,10-9\ \ ,k}$.

Table 3-13: Reference values for the robust mean absolute deviation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.11	—	Very Strong
Config. A	27.7	0.8	Very Strong
Config. B	27.3	0.8	Strong
Config. C	66.8	3.5	Very Strong
Config. D	46.8	3.6	Strong
Config. E	46.5	3.7	Strong

3-2-14- Median Absolute Deviation (stat_medad)

Median absolute deviation is analogous to the mean-based version but measures dispersion from the median:

$$F_{stat.medad} = \frac{1}{N_v} \sum_{k=1}^{N_v} \left| X_{gl,k} - M \right|$$

Here, the median $M = F_{stat.median}$.

Table 3-14: Reference values for the median absolute deviation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.15	—	Very Strong
Config. A	64.3	1	Strong
Config. B	63.8	1	Strong
Config. C	119	4	Strong
Config. D	94.7	3.8	Strong
Config. E	97.9	3.9	Strong

3-2-15- Coefficient of Variation (stat_cov)

The coefficient of variation measures the dispersion of X_{gl}. It is defined as:

$$F_{stat.cov} = \frac{\sigma}{\mu}$$

Here $\sigma = F_{stat.var}{}^{\frac{1}{2}}$ and $\mu = F_{stat.mean}$ are the standard deviation and mean of the intensity distribution, respectively.

Table 3-15: Reference values for the coefficient of variation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.812	—	Very Strong
Config. A	8.9	4.98	Strong
Config. B	10.4	5.2	Strong
Config. C	−4.59	0.29	Strong
Config. D	−7.7	1.01	Strong
Config. E	−8.28	0.95	Strong

3-2-16- Quartile Coefficient of Dispersion (stat_qcod)

This feature is a more robust alternative to the coefficient of variation and it is defined as:

$$F_{stat.qcod} = \frac{P_{75} - P_{25}}{P_{75} + P_{25}}$$

Where P_{25} and P_{75} are the 25th and 75th percentile of X_{gl}, respectively.

Table 3-16: Reference values for the quartile coefficient of dispersion feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.6	—	Very Strong
Config. A	0.636	0.008	Strong
Config. B	0.591	0.008	Strong
Config. C	1.03	0.4	Strong
Config. D	0.74	0.011	Strong
Config. E	0.795	0.337	Strong

3-2-17- Energy (stat_energy)

Energy of X_{gl} is defined as:

$$F_{stat.energy} = \sum_{k=1}^{N_v} X_{gl,k}^2$$

Table 3-17: Reference values for the energy feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	567	—	Very Strong
Config. A	1.65×10^{9}	2×10^{7}	Very Strong
Config. B	3.98×10^{8}	1.1×10^{7}	Strong
Config. C	2.44×10^{9}	1.2×10^{8}	Strong
Config. D	1.48×10^{9}	1.4×10^{8}	Strong
Config. E	1.58×10^{9}	1.4×10^{8}	Strong

3-2-18- Root Mean Square Intensity (stat_rms)

The root mean square (quadratic mean) of X_{gl}:

$$F_{stat.rms} = \sqrt{\frac{\sum_{k=1}^{N_v} X_{gl,k}^{2}}{N_v}}$$

Table 3-18: Reference values for the root mean square feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	2.77	—	Very Strong
Config. A	120	2	Very Strong
Config. B	121	2	Strong
Config. C	230	4	Strong
Config. D	183	7	Strong
Config. E	189	7	Strong

4- Intensity Histogram Features

4-1- Introduction [1]

An intensity histogram is obtained by discretising the original ROI intensity distribution X_{gl} into a set of intensity bins (see Section 2.7 *** for discretisation methods). Let $X_d = \{X_{d,1}, X_{d,2}, \dots, X_{d,N_v}\}$ denote the discretised intensities of the N_v voxels within the ROI intensity mask, taking values on N_g discrete levels. Define the histogram $H = \{n_1, n_2, \dots, n_{N_g}\}$, where n_i counts how often level i occurs in X_d. The occurrence probability for level i is approximated by:

$$p_i = \frac{n_i}{N_v}$$

Aggregating features

We recommend computing intensity-based statistical features on the 3D volume. The per-slice approach with subsequent averaging is not recommended.

4-2- Explanation of Features [1]

4-2-1- Mean Discretised Intensity (ih_mean)

The mean of the discretised intensities is given by:

$$F_{ih.mean} = \frac{1}{N_v} \sum_{k=1}^{N_v} x_{d,k}$$

An equivalent expression uses the histogram probabilities:

$$F_{ih.mean} = \sum_{i=1}^{N_g} i\, p_i$$

Table 4-1: Reference values for the mean feature. [1]

Data	Value	Tol.	Consensus

Dig. phantom	2.15	—	Very Strong
Config. A	21.1	0.1	Strong
Config. B	18.9	0.3	Strong
Config. C	38.6	0.2	Strong
Config. D	18.5	0.5	Strong
Config. E	21.7	0.3	Strong

4-2-2- Discretised Intensity Variance (ih_var)

The variance of X_d is defined by:

$$F_{ih.var} = \frac{1}{N_v} \sum_{k=1}^{N_v} \left(x_{d,k} - \mu\right)^2$$

With $\mu = F_{ih.mean}$. An equivalent form is:

$$F_{ih.var} = \sum_{i=1}^{N_g} (i - \mu)^2 \, p_i$$

Note: No bias correction is applied.

Table 4-2: Reference values for the variance feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	3.05	—	Strong
Config. A	22.8	0.6	Strong
Config. B	18.7	0.2	Strong
Config. C	81.1	2.1	Strong
Config. D	21.7	0.4	Strong
Config. E	30.4	0.8	Strong

4-2-3- Discretised Intensity Skewness (ih_skew)

Skewness of the discretised distribution is:

$$F_{ih.skew} = \frac{\frac{1}{N_v}\sum_{k=1}^{N_v} \left(x_{d,k} - \mu\right)^3}{\left(\frac{1}{N_v}\sum_{k=1}^{N_v} \left(x_{d,k} - \mu\right)^2\right)^{\frac{3}{2}}}$$

With $\mu = F_{ih.mean}$. Equivalently:

$$F_{ih.skew} = \frac{\sum_{i=1}^{N_g} (i-\mu)^3 \, p_i}{\left(\sum_{i=1}^{N_g} (i-\mu)^2 \, p_i\right)^{\frac{3}{2}}}$$

If $F_{ih.var} = 0$, then $F_{ih.skew} = 0$.

Table 4-3: Reference values for the skewness feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.08	—	Very Strong
Config. A	−2.46	0.05	Strong
Config. B	−2.47	0.05	Strong
Config. C	−2.14	0.05	Strong
Config. D	−2.27	0.06	Strong
Config. E	−2.29	0.07	Strong

4-2-4- Discretised Intensity Kurtosis (ih_kurt)

The (excess) kurtosis measuring peakedness of X_d is:

$$F_{ih.kurt} = \frac{\frac{1}{N_v}\sum_{k=1}^{N_v} \left(x_{d,k} - \mu\right)^4}{\left(\frac{1}{N_v}\sum_{k=1}^{N_v} \left(x_{d,k} - \mu\right)^2\right)^{2}} - 3$$

With $\mu = F_{ih.mean}$. An alternative, equivalent formulation is:

$$F_{ih.kurt} = \frac{\sum_{i=1}^{N_g} (i-\mu)^4 \, p_i}{\left(\sum_{i=1}^{N_g} (i-\mu)^2 \, p_i\right)^{2}} - 3$$

A Fisher correction of −3 is used so that a normal distribution maps to 0. If $F_{ih.var} = 0$, then $F_{ih.kurt} = 0$.

Table 4-4: Reference values for the (excess) kurtosis feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	-0.355	—	Very Strong
Config. A	5.9	0.24	Strong
Config. B	5.84	0.24	Strong
Config. C	3.52	0.23	Strong
Config. D	4.31	0.32	Strong
Config. E	4.4	0.33	Strong

4-2-5- Median Discretised Intensity (ih_median)

The sample median of X_d is denoted $F_{ih.median}$.

Table 4-5: Reference values for the median feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	—	Very Strong
Config. A	22	—	Strong
Config. B	20	0.3	Strong
Config. C	42	—	Strong
Config. D	20	0.5	Strong
Config. E	24	0.2	Strong

4-2-6- Minimum Discretised Intensity (ih_min)

The minimum discretised intensity is:

$$F_{ih.min} = min(X_d)$$

For a fixed bin number scheme, $F_{ih.min} = 1$ by definition; fixed bin size may yield $F_{ih.min} > 1$.

Table 4-6: Reference values for the minimum feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	—	Very Strong
Config. A	1	—	Strong

Config. B	1	__	Strong
Config. C	3	0.16	Strong
Config. D	1	__	Strong
Config. E	1	__	Strong

4-2-7- 10th Discretised Intensity Percentile (ih_p10)

This feature is defined as the discretised gray level at which 10% of the cumulative discretised intensity histogram is reached.

Table 4-7: Reference values for the 10th percentile feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	__	Very Strong
Config. A	15	0.4	Strong
Config. B	14	0.5	Strong
Config. C	24	0.7	Strong
Config. D	11	0.7	Strong
Config. E	13	0.7	Strong

4-2-8- 90th Discretised Intensity Percentile (ih_p90)

This feature is defined as the discretised gray level at which 90% of the cumulative discretised intensity histogram is reached.
Note that, for the digital phantom, this percentile may differ across software implementations (e.g., 4.2 instead of 4).

Table 4-8: Reference values for the 90th percentile feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	4	__	Strong
Config. A	24	__	Strong
Config. B	22	0.3	Strong
Config. C	44	__	Strong

Config. D	21	0.5	Strong
Config. E	25	0.2	Strong

4-2-9- Maximum Discretised Intensity (ih_max)

The maximum discretised intensity is:

$$F_{ih.max} = max(X_d)$$

By definition, $F_{ih.max} = N_g$.

Table 4-9: Reference values for the maximum feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	6	__	Very Strong
Config. A	36	0.4	Strong
Config. B	32	__	Strong
Config. C	56	0.5	Strong
Config. D	32	__	Strong
Config. E	32	__	Strong

4-2-10- Intensity Histogram Mode (ih_mode)

$F_{ih.mode}$ is the most frequent discretised level, i.e., the i with the largest n_i. If several bins tie for the highest count, choose the bin nearest to the mean discretised intensity; if two are equidistant, select the bin to the left of the mean.

Table 4-10: Reference values for the mode feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	__	Very Strong
Config. A	23	__	Strong
Config. B	20	0.3	Strong
Config. C	43	0.1	Strong
Config. D	20	0.4	Strong
Config. E	24	0.1	Strong

4-2-11- Discretised Intensity Interquartile Range (ih_iqr)

The interquartile range is:

$$F_{ih.iqr} = P_{75} - P_{25}$$

where P_{25} and P_{75} are the 25th and 75th percentiles of X_d, respectively.

Table 4-11: Reference values for the interquartile range feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	3	—	Very Strong
Config. A	2	—	Strong
Config. B	2	—	Strong
Config. C	3	0.21	Strong
Config. D	2	0.06	Strong
Config. E	1	0.06	Strong

4-2-12- Discretised Intensity Range (ih_range)

The discretised range is:

$$F_{ih.range} = max(X_d) - min(X_d)$$

For a fixed number of bins, the range equals N_g by definition.

Table 4-12: Reference values for the range feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	5	—	Very Strong
Config. A	35	0.4	Strong
Config. B	31	—	Strong
Config. C	53	0.6	Strong
Config. D	31	—	Strong
Config. E	31	—	Strong

4-2-13- Intensity Histogram Mean Absolute Deviation (ih_mad)

The mean absolute deviation is:

$$F_{ih.mad} = \frac{1}{N_v}\sum_{i=1}^{N_v} \left|X_{d,i} - \mu\right|$$

Here $\mu = F_{ih.mean}$.

Table 4-13: Reference values for the mean absolute deviation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.55	—	Very Strong
Config. A	2.94	0.06	Strong
Config. B	2.67	0.03	Strong
Config. C	6.32	0.15	Strong
Config. D	3.15	0.05	Strong
Config. E	3.69	0.1	Strong

4-2-14- Intensity Histogram Robust Mean Absolute Deviation (ih_rmad)

This feature is sensitive to outliers. Therefore, IBSI suggests to restrict the range of considered intensities:

$$X_{d,10-90} = \{x \in X_d \mid P_{10}(X_d) \leq x \leq P_{90}(X_d)\}$$

So that $X_{d,10-90}$ contains $N_{v,10-90} \leq N_v$ elements. The robust mean absolute deviation is:

$$F_{ih.rmad} = \frac{1}{N_{v,10-90}}\sum_{k=1}^{N_{v,10-90}} \left|X_{d,10-90,k} - \underline{X}_{d,10-9}\ \right|$$

Where $X_{d,10-9}$ denotes the sample mean of that subset.

Table 4-14: Reference values for the robust mean absolute deviation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.11	—	Very Strong
Config. A	1.18	0.04	Strong
Config. B	1.03	0.03	Moderate
Config. C	2.59	0.14	Strong
Config. D	1.33	0.06	Strong

Config. E	1.46	0.09	Moderate

4-2-15- Intensity Histogram Median Absolute Deviation (ih_medad)

The median-based analogue is:

$$F_{ih.medad} = \frac{1}{N_v}\sum_{k=1}^{N_v} \left|X_{d,k} - M\right|$$

with median $M = F_{ih.median}$.

Table 4-15: Reference values for the median absolute deviation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.15	—	Very Strong
Config. A	2.58	0.05	Strong
Config. B	2.28	0.02	Strong
Config. C	4.75	0.12	Strong
Config. D	2.41	0.04	Strong
Config. E	2.89	0.07	Strong

4-2-16- Intensity Histogram Coefficient of Variation (ih_cov)

The coefficient of variation for the discretised intensity distribution is:

$$F_{ih.cov} = \frac{\sigma}{\mu}$$

Here $\sigma = {F_{ih.var}}^{\frac{1}{2}}$ is the standard deviation and $\mu = F_{ih.mean}$ is the mean.

Table 4-16: Reference values for the coefficient of variation feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.812	—	Very Strong
Config. A	0.227	0.004	Strong
Config. B	0.229	0.004	Strong
Config. C	0.234	0.005	Strong

Config. D	0.252	0.006	Strong
Config. E	0.254	0.006	Strong

.

4-2-17- Intensity Histogram Quartile Coefficient of Dispersion (ih_qcod)

A robust alternative to the coefficient of variation is:

$$F_{ih.qcod} = \frac{P_{75} - P_{25}}{P_{75} + P_{25}}$$

with P_{25} and P_{75} taken from X_d.

Table 4-17: Reference values for the quartile coefficient of dispersion feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.6	—	Very Strong
Config. A	0.0455	—	Strong
Config. B	0.05	0.0005	Strong
Config. C	0.0361	0.0027	Strong
Config. D	0.05	0.0021	Strong
Config. E	0.0213	0.0015	Strong

4-2-18- Discretised Intensity Entropy (ih_entropy)

Shannon entropy for the discretised intensities is:

$$F_{ih.entropy} = -\sum_{i=1}^{L} p_i \, log_2 \, p_i$$

Entropy is meaningful only for discretised values; for continuous distributions it tends towards $-log_2 N_v$.

Table 4-18: Reference values for the entropy feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1.27	—	Very Strong
Config. A	3.36	0.03	Very Strong
Config. B	3.16	0.01	Strong

Config. C	3.73	0.04	Strong
Config. D	2.94	0.01	Strong
Config. E	3.22	0.02	Strong

4-2-19- Discretised Intensity Uniformity (ih_uniformity)

Uniformity of X_d is given by:

$$F_{ih.uniformity} = \sum_{i=1}^{N_g} p_i^{\,2}$$

When most mass lies in a single bin, uniformity approaches 1; the lower bound is $\frac{1}{N_g}$. This feature is sometimes referred to as energy.

Table 4-19: Reference values for the uniformity feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.512	—	Very Strong
Config. A	0.15	0.002	Very Strong
Config. B	0.174	0.001	Strong
Config. C	0.14	0.003	Strong
Config. D	0.229	0.003	Strong
Config. E	0.184	0.001	Strong

4-2-20- Maximum Histogram Gradient (ih_max_grad)

This feature quantifies the largest absolute change in frequency between adjacent bins. To calculate it, first get the histogram gradient H' of the discretised intensity histogram H. Then, return the maximum value of it.

Let n_i denote the number of voxels in histogram bin i. The maximum histogram gradient is obtained as it is shown below:

$$H'_i = \begin{cases} n_2 - n_1 & i = 1 \\ (n_{i+1} - n_{i-1})/2 & 1 < i < N \\ n_N - n_{N-1} & i = N \end{cases}$$

$$F_{ih.max.grad} = max(H')$$

Table 4-20: Reference values for the maximum histogram gradient feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	8	__	Very Strong
Config. A	1.1×10^4	100	Strong
Config. B	3.22×10^3	50	Strong
Config. C	4.75×10^3	30	Strong
Config. D	7.26×10^3	200	Strong
Config. E	6.01×10^3	130	Strong

4-2-21- Maximum Histogram Gradient Intensity (ih_max_grad_g)

The discretised level corresponding to the maximum histogram gradient is reported as $F_{ih.max.grad.gl}$, i.e., the i where H attains its maximum.

Table 4-21: Reference values for the maximum histogram gradient intensity feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	3	__	Strong
Config. A	21	__	Strong
Config. B	19	0.3	Strong
Config. C	41	__	Strong
Config. D	19	0.4	Strong

Config. E	23	0.2	Moderate

4-2-22- Minimum Histogram Gradient (ih_min_grad)

The minimum histogram gradient is:

$$F_{ih.min.grad} = min(\,H'\,)$$

Table 4-22: Reference values for the minimum histogram gradient feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	-50	—	Very Strong
Config. A	-1.01×10^{4}	100	Strong
Config. B	-3.02×10^{3}	50	Strong
Config. C	-4.68×10^{3}	50	Strong
Config. D	-6.67×10^{3}	230	Strong
Config. E	-6.11×10^{3}	180	Strong

4-2-23- Minimum Histogram Gradient Intensity (ih_min_grad_g)

The discretised level at which the histogram gradient is minimal is reported as $F_{ih.min.grad.gl}$, i.e., the i where H is minimal.

Table 4-23: Reference values for the minimum histogram gradient intensity feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	1	—	Strong
Config. A	24	—	Strong
Config. B	22	0.3	Strong
Config. C	44	—	Strong
Config. D	22	0.4	Strong
Config. E	25	0.2	Strong

5- Intensity-volume Histogram Features

5-1- Introduction [1]

The cumulative intensity–volume histogram (IVH) characterizes the relationship between voxel intensities and the fraction of the ROI volume that contains intensities at or above a certain threshold. For a given ROI, let X_{gl} be the set of voxel intensities. To compute the IVH, intensities are first discretised if required, resulting in a voxel set $X_{d,gl}$. The histogram is then constructed over this discretised set, and its definition depends on the modality and the representation of intensities.

The calculation requires specifying both the total range G of discretised intensities and the discretisation interval w_d. The total range defines the interval of intensities included in the histogram, while the discretisation interval sets the difference between adjacent discretised values.

Discretisation Rules

For modalities with discrete calibrated intensities (e.g. CT), no additional discretisation is required, and therefore $X_{d,gl} = X_{gl}$. In this case, the interval is fixed at $w_d = 1$. The total range is given by the re-segmentation range if provided (see *** 2.5); for half-open ranges, the upper limit is taken as $max(X_{gl})$. Otherwise, $G = [min(X_{gl}), max(X_{gl})]$.

For modalities with continuous calibrated intensities (e.g. PET), a fixed bin size discretisation is recommended to preserve quantitative intensity information (see *** 2.7). This requires defining the minimum and maximum intensities $X_{gl,min}$ and $X_{gl,max}$, together with a bin width w_b. If a re-segmentation range is provided (see *** 2.5), its limits determine $X_{gl,min}$ and $X_{gl,max}$; otherwise, they are set to the observed minimum and maximum values. A suitable example is the use of $w_b = 0.1$ SUV in FDG-PET. After discretisation, the bin indices are replaced with bin centres so that the relationship with the original intensity scale is preserved. The resulting set is defined as:

$$X_{d,gl} = X_{gl,min} + (X_d - 0.5)\, w_b$$

Here, the total range becomes $G = [X_{gl,min} + 0.5\, w_b, X_{gl,max} - 0.5\, w_b]$, and the discretisation interval equals the bin width ($w_d = w_b$).

When intensities lack calibration, use fixed bin number discretisation. IBSI recommends $N_g = 1000$ bins (see *** 2.7). Let $X_{gl,min} = min(X_{gl})$ and $X_{gl,max} = max(X_{gl})$. Then bin width is:

$$w_b = \frac{X_{gl,max} - X_{gl,min}}{N_g}$$

This produces integer bin indices $X_d \in \{1, \dots, N_g\}$. Because of the lack of calibration, set $X_{d,gl} = X_d$, $w_d = 1$ and $G = [1, N_g]$.

Calculating the IVH

Let $G = [min(G), max(G)]$ denote the discrete intensity range used for IVH evaluation. For each discretised intensity $i \in G$ define:

- The fractional volume at intensity i, denoted v_i, as the fraction of ROI voxels with discretised intensity $\geq i$. Using the Iverson bracket $[\,.\,]$ (1 if condition true, 0 otherwise):

$$v_i = 1 - \frac{1}{N_v} \sum_{k=1}^{N_v} \left[X_{d,gl,k} < i \right]$$

- The intensity fraction corresponding to discretised intensity i, denoted γ_i, as the normalised position of i within the total range G:

$$\gamma_i = \frac{i - min(G)}{max(G) - min(G)}$$

Both v_i and γ_i are evaluated for every i in the total range G, including bins that may be absent from $X_{d,gl}$.

