Antenna Array Processing

HW7

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```
clear; clc; close all;
```

یک رادار پالسی با اطلاعات زیر را در نظر بگیرید:

- ✓ فرستنده ی رادار شامل یک آنتن است که در فرکانس $f_c = 150\ MHz$ کار می کند. عرض پالس برابر = $10\ \mu sec$ و $PRI = 0.1\ msec$ است.
- ✓ گیرنده ی رادار یک آرایه ی یکنواخت عمودی شامل M=10 المان است. فاصله ی آنتن اول از زمین برابر یک متر و فاصله ی سایر آنتن ها از یکدیگر نیز برابر یک متر می باشد.
- ✓ فرستنده و گیرنده در یک مکان قرار دارند.
- ✓ زمان ضبط سیگنال ($T_{recording}$) در گیرنده برابر 1 میلی ثانیه و نرخ نمونه برداری برابر $f_s = 1\ MHz$ می باشد.

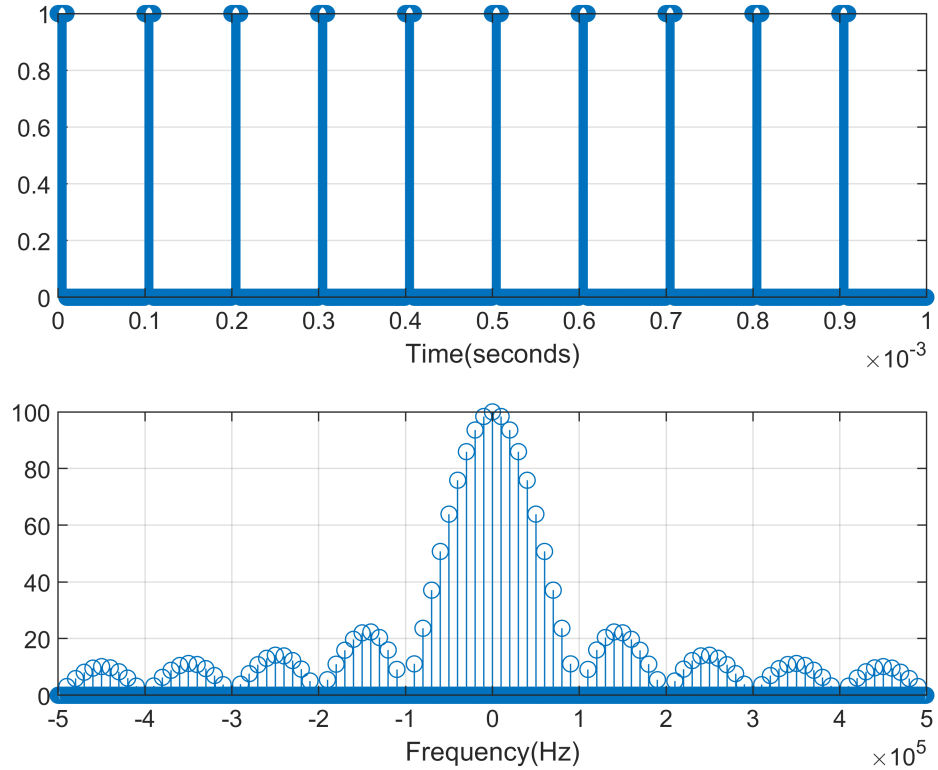
```
fs = 1e6; % 1MHz
ts = 1/fs;

Trecord = 1e-3; % 1ms
delta_f = 1/Trecord;


t = 0 : ts : Trecord-ts;
freq = -fs/2 : delta_f : fs/2-delta_f;

fc = 150e6; % 150MHz

c = 3e8;
Lambda = c/fc;
k = 2*pi*fc/c ; % 2pi/lambda = 2pif/c

Tau  = 10e-6; % 10us
Tau_num = round(Tau/ts);
```

```
PRI  = 0.1e-3; % 0.1ms
PRF = 1/PRI;
PRI_num = round(PRI/ts);
pulse_num = round(Trecord/PRI); % Kolle Pulse ma chanta PRI hast!
sample_num = round(Trecord/ts); % Kolle Pulse ma chanta nemoone hast

