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Convex Optimization

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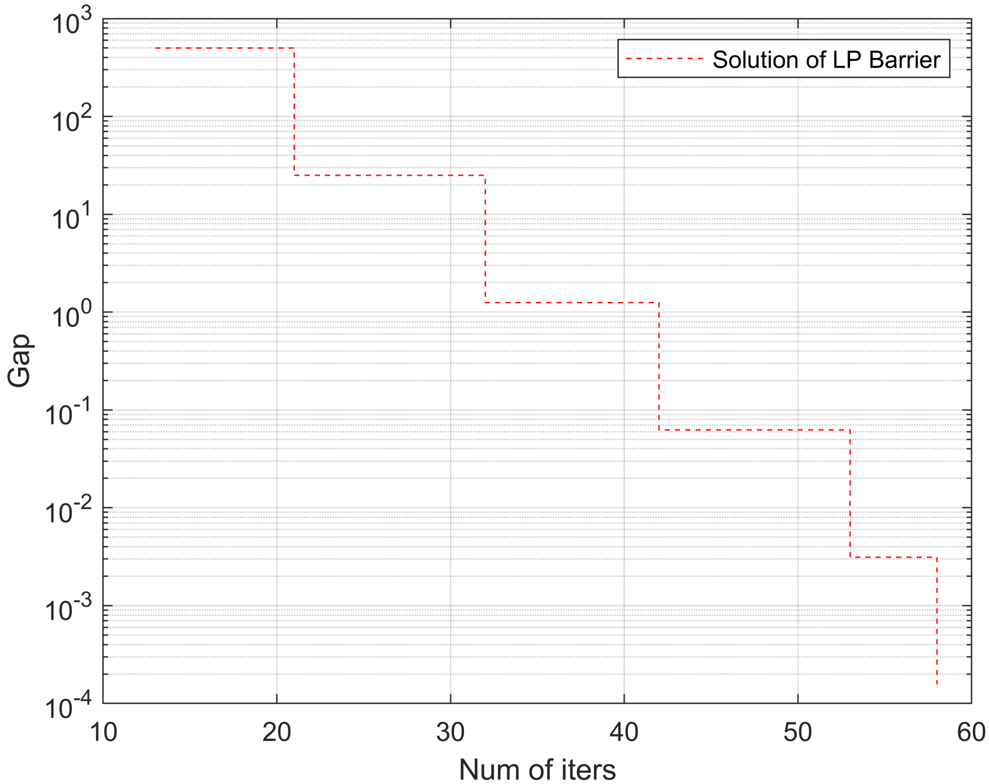
```
% Hw 7 --> Q6
clear ;clc ; close all;

% Load Data:
m = 100;
n = 500;

A= rand(m,n) ;
c= rand(n,1);
x_0 = rand(n,1);

b = A*x_0;
```

solve the LP with barrier:

```
[x_star, Values, gap] = LP_Barrier(A,b,c,x_0);
[xx, yy] = stairs(cumsum(Values(1,:)),Values(2,:));
```

Plot:

```
figure()
semilogy(xx,yy,'r--');
xlabel('Num of iters')
ylabel('Gap')
grid on
legend('Solution of LP Barrier')
```




```
p_star = c'*x_star;
```

