A Study Based on Fourth–Second Order Normalized Cumulant in TDOA Estimation

Hongzhi Wang [1], Jingtao Zhao [2], Liying Qian[3]
[1, 2, 3] College of computer Science and Engineering, Changchun University of Technology, Changchun, China
wanghongzhi@mail.ccut.edu.cn, zjingt517@126.com, coleen3141@yahoo.cn

***Abstract*—This work presents a problem of estimating the difference in arrival times of non-Gaussian signals added Gaussian noises at two separated sensors. The method is based on fourth-second order normalized cumulant and variable-step Least Mean Square algorithm. In this chapter, we investigate the normalized cumulants of the two different received signals by sensors to minimize the mean square error using variable-step Least Mean Square algorithm of time delay estimation. A new objective function is created in adaptive processing. When the adaptive filter converges, the weighted coefficient is calculated, while the value of time delay estimation is also obtained. Simulation results demonstrate that, under two different conditions of the noises, with the normalized cumulants, convergence curve performance via the proposed method is better than the method based on the fourth order cumulant.**

Keywords-TDOA estimation; variable-step LMS algorithm; fourth-second order normalized cumulant; Convergence curve

I. INTRODUCTION

Time delay estimation is a key step in many sensor-array signal processing applications. The estimation of time delay between received signals at two (or more) sensor locations is an important problem in several fields such as sonar, radar, biomedicine and geophysics [1]. Wave in nature propagates at a finite speed, this causes a time delay between the signals arriving at sensors located in different positions in a distributed sensor array. In signal processing literature, this delay is called Time Difference of Arrival (TDOA) [2].

Time delay estimation has been extensively explored in the past years; various methods have been proposed and implemented. See, e.g. Knapp and Carter (1976), Brandstein and Silverman (1997) and Aarabi (2001) [3]. The conventional method in time delay estimation is cross-correlation based on technique introduced by Knapp and Carter (1976) [3]. Non-Gaussian processes contain valuable statistical information in their high order movements (orders greater than two).This information is not used by “conventional” array processing algorithms which, therefore, have suboptimal performance. In fact, it has been shown that estimation methods which exploit the non-Gaussianity of the signals have some inherent advantages over fourth order cumulant method.

II. TIME DIFFERENCE OF ARRIVAL TECHNIQUE

TDOA techniques are based on estimating the difference in the arrival times of the signal from the source at multiple receivers. It is possible to directly determine a position estimate from the received data signals. In TDOA based methods, there must be either two transmitters sending the same signal or two spatially separated receivers measuring the same transmission. Usually only one transmitter is available, hence multiple sensors or receivers must cooperate by sharing data. Time delay estimation measurements are often determined from the generalized cross correlation of the two received signals. The principle of TDOA is shown in Fig.1 [4].

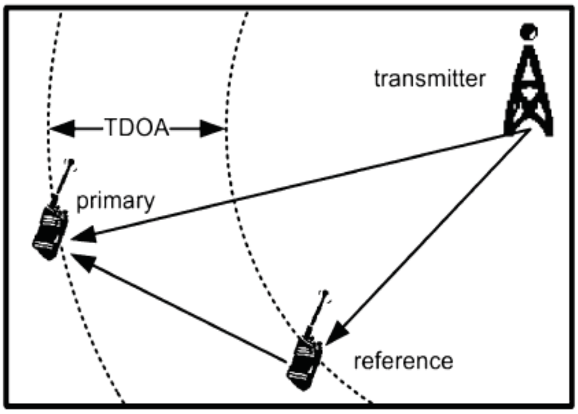

Figure 1. Geometry of TDOA Computation for Source Localization or Navigation

The basic discrete-time model for TDOA estimation can be stated as follows [5]:

$$x_1(n) = s(n) + w_1(n) \tag{1}$$

$$x_2(n) = as(n - D) + w_2(n) \tag{2}$$

Here $s(n)$ is the signal radiating from the source, D is a time delay associated with receiver and additive noise. The signal and noises are assumed to be mutually independent, $x_1(n)$ and $x_2(n)$ are the signals received at the observation points (i.e. sensors). The signals $w_1(n)$ and $w_2(n)$ model the (possibly Gaussian noise) arriving at the first and second sensors, respectively. $a \in (0,1]$ is the relative amplitude gain.

This model assumes that, $w_1(n)$, $w_2(n)$ are real and jointly stationary, zero-mean (time average), random

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processes and that $s(n)$ is uncorrelated with noise $w_1(n)$ and $w_2(n)$. The goal of TDOA estimation is to estimate D given a segment of data obtained from each sensor, without any prior knowledge regarding the source signal $s(n)$ or the noise sources.

