# Rolling Bearing Fault Diagnosis Based on Higher-order Cumulants and BP Neural Network

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**Abstract:** Based on the fact that the rolling bearing fault vibration signals are susceptible to Gauss noise, a fault diagnosis of rolling bearing method using higher-order cumulants and back propagation (BP) neural network is proposed. In this paper, the higher-order statistics of the vibration signals are calculated as feature vectors, including the third-order cumulant and the fourth-order cumulant as well as the second-order cumulant. And a BP neural network is trained to identify the bearing fault by using those features. The effectiveness of the proposed method is verified by four types of rolling bearing, namely ball fault, inner raceway fault, outer raceway fault, and normal bearing. The experimental results show cumulants based fault features have perfect separation. Except the training and test diagnostic accuracy of ball fault are high as 98.75 % and 96.67%, classification accuracies of other faults rate are 100%.

**Key Words:** Rolling Bearings, Vibration Signals, Higher-order Cumulants, BP Neural Network, Fault Diagnosis

## 1 INTRODUCTION

Rolling bearing fault is one of the main causes of rotating machinery equipment failure. Thus, diagnosis of mechanical faults in rolling bearings is very crucial for the reliable operation. It helps us to discover the fault and replace the fault bearings in time, so as to prevent failure and economic losses[1].

Feature extraction is one key issue in rolling bearing fault diagnosis because the performance of a diagnostic methodology greatly depends on the quality of the features being used. In practice, the vibration signals often contain random noise and strong signals from other machine components, and have a low signal-to-noise ratio. When the fault occurs, the weak fault information of bearings is often masked in system noise. So, the feature extracted from those signals is very difficult[2]. In recent years, more attention has been paid to the higher-order statistics (HOS) in dealing with vibration signals. Some studies suggest that the HOS, namely the higher-order cumulants may be useful for fault diagnosis in different types of rotating machines [3-4]. On the one hand, higher-order cumulants are not sensitive to Gaussian noise, so higher-order statistics are applied to deal with non-Gaussian and nonlinear signals. On the other hand, many real world applications, especially rolling bearings, are non-Gaussian. What's more, the noise of the bearing fault vibration signals can be approximated as Gaussian noise. Therefore, more information can be extracted from vibration signals of bearing by using the higher-order statistics[5-6].

In the design of rolling bearing fault diagnosis, it is important to determine if the designed fault diagnosis algorithm is able to correctly classify the different fault conditions[7]. A feed-forward multilayer network with error back propagation, called BP network, is the most widely used type of artificial neural network. BP neural network has a proven ability in the area of nonlinear pattern classification[8]. With one hidden layer, it can realize the arbitrary complex decision boundaries and the logic function[9]. The process of rolling bearing's rotation is a complex nonlinear system, especially in case of failure. Thus, the capacity of BP network to mimic and automate human expertise is what makes it ideally suited for handling rotation system.

In this paper, an approach of fault diagnosis based on higher-order cumulants and BP network is developed. The higher-order statistics (third-order cumulant, fourth-order cumulant) and second-order cumulant of the vibration signals are calculated as feature vectors. The proposed method successfully classified the normal, the ball fault, the inner raceway fault and the outer raceway fault of rolling bearings. Results indicate that the higher-order cumulants combined with BP neural network is effective. It has a high recognition rate.

## 2 HIGHER-ORDER CUMULANTS

Let $x$ denotes the random variable and $f(x)$ denotes the probability density. The first characteristic function is defined as[10]

$$\Phi(\omega) = E[e^{i\omega x}] = \int_{-\infty}^{\infty} f(x)e^{i\omega x}dx \qquad (1)$$

That is to say, the first characteristic function is the Fourier transform of $f(x)$. Where $E[\cdot]$ is the expectation operator, representing the statistical average.

