## Computational Electromagnetics

	_ Hw7-Q1	
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	1401/10/10	

clear; clc; close all;

Q-1):

۱- کنن استدار به طل ۵ رفعاع ۵ از دمط تغذیه می کفید. الف) با دستمه از میتر مل توزیع جریان در آش را دمور

استیر معدر آت رو مید ناشد . که را برابه ۵٬۵۰۱ دنظ میردد.

ب) برن آس در صنی ت ٥=٩ (٥ سفر) و ١٥ عفر) ادبر عات مي درم عات

ع) یافته در خدرا با تنیم در تحس کار سید آمن در دیس انتفار دارید تما به کنید در حنبه در فحلف ریشرف که رعل به آن بری خدر من فائید .

• The Main point in this question, is to find the current over the wire! -- >> The geometry of this problem is defined as a simple dipole on a thined wire with diameter equal to **2a** 

Thined wire is defined as a wire with diameter which is small in comparison to wave length!

The MoM is applied to  $L\{f\} = g$  and results in Current over the wire! -- >> This Current is then used to find the Scattered Field and then the Total Field!

Incident Wave can be modeled as both:

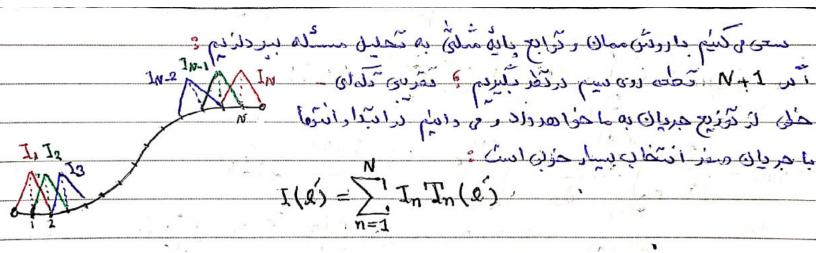
- 1) Gap Generator
- 2) A plane wave propagating in the area

 $L\{f\}$  is defined as:

$$E^{i} = jw\mu_{0}\hat{\ell}.\int I(\ell')\frac{e^{-jkR}}{4\pi R}d\ell' - \frac{1}{jw\epsilon_{0}}\hat{\ell}.\nabla\int\frac{\partial}{\partial\ell'}I(\ell')\frac{e^{-jkR}}{4\pi R}d\ell' ;$$

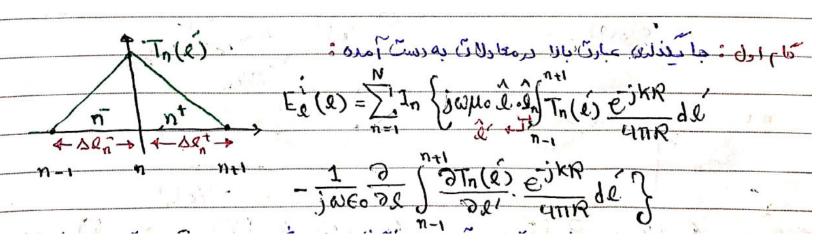
where  $E^i$  equals to the rigth side of the  $L\{f\}=g$  equation!

To apply Method of moments, first we must write our f in terms of corresponding expansion functions!



After choosing Expansion Function, the Second Step is considered Done!

$$I(\mathcal{E}') = \sum_{n=1}^{\infty} I_n T_n(\mathcal{E}')$$



 $\Rightarrow E^i = \mathrm{jw} \mu_0 \widehat{\ell}. \int \sum_{n=1}^{} I_n T_n(\ell') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\ell' - \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\ell}. \nabla \int \frac{\partial}{\partial \ell'} \sum_{n=1}^{} I_n T_n(\ell') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\ell' \ \Rightarrow \text{due to linearity of operators we have:}$ 

 $\Rightarrow E^i = \mathrm{jw} \mu_0 \widehat{\ell} . \sum_n \int I_n T_n(\ell') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\ell' - \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\ell} . \sum_n \nabla \int \frac{\partial}{\partial \ell'} I_n T_n(\ell') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\ell' \Rightarrow -->> \text{Now, it is time to choose } W_m \text{ for the weighting functions at:}$ 

 $< W_m, g > = < W_m, L\{f\} > =>$  For Galerkin we have:

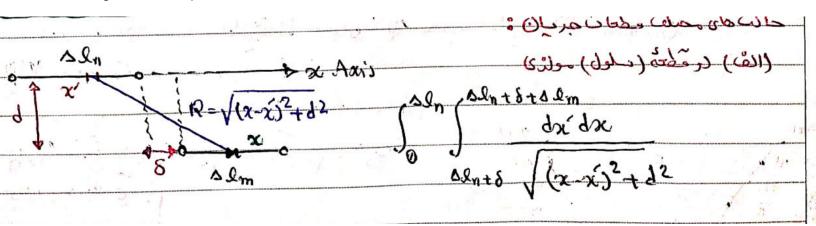
$$< W_m \,, E^i > \ = V_m \, = \, \mathrm{jw} \mu_0 \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \int \, I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C} \, - \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \, \sum_n \int \, W_m \nabla \int \, \frac{\partial}{\partial \mathcal{C}'} I_n T_n(\mathcal{C}') \frac{e^{-\mathrm{jkR}}}{4\pi R} d\mathcal{C}' \, + \frac{1}{\mathrm{jw}\epsilon_0} \widehat{\mathcal{C}} . \,$$

Due to the fact that we are facing an antenna problem, it is desired to have our f value equal to 0 at both ends of the antenna!

