Blind Source Separation

HW6-Section-1

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```
clear ; clc; close all;
```

Section-1:

اول)

ل hw6-part1.mat ، ماتریس دیکشنری D ، مشاهده ی بدون نویز x و مشاهده ی نویزی **x_noisy** قرار داد

```
Data_Q6_p1 = load("hw6-part1.mat"); % Load Given Data: D and x

Dictionary_Part1 =  Data_Q6_p1.D;
X_Part1 =  Data_Q6_p1.x;
X_Noisy_Part1 =  Data_Q6_p1.x_noisy;
```

Part-1:   BP (Basis Pursuit) over x (Not Noisy Data)

الف) روش BP را روی داده های بدون نویز اعمال کنید و بردار s را استخراج کنید.

```
[M,N]=size(Dictionary_Part1);
% Linear Programming
f=ones(2*N,1);
Aeq=[Dictionary_Part1 -Dictionary_Part1];
beq=X_Part1;
lb=zeros(2*N,1);
tic;
yhat = linprog(f,[],[],Aeq,beq,lb,[]); % Linear-Programming: given Cost function: (f) + Equalit
```

```
Optimal solution found.
```

```
% in Matrix Form

splus=   yhat(1:N);
sminus=  yhat(N+1:end);
sBP=     splus-sminus;

posBP=find(abs(sBP)>0.01)'; % Choose Nonzero Elements
delta_t_LinP = toc;
disp("Elapsed Time (LP): "+ delta_t_LinP+"(s)");
```

```
Elapsed Time (LP): 0.051926(s)
```

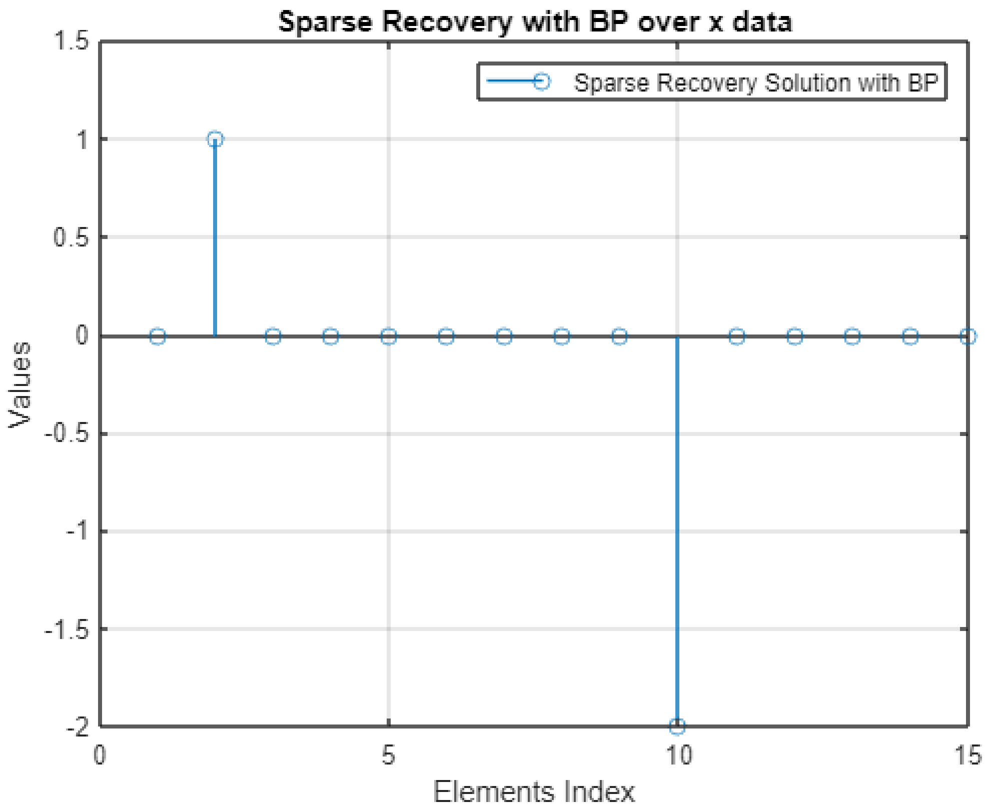
```
disp('BP:')
```

```
BP:
```

```
[posBP;sBP(posBP)']
```

```
ans = 2×2
    2.0000   10.0000
    1.0000   -2.0000
```

```
figure()
stem(sBP)
xlabel("Elements Index")
ylabel("Values")
grid on
legend("Sparse Recovery Solution with BP")
title("Sparse Recovery with BP over x data")
```



Part-2: MP