The second characteristic function is defined logarithm function of $\Phi(\omega)$ ,

$$\Psi(\omega) = \ln\Phi(\omega) \qquad (2)$$

$k$th-order cumulant of $x$ is defined as

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$$c_k = (-i)^k \left. \frac{d^k \Psi(\omega)}{d\omega^k} \right|_{\omega=0} \tag{3}$$

That is to say, the second characteristic function of $x$ in the $k$th-order derivative at the origin is equal to $k$th-order cumulant of $x$.

For $n$-dimension random vector $\{x_1, x_2, \cdots, x_n\}$, $r$th-cumulant $(r = k_1 + k_2 + \cdots + k_n)$ is given by

$$c_{k_1,k_2,\cdots,k_n} = (-i)^r \left. \frac{\partial^r \Psi(\omega_1, \omega_2, \cdots, \omega_n)}{\partial^{k_1}\omega_1 \partial^{k_2}\omega_2 \cdots \partial^{k_n}\omega_n} \right|_{\omega_1=\omega_2=\cdots=\omega_n=0} \tag{4}$$

Especially, when $k_1 = k_2 = \cdots = k_n = 1$, the most common representation of nth-cumulant can be written as

$$cum(x_1, x_2, \cdots, x_n) = c_n = c_{1,1,\cdots,1} \tag{5}$$

In general, $k$th-order cumulant can be further expressed as

$$c_{kx}(\tau_1, \cdots, \tau_{k-1}) = cum\left[x(t), x(t+\tau_1), \cdots, x(t+\tau_{k-1})\right] \tag{6}$$

Where $\tau_1, \tau_2, \cdots, \tau_{k-1}$ are the time delays.

Brilliger is the first man who builds the conversion formula between higher-order cumulants and higher-order moments. For $n$-dimension random vector, its $n$-moments and $n$-cumulant satisfy the following relationship:

$$cum[x_1, x_2, \cdots, x_n] = \sum_{p=1}^{n} (-1)^{p-1}(p-1)! E[\prod_{i \in s_1} x_i] \cdot E[\prod_{i \in s_2} x_i] \cdots E[\prod_{i \in s_p} x_i] \tag{7}$$

Among of them, $(s_1, s_2, \cdots, s_p)$ is a set of $n$ integers of all $p$ blocks, where $p = 1, 2, \cdots, n$.

When the average of the random variable is non-zero, third or more higher-order cumulant expressions will become very complicated. Thus, in practice, doing zero-mean processing on the raw signal at first will make higher-order cumulants simple.

For zero-mean Gaussian random process, the cumulants have the following conclusions:

$$c_{1x} = 0, c_{2x} = \sigma^2, c_{kx} \equiv 0 (k \geq 3) \tag{8}$$

Thus the conclusion shows that when the signal contains additive colored Gaussian noise, higher-order cumulant can completely suppress the influence of noise in theory, then the signal to noise ratio is improved.

## 3 BP NEURAL NETWORK

BP network is a feed-forward network by non-linear transformation units. The architecture of a BP neural network is shown in Figure 1.

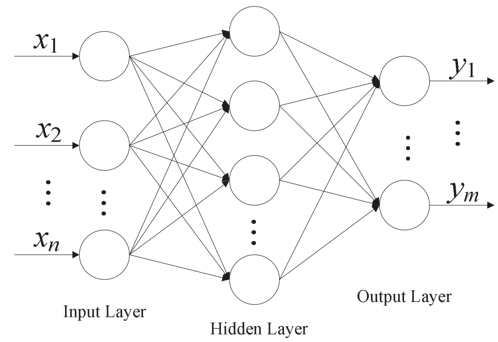


Fig.1 BP neural network

BP neural networks often have a multilayer structure, including input layer, output layer and hidden layer. There are $n$ input nodes, and $m$ output nodes. All input nodes are connected to all hidden nodes through weighted connections, and all hidden nodes are connected to all output nodes through weighted connections. We can compute the number of hidden layer nodes according to the following equation:

$$l = \sqrt{n+m} + a \tag{9}$$

where $l$, $n$ and $m$ are respectively the numbers of hidden layer nodes, input nodes and output nodes; $a$ is a constant between 1-10. Changing $l$ to the same training sample set, we can determine the number of hidden layer nodes from the minimum error of the network.