To calculate the distance one shall use Reff where:

$$\Psi\left(m,n\right) = \frac{e^{-\mathrm{jkR}_{\mathrm{eff}}}}{4\pi R_{\mathrm{eff}}} \text{ where } R_{\mathrm{eff}} = \frac{\mathrm{delta}_{\ell_{m}} * \mathrm{delta}_{\ell_{m}}}{M}$$

For a single wire, M equals to:



#### Assuming:

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad ; \quad \int \sinh^{-1}\left(\frac{x}{a}\right) dx = x \sinh^{-1}\left(\frac{x}{a}\right) - \sqrt{x^2 + a^2} + C$$

$$d = \Delta \ln + \Delta \ln + \delta \quad ; \quad \beta = \Delta \ln + \delta \quad ; \quad \gamma = \Delta \ln + \delta$$

$$\vdots \quad \gamma = \Delta \ln + \Delta \ln + \delta \quad ; \quad \gamma = \Delta \ln + \delta$$

 $\alpha = \mathrm{delta}_{\ell_n} + \mathrm{delta}_{\ell_m + \delta} \ \ \, ; \ \ \, \beta = \mathrm{delta}_{\ell_n} + \delta \; ; \quad \gamma = \mathrm{delta}_{\ell_m} + \delta \; ;$ 

$$M = \alpha \cdot \sinh^{-1}\left(\frac{\alpha}{d}\right) - \beta \cdot \sinh^{-1}\left(\frac{\beta}{d}\right) - \gamma \cdot \sinh^{-1}\left(\frac{\gamma}{d}\right) + \delta \cdot \sinh^{-1}\left(\frac{\delta}{d}\right) - \sqrt{\alpha^2 + d^2} + \sqrt{\beta^2 + d^2} + \sqrt{\gamma^2 + d^2} - \sqrt{\delta^2 + d^2} + \sqrt{\gamma^2 + d$$

For **self-terms** we get:

$$S = \Delta \ln \left\{ \Delta \ln \left( \Delta - \alpha \right) \right\} = \delta \ln \left( \Delta - \alpha \right) + \Delta \ln \left( \Delta \ln \alpha \right) + \Delta \ln \left( \Delta \ln \alpha \right) + \Delta \ln \left( \Delta \ln \alpha \right) + \Delta \ln \alpha \right) + \Delta \ln \left( \Delta \ln \alpha \right) + \Delta \ln \alpha \right]$$

$$\left[ |M| = 2\Delta \ln \delta \ln \ln \left( \Delta \ln \alpha \right) + 2\sqrt{\Delta \ln^2 + \alpha^2} + 2\alpha \left( \delta \ln \delta \ln \alpha \right) \right]$$

