Convex Optimization
Project 3

Spring 1401
Due date: 6th of Khordad

Electric vehicle charging. A group of N electric vehicles need to charge their batteries over the next T time periods. The charging energy for vehicle i in period t is given by $c_{t,i} \geq 0$, for $t = 1, \ldots, T$ and $i = 1, \ldots, N$. In each time period, the total charging energy over all vehicles cannot exceed $C^{\max}$, i.e., $\sum_{i=1}^{N} c_{t,i} \leq C^{\max}$ for $t = 1, \ldots, T$.

The state of charge for vehicle i in period t is denoted $q_{t,i} \geq 0$. The charging dynamics is

$$q_{t+1,i} = q_{t,i} + c_{t,i}, \quad t = 1, \ldots, T, \quad i = 1, \ldots, N.$$

Note that $q_{t,i}$ is defined for $t = T + 1$. The initial vehicle charges $q_{1,i}$ are given. The charging energy and state of charge are given in kWh (kilowatt-hours).

The vehicles have different preferences for how much charge they acquire over time. This is expressed by a target minimum charge level over time, given by $q_{t,i}^{\text{tar}} \in \mathbf{R}_{+}, t = 1, \ldots, T + 1$. These are nondecreasing, i.e., $q_{t+1,i}^{\text{tar}} \geq q_{t,i}^{\text{tar}}$ for $t = 1, \ldots, T, i = 1, \ldots, N$. The charging shortfall in period t for vehicle i is given by

$$s_{t,i} = \left(q_{t,i}^{\text{tar}} - q_{t,i}\right)_{+}, \quad t = 1, \ldots, T + 1, \quad i = 1, \ldots, N,$$

where $(a)_{+} = \max\{a, 0\}$. Our objective is to minimize the mean square shortfall, given by

$$S = \frac{1}{(T+1)N} \sum_{t=1}^{T+1} \sum_{i=1}^{N} s_{t,i}^{2}.$$

This is the same as minimizing the root-mean-square (RMS) shortfall, given by $\sqrt{S}$ (which has units of kWh). Explain how to solve the problem using convex optimization, and solve the following problem instance. We have $N = 4$ vehicles, $T = 90$ time periods, and $C^{\max} = 3$. The initial charges $q_{1,i}$ are $20, 0, 30$, and 25 , respectively. The target minimum charge profiles have the form

$$q_{t,i}^{\text{tar}} = \left(\frac{t}{T+1}\right)^{\gamma_i} q_i^{\text{des}}, \quad t = 1, \ldots, T + 1, \quad i = 1, \ldots, N,$$

with γ values $0.5, 0.3, 2.0, 0.6$ and q_i^{des} values $60, 100, 75, 125$. Note that q_i^{des} gives the final value of the target minimum charge level for vehicle i, and the parameter γ_i sets the 'urgency' of charging, with smaller values indicating more urgency, i.e., a target minimum charge value that rises more quickly. (With the charges all given in kWh, and the time period 5 minutes, these values are all realistic. The total charging period is 7.5 hours, and the maximum charging of 3kWh/ period corresponds to a real power of 36 kW. And no, you do not need to know or understand this to solve the problem.) Give the optimal RMS shortfall, i.e., the squareroot of the optimal objective value. Plot the target minimum charge values and optimal state of charge for each vehicle, with dashed lines showing the target and solid lines showing the optimal charge. Plot the optimal charging energies $c_{t,i}$ over time in a stack plot.

Constant charging. Compare the optimal charging above to a very simple charging policy: Charge each vehicle at a constant energy per period, proportional to $q_i^{\text{des}} - q_{1,i}$, i.e.,

$$c_{t,i} = \theta_i C^{\max}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T$$

with

$$\theta_i = \frac{q_i^{\text{des}} - q_{1,i}}{\sum_{j=1}^{N}\left(q_j^{\text{des}} - q_{1,j}\right)}, \quad i = 1, \ldots, N$$

Give the associated RMS shortfall, and the same plots as above.

Plotting hints. In Python, a basic stack plot is obtained with

```
import matplotlib.pyplot as plt
plt.stackplot(rr, y.T)
```

where rr is a range object (like range (a, b)) with len (list (rr)) == n and y is an $n \times N$ NumPy array. In Julia, a basic stack plot is obtained with

```
using Plots
areaplot(rr, y)
```

where rr is a range object (like a : b) and y is an $n \times N$ Matrix { Float64 } object, with $n = b - a + 1$. For those using Julia, you'll be better off using the solver ECOS, and not SCS or OSQP.

Good Luck!