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Convex Optimization

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Hw8 - Q3:

```
clear;
clc;
close all;
% Load Data:
run('simple_portfolio_data.m');
```

```
% Out problem is a QP with an affine constraint:
%% unconstrained:
cvx_begin
```

```
Warning: A non-empty cvx problem already exists in this scope.
   It is being overwritten.
```

```
    variable x_unconstrained(n)
        minimize(quad_form(x_unconstrained,S))
    % Constraints:
        subject to
        sum(x_unconstrained)    ==     1          ;
        pbar'*x_unconstrained   ==  x_unif' * pbar;
cvx_end
```

```
Calling SDPT3 4.0: 21 variables, 19 equality constraints
   For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

 num. of constraints = 19
 dim. of socp   var  = 21,   num. of socp blk  =  1
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|3.5e+00|2.6e+00|2.4e+01| 0.000000e+00  0.000000e+00| 0:0:00| chol  1  1
 1|1.000|1.000|1.5e-06|4.7e-02|2.1e+00| 1.324089e+00 -6.530277e-01| 0:0:00| chol  1  1
 2|0.996|0.989|2.6e-08|5.2e-03|2.2e-02|-9.923533e-02 -1.197667e-01| 0:0:00| chol  1  1
 3|0.989|0.988|5.9e-09|5.3e-04|2.4e-04|-1.148472e-01 -1.149626e-01| 0:0:00| chol  1  1
 4|0.989|0.987|1.7e-09|5.3e-05|2.8e-06|-1.150222e-01 -1.150120e-01| 0:0:00| chol  1  1
 5|0.989|0.986|1.1e-10|7.2e-07|3.4e-08|-1.150242e-01 -1.150241e-01| 0:0:00| chol  1  1
 6|0.989|0.983|3.0e-12|1.2e-08|4.5e-10|-1.150242e-01 -1.150242e-01| 0:0:00|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   =  6
 primal objective value = -1.15024246e-01
```

```
 dual   objective value = -1.15024243e-01
 gap := trace(XZ)       = 4.47e-10
 relative gap           = 3.64e-10
 actual relative gap    = -2.11e-09
 rel. primal infeas (scaled problem)   = 3.02e-12
 rel. dual     "        "       "      = 1.24e-08
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e-01, 7.6e-01, 7.1e-01
 norm(A), norm(b), norm(C) = 7.5e+00, 1.2e+00, 2.1e+00
 Total CPU time (secs)  = 0.11
 CPU time per iteration = 0.02
 termination code       =  0
 DIMACS: 3.0e-12  0.0e+00  1.3e-08  0.0e+00  -2.1e-09  3.6e-10
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +0.000346223
```

long-only:

```
cvx_begin
    variable x_long(n)
    % Objective:
    minimize(quad_form(x_long,S))

    subject to % Constraints:
        x_long          >= 0;
        sum(x_long)     ==      1       ;
        pbar'*x_long    ==  x_unif'*pbar;
cvx_end
```

```
Calling SDPT3 4.0: 43 variables, 21 equality constraints
