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COnvex Optimization

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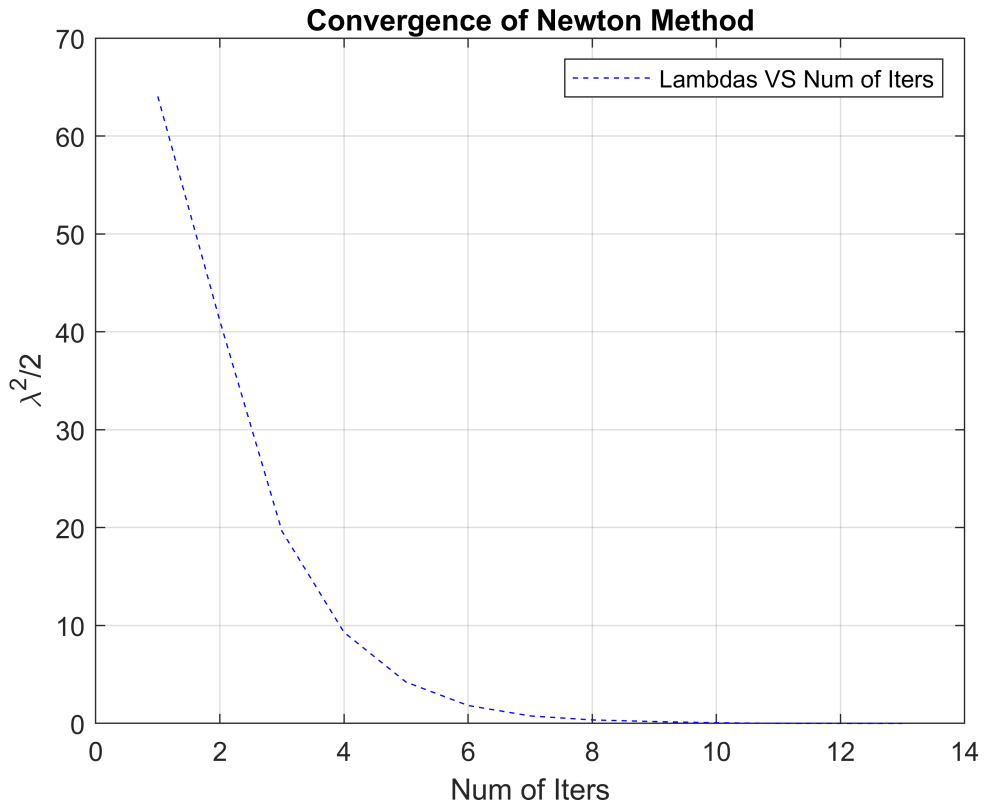
```
% Hw7 - Q-->5
clear; clc; close all;

% Load Random Data:
m = 100;
n = 500;

A= rand(m,n) ;
c= rand(n,1);
x_0 = rand(n,1);

b = A*x_0;
```

```
[x_star, V_opt, Lambdas , iter] = Newton_method_q5(A,b,c,x_0);
```

```
% Plot the results:
figure()
plot(1:iter,Lambdas,'b--')
legend('Lambdas VS Num of Iters');
xlabel('Num of Iters');
ylabel('\lambda^2/2');
title('Convergence of Newton Method')
grid on;
```




```
disp('V_opt is : ')
```

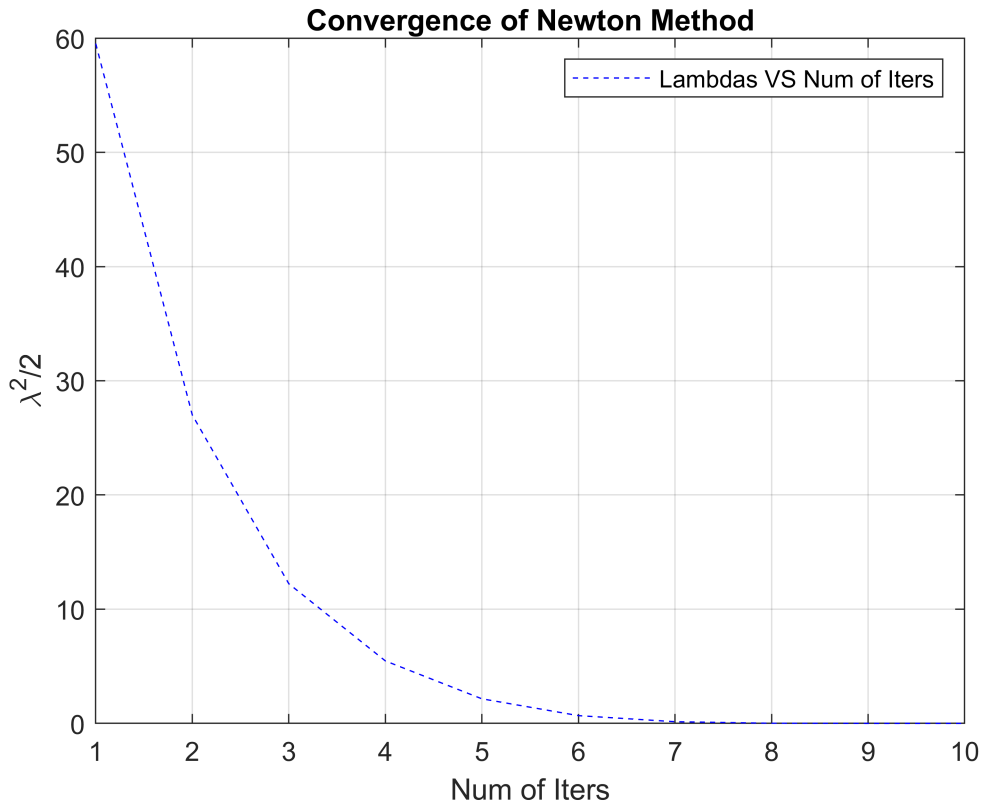
```
V_opt is :
```

```
disp(V_opt')
```

```
  Columns 1 through 6

  -0.022972992172156   0.257392243649177   0.114638436870183  -0.002589169657851  -0.109980334448393   0.0268390523

  Columns 7 through 12

   0.257679940330713  -0.140169538478114   0.264311513257759  -0.087705164013871  -0.032318180523530   0.1251541345

  Columns 13 through 18

   0.377491684303493   0.063362124591734   0.161473918900203   0.018474268663239  -0.177799439135262  -0.1354248051

  Columns 19 through 24

   0.107213895530140   0.022124115346377   0.303507150816960   0.034079131366690  -0.333956166533571  -0.0837849817

  Columns 25 through 30

  -0.049913244918483  -0.087630915780140   0.097562365901992   0.051828399505774  -0.055929477978337   0.3128138783

  Columns 31 through 36

  -0.075407065255637   0.349011181928543  -0.147268442636593  -0.119668152607210   0.099434442796308   0.2017280833

  Columns 37 through 42
```

```
  0.178021083552687   0.254718066811186   0.358486086496091   0.154487241015060   0.036264792032679  -0.1698374317

Columns 43 through 48