```
xr = X_Part1;
N0 = 2;
Chosen_IDX = 100+zeros(1,N0);
Choesn_Value = Chosen_IDX;
[Row_D,Col_D] = size(Dictionary_Part1);
tic;
B = Dictionary_Part1;

for  i=1:N0
    if(i>1)
    B(:,Chosen_IDX(i-1)) = [];
    end
    Corr_matrix = repmat(xr,1,Col_D-i+1).*B;
    Corr_sum    = sum(Corr_matrix,1); % Summation for each Column -->> a Row Vector
    [Value,Idx] = max(abs(Corr_sum)); % Choosing Best Fitted
    if(sum(Idx>Chosen_IDX)>0)
        Idx   = Idx + sum(Idx>Chosen_IDX) ;
    end
    Chosen_IDX(i)    =  Idx;
    Choesn_Value(i) = Value;
    xr = X_Part1 - Value*Dictionary_Part1(:,Idx); % xr = x  - <x,di>di
end
delta_t_MP = toc;
```

```
sMP = zeros(1,Col_D);
sMP(1,Chosen_IDX) = Choesn_Value;
disp("Elapsed Time (MP): "+ delta_t_MP+"(s)");
```

```
Elapsed Time (MP): 0.0054995(s)
```

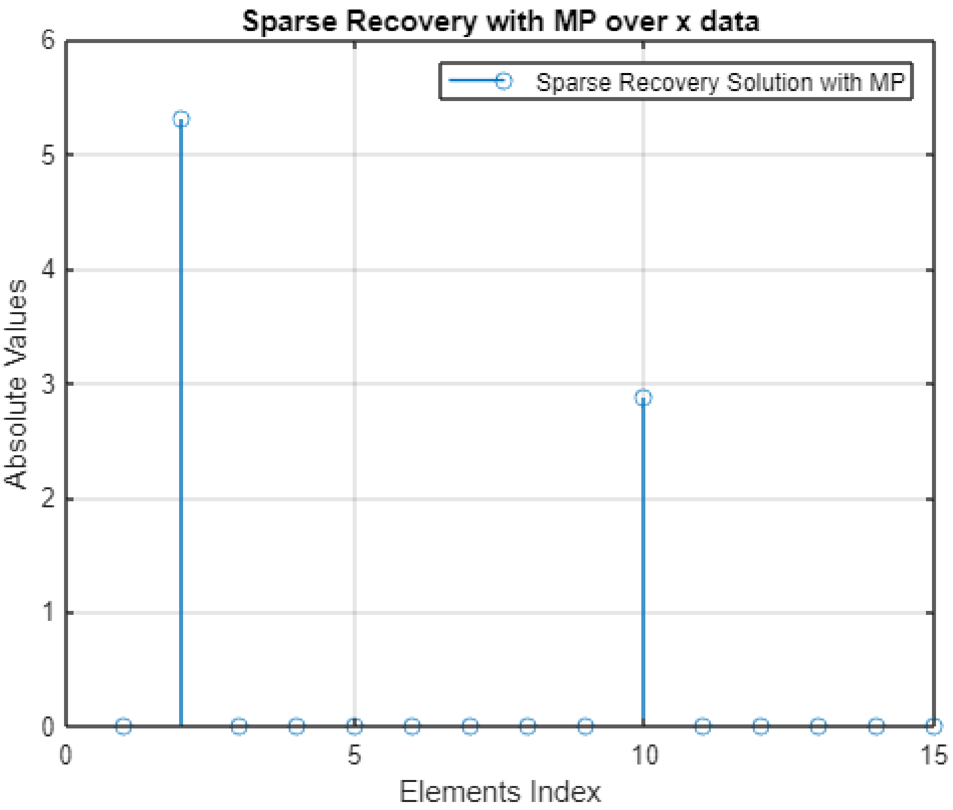
```
disp(" Best Indices Are ")
```

```
 Best Indices Are
```

```
disp(Chosen_IDX)
```

```
    10     2
```

```
figure()
stem( sMP)
xlabel("Elements Index")
ylabel("Absolute Values")
grid on
legend("Sparse Recovery Solution with MP")
title("Sparse Recovery with MP over x data")
```



Part-3: BP over Noisy Data

```

[M,N]=size(Dictionary_Part1);
% Linear Programming
f=ones(2*N,1);
```

```
Aeq=[Dictionary_Part1 -Dictionary_Part1];
beq=X_Noisy_Part1;
lb=zeros(2*N,1);
tic;
yhat_Noisy = linprog(f,[],[],Aeq,beq,lb,[]); % Linear-Programming: given Cost function: (f) + E
```