   For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

 num. of constraints = 21
 dim. of socp   var  = 21,   num. of socp blk  =  1
 dim. of linear var  = 20
 dim. of free   var  =  2 *** convert ublk to lblk
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|3.7e+01|1.7e+01|2.1e+03|-1.563897e-10  0.000000e+00| 0:0:00| chol  1  1
 1|0.990|0.767|3.8e-01|4.0e+00|6.0e+02| 1.528328e+01 -8.971290e-01| 0:0:00| chol  1  1
 2|1.000|0.990|1.4e-06|4.8e-02|1.9e+01| 1.192556e+01 -1.426141e-01| 0:0:00| chol  1  1
 3|0.987|0.838|4.1e-07|8.7e-03|2.0e+00| 1.685324e+00 -1.327458e-01| 0:0:00| chol  1  1
 4|0.945|0.244|6.0e-08|6.6e-03|2.1e-01| 5.262249e-02 -1.309619e-01| 0:0:00| chol  1  1
 5|0.873|0.961|1.5e-08|2.6e-04|2.6e-02|-9.603761e-02 -1.215767e-01| 0:0:00| chol  1  1
 6|0.657|0.257|1.1e-08|2.0e-04|1.4e-02|-1.071348e-01 -1.208118e-01| 0:0:00| chol  1  1
```

```
 7|1.000|0.256|1.1e-08|1.5e-04|6.6e-03|-1.133975e-01 -1.198702e-01| 0:0:00| chol  1  1
 8|0.977|0.486|5.5e-09|7.5e-05|2.7e-03|-1.159578e-01 -1.186293e-01| 0:0:00| chol  1  1
 9|1.000|0.532|1.5e-09|4.1e-05|1.1e-03|-1.168260e-01 -1.178984e-01| 0:0:00| chol  1  1
10|1.000|0.854|1.6e-10|1.0e-05|2.0e-04|-1.171434e-01 -1.173371e-01| 0:0:00| chol  1  1
11|0.986|0.873|9.8e-12|1.7e-06|3.8e-05|-1.172261e-01 -1.172634e-01| 0:0:00| chol  1  1
12|1.000|0.962|2.8e-12|3.1e-07|5.4e-06|-1.172411e-01 -1.172464e-01| 0:0:00| chol  1  1
13|1.000|0.986|5.8e-13|4.4e-08|1.3e-07|-1.172442e-01 -1.172443e-01| 0:0:00| chol  1  1
14|0.997|0.989|8.4e-14|1.1e-09|1.8e-09|-1.172443e-01 -1.172443e-01| 0:0:00|
  stop: max(relative gap, infeasibilities) < 1.49e-08
------------------------------------------------------------------
 number of iterations   = 14
 primal objective value = -1.17244274e-01
 dual   objective value = -1.17244276e-01
 gap := trace(XZ)       = 1.81e-09
 relative gap           = 1.46e-09
 actual relative gap    = 1.36e-09
 rel. primal infeas (scaled problem)   = 8.35e-14
 rel. dual     "        "       "      = 1.09e-09
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.8e-01, 6.0e-01, 7.9e-01
 norm(A), norm(b), norm(C) = 1.0e+01, 1.2e+00, 2.7e+00
 Total CPU time (secs)  = 0.16
 CPU time per iteration = 0.01
 termination code       =  0
 DIMACS: 8.4e-14  0.0e+00  1.5e-09  0.0e+00  1.4e-09  1.5e-09
------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +0.00256626
```

Constrained short Portfolio:

- Limit on total short position: $\mathbf{1}^T(x_-) \leq 0.5$, where $(x_-)_i = \max\{-x_i, 0\}$.