% Uniform Array
M = 10;
h = 1; % height of first element from Ground
d = 1; % distance of each element from the next
D = h + d*(0:M-1);
```

Plot of Low-Pass Signal:

The same as previous HWs:

```
SL_PRI=[ones(1,Tau_num) zeros(1,PRI_num-Tau_num)];
sl=repmat(SL_PRI,1,pulse_num); % Repeat The Baseband SIgnal for each PRI in total Pulse Length

figure(1)
subplot(2,1,1)
stem(t,sl)
xlabel('Time(seconds)')
grid on
subplot(2,1,2)
slf=fftshift(fft(sl));
stem(freq,abs(slf));
xlabel('Frequency(Hz)');
grid on
```



فرض کنید دو هدف متحرک با داپلرهای $f_{d_1} = 2\ KHz$ و $f_{d_2} = 1\ KHz$ و فاصله های $R_1 = R_2 = 6\ Km$ در زوایای ارتفاعی $\theta_1 = 10$ و $\theta_2 = 20$ درجه قرار دارند.

سیگنال باند پایه دریافتی از این دو هدف در آنتن mم به صورت زیر است:

$$y_m = e^{jkd_m \sin(\theta_1)}\, s_l\left(t - \frac{2R_1}{C}\right) e^{j2\pi f_{d_1} t} + e^{jkd_m \sin(\theta_2)}\, s_l\left(t - \frac{2R_2}{C}\right) e^{j2\pi f_{d_2} t} + noise_m$$

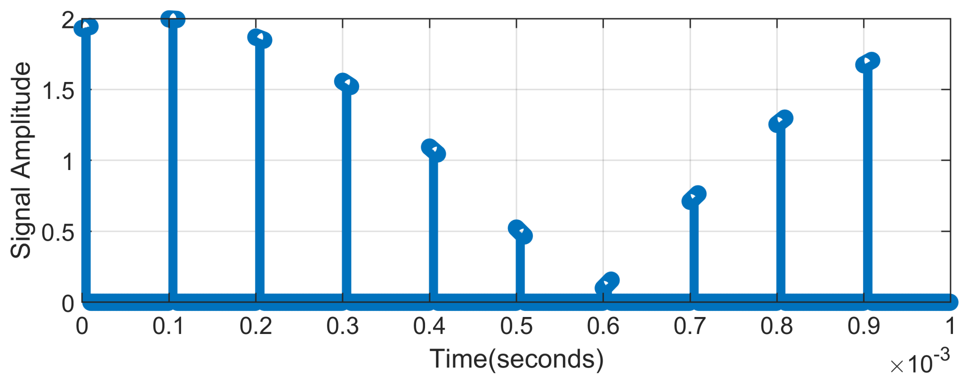
این سیگنال را در هر آنتن تولید کنید. نویز را گوسی (مختلط) و مستقل از منابع با میانگین صفر و واریانس 1 در نظر بگیرید. دامنه ی $s_l(t)$ را هم برابر یک در نظر بگیرید.

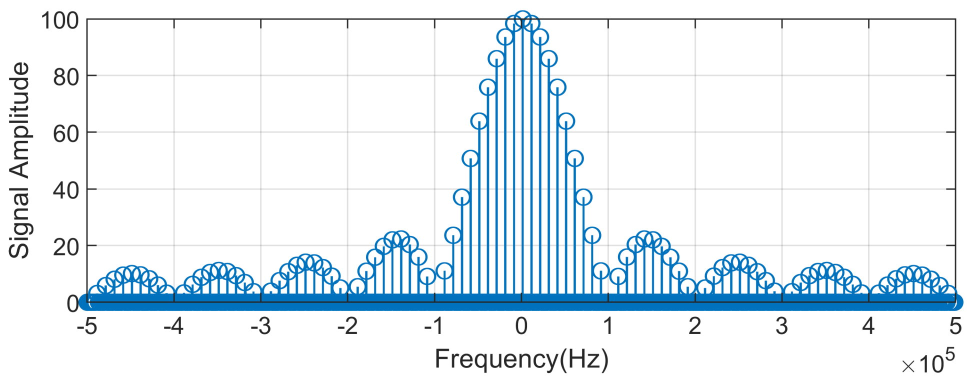
```
% Target 1:
theta_1= 10*pi/180;
fd1 = 2e3; % 2KHz
R1 = 6e3; % 6Km