Feature Aggregation

IBSI recommends computing IVH features from the full 3D volume. Slice-wise computation followed by averaging is not recommended.

5-2- Explanation of Features [1]

Below V_x denotes the volume fraction at intensity fraction x (i.e. percentage $x\%$ expressed as a fraction between 0 and 1), and I_x denotes the intensity at volume fraction x (fraction).

5-2-1- Volume At Intensity Fraction (ivh_v10 / ivh_v90) [1]

The volume at intensity fraction V_x is defined as the largest fractional volume of the ROI that has a corresponding intensity fraction greater than or equal to x. For example, V_{10} measures the proportion of the ROI that remains after excluding the lowest 10% of the intensity fraction range. Similarly, V_{90} corresponds to the remaining volume when excluding all but the top 10% of the intensity fraction range.

Table 5-1: Reference values for the volume fraction at 10% intensity feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.324	__	Very Strong

Config. A	0.978	0.001	Strong
Config. B	0.977	0.001	Strong
Config. C	0.998	0.001	Moderate
Config. D	0.972	0.003	Strong
Config. E	0.975	0.002	Strong

Table 5-2: Reference values for the intensity at 10% volume feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.0946	—	Very Strong
Config. A	6.98×10^{-5}	1.03×10^{-5}	Strong
Config. B	7.31×10^{-5}	1.03×10^{-5}	Strong
Config. C	0.000152	2×10^{-5}	Strong
Config. D	9×10^{-5}	0.000415	Strong
Config. E	0.000157	0.000248	Strong

5-2-2- Intensity At Volume Fraction (ivh_i10 / ivh_i90) [1]

The intensity at volume fraction I_x is defined as the minimum discretised intensity that is present in at most fraction x of the ROI volume. For I_{10}, this corresponds to the lowest discretised intensity such that 10% (or less) of the ROI volume has intensities equal to or below it. For I_{90}, it is the lowest discretised intensity such that 90% (or less) of the ROI volume is included.

Table 5-3: Reference values for the intensity at 10% volume feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	5	—	Very Strong
Config. A	96	—	Strong
Config. B	92	—	Strong

Config. C	88.8	0.2	Moderate
Config. D	87	0.1	Strong
Config. E	770	5	Moderate

Table 5-4: Reference values for the intensity at 90% volume feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	2	—	Very Strong
Config. A	-128	8	Strong
Config. B	-135	8	Strong
Config. C	-421	14	Strong
Config. D	-303	20	Strong
Config. E	399	17	Moderate

5-2-3- Volume Fraction Difference Between Intensity Fractions (ivh_diff_v10_v90) [1]

This feature quantifies the difference in volume fractions between two intensity fraction thresholds: 10% and 90%. Formally, it is defined as:

$$V_{10-90} = V_{10} - V_{90}$$

This provides a measure of how much of the ROI is distributed across the intensity fraction range between 10% and 90%.

Table 5-5: Reference values for the volume fraction difference between 10% and 90% intensity feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	0.23	—	Very Strong
Config. A	0.978	0.001	Strong
Config. B	0.977	0.001	Strong

Config. C	0.997	0.001	Strong
Config. D	0.971	0.001	Strong
Config. E	0.974	0.001	Strong

5-2-4- Intensity Fraction Difference Between Volume Fractions (ivh_diff_i10_i90) [1]

This feature quantifies the spread of discretised intensities across the volume fractions 10% and 90%. It is defined as:

$$I_{10-} = I_{10} - I_{90}$$

This measure reflects the dynamic range of intensity values across the main bulk of the ROI (between the 10% and 90% volume fractions).

Table 5-6: Reference values for the intensity difference between 10% and 90% volume feature. [1]

Data	Value	Tol.	Consensus
Dig. phantom	3	—	Very Strong
Config. A	224	8	Strong
Config. B	227	8	Strong
Config. C	510	14	Strong
Config. D	390	20	Strong
Config. E	371	13	Moderate

5-2-5- Area Under The IVH Curve (ivh_auc) [1]

Note: This feature does not have reference values in the IBSI.

The area under the IVH curve can be approximated by calculating the Riemann sum using the trapezoidal rule.

If the ROI contains only one discretised intensity, the area under the IVH curve is defined as zero $(auc = 0)$.

6- Gray Level Co-occurrence Based Features

6-1- Introduction [1]

In this category, features are generally extracted from discretised image intensities. The gray level co-occurrence matrix (GLCM) then represents how pairs of these discretised gray levels occur among neighbouring pixels (2D) or voxels (3D) along one of the image directions.

For GLCM, the neighbourhood is typically 26-connected in 3D and 8-connected in 2D. In 3D, this reduces to 13 unique direction vectors for Chebyshev distance $\delta = 1$: $(0,0,1)$, $(0,1,0)$, $(1,0,0)$, $(0,1,1)$, $(0,1,-1)$, $(1,0,1)$, $(1,0,-1)$, $(1,1,0)$, $(1,-1,0)$, $(1,1,1)$, $(1,1,-1)$, $(1,-1,1)$, and $(1,-1,-1)$. In 2D, the corresponding unique directions are $(1,0,0)$, $(1,1,0)$, $(0,1,0)$, and $(-1,1,0)$.

The GLCM is computed separately for each direction vector. Let M_m denote the $N_g \times N_g$ gray level co-occurrence matrix, where N_g is the number of discretised gray levels within the ROI, and m is the chosen direction vector. Each element (i, j) of the matrix records how often gray levels i and j appear in neighbouring voxels along the forward direction $m^+ = m$ and the backward direction $m^- = -m$. The full matrix is then obtained by combining both orientations:

$$M_m = M_{m^+} + M_{m^-} = M_{m^+} + (M_{m^+})^T$$

This ensures that the resulting matrix M_m is symmetric. An example of calculating GLCM for each direction is shown in Fig. 6-1.

1	2	2	3
1	2	3	3
4	2	4	1
4	1	2	3

(a) Grey levels

$$\begin{array}{c|cccc} & & j & & \\ \hline & 0 & 3 & 0 & 0 \\ i & 0 & 1 & 3 & 1 \\ & 0 & 0 & 1 & 0 \\ & 2 & 1 & 0 & 0 \end{array}$$

(b) $\mathbf{M}_{\mathbf{m}_+=\rightarrow}$

$$\begin{array}{c|cccc} & & j & & \\ \hline & 0 & 0 & 0 & 2 \\ i & 3 & 1 & 0 & 1 \\ & 0 & 3 & 1 & 0 \\ & 0 & 1 & 0 & 0 \end{array}$$

(c) $\mathbf{M}_{\mathbf{m}_-=\leftarrow}$

Fig. 6-1: Gray levels (a) and corresponding gray level co-occurrence matrices for the 0◦ (b) and 180◦ directions (c). In vector notation these directions are $m_+ = (1, 0)$ and $m_- = (-1, 0)$. To calculate the symmetrical co-occurrence matrix M_m Both matrices are summed by element. [1]

GLCM-based features are computed from the probability distribution of the matrix elements. For example, consider the matrix $M_{m=(1,0)}$ from Fig. 6-1. A probability distribution P_m for gray level co-occurrences is obtained by normalising M_m by the sum of all its elements. Each element p_{ij} of P_m represents the joint probability of gray levels i and j appearing in neighbouring voxels along direction m. The row and column marginal probabilities are then defined as $p_i = \sum_{j=1}^{N_g} p_{ij}$ and $p_j = \sum_{i=1}^{N_g} p_{ij}$, respectively. Since P_m is symmetric by definition, $p_i = p_j$.

Additionally, diagonal and cross-diagonal probabilities, denoted p_{i-j} and p_{i+j}, are defined as sums over all pairs satisfying $|i - j| = k$ and $i + j = k$, respectively (see Fig. 6-2).

$$p_{i-j,k} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_{ij}\,[k = |i-j|] \qquad k = 0,\ldots,N_g - 1$$

$$p_{i+j,k} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_{ij}\,[k = |i+j|] \qquad k = 2,\ldots,2N_g$$

The Iverson bracket is used to include only the pairs meeting the specified condition, assigning 1 when the condition holds and 0 otherwise.

Although GLCMs are often computed with a distance of 1 ($\delta = 1$), larger distances can also be used. For instance, with distance 3 in 3D, the considered neighbouring voxels relative to the center include positions such as $(0,0,3)$, $(0,3,0)$, $(3,0,0)$, $(0,3,3)$, $(0,3,-3)$, $(3,0,3)$, $(3,0,-3)$, $(3,3,0)$, $(3,-3,0)$, $(3,3,3)$, $(3,3,-3)$, $(3,-3,3)$, and $(3,-3,-3)$.

6-2- Feature Aggregation [1]

IBSI [1] recommends aggregating GLCM-based texture features across directions to reduce orientation bias and enhance feature robustness. Here's an overview of each strategy for 2D, 2.5D, and 3D features. These strategies are also illustrated in Fig. 6-3.

1. **2D Averaged (BTW3):** Compute features for each 2D directional GLCM and average them across all directions and image slices.
2. **2D Slice-merged (SUJT):** Merge all 2D directional GLCMs within each slice into a single matrix, compute features, and average across slices.
3. **2.5D Direction-merged (JJUI):** For each direction, merge all slices' GLCMs across that direction, compute the feature, then average across directions.
4. **2.5D Fully Merged (ZW7Z):** Combine all 2D directional GLCMs across all slices into one matrix before feature extraction.
5. **3D Averaged (ITBB):** Extract features for each 3D directional GLCM, then average across all 3D directions.
6. **3D Fully Merged (IAZD):** Merge all 3D directional GLCMs into one aggregate matrix and compute features from it.

	j				$\sum_j$
	0	3	0	2	**5**
	3	2	3	2	**10**
i	0	3	2	0	**5**
	2	2	0	0	**4**
$\sum_i$	**5**	**10**	**5**	**4**	**24**

(a) $\mathbf{M_{m=(1,0)}}$ with margins

	j				$p_{i.}$
	0.00	0.13	0.00	0.08	**0.21**
	0.13	0.08	0.13	0.08	**0.42**
i	0.00	0.13	0.08	0.00	**0.21**
	0.08	0.08	0.00	0.00	**0.17**
$p_{.j}$	**0.21**	**0.42**	**0.21**	**0.17**	**1.00**

(b) $\mathbf{P_{m=(1,0)}}$ with margins

$k = \lvert i-j \rvert$	0	1	2	3
p_{i-j}	0.17	0.50	0.17	0.17

(c) Diagonal probability for $\mathbf{P_{m=(1,0)}}$

$k = i+j$	2	3	4	5	6	7	8
p_{i+j}	0.00	0.25	0.08	0.42	0.25	0.00	0.00

(d) Cross-diagonal probability for $\mathbf{P_{m=(1,0)}}$

Fig. 6-2: Gray level co-occurrence matrix for the 0◦ direction (a); its corresponding probability matrix $P_{m=(1,0)}$ with marginal probabilities $p_{i\,.}$ and $p_{.j}$ (b); the diagonal probabilities p_{i-j} (c); and the cross-diagonal probabilities p_{i+j} (d). Discrepancies in panels b, c, and d are due to rounding errors caused by showing only two decimal places. Also, note that due to GLCM symmetry marginal probabilities $p_{i\,.}$ and $p_{.j}$ are the same in both row and column margins of panel b. [1]

In aggregation approaches 2, 3, 4, and 6, the GLCMs are combined by adding the co-occurrence counts of corresponding elements (i, j) across all relevant matrices. After merging, the resulting GLCM is normalised to obtain a probability distribution, which then serves as the basis for computing GLCM features. It is important to note that the chosen aggregation strategy can have a considerable influence on the resulting feature values.

6-3- Distances and Distance Weighting [1]

Distance weighting of GLCMs can be applied by multiplying the matrix M with a weighting factor ω. The default choice is $\omega = 1$, but alternative schemes may use inverse distance weighting, such as $\omega = \lVert m \rVert^{-1}$, or exponential weighting, for example $\omega = exp\left(-\lVert m \rVert^2\right)$, where $\lVert m \rVert$ represents the length of the direction vector m. The impact of this weighting on feature values is context-dependent. For weighting to take effect, matrices must be merged prior to feature calculation; otherwise, it has no influence. Weighting also does not alter results when the default neighbourhood is used with the Chebyshev norm. Similarly, no changes occur if Manhattan or

Chebyshev norms are employed both in defining a non-standard neighbourhood and in weighting. Moreover, for distance=1, the effect of weighting can vanish depending on the norm used. Due to these limitations and inconsistencies, the use of distance weighting in GLCM computation is generally discouraged.

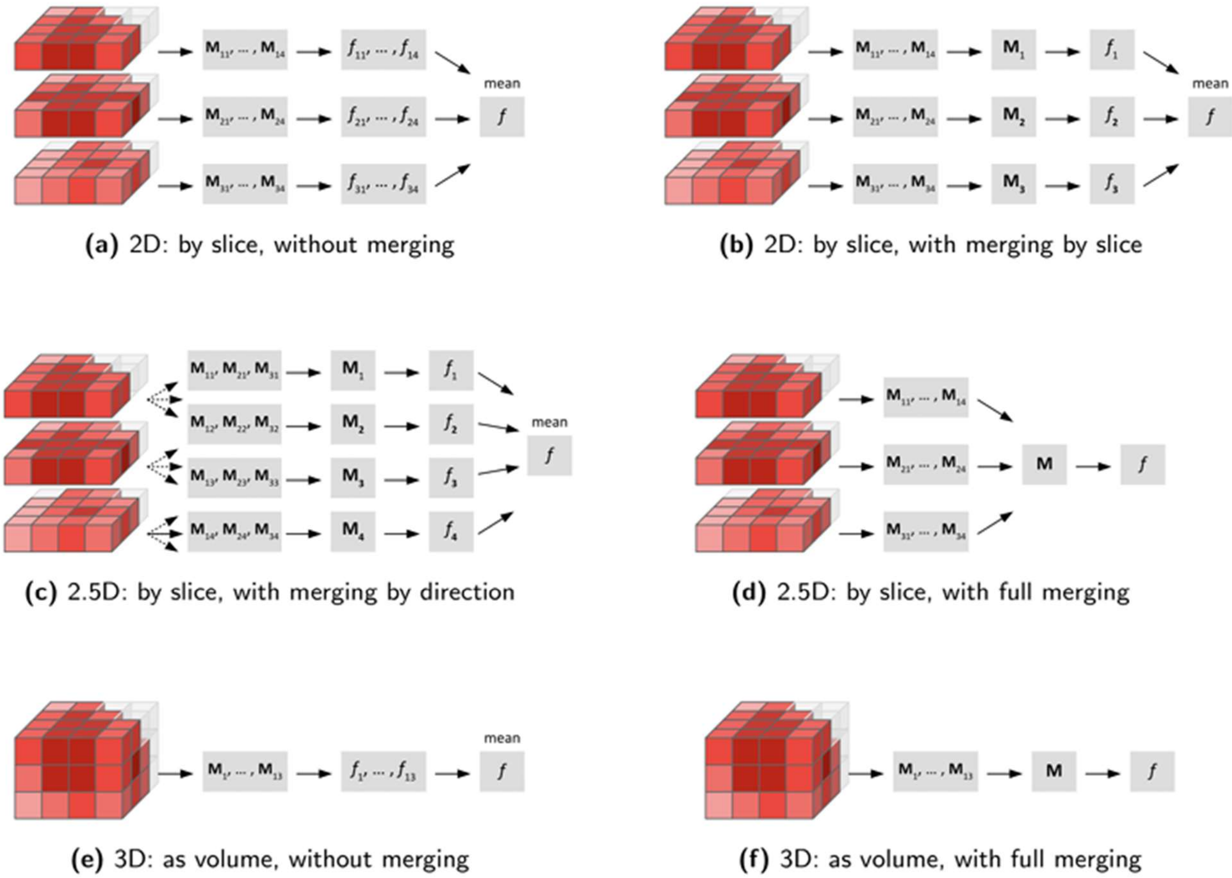

Fig. 6-3: Different approaches of feature aggregation in [1].

6-4- Explanation of Features [1]

Unless noted otherwise, the feature formulas below are identical for 2D, 2.5D, and 3D; only the aggregation strategy differs (see 6-2). We indicate the chosen aggregation in the suffix: **2D_avg**, **2D_comb**, **2_5D_avg**, **2_5D_comb**, **3D_avg**, and **3D_comb**. In the merged variants, GLCMs are summed element-wise before normalisation and feature calculation.

6-4-1- Joint Maximum (cm_joint_max_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Joint maximum is the probability value of the most frequent gray level co-occurrence observed in the GLCM.

$$joint_{max} = maximum\ value\ of\ p_{ij}$$

Table 6-1: Reference values for the intensity difference between 10% and 90% volume feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.519	__	Very Strong
	2D, Slice-merged	0.512	__	Strong
	2.5D, Direction-merged	0.489	__	Strong
	2.5D, Merged	0.492	__	Strong
	3D, Averaged	0.503	__	Very Strong
	3D, Merged	0.509	__	Very Strong
Config. A	2D, Averaged	0.109	0.001	Strong
	2D, Slice-merged	0.109	0.001	Strong
	2.5D, Direction-merged	0.0943	0.0008	Strong
	2.5D, Merged	0.0943	0.0008	Strong
Config. B	2D, Averaged	0.156	0.002	Strong
	2D, Slice-merged	0.156	0.002	Strong
	2.5D, Direction-merged	0.126	0.002	Strong
	2.5D, Merged	0.126	0.002	Strong
Config. C	3D, Averaged	0.111	0.002	Strong
	3D, Merged	0.111	0.002	Very Strong
Config. D	3D, Averaged	0.232	0.007	Strong
	3D, Merged	0.232	0.007	Strong
Config. E	3D, Averaged	0.153	0.003	Moderate

	3D, Merged	0.153	0.003	Strong

6-4-2- Joint Average (cm_joint_avg_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

This feature quantifies the expected gray level value by summing all gray levels weighted by their joint probabilities in the GLCM [1].

$$joint_{avg} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} i\, p_{ij}$$

Since this calculation only reflects the distribution of i, and ignores the distribution of j, it should be applied only to symmetric GLCMs, where $p_{i.} = p_{.j}$, where $i = j$ [3].

Table 6-2: Reference values for the joint average feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	2.14	__	Very Strong
	2D, Slice-merged	2.14	__	Strong
	2.5D, Direction-merged	2.2	__	Strong
	2.5D, Merged	2.2	__	Strong
	3D, Averaged	2.14	__	Very Strong
	3D, Merged	2.15	__	Very Strong
Config. A	2D, Averaged	20.6	0.1	Strong
	2D, Slice-merged	20.6	0.1	Strong
	2.5D, Direction-merged	21.3	0.1	Strong
	2.5D, Merged	21.3	0.1	Strong
Config. B	2D, Averaged	18.7	0.3	Strong
	2D, Slice-merged	18.7	0.3	Strong

	2.5D, Direction-merged	19.2	0.3	Strong
	2.5D, Merged	19.2	0.3	Strong
Config. C	3D, Averaged	39	0.2	Strong
	3D, Merged	39	0.2	Strong
Config. D	3D, Averaged	18.9	0.5	Strong
	3D, Merged	18.9	0.5	Strong
Config. E	3D, Averaged	22.1	0.3	Strong
	3D, Merged	22.1	0.3	Strong

6-4-3- Joint Variance (cm_joint_var_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

This feature (also referred to as sum of squares) measures the spread of gray level values around the joint mean, reflecting the variability of intensity pairs in the GLCM [1]:

$$joint_{var} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \left(i - joint_{avg}\right)^2 p_{ij}$$

Table 6-3: Reference values for the joint variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	2.69	__	Very Strong
	2D, Slice-merged	2.71	__	Strong
	2.5D, Direction-merged	3.22	__	Strong
	2.5D, Merged	3.24	__	Strong
	3D, Averaged	3.1	__	Very Strong

	3D, Merged	3.13	__	Very Strong
Config. A	2D, Averaged	27	0.4	Strong
	2D, Slice-merged	27	0.4	Strong
	2.5D, Direction-merged	18.6	0.5	Strong
	2.5D, Merged	18.6	0.5	Strong
Config. B	2D, Averaged	21	0.3	Strong
	2D, Slice-merged	21	0.3	Strong
	2.5D, Direction-merged	14.2	0.1	Strong
	2.5D, Merged	14.2	0.1	Strong
Config. C	3D, Averaged	73.7	2	Strong
	3D, Merged	73.8	2	Very Strong
Config. D	3D, Averaged	17.6	0.4	Strong
	3D, Merged	17.6	0.4	Strong
Config. E	3D, Averaged	24.4	0.9	Moderate
	3D, Merged	24.4	0.9	Strong

6-4-4- Joint Entropy (cm_joint_entr_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

This feature is defined as [1]:

$$joint_{entropy} = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_{ij} \, log_2 \, p_{ij}$$

Table 6-4: Reference values for the joint entropy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	2.05	__	Very Strong
	2D, Slice-merged	2.24	__	Strong
	2.5D, Direction-merged	2.48	__	Strong
	2.5D, Merged	2.61	__	Strong
	3D, Averaged	2.4	__	Very Strong
	3D, Merged	2.57	__	Very Strong
Config. A	2D, Averaged	5.82	0.04	Strong
	2D, Slice-merged	5.9	0.04	Strong
	2.5D, Direction-merged	5.78	0.04	Strong
	2.5D, Merged	5.79	0.04	Strong
Config. B	2D, Averaged	5.26	0.02	Strong
	2D, Slice-merged	5.45	0.01	Strong
	2.5D, Direction-merged	5.45	0.01	Strong
	2.5D, Merged	5.46	0.01	Strong
Config. C	3D, Averaged	6.39	0.06	Strong
	3D, Merged	6.42	0.06	Very Strong
Config. D	3D, Averaged	4.95	0.03	Strong
	3D, Merged	4.96	0.03	Strong

Config. E	3D, Averaged	5.6	0.03	Moderate
	3D, Merged	5.61	0.03	Strong

6-4-5- Difference Average (cm_diff_avg_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Difference average (computed from the diagonal probabilities) is defined as:

$$diff_{avg} = \sum_{k=0}^{N_g-1} k\, p_{i-j}(k)$$

By definition, the difference average is equivalent to the dissimilarity feature [1].