$$\delta = \delta_{\ell_n}$$
,  $delta_{\ell_m} = delta_{\ell_n}$ ;  $d = a$ ;

```
\Rightarrow \beta = \gamma = 0; \ \alpha = \text{delta}_{\ell};
M = 2\text{delta}_{\ell}. \sinh^{-1}\left(\frac{\text{delta}_{\ell}}{a}\right) - 2\sqrt{\text{delta}_{\ell}^2 + a^2} + 2a
```

```
c = 3e+08;  % Electromagnetics wave's speed in Air
f = 10e+06; % 10MHz

Lambda = c/f; % Wavelength

a = 1e-3*Lambda;
r = a;  % Radius of Wires
D = 2*a; % Diameter of each Wire
L = Lambda/2; % Antenna Length (sum of Dipole's length)
```

```
N = 7;

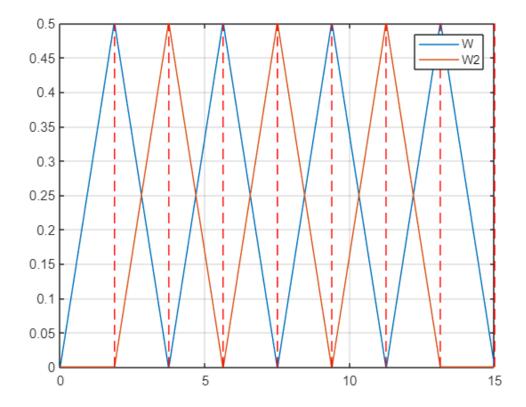
delta_l = L/(N+1);
delta_l_index = 100; % Each Triangle Delta_l is equal to 100 indexes in l_vec

l_vec = linspace(0,L,(N+1)*delta_l_index);
```

```
[W,W2] = W_calc(N,delta_l_index,l_vec);
```

```
% close 1
figure(1)
plot(1_vec,W);

grid on
hold on
plot(1_vec , W2)
for i=1:N+1
    plot( i*delta_l*ones(1,10) , linspace(0,max(W),10),'r--');
end
legend("W","W2")
```



```
V = zeros(N+1,1);
E_i = zeros(1,(N+1)*delta_l_index) ;

mid_point = floor(length(E_i)/2) ; % Tahrik az vasat

E_i(mid_point) = 1 ;
for m = 1:N+1
    V(m) = G_m_calc(W , E_i , m ,l_vec , delta_l_index);
```

```
end
M = zeros(N+1,N+1);
Z = M;
PSAI = M;
PSAI f = PSAI;
d = a;
k = 2*pi/Lambda; % wave number
w = 2*pi*f; % Rad/m
mu0 = 4*pi*1e-07; % H/m
eps0 = 8.85*1e-12; % F/m
for m = 1:N+1
   for n=1:N+1
       M(m,n) = M_calc( m , n , delta_l , d );
       Reff = (delta_l*delta_l)/M(m,n);
       PSAI(m,n) = exp(-1j*k*Reff)/(4*pi*Reff);
       % PSAI_f(m,n) =
       if( (m==N+1) || (n==N+1) )
           Z(m,n) = 1j*w*mu0*delta_l*delta_l*PSAI(m,n) + ...
               (1/(1j*w*eps0)*(0+PSAI(m,n)-0-0));
       else
                  = 1j*w*mu0*delta_l*delta_l*PSAI(m,n) +...
           Z(m,n)
               (1/(1j*w*eps0)*(PSAI(m+1,n+1)+PSAI(m,n)-PSAI(m+1,n)-PSAI(m,n+1));
       end
   end
end
```

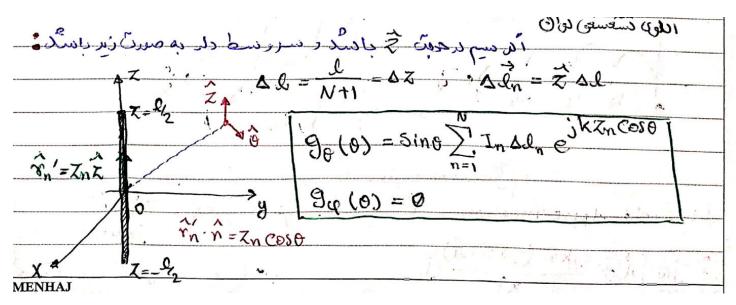
```
I = inv(Z)*V;

Z_in = V(floor(mid_point/delta_l_index))/I(floor(mid_point/delta_l_index));

disp((Z_in))
```