% Target 2:
theta_2  = 20*pi/180;
```

```
fd2 = 1e3; % 1KHz
R2 = 6e3; % 6Km



% Noise Generation:  Independent from Signals! -- Gaussian --- Zero Mean:
Noise = randn(M, length(t)) + 1j*randn(M, length(t));  % M*T Noise Matrix for M elements of Ant
Noise = Noise/sqrt(2);


Y =  exp(1j*k*D'*sin(theta_1) ) .* circshift(sl , floor(2*R1/c) ) .* exp(1j*2*pi*fd1*t)  + ...
     exp(1j*k*D'*sin(theta_2) ) .* circshift(sl , floor(2*R2/c) ) .* exp(1j*2*pi*fd2*t);


Y_noisy = Y + Noise;
```

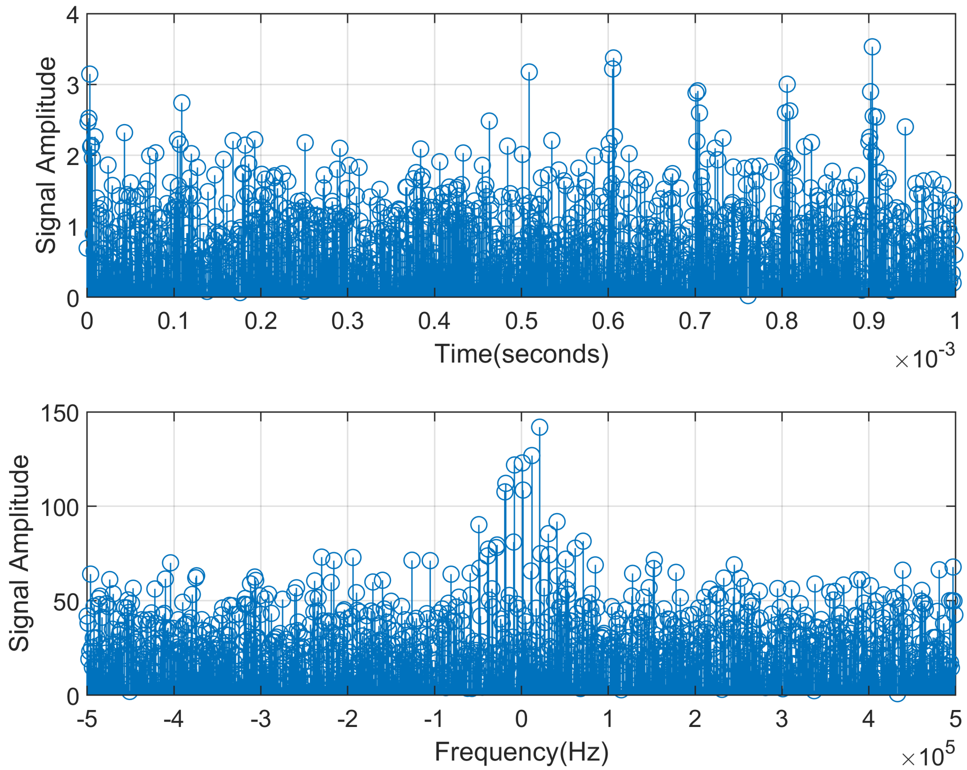
Plotting the Received Signal At Terminals:

```
figure(2)
y_in=Y(randi(10,1), :); % Choose a random Terminal to plot its received Signal
y_in_f=fftshift(fft(y_in));


subplot(2,1,1)
stem(t,abs(y_in))
xlabel('Time(seconds)')
ylabel("Signal Amplitude")
grid on

subplot(2,1,2)
stem(freq,abs(y_in_f))
xlabel('Frequency(Hz)')
ylabel("Signal Amplitude")
grid on
```






```
figure(3)
y_in_noisy=Y_noisy(randi(10,1), :); % Choose a random Terminal to plot its received Signal
y_in_noisy_f=fftshift(fft(y_in_noisy));


subplot(2,1,1)
stem(t,abs(y_in_noisy))
xlabel('Time(seconds)')
ylabel("Signal Amplitude")
grid on

subplot(2,1,2)
stem(freq,abs(y_in_noisy_f))
xlabel('Frequency(Hz)')
ylabel("Signal Amplitude")
grid on
```



BeamForming:

The Projection of Steering Vectors on Signal Space! --> Maximize this Likelihood and the answer return the corresponding angle where the target is located.

