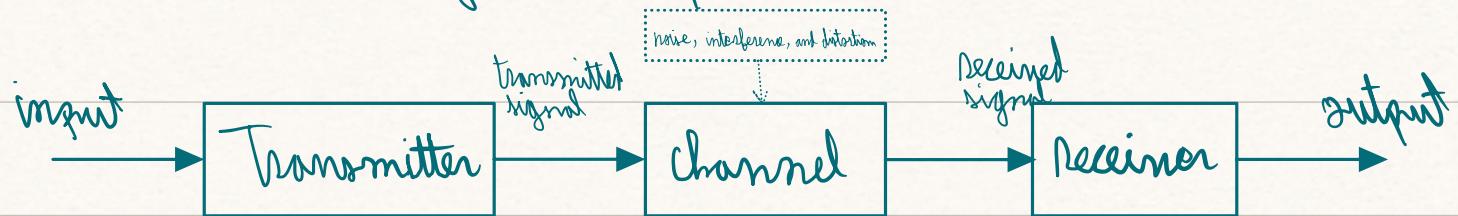


# Chapter 1

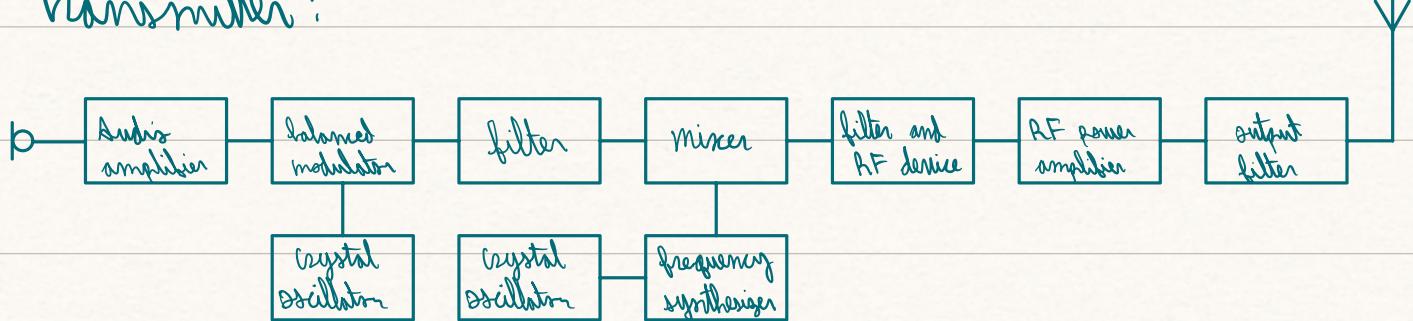
\* RF (communication) electronics: branch of electronics that deals with the study and design of devices, circuits, and systems operating in the radio frequency band

\* RF bands: electromagnetic signals that cover a specific wide range of frequencies  
these bands are allocated by the ITU

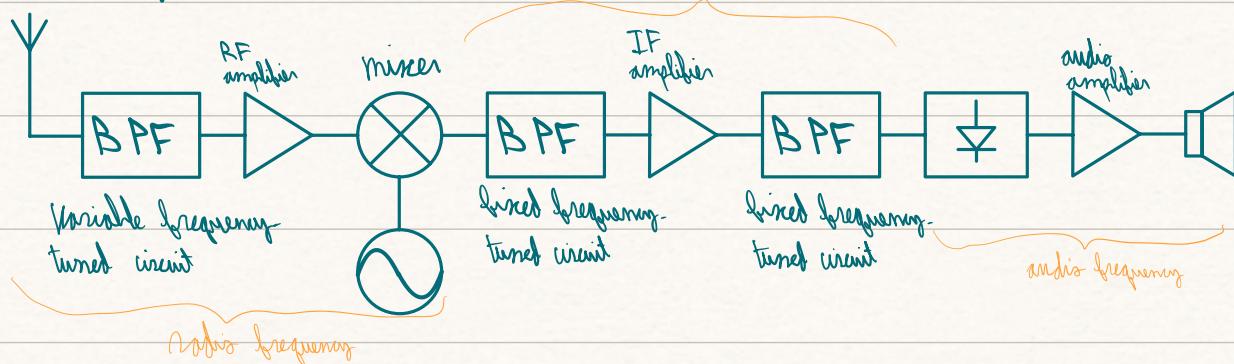
\* Communication system components:



+ Transmitter:



+ Receiver:



- at high frequencies, stray capacitances and inductances begin having noticeable effects and altering the impedances, thereby changing the matching conditions.
- wires and board tracks begin having large resistances at high frequencies due to the skin effect. additionally, they begin radiating electromagnetic waves as the wires' electrical length become short relative to the wavelength.
- finite wavelength effects produce propagation delays and phase shifts, which may result in destructive interference between signals from different paths.
- since low-power signal are received, the noise generated at the receiver must be taken into account, as it determines the minimum power the transmitter should transmit.

## Chapter 2

- noise cannot be removed from the received signal, but it can be minimised by using low-noise devices and proper filters
- another problem at the receiver is intermodulation distortion, which is caused by nonlinear devices.

\* noise: Random unwanted electrical signal that is added (AWGN) to the desired signal.

+ noise is generated by:

- internal sources: Components within the system (resistors, diodes, transistors, etc.)
- external sources: Sun, stars, meteors, etc.
- noise is defined by its statistical properties such as mean, pdf, etc.
- total average noise delivered to a load:  $P = \int_{f_1}^{f_2} f(f_f) df$   
spectral density function (W/Hz)

+ types of internal noise:

- thermal noise
- shot noise
- flicker noise

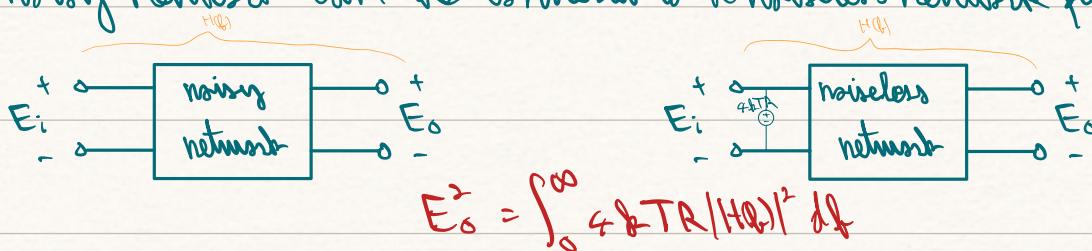
\* Thermal noise:

- generated in resistors, ideal capacitors and conductors do not generate
- mean-square spectral density:  $E^2 = 4 k T R$   
Temperature (K) resistance (Ω) Boltzmann Constant  $V^2/Hz$  or  $W/Hz$
- noisy resistor is equivalent to noiseless resistance plus noisy voltage source



- total mean-square power:  $E^2 = \int_0^\infty 4kT A(f) df$   $V^2 \text{ or } W$