```
Optimal solution found.
```

```

% in Matrix Form

splus_Noisy=   yhat_Noisy(1:N);
sminus_Noisy=  yhat_Noisy(N+1:end);
sBP_Noisy=     splus_Noisy-sminus_Noisy;

posBP_Noisy=find(abs(sBP_Noisy)>0.01)'; % Choose Nonzero Elements
delta_t_LinP = toc;
disp("Elapsed Time (LP): "+ delta_t_LinP+"(s)");
```

```
Elapsed Time (LP): 0.16902(s)
```

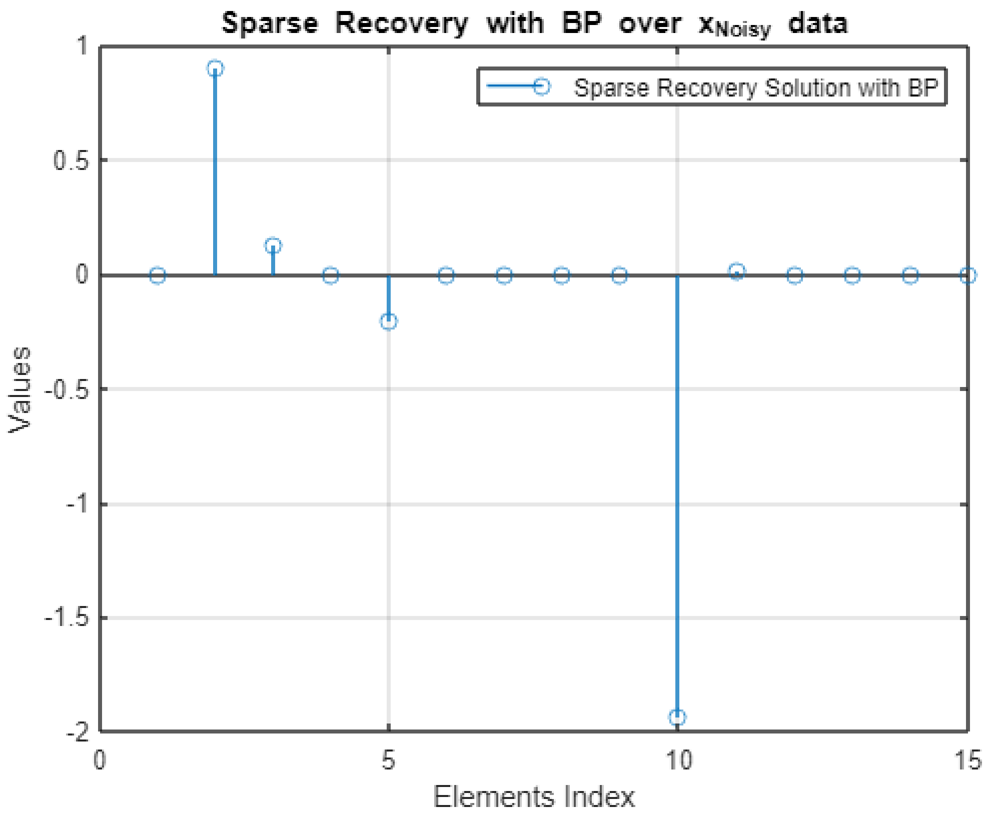
```
disp('BP:')
```

```
BP:
```

```
[posBP_Noisy;sBP_Noisy(posBP_Noisy)']
```

```
ans = 2×5
    2.0000    3.0000    5.0000   10.0000   11.0000
    0.9013    0.1304   -0.2066   -1.9396    0.0148
```