```
cvx_begin

variable x_shortconstr(n)
% Objective:
minimize(quad_form(x_shortconstr,S))

subject to % Constraints:
    sum(pos(-x_shortconstr))   <=    0.5        ;
    sum(x_shortconstr)         ==    1          ;
    pbar'*x_shortconstr        ==   x_unif'*pbar;
cvx_end
```

```
Calling SDPT3 4.0: 64 variables, 41 equality constraints
   For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

 num. of constraints = 41
 dim. of socp   var  = 21,   num. of socp blk  =  1
 dim. of linear var  = 41
 dim. of free   var  =  2 *** convert ublk to lblk
```

```
*****************************************************************
   SDPT3: Infeasible path-following algorithms
*****************************************************************
 version  predcorr  gam  expon  scale_data
    NT      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-----------------------------------------------------------------
 0|0.000|0.000|5.7e+01|2.4e+01|4.4e+03| 5.000000e+00  0.000000e+00| 0:0:00| chol  1  1
 1|0.981|0.719|1.1e+00|6.9e+00|1.5e+03| 2.786607e+01 -1.085812e+00| 0:0:00| chol  1  1
 2|1.000|0.986|2.5e-06|1.1e-01|4.7e+01| 2.433295e+01 -1.674058e-01| 0:0:00| chol  1  1
 3|0.987|0.831|6.5e-07|1.9e-02|7.4e+00| 5.921714e+00 -1.474855e-01| 0:0:00| chol  1  1
 4|0.990|0.191|7.7e-08|1.6e-02|1.1e+00| 7.461773e-01 -1.450168e-01| 0:0:00| chol  1  1
 5|0.962|0.876|6.4e-09|2.0e-03|1.1e-01|-1.734550e-02 -1.244846e-01| 0:0:00| chol  1  1
 6|0.934|0.418|4.4e-09|1.2e-03|4.4e-02|-7.872457e-02 -1.210989e-01| 0:0:00| chol  1  1
 7|0.940|0.443|1.5e-09|6.5e-04|1.6e-02|-1.033313e-01 -1.188514e-01| 0:0:00| chol  1  1
 8|0.763|0.405|1.2e-09|3.9e-04|1.1e-02|-1.052388e-01 -1.164977e-01| 0:0:00| chol  1  1
 9|1.000|0.415|6.4e-11|2.3e-04|3.3e-03|-1.129110e-01 -1.161726e-01| 0:0:00| chol  1  1
10|1.000|0.530|6.4e-11|9.9e-05|1.6e-03|-1.140886e-01 -1.156750e-01| 0:0:00| chol  1  1
11|1.000|0.818|9.4e-11|2.0e-05|6.0e-04|-1.147180e-01 -1.153134e-01| 0:0:00| chol  1  1
12|0.830|0.937|2.3e-10|3.4e-06|2.5e-04|-1.149765e-01 -1.152256e-01| 0:0:00| chol  1  1
13|1.000|0.949|1.3e-10|1.4e-06|6.4e-05|-1.150815e-01 -1.151450e-01| 0:0:00| chol  2  1
14|0.957|0.952|6.7e-11|3.5e-07|1.6e-05|-1.151109e-01 -1.151267e-01| 0:0:00| chol  2  2
15|1.000|0.974|1.8e-11|8.6e-08|1.5e-06|-1.151181e-01 -1.151196e-01| 0:0:00| chol  1  1
16|1.000|0.987|1.2e-11|8.2e-09|4.2e-08|-1.151188e-01 -1.151188e-01| 0:0:00| chol  1  2
