# Convex Optimization

## Project 5

Spring 1401
Due date: 24th of Khordad

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1. *Estimating mixture coefficients.* We are given $N$ IID samples $x_1, \ldots, x_N \in \mathbf{R}^m$ from a distribution with mixture density

$$p(x; \lambda) = \sum_{j=1}^{k} \lambda_j p_j(x),$$

where $\lambda \in \mathbf{R}_+^k$, with $\mathbf{1}^T \lambda = 1$, are the mixture coefficients, and $p_1, \ldots, p_k$ are given densities on $\mathbf{R}^m$.

(a) Explain how to use convex optimization to find the maximum likelihood estimate of the mixture coefficients $\lambda^{\text{ml}} \in \mathbf{R}_+^k$. (You can assume that the maximum likelihood problem is well posed, i.e., there is an optimal $\lambda^{\text{ml}} \in \mathbf{R}_+^k$.) If you change variables, or form a relaxation, be sure to fully justify it.

*Note.* We will not accept methods or algorithms from other courses or fields, even if they work.

(b) The data files mixture_coeffs_data.* contain code that generates $N = 100$ samples from a mixture of $k = 3$ distributions on $\mathbf{R}$,

$$\mathcal{N}(3,4), \quad \mathcal{U}(-1,2), \quad \mathcal{L}(-2,3),$$

with mixture coefficients $\lambda^{\text{true}} = (0.3, 0.5, 0.2)$. The first distribution is Gaussian with mean 3 and variance 4 ; the second is a uniform distribution on $[-1, 2]$, and the third is a Laplace or double-sided exponential distribution with mean $-2$ and shape parameter 3 , which has density $p(x) = \frac{1}{6}\exp(-|x+2|/3)$. The data file contains code for evaluating the density values at the sample points, i.e., $p_j(x_i), j = 1,2,3$ and $i = 1, \ldots, N$.

Carry out the method of part (a) on this data. Compare the ML estimate of the mixture coefficients with their true values. Plot the true and estimated mixture densities on the same plot. The data file also contains code for these plots; you just have to plug in your $\lambda^{\text{ml}}$. (Of course, in any real problem you would not have a 'true' distribution.)

**Good Luck!**