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ON IMPROVING THE CONVERGENCE AND ACCURACY OF THE CUMULANT METHOD OF CALCULATING RELIABILITY AND PRODUCTION COST

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ABSTRACT

Two techniques for improving the convergence and accuracy of the Cumulant method are presented. The Large Deviations Approach that has already been used in a three-cumulant version with excelent results is formulated recursively thus allowing to effectively use any number of cumulants. A new expansion centering technique that significantly improves the accuracy of the method is also presented and implemented recursively. Numerical results show that these enhancements are quite effective.

INTRODUCTION

Two of the most important problems that appear in generation planning are the evaluation of reliability indices and the estimation of production costs. Direct computational techniques for such calculations [17] often become too time-consuming to be effectively used for expansion planning. The method of cumulants has been used as an approximate, fast and simple way of calculating LOLP in generation planning studies for over two decades [1-3]. Recently this method has also been used to calculate other reliability indices such as EUE and frequency and duration of outages, for production costing in single and interconnected power systems and for maintenance scheduling [4-12].

Although the accuracy of the cumulant method has been found to be acceptable in some systems [13], the results for other smaller systems has not been quite satisfactory [14-16]. The convergence behavior and accuracy of the method has been studied in [18] using a continuous probability model that approximates as closely as desired the discrete probability distribution of the available generating capacity by replacing the impulses by gaussians of given standard deviatin h. The Recursive formulation presented in the same paper allowed to study the convergence characteristics of the method both as the number of cumulants was increased and as the parameter h approached zero. The behavior of the expansions was observed to be very sensitive to the value of h. For values of h small compared with the size of the biggest unit the expansion diverged in all cases tested as the number of cumulants was increased.

The technique of Large Deviations [19-20], also based on the use of cumulants, has recently been shown to yield surprisingly accurate results compared with the conventional method.

This paper presents the following improvements to the cumulant method for calculating reliability and production costs:

- The recursive formulation of the method [18] is modified to include the Large Deviations technique as a natural extension thus making it possible to have the advantages of both.
- An expansion centering technique that additionally improves the accuracy of the methods is also presented.

The convergence behavior and accuracy of the improved recursive formulation is then studied as the number of cumulants increases and as the parameter h tends to zero. The results obtained show that many of the convergence problems of the cumulant method can be effectively solved using the techniques described in the paper.

NOTATION

q_k = outage rate of unit k

p_k = availability of unit k

C_k = capacity of unit k

NU = number of generating units

L = load level

h = continuous model parameter, this is, either standard deviation of gaussians that replace impulses in discrete probability distribution of available capacity or standard deviation of load

s = Large Deviation parameter

y = Expansion Centering parameter

N(x)= Standard normal probability density function

$N_{-1}(z)$= Integral of N(x) from $-\infty$ to z

NC = Number of Cumulants in Expansion

K = Normalization Factor of associated probability distribution

A_k = Cumulant of order k

B_k = k^{th} Fourier coefficient in expansion

$H_k(x)$ = Hermite polynomial of order k

u = A_1 = mean value of reserve margin

σ = $A_2^{1/2}$ = standard deviation of reserve margin

LOLP = Loss-of-Load Probability

EUE = Expected Unserved Energy

BASIS FOR IMPROVING THE METHOD

Reliability and production cost calculations require computations of the following typical expressions:

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$$\text{LOLP} = \int_{-\infty}^{0} f(x)dx \text{ and} \qquad (1)$$

$$\text{EUE} = \int_{-\infty}^{0} xf(x)dx \qquad (2)$$

where $f(x)$ is the probability density function of the reserve margin x. Accurate estimation of these variables requires $f(x)$ to be approximated as close as possible within the integration interval $(-\infty,0)$. This region is just the left tail of $f(x)$. Approximation errors outside the integration interval do not have impact on the accuracy of LOLP and EUE calculations.

Accuracy improvements in the cumulant method have been obtained by making effective use of the two following results:

Expansion Centering

The coefficients B_k in the Gram-Charlier expansion

$$f(x) = \sum_{k=0}^{NC} B_k H_k(x)N(x) \qquad (3)$$

minimize the weighted least square error

$$\int_{-\infty}^{\infty} (f(x) - \sum_{k=0}^{NC} B_k H_k(x)N(x))^2 w(x)dx \qquad (4)$$

with

$$w(x) = \exp(1/2x^2) \qquad (5)$$

This result is shown in Appendix I. According to this result the expansion error is not expected to be uniform throughout the interval $(-\infty,\infty)$ the error being larger in the vicinity of $x = 0$, where the weight is minimum. It follows that by appropiately centering the expansion with respect to the integration interval significant accuracy improvements can be achieved.