-0.165734184067978  -0.105247758633532  -0.016434037412439   0.271190330387627   0.114913293013628  -0.3035691620

Columns 49 through 54

-0.653933969783423  -0.076449180295869   0.151611116136856   0.241353153345521  -0.177351354924274  -0.4908377032

Columns 55 through 60

-0.047777356006099  -0.026149334244971   0.035216896925807   0.075842253015648   0.147718156961624   0.1548205509

Columns 61 through 66

-0.159615356665662  -0.142626679151218   0.064544492323424   0.165692497137224   0.259539042785198  -0.0979076796

Columns 67 through 72

-0.160140924252197   0.067238527561121   0.041003868093497   0.314098926488893   0.039642669180460   0.1835253706

Columns 73 through 78

 0.067352079849184  -0.142372092659506   0.062913059421753  -0.272481059478832   0.075482940737750   0.3104885906

Columns 79 through 84

 0.041342406475122  -0.182358817059120  -0.083889042621303   0.156464282400288   0.209805814438181  -0.2000850064

Columns 85 through 90

 0.055791741366122   0.213665180995938   0.096390601643372   0.222190650480826   0.344728837451732   0.0300782267

Columns 91 through 96

-0.101215415800360   0.170768643730221  -0.025226612374215   0.194329868207052  -0.132620114644636  -0.2252413848

Columns 97 through 100

 0.117142649307907   0.019943306909368   0.155633673772876   0.084730194954155
```

```
% Another random point:
A= rand(m,n) ;
c= rand(n,1);
x_0 = rand(n,1);

b = A*x_0;
[x_star2, V_opt2, Lambdas2 , iter2] = Newton_method_q5(A,b,c,x_0);
% Plot the results:
figure()
plot(1:iter2,Lambdas2,'b--')
legend('Lambdas VS Num of Iters');
xlabel('Num of Iters');
ylabel('\lambda^2/2');
title('Convergence of Newton Method')
grid on;
```

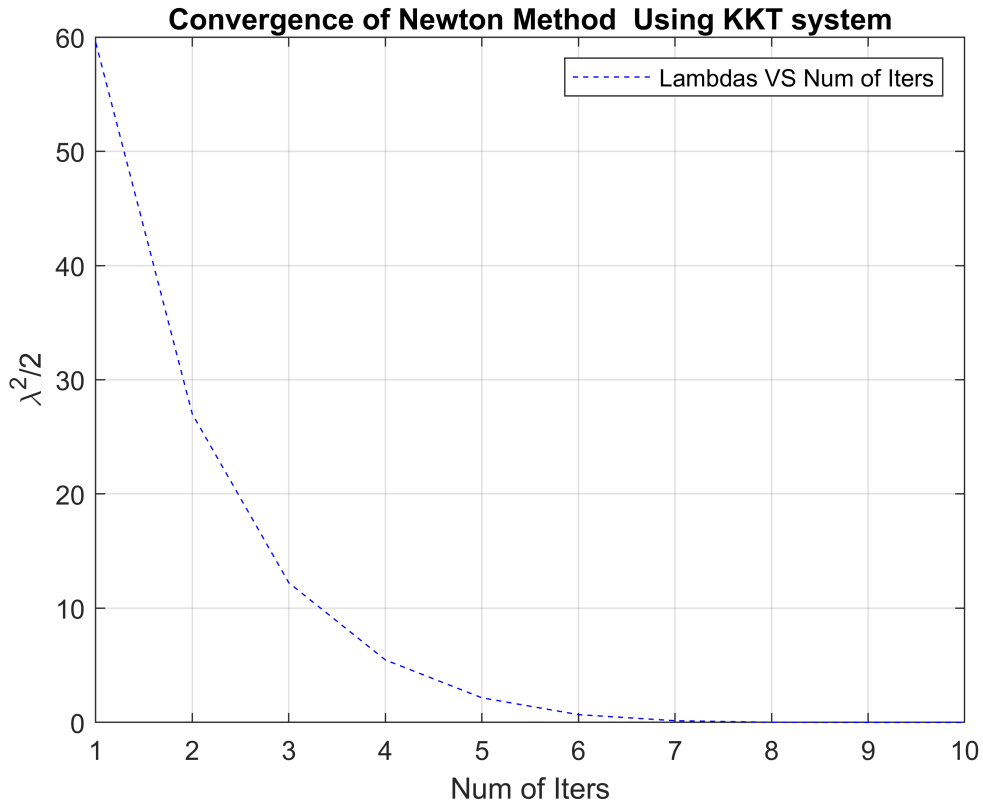



```
% Using KKT System:

[x_star3, V_opt3, Lambdas3, iter3] = Newton_method_q5_KKT(A,b,c,x_0)