```
[U, S, V] = svd(Y_noisy); % SVD Decomposition!
disp(S(:,1:20)) % M*T Matrix! --> Diagnal Elements have Values
```

```
  Columns 1 through 9

   47.6796         0         0         0         0         0         0         0         0
         0   42.3816         0         0         0         0         0         0         0
         0         0   33.4834         0         0         0         0         0         0
         0         0         0   32.8863         0         0         0         0         0
         0         0         0         0   31.9854         0         0         0         0
         0         0         0         0         0   31.0141         0         0         0
         0         0         0         0         0         0   30.5152         0         0
         0         0         0         0         0         0         0   30.3211         0
         0         0         0         0         0         0         0         0   29.6662
         0         0         0         0         0         0         0         0         0

  Columns 10 through 18
```

```
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
   28.9354         0         0         0         0         0         0         0         0

  Columns 19 through 20

         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
```

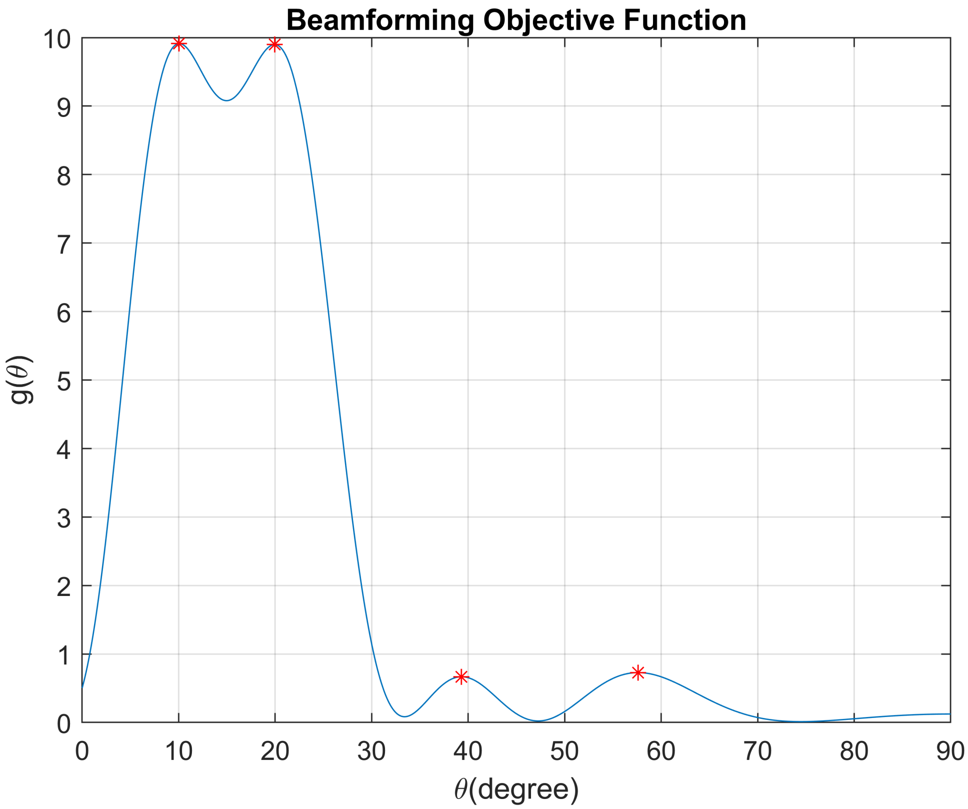
It is obvious that only the first 2 values originate from existance of targets and the rest are the result of having Noise in our Terminals! < K=2; also it is obvious that Noise levels are kind of the same in power/Energy represented on diagnal elements of S! >

```
K = 2;  % Number of Targets   ----  Known but chosen wisely from SVD result

Usig = U(:, 1:K);  % M*K
Unull = U(:, K+1:end); % M* (M-K)