III. NEW TDOA ESTIMATION METHOD

The important estimation value in TDOA problem is the time delay D [6]. How to reduce the noise in the source signal and other interference factors, improve signal to noise ratio (SNR) is a key factor. Variable-step Least Mean Square Method, because of its wide applicability high reliability, less computation and good ability for real-time has been widely used in TDOA estimation [7].

Considering high order cumulants of blindness to Gaussian noise, in this chapter, we will study variable-step least mean square time delay estimation method based on high order cumulants. The new algorithm has the peculiarities of less computation and simple principium.

A. Variable-step Least Mean Square Algorithm

Least Mean Square (LMS) algorithm using an adaptive filter adjusts its coefficients to minimize the mean square error between its output and that of the output result referred to as the desired signal [8].

On the basis of this measure, the variable-step adaptive filter will change its coefficients in an attempt to reduce the error which is computed as the desired signal minuses the output of the adaptive filter real time. The LMS algorithm is often called a stochastic gradient algorithm; the gradient is a vector pointing in the direction of the change in filter coefficients that will cause the greatest increase in the error signal [8]. After repeatedly adjusting each coefficient in the direction opposite to the gradient of the error, the variable-step adaptive filter firstly diminishes the search area, secondly converges fast; that is, the difference between the unknown and adaptive systems should get smaller and smaller.

When the variable-step adaptive filter converges, the error will be zero; the result is minimized, and estimates the delay value $\hat{D}$ is the closest to the real delay value D.

B. High Order Cumulants

Let $v = col(v_1, v_2, ..., v_k)$ and $x = col(x_1, x_2, ..., x_k)$, where $(x_1, x_2, ..., x_k)$ denote a collection of random variables (the material in this section is taken, for the most part, from). The k th-order cumulant of these random variables is defined as the coefficient of $(v_1, v_2, ..., v_k)$ in the Taylor series expansion (provided it exists) of the cumulant-generating function [9]

$$K(v) = \ln E\left\{\exp(jv'x)\right\} \tag{3}$$

The k th-order cumulant is therefore defined in terms of its joint moments of orders up to k. See Appendix A, (A-l), for the explicit relationship between cumulants and moments [9].

For zero-mean real random variables, the second-, third-, and fourth-order cumulants are given by [9]

$$\mathrm{cum}(x_1, x_2) = E\left\{x_1 x_2\right\} \tag{4}$$

$$\mathrm{cum}(x_1, x_2, x_3) = E\left\{x_1 x_2 x_3\right\} \tag{5}$$

$$\begin{aligned}\mathrm{cum}(x_1, x_2, x_3, x_4) &= E\left\{x_1 x_2 x_3 x_4\right\} - E\left\{x_1 x_2\right\} E\left\{x_3 x_4\right\} \\ &- E\left\{x_1 x_3\right\} E\left\{x_2 x_4\right\} - E\left\{x_1 x_4\right\} E\left\{x_2 x_3\right\}\end{aligned} \tag{6}$$

Let $\{x(t)\}$ be a zero-mean k th-order stationary random process. The k th-order cumulant of this process, denoted $C_{k,\tau}(\tau_1, \tau_2, ..., \tau_{k-1})$ is defined as the joint k th-order cumulant of the random variables $x(t)$, $x(t+\tau_1)$, ..., $x(t+\tau_{k-1})$, i.e., [9]

$$C_{k,\tau}(\tau_1, \tau_2, ..., \tau_{k-1}) = \mathrm{cum}(x(t), x(t+\tau_1), ..., x(t+\tau_{k-1})) \tag{7}$$

$$C_{2,x}(\tau) = E\left\{x(t)x(t+\tau)\right\} \tag{8}$$

$$C_{3,x}(\tau_1, \tau_2) = E\left\{x(t)x(t+\tau_1)x(t+\tau_2)\right\} \tag{9}$$

$$\begin{aligned}C_{4,x}(\tau_1, \tau_2, \tau_3) &= E\left\{x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)\right\} \\ &- C_{2,x}(\tau_1)C_{2,x}(\tau_2-\tau_3) - C_{2,x}(\tau_2)C_{2,x}(\tau_3-\tau_1) \\ &- C_{2,x}(\tau_3)C_{2,x}(\tau_1-\tau_2)\end{aligned} \tag{10}$$

C. New Algorithm Based on Normalized Cumulants

We assume that the source signal is $s(n)$ and its time delay signal is $s(n-D)$, the signals of being received from different sensors are $x_1(n)$ and $x_2(n)$, which added Gaussian noises.