```

```
x_star3 = 500×1
   0.580950884106911
   0.384919681597391
   0.621084744126251
   0.910334217433320
   0.695994006660801
   0.493436305558372
   0.454076261860091
   0.346822559183342
   0.905628874456968
   0.334332236621497
      ⋮
V_opt3 = 100×1
  -0.001279949180280
   0.564618811769372
   0.081422214132736
   0.378430335227760
   0.248812682551641
   0.151601646521170
  -0.011121959055077
   0.157025615350592
  -0.190430797466370
  -0.182561414405442
      ⋮
Lambdas3 = 1×10
```

```
  59.522971167459261  27.008351415366697  12.226089457588960   5.470696575491893 ···
iter3 =
    10
```

```
figure()
plot(1:iter2,Lambdas2,'b--')
legend('Lambdas VS Num of Iters');
xlabel('Num of Iters');
ylabel('\lambda^2/2');
title('Convergence of Newton Method  Using KKT system')
grid on;
```




```
function [x_star, V_opt, Lambdas, iter] = Newton_method_q5_KKT(A,b,c,x_0)


    max_Count = 100;
    m = length(b);
    n = length(x_0);
    x = x_0;
    Lambdas = [];
    alpha = 0.01;
    beta = 0.5;
    eps = 10^-6;


    if (min(x_0) <= 0) || (norm(A*x_0 - b) > 1e-3) % check feasibility of x_0
```

```
        fprintf('Not Feasible');
        V_opt = []; x_star = []; Lambdas=[];
        return;
    end

for iter = 1:max_Count

    H = diag(x.^(-2));
    g = c - x.^(-1);


% Newton step via whole KKT system
        M = [ H A'; A zeros(m,m)];
        d = M\[-g; zeros(m,1)];
        dx = d(1:n);
        w = d(n+1:end);


    lambdasqr = -g'*dx;          % dx'*H*dx;
    Lambdas = [Lambdas lambdasqr/2];


    if lambdasqr/2 <= eps
        break;
    end


    % backtracking line search
    % first bring the point inside the domainTheta
    t = 1;
    while min(x+t*dx) <= 0
        t = beta*t;
    end