## 4 FAULT DIAGNOSIS BASED ON HIGHER-ORDER CUMULANTS AND BP NETWORK

Based on the above analysis, this paper presents the method of higher-order cumulants-BP neural network for fault diagnosis of rolling bearings. The framework of the fault diagnosis methodology is shown in Figure 2.

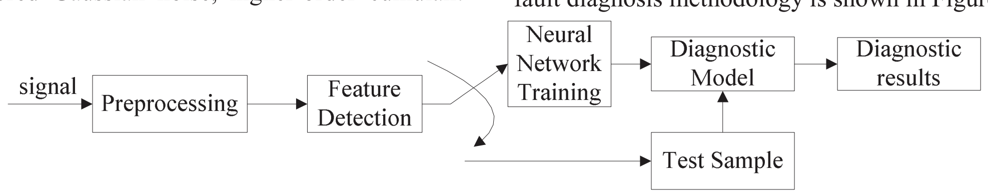


Fig.2 Framework of the fault diagnosis methodology

The detailed steps of the proposed diagnostic method are as following:

**Step 1:** Higher-order cumulants based fault feature extraction

Firstly, before feature extraction, zero-mean normalization is applied to the vibration acceleration signals of rolling bearings. Then, according to the following equations (10)-(12), second-order cumulant, third-order cumulant and fourth-order cumulant under four conditions of rolling bearings are calculated after the DC signals have been removed. A 3-dimantion fault feature vector $[c_{2x}\ c_{3x}\ c_{4x}]$ is obtained.

$$c_{2x}(\tau) = E\{x(t)x(t+\tau)\} = R_x(t) \tag{10}$$

$$c_{3x}(\tau_1, \tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\} \tag{11}$$

$$c_{4x}(\tau_1,\tau_2,\tau_3)=E\{x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)\}-R_x(\tau_1)R_x(\tau_3-\tau_2) \\ -R_x(\tau_2)R_x(\tau_3-\tau_1)-R_x(\tau_3)R_x(\tau_2-\tau_1) \quad (12)$$

Where $R_x(t)$ is the autocorrelation function.

**Step 2:** BP network training

Feature vector $[c_{2x}\ c_{3x}\ c_{4x}]$ of each training sample is the inputs of BP, and the target vector of each sample is the outputs, then the network trained.

At the beginning of the BP network's training, the fault diagnosis of the bearing's condition will not be accurate. An error quantity is measured and used to adjust the internal parameters of the BP network in order to produce an accurate output. This process is repeated until a suitable error is achieved. Finally, the network's parameters are saved.

**Step 3:** Fault determination

After the BP network training, the exact actual outputs are got. But, with those exact data, the classification results of the rolling bearings cannot be got. That is to say, in order to make the exact actual outputs can be automatically distinguished, some values are set up. If the actual output value is greater than 0.9, the value is set to 1. If the actual output value is less than 0.1, the value is set to 0. If the actual output value between 0.9 and 0.1, the value is set to 0.5, which is regarded as the wrong diagnosis results. Comparing with the target vectors, the fault category will be recognized.

Once the network has been established, it can be used to identify the faults of rolling bearing. Firstly, new signals of rolling bearing are processed according to Step 1. Secondly, through the established diagnostic model, we can get the actual output of the new signals. Finally, after the fault determination, the results of fault diagnosis are obtained.

## 5 EXPERIMENT

In this work, the experimental data comes from the Case Western Reserve University's (CWRU) bearing data center. There are three types of bearing faults, namely ball fault, inner raceway fault and outer raceway fault. All the faults are single point faults which were introduced by using electro-discharge machining and fault diameter is 0.021 inches. Motor speed is 1797 rpm. Data was collected at 12,000 samples per second.

