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Convex Optimization

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Hw8 - Q4:

```
clear;
clc;
close all;
% Load Data:
run('max_alg_conn_data.m');
```

maximizing algebraic connectivity of a graph

```
Q = null(ones(1,n)); % columns of Q are orthonormal basis for N(1)
cvx_begin
    variable w(m)

        L = A*diag(w)*A';
        maximize (lambda_min(Q'*L*Q))

        subject to
            w >= 0;
            F*w <= g;
cvx_end
```

```
Calling SDPT3 4.0: 1426 variables, 201 equality constraints
   For improved efficiency, SDPT3 is solving the dual problem.
------------------------------------------------------------

 num. of constraints = 201
 dim. of sdp    var  = 49,   num. of sdp  blk  =  1
 dim. of linear var  = 201
*******************************************************************
   SDPT3: Infeasible path-following algorithms
*******************************************************************
 version  predcorr  gam  expon  scale_data
   HKM      1      0.000   1        0
it pstep dstep pinfeas dinfeas  gap      prim-obj      dual-obj    cputime
-------------------------------------------------------------------
 0|0.000|0.000|4.6e+02|1.1e+02|8.9e+04| 2.835489e+01  0.000000e+00| 0:0:00| chol  1  1
 1|0.990|0.990|4.7e+00|1.2e+00|1.0e+03| 2.832365e+01 -9.819784e+00| 0:0:00| chol  1  1
 2|1.000|1.000|9.8e-06|1.0e-02|4.1e+01| 2.620049e+01 -7.860970e+00| 0:0:00| chol  1  1
 3|1.000|0.986|2.0e-04|1.1e-03|1.1e+00| 9.698185e-01 -8.653272e-02| 0:0:01| chol  1  1
 4|0.952|0.722|9.8e-06|3.9e-04|9.3e-02| 9.106602e-02 -9.813737e-04| 0:0:01| chol  1  1
 5|0.446|1.000|5.4e-06|1.2e-05|8.2e-02| 5.999893e-02 -2.166734e-02| 0:0:01| chol  1  1
 6|0.923|0.913|4.2e-07|3.0e-06|1.4e-02| 1.622326e-02  1.931004e-03| 0:0:01| chol  1  1
 7|0.727|0.911|1.1e-07|4.5e-07|9.4e-03| 1.314581e-02  3.788538e-03| 0:0:01| chol  1  1
 8|0.963|1.000|4.2e-09|3.3e-08|4.5e-03| 8.641636e-03  4.154362e-03| 0:0:01| chol  1  1
 9|0.955|0.959|1.9e-10|3.2e-09|1.3e-03| 5.962881e-03  4.632957e-03| 0:0:01| chol  1  1
10|0.808|1.000|3.7e-11|1.4e-10|8.1e-04| 5.601064e-03  4.788713e-03| 0:0:01| chol  1  1
```

```
11|1.000|1.000|4.8e-15|1.7e-11|2.5e-04| 5.182322e-03  4.928013e-03| 0:0:01| chol  1  1
12|1.000|1.000|4.9e-15|2.0e-12|1.1e-04| 5.087047e-03  4.974593e-03| 0:0:01| chol  1  1
13|1.000|1.000|1.5e-14|1.1e-12|3.6e-05| 5.039001e-03  5.003404e-03| 0:0:01| chol  1  1
14|1.000|1.000|5.1e-14|1.0e-12|9.8e-06| 5.023628e-03  5.013864e-03| 0:0:01| chol  1  1
15|1.000|1.000|2.1e-13|1.0e-12|2.2e-06| 5.019212e-03  5.017043e-03| 0:0:01| chol  1  1
16|1.000|1.000|2.5e-13|1.0e-12|3.5e-07| 5.018163e-03  5.017811e-03| 0:0:01| chol  1  1
17|1.000|1.000|9.6e-13|1.0e-12|2.8e-08| 5.017976e-03  5.017949e-03| 0:0:01| chol  2  2
18|1.000|1.000|1.7e-13|1.0e-12|5.5e-10| 5.017961e-03  5.017960e-03| 0:0:01|
  stop: max(relative gap, infeasibilities) < 1.49e-08
-------------------------------------------------------------------
 number of iterations   = 18
 primal objective value =  5.01796059e-03
 dual   objective value =  5.01796005e-03
 gap := trace(XZ)       = 5.47e-10
 relative gap           = 5.41e-10
 actual relative gap    = 5.41e-10
 rel. primal infeas (scaled problem)   = 1.75e-13
 rel. dual     "        "       "      = 1.00e-12
 rel. primal infeas (unscaled problem) = 0.00e+00
 rel. dual     "        "       "      = 0.00e+00
 norm(X), norm(y), norm(Z) = 7.4e-01, 1.3e-01, 3.8e-01
 norm(A), norm(b), norm(C) = 3.6e+01, 2.0e+00, 2.0e+00
 Total CPU time (secs)  = 1.24
 CPU time per iteration = 0.07
 termination code       =  0
 DIMACS: 1.7e-13  0.0e+00  1.0e-12  0.0e+00  5.4e-10  5.4e-10
-------------------------------------------------------------------

------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +0.00501796
```

```
% Now Omitting those with weak Connections:
w(abs(w) < 1e-4) = 0;
```