- noisy network can be converted to a noiseless network plus noise source



- all the noise power is given by  $A$ :

$$E_o^2 = 4kT A \cdot B_n \quad \therefore \int_0^\infty (H(f))^2 df = B_n$$

\* shot noise: due to random fluctuations in current passing an junction

- power spectral density:  $i_n^2 = 2q I_0$   $\frac{A^2}{Hz}$   
electron charge

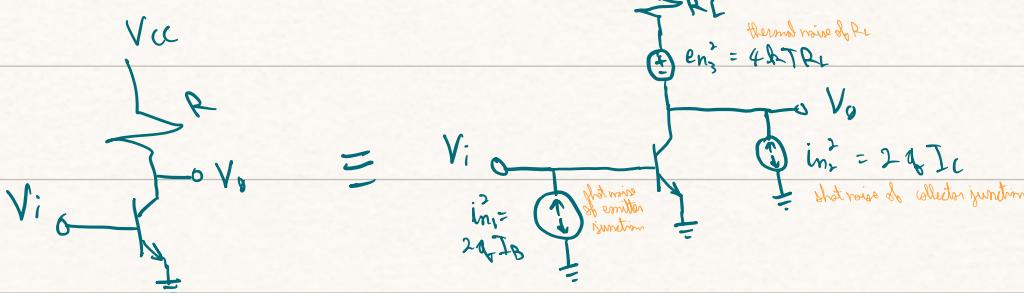
- total mean-squared shot noise:  $I_n^2 = 2q I_0 B$   $\frac{A^2}{Hz}$   
bandwidth

- thermal and shot noise are called "white" since their power spectral densities are independent of frequency
- thermal noise is additive since it is independent of voltage whereas shot noise is multiplicative.

\* flicker noise: low frequency phenomenon where the power spectral density is inversely proportional to frequency

- in a transistor amplifier, noise is generated by the resistor (thermal) and pn junction (shot)

- total noise power can be found by connecting all noises to voltages and referring them to the amplifier's input.

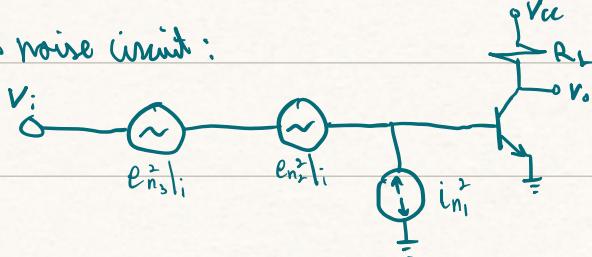


- equivalent thermal noise at input:  $e_{n,i}^2 |_i = \frac{4kTR_L}{A_V^2}$  → voltage Amplification
- equivalent collector shot noise at input:  $e_{n,r}^2 |_i = \frac{i_{n,r}^2 \cdot R_L}{A_V^2}$  → connected to voltage

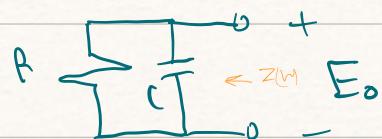
$$\therefore A_V = g_m R_L \quad (\text{for common emitter})$$

$$\therefore e_{n,i}^2 |_i = \frac{4kT}{g_m^2 R_L} \quad \& \quad e_{n,r}^2 |_i = \frac{2qI_c}{g_m^2 \cdot R_L}$$

equivalent noise circuit:



- Example 3.1:



Noise is generated by real part of impedance

$$Z(w) = \frac{R + \frac{1}{j\omega C}}{R + \frac{1}{j\omega L}} = \frac{R}{j\omega RL + 1} \times \frac{1 - j\omega RL}{1 + j\omega RL}$$

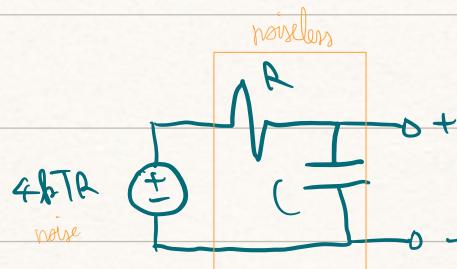
$$\rightarrow Z(w) = \boxed{\frac{R}{1 + w^2 R^2 C^2}} - \frac{j\omega RC}{1 + w^2 R^2 C^2}$$

$$\therefore \overline{E_o^2} = \int_0^\infty 4kT R(f) df = \int_0^\infty 4kT \cdot \frac{R}{1 + 4\pi^2 f^2 C^2} df$$

$$\rightarrow \overline{E_o^2} = \frac{kT}{C}$$

or (Desent approach)

$$\therefore \overline{E_o^2} = V_L$$



$$\rightarrow \text{Voltage division: } \overline{E_o^2} = 4kT R \cdot \frac{V_{noise}}{R + X_{gwc}}$$

$$\therefore H(f) = \frac{1}{1 + j\omega RC}$$

$$\therefore G(f) = H(f) \cdot F(f) = \frac{1}{1 + j\omega RC} \cdot \frac{1}{1 - j\omega RC}$$

$$\rightarrow \frac{1}{(1 + \omega^2 C^2 R^2)^2}$$

$$\rightarrow \overline{E_o^2} = 4kT R \int_0^\infty \frac{1}{1 + 4\pi^2 f^2 C^2 R^2} df = \frac{kT}{C}$$

- the noise power was found independent of  $R$  since it is dependent on the noise bandwidth, which is inversely proportional to  $R$ , as follows:

$$B_n = \int_0^\infty G(f) df = \frac{kT}{C} \cdot \frac{1}{4kTR} = \frac{1}{4RC}$$

Therefore, increasing  $R$  will increase the noise's power spectral density but decrease its bandwidth.

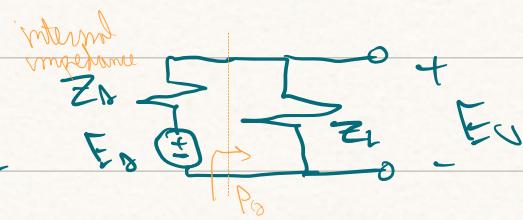
- The 3-dB bandwidth can be found from the frequency at which the transfer function squared (i.e.,  $G(f)$ ) is equal to  $\frac{1}{2}$

$$\rightarrow f_{3-dB} : \frac{1}{(1 + \omega^2 R^2 C^2)^2} = \frac{1}{2} \rightarrow 4\pi^2 f^2 R^2 C^2 = 1$$

\* Available power: maximum power that can be delivered to a load

- for max. power transfer:  $R_L = R_S$

$$\rightarrow Z_L = Z_S^* \quad (\text{matching})$$



if matched  $\rightarrow$  Voltage is evenly divided (i.e.,  $E_O = \frac{E_S}{2}$ )

$$\therefore P_O = \frac{(E_S/2)^2}{R_L} = \frac{E_S^2}{4R_L} = \frac{E_S^2}{4R_S} \quad \text{only valid for matched}$$

- noise factor is:  $F = \frac{\text{available output noise power}}{\text{available output noise power due to source}}$

$$1 \leq F < \infty \rightarrow 0 \leq NF < \infty$$

- noise figure: noise factor in dB  $\rightarrow NF = 10 \log_{10}(F)$

$$F = \frac{N_o}{(N_i)_o} = \frac{(N_o)_i}{N_i} = \frac{N_i + N_o}{N_i}$$

referred to input  
noise added by network is normally  
proportional to the input

available noise from the source

$S_i$ : noisy network  $N_o, G_r = A_v^2$   
 $N_i$ : noise power added by network  
 $N_o$ : noise component at output