A good intuitive strategy appears to be to center the expansion outside the integration interval near the mean value of $f(x)$. This has been born out by the numerical results.

Large Deviations [19,20]

The associated distribution of $f(x)$ is defined by:

$$v(x) = \exp(sx)f(x)/K \qquad (6)$$

where K is a normalization factor. It can easily be shown that if $f(x)$ is normal then $v(x)$ is also normal and identical to $f(x)$ but displaced by an amount equal to s times the variance of $f(x)$. By using the associated distribution of the reserve margin the original function can be displaced so that it becomes centered on the integration interval. This, in turn, makes the expansion significantly more accurate.

COMPUTATIONAL PROCEDURE

1. Calculate Large Deviation parameter s
   Find s from Eq. (7) so that the mean of the associated distribution of the reserve margin is zero:

$$sh^2 + \sum_{k=1}^{NU} \{p_k C_k/(p_k + q_k \exp(-sC_k))\} - L = 0 \qquad (7)$$

using Newton's method as in [19]

2. Calculate Cumulants of Reserve Margin
   a) calculate cumulants for each unit:

   $K = p_k \exp(sC_k) + q_k$

   $A_1 = p_k C_k/K$

   $T_2 = q_k p_k C_k^2/K^2$; set other T's to zero

   $A_2 = T_2$

   For i = 3 to NC:

       $A_i = 0$

       For j = 1 to $i-1$:

           $T_{i-j+1} = \{(i-j)q_k T_{i-j+1} - jp_k T_{i-j}\}C_k/K$

           $A_i = A_i + T_{i-j+1}$

       next j

   next i

   b) calculate load cumulants:

   $K = \exp(-sL + 1/2s^2h^2)$

   $A_1 = sh^2 - L$

   $A_2 = h^2$

   c) Add cumulants and multiply normalization factors to obtain corresponding parameters of reserve margin distribution.

3. Normalize and factorialize cumulants

   $a_k = A_k/(k!\sigma^k)$ for k = 3 to NC

   where $\sigma^2 = A_2$

4. Calculate expansion coefficients

   a) Initialization:
   For a given expansion centering parameter y:

   $B_k = y^k/k!$ for k = 0 to NC

   b) Recursive update:

   For n = 3 to NC:

       Set $T_k = B_k$ for k = 0 to NC

       For k = 1 to Integer part of NC/n:

           For i = kn to NC:

               $T_i = B_{i-kn}a_n^k/k! + T_i$

           next i

       next k

       Set $B_k = T_k$ for k = 0 to NC

   next n

5. Calculate LOLP and/or EUE

a) Initialize:

$$s = s\,\sigma$$

$$z = y - u/\sigma$$

$$a = \exp(-sz)$$

$$H_{-1} = 0$$

$$H_o = N(z)$$

$$D_o = \exp(1/2s^2)N_{-1}(s+z)$$

$$E_o = -aH_o - sD_o$$

b) update for k = 1 to NC:

$$H_k = zH_{k-1} - (k-1)H_{k-2}$$

$$D_k = -aH_{k-1} - sD_{k-1}$$

$$E_k = -zaH_{k-1} + D_{k-1} - sE_{k-1}$$

c) calculate indices:

$$LOLP = \left(\sum_{k=0}^{NC} B_k D_k\right)(K/a)$$

$$EUE = \sigma\left\{z\ LOLP - \left(\sum_{k=0}^{NC} B_k E_k\right)(K/a)\right\}$$

NUMERICAL RESULTS

The enhancements to the Recursive Cumulant method presented in this paper were tried in a variety of genrating systems. The convergence behavior and accuracy of the method were observed to be very sensitive to the three control parameters: a) generating unit model precision, or load standard deviation h; b) large deviations parameter s; and c) expansion centering parameter y. The patterns of behavior showed to be very much system independent. The results obtained with the TEXAS ELECTRIC case with a load of 3800 MW are used to illustrate these patterns. Only results for LOLP calculations are shown. As far as convergence and accuracy results for EUE are similar. Table I shows the corresponding unit data.