compare algebraic connectivities:

```
L_unif = (1/m)*A*A';
dunif = eig(L_unif); % eigen value and eigen vectors using eig(A)
dopt = eig(L);
fprintf(1, 'Algebraic connectivity of L_unif: %f\n', dunif(2));
```

```
Algebraic connectivity of L_unif: 0.002204
```

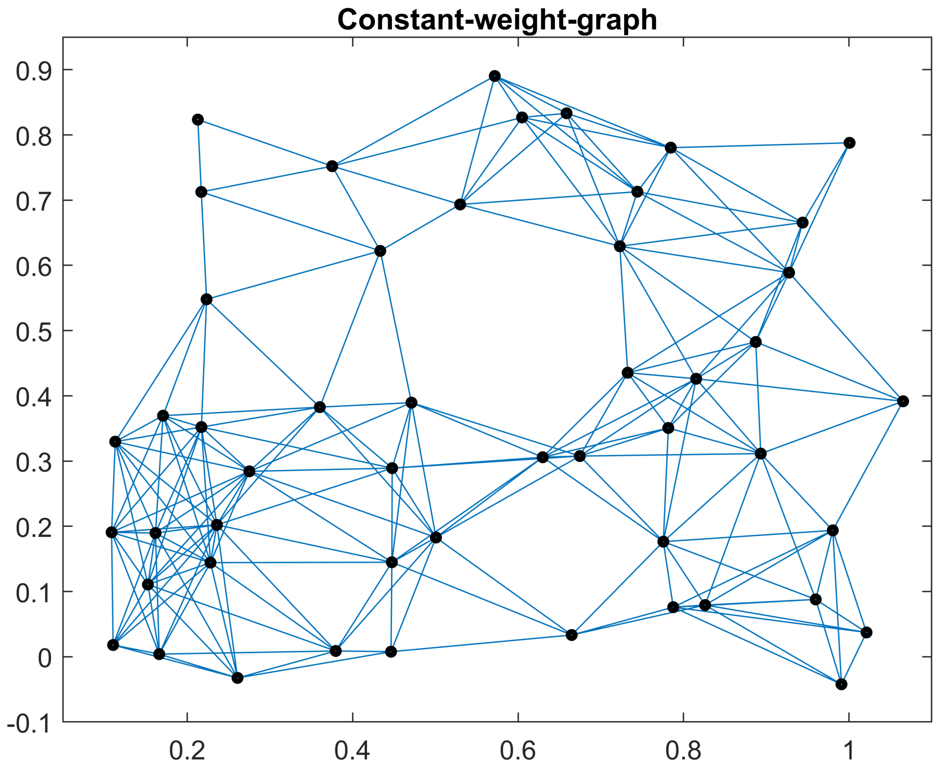
```
fprintf(1, 'Algebraic connectivity of L_opt: %f\n', dopt(2));
```

```
Algebraic connectivity of L_opt: 0.005018
```