- in terms of input and output SNR:

$$\therefore F = \frac{(N_o)_i}{N_i} = \frac{S_o}{S_i} \quad \text{and } S_o = G_r S_i \rightarrow F = \frac{(N_o)_i G_r S_i}{N_i S_o} \quad \therefore F = \frac{S_i / N_i}{S_o / N_o}$$

- noise factor is always equal to or greater than one.  $\frac{(S/N)_i}{(S/N)_o} \gg 1 \quad (S/N)_i \geq (S/N)_o$

- the minimum noise power ( $kT$ ) available from a source:

$$N_i = \frac{kT_A}{\frac{4P_S}{Z_S}} = kT$$

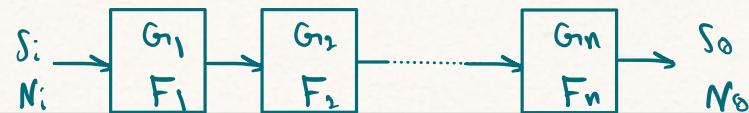
matched

$$\therefore F = 1 + \frac{N_o}{N_i} = 1 + \frac{N_o}{kT}$$

- Therefore, the noise added by the network is:

$$N_o = (F - 1) kT$$

+ for cascaded networks :



- total power gain is the product of all gains :  $G = G_1 \cdot G_2 \cdot \dots \cdot G_n \rightarrow S_o = G S_i$ .

- total noise factor is :  $F = \frac{N_i + N_o}{N_i}$  noise added by all stages

1  $N_o$  : noise added by each stage referred to the input of the network.

$$\rightarrow F = \frac{N_i + (F_1 - 1)N_i + (F_2 - 1)\frac{N_i}{G_1} + (F_3 - 1)\frac{N_i}{G_1 \cdot G_2} + \dots + (F_n - 1)\frac{N_i}{G_1 \cdot G_2 \cdot \dots \cdot G_{n-1}}}{N_i}$$

$$\therefore F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \dots + \frac{F_n - 1}{G_1 \cdot G_2 \cdot \dots \cdot G_{n-1}}$$

- since the denominator keeps getting larger, then the contribution of each stage keeps getting smaller. Therefore, the first stage gives the largest contribution, implying that  $F_1$  is the most important noise factor to minimize. Thus, low-noise amplifiers (LNA) are usually used. at the input

+ noise Temperature : often used to provide higher accuracy and less

rounding than noise power per Hz

$$\therefore F = 1 + \frac{N_o}{N_i} = 1 + \frac{kT_o}{kT} \rightarrow F = 1 + \frac{T_o}{T} \quad \text{under range}$$

$$\therefore T_o = (F - 1) \cdot T \quad \text{room temperature}$$

\* receiver sensitivity (noise floor  $N_f$ ) : the required available input signal power to achieve a certain output SNR

$$\therefore \left(\frac{S}{N}\right)_i = F \cdot \left(\frac{S}{N}\right)_o \rightarrow S_i = F N_i (S/N)_o$$

$$\therefore (S_i)_{\min} = F k T_B \left(\frac{S}{N}\right)_o$$

= noise floor  $N_f$

specified

- if the input  $s_i$  is less than the sensitivity, the output will not satisfy the performance measure (noisy, distorted, etc.)

- the minimum detectable signal is the sensitivity in V

$$\text{for matched: } s_i = \frac{E_i^2}{4R_s} \rightarrow E_i = \sqrt{4R_s s_i}$$

- example 3.3:

$$G_r = 12 + 15 = 27 \text{ dB}, \quad F_r = 10^{0.2}, \quad F_n = 10^{0.6}$$

$$\rightarrow F = 10^{0.2} + \frac{10^{0.6}-1}{10^{1.2}} = 1.973$$

- example 3.4:

$$\therefore (N_o)_i = N_i + N_o \quad \wedge \quad N_o = (F - 1) \cdot N_i$$

$$\rightarrow (N_o)_i = F_{kTB} \rightarrow N_o = F_{kTB} \cdot G_r = F_{kTB} \cdot G_1 \cdot G_2$$

$$\therefore N_o = 3.39 \times 10^{-15} \text{ W}$$

- example 3.5:

$$F = [1, 1.6] \rightarrow T_a = [0, 194] \text{ K}$$

$$\therefore T_a = (F - 1) T$$

- example 3.6:  $\therefore NF = 8 \text{ dB}, R_s = 50 \Omega, (SNR)_o = 0 \text{ dB}, B = 2.1 \text{ kHz}$

$$\therefore s_i = F_{kTB} \cdot (\frac{1}{N})_o = NF + (kTB)_{dB} + (SNR)_o, s_B = -16 \text{ dB}$$

$$\therefore s_i = 5.3 \times 10^{-19} \text{ W} \rightarrow E_i = 0.1 \text{ mV}$$

- example 3.7:

$$\text{If } (SNR)_o = 10 \text{ dB} \rightarrow s_i = 152.9 \text{ dBW} = 5.37 \times 10^{-16} \text{ W}$$

$$\rightarrow E_i = 0.329 \text{ mV}$$

- example 3.8:  $\text{NF} = 4 \text{ dB}$ ,  $R_s = 50 \Omega$ ,  $B = 32 \text{ Hz}$ ,  $(\text{SNR})_b = 10 \text{ dB}$

a)  $S_i = 4 + (\text{dTB})_{\text{dB}} + 10 = -155.2 \text{ dBW}$

$$E_i = \sqrt{R_s \cdot 4 \cdot 10^{\frac{-155.2}{10}}} = 2.45 \times 10^{-9} \text{ V}$$

b) antenna noise figure =  $20 \text{ dB}$  :  $\frac{S_i}{N_i} \rightarrow \boxed{G=0 \text{ dB}, \text{NF}=20} \rightarrow \boxed{G=1, \text{NF}=4} \rightarrow \frac{S_o}{N_o}$

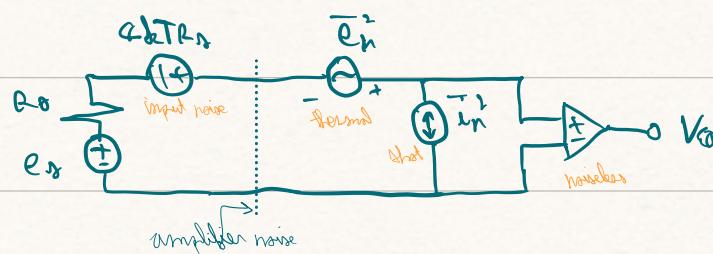
$$\text{so } G_2 = 0 + ? \quad \wedge \quad \text{NF}_{\text{total}} = \left( F_1 + \frac{F_2 - 1}{G_1} \right)_{\text{dB}} = 20.0 \text{ dB}$$

$$\text{so } S_i = \text{NF} + (\text{dTB})_{\text{dB}} + (\text{SNR})_b = -139 \text{ dBW}$$

$$\therefore E_i = 1.56 \text{ uV}$$

## \* Design of low noise networks:

- any linear noisy amplifier can be represented as a noiseless amplifier in addition to two noise sources at its input (thermal and shot)



$$\rightarrow N_i = 4kTR_s, \quad N_o = \bar{e}_n^2 + \bar{i}_n^2 R_a^2$$

$$\wedge F = \frac{N_i + N_o}{N_i} = \frac{4kTR_s + \bar{e}_n^2 + \bar{i}_n^2 R_a^2}{4kTR_s}$$

- since  $R_s$  is the only variable, it must be reduced to reduce the noise.