Vsig = V(:, 1:K); % T * K
Vnull = V(:, K+1:end); % T * (M-K)
```

Beamforming --- >>>> maximize this objective function to find Direction of Arrival!

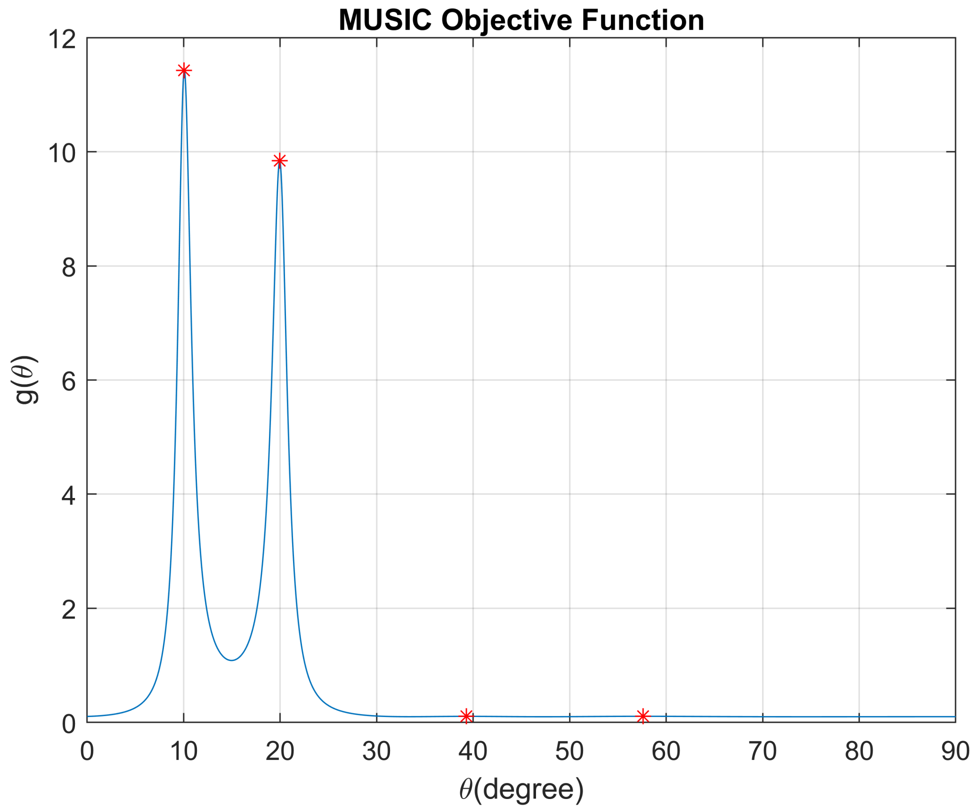
$g(\theta) = |a(\theta)' * U_{\text{sig}}|_2^2$

```
theta = 0 : 0.5e-1 : 90;
a = exp(1j*k*D'*sind(theta)); % Steering Vector
g = sum(abs(a'*Usig).^2, 2); % g(theta)

figure(4)
plot(theta, g)
xlabel('\theta(degree)')
ylabel('g(\theta)')
grid on
title("Beamforming Objective Function")
[Beamforming_peaks , Beamformingidx ] = findpeaks(g);
hold on
plot(theta(Beamformingidx),Beamforming_peaks,"r*")
```



MUSIC:

```
theta = 0 : 0.5e-1 : 90;
a = exp(1j*k*D'*sind(theta)); % Steering Vector
f = 1./sum(  abs(a'*Unull).^2 ,2 ); % f(theta)

figure(5)
close all;
plot(theta, f)
xlabel('\theta(degree)')
ylabel('g(\theta)')
grid on
title("MUSIC Objective Function")
[Music_peaks , Music_idx ] = findpeaks(f);
hold on
plot(theta(Music_idx),Music_peaks,"r*")
```



It is clear, that the peaks in MUSIC, are sharper than those in Beamforming Method due to Dimension Dominance of the Unull over the Usig!

ج) از روی مشاهدات و با فرض معلوم بودن رنج اهداف (R=6 km)، با استفاده از روش beamforming داپلر اهداف را تخمین بزنید.

د) از روی مشاهدات و با فرض معلوم بودن رنج اهداف (R=6 km)، با استفاده از روش MUSIC داپلر اهداف را تخمین بزنید.