Table 6-5: Reference values for the difference average feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	1.42	__	Very Strong
	2D, Slice-merged	1.4	__	Strong
	2.5D, Direction-merged	1.46	__	Strong
	2.5D, Merged	1.44	__	Strong
	3D, Averaged	1.43	__	Very Strong
	3D, Merged	1.38	__	Very Strong
Config. A	2D, Averaged	1.58	0.03	Strong
	2D, Slice-merged	1.57	0.03	Strong
	2.5D, Direction-merged	1.35	0.03	Strong
	2.5D, Merged	1.35	0.03	Strong
Config. B	2D, Averaged	1.81	0.01	Strong
	2D, Slice-merged	1.81	0.01	Strong

	2.5D, Direction-merged	1.47	0.01	Strong
	2.5D, Merged	1.47	0.01	Strong
Config. C	3D, Averaged	2.17	0.05	Strong
	3D, Merged	2.16	0.05	Strong
Config. D	3D, Averaged	1.29	0.01	Strong
	3D, Merged	1.29	0.01	Strong
Config. E	3D, Averaged	1.7	0.01	Strong
	3D, Merged	1.7	0.01	Strong

6-4-6- Difference Variance (cm_diff_var_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Difference variance (based on the diagonal probabilities) is defined as [1]:

$$diff_{var} = \sum_{k=0}^{N_g-1} \left(k - diff_{avg}\right)^2 p_{i-j,k}$$

Table 6-6: Reference values for the difference variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	2.9	__	Very Strong
	2D, Slice-merged	3.06	__	Strong
	2.5D, Direction-merged	3.11	__	Strong
	2.5D, Merged	3.23	__	Strong
	3D, Averaged	3.06	__	Very Strong
	3D, Merged	3.21	__	Very Strong

Config. A	2D, Averaged	4.94	0.19	Strong
	2D, Slice-merged	4.96	0.19	Strong
	2.5D, Direction-merged	4.12	0.2	Strong
	2.5D, Merged	4.14	0.2	Strong
Config. B	2D, Averaged	7.74	0.05	Strong
	2D, Slice-merged	7.76	0.05	Strong
	2.5D, Direction-merged	6.48	0.06	Strong
	2.5D, Merged	6.48	0.06	Strong
Config. C	3D, Averaged	14.4	0.5	Strong
	3D, Merged	14.4	0.5	Strong
Config. D	3D, Averaged	5.37	0.11	Strong
	3D, Merged	5.38	0.11	Strong
Config. E	3D, Averaged	8.22	0.06	Strong
	3D, Merged	8.23	0.06	Strong

6-4-7- Difference Entropy (cm_diff_entr_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Difference entropy (computed from the diagonal probabilities) is defined as [1]:

$$diff_{entropy} = -\sum_{k=0}^{N_g-1} p_{i-j,k}\, log_2\, p_{i-j,k}$$

Table 6-7: Reference values for the difference entropy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	1.4	__	Very Strong
	2D, Slice-merged	1.49	__	Strong
	2.5D, Direction-merged	1.61	__	Strong
	2.5D, Merged	1.67	__	Strong
	3D, Averaged	1.56	__	Very Strong
	3D, Merged	1.64	__	Very Strong
Config. A	2D, Averaged	2.27	0.03	Strong
	2D, Slice-merged	2.28	0.03	Strong
	2.5D, Direction-merged	2.16	0.03	Strong
	2.5D, Merged	2.16	0.03	Strong
Config. B	2D, Averaged	2.35	0.01	Strong
	2D, Slice-merged	2.38	0.01	Strong
	2.5D, Direction-merged	2.24	0.01	Moderate
	2.5D, Merged	2.24	0.01	Strong
Config. C	3D, Averaged	2.64	0.03	Strong
	3D, Merged	2.64	0.03	Very Strong
Config. D	3D, Averaged	2.13	0.01	Strong
	3D, Merged	2.14	0.01	Strong
Config. E	3D, Averaged	2.39	0.01	Strong

	3D, Merged	2.4	0.01	Strong

6-4-8- Sum Average (cm_sum_avg_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Sum average (calculated from the cross-diagonal probabilities) is defined as:

$$sum_{avg} = \sum_{k=2}^{2N_g} k \, p_{i+j,k}$$

By definition, this feature equals twice the joint average (see 6-4-2) [1]:

$$sum_{avg} = 2 \, joint_{avg}$$

Table 6-8: Reference values for the sum average feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	4.28	__	Very Strong
	2D, Slice-merged	4.29	__	Strong
	2.5D, Direction-merged	4.41	__	Strong
	2.5D, Merged	4.41	__	Strong
	3D, Averaged	4.29	__	Very Strong
	3D, Merged	4.3	__	Very Strong
Config. A	2D, Averaged	41.3	0.1	Strong
	2D, Slice-merged	41.3	0.1	Strong
	2.5D, Direction-merged	42.7	0.1	Strong
	2.5D, Merged	42.7	0.1	Strong
Config. B	2D, Averaged	37.4	0.5	Strong

	2D, Slice-merged	37.4	0.5	Strong
	2.5D, Direction-merged	38.5	0.6	Strong
	2.5D, Merged	38.5	0.6	Strong
Config. C	3D, Averaged	78	0.3	Strong
	3D, Merged	78	0.3	Strong
Config. D	3D, Averaged	37.7	0.8	Strong
	3D, Merged	37.7	0.8	Strong
Config. E	3D, Averaged	44.3	0.4	Strong
	3D, Merged	44.3	0.4	Strong

6-4-9- Sum Variance (cm_sum_var_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Sum variance (based on the cross-diagonal probabilities) is defined as:

$$sum_{var} = \sum_{k=2}^{2N_g} \left(k - sum_{avg}\right)^2 p_{i+j,k}$$

Where sum_{avg} is the previously defined sum average. By definition, the sum variance is mathematically equivalent to the cluster tendency feature [1].

Table 6-9: Reference values for the sum variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus

Dig. phantom	2D, Averaged	5.47	__	Very Strong
	2D, Slice-merged	5.66	__	Strong
	2.5D, Direction-merged	7.48	__	Strong
	2.5D, Merged	7.65	__	Strong
	3D, Averaged	7.07	__	Very Strong
	3D, Merged	7.41	__	Very Strong
Config. A	2D, Averaged	100	1	Strong
	2D, Slice-merged	100	1	Strong
	2.5D, Direction-merged	68.5	1.3	Strong
	2.5D, Merged	68.5	1.3	Strong
Config. B	2D, Averaged	72.1	1	Strong
	2D, Slice-merged	72.3	1	Strong
	2.5D, Direction-merged	48.1	0.4	Strong
	2.5D, Merged	48.1	0.4	Strong
Config. C	3D, Averaged	276	8	Strong
	3D, Merged	276	8	Very Strong
Config. D	3D, Averaged	63.4	1.3	Strong
	3D, Merged	63.5	1.3	Strong
Config. E	3D, Averaged	86.6	3.3	Moderate
	3D, Merged	86.7	3.3	Strong

6-4-10- Sum Entropy (cm_sum_entr_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Sum entropy (derived from the cross-diagonal probabilities) is defined as:

$$sum_{entropy} = -\sum_{k=2}^{2N_g} p_{i+j,k} \, log_2 \, p_{i+j,k}$$

This feature measures the uncertainty or randomness in the distribution of gray level sums in the GLCM.

Table 6-10: Reference values for the sum entropy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	1.6	__	Very Strong
	2D, Slice-merged	1.79	__	Strong
	2.5D, Direction-merged	2.01	__	Strong
	2.5D, Merged	2.14	__	Strong
	3D, Averaged	1.92	__	Very Strong
	3D, Merged	2.11	__	Very Strong
Config. A	2D, Averaged	4.19	0.03	Strong
	2D, Slice-merged	4.21	0.03	Strong
	2.5D, Direction-merged	4.17	0.03	Strong
	2.5D, Merged	4.18	0.03	Strong
Config. B	2D, Averaged	3.83	0.01	Strong
	2D, Slice-merged	3.89	0.01	Strong
	2.5D, Direction-merged	3.91	0.01	Strong
	2.5D, Merged	3.91	0.01	Strong

Config. C	3D, Averaged	4.56	0.04	Strong
	3D, Merged	4.45	0.04	Very Strong
Config. D	3D, Averaged	3.68	0.02	Strong
	3D, Merged	3.68	0.02	Strong
Config. E	3D, Averaged	3.96	0.02	Moderate
	3D, Merged	3.97	0.02	Strong

6-4-11- Angular Second Moment (cm_energy_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

This feature quantifies the uniformity or orderliness of the GLCM and is defined as:

$$energy = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_{ij}^2$$

This feature is also referred to as Energy or Uniformity.

Table 6-11: Reference values for the angular second moment feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.368	__	Very Strong
	2D, Slice-merged	0.352	__	Strong
	2.5D, Direction-merged	0.286	__	Strong
	2.5D, Merged	0.277	__	Strong
	3D, Averaged	0.303	__	Very Strong
	3D, Merged	0.291	__	Very Strong
Config. A	2D, Averaged	0.045	0.0008	Strong
	2D, Slice-merged	0.0446	0.0008	Strong

	2.5D, Direction-merged	0.0429	0.0007	Strong
	2.5D, Merged	0.0427	0.0007	Strong
Config. B	2D, Averaged	0.0678	0.0006	Strong
	2D, Slice-merged	0.0669	0.0006	Strong
	2.5D, Direction-merged	0.0581	0.0006	Strong
	2.5D, Merged	0.058	0.0006	Strong
Config. C	3D, Averaged	0.045	0.001	Strong
	3D, Merged	0.0447	0.001	Very Strong
Config. D	3D, Averaged	0.11	0.003	Strong
	3D, Merged	0.109	0.003	Strong
Config. E	3D, Averaged	0.0638	0.0009	Strong
	3D, Merged	0.0635	0.0009	Strong

6-4-12- Contrast (cm_contrast_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Contrast measures the intensity variation in the GLCM. Matrix elements that correspond to large gray-level differences are weighted more heavily. It is defined as:

$$contrast = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i-j)^2\, p_{ij}$$

Although Haralick's original definition appeared more complex, IBSI [1] simplifies it to the formulation shown above.

Table 6-12: Reference values for the contrast feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	5.28	__	Very Strong

	2D, Slice-merged	5.19	—	Strong
	2.5D, Direction-merged	5.39	—	Strong
	2.5D, Merged	5.29	—	Strong
	3D, Averaged	5.32	—	Very Strong
	3D, Merged	5.12	—	Very Strong
Config. A	2D, Averaged	7.85	0.26	Strong
	2D, Slice-merged	7.82	0.26	Strong
	2.5D, Direction-merged	5.96	0.27	Strong
	2.5D, Merged	5.95	0.27	Strong
Config. B	2D, Averaged	11.9	0.1	Strong
	2D, Slice-merged	11.8	0.1	Strong
	2.5D, Direction-merged	8.66	0.09	Strong
	2.5D, Merged	8.65	0.09	Strong
Config. C	3D, Averaged	19.2	0.7	Strong
	3D, Merged	19.1	0.7	Very Strong
Config. D	3D, Averaged	7.07	0.13	Strong
	3D, Merged	7.05	0.13	Strong
Config. E	3D, Averaged	11.1	0.1	Strong
	3D, Merged	11.1	0.1	Strong

6-4-13- Dissimilarity (cm_dissimilarity_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Dissimilarity quantifies differences in gray levels within the GLCM and is closely related to the contrast feature. It is defined as:

$$dissimilarity = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} |i-j|\, p_{ij}$$

By definition, dissimilarity is equivalent to the difference average feature [1].

Table 6-13: Reference values for the dissimilarity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	1.42	__	Very Strong
	2D, Slice-merged	1.4	__	Strong
	2.5D, Direction-merged	1.46	__	Strong
	2.5D, Merged	1.44	__	Strong
	3D, Averaged	1.43	__	Very Strong
	3D, Merged	1.38	__	Very Strong
Config. A	2D, Averaged	1.58	0.03	Strong
	2D, Slice-merged	1.57	0.03	Strong
	2.5D, Direction-merged	1.35	0.03	Strong
	2.5D, Merged	1.35	0.03	Strong
Config. B	2D, Averaged	1.81	0.01	Strong
	2D, Slice-merged	1.81	0.01	Strong

	2.5D, Direction-merged	1.47	0.01	Strong
	2.5D, Merged	1.47	0.01	Strong
Config. C	3D, Averaged	2.17	0.05	Strong
	3D, Merged	2.16	0.05	Very Strong
Config. D	3D, Averaged	1.29	0.01	Strong
	3D, Merged	1.29	0.01	Strong
Config. E	3D, Averaged	1.7	0.01	Strong
	3D, Merged	1.7	0.01	Strong

6-4-14- Inverse Difference (cm_inv_diff_2D_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Inverse difference measures texture homogeneity by down-weighting gray-level pairs with large intensity differences. Its value is maximal when all gray levels are identical:

$$diff_{inv} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{p_{ij}}{1 + |i - j|}$$

This can also be expressed using diagonal probabilities:

$$diff_{inv} = \sum_{k=0}^{N_g - 1} \frac{p_{i-j,k}}{1 + k}$$

Table 6-14: Reference values for the inverse difference feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.678	__	Very Strong
	2D, Slice-merged	0.683	__	Strong
	2.5D, Direction-merged	0.668	__	Strong

	2.5D, Merged	0.673	__	Strong
	3D, Averaged	0.677	__	Very Strong
	3D, Merged	0.688	__	Very Strong
Config. A	2D, Averaged	0.581	0.003	Strong
	2D, Slice-merged	0.581	0.003	Strong
	2.5D, Direction-merged	0.605	0.003	Strong
	2.5D, Merged	0.605	0.003	Strong
Config. B	2D, Averaged	0.592	0.001	Strong
	2D, Slice-merged	0.593	0.001	Strong
	2.5D, Direction-merged	0.628	0.001	Strong
	2.5D, Merged	0.628	0.001	Strong
Config. C	3D, Averaged	0.582	0.004	Strong
	3D, Merged	0.583	0.004	Very Strong
Config. D	3D, Averaged	0.682	0.003	Strong
	3D, Merged	0.682	0.003	Strong
Config. E	3D, Averaged	0.608	0.001	Strong
	3D, Merged	0.608	0.001	Strong

6-4-15- Normalised Inverse Difference (cm_inv_diff_norm_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

IBSI uses the normalized inverse difference proposed by [4]:

$$diff_{inv.norm} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{p_{ij}}{1 + \frac{|i-j|}{N_g}}$$

Equivalent form using diagonal probabilities:

$$diff_{inv.norm} = \sum_{k=0}^{N_g-1} \frac{p_{i-j,k}}{1 + \frac{k}{N_g}}$$

Note: Clausi's original definition used $|i-j|^2/N_g^2$,which seems to be a misprint since it matches the normalized inverse difference moment instead [1].

Table 6-15: Reference values for the normalised inverse difference feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.851	__	Very Strong
	2D, Slice-merged	0.854	__	Strong
	2.5D, Direction-merged	0.847	__	Strong
	2.5D, Merged	0.85	__	Strong
	3D, Averaged	0.851	__	Very Strong
	3D, Merged	0.856	__	Very Strong
Config. A	2D, Averaged	0.961	0.001	Strong
	2D, Slice-merged	0.961	0.001	Strong
	2.5D, Direction-merged	0.966	0.001	Strong

	2.5D, Merged	0.966	0.001	Strong
Config. B	2D, Averaged	0.952	0.001	Strong
	2D, Slice-merged	0.952	0.001	Strong
	2.5D, Direction-merged	0.96	0.001	Strong
	2.5D, Merged	0.96	0.001	Strong
Config. C	3D, Averaged	0.966	0.001	Strong
	3D, Merged	0.966	0.001	Very Strong
Config. D	3D, Averaged	0.965	0.001	Strong
	3D, Merged	0.965	0.001	Strong
Config. E	3D, Averaged	0.955	0.001	Strong
	3D, Merged	0.955	0.001	Strong

6-4-16- Inverse Difference Moment (cm_inv_diff_mom_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Inverse Difference Moment (IDM), also called homogeneity, is similar to inverse difference but applies a stronger down-weighting for larger intensity differences [1]:

$$diff_{inv.mom} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{p_{ij}}{1+(i-j)^2}$$

Alternative form with diagonal probabilities [1]:

$$diff_{inv.mom} = \sum_{k=0}^{N_g-1} \frac{p_{i-j,k}}{1+k^2}$$

Table 6-16: Reference values for the inverse difference moment feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.619	__	Very Strong
	2D, Slice-merged	0.625	__	Strong
	2.5D, Direction-merged	0.606	__	Strong
	2.5D, Merged	0.613	__	Strong
	3D, Averaged	0.618	__	Very Strong
	3D, Merged	0.631	__	Very Strong
Config. A	2D, Averaged	0.544	0.003	Strong
	2D, Slice-merged	0.544	0.003	Strong
	2.5D, Direction-merged	0.573	0.003	Strong
	2.5D, Merged	0.573	0.003	Strong
Config. B	2D, Averaged	0.557	0.001	Strong
	2D, Slice-merged	0.558	0.001	Strong
	2.5D, Direction-merged	0.6	0.001	Strong
	2.5D, Merged	0.6	0.001	Strong
Config. C	3D, Averaged	0.547	0.004	Strong
	3D, Merged	0.548	0.004	Very Strong
Config. D	3D, Averaged	0.656	0.003	Strong
	3D, Merged	0.657	0.003	Strong
Config. E	3D, Averaged	0.576	0.001	Strong

	3D, Merged	0.577	0.001	Strong

6-4-17- Normalised Inverse Difference Moment (cm_inv_diff_mom_norm_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

IBSI [1] normalizes IDM as it is defined in [4]:

$$diff_{inv.mom.norm} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{p_{ij}}{1 + \frac{(i-j)^2}{N_g{}^2}}$$

Diagonal form:

$$diff_{inv.mom.norm} = \sum_{k=0}^{N_g-1} \frac{p_{i-j,k}}{1 + \left(\frac{k}{N_g}\right)^2}$$

Table 6-17: Reference values for the normalised inverse difference moment feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.899	__	Very Strong
	2D, Slice-merged	0.901	__	Strong
	2.5D, Direction-merged	0.897	__	Strong
	2.5D, Merged	0.899	__	Strong
	3D, Averaged	0.898	__	Very Strong
	3D, Merged	0.902	__	Very Strong
Config. A	2D, Averaged	0.994	0.001	Strong
	2D, Slice-merged	0.994	0.001	Strong
	2.5D, Direction-merged	0.996	0.001	Strong

	2.5D, Merged	0.996	0.001	Strong
Config. B	2D, Averaged	0.99	0.001	Strong
	2D, Slice-merged	0.99	0.001	Strong
	2.5D, Direction-merged	0.992	0.001	Strong
	2.5D, Merged	0.992	0.001	Strong
Config. C	3D, Averaged	0.994	0.001	Strong
	3D, Merged	0.994	0.001	Very Strong
Config. D	3D, Averaged	0.994	0.001	Strong
	3D, Merged	0.994	0.001	Strong
Config. E	3D, Averaged	0.99	0.001	Strong
	3D, Merged	0.99	0.001	Strong

6-4-18- Inverse Variance (cm_inv_var_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

This feature is defined as [1]:

$$var_{inv} = 2\sum_{i=1}^{N_g} \sum_{j>i}^{N_g} \frac{p_{ij}}{(i-j)^2}$$

Alternative form with diagonal probabilities [1]:

$$var_{inv} = \sum_{k=1}^{N_g-1} \frac{p_{i-j,k}}{k^2}$$

Note that in this equation, k starts from 1 instead of 0.

Table 6-18: Reference values for the inverse variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus

Dig. phantom	2D, Averaged	0.0567	__	Very Strong
	2D, Slice-merged	0.0553	__	Strong
	2.5D, Direction-merged	0.0597	__	Strong
	2.5D, Merged	0.0582	__	Strong
	3D, Averaged	0.0604	__	Very Strong
	3D, Merged	0.0574	__	Very Strong
Config. A	2D, Averaged	0.441	0.001	Strong
	2D, Slice-merged	0.441	0.001	Strong
	2.5D, Direction-merged	0.461	0.002	Strong
	2.5D, Merged	0.461	0.002	Strong
Config. B	2D, Averaged	0.401	0.002	Strong
	2D, Slice-merged	0.401	0.002	Strong
	2.5D, Direction-merged	0.424	0.003	Strong
	2.5D, Merged	0.424	0.003	Strong
Config. C	3D, Averaged	0.39	0.003	Strong
	3D, Merged	0.39	0.003	Very Strong
Config. D	3D, Averaged	0.341	0.005	Strong
	3D, Merged	0.34	0.005	Strong
Config. E	3D, Averaged	0.41	0.004	Strong
	3D, Merged	0.41	0.004	Strong

6-4-19- Correlation (cm_corr_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Correlation measures the linear dependency of gray levels in the GLCM [5]. Using row and column marginal probabilities, we can calculate this feature using these equations [1]:

$$correlation = \frac{1}{\sigma_{i.}^2}\left(-\mu_{i.}^2 + \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} i\, j\, p_{ij}\right)$$

An equivalent formulation [1]:

$$correlation = \frac{1}{\sigma_{i.}\,\sigma_{.j}}\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} (i-\mu_{i.})(j-\mu_{.j})\, p_{ij}$$

Due to matrix symmetry, we can simplify it to [1]:

$$correlation = \frac{1}{\sigma_{i.}^2}\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} (i-\mu_{i.})(j-\mu_{i.})\, p_{ij}$$

Where $\mu_{i.} = \sum_{i=1}^{N_g} i\, p_{i.}$ and $\sigma_{i.} = \left(\sum_{i=1}^{N_g} (i-\mu_{i.})^2\, p_{i.}\right)^{\frac{1}{2}}$ are the mean and standard deviation of row marginal probability $p_{i.}$. The mean and standard deviation of column marginal probability $p_{.j}$, i.e. $\mu_{.j}$ and $\sigma_{.j}$, are defined similarly [1].