    % Backtracking line search:
        while c'*(t*dx)-sum(log(x+t*dx))+sum(log(x))-alpha*t*g'*dx> 0
            t = beta*t;
            end
            x = x + t*dx;
 end

    if iter == max_Count % max_Count reached
        disp('Did not Coverged!');
        x_star = []; V_opt = [];
        else
        x_star = x;
        V_opt = w;
    end

end
```

```
%%

function [x_star, V_opt, Lambdas, iter] = Newton_method_q5(A,b,c,x_0)


    max_Count = 100;
    m = length(b);
    n = length(x_0);
    x = x_0;
    Lambdas = [];
    alpha = 0.01;
    beta = 0.5;
    eps = 10^-6;


    if (min(x_0) <= 0) || (norm(A*x_0 - b) > 1e-3) % check feasibility of x_0
        fprintf('Not Feasible');
        V_opt = []; x_star = []; Lambdas=[];
        return;
    end

for iter = 1:max_Count

    H = diag(x.^(-2));
    g = c - x.^(-1);


% Newton step via whole KKT system
        % M = [ H A'; A zeros(m,m)];
        % d = M\[-g; zeros(m,1)];
        % dx = d(1:n);
        % w = d(n+1:end);

    % Newton Step by elimination method
    w = (A*diag(x.^2)*A')\(-A*diag(x.^2)*g);
    dx = -diag(x.^2)*(A'*w + g);
    lambdasqr = -g'*dx;          % dx'*H*dx;
    Lambdas = [Lambdas lambdasqr/2];


    if lambdasqr/2 <= eps
        break;
    end


    % backtracking line search
    % first bring the point inside the domainTheta
    t = 1;
    while min(x+t*dx) <= 0
        t = beta*t;
```

```
    end

    % Backtracking line search:
        while c'*(t*dx)-sum(log(x+t*dx))+sum(log(x))-alpha*t*g'*dx> 0
            t = beta*t;
            end
            x = x + t*dx;
 end

    if iter == max_Count % max_Count reached
        disp('Did not Coverged!');
        x_star = []; V_opt = [];
        else
        x_star = x;
        V_opt = w;
    end

end
```