```
figure()
stem(sBP_Noisy)
xlabel("Elements Index")
ylabel("Values")
grid on
legend("Sparse Recovery Solution with BP")
title("Sparse Recovery with BP over x_{Noisy} data")
```

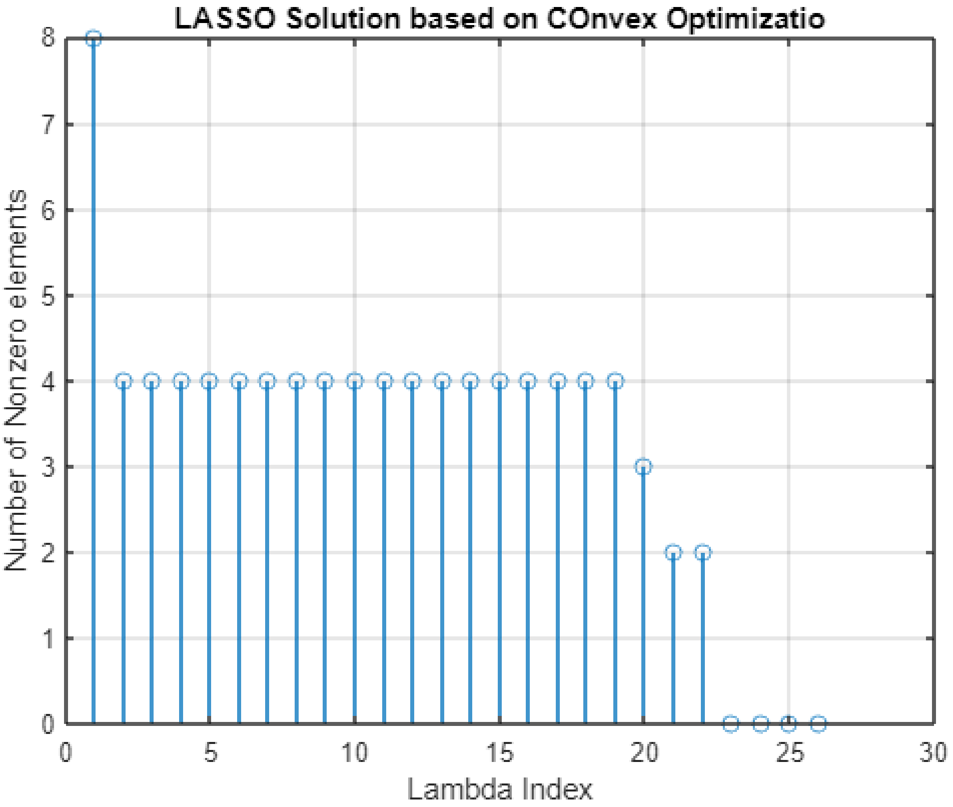


Part-4: LASSO (Least Absolute Shrinkage and selection Operator): with Convex Optimization

LASSO را با استفاده از داده های نویزی فرمول بندی کنید. به ازای λ های مختلف مساله را حل کنید. مقداری از λ
آن) که منجر به جواب درست می شود را گزارش کنید.

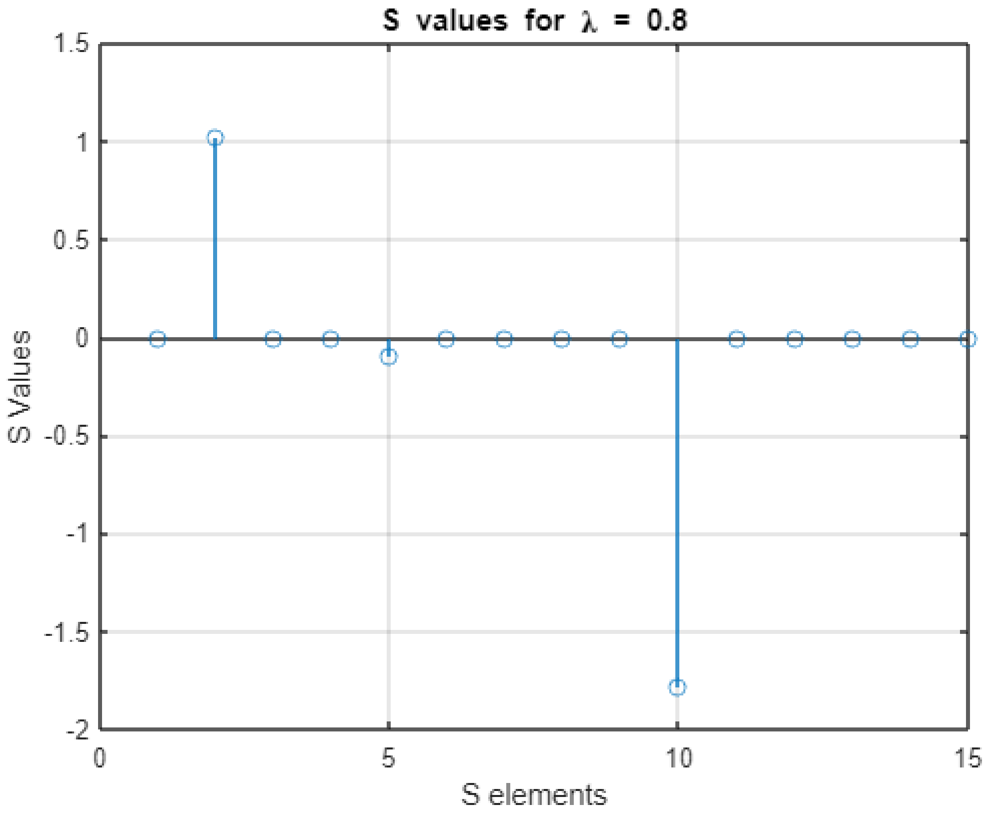
```
% Convex Optimization:

Lambda_Vec = [ 1e-8 , 1e-7, 1e-6 , 1e-5, 2e-5, 5e-5 , 8e-5 ,1e-4, 3e-4, 5e-4, 8e-4 ,1e-3, 1e-2
               1e-1, 2e-1, 5e-1, 8e-1, 9e-1, 9.5e-1 ,1 , 1e01 , 1e02 , 1e03 ];
Sparsity_Level = zeros(1,length(Lambda_Vec));
SParse_Values = cell(1,length(Lambda_Vec));
SparseIndices = SParse_Values;

for i=1:length(Lambda_Vec)
    lambda_COnvex = Lambda_Vec(i);

    % Set up and solve the LASSO problem using CVX
    cvx_begin quiet
        variable s_Convex(N)
        minimize(  norm(X_Noisy_Part1 - Dictionary_Part1* s_Convex, 2) + lambda_COnvex * norm(
    cvx_end

    % Print the results
    % disp(cvx_optval)
    % disp(s_Convex(abs(s_Convex)>1e-3))
    Sparsity_Level(i)  = length(s_Convex(abs(s_Convex)>1e-1));
    SParse_Values{1,i} =  s_Convex;
    SparseIndices{1,i} =  find( abs(s_Convex)>1e-1  );
    clear s_Convex;

end
```

```
figure()
stem(Sparsity_Level)
grid on
xlabel("Lambda Index");
ylabel("Number of Nonzero elements")
title("LASSO Solution based on COnvex Optimizatio")
```



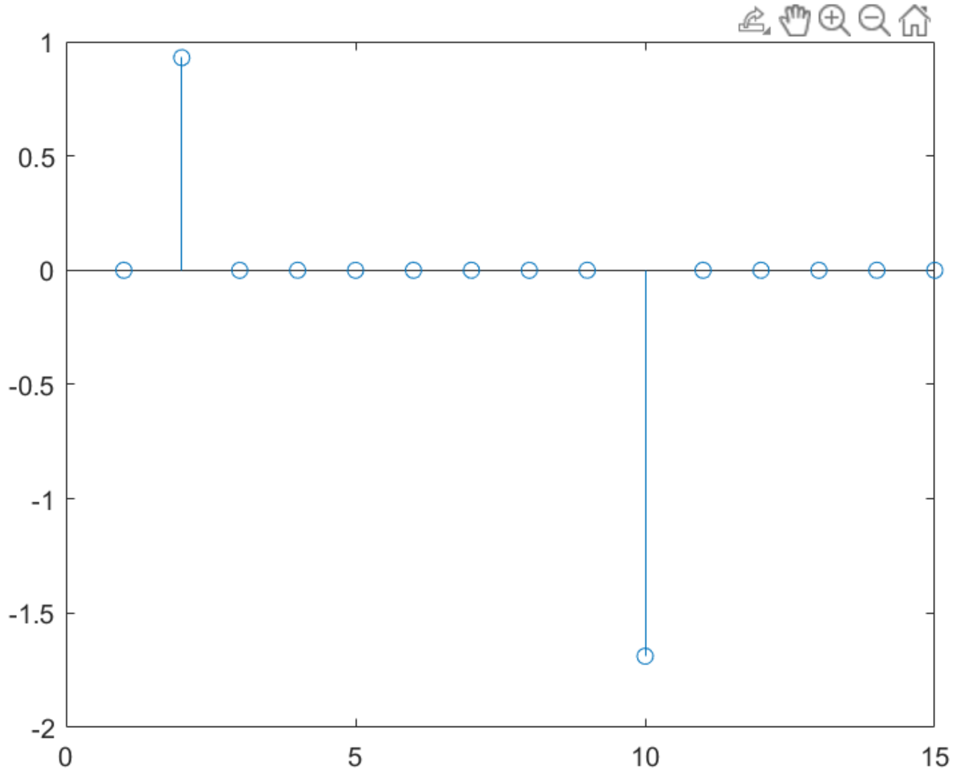
```
Index_to_Show_Lambda = 20;
figure()
stem(SParse_Values{1,Index_to_Show_Lambda})
grid on
xlabel("S elements")
```