Table 6-19: Reference values for the correlation feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	−0.0121	__	Very Strong
	2D, Slice-merged	0.0173	__	Strong
	2.5D, Direction-merged	0.178	__	Strong
	2.5D, Merged	0.182	__	Strong
	3D, Averaged	0.157	__	Very Strong
	3D, Merged	0.183	__	Very Strong
Config. A	2D, Averaged	0.778	0.002	Strong

	2D, Slice-merged	0.78	0.002	Strong
	2.5D, Direction-merged	0.839	0.003	Strong
	2.5D, Merged	0.84	0.003	Strong
Config. B	2D, Averaged	0.577	0.002	Strong
	2D, Slice-merged	0.58	0.002	Strong
	2.5D, Direction-merged	0.693	0.003	Strong
	2.5D, Merged	0.695	0.003	Strong
Config. C	3D, Averaged	0.869	0.001	Strong
	3D, Merged	0.871	0.001	Strong
Config. D	3D, Averaged	0.798	0.005	Strong
	3D, Merged	0.8	0.005	Strong
Config. E	3D, Averaged	0.771	0.006	Moderate
	3D, Merged	0.773	0.006	Strong

6-4-20- Autocorrelation (cm_auto_corr_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

For this feature, IBSI [1] uses following definition which measures the weighted average of co-occurring gray-level pairs:

$$autocorrelation = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} i\, j\, p_{ij}$$

Table 6-20: Reference values for the autocorrelation feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus

Dig. phantom	2D, Averaged	5.09	__	Very Strong
	2D, Slice-merged	5.14	__	Strong
	2.5D, Direction-merged	5.4	__	Strong
	2.5D, Merged	5.45	__	Strong
	3D, Averaged	5.06	__	Very Strong
	3D, Merged	5.19	__	Very Strong
Config. A	2D, Averaged	455	2	Strong
	2D, Slice-merged	455	2	Strong
	2.5D, Direction-merged	471	2	Strong
	2.5D, Merged	471	2	Strong
Config. B	2D, Averaged	369	11	Strong
	2D, Slice-merged	369	11	Strong
	2.5D, Direction-merged	380	11	Strong
	2.5D, Merged	380	11	Strong
Config. C	3D, Averaged	1.58×10^{3}	10	Strong
	3D, Merged	1.58×10^{3}	10	Strong
Config. D	3D, Averaged	370	16	Strong
	3D, Merged	370	16	Strong
Config. E	3D, Averaged	509	8	Strong
	3D, Merged	509	8	Strong

6-4-21- Cluster Tendency (cm_clust_tend_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Cluster tendency (sometimes referred to as the *sum variance* feature) measures the degree to which gray-level pairs group around the mean of the distribution. It is defined as:

$$cluster_{tend} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \left(i + j - \mu_{i.} - \mu_{.j}\right)^2 p_{ij}$$

Where $\mu_{i.} = \sum_{i=1}^{N_g} i\, p_{i.}$ and $\mu_{.j} = \sum_{j=1}^{N_g} j\, p_{.j}$. Since the GLCM $P_{..}$ is symmetric, the expression can also be written as:

$$cluster_{tend} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i + j - 2\mu_{i.})^2\, p_{ij}$$

Table 6-21: Reference values for the cluster tendency feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	5.47	__	Very Strong
	2D, Slice-merged	5.66	__	Strong
	2.5D, Direction-merged	7.48	__	Strong
	2.5D, Merged	7.65	__	Strong
	3D, Averaged	7.07	__	Very Strong
	3D, Merged	7.41	__	Very Strong
Config. A	2D, Averaged	100	1	Strong
	2D, Slice-merged	100	1	Strong
	2.5D, Direction-merged	68.5	1.3	Strong
	2.5D, Merged	68.5	1.3	Strong

Config. B	2D, Averaged	72.1	1	Strong
	2D, Slice-merged	72.3	1	Strong
	2.5D, Direction-merged	48.1	0.4	Strong
	2.5D, Merged	48.1	0.4	Strong
Config. C	3D, Averaged	276	8	Strong
	3D, Merged	276	8	Very Strong
Config. D	3D, Averaged	63.4	1.3	Strong
	3D, Merged	63.5	1.3	Strong
Config. E	3D, Averaged	86.6	3.3	Moderate
	3D, Merged	86.7	3.3	Strong

6-4-22- Cluster Shade (cm_clust_shade_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

This feature is defined as:

$$cluster_{shade} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \left(i + j - \mu_{i\,.} - \mu_{.j}\right)^3 p_{ij}$$

Where $\mu_{i\,.} = \sum_{i=1}^{N_g} i\, p_{i\,.}$ and $\mu_{.j} = \sum_{j=1}^{N_g} j\, p_{.j}$. Since $P_{..}$ is symmetric, the formulation simplifies to:

$$cluster_{shade} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i + j - 2\mu_{i\,.})^3 p_{ij}$$

Table 6-22: Reference values for the cluster shade feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	7	—	Very Strong
	2D, Slice-merged	6.98	—	Strong
	2.5D, Direction-merged	16.6	—	Strong
	2.5D, Merged	16.4	—	Strong
	3D, Averaged	16.6	—	Very Strong
	3D, Merged	17.4	—	Very Strong
Config. A	2D, Averaged	-1.04×10^{3}	20	Strong
	2D, Slice-merged	-1.05×10^{3}	20	Strong
	2.5D, Direction-merged	-1.49×10^{3}	30	Strong
	2.5D, Merged	-1.49×10^{3}	30	Strong
Config. B	2D, Averaged	-668	17	Strong
	2D, Slice-merged	-673	17	Strong
	2.5D, Direction-merged	-905	19	Strong
	2.5D, Merged	-906	19	Strong
Config. C	3D, Averaged	-1.06×10^{4}	300	Strong
	3D, Merged	-1.06×10^{4}	300	Very Strong

Config. D	3D, Averaged	-1.27×10^{3}	40	Strong
	3D, Merged	-1.28×10^{3}	40	Strong
Config. E	3D, Averaged	-2.07×10^{3}	70	Moderate
	3D, Merged	-2.08×10^{3}	70	Strong

6-4-23- Cluster Prominence (cm_clust_prom_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

This feature is defined as:

$$cluster_{prom} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \left(i + j - \mu_{i.} - \mu_{.j}\right)^4 p_{ij}$$

As before, $\mu_{i.} = \sum_{i=1}^{N_g} i\, p_{i.}$ and $\mu_{.j} = \sum_{j=1}^{N_g} j\, p_{.j}$. Since $P_{..}$ is symmetric, the formulation simplifies to:

$$cluster_{prom} = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} (i + j - 2\mu_{i.})^4 p_{ij}$$

Table 6-23: Reference values for the cluster prominence feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	79.1	__	Very Strong
	2D, Slice-merged	80.4	__	Strong
	2.5D, Direction-merged	147	__	Strong
	2.5D, Merged	142	__	Strong
	3D, Averaged	145	__	Very Strong
	3D, Merged	147	__	Very Strong
Config. A	2D, Averaged	5.27×10^{4}	500	Strong

	2D, Slice-merged	5.28×10^{4}	500	Strong
	2.5D, Direction-merged	4.76×10^{4}	700	Strong
	2.5D, Merged	4.77×10^{4}	700	Strong
Config. B	2D, Averaged	2.94×10^{4}	1.4×10^{3}	Strong
	2D, Slice-merged	2.95×10^{4}	1.4×10^{3}	Strong
	2.5D, Direction-merged	2.52×10^{4}	1×10^{3}	Strong
	2.5D, Merged	2.53×10^{4}	1×10^{3}	Strong
Config. C	3D, Averaged	5.69×10^{5}	1.1×10^{4}	Strong
	3D, Merged	5.7×10^{5}	1.1×10^{4}	Very Strong
Config. D	3D, Averaged	3.57×10^{4}	1.4×10^{3}	Strong
	3D, Merged	3.57×10^{4}	1.5×10^{3}	Strong
Config. E	3D, Averaged	6.89×10^{4}	2.1×10^{3}	Moderate
	3D, Merged	6.9×10^{4}	2.1×10^{3}	Strong

6-4-24- Information Correlation 1 (cm_info_corr1_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

The first information theoretic correlation measure quantifies the dependency between gray-levels based on entropy. It is defined as:

$$info\_corr_1 = \frac{HXY - HXY_1}{HX}$$

Where HXY is the joint entropy, and HX is the entropy for the row marginal probability:

$$HXY = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_{ij} \, log_2 \, p_{ij}$$

$$HX = -\sum_{i=1}^{N_g} p_{i\,.}\, log_2\, p_{i\,.}$$

HXY_1 is a type of entropy that is defined as:

$$HXY_1 = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_{ij}\, log_2\left(p_{i\,.}\, p_{.j}\right)$$

Table 6-24: Reference values for the information correlation 1 feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	-0.155	__	Very Strong
	2D, Slice-merged	-0.0341	__	Strong
	2.5D, Direction-merged	-0.124	__	Strong
	2.5D, Merged	-0.0334	__	Strong
	3D, Averaged	-0.157	__	Very Strong
	3D, Merged	-0.0288	__	Very Strong
Config. A	2D, Averaged	-0.236	0.001	Strong
	2D, Slice-merged	-0.214	0.001	Strong

	2.5D, Direction-merged	-0.231	0.001	Strong
	2.5D, Merged	-0.228	0.001	Strong
Config. B	2D, Averaged	-0.239	0.001	Strong
	2D, Slice-merged	-0.181	0.001	Strong
	2.5D, Direction-merged	-0.188	0.001	Strong
	2.5D, Merged	-0.185	0.001	Strong
Config. C	3D, Averaged	-0.236	0.001	Strong
	3D, Merged	-0.228	0.001	Strong
Config. D	3D, Averaged	-0.231	0.003	Strong
	3D, Merged	-0.225	0.003	Strong
Config. E	3D, Averaged	-0.181	0.003	Moderate
	3D, Merged	-0.175	0.003	Strong

6-4-25- Information Correlation 2 (cm_info_corr2_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

The second entropy-based correlation measure is given by:

$$info\_corr_2 = \sqrt{1 - exp\big(-2(HXY_2 - HXY)\big)}$$

HXY is defined in 6-4-24, and HXY_2 is a type of entropy defined as:

$$HXY_2 = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_{i\,.}\, p_{.j}\, log_2\left(p_{i\,.}\, p_{.j}\right)$$

Table 6-25: Reference values for the information correlation 2 feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.487	—	Strong

	2D, Slice-merged	0.263	__	Strong
	2.5D, Direction-merged	0.487	__	Strong
	2.5D, Merged	0.291	__	Strong
	3D, Averaged	0.52	__	Very Strong
	3D, Merged	0.269	__	Very Strong
Config. A	2D, Averaged	0.863	0.003	Strong
	2D, Slice-merged	0.851	0.002	Strong
	2.5D, Direction-merged	0.879	0.001	Strong
	2.5D, Merged	0.88	0.001	Strong
Config. B	2D, Averaged	0.837	0.001	Strong
	2D, Slice-merged	0.792	0.001	Strong
	2.5D, Direction-merged	0.821	0.001	Strong
	2.5D, Merged	0.819	0.001	Strong
Config. C	3D, Averaged	0.9	0.001	Strong
	3D, Merged	0.889	0.001	Strong
Config. D	3D, Averaged	0.845	0.003	Strong
	3D, Merged	0.846	0.003	Strong
Config. E	3D, Averaged	0.813	0.004	Moderate
	3D, Merged	0.813	0.004	Strong

7- Gray Level Run Length Based Features

7-1- Introduction [1]

Gray level run length features are computed from the Gray Level Run Length Matrix (GLRLM), which captures contiguous sequences ("runs") of equal gray level along a specified direction m (see Fig. 7-1).

A run length is the number of consecutive pixels/voxels with the same discretised gray level encountered when stepping along direction m.

In a GLRLM M_m of size $N_g \times N_r$, element r_{ij} counts the number of runs of gray level i and length j observed in the ROI along direction m.

Marginals are defined over gray levels and run lengths: $r_{i.} = \sum_{j=1}^{N_r} r_{ij}$ (runs per gray level) and $r_{.j} = \sum_{i=1}^{N_g} r_{ij}$ (runs per length); the total number of runs is $N_s = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} r_{ij}$.

1	2	2	3
1	2	3	3
4	2	4	1
4	1	2	3

(a) Grey levels

		Run length j			
		1	2	3	4
i	1	4	0	0	0
	2	3	1	0	0
	3	2	1	0	0
	4	3	0	0	0

(b) $\mathbf{M_{m=\rightarrow}}$

		Run length j			
		1	2	3	4
i	1	4	0	0	0
	2	3	1	0	0
	3	2	1	0	0
	4	3	0	0	0

(c) $\mathbf{M_{m=\nearrow}}$

Fig 7-1: Gray level run length matrices of (a) for 0∘ (b) and 45∘ (c) directions. These directions are associated with vectors $m = (1,0)$ and $m = (1,1)$, respectively. [1]

7-2- Feature Aggregation

Feature aggregation for GLRLM follows the same six strategies defined in 6-2; when a merged strategy is selected, run counts are summed element-wise before normalization.

7-3- Distances and Distance weighting

Distance weighting for GLRLM follows the same scheme defined in 6-3.

7-4- Explanation of Features

Unless specified otherwise, formulas are identical for 2D, 2.5D, and 3D; only the aggregation differs (see 7-2). Suffixes indicate aggregation: 2D_avg, 2D_comb, 2_5D_avg, 2_5D_comb, 3D_avg, 3D_comb. In merged variants, GLRLMs are summed element-wise before normalisation and feature calculation.

7-4-1- Short Runs Emphasis (rlm_sre_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Captures how strongly the texture is dominated by short runs; higher SRE indicates finer, more rapidly changing patterns with frequent run breaks [6]. It is defined as [1]:

$$sre = \frac{1}{N_s}\sum_{j=1}^{N_r} \frac{r_{.j}}{j^2}$$

Table 7-1: Reference values for the short runs emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.641	__	Very Strong
	2D, Slice-merged	0.661	__	Strong
	2.5D, Direction-merged	0.665	__	Strong
	2.5D, Merged	0.68	__	Strong
	3D, Averaged	0.705	__	Very Strong
	3D, Merged	0.729	__	Very Strong
Config. A	2D, Averaged	0.785	0.003	Strong
	2D, Slice-merged	0.786	0.003	Strong
	2.5D, Direction-merged	0.768	0.003	Strong
	2.5D, Merged	0.769	0.003	Strong
Config. B	2D, Averaged	0.781	0.001	Strong

	2D, Slice-merged	0.782	0.001	Strong
	2.5D, Direction-merged	0.759	0.001	Strong
	2.5D, Merged	0.759	0.001	Strong
Config. C	3D, Averaged	0.786	0.003	Strong
	3D, Merged	0.787	0.003	Strong
Config. D	3D, Averaged	0.734	0.001	Strong
	3D, Merged	0.736	0.001	Strong
Config. E	3D, Averaged	0.776	0.001	Moderate
	3D, Merged	0.777	0.001	Strong

7-4-2- Long Runs Emphasis (rlm_lre_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb)

Highlights the presence of longer, uninterrupted runs; higher LRE reflects coarser structure [6]. This feature is defined as follows [1]:

$$lre = \frac{1}{N_s}\sum_{j=1}^{N_r} j^2 \, r_{.j}$$

Table 7-2: Reference values for the long runs emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	3.78	__	Very Strong
	2D, Slice-merged	3.51	__	Strong
	2.5D, Direction-merged	3.46	__	Strong
	2.5D, Merged	3.27	__	Strong
	3D, Averaged	3.06	__	Very Strong

	3D, Merged	2.76	__	Very Strong
Config. A	2D, Averaged	2.91	0.003	Strong
	2D, Slice-merged	2.89	0.003	Strong
	2.5D, Direction-merged	3.09	0.003	Strong
	2.5D, Merged	3.08	0.003	Strong
Config. B	2D, Averaged	3.52	0.04	Strong
	2D, Slice-merged	3.5	0.04	Strong
	2.5D, Direction-merged	3.82	0.05	Strong
	2.5D, Merged	3.81	0.05	Strong
Config. C	3D, Averaged	3.31	0.04	Strong
	3D, Merged	3.28	0.04	Strong
Config. D	3D, Averaged	6.66	0.18	Strong
	3D, Merged	6.56	0.18	Strong
Config. E	3D, Averaged	3.55	0.07	Strong
	3D, Merged	3.52	0.07	Strong

7-4-3- Low Gray Level Run Emphasis (rlm_lgre_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Gray-level analogue of SRE; emphasises low gray levels.

$$lgre = \frac{1}{N_s} \sum_{i=1}^{N_g} \frac{r_{i.}}{i^2}$$

Table 7-3: Reference values for the low gray level run emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	3.78	—	Very Strong
	2D, Slice-merged	3.51	—	Strong
	2.5D, Direction-merged	3.46	—	Strong
	2.5D, Merged	3.27	—	Strong
	3D, Averaged	3.06	—	Very Strong
	3D, Merged	2.76	—	Very Strong
Config. A	2D, Averaged	2.91	0.0003	Strong
	2D, Slice-merged	2.89	0.0003	Strong
	2.5D, Direction-merged	3.09	0.0004	Strong
	2.5D, Merged	3.08	0.0004	Strong
Config. B	2D, Averaged	3.52	0.0006	Strong
	2D, Slice-merged	3.5	0.0006	Strong
	2.5D, Direction-merged	3.82	0.0006	Strong
	2.5D, Merged	3.81	0.0006	Strong
Config. C	3D, Averaged	3.31	5×10^{-5}	Strong
	3D, Merged	3.28	5×10^{-5}	Strong
Config. D	3D, Averaged	6.66	0.0012	Strong

	3D, Merged	6.56	0.0012	Strong
Config. E	3D, Averaged	3.55	0.0008	Moderate
	3D, Merged	3.52	0.0008	Strong

7-4-4- High Gray Level Run Emphasis (rlm_hgre_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Gray-level analogue of LRE; emphasises high gray levels:

$$hgre = \frac{1}{N_s} \sum_{i=1}^{N_g} \quad i^2 \, r_i \, .$$

Table 7-4: Reference values for the high gray level run emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	9.82	__	Very Strong
	2D, Slice-merged	9.74	__	Strong
	2.5D, Direction-merged	10.3	__	Strong
	2.5D, Merged	10.2	__	Strong
	3D, Averaged	9.7	__	Very Strong
	3D, Merged	9.64	__	Very Strong

Config. A	2D, Averaged	428	3	Strong
	2D, Slice-merged	428	3	Strong
	2.5D, Direction-merged	449	3	Strong
	2.5D, Merged	449	3	Strong
Config. B	2D, Averaged	342	11	Strong
	2D, Slice-merged	342	11	Strong
	2.5D, Direction-merged	356	11	Strong
	2.5D, Merged	356	11	Strong
Config. C	3D, Averaged	1.47×10^{3}	10	Strong
	3D, Merged	1.47×10^{3}	10	Strong
Config. D	3D, Averaged	326	17	Strong
	3D, Merged	326	17	Strong
Config. E	3D, Averaged	471	9	Strong
	3D, Merged	471	9	Strong

7-4-5- Short Run Low Gray Level Emphasis (rlm_srlge_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Emphasises runs in the upper-left of the GLRLM (short runs + low gray levels):

$$srlge = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \frac{r_{ij}}{i^2 j^2}$$

Table 7-5: Reference values for the short run low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.294	__	Very Strong
	2D, Slice-merged	0.311	__	Strong
	2.5D, Direction-merged	0.296	__	Strong
	2.5D, Merged	0.312	__	Strong
	3D, Averaged	0.352	__	Very Strong
	3D, Merged	0.372	__	Very Strong
Config. A	2D, Averaged	0.0243	0.0003	Strong
	2D, Slice-merged	0.0243	0.0003	Strong
	2.5D, Direction-merged	0.0135	0.0004	Strong
	2.5D, Merged	0.0135	0.0004	Strong
Config. B	2D, Averaged	0.0314	0.0006	Strong
	2D, Slice-merged	0.0313	0.0006	Strong
	2.5D, Direction-merged	0.0181	0.0006	Strong
	2.5D, Merged	0.0181	0.0006	Strong
Config. C	3D, Averaged	0.00136	5×10^{-5}	Strong
	3D, Merged	0.00136	5×10^{-5}	Strong
Config. D	3D, Averaged	0.0232	0.001	Strong
	3D, Merged	0.0232	0.001	Strong
Config. E	3D, Averaged	0.0187	0.0007	Moderate