Check Versus CVX:

```
% solve LP using cvx for comparison
cvx_begin
        variable x(n)
            minimize(c'*x)
            subject to
                A*x == b
                x >= 0
cvx_end
```

```
Calling SDPT3 4.0: 500 variables, 100 equality constraints
------------------------------------------------------------

 num. of constraints = 100
 dim. of linear var  = 500
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|4.2e+02|3.6e+01|2.4e+06| 5.179399e+04  0.000000e+00| 0:0:00| chol  1  1
 1|0.972|1.000|1.2e+01|3.0e-01|7.3e+04| 1.575509e+03 -5.551456e+03| 0:0:00| chol  1  1
 2|1.000|1.000|1.4e-08|3.0e-02|4.8e+03| 1.221327e+02 -4.690140e+03| 0:0:00| chol  1  1
```

```
 3|1.000|0.954|1.1e-09|4.2e-03|2.2e+02| 1.204767e+02 -1.003720e+02| 0:0:00| chol  1  1
 4|0.948|1.000|6.1e-10|3.0e-04|1.3e+02| 8.225414e+01 -5.098626e+01| 0:0:00| chol  1  1
 5|1.000|1.000|3.7e-11|3.0e-05|5.1e+01| 5.971778e+01  8.495231e+00| 0:0:00| chol  1  1
 6|0.711|0.956|1.8e-11|4.2e-06|2.6e+01| 4.772946e+01  2.142844e+01| 0:0:00| chol  1  1
 7|1.000|1.000|4.5e-12|3.0e-07|1.0e+01| 3.867463e+01  2.819146e+01| 0:0:00| chol  1  1
 8|1.000|0.910|2.5e-12|5.4e-08|3.1e+00| 3.398557e+01  3.088122e+01| 0:0:00| chol  1  1
 9|0.938|0.955|9.1e-13|5.3e-09|9.3e-01| 3.257465e+01  3.164401e+01| 0:0:00| chol  1  1
10|0.910|0.955|3.2e-14|5.3e-10|3.0e-01| 3.209141e+01  3.179621e+01| 0:0:00| chol  1  1
11|1.000|1.000|5.8e-12|3.1e-11|1.3e-01| 3.196065e+01  3.182869e+01| 0:0:00| chol  1  1
12|1.000|0.996|8.1e-13|4.3e-12|2.7e-02| 3.187604e+01  3.184950e+01| 0:0:00| chol  1  1
13|0.897|0.956|3.0e-11|1.5e-12|3.5e-03| 3.185787e+01  3.185437e+01| 0:0:00| chol  2  2
14|1.000|0.816|4.9e-12|1.8e-12|1.1e-03| 3.185569e+01  3.185459e+01| 0:0:00| chol  2  2
15|0.983|0.983|7.2e-13|1.0e-12|1.9e-05| 3.185486e+01  3.185484e+01| 0:0:00| chol  2  2
16|0.995|0.999|5.6e-13|1.0e-12|3.2e-07| 3.185484e+01  3.185484e+01| 0:0:00|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 16
 primal objective value =  3.18548411e+01
 dual   objective value =  3.18548408e+01
 gap := trace(XZ)       = 3.16e-07
 relative gap           = 4.88e-09
 actual relative gap    = 4.88e-09
 rel. primal infeas (scaled problem)   = 5.63e-13
 rel. dual     "        "       "      = 1.00e-12
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 3.1e+01, 6.7e-01, 1.1e+01
 norm(A), norm(b), norm(C) = 1.3e+02, 1.3e+03, 1.4e+01
 Total CPU time (secs)  = 0.27
 CPU time per iteration = 0.02
 termination code       =  0
 DIMACS: 5.2e-12  0.0e+00  6.8e-12  0.0e+00  4.9e-09  4.9e-09
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +31.8548
```

```
disp('Optimal value found by LP_barrier :');
```

```
Optimal value found by LP_barrier :
```

```
p_star
```

```
p_star =
  31.854965387543849
```

```
disp('Duality Gap from LP_barrier :');
```

```
Duality Gap from LP_barrier :
```

```
gap
```

```
gap =
     1.562500000000000e-04
```

```
disp('Optimal value found by CVX_MATLAB :');
```

```
Optimal value found by CVX_MATLAB :
```

```
cvx_optval
```

```
cvx_optval =
  31.854841088424255
```

```
disp("difference between CVX and LP_Barrier : ")
```

```
difference between CVX and LP_Barrier :
```

```
disp(abs(p_star-cvx_optval)) % Which is sooo small!
```

```
    1.242991195944398e-04
```

```
function [x_star, Values, gap] = LP_Barrier(A,b,c,x_0)
% solves standard form LP
% min c'*x
% subject to Ax = b, x >=0;

    T_0 = 1;
    V = 20;
    n = length(x_0);
    t = T_0;
    x = x_0;
    Values = [];


EPSILON = 1e-3; % Duality Gap Stopping condition
    while(1)
        [x_star, V_opt, lambda_hist] = Newton_method_q5(A,b,t*c,x);
        x = x_star;
        gap = n/t;
        Values = [Values [length(lambda_hist); gap]];
            if gap < EPSILON
                break;
            end
        t = V*t;
    end

end


function [x_star, V_opt, Lambdas, iter] = Newton_method_q5(A,b,c,x_0)
```

```
    max_Count = 100;
    m = length(b);
    n = length(x_0);
    x = x_0;
    Lambdas = [];
    alpha = 0.01;
    beta = 0.5;
    eps = 10^-6;


    if (min(x_0) <= 0) || (norm(A*x_0 - b) > 1e-3) % check feasibility of x_0
        fprintf('Not Feasible');
        V_opt = []; x_star = []; Lambdas=[];
        return;
    end

for iter = 1:max_Count

    H = diag(x.^(-2));
    g = c - x.^(-1);


% Newton step via whole KKT system
        % M = [ H A'; A zeros(m,m)];
        % d = M\[-g; zeros(m,1)];
        % dx = d(1:n);
        % w = d(n+1:end);

    % Newton Step by elimination method
    w = (A*diag(x.^2)*A')\(-A*diag(x.^2)*g);
    dx = -diag(x.^2)*(A'*w + g);
    lambdasqr = -g'*dx;          % dx'*H*dx;
    Lambdas = [Lambdas lambdasqr/2];


    if lambdasqr/2 <= eps
        break;
    end


    % backtracking line search
    % first bring the point inside the domainTheta
    t = 1;
    while min(x+t*dx) <= 0
        t = beta*t;
    end

    % Backtracking line search:
        while c'*(t*dx)-sum(log(x+t*dx))+sum(log(x))-alpha*t*g'*dx> 0
            t = beta*t;
```

```
            end
            x = x + t*dx;
 end

    if iter == max_Count % max_Count reached
        disp('Did not Coverged!');
        x_star = []; V_opt = [];
        else
        x_star = x;
        V_opt = w;
    end

end
```