```
ylabel("S Values")
title("S values for \lambda = "+Lambda_Vec(Index_to_Show_Lambda))
```

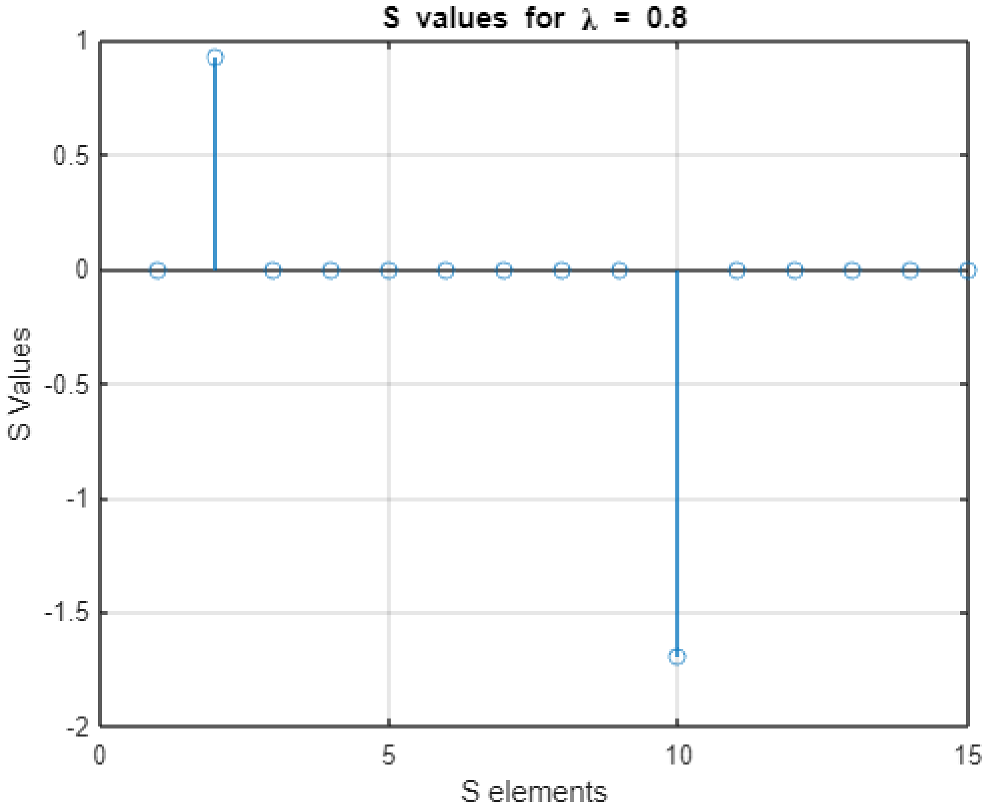


Part-4: LASSO (Least Absolute Shrinkage and selection Operator): with Alternation Minimization

```
% Update each S for different Iterations:

IterMax = 20;
[M,N]=size(Dictionary_Part1);
s_Alter = ones(1,N);
Lambda_Alter  =8e-1;

for i=1:IterMax
    for Indice = 1:length(s_Alter)
        d_i               =           Dictionary_Part1(:,Indice);
        indices_Chosen    = 1:length(s_Alter);
        indices_Chosen(Indice) = [] ;
        d_n               = Dictionary_Part1(:,indices_Chosen);
        s_n               = s_Alter(1,indices_Chosen)';

        r_i = X_Noisy_Part1 - d_n*s_n;
        RHO = r_i'*d_i;
        s_Alter(Indice) = My_soft(RHO, Lambda_Alter/2) ;
        drawnow
        stem(s_Alter)
    end

end
```



```
figure()
stem(s_Alter)
grid on
xlabel("S elements")
ylabel("S Values")
title("S values for \lambda = "+Lambda_Alter   )
```



```matlab
function out = My_soft(a,b)

if(length(a)>1)
    out = zeros(size(a));
    outside_idx = a <= -b/2 | a >= b/2;
    out(outside_idx) = a(outside_idx) - sign(a(outside_idx))*b/2;
    out(~outside_idx) = 0;
else
    out = 0;
    if( a < -abs(b))
        out = a + abs(b) ;
    elseif (a > abs(b))
        out = a - abs(b);
    end

end

end
```