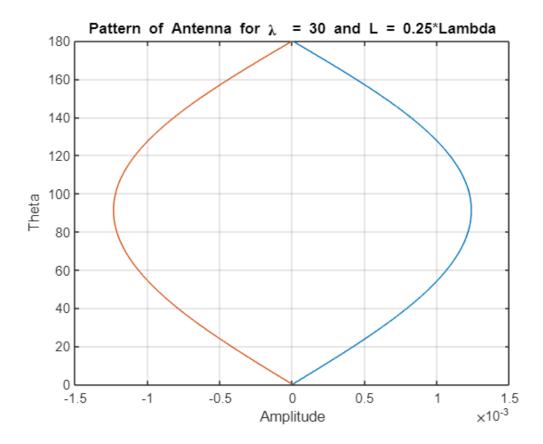
```
-9.9462e+00 - 4.6961e+02i
```

#### Part B: Draw Pattern

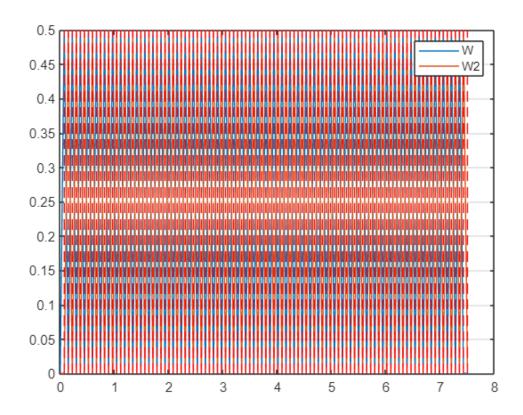


```
theta = 0.01: 0.1 :180 ;
zn = linspace(-L/2,L/2,length(I))';
Pattern = sind(theta).*sum( delta_l*I.*exp(1j*k*zn*cosd(theta)) ) ;

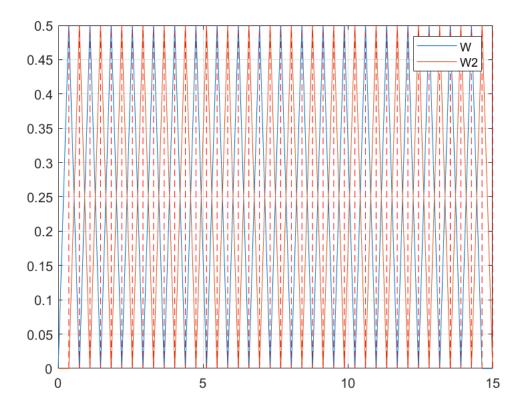
figure(3)
plot( abs(Pattern) , theta )
hold on
plot( -abs(Pattern) , theta )
title("Pattern of Antenna for \lambda = "+Lambda+" and L = "+L/Lambda+"*Lambda")
grid on
xlabel("Amplitude")
ylabel("Theta")
```



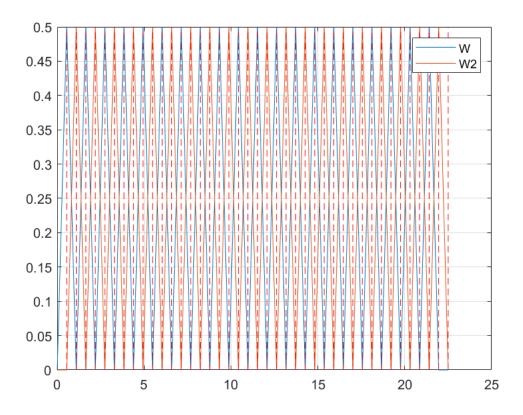
```
% For N = 100;
N =40;
L1 = Lambda/4;
A_L1 = Total_Worker(N,L1,a,f,c,0);
```



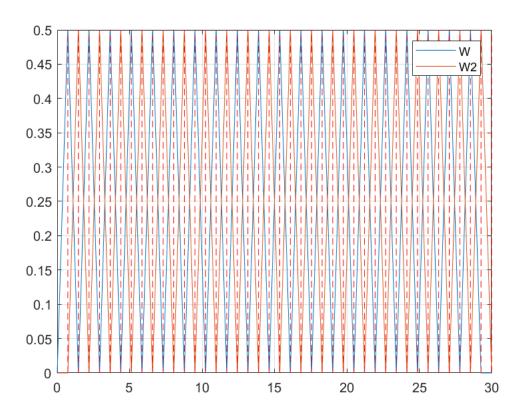
```
L2 = Lambda/2;
A_L2 = Total_Worker(N,L2,a,f,c,0);
```



```
L3 = 3*Lambda/4;
A_L3 = Total_Worker(N,L3,a,f,c,0);
```

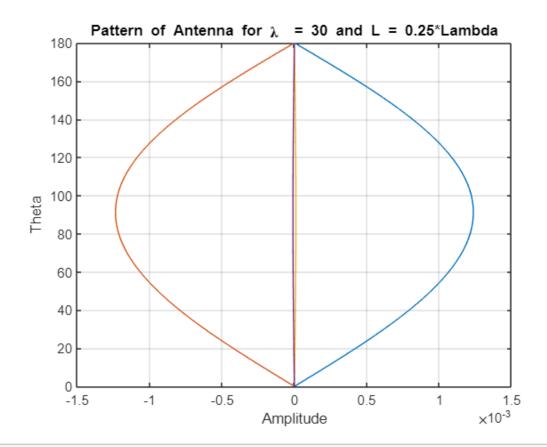


```
L4 = 1*Lambda;
A_L4 = Total_Worker(N,L4,a,f,c,0);
```



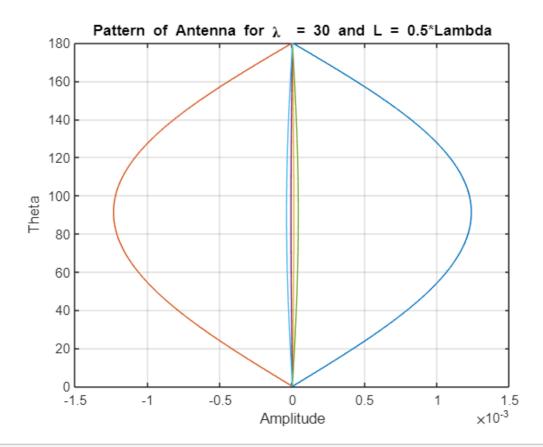
# L = Lambda/4 Pattern:

Pattern\_draw(A\_L1);