	3D, Merged	0.0186	0.0007	Strong

7-4-6- Short Run High Gray Level Emphasis (rlm_srhge_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Emphasises runs in the lower-left (short runs + high gray levels):

$$srhge = \frac{1}{N_s}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r}\frac{i^{\,2}\,r_{ij}}{j^{\,2}}$$

Table 7-6: Reference values for the short run high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	8.57	__	Very Strong
	2D, Slice-merged	8.67	__	Strong
	2.5D, Direction-merged	9.03	__	Strong
	2.5D, Merged	9.05	__	Strong
	3D, Averaged	8.54	__	Very Strong
	3D, Merged	8.67	__	Very Strong
Config. A	2D, Averaged	320	1	Strong
	2D, Slice-merged	320	1	Strong
	2.5D, Direction-merged	332	1	Strong
	2.5D, Merged	333	1	Strong
Config. B	2D, Averaged	251	8	Strong
	2D, Slice-merged	252	8	Strong
	2.5D, Direction-merged	257	9	Strong
	2.5D, Merged	258	9	Strong

Config. C	3D, Averaged	1.1×10^{3}	10	Strong
	3D, Merged	1.1×10^{3}	10	Strong
Config. D	3D, Averaged	219	13	Strong
	3D, Merged	219	13	Strong
Config. E	3D, Averaged	346	7	Strong
	3D, Merged	347	7	Strong

7-4-7- Long Run Low Gray Level Emphasis (rlm_lrlge_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Emphasises runs in the upper-right (long runs + low gray levels):

$$lrlge = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} \frac{j^{2} r_{ij}}{i^{2}}$$

Table 7-7: Reference values for the long run low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	3.14	—	Very Strong
	2D, Slice-merged	2.92	—	Strong
	2.5D, Direction-merged	2.79	—	Strong
	2.5D, Merged	2.63	—	Strong
	3D, Averaged	2.39	—	Very Strong
	3D, Merged	2.16	—	Very Strong
Config. A	2D, Averaged	0.0386	0.0003	Strong
	2D, Slice-merged	0.0385	0.0003	Strong
	2.5D, Direction-merged	0.0229	0.0004	Strong

	2.5D, Merged	0.0228	0.0004	Strong
Config. B	2D, Averaged	0.0443	0.0008	Strong
	2D, Slice-merged	0.0442	0.0008	Strong
	2.5D, Direction-merged	0.0293	0.0009	Strong
	2.5D, Merged	0.0292	0.0009	Strong
Config. C	3D, Averaged	0.00317	4×10^{-5}	Strong
	3D, Merged	0.00314	4×10^{-5}	Strong
Config. D	3D, Averaged	0.0484	0.0031	Strong
	3D, Merged	0.0478	0.0031	Strong
Config. E	3D, Averaged	0.0313	0.0016	Moderate
	3D, Merged	0.0311	0.0016	Strong

7-4-8- Long Run High Gray Level Emphasis (rlm_lrhge_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Emphasises runs in the lower-right (long runs + high gray levels):

$$lrhge = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} i^2 j^2 r_{ij}$$

Table 7-8: Reference values for the long run high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	17.4	__	Very Strong
	2D, Slice-merged	16.1	__	Strong
	2.5D, Direction-merged	17.9	__	Strong
	2.5D, Merged	17	__	Strong

	3D, Averaged	17.6	__	Very Strong
	3D, Merged	15.6	__	Very Strong
Config. A	2D, Averaged	1.41×10^{3}	20	Strong
	2D, Slice-merged	1.4×10^{3}	20	Strong
	2.5D, Direction-merged	1.5×10^{3}	20	Strong
	2.5D, Merged	1.5×10^{3}	20	Strong
Config. B	2D, Averaged	1.39×10^{3}	30	Strong
	2D, Slice-merged	1.38×10^{3}	30	Strong
	2.5D, Direction-merged	1.5×10^{3}	30	Strong
	2.5D, Merged	1.5×10^{3}	30	Strong
Config. C	3D, Averaged	5.59×10^{3}	80	Strong
	3D, Merged	5.53×10^{3}	80	Strong
Config. D	3D, Averaged	2.67×10^{3}	30	Strong
	3D, Merged	2.63×10^{3}	30	Strong
Config. E	3D, Averaged	1.9×10^{3}	20	Moderate
	3D, Merged	1.89×10^{3}	20	Strong

7-4-9- Gray Level Non-uniformity (rlm_glnu_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

This feature assesses distribution of runs across gray levels; lower when runs are evenly distributed over gray levels.

$$glnu = \frac{1}{N_s} \sum_{i=1}^{N_g} r_i^{\,2}.$$

Table 7-9: Reference values for the gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	5.2	—	Very Strong
	2D, Slice-merged	20.5	—	Strong
	2.5D, Direction-merged	19.5	—	Strong
	2.5D, Merged	77.1	—	Strong
	3D, Averaged	21.8	—	Very Strong
	3D, Merged	281	—	Very Strong
Config. A	2D, Averaged	432	1	Strong
	2D, Slice-merged	1.73×10^3	10	Strong
	2.5D, Direction-merged	9.85×10^3	10	Strong
	2.5D, Merged	3.94×10^4	100	Strong
Config. B	2D, Averaged	107	1	Strong
	2D, Slice-merged	427	1	Strong
	2.5D, Direction-merged	2.4×10^3	10	Strong
	2.5D, Merged	9.6×10^3	20	Strong
Config. C	3D, Averaged	3.18×10^3	10	Strong
	3D, Merged	4.13×10^4	100	Strong

Config. D	3D, Averaged	3.29×10^{3}	10	Strong
	3D, Merged	4.28×10^{4}	200	Strong
Config. E	3D, Averaged	4×10^{3}	10	Moderate
	3D, Merged	5.19×10^{4}	200	Strong

7-4-10- Normalised Gray Level Non-uniformity (rlm_glnu_norm_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Normalised version of GLNU.

$$glnu_{norm} = \frac{1}{N_s^{2}} \sum_{i=1}^{N_g} r_{i.}^{2}$$

Table 7-10: Reference values for the normalised gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.46	__	Very Strong
	2D, Slice-merged	0.456	__	Strong
	2.5D, Direction-merged	0.413	__	Strong
	2.5D, Merged	0.412	__	Strong
	3D, Averaged	0.43	__	Very Strong
	3D, Merged	0.43	__	Very Strong
Config. A	2D, Averaged	0.128	0.003	Strong
	2D, Slice-merged	0.128	0.003	Strong
	2.5D, Direction-merged	0.126	0.003	Strong

	2.5D, Merged	0.126	0.003	Strong
Config. B	2D, Averaged	0.145	0.001	Strong
	2D, Slice-merged	0.145	0.001	Strong
	2.5D, Direction-merged	0.137	0.001	Strong
	2.5D, Merged	0.137	0.001	Strong
Config. C	3D, Averaged	0.102	0.003	Strong
	3D, Merged	0.102	0.003	Very Strong
Config. D	3D, Averaged	0.133	0.002	Strong
	3D, Merged	0.134	0.002	Strong
Config. E	3D, Averaged	0.135	0.003	Strong
	3D, Merged	0.135	0.003	Strong

7-4-11- Run Length Non-uniformity (rlm_rlnu_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Assesses distribution of runs across run lengths; lower when runs are evenly distributed.

$$rlnu = \frac{1}{N_s} \sum_{j=1}^{N_r} r_{.j}^{2}$$

Table 7-11: Reference values for the run length non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	6.12	__	Very Strong
	2D, Slice-merged	21.6	__	Strong

	2.5D, Direction-merged	22.3	—	Strong
	2.5D, Merged	83.2	—	Strong
	3D, Averaged	26.9	—	Very Strong
	3D, Merged	328	—	Very Strong
Config. A	2D, Averaged	1.65×10^{3}	10	Strong
	2D, Slice-merged	6.6×10^{3}	30	Strong
	2.5D, Direction-merged	4.27×10^{4}	200	Strong
	2.5D, Merged	1.71×10^{5}	1×10^{3}	Strong
Config. B	2D, Averaged	365	3	Strong
	2D, Slice-merged	1.46×10^{3}	10	Strong
	2.5D, Direction-merged	9.38×10^{3}	70	Strong
	2.5D, Merged	3.75×10^{4}	300	Strong
Config. C	3D, Averaged	1.8×10^{4}	500	Strong
	3D, Merged	2.34×10^{5}	6×10^{3}	Strong
Config. D	3D, Averaged	1.24×10^{4}	200	Strong
	3D, Merged	1.6×10^{5}	3×10^{3}	Strong
Config. E	3D, Averaged	1.66×10^{4}	300	Strong
	3D, Merged	2.15×10^{5}	4×10^{3}	Strong

7-4-12- Normalised Run Length Non-uniformity (rlm_rlnu_norm_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Normalised version of RLNU.

$$rlnu_{norm} = \frac{1}{{N_s}^2} \sum_{j=1}^{N_r} r_{.j}^2$$

Table 7-12: Reference values for the normalised run length non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.492	__	Very Strong
	2D, Slice-merged	0.441	__	Strong
	2.5D, Direction-merged	0.461	__	Strong
	2.5D, Merged	0.445	__	Strong
	3D, Averaged	0.513	__	Very Strong
	3D, Merged	0.501	__	Very Strong
Config. A	2D, Averaged	0.579	0.003	Strong
	2D, Slice-merged	0.579	0.003	Strong
	2.5D, Direction-merged	0.548	0.003	Strong

	2.5D, Merged	0.548	0.003	Strong
Config. B	2D, Averaged	0.578	0.001	Strong
	2D, Slice-merged	0.578	0.001	Strong
	2.5D, Direction-merged	0.533	0.001	Strong
	2.5D, Merged	0.534	0.001	Strong
Config. C	3D, Averaged	0.574	0.004	Strong
	3D, Merged	0.575	0.004	Strong
Config. D	3D, Averaged	0.5	0.001	Strong
	3D, Merged	0.501	0.001	Strong
Config. E	3D, Averaged	0.559	0.001	Moderate
	3D, Merged	0.56	0.001	Strong

7-4-13- Run Percentage (rlm_r_perc_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

This feature is the fraction of realised runs to the maximum possible runs; strongly linear or highly uniform ROIs yield low values.

$$r\_perc = \frac{N_s}{N_v}$$

Note: If this feature is computed from merged GLRLMs, sum N_v across underlying matrices [1].

Table 7-13: Reference values for the run percentage feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.627	—	Very Strong
	2D, Slice-merged	0.627	—	Strong

	2.5D, Direction-merged	0.632	__	Strong
	2.5D, Merged	0.632	__	Strong
	3D, Averaged	0.68	__	Very Strong
	3D, Merged	0.68	__	Very Strong
Config. A	2D, Averaged	0.704	0.003	Strong
	2D, Slice-merged	0.704	0.003	Strong
	2.5D, Direction-merged	0.68	0.003	Strong
	2.5D, Merged	0.68	0.003	Strong
Config. B	2D, Averaged	0.681	0.002	Strong
	2D, Slice-merged	0.681	0.002	Strong
	2.5D, Direction-merged	0.642	0.002	Strong
	2.5D, Merged	0.642	0.002	Strong
Config. C	3D, Averaged	0.679	0.003	Strong
	3D, Merged	0.679	0.003	Strong
Config. D	3D, Averaged	0.554	0.005	Strong
	3D, Merged	0.554	0.005	Strong
Config. E	3D, Averaged	0.664	0.003	Moderate
	3D, Merged	0.664	0.003	Strong

7-4-14- Gray Level Variance (rlm_gl_var_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Variance of runs over gray levels. Here we use the joint probability estimate $p_{ij} = \frac{r_{ij}}{N_s}$ to find discretised graylevel i with run length j. This feature is defined as:

$$gl\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} (i - \mu)^2 \, p_{ij}$$

Here, $\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} i \, p_{ij}$.

Table 7-14: Reference values for the gray level variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	3.35	__	Very Strong
	2D, Slice-merged	3.37	__	Strong
	2.5D, Direction-merged	3.58	__	Strong
	2.5D, Merged	3.59	__	Strong
	3D, Averaged	3.46	__	Very Strong
	3D, Merged	3.48	__	Very Strong
Config. A	2D, Averaged	33.7	0.6	Strong
	2D, Slice-merged	33.7	0.6	Strong
	2.5D, Direction-merged	29.1	0.6	Strong
	2.5D, Merged	29.1	0.6	Strong
Config. B	2D, Averaged	28.3	0.3	Strong
	2D, Slice-merged	28.3	0.3	Strong

	2.5D, Direction-merged	25.7	0.2	Strong
	2.5D, Merged	25.7	0.2	Strong
Config. C	3D, Averaged	101	3	Strong
	3D, Merged	101	3	Very Strong
Config. D	3D, Averaged	31.5	0.4	Strong
	3D, Merged	31.4	0.4	Strong
Config. E	3D, Averaged	39.8	0.9	Moderate
	3D, Merged	39.7	0.9	Strong

7-4-15- Run Length Variance (rlm_rl_var_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Variance of runs over run lengths. As before, we use the joint probability estimate $p_{ij} = \frac{r_{ij}}{N_s}$. The feature is then defined as:

$$rl\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} (j-\mu)^2\, p_{ij}$$

Where, $\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} j\, p_{ij}$.

Table 7-15: Reference values for the run length variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	0.761	__	Very Strong
	2D, Slice-merged	0.778	__	Strong

	2.5D, Direction-merged	0.758	__	Strong
	2.5D, Merged	0.767	__	Strong
	3D, Averaged	0.574	__	Very Strong
	3D, Merged	0.598	__	Very Strong
Config. A	2D, Averaged	0.828	0.008	Strong
	2D, Slice-merged	0.826	0.008	Strong
	2.5D, Direction-merged	0.916	0.011	Strong
	2.5D, Merged	0.914	0.011	Strong
Config. B	2D, Averaged	1.22	0.03	Strong
	2D, Slice-merged	1.21	0.03	Strong
	2.5D, Direction-merged	1.39	0.03	Strong
	2.5D, Merged	1.39	0.03	Strong
Config. C	3D, Averaged	1.12	0.02	Strong
	3D, Merged	1.11	0.02	Strong
Config. D	3D, Averaged	3.35	0.14	Strong
	3D, Merged	3.29	0.13	Strong
Config. E	3D, Averaged	1.26	0.05	Strong
	3D, Merged	1.25	0.05	Strong

7-4-16- Run Entropy (rlm_rl_entr_2D_avg / 2D_comb / 2_5D_avg / 2_5D_comb / 3D_avg / 3D_comb) [1]

Again, let $p_{ij} = \frac{r_{ij}}{N_s}$. The run entropy is then defined as:

$$rl\_entropy = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_r} p_{ij} \, log_2 \, p_{ij}$$

Table 7-16: Reference values for the run entropy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D, Averaged	2.17	__	Very Strong
	2D, Slice-merged	2.57	__	Strong
	2.5D, Direction-merged	2.52	__	Strong
	2.5D, Merged	2.76	__	Strong
	3D, Averaged	2.43	__	Very Strong
	3D, Merged	2.62	__	Very Strong
Config. A	2D, Averaged	4.73	0.02	Strong
	2D, Slice-merged	4.76	0.02	Strong
	2.5D, Direction-merged	4.87	0.01	Strong
	2.5D, Merged	4.87	0.01	Strong

Config. B	2D, Averaged	4.53	0.02	Strong
	2D, Slice-merged	4.58	0.01	Strong
	2.5D, Direction-merged	4.84	0.01	Strong
	2.5D, Merged	4.84	0.01	Strong
Config. C	3D, Averaged	5.35	0.03	Strong
	3D, Merged	5.35	0.03	Very Strong
Config. D	3D, Averaged	5.08	0.02	Strong
	3D, Merged	5.08	0.02	Strong
Config. E	3D, Averaged	4.87	0.03	Strong
	3D, Merged	4.87	0.03	Strong

8- Gray Level Size Zone Based Features

8-1- Introduction [1]

The Gray Level Size Zone Matrix (GLSZM) counts groups ("zones") of linked voxels that share the same discretised gray level. Two voxels are linked if a neighbouring voxel has the same gray level, with "neighbour" determined by the chosen connectedness.
In 3D, zones are identified with 26-connectedness (a voxel may link to any of its 26 neighbours); in 2D, zones use 8-connectedness.
A potential issue with a purely 2D approach is that voxels that would form one continuous zone when linking across slices may appear as two or more separate zones within a slice; whether this impacts predictive performance or reproducibility of GLSZM features is undetermined.
Let M denote the $N_g \times N_z$ GLSZM, where N_g is the number of discretised gray levels and N_z the maximum zone size observed. Element s_{ij} is the number of zones with gray level i and size j (see Fig. 8-1). Define N_v as the voxel count in the ROI and $N_s = \sum_{i=1}^{N_g} \sum_{j=1}^{N_z} s_{ij}$ as the total number of zones. Marginals are: $s_{i.} = \sum_{j=1}^{N_z} s_{ij}$ (zones per gray level) and $s_{.j} = \sum_{i=1}^{N_g} s_{ij}$ (zones per size).

1	2	2	3
1	2	3	3
4	2	4	1
4	1	2	3

(a) Grey levels

		Zone size j				
		1	2	3	4	5
i	1	2	1	0	0	0
	2	0	0	0	0	1
	3	1	0	1	0	0
	4	1	1	0	0	0

(b) Grey level size zone matrix

Fig. 8-1: An example of calculating GLSZM [1].

8-2- Feature Aggregation [1]

IBSI [1] explains three methods for aggregating features to a single value. These approaches are defined as follows; and an example of each is illustrated in Fig. 8-1.

1. 2D (per-slice, averaged): compute GLSZM per slice and average feature values across slices. This keeps slice specificity but may split zones that would connect across slices.
2. 2.5D (per-slice, merged): merge the per-slice GLSZMs (sum zone counts) into a single matrix before computing the feature, partially restoring cross-slice information without full 3D connectivity.
3. 3D (full volume): compute a single 3D GLSZM using 26-connectedness over the entire volume.

Note: When matrices are merged, N_v must be summed as well (for features that normalise by voxel count). Feature values can depend strongly on the chosen aggregation mode.

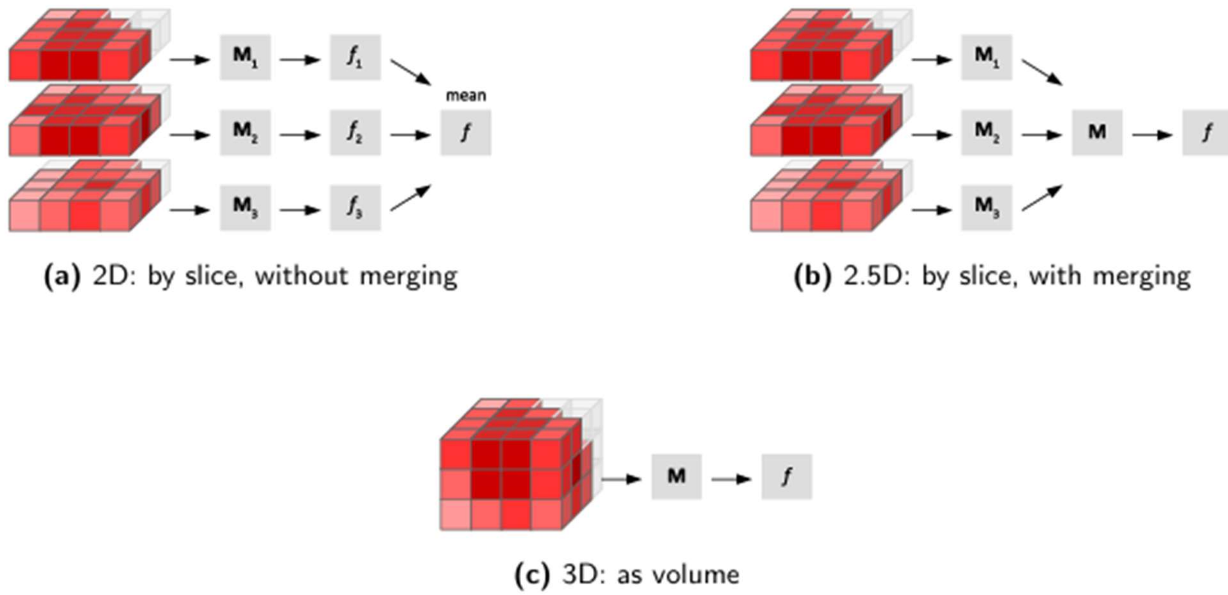

Fig. 8-2: Different approaches of feature aggregation are illustrated here [1].

8-3- Distances and Distance Weighting [1]

The default neighbourhood for GLSZM uses Chebyshev distance $\delta = 1$. Using Manhattan or Euclidean norms yields 6-connected (3D) and 4-connected (2D) neighbourhoods. Larger distances are technically possible, but may cause separate zones with the same intensity to be treated as a single zone. Choosing non-default neighbourhoods is non-standard and cautioned due to potential reproducibility issues.

8-4- Explanation of Features [1]

Unless stated otherwise, the definitions of this section apply identically in 2D, 2.5D, and 3D; only the aggregation choice (8-2) differs. In merged modes, GLSZMs are summed element-wise first and normalised second.
Note that GLSZM features are based on GLRLM features. Hence, you may find references in section 7-4.