- To find the minimum, differentiate with respect to the variable

$$\rightarrow \frac{dF}{dR_{\text{S}}} = \frac{1}{4kT} \left[ i_n^2 - \frac{\bar{e}_n^2}{R_{\text{S}}^2} \right] = 0$$

$$\rightarrow R_{\text{S}}^2 = \frac{\bar{e}_n^2}{i_n^2} \quad \therefore R_{\text{S}} = \frac{\bar{e}_n}{i_n}$$

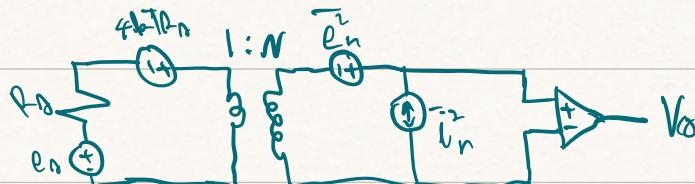
- minimizing the noise factor does not necessarily maximize the output SNR because of mismatching.

- since all noises are referred to the input, the output SNR can be

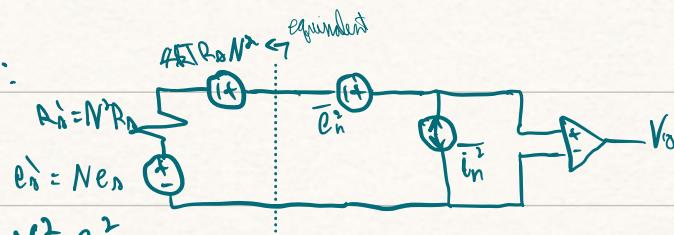
found as follows :  $(\frac{S}{N})_0 = \frac{\bar{e}_n^2}{4kTR_{\text{S}} + \bar{e}_n^2 + i_n^2 \cdot R_{\text{S}}}$

- hence the minimum SNR is when  $R_{\text{S}}$  is equal to zero
- Therefore, there is a conflict between minimizing SNR and maximizing it.

- This conflict is resolved by adding a transformer between the source and noisy network to isolate  $R_{\text{S}}$



- taking the equivalent circuit of the above transformer referred to the  $N$  turns side :



$$\rightarrow (\text{SNR})_0 = \frac{N^2 \bar{e}_n^2}{4kTR_S N^2 + \bar{e}_n^2 + i_n^2 \cdot N^4 \cdot R_S^2}$$

- the only variable in the above equation is  $N$ , assuming  $R_S$  is fixed for matching. Thus, the maximum SNR is found from:

$$\frac{d(\text{SNR})_o}{dN} = 0 \rightarrow R_S' = N^2 R_S = \frac{\ln}{\text{in}} \xrightarrow{\text{optimum}} N = \sqrt{\frac{\ln}{\text{in}} \cdot \frac{1}{R_S}}$$

- adding this transformer therefore maximizes the  $(\text{SNR})_o$  and minimizes the noise factor.

## \* intermodulation distortion:

- since some sections of a receiver may operate in the saturation region then the receiver is not entirely linear.

- the non-linearity can be described by the following Taylor series expansion:  $y(x) = \underbrace{k_1 f(x)}_{\text{linear}} + \underbrace{k_2 f^2(x)}_{\text{non-linear}} + \underbrace{k_3 f^3(x)}_{\text{non-linear}}$

- If two adjacent signals enter the receiver, then:

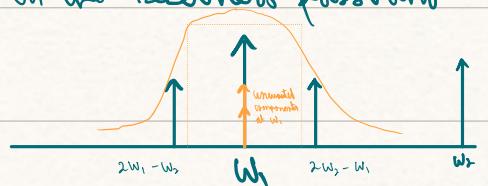
$$f(x) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$$

$$\rightarrow y(t) = k_1 [A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)] + k_2 [A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)]^2 + k_3 [A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)]^3$$

$$\begin{aligned} \rightarrow y(t) &= k_1 A_1 \cos(\omega_1 t) + k_1 A_2 \cos(\omega_2 t) + k_2 [A_1^2 \cos^2(\omega_1 t) + 2A_1 A_2 \cos(\omega_1 t) \cdot \\ &\quad \cos(\omega_2 t) + A_2^2 \cos^2(\omega_2 t)] + k_3 [A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)] \cdot [A_1^2 \cos^2(\omega_1 t) + \\ &\quad + 2A_1 A_2 \cos(\omega_1 t) \cos(\omega_2 t) + A_2^2 \cos^2(\omega_2 t)] \end{aligned}$$

$$\rightarrow y(t) = k_1 A_1 \cos(\omega_1 t) + k_2 A_2 \cos(\omega_2 t) + k_3 \left[ \frac{A_1^2}{2} + \frac{A_2^2}{2} \cos(2\omega_1 t) + A_1 A_2 [\cos(\omega_1 - \omega_2)t) + \cos(\omega_1 + \omega_2)t)] + \frac{A_1^2}{2} + \frac{A_2^2}{2} \cos(2\omega_2 t) \right] + k_3 \left[ \frac{A_1^2}{2} + \frac{A_2^2}{2} \cos(2\omega_1 t) + A_1 A_2 [\cos(\omega_1 - \omega_2)t) + \cos(\omega_1 + \omega_2)t)] + \frac{A_1^2}{2} + \frac{A_2^2}{2} \cos(2\omega_2 t) \right] \cdot [A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)]$$

- this gives many frequency components with various amplitudes, but the most dangerous are the ones closest in frequency to our desired signal, so they may be included in the receiver's passband.