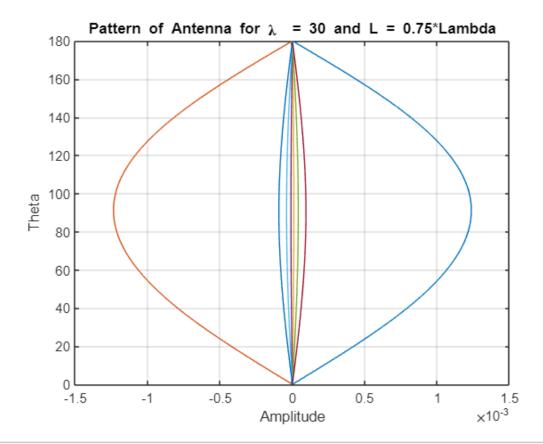
# L = Lambda/2 Pattern

Pattern\_draw(A\_L2);



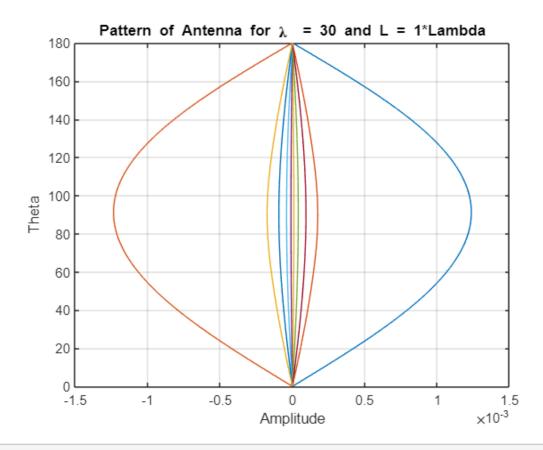
# L = 3\*Lambda/4 Pattern

Pattern\_draw(A\_L3);

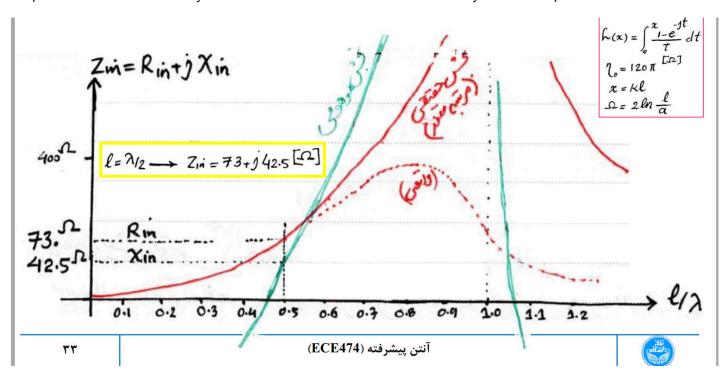


### L = Lambda Pattern:

Pattern\_draw(A\_L4);



**Part C:**Compare our results with analyticall solution obtained from classical analysis of the dipole antenna:



From Balanis we have also:

V

$$R_{in} = \left[\frac{I_0}{I_{in}}\right]^2 R_r \tag{4-77a}$$

where

 $R_{in}$  = radiation resistance at input (feed) terminals

 $R_r$  = radiation resistance at current maximum Eq. (4-70)

 $I_0 = \text{current maximum}$ 

 $I_{in} = \text{current at input terminals}$ 

where the radiation resistance of dipole antenna is considered:

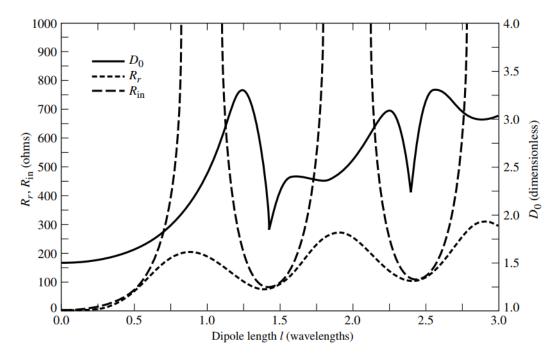
The radiation resistance can be obtained using (4-18) and (4-68) and can be written as

$$R_{r} = \frac{2P_{\text{rad}}}{|I_{0}|^{2}} = \frac{\eta}{2\pi} \{C + \ln(kl) - C_{i}(kl) + \frac{1}{2}\sin(kl) \times [S_{i}(2kl) - 2S_{i}(kl)] + \frac{1}{2}\cos(kl) \times [C + \ln(kl/2) + C_{i}(2kl) - 2C_{i}(kl)] \}$$

$$(4-70)$$

Shown in Figure 4.9 is a plot of  $R_r$  as a function of l (in wavelengths) when the antenna is radiating into free-space ( $\eta \simeq 120\pi$ ).

and for different values of  $\ell$ , in figure 4.9 of balanis book we get to see the variation of Radiation resistance and input resistance of the dipole antenna:



**Figure 4.9** Radiation resistance, input resistance and directivity of a thin dipole with sinusoidal current distribution.