plot topology of constant-weight graph

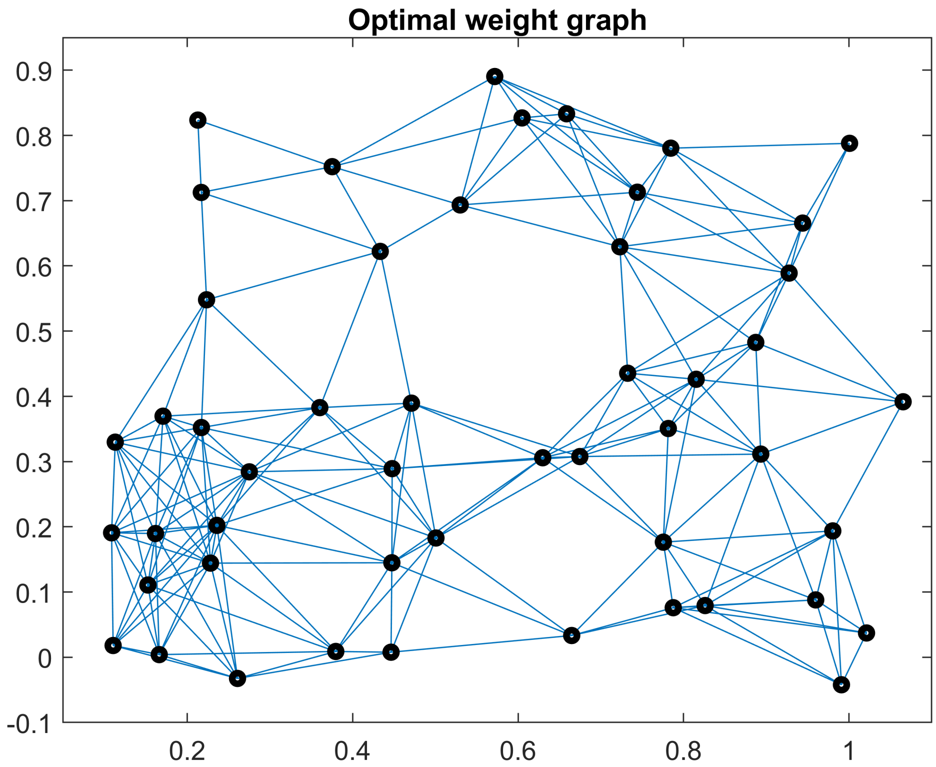
```

Fig1 = figure(); clf;
gplot(L_unif,xy); % edges
hold on;
plot(xy(:,1), xy(:,2), 'ko','LineWidth',2, 'MarkerSize',2.5); % Points [Nodes]
axis([0.05 1.1 -0.1 0.95]);
```

```
title('Constant-weight-graph')
```



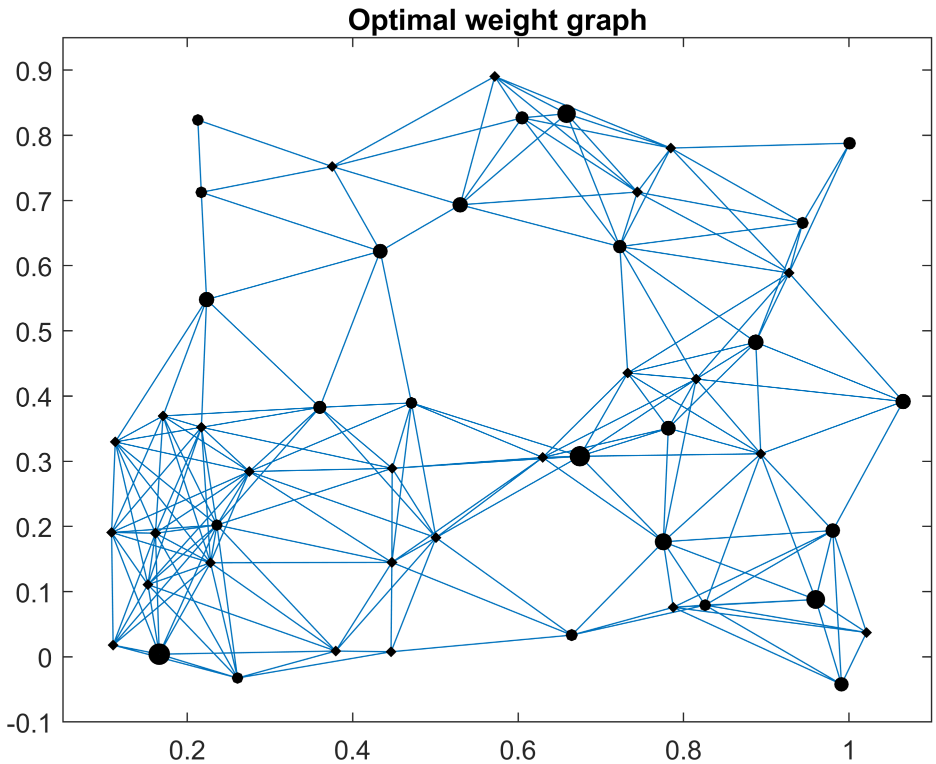
plot topology of optimal weight graph

```
Fig2 = figure(2); clf;
gplot(L,xy);
hold on;
plot(xy(:,1), xy(:,2), 'ko','LineWidth',2.5, 'MarkerSize',4);
axis([0.05 1.1 -0.1 0.95]);
title('Optimal weight graph')
```