17|0.995|0.988|1.3e-13|2.3e-10|8.5e-10|-1.151188e-01 -1.151188e-01| 0:0:00|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-----------------------------------------------------------------
 number of iterations   = 17
 primal objective value = -1.15118796e-01
 dual   objective value = -1.15118797e-01
 gap := trace(XZ)       = 8.54e-10
 relative gap           = 6.94e-10
 actual relative gap    = 6.87e-10
 rel. primal infeas (scaled problem)   = 1.29e-13
 rel. dual     "        "       "      = 2.26e-10
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 1.6e-01, 7.9e-01, 9.1e-01
 norm(A), norm(b), norm(C) = 1.5e+01, 1.2e+00, 2.8e+00
 Total CPU time (secs)  = 0.20
 CPU time per iteration = 0.01
 termination code       =  0
 DIMACS: 1.3e-13  0.0e+00  3.2e-10  0.0e+00  6.9e-10  6.9e-10
-----------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +0.000440777
```

```
disp("unconstrained sd:  " + num2str(sqrt(quad_form(x_unconstrained,S))  ))
```

```
unconstrained sd:  0.018607
```

```
disp("long only sd:  " + num2str(sqrt(quad_form(x_long,S)))  )
```

```
long only sd:  0.050658
```

```
disp("constrained short sd:  " + num2str(sqrt(quad_form(x_shortconstr,S)))  )
```

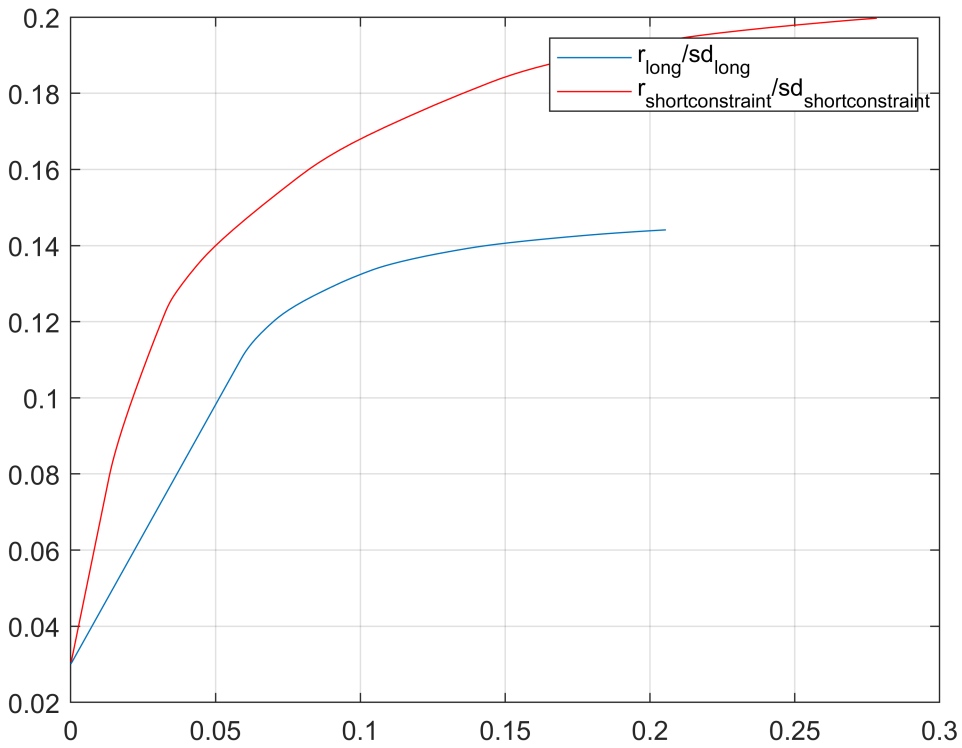
```
constrained short sd:  0.020995
```

```
disp("x_unif sd:  " + num2str(sqrt(quad_form(x_unif,S)))  )
```

```
x_unif sd:  0.087036
```

```
novals=100;
r_long = [];
r_shortconstr = [];
sd_long = [];
sd_shortconstr = [];
muvals = logspace(-1,4,novals);
for i=1:novals
mu = muvals(i);
%long only
    cvx_begin
    cvx_quiet(true)
        variable x(n)
        maximize(pbar'*x - mu*quad_form(x,S))
        subject to
        x        >=   0;
        sum(x)   ==   1;
    cvx_end
    r_long = [r_long, pbar'*x];
    sd_long = [sd_long, sqrt(x'*S*x) ];
%constrained short
    cvx_begin
    cvx_quiet(true)
        variables x(n)
        maximize(pbar'*x - mu*quad_form(x,S))
        subject to
        sum(x)       ==   1 ;
        sum(pos(-x)) <=  0.5;
    cvx_end
    r_shortconstr = [r_shortconstr, pbar'*x];
    sd_shortconstr = [sd_shortconstr, sqrt(x'*S*x)];
end
```

```
Fig7=figure();
plot(sd_long, r_long);
hold on;
plot(sd_shortconstr, r_shortconstr, 'r');
grid on
legend('r_{long}/sd_{long}','r_{shortconstraint}/sd_{shortconstraint}')
```


0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.05
0.1
0.15
0.2
0.25
0.3
r_long/sd_long
r_shortconstraint/sd_shortconstraint