+ gain compression:

- by expanding the  $k_3$  term, some components at  $\omega_1$  can be found:

$$\begin{aligned} & k_3 \left[ \frac{A_1^2}{2} + \frac{A_2^2}{2} \cos(2\omega_1 t) + A_1 A_2 [\cos(\omega_1 - \omega_2)t) + \cos(\omega_1 + \omega_2)t)] + \frac{A_1^2}{2} + \frac{A_2^2}{2} \cos(2\omega_2 t) \right] \\ & \cdot [A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)] \\ \rightarrow & k_3 \left[ \underbrace{\frac{A_1^3}{2} \cos(\omega_1 t)}_{\frac{A_1^3}{2} \cos(\omega_1 t)} + \underbrace{\frac{A_1^3}{2} \cos(2\omega_1 t) \cos(\omega_1 t)}_{\cos(\omega_1 t) \cos(\omega_1 t)} + \underbrace{A_1^2 A_2 [\cos(\omega_1 - \omega_2)t) + \cos(\omega_1 + \omega_2)t]}_{\cos(\omega_1 t) \cos(\omega_1 t)} \right. \\ & + \underbrace{\frac{A_1 A_2^2}{2} \cos(\omega_1 t)}_{\frac{A_1 A_2^2}{2} \cos(\omega_1 t)} + \underbrace{\frac{A_1 A_2^2}{2} \cos(2\omega_2 t) \cos(\omega_1 t)}_{\frac{A_1 A_2^2}{2} \cos(\omega_1 t)} + k_3 \left[ \frac{A_1^2 A_2}{2} \cos(\omega_2 t) + \right. \\ & \left. \left. \frac{A_1^2 A_2}{2} \cos(2\omega_1 t) \cos(\omega_2 t) + A_1 A_2 [\cos((\omega_1 - \omega_2)t) \cos(\omega_2 t) + \cos((\omega_1 + \omega_2)t) \cdot \right. \right. \\ & \left. \left. \cos(\omega_2 t)] + \frac{A_2^3}{2} \cos(\omega_2 t) + \frac{A_2^3}{2} \cos(2\omega_1 t) \cos(\omega_2 t) \right] \\ \rightarrow & k_3 \left[ \frac{A_1^3}{2} \cos(\omega_1 t) + \frac{A_1^3}{4} \cos(\omega_1 t) + \frac{A_1 A_2^2}{2} \cos(\omega_1 t) + A_1 A_2^2 \cos(\omega_1 t) \right] \\ \therefore & \omega_1 \text{ components : } k_3 \left[ \frac{3}{4} A_1^3 \cos(\omega_1 t) + \frac{3}{2} A_1 A_2^2 \cos(\omega_1 t) \right] \end{aligned}$$

- these components will be added to the signal at  $w_1$  and distort it:

$$A'_1 \cos(w_1 t) = \underbrace{k_1 A_1 \cos(w_1 t)}_{\text{desired}} + \underbrace{k_3 \left( \frac{3}{4} A_1^3 + \frac{3}{2} A_1 A_2^2 \right) \cos(w_1 t)}_{\text{undesired}}$$

- $k_3$  is usually negative, thus it will weaken the received signal.
- if  $k_3$  is positive it will distort the signal.
- if the adjacent signals amplitude is strong, the desired signal may be lost, especially since the undesired signal level is proportional with the square of the adjacent signals amplitude.

### + single tone compression:

- occurs when the adjacent signal is negligible and the desired signal compresses itself.

ratio of received signal to desired signal

$$\text{gain compression factor: } \frac{A'_1}{k_1 A_1} = \frac{k_1 A_1 + \frac{3}{4} k_3 A_1^3}{k_1 A_1}$$

- the signal can therefore cancel itself completely if  $k_1 = -\frac{3}{4} k_3 A_1^2$

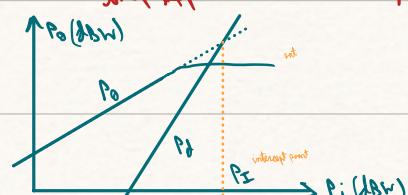
### + intermodulation distortion (IMD)

- if  $w_1$  and  $w_2$  are adjacent, then either one of the intermodulation components ( $2w_1 - w_2$ ) or ( $2w_2 - w_1$ ) may be within the receivers

only one

passband of  $w_1$ .  $y(t) = \underbrace{k_1 A_1 \cos(w_1 t)}_{\text{desired signal}} + \underbrace{\frac{3}{4} k_3 A_1^2 A_2 \cos((2w_1 - w_2)t)}_{\text{undesired intermodulation distortion component}}$

- intermodulation distortion ratio:  $\text{IMR} = \frac{\frac{3}{4} k_3 A_1^2 A_2}{k_1 A_1} = \frac{3 k_3 A_1 A_2}{4 k_1}$



+ intercept point:

- the value of input signal power that gives an output power equal to the IMD power

$$\therefore P_d = \frac{(\frac{3}{4} k_3 A_1^2 A_2)^2}{2}, \text{ if } A_1 \approx A_2 \rightarrow P_d = \frac{(\frac{3}{4} k_3 A_1^3)^2}{2}$$

$$\therefore P_i = \frac{A_1^2}{2} \rightarrow P_d = (\frac{3}{4} k_3 P_i)^2 \rightarrow P_d \propto P_i^3$$

$$\rightarrow P_{IMA} = \frac{P_d}{P_0} \quad \therefore P_0 = \frac{(k_3 A_1)^2}{2} \rightarrow P_0 = k_3^2 P_i$$

$$\therefore P_{IMA} = (\frac{3}{4} k_3 P_i)^2 \quad \text{s.t., } k_3 = \frac{k_3^2}{P_0}$$

- to find the intercept point:  $P_d = P_0 \rightarrow \frac{3}{4} k_3 P_i^2 = 1 \quad \therefore P_i = P_I$

$$\therefore P_I = \frac{1}{k_3} \rightarrow P_{IMA} = \left( \frac{P_i}{P_I} \right)^2$$

- if  $k_3$  is 0, then the intercept point  $P_I$  is  $\infty$ .

+ dynamic range:

- the range from the minimum signal level (sensitivity) to the maximum input signal level.

- the maximum input signal level depends on distortion, rather than noise.
- the maximum input signal is the input signal at which the intermodulation distortion, referred to the input, is equal to the sensitivity.

$$(\Delta_i)_{\text{max}} @ (P_d)_i = (\Delta_i)_{\text{min}}$$

$$\therefore P_{IMA} = \frac{P_d}{P_0} = \frac{P_d}{P_i \cdot k_3} = \frac{P_d/k_3}{P_i} = \frac{(P_d)_i}{P_i} = \left( \frac{P_i}{P_I} \right)^2$$

$$\therefore \frac{N_r}{(\Delta_i)_{\text{max}}} = \left( \frac{(\Delta_i)_{\text{max}}}{P_I} \right)^2 \rightarrow (\Delta_i)_{\text{max}} = \sqrt[3]{P_I^2 N_r}$$

$$\therefore \text{dynamic range (DR)} : \frac{(\delta_i)_{\text{max}}}{(\delta_i)_{\text{min}}} = \left( \frac{P_I}{N_f} \right)^{2/3}$$

- in normal power units, the dynamic range is a ratio, whereas in dB

$$\text{it is } (\delta_i)_{\text{max}} \text{ dBW} - (\delta_i)_{\text{min}} \text{ dBW} = \frac{2}{3} [P_I - N_f] \text{ dB}$$

$$- \text{example 3.10: } \because R_d = 10k, \bar{e_n^2} = 8 \times 10^{-16} \text{ V}^2/\text{Hz}, i_m = 4 \times 10^{-25} \text{ A}^2/\text{Hz}$$

$$\rightarrow F = \frac{4kT R_d + \bar{e_n^2} + i_m^2 R_d^2}{4kT R_d} = 6.56$$

$$- \text{example 3.11: } \because (R_d)_{\text{opt}} = \sqrt{\frac{\bar{e_n^2}}{i_m^2}} = 29.8 \text{ k}\Omega$$