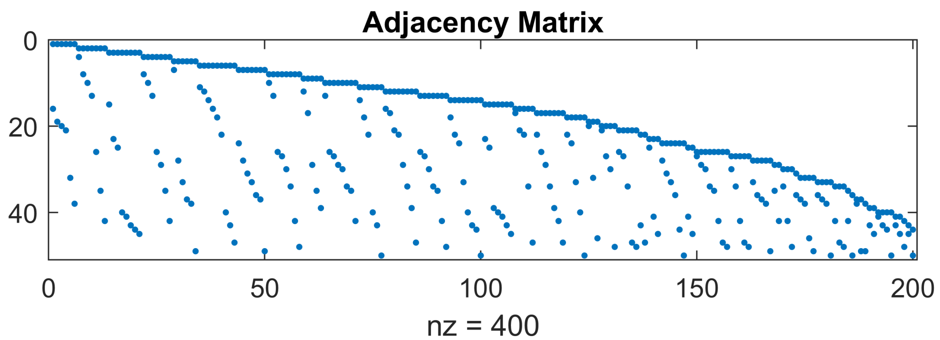
plot optimal weight graph with edge thickness proportional to weight

```
Fig5 = figure(); clf;
gplot(L,xy);
hold on;
for i=1:length(L)
plot(xy(i,1), xy(i,2), 'ko','LineWidth',4, 'MarkerSize',abs(w(i)*4/max(w))+0.1);
end
axis([0.05 1.1 -0.1 0.95]);
title('Optimal weight graph')
```



Sparsity Pattern of A:

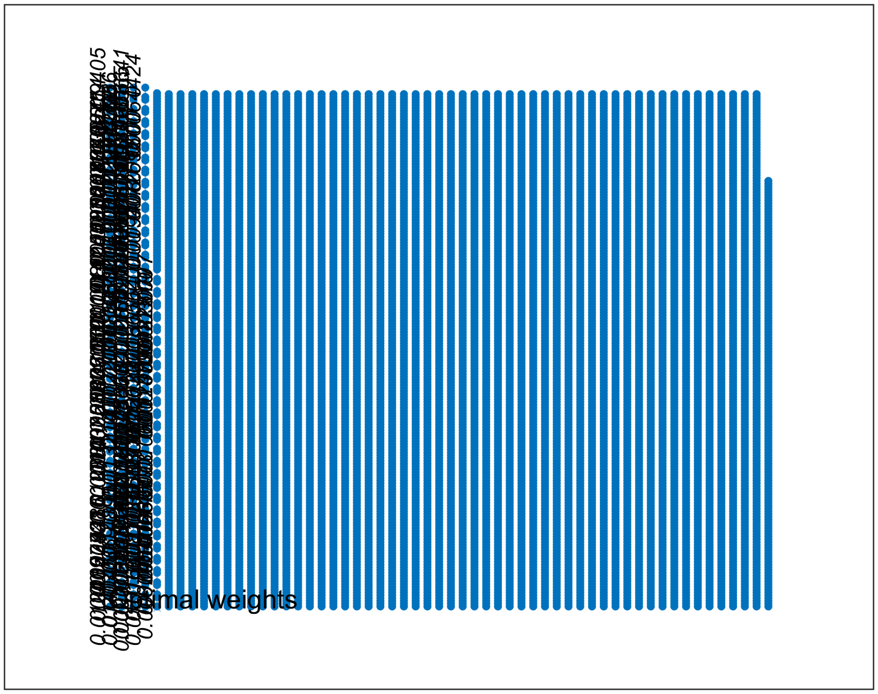
```
spy(A)
title('Adjacency Matrix')
```




```
s = find(A_full > 0.5);
t = find(A_full < -0.5);
g = digraph(s,t,w)
```

```
g = 
  digraph with properties:

    Edges: [200×2 table]
    Nodes: [10000×0 table]
```

```
plot(g,'EdgeLabel',g.Edges.Weight)
text(0.3,1.05,'Optimal weights')
```


Optimal weights