In the Name of God

**Convex Optimization Course**

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# Problem:

Consider a vector time series xt ∈ Rn, t = 1, 2, . . . . We want to ft a model of the form xt ∼N (0, atΣ), where Σ ∈ Sn ++ is given, and > 0. (We assume xt and xs are independent for t ̸= s.)  
Roughly speaking, the covariance matrix of xt scales up and down with time; at is scale factor at  
time t.  
We are given the base covariance matrix Σ, and a sample sequence x1, . . . , xT . We are to fnd the  
scale factor time series , t = 1, . . . , TWe will ft the scale factor times series by minimizing the negative log likelihood, plus a term that  
regularizes the variation in a(t),  
  
,

where λ > 0 is a given hyper-parameter. (Note that log at+1 - log at can be interpreted as the  
fractional change in the scaling parameter from t to t + 1.)

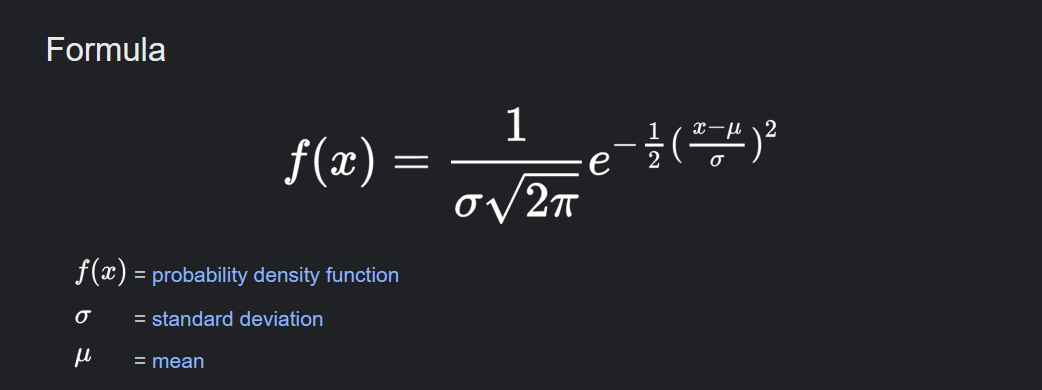
**a)** Show how to solve this ftting problem using convex or quasiconvex optimization. Fully justify  
any changes of variables, or relaxations, that your method uses.  
**b)** Carry out your method on the data given in covar series data.⋆, for the three hyper parameter  
values λ = 0.01, λ = 1, λ = 100. (This gives three diﬀerent estimates of the scale factor time  
series.) Plot these three estimates versus t.  
**C)** Validation. The data covar series data.∗ contains another time series y1, . . . , yT from the same  
source. Evaluate the negative log likelihood of your three models obtained in part (b) on this  
validation data set. Which of the three hyper-parameter values achieves the smallest negative  
log-likelihood?

# Solution:

We are asked to provide the likelihood function, considering given constraints and the objective function mixed with regularization term we have:

## Part (a):

Gaussian Distribution PDF



Turning scalar form into vectorized form:

We are told, xt and xs for t~=s are independent! =>

The joint pdf distribution will be the product of each single pdf:

* We consider the log-likelihood due to ease of calculations!

With sum simplifications we derive the equation below:

* Objective function with regularization Term:

Cost function = J =

Min J

s.t. for t =1,…,T

Before going to matlab, to solve the problem using CVX, we need to perform a change of variable:

Converting J into Vectorized form will yield:

For:

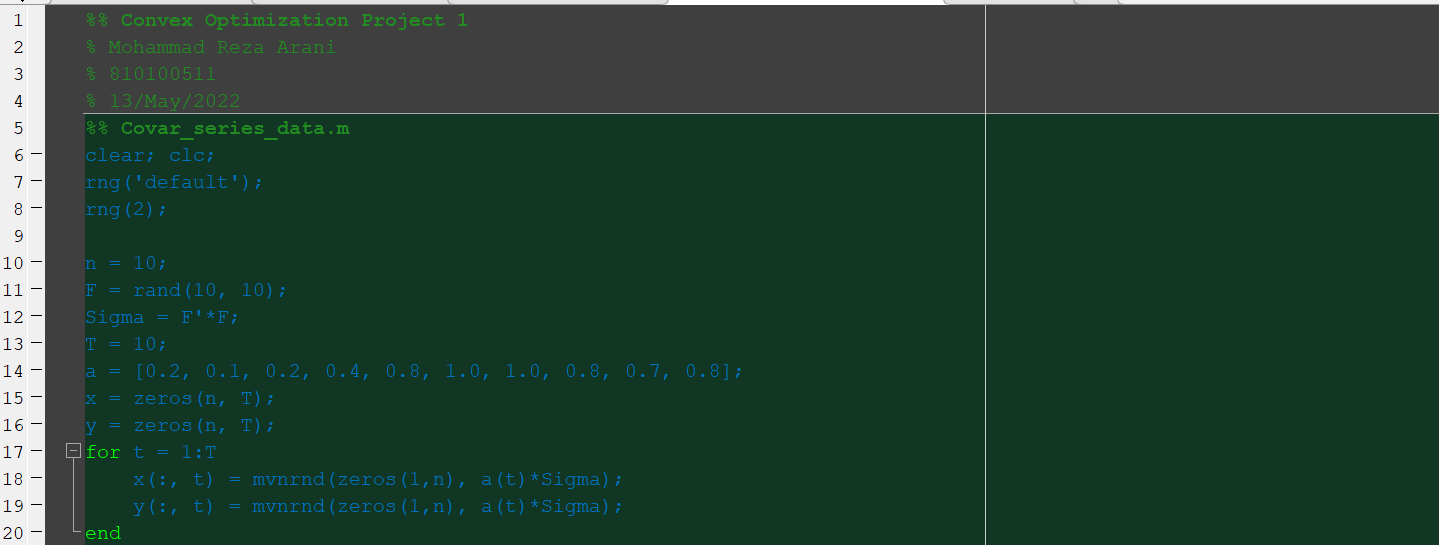
&

P as the differentiation matrix is like a Toeplitz matrix:

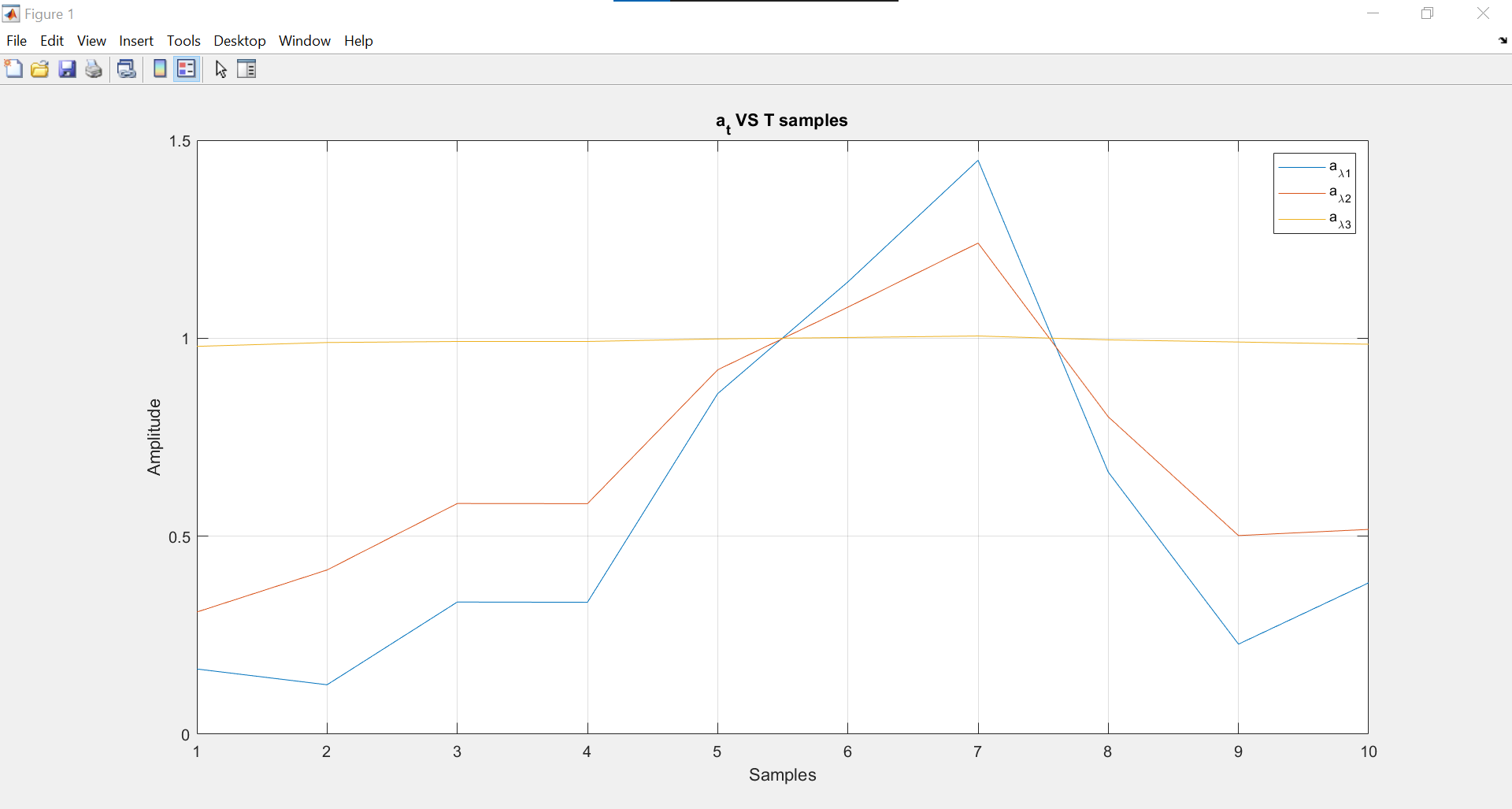
* Summation of convex elements is convex.
* Quadratic form is also convex considering P being PSD.