Experimental vibration data was taken from normal bearings and three kinds of fault bearings. Each 1024 sample points are a group, so there are 110 groups for each fault, totally 440 groups. The first 80 sets of data in each fault are the training samples, and another 30 sets of data are the test samples. A single hidden layer of the BP neural network was used for the fault classifier. Second-order, third-order and fourth-order cumulants are the inputs of BP network, so the number of input layer neuron is 3. The number of output layer neuron is 3, and the ideal outputs and the corresponding category are shown in Table 1. Here, the number of hidden layer neuron is set to 12. The BP network is trained using Levenberg-Marquardt and the learning rate is 0.01.

Table1. Training samples and Ideal Output

| Category | Training Samples | Ideal Output |
|---|---|---|
| 1 | Normal | (0,0,0) |
| 2 | Ball fault | (1,0,0) |
| 3 | Inner raceway fault | (0,1,0) |
| 4 | Outer raceway fault | (0,0,1) |

Part of the training samples and the actual outputs are shown in Table 2.

Table2. Training features and Actual Outputs

| | $c_{2x}$ | $c_{3x}$ | $c_{4x}$ | Actual Output |
|---|---|---|---|---|
| 1 | 0.0056 | -0.0000 | -0.0000 | (-0.0017,0.0017,-0.0028) |
| | 0.0049 | -0.0000 | -0.0000 | (-0.0017,0.0017,-0.0015) |
| | 0.0059 | 0.0000 | -0.0000 | (-0.0018,0.0018,0.0053) |
| | 0.0062 | -0.0000 | -0.0000 | (-0.0017,0.0018,0.0113) |
| | 0.0054 | -0.0000 | -0.0000 | (-0.0017,0.0018,-0.0006) |
| 2 | 0.2896 | -0.0228 | 0.2434 | (1.0011,-0.0004,-0.0003) |
| | 0.3278 | 0.0028 | 0.9179 | (1.0010,-0.0004,-0.0002) |
| | 0.3822 | -0.0185 | 0.4184 | (1.0013,-0.0004,-0.0003) |
| | 0.2747 | -0.0366 | 0.3035 | (1.0001,-0.0005,-0.0002) |
| | 0.4640 | 0.0016 | 0.2742 | (0.9964,0.0044,-0.0002) |
| 3 | 0.2713 | 0.0094 | 0.0221 | (0.0232,0.9808,-0.0001) |
| | 0.2636 | 0.0114 | -0.0434 | (0.0214,0.9807,-0.0001) |
| | 0.2779 | 0.0162 | 0.1194 | (0.0208,0.9806,-0.0001) |
| | 0.2704 | 0.0138 | 0.1299 | (0.0229,0.9808,-0.0001) |
| | 0.2428 | 0.0121 | 0.0019 | (0.0200,0.9806,-0.0001) |
| 4 | 0.0574 | 0.0000 | 0.0027 | (-0.0014,0.0024,1.0022) |
| | 0.0687 | -0.0000 | 0.0106 | (-0.0000,0.0000,1.0022) |
| | 0.0963 | 0.0000 | 0.0042 | (-0.0000,0.0023,1.0022) |
| | 0.0947 | 0.0000 | 0.0086 | (-0.0035,0.0053,1.0022) |
| | 0.0653 | 0.0000 | -0.0035 | (-0.0018,0.0000,1.0021) |

As seen from Table 2, the third-order cumulants and the fourth-order cumulants of a defect-free bearing are zeros, so vibration signals belong to the Gaussian signal. For ball fault and inner raceway faults, third-order and fourth-order cumulants are not equal to zero. Although, the third-order cumulants of outer raceway fault are equal to zero, the fourth-order cumulants are not. Therefore, for the three types of faults, their vibration signals are non Gaussian signals. Obviously, more fault features can be extracted by using higher-order cumulants.

