Convex Optimization

Project 1

Spring 1401

Due date: 23rd of Ordibehesht

Consider a vector time series $x_t \in \mathbf{R}^n$, $t = 1, 2, \ldots$. We want to fit a model of the form $x_t \sim \mathcal{N}(0, a_t\Sigma)$, where $\Sigma \in \mathbf{S}^n_{++}$ is given, and $a_t > 0$. (We assume x_t and x_s are independent for $t \neq s$.) Roughly speaking, the covariance matrix of x_t scales up and down with time; a_t is scale factor at time t.

We are given the base covariance matrix Σ, and a sample sequence $x_1, \ldots, x_T$. We are to find the scale factor time series a_t, $t = 1, \ldots, T$

We will fit the scale factor times series by minimizing the negative log likelihood, plus a term that regularizes the variation in a(t),

$$\lambda \sum_{t=1}^{T-1} (\log a_{t+1} - \log a_t)^2,$$

where $\lambda > 0$ is a given hyper-parameter. (Note that $\log a_{t+1} - \log a_t$ can be interpreted as the fractional change in the scaling parameter from t to $t + 1$.)

a Show how to solve this fitting problem using convex or quasiconvex optimization. Fully justify any changes of variables, or relaxations, that your method uses.

b Carry out your method on the data given in covar_series_data.⋆, for the three hyper parameter values $\lambda = 0.01$, $\lambda = 1$, $\lambda = 100$. (This gives three different estimates of the scale factor time series.) Plot these three estimates versus t.

c *Validation.* The data covar_series_data.∗ contains another time series $y_1, \ldots, y_T$ from the same source. Evaluate the negative log likelihood of your three models obtained in part (b) on this validation data set. Which of the three hyper-parameter values achieves the smallest negative log-likelihood?