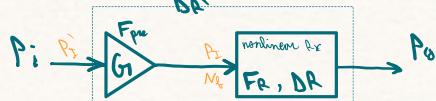
$$\rightarrow F = 4.35$$

$$- \text{example 3.14: } \because P_I = 20 \text{ dBm} \quad \& \quad N_f = -123 \text{ dBm}$$

$$\rightarrow DR = \frac{2}{3} [143] = 95.3 \text{ dB}$$

+ if a linear amplifier is connected before the non-linear receiver:

$$\because DR = \left( \frac{P_I}{N_f} \right)^{2/3}$$

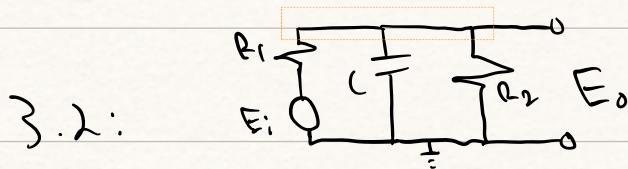


$$\text{for overall system, } DR' = \left( \frac{P_I'}{N_f'} \right)^{2/3} \quad \& \quad P_I' = \frac{P_I}{G_A}$$

$$\because F_{\text{total}} = F_{\text{pre}} + \frac{F_R - 1}{G_A} \rightarrow N_f' = F_{\text{total}} kT B \left( \frac{S}{N} \right)_0$$

$$\rightarrow DR' = \left( \frac{P_I}{G_A N_f'} \right)^{2/3}$$

## EE 524: homework #1



applying KCL:  $\frac{E_o}{R_2} + E_o \cdot j\omega C = \frac{E_i - E_o}{R_1}$

$$\rightarrow E_o \left[ \frac{1}{R_2} + j\omega C + \frac{1}{R_1} \right] = E_i \cdot \frac{1}{R_1}$$

$$\therefore H(\omega) = \frac{E_o}{E_i} = \frac{1}{\frac{1}{R_1} + j\omega C R_1 + 1} = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C}$$

$$\rightarrow |H(\omega)|^2 = H(\omega) \cdot H^*(\omega) = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C} \cdot \frac{R_2}{R_1 + R_2 - j\omega R_1 R_2 C}$$

$$\rightarrow |H(\omega)|^2 = \frac{R_2^2}{(R_1 + R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}$$

$$B_n = \int_0^\infty |H(\omega)|^2 d\omega = \boxed{\frac{1}{4R_1 R_2 (R_1 + R_2) C}}$$

$$\boxed{\frac{R_1 + R_2}{4R_1 R_2 C}}$$

at  $|H(\omega_{3-\text{dB}})|^2 = \frac{1}{2} \text{ max}[|H(\omega)|^2]$

$$\therefore |H(\omega)|^2 \text{ max at } \omega = 0 = \left( \frac{R_2}{R_1 + R_2} \right)^2$$

$$\rightarrow |H(\omega_{3-\text{dB}})| = \frac{R_2}{(R_1 + R_2)^2 + \omega_{3-\text{dB}}^2 R_1^2 R_2^2 C^2} = \frac{1}{2} \left( \frac{R_2}{R_1 + R_2} \right)^2$$

$$\rightarrow 4\pi^2 f_{3-\text{dB}}^2 \cdot R_1^2 \cdot R_2^2 \cdot C^2 = (R_1 + R_2)^2$$

$$\therefore f_{3-\text{dB}} = \boxed{\frac{R_1 + R_2}{2\pi R_1 R_2 C}}$$

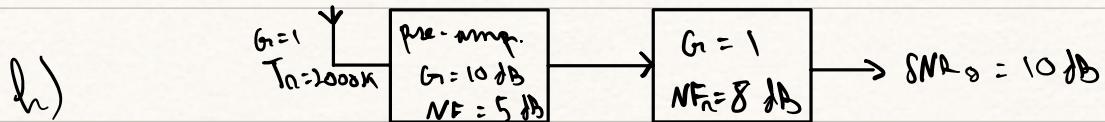
3.7: a)

$$\begin{array}{l} G=1 \\ T_n=2000\text{K} \end{array} \boxed{G=1 \\ NFe=8\text{ dB}} \rightarrow SNR_0 = 10\text{ dB}$$

$$\therefore T_n = 2000 \text{ K} \rightarrow F = 7.896 \quad \text{and} \quad F_n = 10^{0.8}$$

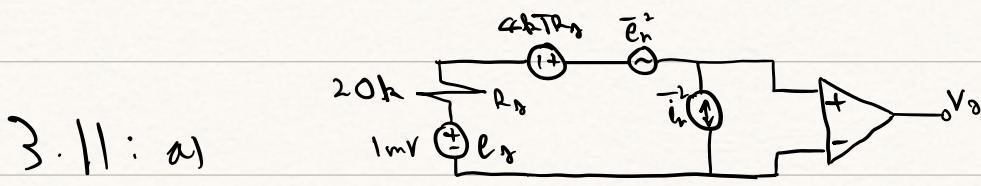
$$\rightarrow F_{\text{total}} = 7.896 + \frac{10^{0.8} - 1}{1} = 13.2$$

$$\therefore (S_i)_{\min} = F_{\text{total}} k_B T B \left(\frac{S}{N}\right)_0 = 1.585 \times 10^{-15} \text{ W}$$



$$F_{\text{total}} = 7.896 + \frac{10^{0.5} - 1}{1} + \frac{10^{0.8} - 1}{10} = 10.59$$

$$\rightarrow (S_i)_{\min} = F_{\text{total}} k_B T B_{\text{receiver}} \left(\frac{S}{N}\right)_0 = 1.271 \times 10^{-15} \text{ W}$$



$$\left(\frac{S}{N}\right)_0 = \frac{e_n^2}{4kT R_o + \bar{e}_n^2 + \bar{i}_n^2 \cdot R_o^2}$$

$$\because f_L = 1 \text{ kHz} \rightarrow \bar{e}_n^2 = 8 \times 10^{-16} \text{ V}^2/\text{Hz} \quad \lambda \bar{i}_n^2 = 9 \times 10^{-24} \text{ A}^2/\text{Hz}$$

$$\rightarrow \left(\frac{S}{N}\right)_0 = 676 \times 10^6 \quad \text{per Hz}$$

b)

$$R_o = \frac{e_n}{i_n} = 29.814 \text{ k}\Omega \rightarrow \left(\frac{S}{N}\right)_0 = 481 \times 10^6$$

3.13: a)  $\therefore (S_i)_{\min} = F k_B T B \left(\frac{S}{N}\right)_0 = 7.57 \times 10^{-16} \text{ W}$

b)  $\therefore P_I = 20 \text{ dBm} = 100 \text{ mW} \rightarrow DR = \left(\frac{P}{N_B}\right)^{2/3}$