```
% To find Doppler of a target, we must perform our operation over S
% obtained in SVD decomposition and use projection over Vsig , Vnull to
% decide Range+Doppler for a target!
```

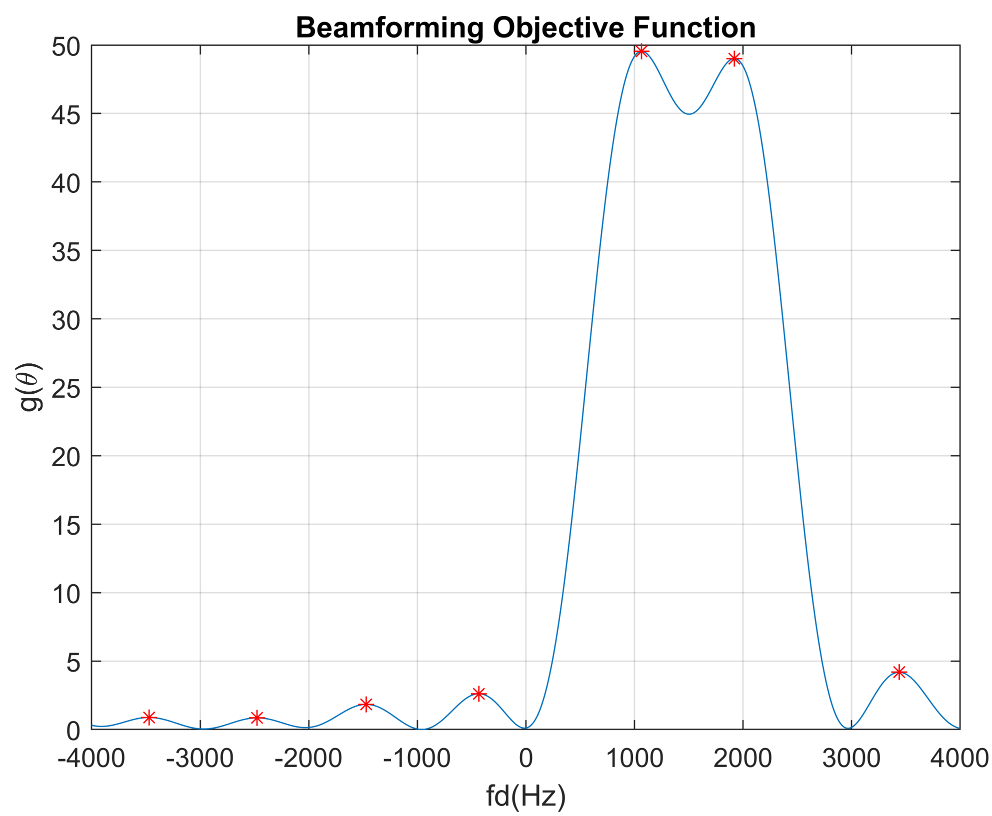
Maximize Below objective function

$g(\theta) = |s(R, f_d)' * V_{\text{sig}}|_2^2$

gives Beamforming result for the Doppler! of a target

```
fd = (-4e3 : 1 : 4e3)'; % The Value to be found is the doppler of the targets!
s = circshift(sl , floor(2*R1/c) ).* exp(1j*2*pi*fd*t); % Signal Space with given Range
```

```
g = sum(  abs(s*Vsig).^2 ,2 ); % g(fd)

figure(6)
close all;
plot(fd , g)
xlabel('fd(Hz)')
ylabel('g(\theta)')
grid on
title("Beamforming Objective Function")
[Beamforming_peaks , Beamformingidx ] = findpeaks(g);
hold on
plot(fd(Beamformingidx),Beamforming_peaks,"r*")
```



Fd is obtained correctly -->>> 1KHz and 2KHz!

```
fd = (-4e3 : 1 : 4e3)'; % The Value to be found is the doppler of the targets!
s = circshift(sl , floor(2*R1/c) ).* exp(1j*2*pi*fd*t); % Signal Space with given Range
f = 1./sum(  abs(s*Vnull).^2 ,2 ); % f(fd)

figure(6)
close all;
plot(fd , f)
xlabel('fd(Hz)')
```

```
ylabel('f(fd)')
grid on
title("MUSIC Objective Function")
[Music_peaks , Music_idx ] = findpeaks(f);
hold on
plot(fd(Music_idx),Music_peaks,"r*")
```