and for the Imaginary part of the input impedance, we can use the below equation:

$$X_{m} = \frac{\eta}{4\pi} \left\{ 2S_{i}(kl) + \cos(kl)[2S_{i}(kl) - S_{i}(2kl)] - \sin(kl) \left[ 2C_{i}(kl) - C_{i}(2kl) - C_{i}\left(\frac{2ka^{2}}{l}\right) \right] \right\}$$
(4-70a)

in above equations, Ci(x) and Si(x) are used which are fresnel cosine and sine integrals defined as:

$$C_i(x) = -\int_{x}^{\infty} \frac{\cos y}{y} \, dy = \int_{\infty}^{x} \frac{\cos y}{y} \, dy \tag{4-68a}$$

$$S_i(x) = \int_0^x \frac{\sin y}{y} \, dy \tag{4-68b}$$

For I/Lambda from 0 to 3 we get the figures below to express the difference between input resistance and the radiation resistance:

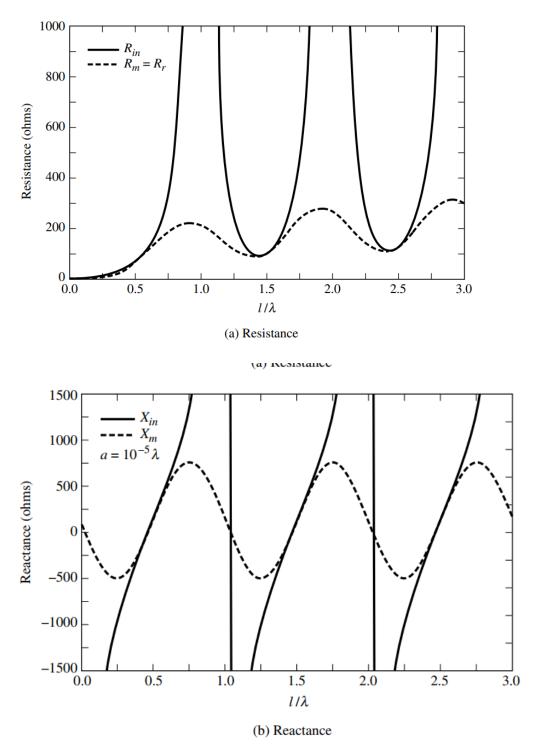


Figure 8.16 Self-resistance and self-reactance of dipole antenna with wire radius of  $10^{-5} \lambda$ .

```
Antenna_Length = [Lambda/4 , Lambda/2 , 3*Lambda/4 , Lambda ];

Z_in = zeros(length(Antenna_Length) , 1);

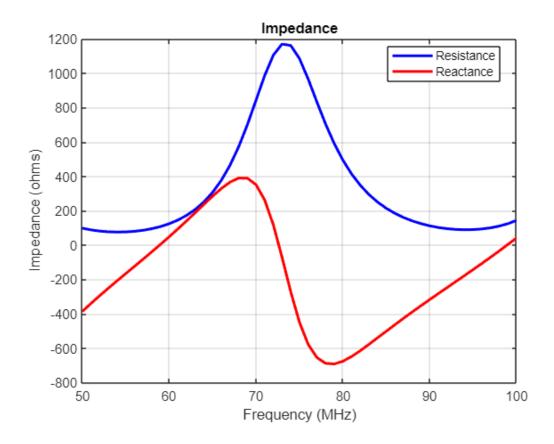
for i=1:length(Antenna_Length)
    Z_in(i) = Dipole_Antenna_exact_Z(Antenna_Length(i),a,Lambda);
    disp("For Dipole with length equal to: "+Antenna_Length(i) )
    disp(Z_in(i));

end

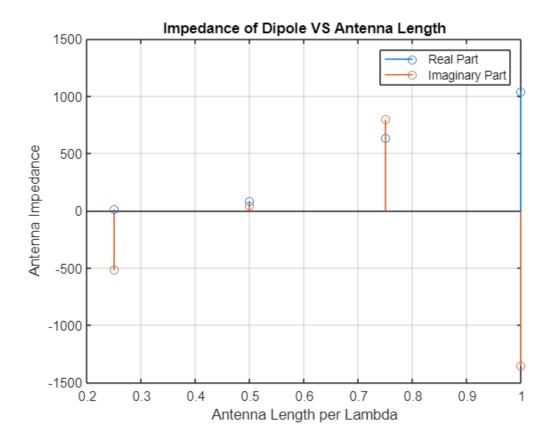
For Dipole with length equal to: 7.5
    33.7808 +64.3415i
For Dipole with length equal to: 15
    58.4540 +13.9715i
For Dipole with length equal to: 22.5
    2.3536e+02 + 9.3291e+01i
For Dipole with length equal to: 30
```

```
% Also we have from MATLAB Antenna ToolBox:
MATLAB_Antenna = dipole('Length',L,'Width',a);
figure(7)
impedance(MATLAB_Antenna,linspace(50e6,100e6,51));
```