Here provides a visual expression of the classification results by using $[c_{2x}\ c_{3x}\ c_{4x}]$ as feature vectors. All cumulants of training samples are shown in Figure 3. The x, y, z axes are respectively named the second-order cumulant, the third-order cumulant and the fourth-order cumulant. As we can see from Figure 3, a clear separation of the normal bearings and the outer raceway fault bearings from other bearings. Also, from Figure 3, between the ball fault bearings and the inner raceway fault, only a few points are overlapped. In general, for all the feature vectors, normal, ball fault, inner raceway fault and outer raceway fault of the rolling bearings can be separated from each other by using the higher-order cumulant method.

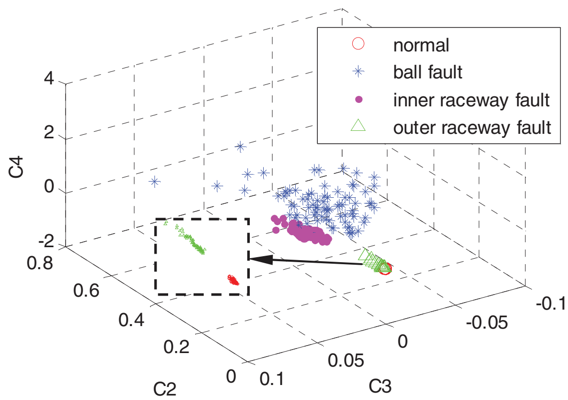


Fig.3 All cumulants of training samples' classification results

The performance of the fault diagnostic model is shown in Table 3. The diagnostic rate (%) is defined as the percentage of data samples of a fault type that were classified. As seen from Table 3, except the ball fault training samples' diagnostic accuracy rate is 98.75 % and the test diagnostic accuracy rate of ball fault is 96.67%, classification accuracies of other faults rate are 100%.

Table3. Diagnosis Results

| | | Number of Samples | Correct Samples | Diagnostic Rate (%) |
|---|---|---|---|---|
| 1 | Training | 80 | 80 | 100 |
| | Test | 30 | 30 | 100 |
| 2 | Training | 80 | 79 | 98.75 |
| | Test | 30 | 29 | 96.67 |
| 3 | Training | 80 | 80 | 100 |
| | Test | 30 | 30 | 100 |
| 4 | Training | 80 | 80 | 100 |
| | Test | 30 | 30 | 100 |

Through the experiment we can find that one of the training ball faults and one of the test ball faults are misdiagnosed as inner raceway faults. One possible explanation is that the ball fault bearings and the inner raceway fault bearings are overlapped in the classification, which we can observe from Figure 3. In order to have an overall diagnostic result, the diagnostic accuracy of total training samples and total testing samples and the mean square error by the neural network are shown in Table 4.

Table4. MSE and identification results

| | Total Samples | MSE | Correct Samples | Diagnostic Rate (%) |
|---|---|---|---|---|
| Training | 320 | 0.0021 | 319 | 99.69 |
| Test | 120 | 0.0029 | 119 | 99.17 |

From Table 4, one can see that if the second-order cumulant, the third-order cumulant and the fourth-order cumulant as feature vectors, the overall fault classification of training samples accuracy rate is 99.69%, and 99.17% for test samples. The results show that the method of fault diagnosis is able to accurately diagnose the three faults of rolling bearings. And the diagnostic accuracy rate is high.

## 6 CONCLUSION

In this paper, the higher-order cumulant features of rolling bearing vibration signals are applied to BP network to build an automatic fault diagnosis machine. The effectiveness of the developed fault diagnostic method was validated using CWRU's bearing data. The validation results have shown that when the second-order cumulant, the third-order cumulant and the fourth-order cumulant as feature vectors of the rolling bearing vibration signals, the fault of rolling bearings can be separated clearly, and the diagnostic accuracy rate is high. Thus, this paper provides an effective way to the fault feature vectors' construction and rolling bearings' fault diagnosis.

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