TABLE I

Generating Unit Data of Test System (*)

No. of Units	Capacity(MW)	No. of Units	Capacity(MW)
1	536	1	115
1	518	1	81
2	455	2	75
1	405	1	46
1	396	1	44
1	387	1	20
1	248	2	18
1	170	1	15
1	123	6	13
		26	4278

* Forced Outage Rate for all units = .02

The accuracy of the expansions has been measured by the relative expansion error, this is, absolute value of (expansion error/exact value). The exact value was obtained using a 1-MW step conventional recursive procedure [17]. The following stopping rule and accuracy improvements suggested in [18] have been used with some enhancements to detect expansion divergence:

stopping rule: monitor consecutive maximum and minimum values of the expansion as new terms are added; stop when their difference begins to grow(onset of divergence).

accuracy improvement: use as final result the average of the last consecutive maximum and minimum expansion values obtained before the onset of divergence.

Two clearly different cases occur: a) h is small compared with the size of the biggest unit and b) h is comparable to the biggest unit size. The first case is illustrated using h=0; the second one using h=360 MW or approximately 2/3 the size of the biggest unit. The value s^* for which the mean of the associated probability distribution of the reserve margin becomes zero was first calculated. Four different values of s were then considered: a) $s=-s^*$; b) s=0, c) $s=s^*$, and d) $s=2s^*$. For each one of these subcases the expansion centering parameter was finally varied between -1 (equivalent to centering the expansion one standard deviation to the left of the mean) and 2 (two standard deviations to the right).

For small h the convergence of the expansion was very sensitive to the value of s. With s equal to $-s^*$ or 0 the expansion clearly diverged while for values equal to or grater than s^* it did not diverge but exhibited a small (arround 13% of the correct value) but persistent oscillation as new terms were added to the expansion.

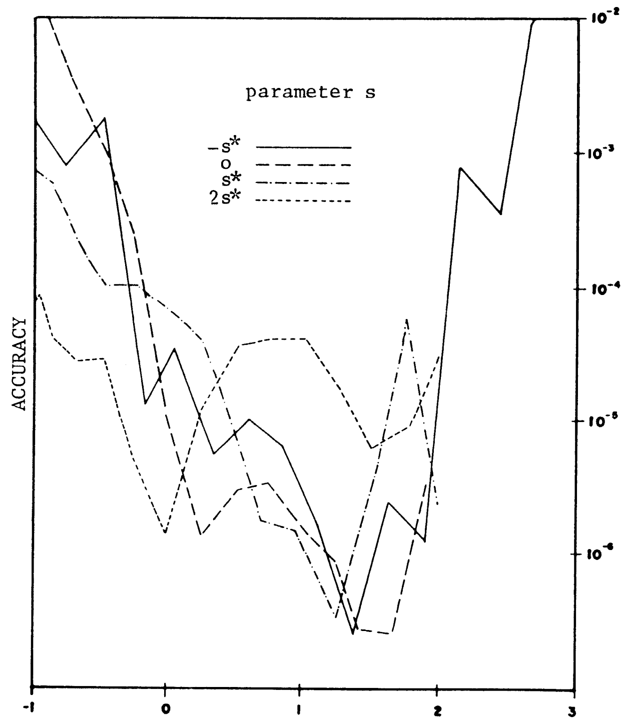
For values of h comparable to the size of the biggest unit the situation was different. To start with, the conventional cumulant method (s=0, y=0) converged to an accuracy better than 10 E-4 with 12 cumulants; then slowly diverged as additional cumulants were introduced. The centering parameter turned to have in this case signicantly more impact on improving the final accuracy than the large deviations parameter. Fig 1. shows the accuracies obtained with various values of s and y. It is clear that the best accuracy (better than .5x10 E-6) is obtained with nearly the same value of y (arround 1.4) for all values of s except $s=2s^*$. It can also be observed that the impact of s on the accuracy markedly depends on the value of y without following an easily recognizable pattern.

Table II shows the number of terms required to attain the highest accuracy as well as a more practical value of 1% in each case. For the particular value of h=360 MW and for practical requirements it is not very clear that going to values different than s=0 or y=0 results in significant improvements. For smaller values however the advantages of using a large deviations parameter close to s are apparent.

TABLE II

Number of Terms Required to attain Indicated Accuracy

Value of s	Highest Accuracy	1 % Accuracy
$-s^*$	18	9
0_*	19	7
s_*	26	7
$2s^*$	31	11



Expansion Centering Parameter y

Figure 1. Best Accuracy as a function of s and y.

Further work and experimentation is required to arrive at effective strategies for using the parameters s and y. The implementation effort necessary to introduce the corresponding enhancements to the Recursive Cumulant method however, is small compared to their potential advantages.