However, $x_1(n)$ and $x_2(n)$ are real and jointly stationary, zero-mean (time average) and random processes. The fourth order cumulant of $s(n)$ is defined as c_{4s}, while $s(n-D)$ is defined as c_{4s_D}, the fourth order cross-cumulant of $s(n)$ and $s(n-D)$ is defined as c_{4ss_D}. Then, the second order cumulant of $s(n)$ is defined as c_{2s}, $s(n-D)$ is defined as c_{2s_D}, the second order cross-cumulant of $s(n)$ and $s(n-D)$ is defined as c_{2ss_D}. Let

$c_{4x_1x_2}$ denote the fourth order cross-cumulant of $x_1(n)$ and $x_2(n)$, $c_{2x_1x_2}$ denote the second order cross-cumulant of $x_1(n)$ and $x_2(n)$.

According to BBR function, the fourth-second order cross-cumulant of $s(n)$ and $s(n-D)$ is given by

$$k_s(4,2) = \frac{c_{4ss_D}}{c_{2ss_D}^{4/2}} \quad \text{if } c_{2ss_D} \neq 0 \tag{11}$$

While the fourth-second order cross-cumulant of $x_1(n)$ and $x_2(n)$ is

$$k_x(4,2) = \frac{c_{4x_1x_2}}{c_{2x_1x_2}^{4/2}} \quad \text{if } c_{2x_1x_2} \neq 0 \tag{12}$$

This method is enabled by the fact that the functions from (11) to (12) based on the high order cumulants time delay estimation combining variable-step LMS algorithm to create a new objective function, $k_s(4,2)$ has the only one least value in the whole scale , when the least value is computed , the time delay value D is also resolved. The new formula is based on the fourth-second order cross-cumulant tessellates the researching scale, improves the speed of the convergence. Fig.2 shows the method system model.

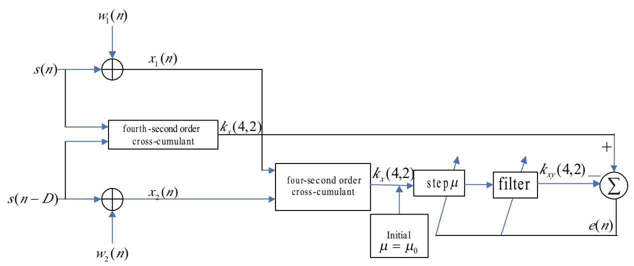


Figure 2. The system model for new TDOA Estimation

The new TDOA estimation method computation is a three-step process:

S1) (high order cumulants): Calculate the fourth-second order cross-cumulant of the pure source signal $s(n)$ and its time delay signal $s(n-D)$ which is $k_s(4,2)$, then compute the received signals the fourth-second order cross-cumulant of $x_1(n)$ and $x_2(n)$ that is $k_x(4,2)$.

S2) (adaptive processing): Set initial value of $\mu = \mu_0$, send $k_x(4,2)$ to the adaptive filter, generate another step-size array $k_{xy}(4,2)$, the error is $e(n)$, which is computed as $e(n) = k_s(4,2) - k_{xy}(4,2)$, use $e(n)$ and the variable step size μ to update the adaptive filter coefficient to make the filter converges.

S3) (compare data): Plot the convergence curve and calculate the slew rate of the curve, then analyze the performance of the proposed method.