## Part (b):

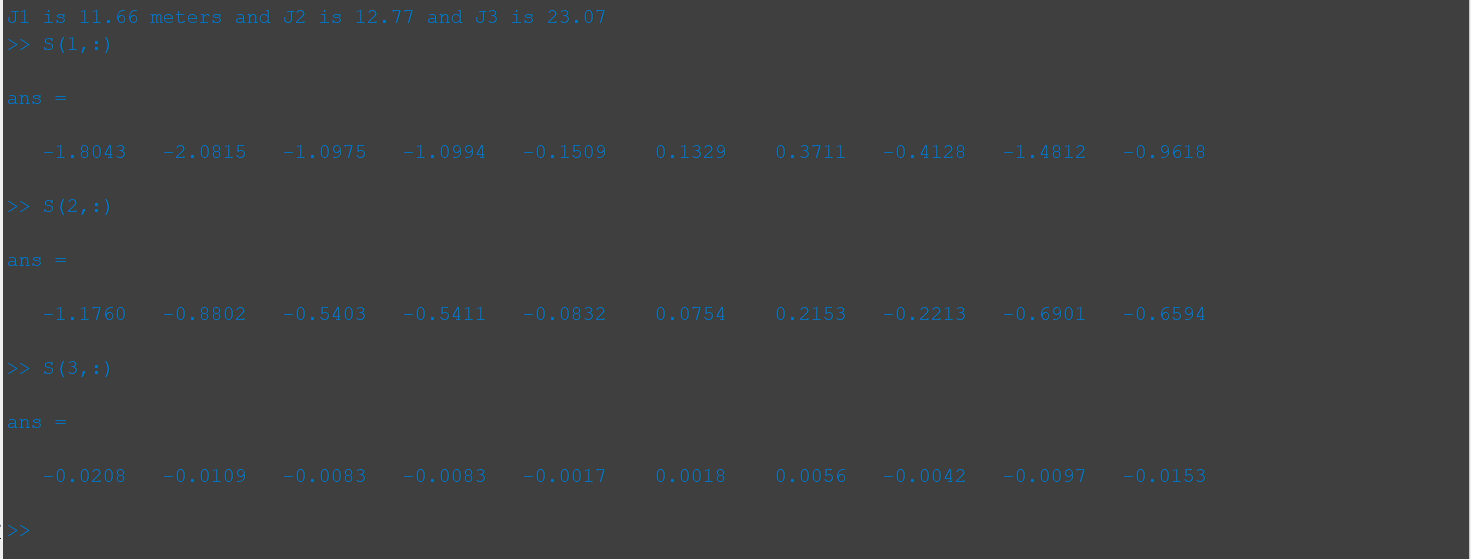
Loading data in matlab:



Figure



Figure



Figure

## Part (c):

Done in Matlab