1.5614e+08 + 2.6760e+33i



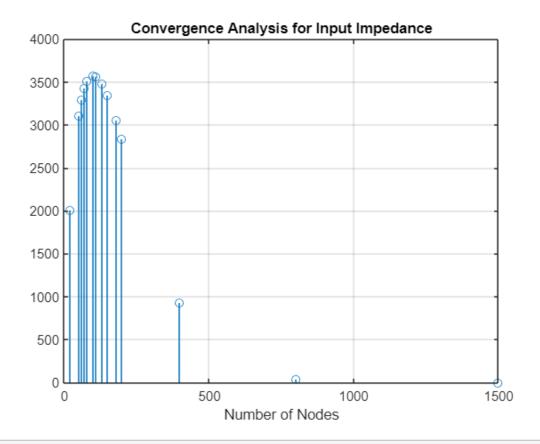
```
Z Matlab in = zeros(length(Antenna Length),1);
MATLAB_Antenna_mine = cell(length(Antenna_Length),1);
for i=1:length(Antenna Length)
    MATLAB_Antenna_mine{i} = dipole('Length', Antenna_Length(i), 'Width', 2*a');
    Z_Matlab_in(i) = impedance(MATLAB_Antenna_mine{i},f);
end
close 8
figure(8)
stem(Antenna_Length/Lambda , real(Z_Matlab_in));
hold on
stem(Antenna_Length/Lambda , imag(Z_Matlab_in));
legend("Real Part" ,"Imaginary Part" );
grid on
title("Impedance of Dipole VS Antenna Length");
xlabel("Antenna Length per Lambda");
ylabel("Antenna Impedance")
```



### **Convergence Test:**

```
L = Lambda/4;
N_{\text{vec}} = [20, 50, 60, 70, 80, 100, 110, 130, 150, 180, 200, 400, 800, 1500];
N = N_{vec}(1);
A1_N1 = Total_Worker(N,L,a,f,c,0);
N = N \text{ vec}(2);
A1_N2 = Total_Worker(N,L,a,f,c,0);
N = N \text{ vec}(3);
A1_N3 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(4);
A1_N4 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(5);
A1_N5 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(6);
A1_N6 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(7);
A1_N7 = Total_Worker(N,L,a,f,c,0);
```

```
N = N_{vec}(8);
A1_N8 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(9);
A1_N9 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(10);
A1_N10 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(11);
A1_N11 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(12);
A1_N12 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(13);
A1_N13 = Total_Worker(N,L,a,f,c,0);
N = N_{vec}(14);
A1_N14 = Total_Worker(N,L,a,f,c,0);
% N = N_{vec}(15);
% A1_N15 = Total_Worker(N,L,a,f,c,0);
A1_vec = { A1_N1 , A1_N2 , A1_N3 , A1_N4 ,A1_N5 ,A1_N6 , A1_N7 ,A1_N8 ,A1_N9 ,A1_N10 , A1_N11
Z_A1 = zeros(length(N_vec),1);
for i=1:length(N_vec)
    Temp = A1_vec{i};
    Z_A1(i) = Temp.Z_in;
end
figure()
stem( N_vec, abs(Z_A1))
title("Convergence Analysis for Input Impedance")
xlabel("Number of Nodes")
grid on
```