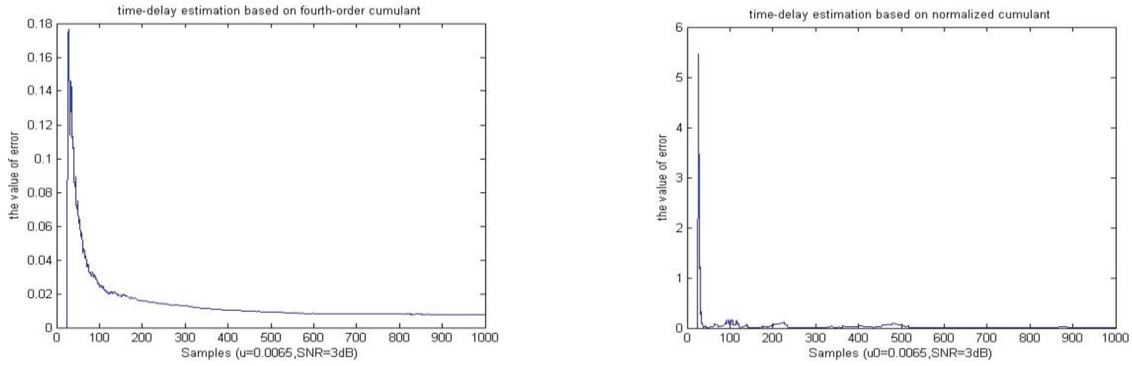
IV. SIMULATIONS AND ANALYSIS

This section provides performance analysis via simulations. The source signal is square sequence with the amplitude between 1 and 0, its duty cycle is 20%, the transmitter uses phase modulation of the carrier frequency is 500Hz, base band frequency is 50Hz, and sampling frequency is 1000Hz. Two noises are real and jointly stationary, zero-mean, random processes, their row column matrix of N (N=1000), the noises are random Gaussian noises. We also assume $D=10$. For simplicity the noise powers are the same at both sensors, hence the SNR at either sensor is $SNR = \sigma_x^2 / \sigma_{ref}^2$. Table Ⅰ shows performance of the method based on the fourth-second order normalized cumulant and the fourth order cumulant under different conditions. Calculate the slew rate of the convergence curves of two methods.

TABLE I. TWO METHODS COMPARISION

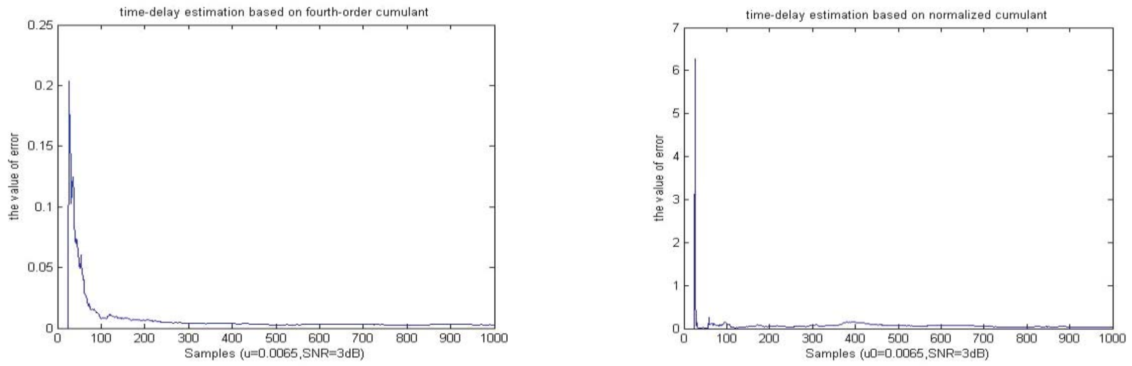
Step Size	Slew Rate of Convergence curve		
	Gaussian noise	***uncorrelated***	***correlated***
fixed step	Fourth order cumulant	0.0016	0.0018
variable step	Normalized cumulants	0.038	0.053

Here we use the methods based on the fourth order cumulant and the fourth-second order normalized cumulant to solve the slew rate of the convergence curve, the comparison of two methods of slew rate value in Table Ⅰ . A plot of the probability of error based on the fourth order cumulant with LMS algorithm and the new algorithm in TDOA estimation are given in Fig.3. In Fig.3 the SNR is 3dB. In Fig.3 (a), the step-size μ is fixed, $\mu = 0.0065$ to make step 2 limiting factors in performance. The step-size is variable in Fig.3 (b), the initial value is $\mu_0 = 0.0065$, and in the adaptive processing the step-size realizes to change every moment and diminishes the research region, so it can improve the speed of the convergence curve. While Fig.4 shows the astringency of convergence curves under correlated Gaussian noises. The SNR is also 3dB and the step-size is $\mu = 0.0065$ based on the fourth order cumulant in Fig.4 (a), Fig.4 (b) is based on normalized cumulants using variable step-size. Let $w_1(n) = w_2(n)$ to make the correlated Gaussian noises in Fig.4. The results indicate the new algorithm has greater convergence of values, it means that it obtains faster convergence speed, and also suppresses Gaussian noise or symmetric distributed noise effectively as the method based on the fourth order cumulant.



(a) (b)

Figure 3. The convergence curve under Gaussian noise:(a) this is based on the fourth order cumulant (b) this is based on nomalized cumulants



(a) (b)

Figure4. The convergence curve under correlated Gaussian noise: (a) this is based on the fourth order cumulants (b) this is based on nomalized cumulants

V. CONCLUSION

A new algorithm approach to time delay estimation using normalized cumulants of two sensors was considered. The proposed method can improve SNR, reduce interference of Gaussian noise, correlated Gaussian noise or symmetric distributed noise, and meanwhile reduce the calculation and improve the convergence rate. This new method increase an order of magnitude in convergence speed than the method based on the fourth order cumulant, and its realization is simple.

ACKNOWLEDGMENT

The authors wish to thank all the reviews for their constructive suggestions and valuable comments.

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