CONCLUSION

The enhancements introduced in the paper to the Recursive Cumulant method of calculating reliability and production cost showed to be effective as a way of improving its convergence characteristics and accuaracy. The improvements were incorporated as natural extensions of the recursive algorithm requiring minimal implementation effort.

The large deviations technique showed to have a significant impact on improving the convergence behavior of the method for small values of the parameter h compared with the size of the biggest unit. The expansion centering method turned out to be of more significance in improving the accuracy for larger values of h.

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APPENDIX I

GRAM-CHARLIER EXPANSION AND WEIGHTED LEAST SQUARES

Consider the weighted least square problem

$$\min_{B_k} \int_{-\infty}^{\infty} \left(f(x) - \sum_{k=0}^{NC} B_k H_k(x)N(x)\right)^2 w(x)dx \quad (A1)$$

Taking derivatives with respect to B_j and equating to zero:

$$\int_{-\infty}^{\infty} f(x)H_j(x)N(x)w(x)dx = \sum \int_{-\infty}^{\infty} B_k\, H_j(x)H_k(x)N^2(x)w(x)dx \quad (A2)$$

Since the Hermite polynomials are orthogonal in $(-\infty,\infty)$ with respect to the weight $\exp(-1/2x^2)$ [21], one chooses

$$N^2(x)w(x) = \exp(-1/2x^2) \quad (A3)$$

and $w(x) = \exp(1/2x^2)$ to a scale factor.

APPENDIX II

DERIVATION OF COMPUTATIONAL PROCEDURES

The basic result used in the derivation of the computational procedures is the following:

Let $\Phi_x(j\omega)$ be the characteristic function c.f. (Fourier Transform) of the probability density function $f(x)$:

$$\Phi_x(j\omega) = \int_{-\infty}^{\infty} \exp(j\omega x)f(x)dx \quad (A4)$$

then, a) the c.f. of $\exp(sx)f(x)$ is:

$$\Phi_x(j\omega+s) \quad (A5)$$

and b) the c.f. of $f(x+y)$ is:

$$\exp(j\omega y)\ \Phi_x(j\omega) \quad (A6)$$

Cumulants of generating unit available capacity

The c.f. of the available capacity of a two-state unit is:

$$p\ \exp(j\omega C) + q$$

where p and C are the availability and capacity of the unit. The c.f. of the associated distribution is then:

$$p\ \exp(j\omega+s) + q = p\ \exp(sC)\ \exp(j\omega C) + q$$

Derivation of the recursive procedure is identical to the one given in App. I of [18] with p replaced by p exp(sC). Since with this substitution p + q is no longer equal to one, the recursive Eq. requires an additional factor of $1/(p+q)^n$.

The multi-state case is a straight-forward generalization of the derivation given above.

The normalization factor is obtained by setting $\omega=0$:

$$K = p\ \exp(sC) + q$$

Cumulants of load distribution

The c.f. of a load normally distributed with mean -L and standard deviation h is:

$$\exp(-j\omega L + 1/2(j\omega)^2h^2)$$

the c.f. of the associated distribution is then:

$$\exp\{-(j\omega+s)L + 1/2(j\omega+s)^2h^2\} =$$

$$\exp(-sL + 1/2s^2h^2)\ \exp\{j\omega(sh^2-L) + 1/2(j\omega h)^2\}$$

the normalization factor is obtained again by setting $\omega=0$.

Recursive procedure for expansion coefficients

Once the cumulants of all units and the load have been calculated and added toghether and the corresponding normalization factors multiplied the c.f. of the associated distribution of the reserve margin (after standarization of the variable) is:

$$\Phi_x(j\omega) = \exp(-1/2\omega^2 + \sum_{k=3}^{NC} a_k(j\omega)^k/k!)/K$$

introducing the centering parameter y, after using expression (A6) one obtains:

$$\exp(j\omega y)\ \Phi_x(j\omega) = (\sum_{n=0}^{NC} y^n(j\omega)^n/n!)\ \Phi_x(j\omega)$$

Calculation of the G-C expansion coefficients can thus be carried out with the same procedure derived in App. II of [18] using the following initialization:

$$B_n = y^n/n! \text{ for } n = 0 \text{ to } NC.$$

Recursive Equations for LOLP and EUE

To derive expressions for LOLP and EUE define:

$$H_k = H_k(z)N(z) \quad (A7)$$

$$D_k = \int_{-\infty}^{z} \exp(-sx)H_k(x)N(x)dx \quad (A8)$$

$$E_k = \int_{-\infty}^{z} x\ \exp(-sx)H_k(x)N(x)dx \quad (A9)$$

Integration by parts leads to the recursive equations.