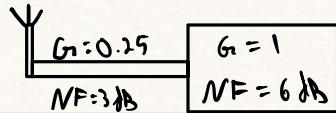
$$\therefore DR = 2.59 \times 10^9 = 94.14 \text{ dB}$$

c)  $F_{\text{total}} = 0 + \frac{10^{0.8} - 1}{100} = 0.531 \rightarrow N_f = 6.395 \times 10^{-17} \text{ W}$

$$1.25 \times 10^{-16} \text{ W}$$

$P = 1 \text{ mW} \rightarrow DR = 101.3 \text{ dB}$

3.16:

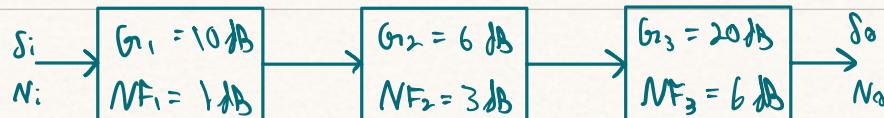


$$F_{\text{total}} = 10^{0.3} + \frac{10^{0.6} - 1}{0.25} = 13.92 \quad \therefore (S_i)_{\min} = 1.69 \times 10^{-15} \text{ W}$$

$$\text{If antenna } T_a = 300 \text{ K} \rightarrow F = 11.35 \rightarrow F_{\text{total}} = 11.35 + 10^{0.3} - 1 + \frac{10^{0.6} - 1}{0.25}$$

$$\therefore F_{\text{total}} = 24.27 \rightarrow (S_i)_{\min} = 2.914 \times 10^{-15} \text{ W}$$

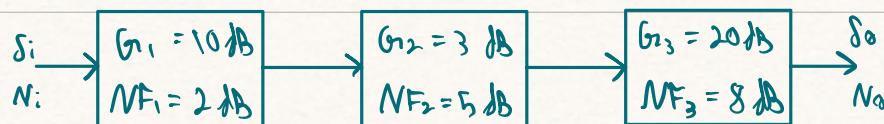
+ Quiz #1 practice:



$$G_{\text{total}} = 36 \text{ dB}, F_{\text{total}} = 10^{0.1} + \frac{10^{0.3}-1}{10} + \frac{10^{0.6}-1}{10 \cdot 10^{0.6}} = 1.433$$

$$(S_i)_{\min} = F_{\text{total}} kTB \left(\frac{f}{N}\right)_0 = 1.147 \times 10^{-13} \text{ W} = -99.4 \text{ dBm}$$

$$\rightarrow (E_i)_{\min} = \sqrt{4 \pi R_0 (S_i)_{\min}} = 4.99 \text{ mV}$$



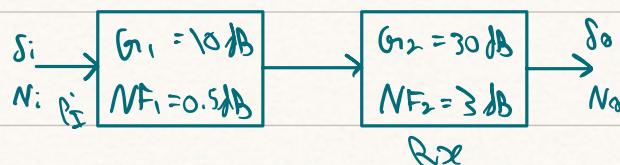
$$G_{\text{total}} = 33 \text{ dB} = 1995.3$$

$$F_{\text{total}} = 10^{0.2} + \frac{10^{0.5}-1}{10} + \frac{10^{0.8}-1}{10 \cdot 10^{0.5}} = 2.07$$

$$\rightarrow NF_{\text{total}} = 3.16 \text{ dB}$$

$$\lambda (S_i)_{\min} = F_{\text{FTB}} \cdot 10^3 = 8.28 \times 10^{-14} \text{ W} = -100.8 \text{ dBm}$$

$$\rightarrow (E_i)_{\min} = \sqrt{4 \pi R_0 \cdot (S_i)_{\min}} = 4.09 \text{ mV}$$



$$\text{a)} (S_i)_{\min, \text{Rx}} = 10^{0.3} \cdot kT \cdot 10 \Omega \cdot 10^2 = 7.985 \times 10^{-15} \text{ W}$$

$$\text{oo } P_{I, \text{Rx}} = 20 \text{ dBm} \quad \lambda DR = \left(\frac{P_I}{N_0}\right)^{2/3} \quad \text{oo } (S_i)_{\min} = \sqrt[3]{P_I^2 N_0}$$

$$\rightarrow DR = 5.39 \times 10^8 = 87.3 \text{ dB}$$

$$\text{b)} F_{\text{total}} = 10^{0.05} + \frac{10^{0.3}-1}{10} = 1.22 \rightarrow (S_i)_{\min} = 4.88 \times 10^{-15} \text{ W}$$

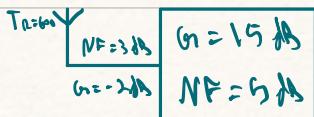
$$\therefore P_I' = \frac{10^2 \text{ mW}}{10} = 10 \text{ mW} \rightarrow DR = \left( \frac{10 \text{ mW}}{4.88 \times 10^{-15} \text{ W}} \right)^{2/3}$$

$$\rightarrow DR = 82.1 \text{ dB}$$

+ first exam practice:

11/2012

Q1)



$$F_{\text{antenna}} = 1 + \frac{600}{200} = 3.07$$

$$\rightarrow F_{\text{total}} = 3.07 + \frac{10^{0.3} - 1}{10^{0.2}} + \frac{10^{0.5} - 1}{10^{0.2}} = 7.49$$

$$\text{a) } (S_i)_{\min} = F_{\text{total}} kT_B \left(\frac{S}{N}\right)_0 = 2.94 \times 10^{-14} \text{ W}$$

$$\rightarrow (S_i)_{\min} = -105.2 \text{ dBm}$$

$$\lambda(E_i)_{\min} = 2.445 \text{ mW}$$

$$\text{b) } \text{a)} (SNR)_0 = 20 \text{ dB} \quad \lambda(S_i)_{\min}$$

$$\rightarrow S_0 = (S_i)_{\min} \cdot 10^{13} \rightarrow N_0 = 5.99 \times 10^{-15} \text{ W}$$

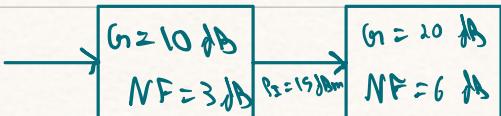
$$\text{c) } N_0 = (F kT_B) \cdot G_1 G_2$$

$$\text{Q2) a) } \text{DR} = \left( \frac{P_I}{N_0} \right)^{2/3} \rightarrow DR = \frac{2}{3} [15 + 100] = 76.7 \text{ dB}$$

$$\text{b) } F kT_B (S_i)_0 = -100 \text{ dBm}$$

$$\rightarrow S_i)_0 = 6.28 \times 10^6$$

b)



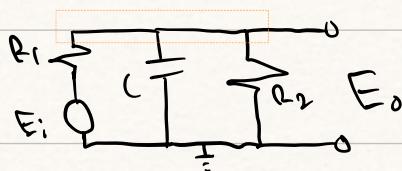
$$P_I' = 5 \text{ dBm}, \quad F_{\text{total}} = 10^{0.3} + \frac{10^{0.6} - 1}{10} = 2.29 \rightarrow 3.59 \text{ dB}$$

$$\rightarrow (S_i)_{\min} = F_{\text{total}} kT_B \left(\frac{S}{N}\right)_0 = -102.4 \text{ dBm} \rightarrow DR = 71.6 \text{ dB}$$

