Boundary Value Problems!

Shooting Method

تمرین ۴. معادله‌ی دیفرانسیل توزیع دما در یک پره به صورت تابعی از طول پره، در شرایط پایا، به‌دست آمده است. معادله‌ی دیفرانسیل و شرایط مرزی پس از بی‌بعدسازی و مرتب کردن آن، به‌صورت زیر حاصل شده است:

$$\frac{d^2\theta}{dz^2} - \varphi^2\theta^{\frac{4}{3}} = 0, \quad \varphi^2 = m^2w^2(T_s - T_a)^{\frac{1}{3}}$$

$$\begin{cases} z=0 \ : \ \theta=1 \\ z=1 \ : \ \dfrac{d\theta}{dz} = -\psi^2\theta^{\frac{4}{3}}, \quad \psi^2 = \dfrac{w\alpha}{k}(T_s - T_a)^{\frac{1}{3}} \end{cases}$$

الف- این معادله را با برنامه‌نویسی در محیط نرم‌افزار MATLAB به روش پرتابی حل کنید؛

ب- با کمک روش اختلاف‌های محدود در محیط MATLAB، مسئله را حل کرده و نتایج به دست آمده را با نتایج حاصل از بند الف مقایسه کنید.

- $$y''(x,t) = f(x, y(x,t), y'(x,t)) \qquad \begin{cases} y(a,t) = \alpha & boundary\ condition \\ y'(a,t) = t & guess \end{cases} \qquad (I)$$

- با انتخاب متغیر Z به صورت: $Z = \frac{dy}{dt}(x,t)$ و قرار دادن آن در معادله بالا:

$$\begin{cases} Z'' = \dfrac{\partial f}{\partial y}Z + \dfrac{\partial f}{\partial y'}Z' \\ Z(a,t) = 0 \\ Z'(a,t) = 1 \end{cases} \qquad (II)$$

- **نکته:**
- ۱- اگر حدس مسئله مشتق تابع در نقطه x=a باشد ($y'(a)=t$) در نتیجه:

$$\begin{cases} Z(a,t)=0 \\ Z'(a,t)=1 \end{cases}$$

- ۴- اگر لازم باشد نتایج حاصل با $y'(b)=\beta'$ مقایسه شود، معادله انترپولاسیون:

$$t_{i+1}=t_i-\frac{y'(b,t_i)-\beta'}{Z'(b,t_i)}$$

We need to solve this Problem:

theta -- >> y ::::: Change of Variable Names for simplicity!

z -- >> x

$$y''=m^2w^2(T_s-T_a)^{\frac{1}{3}}.y^{\frac{4}{3}} \;;$$

for $y(a)=1$; $y'(1)=w\frac{\alpha}{k}(T_s-T_a)^{\frac{1}{3}}.y^{\frac{4}{3}}$;

$$y''=f(x,y)=m^2w^2(T_s-T_a)^{\frac{1}{3}}\;y^{\frac{4}{3}};\;\Longrightarrow\;\text{if}\;\;y'=w\,;\;w'=f\;--\rightarrow$$

$$\begin{cases} y'=W & ;\;\; y(0)=1; \\ W'=m^2w^2(T_s-T_a)^{\frac{1}{3}}y^{\frac{4}{3}} & ;\;\; y'(1,t1)=t1\;\;--\text{guess} \end{cases} ; \quad \begin{cases} Z'=u; & :\;\; Z(0,t1)=0\,; \\ u'=\frac{4}{3}m^2w^2(T_s-T_a)^{\frac{1}{3}}y^{\frac{1}{3}}Z; & ;\;\; Z'(1,t1)=1; \end{cases}$$

To make it simpler in form we can use:

$y--\rightarrow$ y1
$W--\rightarrow$y2
$Z--\rightarrow$ y3
$u--\rightarrow$ y4

and then we have:

$$y1' = y2;$$

$$y2' = \text{MULTING} * y1^{\frac{4}{3}};$$

$$y3' = y4;$$

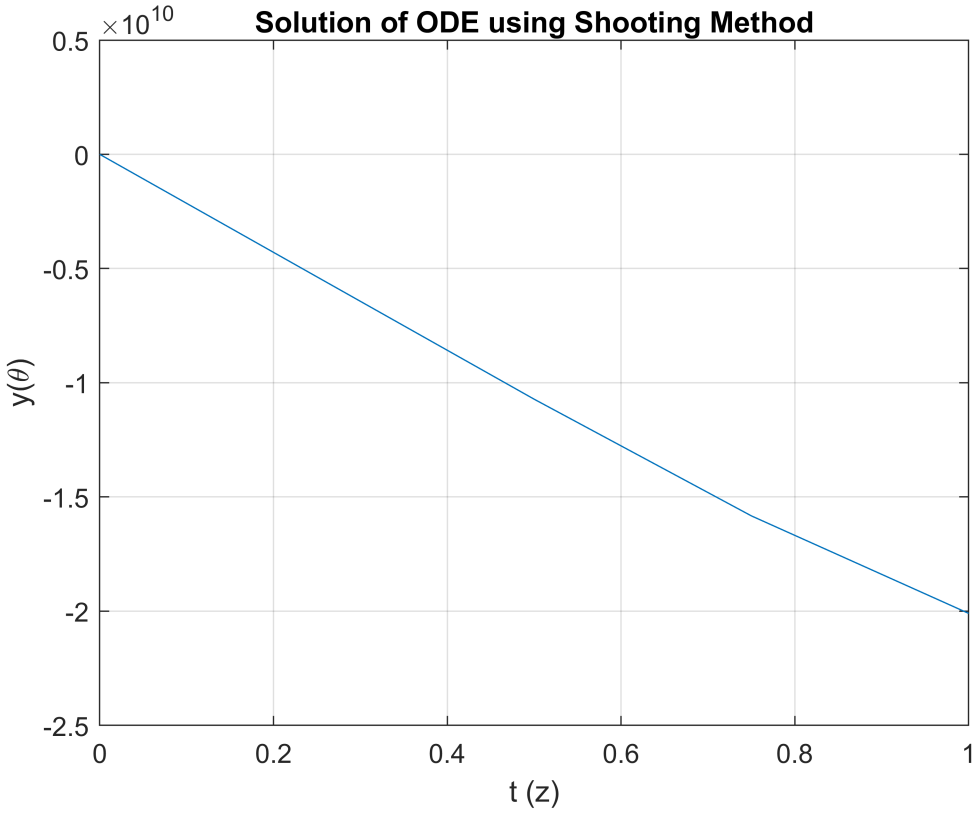
$$y4' = \frac{4}{3} * \text{MULTING} * y1^{\frac{1}{3}} * y3$$

```
clear; clc; close all;

T = zeros(20,1);

m = 1;   w = 2; Ts  =210; Ta = 180; alpha = 0.2; k = 1;

global MULTING

MULTING = m^2 * w^2 * (Ts - Ta)^(1/3);

a = 0;
b =1;


T(1) = b*randi(1);  % Guess
tspan = [0.1  1];

opts = odeset('RelTol',1e-2,'AbsTol',1e-4);


for i=1:length(T)
    ic = [1 T(i) 1 1];
    [t,Y] = ode45(@(t,y) ODE_W(t,y), tspan, ic, opts);
 %%% Update t
     y_prime_sol = Y(end,2);
     Z_prime_sol = Y(end,4);
     beta_guess  = Y(end,1);
     if (i>2) % we are comparing y'(b) = Beta' --> so we choose the interpolation formula of:
        T(i+1) = T(i) - (y_prime_sol-beta_guess )/Z_prime_sol; % Interpolation
     else
        T(i+1) = randi(1); % Second Guess
     end

end
```

```
Warning: Failure at t=6.468315e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (1.776357e-15) at time t.
```

```
Warning: Failure at t=6.468315e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (1.776357e-15) at time t.
Warning: Failure at t=6.468315e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (1.776357e-15) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=8.075849e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (1.776357e-15) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
Warning: Failure at t=1.000000e-01.  Unable to meet integration tolerances without reducing the step size
below the smallest value allowed (2.220446e-16) at time t.
```

```
t = linspace(0,1,length(Y(:,1)));
figure(1)
plot(t,Y(:,1));
grid on
ylabel("y(\theta)")
xlabel("t (z)")
title("Solution of ODE using Shooting Method")
```




```
function  dy = ODE_W(t,y)
dy = zeros(4,1);

global MULTING
dy(1)  =  y(2);
dy(2)  = MULTING*(y(1).^4/3);
dy(3)  =  y(4);
dy(4)  =  4/3*MULTING*(y(1).^1/3)*y(3) ;

end
```