8-4-1- Small Zone Emphasis (szm_sze_2D / 2_5D / 3D) [1]

Emphasises small zones; increases when many small sized zones are present.

$$sze = \frac{1}{N_s} \sum_{j=1}^{N_z} \frac{s_{.j}}{j^{2}}$$

Table 8-1: Reference values for the small zone emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.363	__	Strong
	2.5D	0.368	__	Strong
	3D	0.255	__	Very Strong
Config. A	2D	0.688	0.003	Strong
	2.5D	0.68	0.003	Strong
Config. B	2D	0.745	0.003	Strong
	2.5D	0.741	0.003	Strong
Config. C	3D	0.695	0.001	Strong
Config. D	3D	0.637	0.005	Strong
Config. E	3D	0.676	0.003	Strong

8-4-2- Large Zone Emphasis (szm_lze_2D / 2_5D / 3D) [1]

Emphasises large zones; increases when sizeable homogeneous regions dominate.

$$lze = \frac{1}{N_s} \sum_{j=1}^{N_z} j^{2} s_{.j}$$

Table 8-2: Reference values for the large zone emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	43.9	__	Strong
	2.5D	34.2	__	Strong
	3D	550	__	Very Strong

Config. A	2D	625	9	Strong
	2.5D	675	8	Strong
Config. B	2D	439	8	Strong
	2.5D	444	8	Strong
Config. C	3D	3.89×10^{4}	900	Strong
Config. D	3D	9.91×10^{4}	2.8×10^{3}	Strong
Config. E	3D	5.86×10^{4}	800	Strong

8-4-3- Low Gray Level Zone Emphasis (szm_lgze_2D / 2_5D / 3D) [1]

This feature is gray level analogue to SZE, emphasises low gray levels, irrespective of zone size.

$$lgze = \frac{1}{N_s} \sum_{i=1}^{N_g} \frac{s_{i.}}{i^2}$$

Table 8-3: Reference values for the low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.371	—	Strong
	2.5D	0.368	—	Strong
	3D	0.253	—	Very Strong
Config. A	2D	0.0368	0.0005	Strong
	2.5D	0.0291	0.0005	Strong
Config. B	2D	0.0475	0.001	Strong
	2.5D	0.0387	0.001	Strong

Config. C	3D	0.00235	2.8×10^{3}	Strong
Config. D	3D	0.0409	0.0005	Strong
Config. E	3D	0.034	0.0004	Strong

8-4-4- High Gray Level Zone Emphasis (szm_hgze_2D / 2_5D / 3D) [1]

This feature is gray level analogue to SZE, emphasises high gray levels, irrespective of zone size.

$$hgze = \frac{1}{N_s} \sum_{i=1}^{N_g} i^2 s_i .$$

Table 8-4: Reference values for the high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	16.4	__	Strong
	2.5D	16.2	__	Strong
	3D	15.6	__	Very Strong
Config. A	2D	363	3	Strong
	2.5D	370	3	Strong
Config. B	2D	284	11	Strong
	2.5D	284	11	Strong
Config. C	3D	971	7	Strong
Config. D	3D	188	10	Strong
Config. E	3D	286	6	Strong

8-4-5- Small Zone Low Gray Level Emphasis (szm_szlge_2D / 2_5D / 3D) [1]

This feature targets the upper-left of the GLSZM (small sizes and low gray levels).

$$szlge = \frac{1}{N_s}\sum_{i=1}^{N_g}\sum_{j=1}^{N_z}\frac{s_{ij}}{i^2 j^2}$$

Table 8-5: Reference values for the small zone low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.0259	__	Strong
	2.5D	0.0295	__	Strong
	3D	0.0256	__	Very Strong
Config. A	2D	0.0298	0.0005	Strong
	2.5D	0.0237	0.0005	Strong
Config. B	2D	0.0415	0.0008	Strong
	2.5D	0.0335	0.0009	Strong
Config. C	3D	0.0016	4×10^{-5}	Strong
Config. D	3D	0.0248	0.0004	Strong
Config. E	3D	0.0224	0.0004	Strong

8-4-6- Small Zone High Gray Level Emphasis (szm_szhge_2D / 2_5D / 3D) [1]

Targets the lower-left of the GLSZM (small sizes and high gray levels).

$$szhge = \frac{1}{N_s}\sum_{i=1}^{N_g}\sum_{j=1}^{N_z}\frac{i^2 s_{ij}}{j^2}$$

Table 8-6: Reference values for the small zone high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	10.3	__	Strong
	2.5D	9.87	__	Strong
	3D	2.76	__	Very Strong
Config. A	2D	226	1	Strong
	2.5D	229	1	Strong
Config. B	2D	190	7	Strong
	2.5D	190	7	Strong
Config. C	3D	657	4	Strong
Config. D	3D	117	7	Strong
Config. E	3D	186	4	Strong

8-4-7- Large Zone Low Gray Level Emphasis (szm_lzlge_2D / 2_5D / 3D) [1]

Targets the upper-right of the GLSZM (large sizes and low gray levels).

$$lzlge = \frac{1}{N_s}\sum_{i=1}^{N_g}\sum_{j=1}^{N_z}\frac{j^2 s_{ij}}{i^2}$$

Table 8-7: Reference values for the large zone low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	40.4	__	Strong

	2.5D	30.6	__	Strong
	3D	503	__	Very Strong
Config. A	2D	1.35	0.03	Strong
	2.5D	1.44	0.02	Strong
Config. B	2D	1.15	0.04	Strong
	2.5D	1.16	0.04	Strong
Config. C	3D	21.6	0.5	Strong
Config. D	3D	241	14	Strong
Config. E	3D	105	4	Strong

8-4-8- Large Zone High Gray Level Emphasis (szm_lzhge_2D / 2_5D / 3D) [1]

Targets the lower-right of the GLSZM (large sizes and high gray levels).

$$lzhge = \frac{1}{N_s}\sum_{i=1}^{N_g}\sum_{j=1}^{N_z} i^2 j^2 s_{ij}$$

Table 8-8: Reference values for the large zone high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	113	__	Strong

	2.5D	107	__	Strong
	3D	1.49×10^{3}	__	Very Strong
Config. A	2D	3.16×10^{5}	5×10^{3}	Strong
	2.5D	3.38×10^{5}	5×10^{3}	Strong
Config. B	2D	1.81×10^{5}	3×10^{3}	Strong
	2.5D	1.81×10^{5}	3×10^{3}	Strong
Config. C	3D	7.07×10^{7}	1.5×10^{6}	Strong
Config. D	3D	4.14×10^{7}	3×10^{5}	Strong
Config. E	3D	3.36×10^{7}	3×10^{5}	Strong

8-4-9- Gray Level Non-uniformity (szm_glnu_2D / 2_5D / 3D) [1]

Assesses distribution of zone counts across gray levels; lower values indicate more even distribution over intensities.

$$glnu = \frac{1}{N_s} \sum_{i=1}^{N_g} s_{i.}^2$$

Table 8-9: Reference values for the gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	1.41	__	Strong
	2.5D	5.44	__	Strong
	3D	1.4	__	Very Strong
Config. A	2D	82.2	0.1	Strong
	2.5D	1.8×10^3	10	Strong
Config. B	2D	20.5	0.1	Strong
	2.5D	437	3	Strong
Config. C	3D	195	6	Strong
Config. D	3D	212	6	Very Strong
Config. E	3D	231	6	Strong

8-4-10- Normalised Gray Level Non-uniformity (szm_glnu_norm_2D / 2_5D / 3D) [1]

Normalised version of GLNU (by N_s).

$$glnu_{norm} = \frac{1}{{N_s}^2} \sum_{i=1}^{N_g} s_i^2 .$$

Table 8-10: Reference values for the normalised gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.323	__	Strong
	2.5D	0.302	__	Strong
	3D	0.28	__	Very Strong

Config. A	2D	0.0728	0.0014	Strong
	2.5D	0.0622	0.0007	Strong
Config. B	2D	0.0789	0.001	Strong
	2.5D	0.0613	0.0005	Strong
Config. C	3D	0.0286	0.0003	Strong
Config. D	3D	0.0491	0.0008	Strong
Config. E	3D	0.0414	0.0003	Strong

8-4-11- Zone Size Non-uniformity (szm_zsnu_2D / 2_5D / 3D) [1]

This feature assesses distribution of zone counts across sizes; lower values indicate more even spread over sizes.

$$zsnu = \frac{1}{N_s} \sum_{j=1}^{N_z} s_{.j}^{2}$$

Table 8-11: Reference values for the zone size non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	1.49	__	Strong
	2.5D	3.44	__	Strong
	3D	1	__	Very Strong
Config. A	2D	479	4	Strong
	2.5D	1.24×10^{4}	100	Strong
Config. B	2D	140	3	Strong

	2.5D	3.63×10^{3}	70	Strong
Config. C	3D	3.04×10^{3}	100	Strong
Config. D	3D	1.63×10^{3}	10	Strong
Config. E	3D	2.37×10^{3}	40	Strong

8-4-12- Normalised Zone Size Non-uniformity (szm_zsnu_norm_2D / 2_5D / 3D) [1]

Normalised version of ZSNU (by N_s).

$$zsnu_{norm} = \frac{1}{{N_s}^2} \sum_{i=1}^{N_z} s_{.j}^2$$

Table 8-12: Reference values for the normalised zone size non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.333	__	Strong
	2.5D	0.191	__	Strong
	3D	0.2	__	Very Strong
Config. A	2D	0.44	0.004	Strong
	2.5D	0.427	0.004	Strong
Config. B	2D	0.521	0.004	Strong
	2.5D	0.509	0.004	Strong
Config. C	3D	0.447	0.001	Strong
Config. D	3D	0.377	0.006	Strong

Config. E	3D	0.424	0.004	Strong

8-4-13- Zone Percentage (szm_z_perc_2D / 2_5D / 3D) [1]

This feature is a fraction of realised zones relative to the number of voxels; highly uniform ROIs tend to yield low z_perc.

$$z\_perc = \frac{N_s}{N_v}$$

Table 8-13: Reference values for the zone percentage feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.24	__	Strong
	2.5D	0.243	__	Strong
	3D	0.0676	__	Very Strong
Config. A	2D	0.3	0.003	Strong
	2.5D	0.253	0.004	Strong
Config. B	2D	0.324	0.001	Strong
	2.5D	0.26	0.002	Strong
Config. C	3D	0.148	0.003	Very Strong
Config. D	3D	0.0972	0.0007	Strong
Config. E	3D	0.126	0.001	Strong

8-4-14- Gray Level Variance (szm_gl_var_2D / 2_5D / 3D) [1]

This feature estimates variance of zone counts over gray levels using $P_{ij} = \frac{s_{ij}}{N_s}$:

$$gl\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_z} (i-\mu)^2 \, p_{ij}$$

Where, $\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_z} i \, p_{ij}$.

Table 8-14: Reference values for the gray level variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	3.97	__	Strong
	2.5D	3.92	__	Strong
	3D	2.64	__	Very Strong
Config. A	2D	42.7	0.7	Strong
	2.5D	47.9	0.4	Strong
Config. B	2D	36.1	0.3	Strong
	2.5D	41	0.7	Strong
Config. C	3D	106	1	Strong

Config. D	3D	32.7	1.6	Strong
Config. E	3D	50.8	0.9	Strong

8-4-15- Zone Size Variance (szm_zs_var_2D / 2_5D / 3D) [1]

This feature estimates variance of zone counts over zone sizes using $P_{ij} = \frac{s_{ij}}{N_s}$; requires the mean zone size.

$$zs\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_z} (j-\mu)^2 \, p_{ij}$$

Mean zone size is defined as $\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_z} j \, p_{ij}$.

Table 8-15: Reference values for the zone size variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	21	__	Strong
	2.5D	17.3	__	Strong
	3D	331	__	Very Strong
Config. A	2D	609	9	Strong
	2.5D	660	8	Strong
Config. B	2D	423	8	Strong

	2.5D	429	8	Strong
Config. C	3D	3.89×10^4	900	Strong
Config. D	3D	9.9×10^4	2.8×10^3	Strong
Config. E	3D	5.85×10^4	800	Strong

8-4-16- Zone Size Entropy (szm_zs_entr_2D / 2_5D / 3D) [1]

As before, let $P_{ij} = \frac{s_{ij}}{N_s}$; zone size entropy is then defined as:

$$zs\_entr = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_z} p_{ij} \, log_2 \, p_{ij}$$

Table 8-16: Reference values for the zone size entropy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	1.93	__	Strong
	2.5D	3.08	__	Strong
	3D	2.32	__	Very Strong
Config. A	2D	5.92	0.02	Strong

	2.5D	6.39	0.01	Strong
Config. B	2D	5.29	0.01	Strong
	2.5D	5.98	0.02	Strong
Config. C	3D	7	0.01	Strong
Config. D	3D	6.52	0.01	Strong
Config. E	3D	6.57	0.01	Strong

9- Neighbourhood Gray Tone Difference Based Features

9-1- Introduction [1]

According to IBSI, NGTDM is an alternative to GLCM. For each discretised gray level i, it accumulates the absolute difference between i and the local neighbourhood average (within a Chebyshev radius δ) over all voxels whose intensity equals i. Formally, with $X_{d,k}$ the discretised intensity at voxel $k = (k_x, k_y, k_z)$, the neighbourhood mean (excluding the center) is:

$$\underline{X_k} = \frac{1}{W}\sum_{m_z=-\delta}^{\delta}\sum_{m_y=-\delta}^{\delta}\sum_{m_x=-\delta}^{\delta} X_d(k_x+m_x, k_y+m_y, k_z+m_z) \qquad (m_x, m_y, m_z) \neq (0,0,0)$$

With $W = (2\delta+1)^3 - 1$ in 3D and $W = (2\delta+1)^2 - 1$ in 2D. The gray tone difference for level i is:

$$s_i = \sum_{k}^{N_v} \left|i - \underline{X_k}\right| \, [\, X_d(K) = i \; and \; k \; has \; a \; valid \; neighbourhood]$$

Using $[\ldots]$ as the Iverson bracket. IBSI generalises "valid neighbourhood" so that a neighbourhood exists if at least one neighbour lies inside the ROI; in that case:

$$\underline{X_k} = \frac{1}{W_k}\sum_{m_z=-\delta}^{\delta}\sum_{m_y=-\delta}^{\delta}\sum_{m_x=-\delta}^{\delta} X_d(k+m)[\, m \neq 0 \; and \; k \; + m \; in \; ROI]$$

$$W_k = \sum_{m_z=-\delta}^{\delta}\sum_{m_y=-\delta}^{\delta}\sum_{m_x=-\delta}^{\delta} [\, m \neq 0 \; and \; k + m \; in \; ROI]$$

Then:

$$s_i = \sum_{k}^{N_v} \left|i - \underline{X_k}\right| \, [\, X_d(K) = i \; and \; W_k \neq 0]$$

Define n_i as the number of voxels with gray level i that have $W_k \neq 0$. The gray-level probabilities used by NGTDM features are then $p_i = \frac{n_i}{N_{v,c}}$. Where $N_{v,c} = \sum n_i$. Also let N_g be the number of discretised levels in the ROI mask and $N_{g,p} \leq N_g$ the number of discretised levels with $p_i > 0$.

Fig. 9-1 shows an example of calculating NGTDM. Consider pixels inside the rectangle in (a) as $N_{v,c}$ with valid neighbours at $\delta = 1$.

Note: IBSI suggests to consider $N_{v,c}$ even if the pixels/voxels have one neighbour in the ROI.

1	2	2	3
1	2	3	3
4	2	4	1
4	1	2	3

(a) Grey levels

		n_i	p_i	s_i
i	1	0	0.00	0.000
	2	2	0.50	1.000
	3	1	0.25	0.625
	4	1	0.25	1.875

(b) Neighbourhood grey tone difference matrix

Fig 9-1: An example of calculating NGTDM. (a) The pixels with valid neighbours are located inside the triangle. (b) The NGTDM.

9-2- Feature Aggregation

The aggregation strategy is the same as 8-2.

NGTDM-specific notes: When merging NGTDMs across slices, sum the neighbourhood gray-tone differences s_i and the counts n_i (number of voxels at gray level i with a valid neighbourhood) element-wise across slices. After merging, recompute $N_{v,c}$ and p_i, from the merged NGTDM before calculating features. As with other families, feature values may depend strongly on the chosen aggregation method.

9-3- Distances and Distance Weighting [1]

The neighbourhood is defined by the Chebyshev norm. Alternative norms (Manhattan or Euclidean) may also be used. In general, the neighbourhood average gray level at voxel k is:

$$\underline{X_k} = \frac{1}{W_k} \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} X_d(k+m)\left[\,||m|| \leq \delta \; and \; m \neq 0 \; and \; k+m \; in \; ROI\right]$$

with neighbourhood size:

$$W_k = \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} \left[\,||m|| \leq \delta \; and \; m \neq 0 \; and \; k+m \; in \; ROI\right]$$

Here, $X_d(.)$ is the discretised intensity, $m = (m_x, m_y, m_z)$ is the offset vector, δ is the neighbourhood radius, $||.||$ is the chosen norm, and $[.]$ denotes the Iverson bracket (1 if the condition is true, 0 otherwise).

Distance weighting for NGTDM is straightforward. Let w be a weight that depends on the offset, e.g. $w = ||m||^{-1}$ or $w = exp\left(-||m||^2\right)$. Then:

$$\underline{X_k} = \frac{1}{W_k} \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} w(m) X_d(k+m) \left[\|m\| \leq \delta \text{ and } m \neq 0 \text{ and } k+m \text{ in } ROI \right]$$

With the corresponding (weighted) normaliser:

$$W_k = \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} w(m) \left[\|m\| \leq \delta \text{ and } m \neq 0 \text{ and } k+m \text{ in } ROI \right]$$

IBSI flags alternative norms/weighting as non-standard and cautions about reproducibility.

9-4- Explanation of Features

Let $p_i = \frac{n_i}{N_{v,c}}$ be the gray-level probabilities over contributing voxels, s_i the neighbourhood gray-tone differences, N_g the number of discretised gray levels in the ROI, and $N_{g,p} = \#\{i: p_i > 0\}$ [1]. Unless noted otherwise, the feature formulas are identical in 2D, 2.5D, and 3D; the only difference is the aggregation strategy.

9-4-1- Coarseness (ngt_coarseness_2D / 2_5D / 3D) [1]

Gray-level differences in coarse textures are generally small due to the dominance of large-scale patterns. Summing these differences provides an indication of the spatial rate of intensity change. The feature is defined as:

$$F_{ngt.coarseness} = \frac{1}{\sum_{i=1}^{N_g} \quad p_i \, s_i}$$

Since $\sum_{i=1}^{N_g} \quad p_i \, s_i$ may evaluate 0, the maximum coarseness value is set to an arbitrary constant of 10^6.

Table 9-1: Reference values for the coarseness feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.121	__	Strong
	2.5D	0.0285	__	Strong
	3D	0.0296	__	Very Strong

Config. A	2D	0.00629	0.00046	Strong
	2.5D	9.06×10^{-5}	3.3×10^{-6}	Strong
Config. B	2D	0.0168	0.0005	Strong
	2.5D	0.000314	4×10^{-6}	Strong
Config. C	3D	0.000216	4×10^{-6}	Strong
Config. D	3D	0.000208	4×10^{-6}	Strong
Config. E	3D	0.000188	4×10^{-6}	Strong

9-4-2- Contrast (ngt_contrast_2D / 2_5D / 3D) [1]

Contrast depends both on the dynamic range of gray levels and the spatial frequency of intensity changes. It is defined as:

$$F_{ngt.contrast} = \left(\frac{1}{N_{g,p}\left(N_{g,p} - 1\right)} \sum_{i_1=1}^{N_g} \sum_{i_2=1}^{N_g} p_{i_1} p_{i_2} (i_1 - i_2)^2 \right) \left(\frac{1}{N_{v,c}} \sum_{i=1}^{N_g} s_i \right)$$

Here, p_{i_1} and p_{i_2} are gray level probabilities with different iterators, i.e. $p_{i_1} = p_{i_2}$ for $i_1 = i_2$. The first term reflects the gray level dynamic range. The second term measures intensity variation within the ROI. If $N_g^p = 1$, then $contrast = 0$.

Table 9-2: Reference values for the contrast feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.925	—	Strong
	2.5D	0.601	—	Strong
	3D	0.584	—	Very Strong
Config. A	2D	0.107	0.002	Strong
	2.5D	0.0345	0.0009	Strong
Config. B	2D	0.181	0.001	Strong

	2.5D	0.0506	0.0005	Strong
Config. C	3D	0.0873	0.0019	Strong
Config. D	3D	0.046	0.0005	Strong
Config. E	3D	0.0752	0.0019	Moderate

9-4-3- Busyness (ngt_busyness_2D / 2_5D / 3D) [1]

Busy textures show rapid gray level changes between neighbouring voxels. IBSI defines this feature as:

$$F_{ngt.busyness} = \frac{\sum_{i=1}^{N_g} p_i \, s_i}{\sum_{i_1=1}^{N_g} \sum_{i_2=1}^{N_g} \left| i_1 p_{i_1} - i_2 p_{i_2} \right|}, \qquad p_{i_1} \neq 0 \; and \; p_{i_2} \neq 0$$

If $N_g^p = 1$, then $busyness = 0$.

Table 9-3: Reference values for the busyness feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	2.99	—	Strong
	2.5D	6.8	—	Strong
	3D	6.54	—	Very Strong
Config. A	2D	0.489	0.001	Strong
	2.5D	8.84	0.01	Strong
Config. B	2D	0.2	0.005	Strong
	2.5D	3.45	0.07	Strong
Config. C	3D	1.39	0.01	Very Strong
Config. D	3D	5.14	0.14	Strong

Config. E	3D	4.65	0.1	Strong

9-4-4- Complexity (ngt_complexity_2D / 2_5D / 3D) [1]

Complex textures are characterised by non-uniform gray level distributions and frequent rapid changes. Texture complexity is defined as:

$$F_{ngt.complexity} = \frac{1}{N_{v,c}} \sum_{i_1=1}^{N_g} \sum_{i_2=1}^{N_g} |i_1 - i_2| \frac{p_{i_1}s_{i_1} + p_{i_2}s_{i_2}}{p_{i_1} + p_{i_2}}, \qquad p_{i_1} \neq 0 \; and \; p_{i_2} \neq 0$$

With $p_{i_1}, p_{i_2} \neq 0$. Also, $p_{i_1} = p_{i_2}$ and $s_{i_1} = s_{i_2}$ if $i_1 = i_2$.