#### **Functions:**

```
Beta = delta l + delta;
Gamma = delta_l + delta;
if(m==n)% self Term:
                    M = 2*delta_1*asinh(delta_1/d) - 2*sqrt(delta_1^2+ d^2) + 2*d;
else
                    M = alpha*asinh(alpha/d) - Beta*asinh(Beta/d) - Gamma*asinh(Gamma/d) + delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta*asinh(delta
                               - sqrt(alpha^2 +d^2) + sqrt(Beta^2 +d^2) + sqrt(Gamma^2 +d^2) - sqrt(delta^2 +d^2
end
end
function delta = delta_calc(m,n,delta_1)
         delta = (abs(m-n)-1)*delta l;
end
function Z_in = Dipole_Antenna_exact_Z(1,a,Lambda)
         eta = 120*pi;
         k = 2*pi/Lambda;
         C = 0.5772;
         X = \frac{eta}{4*pi} * (2*fresnels(k*1) + cos(k*1)*(2*fresnels(k*1) - fresnels(2*k*1)) ...
                                          -\sin(k*1)*(2*fresnelc(k*1)-fresnelc(2*k*1)-fresnelc(2*k*a^2/1));
         Rr = eta/(2*pi) * ( ... 
                     C + \log(k*1) - fresnelc(k*1) \dots
                     + 1/2*sin(k*1)* (fresnels(2*k*1)-2*fresnels(k*1)) ...
                     + 1/2*cos(k*1)* (C + log(k*1/2) + fresnelc(2*k*1) - 2*fresnelc(k*1)) ...
                                                   ) ; % C = 0.5772 (Euler s constant)
         R_{in} = Rr/(1e-3+sin(k*1/2))^2;
         X \text{ in } = X/(\sin(k*1/2))^2;
         Z_{in} = R_{in} + 1j*X_{in};
end
function [W,W2] = W_calc(N,delta_l_index,l_vec)
W = zeros(1,length(l_vec));
for i=1 : N+1
         if(mod(i,2)==1)
                  W(1 + (i-1)*delta_l_index:i*delta_l_index) = 0.5*linspace(0,1,delta_l_index);
```

```
else
       W(1 + (i-1)*delta_l_index:(i)*delta_l_index) = -0.5*(linspace(1,2,delta_l_index))
   end
end
W2 = circshift([W(1:end-delta_l_index),zeros(1,delta_l_index)],delta_l_index);
if(mod(N,2)==0)
W(end-delta l index:end) = 0;
else
W2(end-delta_l_index:end) = 0;
end
function Pattern_draw(Object_Antenna)
L =Object_Antenna.L;
I = Object_Antenna.I;
Lambda = Object Antenna.Lambda;
delta_1 =Object_Antenna.delta_1;
k =Object Antenna.k;
theta = 0.01: 0.1 :180 ;
zn = linspace(-L/2,L/2,length(I))';
Pattern = sind(theta).*sum( delta_l*I.*exp(1j*k*zn*cosd(theta)) );
figure(3)
plot( abs(Pattern) , theta )
hold on
plot( -abs(Pattern) , theta )
title("Pattern of Antenna for \lambda = "+Lambda+" and L = "+L/Lambda+"*Lambda")
grid on
xlabel("Amplitude")
ylabel("Theta")
end
function Total_Object = Total_Worker(N,L,a,f,c,draw)
Lambda = c/f;
delta l = L/(N+1);
delta_l_index = 100; % Each Triangle Delta_l is equal to 100 indexes in l_vec
l_vec = linspace(0,L,(N+1)*delta_l_index);
Total_Object = struct();
```

```
Total_Object.Lambda = Lambda;
Total_Object.delta_l = delta_1;
Total_Object.delta_l_index = delta_l_index ;
Total_Object.l_vec = l_vec;
Total_Object.draw = draw;
Total_Object.N = N;
Total Object.L = L;
Total_Object.f = f;
[W,W2] = W_calc(N,delta_l_index,l_vec);
if(draw==1)
    figure()
    plot(l_vec,W);
    grid on
    hold on
    plot(1 vec , W2)
    for i=1:N+1
        plot( i*delta_l*ones(1,10) , linspace(0,max(W),10),'r--');
    end
    legend("W","W2")
end
Total_Object.W = W;
Total_Object.W2 = W2;
V = zeros(N+1,1);
E_i = zeros(1,(N+1)*delta_l_index) ;
mid_point = floor(length(E_i)/2); % Tahrik az vasat
E_i(mid_point) = 1 ;
for m = 1:N+1
    V(m) = G_m_calc(W , E_i , m ,l_vec , delta_l_index);
end
Total_Object.mid_point = mid_point;
Total_Object.E_i = E_i;
Total_Object.V = V;
M = zeros(N+1,N+1);
Z = M;
PSAI = M;
```

```
% PSAI_f = PSAI;
d = a;
k = 2*pi/Lambda; % wave number
w = 2*pi*f; % Rad/m
mu0 = 4*pi*1e-07; % H/m
eps0 = 8.85*1e-12; % F/m
Total_Object.eps0 = eps0;
Total Object.mu0 = mu0;
Total Object.w = w;
Total_Object.d = a;
Total_Object.k = k;
for m = 1:N+1
    for n=1:N+1
       M(m,n) = M_calc( m , n , delta_l , d );
       Reff = (delta_l*delta_l)/M(m,n);
       PSAI(m,n) = exp(-1j*k*Reff)/(4*pi*Reff);
       % PSAI_f(m,n) =
       if( (m==N+1) || (n==N+1) )
            Z(m,n) = 1j*w*mu0*delta_l*delta_l*PSAI(m,n) + ...
                (1/(1j*w*eps0)*(0+PSAI(m,n)-0-0));
       else
                     = 1j*w*mu0*delta_l*delta_l*PSAI(m,n) +...
                (1/(1j*w*eps0)*(PSAI(m+1,n+1)+PSAI(m,n)-PSAI(m+1,n)-PSAI(m,n+1));
       end
    end
end
Total_Object.M =M;
Total_Object.PSAI = PSAI;
I = inv(Z)*V;
Z in = V(floor(mid point/delta l index))/I(floor(mid point/delta l index));
% disp("Impedance for Antenna with L = "+L/Lambda+"*Lambda: ")
% disp((Z_in))
Total_Object.Z_in = Z_in;
Total Object.I = I;
```

end