Table 9-4: Reference values for the complexity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	10.4	—	Strong
	2.5D	14.1	—	Strong
	3D	13.5	—	Very Strong
Config. A	2D	438	9	Strong
	2.5D	580	19	Strong
Config. B	2D	391	7	Strong
	2.5D	496	5	Strong
Config. C	3D	1.81×10^{3}	60	Very Strong
Config. D	3D	400	5	Strong
Config. E	3D	574	1	Moderate

9-4-5- Strength (ngt_strength_2D / 2_5D / 3D) [1]

This feature measures how strongly gray level pairs contribute to large differences, and it is defined as:

$$F_{ngt.strength} = \frac{\sum_{i=1}^{N_g} \sum_{i_2=1}^{N_g} (p_{i_1}+p_{i_2})(i_1-i_2)^2}{\sum_{i=1}^{N_g} s_i}, \qquad p_{i_1} \neq 0 \; and \; p_{i_2} \neq 0$$

If $\sum_{i=1}^{N_g} s_i = 0$, then $strength = 0$.

Table 9-5: Reference values for the strength feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	2.88	__	Strong
	2.5D	0.741	__	Strong
	3D	0.763	__	Very Strong
Config. A	2D	3.33	0.08	Strong
	2.5D	0.0904	0.0027	Strong
Config. B	2D	6.02	0.23	Strong
	2.5D	0.199	0.009	Strong
Config. C	3D	0.651	0.015	Strong
Config. D	3D	0.162	0.008	Strong
Config. E	3D	0.167	0.006	Strong

10- Neighbouring Gray Level Dependence Based Features

10-1- Introduction [1]

Sun and Wee [7] first introduced the Neighbouring Gray Level Dependence Matrix (NGLDM) as an alternative to the gray level co-occurrence matrix. Its purpose is to describe texture coarseness in an image while maintaining rotational invariance.

The NGLDM is built using the concept of a neighbourhood around each voxel. All voxels within a specified Chebyshev distance are considered neighbours of the central voxel. For a voxel at position k with discretised intensity $X_d(k)$, and a neighbour at position $k + m$ with value $X_d(k + m)$, the two voxels are considered dependent if their intensity difference does not exceed a non-negative coarseness parameter α.

The number of dependent neighbours, denoted j_k, is defined as:

$$j_k = 1 + \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} [\,|X_d(k) - X_d(k+m)| \leq \alpha \; and \; m \neq 0]$$

$$j_k = \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} [\,|X_d(k) - X_d(k+m)| \leq \alpha]$$

The dependence j_k is calculated for each voxel k inside the ROI mask. These values are then summarised in the NGLDM matrix M, which has size $N_g \times N_n$, where N_g is the number of gray levels present and $N_n = max(j_k)$ is the maximum dependence count observed. Each element s_{ij} in M represents the number of neighbourhoods with centre gray level i and dependence count j.

Let N_v be the total number of voxels in the ROI mask, and $N_s = \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} s_{ij}$.

Marginal sums are also defined as:

- $s_i = \sum_{j=1}^{N_n} s_{ij}$: the number of neighbourhoods with centre gray level i,
- $s_j = \sum_{i=1}^{N_g} s_{ij}$: the number of neighbourhoods with dependence j.

A two-dimensional illustration is shown in Fig. 10-1.

1	2	2	3
1	2	3	3
4	2	4	1
4	1	2	3

(a) Grey levels

		dependence k			
		0	1	2	3
i	1	0	0	0	0
	2	0	0	1	1
	3	0	0	1	0
	4	1	0	0	0

(b) Neighbouring grey level dependence matrix

Fig 10-1: An example of calculating NGLDM. [1]

This implementation differs slightly from the original definition by [7]. In practice, ROIs are often irregular and not cuboidal, so excluding neighbourhoods that extend outside the ROI can lead to inconsistencies, particularly at larger neighbourhood sizes. Therefore, the definition used here evaluates every voxel inside the ROI mask as a centre, and sets $N_v = N_s$. Neighbouring voxels located outside the ROI are ignored in the calculation:

$$j_k = \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} [\ |X_d(k) - X_d(k+m)| \leq \ \alpha \ and \ k+m \ in \ ROI]$$

Typically, $\alpha = 0$ is chosen for the coarseness parameter, and $\delta = 1$ for the neighbourhood radius, though other values can also be used.

10-2- Feature Aggregation

Feature aggregation for NGLDM follows the same three strategies defined in 8-2.

10-3- Distances and Distance Weighting [1]

Neighbourhoods are usually defined by the Chebyshev norm, but other distance norms may also be applied. The dependence count formula can then be generalised as:

$$j_k = \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} \left[\left||m|\right| \leq \delta \ and \ |X_d(k) - X_d(k+m)| \leq \ \alpha \ and \ k+m \ in \ ROI \right]$$

where m is the vector between voxel k and its neighbour, and $|m|$ is its length under the chosen norm.

Dependence can also be weighted by distance. For example, with $w = \left||m|\right|^{-1}$ or $w = exp\left(-\left||m|\right|^2\right)$, the dependence count becomes:

$$j_k = \sum_{m_z=-\delta}^{\delta} \sum_{m_y=-\delta}^{\delta} \sum_{m_x=-\delta}^{\delta} w(m) \left[\left||m|\right| \leq \delta \ and \ |X_d(k) - X_d(k+m)| \leq \ \alpha \ \ and \ k+m \ in \ ROI \right]$$

However, using alternative norms or distance weighting is regarded as non-standard, since it can introduce reproducibility issues.

10-4- Explanation of Features

The NGLDM structure parallels GLRLM, GLSZM and GLDZM; many NGLDM features are therefore defined similarly to GLRLM features (see section 7), except those originally defined by [7].

Unless noted otherwise, the feature formulas are identical in 2D, 2.5D, and 3D; the only difference is the aggregation strategy.

10-4-1- Low Dependence Emphasis (ngl_lde_2D / 2_5D / 3D) [1]

This feature highlights regions where voxels have small dependence counts. It corresponds to the “small number emphasis” feature originally proposed by [7].

$$F_{ngl.lde} = \frac{1}{N_s} \sum_{j=1}^{N_n} \frac{s_{.j}}{j^2}$$

Table 10-1: Reference values for the low dependence emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.158	__	Strong
	2.5D	0.159	__	Strong
	3D	0.045	__	Very Strong
Config. A	2D	0.281	0.003	Strong
	2.5D	0.243	0.004	Strong
Config. B	2D	0.31	0.001	Strong
	2.5D	0.254	0.002	Strong
Config. C	3D	0.137	0.003	Very Strong
Config. D	3D	0.0912	0.0007	Strong
Config. E	3D	0.118	0.001	Strong

10-4-2- High Dependence Emphasis (ngl_hde_2D / 2_5D / 3D) [1]

This feature emphasises large dependence counts. It corresponds to the "large number emphasis" feature in [7].

$$F_{ngl.hde} = \frac{1}{N_s} \sum_{j=1}^{N_n} j^2 s_{.j}$$

Table 10-2: Reference values for the high dependence emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	19.2	—	Strong
	2.5D	18.8	—	Strong
	3D	109	—	Very Strong
Config. A	2D	14.8	0.1	Strong
	2.5D	16.1	0.2	Strong
Config. B	2D	17.3	0.2	Strong
	2.5D	19.6	0.2	Strong
Config. C	3D	126	2	Strong
Config. D	3D	223	5	Strong
Config. E	3D	134	3	Strong

10-4-3- Low Gray Level Count Emphasis (ngl_lgce_2D / 2_5D / 3D) [1]

This feature emphasises low gray levels. It is the gray-level analogue of low dependence emphasis.

$$F_{ngl.lgce} = \frac{1}{N_s} \sum_{i=1}^{N_g} \frac{s_{i.}}{i^2}$$

Table 10-3: Reference values for the low gray level count emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.702	—	Strong
	2.5D	0.693	—	Strong
	3D	0.693	—	Very Strong
Config. A	2D	0.0233	0.0003	Strong
	2.5D	0.0115	0.0003	Strong
Config. B	2D	0.0286	0.0004	Strong
	2.5D	0.0139	0.0005	Strong
Config. C	3D	0.0013	4×10^{-5}	Strong
Config. D	3D	0.0168	0.0009	Strong
Config. E	3D	0.0154	0.0007	Strong

10-4-4- High Gray Level Count Emphasis (ngl_hgce_2D / 2_5D / 3D) [1]

This feature emphasises high gray levels. It is the gray-level analogue of high dependence emphasis.

$$F_{ngl.hgce} = \frac{1}{N_s} \sum_{i=1}^{N_g} i^2 s_i .$$

Table 10-4: Reference values for the high gray level count emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus

Dig. phantom	2D	7.49	—	Strong
	2.5D	7.66	—	Strong
	3D	7.66	—	Very Strong
Config. A	2D	446	2	Strong
	2.5D	466	2	Strong
Config. B	2D	359	10	Strong
	2.5D	375	11	Strong
Config. C	3D	1.57×10^{3}	10	Strong
Config. D	3D	364	16	Strong
Config. E	3D	502	8	Strong

10-4-5- Low Dependence Low Gray Level Emphasis (ngl_ldlge_2D / 2_5D / 3D) [1]

This feature highlights regions with both low dependence counts and low gray levels. It corresponds to the upper-left quadrant of the NGLDM.

$$F_{ngl.ldlge} = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} \frac{s_{ij}}{i^2 j^2}$$

Table 10-5: Reference values for the low dependence low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.0473	—	Strong
	2.5D	0.0477	—	Strong
	3D	0.00963	—	Very Strong
Config. A	2D	0.0137	0.0002	Strong

	2.5D	0.00664	0.0002	Strong
Config. B	2D	0.0203	0.0003	Strong
	2.5D	0.00929	0.00026	Strong
Config. C	3D	0.000306	1.2×10^{-5}	Strong
Config. D	3D	0.00357	4×10^{-5}	Strong
Config. E	3D	0.00388	4×10^{-5}	Strong

10-4-6- Low Dependence High Gray Level Emphasis (ngl_ldhge_2D / 2_5D / 3D) [1]

This feature highlights regions with low dependence counts combined with high gray levels. It corresponds to the lower-left quadrant of the NGLDM.

$$F_{ngl.ldhge} = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} \frac{i^2 s_{ij}}{j^2}$$

Table 10-6: Reference values for the low dependence high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	3.06	—	Strong
	2.5D	3.07	—	Strong
	3D	0.736	—	Very Strong
Config. A	2D	94.2	0.4	Strong
	2.5D	91.9	0.5	Strong
Config. B	2D	78.9	2.2	Strong
	2.5D	73.4	2.1	Strong
Config. C	3D	141	2	Strong

Config. D	3D	18.9	1.1	Strong
Config. E	3D	36.7	0.5	Strong

10-4-7- High Dependence Low Gray Level Emphasis (ngl_hdlge_2D / 2_5D / 3D) [1]

This feature highlights regions with high dependence counts combined with low gray levels. It corresponds to the upper-right quadrant of the NGLDM.

$$F_{ngl.hdlge} = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} \frac{j^2 s_{ij}}{i^2}$$

Table 10-7: Reference values for the high dependence low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	17.6	__	Strong
	2.5D	17.2	__	Strong
	3D	102	__	Very Strong
Config. A	2D	0.116	0.001	Strong
	2.5D	0.0674	0.0004	Strong
Config. B	2D	0.108	0.003	Strong
	2.5D	0.077	0.0019	Strong
Config. C	3D	0.0828	0.0003	Strong
Config. D	3D	0.798	0.072	Strong
Config. E	3D	0.457	0.031	Strong

10-4-8- High Dependence High Gray Level Emphasis (ngl_hdhge_2D / 2_5D / 3D) [1]

This feature highlights regions with both high dependence counts and high gray levels. It corresponds to the lower-right quadrant of the NGLDM.

$$F_{ngl.hdhg} = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} i^2 j^2 s_{ij}$$

Table 10-8: Reference values for the high dependence high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	49.5	—	Strong
	2.5D	50.8	—	Strong
	3D	235	—	Very Strong
Config. A	2D	7.54×10^3	60	Strong
	2.5D	8.1×10^3	60	Strong
Config. B	2D	7.21×10^3	130	Strong
	2.5D	7.97×10^3	150	Strong
Config. C	3D	2.27×10^5	3×10^3	Strong
Config. D	3D	9.28×10^4	1.3×10^3	Strong
Config. E	3D	7.6×10^6	600	Strong

10-4-9- Gray Level Non-uniformity (ngl_glnu_2D / 2_5D / 3D) [1]

This feature measures the variability of dependence counts across gray levels. Lower values indicate more uniform gray level distribution.

$$F_{ngl.glnu} = \frac{1}{N_s} \sum_{i=1}^{N_g} s_{i.}^2$$

Table 10-9: Reference values for the gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	10.2	—	Strong

	2.5D	37.9	__	Strong
	3D	37.9	__	Very Strong
Config. A	2D	757	1	Strong
	2.5D	1.72×10^{4}	100	Strong
Config. B	2D	216	3	Strong
	2.5D	4.76×10^{3}	50	Strong
Config. C	3D	6.42×10^{3}	10	Strong
Config. D	3D	1.02×10^{4}	300	Strong
Config. E	3D	8.17×10^{3}	130	Strong

10-4-10- Normalised Gray Level Non-uniformity (ngl_glnu_norm_2D / 2_5D / 3D) [1]

This feature is the normalised version of gray level non-uniformity. It is mathematically equivalent to intensity histogram uniformity when computed from a single 3D NGLDM.

$$F_{ngl.glnu.norm} = \frac{1}{N_s^2} \sum_{i=1}^{N_g} s_{i.}^2$$

Table 10-10: Reference values for the normalised gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.562	__	Strong
	2.5D	0.512	__	Strong
	3D	0.512	__	Very Strong
Config. A	2D	0.151	0.003	Strong

	2.5D	0.15	0.002	Strong
Config. B	2D	0.184	0.001	Strong
	2.5D	0.174	0.001	Strong
Config. C	3D	0.14	0.003	Very Strong
Config. D	3D	0.229	0.003	Strong
Config. E	3D	0.184	0.001	Strong

10-4-11- Dependence Count Non-uniformity (ngl_dcnu_2D / 2_5D / 3D) [1]

This feature measures the variability of voxel counts across dependence counts. Lower values indicate a more even distribution.

$$dcnu = \frac{1}{N_s} \sum_{j=1}^{N_n} s_{.j}^{2}$$

Table 10-11: Reference values for the dependence count non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	3.96	—	Strong
	2.5D	12.4	—	Strong
	3D	4.86	—	Very Strong
Config. A	2D	709	2	Strong
	2.5D	1.75×10^{4}	100	Strong
Config. B	2D	157	1	Strong
	2.5D	3.71×10^{3}	30	Strong
Config. C	3D	2.45×10^{3}	60	Strong

Config. D	3D	1.84×10^3	30	Strong
Config. E	3D	2.25×10^3	30	Strong

10-4-12- Normalised Dependence Count Non-uniformity (ngl_dcnu_norm_2D / 2_5D / 3D) [1]

This feature is the normalised version of dependence count non-uniformity.

$$dcnu_{norm} = \frac{1}{N_s^2} \sum_{i=1}^{N_n} s_{.j}^2$$

Table 10-12: Reference values for the normalised dependence count non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.212	—	Strong
	2.5D	0.167	—	Strong
	3D	0.0657	—	Very Strong
Config. A	2D	0.175	0.001	Strong
	2.5D	0.153	0.001	Strong
Config. B	2D	0.179	0.001	Strong
	2.5D	0.136	0.001	Strong
Config. C	3D	0.0532	0.0005	Strong
Config. D	3D	0.0413	0.0003	Strong
Config. E	3D	0.0505	0.0003	Strong

10-4-13- Dependence Count percentage (ngl_dc_perc_2D / 2_5D / 3D) [1]

This feature measures the fraction between the number of realised neighbourhoods and the maximum possible number of neighbourhoods. Under the IBSI definition, where complete neighbourhoods are not required, the feature always evaluates to 1 and can therefore be omitted.

$$dc\_perc = \frac{N_s}{N_v}$$

Table 10-13: Reference values for the dependence count percentage feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	1	__	Strong
	2.5D	1	__	Moderate
	3D	1	__	Strong
Config. A	2D	1	__	Moderate
	2.5D	1	__	Strong
Config. B	2D	1	__	Moderate
	2.5D	1	__	Moderate
Config. C	3D	1	__	Strong
Config. D	3D	1	__	Strong
Config. E	3D	1	__	Moderate

10-4-14- Gray Level Variance (ngl_gl_var_2D / 2_5D / 3D) [1]

This feature reflects the variance of dependence counts across gray levels. It is computed from the joint probability $p_{ij} = \frac{s_{ij}}{N_s}$, which represents the probability of encountering gray level i with dependence j.

$$gl\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} (i - \mu)^2 p_{ij}$$

Where:

$$\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} i\, p_{ij}$$

Table 10-14: Reference values for the gray level variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	2.7	__	Strong
	2.5D	3.05	__	Strong
	3D	3.05	__	Very Strong
Config. A	2D	31.1	0.5	Strong
	2.5D	22.8	0.6	Strong
Config. B	2D	25.3	0.4	Strong
	2.5D	18.7	0.2	Strong
Config. C	3D	81.1	2.1	Very Strong
Config. D	3D	21.7	0.4	Strong

Config. E	3D	30.4	0.8	Strong

10-4-15- Dependence Count Variance (ngl_dc_var_2D / 2_5D / 3D) [1]

This feature estimates the variance of dependence counts across all possible dependence levels. It is based on the joint probability $p_{ij} = \frac{s_{ij}}{N_s}$.

$$dc\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} (j - \mu)^2 p_{ij}$$

Where the mean dependence count is:

$$\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} j\, p_{ij}$$

Table 10-15: Reference values for the dependence count variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	2.73	__	Strong
	2.5D	3.27	__	Strong
	3D	22.1	__	Very Strong
Config. A	2D	3.12	0.02	Strong
	2.5D	3.37	0.01	Strong
Config. B	2D	4.02	0.05	Strong
	2.5D	4.63	0.06	Strong

Config. C	3D	39.2	0.1	Strong
Config. D	3D	63.9	1.3	Strong
Config. E	3D	39.4	1	Strong

10-4-16- Dependence Count Entropy (ngl_dc_entr_2D / 2_5D / 3D) [1]

This feature measures the randomness in the distribution of dependence counts. It corresponds to the feature termed entropy in [7]. Using the joint probability $p_{ij} = \frac{s_{ij}}{N_s}$, it is defined as:

$$dc\_entr = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_n} p_{ij} \, log_2 \, p_{ij}$$

This formulation corrects an error in the original definition by [7], where dependence counts s_{ij} were incorrectly used inside the logarithm instead of the probability p_{ij} .

Table 10-16: Reference values for the dependence count entropy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	2.71	__	Strong
	2.5D	3.36	__	Strong
	3D	4.4	__	Very Strong
Config. A	2D	5.76	0.02	Strong

	2.5D	5.93	0.02	Strong
Config. B	2D	5.38	0.01	Strong
	2.5D	5.78	0.01	Strong
Config. C	3D	7.54	0.03	Very Strong
Config. D	3D	6.98	0.01	Strong
Config. E	3D	7.06	0.02	Strong

10-4-17- Dependence Count Energy (ngl_dc_energy_2D / 2_5D / 3D) [1]

This feature reflects the uniformity of the dependence count distribution. It is also referred to as the second moment by [7]. With $p_{ij} = \frac{s_{ij}}{N_s}$, it is computed as:

$$dc\_energy = \sum_{i=1}^{N_g} \sum_{j=1}^{N_n} p_{ij}^2$$

This definition also corrects an error in the original work [7], where squared dependence counts s_{ij}^2 were only normalised by N_s, introducing a strong dependency on image volume. The revised formulation uses the probability p_{ij} , which normalises by $N_s{}^2$.

Table 10-17: Reference values for the dependence count energy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.17	__	Strong
	2.5D	0.122	__	Strong

	3D	0.0533	__	Very Strong
Config. A	2D	0.0268	0.0004	Strong
	2.5D	0.0245	0.0003	Strong
Config. B	2D	0.0321	0.0002	Strong
	2.5D	0.0253	0.0001	Moderate
Config. C	3D	0.00789	0.00011	Strong
Config. D	3D	0.0113	0.0002	Strong
Config. E	3D	0.0106	0.0001	Strong

11- Gray Level Distance Zone Based Features

11-1- Introduction [1]

The Gray Level Distance Zone Matrix (GLDZM) counts the number of connected voxel groups (zones) that share the same discretised gray level and have the same distance to the ROI boundary. This matrix captures the combined relationship between gray level intensity and spatial location within the ROI.

To compute the GLDZM, two input maps are required:

1. A gray level zone map (as defined for GLSZM, section 8).
2. A distance map describing voxel distance to the ROI edge.

Neighbouring voxels are grouped if they have the same gray level, using 26-connectedness in 3D and 8-connectedness in 2D. Distances to the ROI boundary are defined using 6-connectedness in 3D and 4-connectedness in 2D. The distance of a voxel corresponds to the minimum number of steps through neighbouring voxels required to reach the boundary. The distance assigned to a zone equals the minimum distance among all its voxels.

This definition differs from the original by [8], which assumed rectangular 2D images. Since ROIs are rarely rectangular, Chamfer maps are not practical. Instead, distance is determined iteratively as follows:

1. The ROI mask is eroded using the chosen connectivity (6 or 4-connected).
2. The distance map values for eroded voxels are incremented by 1.
3. Steps 1–2 are repeated until the ROI mask is empty.

A further difference is that the minimum distance is set to 1 (rather than 0 for boundary voxels) to avoid division by zero in some feature definitions.

Let M denote the GLDZM with dimensions $N_g \times N_d$, where N_g is the number of gray levels and N_d the largest distance in the ROI. The element d_{ij} represents the number of zones of gray level i and distance j. Marginal sums are:

- $d_{i.} = \sum_{j=1}^{N_d} d_{ij}$: total zones for gray level i.
- $d_{.j} = \sum_{i=1}^{N_g} d_{ij}$: total zones at distance j.

The total number of zones is $N_s = \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} d_{ij}$, and the number of voxels is N_v. An example of calculating GLDZM is illustrated in Fig. 11-1.

GLDZM uses both ROI masks: the morphological mask is applied to compute distances, while the intensity mask is used to determine zones.

11-2- Feature Aggregation

Feature aggregation for GLDZM follows the same three strategies defined in 8-2. When merging matrices, both d_{ij} and N_v must be summed to ensure consistency. Results may vary significantly depending on the aggregation method.

11-3- Distances and Distance Weighting [1]

By default, the Manhattan norm is used for efficiency. Alternatives include Chebyshev (changing voxel connectivity) and Euclidean norms. The latter is less efficient and may cause variability in j values due to rounding. IBSI discourages non-standard norms due to reproducibility concerns.

$$\begin{array}{cccc} 1 & 2 & 2 & 3 \\ 1 & 2 & 3 & 3 \\ 4 & 2 & 4 & 1 \\ 4 & 1 & 2 & 3 \end{array}$$

(a) Grey levels

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{array}$$

(b) Distance map

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{j} \\ & & 1 & 2 \\ \hline & 1 & 3 & 0 \\ i & 2 & 2 & 0 \\ & 3 & 2 & 0 \\ & 4 & 1 & 1 \end{array}$$

(c) Grey level distance zone matrix

Fig 11-1: An example of calculating GLDZM [1].

11-4- Explanation of Features [1]

Unless noted otherwise, the feature formulas are identical in 2D, 2.5D, and 3D; the only difference is the aggregation strategy.

11-4-1- Small Distance Emphasis (dzm_sde_2D / 2_5D / 3D)

This feature emphasizes short distances between zones and the ROI boundary.

$$sde = \frac{1}{N_s} \sum_{j=1}^{N_d} \frac{d_{.j}}{j^2}$$

Table 11-1: Reference values for the small distance emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.946	__	Strong

	2.5D	0.917	__	Moderate
	3D	1	__	Very Strong
Config. A	2D	0.192	0.006	Strong
	2.5D	0.168	0.005	Strong
Config. B	2D	0.36	0.005	Strong
	2.5D	0.329	0.004	Strong
Config. C	3D	0.531	0.006	Strong
Config. D	3D	0.579	0.004	Strong
Config. E	3D	0.527	0.004	Moderate

11-4-2- Large Distance Emphasis (dzm_lde_2D / 2_5D / 3D)

This feature highlights long distances to the ROI boundary.

$$lde = \frac{1}{N_s} \sum_{j=1}^{N_d} j^2 d_{.j}$$

Table 11-2: Reference values for the large distance emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	1.21	__	Strong
	2.5D	1.33	__	Moderate
	3D	1	__	Very Strong

Config. A	2D	161	1	Moderate
	2.5D	178	1	Moderate
Config. B	2D	31.6	0.2	Moderate
	2.5D	34.3	0.2	Moderate
Config. C	3D	11	0.3	Strong
Config. D	3D	10.3	0.1	Strong
Config. E	3D	12.6	0.1	Moderate

11-4-3- Low Gray Level Zone Emphasis (dzm_lgze_2D / 2_5D / 3D)

This feature emphasizes zones with low gray levels, analogous to Small Distance Emphasis but along the gray level axis.

$$lgze = \frac{1}{N_s} \sum_{i=1}^{N_g} \frac{d_{i.}}{i^2}$$

Table 11-3: Reference values for the low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.371	__	Strong
	2.5D	0.368	__	Moderate
	3D	0.253	__	Very Strong
Config. A	2D	0.0368	0.0005	Strong
	2.5D	0.0291	0.0005	Strong
Config. B	2D	0.0475	0.001	Strong

	2.5D	0.0387	0.001	Strong
Config. C	3D	0.00235	6×10^{-5}	Strong
Config. D	3D	0.0409	0.0005	Strong
Config. E	3D	0.034	0.0004	Moderate

11-4-4- High Gray Level Zone Emphasis (dzm_hgze_2D / 2_5D / 3D)

This feature emphasizes zones with high gray levels, analogous to Large Distance Emphasis.

$$hgze = \frac{1}{N_s} \sum_{i=1}^{N_g} i^2 d_i .$$

Table 11-4: Reference values for the high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	16.4	__	Strong
	2.5D	16.2	__	Moderate
	3D	15.6	__	Very Strong
Config. A	2D	363	3	Strong
	2.5D	370	3	Strong
Config. B	2D	284	11	Strong
	2.5D	284	11	Strong
Config. C	3D	971	7	Strong
Config. D	3D	188	10	Strong
Config. E	3D	286	6	Strong

11-4-5- Small Distance Low Gray Level Emphasis (dzm_sdlge_2D / 2_5D / 3D)

This feature emphasizes zones with both low gray levels and short distances (upper-left quadrant of the GLDZM).

$$sdlge = \frac{1}{N_s}\sum_{i=1}^{N_g}\sum_{j=1}^{N_d}\frac{d_{ij}}{i^2 j^2}$$

Table 11-5: Reference values for the small distance low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.367	—	Strong
	2.5D	0.362	—	Moderate
	3D	0.253	—	Very Strong
Config. A	2D	0.00913	0.00023	Strong
	2.5D	0.00788	0.00022	Strong
Config. B	2D	0.0192	0.0005	Strong
	2.5D	0.0168	0.0005	Strong
Config. C	3D	0.00149	4×10^{-5}	Strong
Config. D	3D	0.0302	0.0006	Strong

Config. E	3D	0.0228	0.0003	Moderate

11-4-6- Small Distance High Gray Level Emphasis (dzm_sdhge_2D / 2_5D / 3D)

This feature emphasizes zones with high gray levels and short distances (lower-left quadrant of the GLDZM).

$$sdhge = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} \frac{i^2 d_{ij}}{j^2}$$

Table 11-6: Reference values for the small distance high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	15.2	__	Strong
	2.5D	14.3	__	Moderate
	3D	15.6	__	Very Strong
Config. A	2D	60.1	3.3	Strong
	2.5D	49.5	2.8	Strong
Config. B	2D	95.7	5.5	Strong
	2.5D	81.4	4.6	Strong
Config. C	3D	476	11	Strong
Config. D	3D	99.3	5.1	Strong
Config. E	3D	136	4	Moderate

11-4-7- Large Distance Low Gray Level Emphasis (dzm_ldlge_2D / 2_5D / 3D)

This feature emphasizes zones with low gray levels and long distances (upper-right quadrant of the GLDZM).

$$ldlge = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} \frac{j^2 d_{ij}}{i^2}$$

Table 11-7: Reference values for the large distance low gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.386	—	Strong
	2.5D	0.391	—	Moderate
	3D	0.253	—	Very Strong
Config. A	2D	2.96	0.02	Moderate
	2.5D	2.31	0.01	Moderate
Config. B	2D	0.934	0.018	Moderate
	2.5D	0.748	0.017	Moderate
Config. C	3D	0.0154	0.0005	Strong
Config. D	3D	0.183	0.004	Strong
Config. E	3D	0.179	0.004	Moderate

11-4-8- Large Distance High Gray Level Emphasis (dzm_ldhge_2D / 2_5D / 3D)

This feature emphasizes zones with both high gray levels and long distances (lower-right quadrant of the GLDZM).

$$ldhge = \frac{1}{N_s} \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} i^2 j^2 d_{ij}$$

Table 11-8: Reference values for the large distance high gray level emphasis feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	21.3	—	Strong
	2.5D	23.7	—	Moderate
	3D	15.6	—	Very Strong
Config. A	2D	7.01×10^4	100	Moderate
	2.5D	7.95×10^4	100	Moderate
Config. B	2D	1.06×10^4	300	Strong
	2.5D	1.16×10^4	400	Strong
Config. C	3D	1.34×10^4	200	Strong
Config. D	3D	2.62×10^3	110	Strong
Config. E	3D	4.85×10^3	60	Moderate

11-4-9- Gray Level Non-uniformity (dzm_glnu_2D / 2_5D / 3D)

This feature measures the variability of zone counts across gray levels. A low value indicates an even distribution.

$$glnu = \frac{1}{N_s} \sum_{i=1}^{N_g} d_i^2.$$

Table 11-9: Reference values for the gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	1.41	—	Strong
	2.5D	5.44	—	Moderate
	3D	1.4	—	Very Strong
Config. A	2D	82.2	0.1	Strong
	2.5D	1.8×10^{3}	10	Strong
Config. B	2D	20.5	0.1	Strong
	2.5D	437	3	Strong
Config. C	3D	195	6	Strong
Config. D	3D	212	6	Strong
Config. E	3D	231	6	Moderate

11-4-10- Normalised Gray Level Non-uniformity (dzm_glnu_norm_2D / 2_5D / 3D)

This is the normalised version of Gray Level Non-Uniformity.

$$glnu_{norm} = \frac{1}{N_s^2} \sum_{i=1}^{N_g} d_{i.}^2$$

Table 11-10: Reference values for the normalised gray level non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.323	—	Strong
	2.5D	0.302	—	Moderate

	3D	0.28	__	Very Strong
Config. A	2D	0.0728	0.0014	Strong
	2.5D	0.0622	0.0007	Strong
Config. B	2D	0.0789	0.001	Strong
	2.5D	0.0613	0.0005	Strong
Config. C	3D	0.0286	0.0003	Strong
Config. D	3D	0.0491	0.0008	Strong
Config. E	3D	0.0414	0.0003	Moderate

11-4-11- Zone Distance Non-uniformity (dzm_zdnu_2D / 2_5D / 3D)

This feature measures the variability of zone counts across distances. A low value indicates equal distribution.

$$zdnu = \frac{1}{N_s}\sum_{j=1}^{N_d} d_{.j}^{2}$$

Table 11-11: Reference values for the zone distance non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	3.79	__	Strong
	2.5D	14.4	__	Moderate
	3D	5	__	Very Strong
Config. A	2D	64	0.4	Moderate
	2.5D	1.57×10^{3}	10	Strong

Config. B	2D	39.8	0.3	Moderate
	2.5D	963	6	Moderate
Config. C	3D	1.87×10^{3}	40	Strong
Config. D	3D	1.37×10^{3}	20	Strong
Config. E	3D	1.5×10^{3}	30	Moderate

11-4-12- Normalised Zone Distance Non-uniformity (dzm_zdnu_norm_2D / 2_5D / 3D)

This is the normalised version of Zone Distance Non-Uniformity.

$$zdnu_{norm} = \frac{1}{N_s^2} \sum_{j=1}^{N_d} d_{.j}^2$$

Table 11-12: Reference values for the normalised zone distance non-uniformity feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.898	—	Strong
	2.5D	0.802	—	Moderate
	3D	1	—	Very Strong
Config. A	2D	0.0716	0.0022	Strong
	2.5D	0.0543	0.0014	Strong
Config. B	2D	0.174	0.003	Strong
	2.5D	0.135	0.001	Strong
Config. C	3D	0.274	0.005	Strong
Config. D	3D	0.317	0.004	Strong

Config. E	3D	0.269	0.003	Moderate

11-4-13- Zone Percentage (dzm_z_perc_2D / 2_5D / 3D)

This feature measures the ratio of the number of observed zones to the maximum possible number of zones. Homogeneous ROIs yield lower percentages.

$$z\_perc = \frac{N_s}{N_v}$$

Table 11-13: Reference values for the zone percentage feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.24	__	Strong
	2.5D	0.243	__	Moderate
	3D	0.0676	__	Very Strong
Config. A	2D	0.3	0.003	Strong
	2.5D	0.253	0.004	Moderate
Config. B	2D	0.324	0.001	Strong
	2.5D	0.26	0.002	Moderate
Config. C	3D	0.148	0.003	Strong

Config. D	3D	0.0972	0.0007	Strong
Config. E	3D	0.126	0.001	Moderate

11-4-14- Gray Level Variance (dzm_gl_var_2D / 2_5D / 3D)

This feature estimates the variance of zone counts across gray levels. With $p_{ij} = \frac{d_{ij}}{N_s}$,

$$gl\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} (i-\mu)^2 \, p_{ij}$$

Where $\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} i \, p_{ij}$.

Table 11-14: Reference values for the gray level variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	3.97	__	Strong
	2.5D	3.92	__	Moderate
	3D	2.64	__	Very Strong
Config. A	2D	42.7	0.7	Moderate
	2.5D	47.9	0.4	Strong
Config. B	2D	36.1	0.3	Moderate
	2.5D	41	0.7	Strong
Config. C	3D	106	1	Strong
Config. D	3D	32.7	1.6	Strong
Config. E	3D	50.8	0.9	Strong

11-4-15- Zone Distance Variance (dzm_zd_var_2D / 2_5D / 3D)

This feature estimates the variance of zone counts across distances. With $p_{ij} = \frac{d_{ij}}{N_s}$,

$$zd\_var = \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} (j - \mu)^2 \, p_{ij}$$

Where $\mu = \sum_{i=1}^{N_g} \sum_{j=1}^{N_d} j \, p_{ij}$.

Table 11-15: Reference values for the zone distance variance feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	0.051	__	Strong
	2.5D	0.0988	__	Moderate
	3D	0	__	Very Strong
Config. A	2D	69.4	0.1	Moderate
	2.5D	78.9	0.1	Moderate
Config. B	2D	13.5	0.1	Moderate
	2.5D	15	0.1	Moderate
Config. C	3D	4.6	0.06	Strong
Config. D	3D	4.61	0.04	Strong
Config. E	3D	5.56	0.05	Strong

11-4-16- Zone Distance Entropy (dzm_zd_entr_2D / 2_5D / 3D)

This feature measures the randomness of the joint *gray level-distance* distribution. With $p_{ij} = \frac{d_{ij}}{N_s}$
:

$$zd\_entr = -\sum_{i=1}^{N_g} \sum_{j=1}^{N_d} p_{ij} \log_2 p_{ij}$$

Table 11-16: Reference values for the zone distance entropy feature. [1]

Data	Aggr. Method	Value	Tol.	Consensus
Dig. phantom	2D	1.73	—	Strong
	2.5D	2	—	Moderate
	3D	1.92	—	Very Strong
Config. A	2D	8	0.04	Strong
	2.5D	8.87	0.03	Strong
Config. B	2D	6.47	0.03	Strong
	2.5D	7.58	0.01	Moderate
Config. C	3D	7.56	0.03	Strong
Config. D	3D	6.61	0.03	Strong
Config. E	3D	7.06	0.01	Moderate

12- 3D moment Invariants

12-1- Introduction

In this category, 3D Moment Invariants are calculated. These invariants are built from raw moments and central moments, which are invariant to translation, rotation, and scale. The papers [9][10] introduce these moments and outlines how central moments are formed by subtracting the centroid from the raw moments, ensuring that they are translation-invariant. The normalization step removes the effect of size (scale) by dividing the moments by appropriate powers of the zeroth moment.

The central moments μ_{pqr} of order $p+q+r$ are defined as follows:

$$\mu_{pqr} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(x_1 - \underline{x}_1\right)^p \left(x_2 - \underline{x}_2\right)^q \left(x_3 - \underline{x}_3\right)^r \rho(x_1, x_2, x_3)\, dx_1 dx_2 dx_3$$ [9]

Where x_1, x_2, x_3 are the voxel coordinates, and $\underline{x}_1, \underline{x}_2, \underline{x}_3$ are the centroids (mean coordinates). The centroid is calculated as:

$$\underline{x}_1 = \frac{m_{100}}{m_{000}}\ ;\quad \underline{x}_2 = \frac{m_{010}}{m_{000}}\ ;\quad \underline{x}_3 = \frac{m_{001}}{m_{000}}$$ [9]

Where m_{pqr} are the raw moments given by:

$$m_{pqr} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1^p\, x_2^q\, x_3^r\, \rho(x_1, x_2, x_3)\, dx_1 dx_2 dx_3$$ [9]

The normalized central moments are then obtained by dividing the central moments by the zeroth moment raised to an appropriate power:

$$n_{pqr} = \frac{\mu_{pqr}}{\mu_{000}^{1+\frac{p+q+r}{3}}}$$

For second-order moments $(p+q+r=2)$, the exponent is $\frac{5}{3}$, and for third-order moments $(p+q+r=3)$, the exponent is 2. These normalized moments form the basis for the rotation and scale-invariant features. [9]

Aggregating features

No feature aggregation is required.

Distances and Distance Weighting

No distance weighting is required.

12-4- Explanation of Features

We are calculating features of this category once for the intensity-weighted version of the image and once again for the binary mask (shape-only). Formulations are identical in both cases; only the inputs differ.

Note that reference [11] is cited in [10], and includes additional Moment Invariant which were not defined in the original paper.

12-4-1- First Principal Invariant (mu_FPI_intensity / binary)

The first invariant is the sum of the diagonal components of the second-order moment tensor, which corresponds to the sum of the second-order central moments. This feature is calculated as:

$$J_1 = n_{200} + n_{020} + n_{002}$$ [9]

12-4-2- Second-order Frobenius Norm Squared (mu_SFNS_intensity / binary)

This feature is derived from the second-order central moments and is a measure of the Frobenius norm of the moment tensor. It is used to describe the "spread" of the shape's second-order features. This feature is calculated as follows:

$$Q = n_{200}^2 + n_{020}^2 + n_{002}^2 + 2\,(n_{110}^2 + n_{101}^2 + n_{011}^2)$$ [10]

12-4-2- Second Principal Invariant (mu_SPI_intensity / binary)

The second moment invariant is calculated as:

$$J_2 = n_{020}\, n_{002} - n_{011}^2 + n_{200}\ \ n_{002} - n_{101}^2 + n_{200}\ \ n_{020} - n_{110}^2$$ [9]

12-4-3- Third Principal Invariant (mu_TPI_intensity / binary)

The third invariant is a more complex combination of second-order central moments, which reflects the determinant-like properties of the moment tensor:

$$J_3 = n_{200}\, n_{020}\, n_{002} + 2\, n_{110}\, n_{01}\ \ n_{101} - n_{020}\, n_{101}^2 - n_{011}^2\, n_{200} - n_{110}^2\, n_{002}$$ [9]

12-4-5- Third-order Frobenius Norm Squared (mu_TFNS_intensity / binary)

This feature is calculated as follows:

$$B_3 = n_{300}^2 + n_{030}^2 + n_{003}^2 + 3\,(n_{210}^2 + n_{201}^2 + n_{120}^2 + n_{102}^2 + n_{021}^2 + n_{012}^2) + 6\,(n_{111}^2)$$ [11]

References

[1] Zwanenburg, A., Vallières, M., Abdalah, M.A. *et al.* (2020). The Image Biomarker Standardisation Initiative: Standardised Quantitative Radiomics for High-Throughput Image-based Phenotyping. *Radiology*, 295(2), 328–338. doi:10.1148/radiol.2020191145.

[2] Vallières, M., Freeman, C. R., Skamene, S. R., & El Naqa, I. (2015). A radiomics model from joint FDG-PET and MRI texture features for the prediction of lung metastases in soft-tissue sarcomas of the extremities. Physics in Medicine & Biology, 60(14), 5471.

[3] Van Griethuysen, J. J., Fedorov, A., Parmar, C., Hosny, A., Aucoin, N., Narayan, V., ... & Aerts, H. J. (2017). Computational radiomics system to decode the radiographic phenotype. Cancer research, 77(21), e104-e107.

[4] Clausi, D. A. (2002). An analysis of co-occurrence texture statistics as a function of gray level quantization. Canadian Journal of remote sensing, 28(1), 45-62.

[5] Haralick, R. M., Shanmugam, K., & Dinstein, I. H. (2007). Textural features for image classification. IEEE Transactions on systems, man, and cybernetics, (6), 610-621.

[6] Van Griethuysen, J. J., Fedorov, A., Parmar, C., Hosny, A., Aucoin, N., Narayan, V., ... & Aerts, H. J. (2017). Computational radiomics system to decode the radiographic phenotype. Cancer research, 77(21), e104-e107.

[7] Sun, C., & Wee, W. G. (1983). Neighboring gray level dependence matrix for texture classification. Computer Vision, Graphics, and Image Processing, 23(3), 341-352.

[8] Thibault, G., Angulo, J., & Meyer, F. (2013). Advanced statistical matrices for texture characterization: application to cell classification. IEEE Transactions on Biomedical Engineering, 61(3), 630-637.

[9] Sadjadi, F. A., & Hall, E. L. (1980). Three-dimensional moment invariants. IEEE Transactions on Pattern Analysis and Machine Intelligence, (2), 127-136.

[10] Suk, T., & Flusser, J. (2011, August). Tensor method for constructing 3D moment invariants. In International Conference on Computer Analysis of Images and Patterns (pp. 212-219). Berlin, Heidelberg: Springer Berlin Heidelberg.

[11] Department of Image Processing: 3D rotation moment invariants,
http://zoi.utia.cas.cz/3DRotationInvariants