# + Homework #1 revisited:

3.2:

initial solution:



$$\text{applying KCL: } \frac{E_o}{R_2} + E_o \cdot j\omega C = \frac{E_i - E_o}{R_1}$$

$$\rightarrow E_o \left[ \frac{1}{R_2} + j\omega C + \frac{1}{R_1} \right] = E_i \cdot \frac{1}{R_1}$$

$$\therefore H(\omega) = \frac{E_o}{E_i} = \frac{1}{\frac{R_1}{R_1 + j\omega C R_1 + 1}} = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C}$$

$$\rightarrow |H(\omega)|^2 = H(\omega) \cdot H^*(\omega) = \frac{R_2}{R_1 + R_2 + j\omega R_1 R_2 C} \cdot \frac{R_2}{R_1 + R_2 - j\omega R_1 R_2 C}$$

$$\rightarrow |H(\omega)|^2 = \frac{R_2^2}{(R_1 + R_2)^2 + \omega^2 R_1^2 R_2^2 C^2}$$

$$B_n = \int_0^\infty |H(\omega)|^2 d\omega = \boxed{\frac{1}{4R_1 R_2} \frac{R_2^2}{(R_1 + R_2) C}}$$

prob. massur  
solution ↓

$$\boxed{\frac{R_1 + R_2}{4 R_1 R_2 C}}$$

$$\lambda \text{ for } 3\text{-dB} \text{ at } |H(\omega_{3-\text{dB}})|^2 = \frac{1}{2} \text{ max}[|H(\omega)|^2]$$

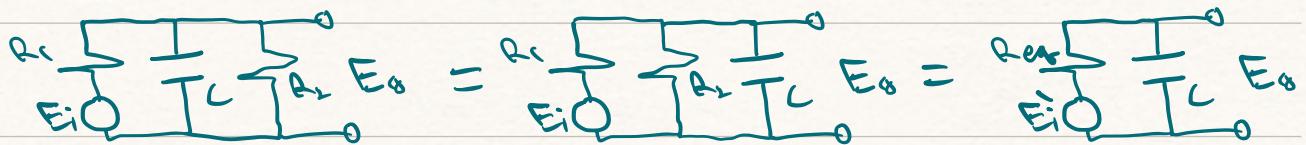
$$\because |H(\omega)|^2 \text{ max at } \omega = 0 = \left( \frac{R_2}{R_1 + R_2} \right)^2$$

$$\rightarrow |H(\omega_{3-\text{dB}})| = \frac{R_2}{\sqrt{(R_1 + R_2)^2 + (\omega_{3-\text{dB}})^2 R_1^2 R_2^2 C^2}} = \frac{1}{2} \left( \frac{R_2}{R_1 + R_2} \right)$$

$$\rightarrow 4\pi^2 f_{3-\text{dB}}^2 \cdot R_1^2 \cdot R_2^2 \cdot C^2 = (R_1 + R_2)^2$$

$$\therefore f_{3-\text{dB}} = \boxed{\frac{R_1 + R_2}{2\pi R_1 R_2 C}}$$

+ by following prof. mansoor method of finding  
the Thevenin equivalent:



$$\therefore R_{eq} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2} \quad \text{and} \quad E'_i = E_i \cdot \frac{R_2}{R_1 + R_2}$$

- from the solved example:  $H(w) = \frac{1}{jwRC + 1}$

$$\rightarrow \frac{E_o}{E'_i} = \frac{1}{jwR_{eq} + 1} = H(w)$$

$$\therefore \frac{E_o}{E'_i} = \frac{E_o}{E_i} \cdot \frac{R_1 + R_2}{R_2} \rightarrow H'(w) = H(w) \cdot \frac{R_1 + R_2}{R_2}$$

$$\therefore H(w) = \frac{R_2}{R_1 + R_2} \cdot \frac{1}{jw \frac{R_1 R_2}{R_1 + R_2} C + 1} = \boxed{\frac{R_2}{jw R_1 R_2 C + R_1 + R_2}}$$

$$\rightarrow H(w) \cdot H^*(w) = \frac{R_2^2}{(R_1 + R_2)^2 + (w R_1 R_2 C)^2}$$

$$\therefore \int_0^\infty G(\beta) d\beta = \boxed{\frac{A_s}{4 R_1 (R_1 + R_2) C}} \quad H_\gamma = B_n$$

$$3.13: \text{ a) } \because (S_i)_{\min} = F k T_B \cdot \left(\frac{S}{N}\right)_0 = 10^{0.8} \cdot k \cdot T \cdot 3 \cdot 10 = 7.57 \times 10^{-16} \text{ W}$$

$$\rightarrow (E_i)_{\min} = \sqrt{4 k T_B (S_i)_{\min}} = 3.89 \times 10^{-7} \text{ V}$$

$$\text{b) } \because N_f = 7.57 \times 10^{-16} \text{ W}, P_I = 100 \text{ mW}$$

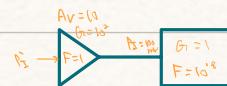
$$\rightarrow DR = 94.14 \text{ dB}$$

c) wireless  $\rightarrow NF = 0 \text{ dB} \rightarrow F = 1$

$$F' = 1 + \frac{10^{0.8} - 1}{100} = 1.0531, P'_I = \frac{P_I}{100} = 1 \text{ mW}$$

$$\rightarrow N'_f = F' k T_B \left(\frac{S}{N}\right)_0 = 1.264 \times 10^{-16} \text{ W}$$

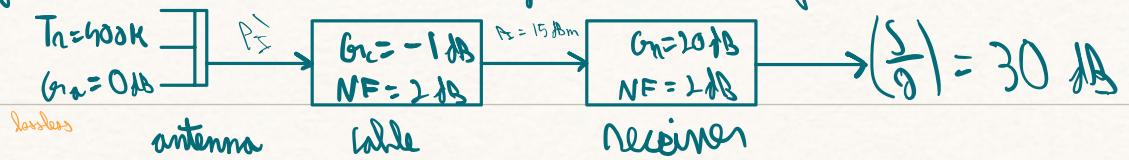
$$\therefore DR' = 85.9 \text{ dB}$$



+ first exam practice continued:

12/2020:

Q1: first draw the block diagram of the system:



$$a) G_{\text{total}}(\text{dB}) = G_{\text{ra}}(\text{dB}) + G_{\text{rc}}(\text{dB}) + G_{\text{ra}}(\text{dB}) = 19 \text{ dB}$$

$$F_{\text{total}} = F_a + \frac{F_c - 1}{G_{\text{ra}}} + \frac{F_n - 1}{G_{\text{ra}} G_{\text{rc}}}$$

$$\therefore T_a = 400 \text{ K} \rightarrow F_a = 1 + \frac{T_a}{T} = 1 + \frac{400}{290} = 2.71$$

$$\rightarrow F_{\text{total}} = 2.71 + \frac{10^{0.2} - 1}{1} + \frac{10^{0.2} - 1}{10^{0.1}} = 4.04$$

$$\therefore NF_{\text{total}} = 6.06 \text{ dB}$$

$$\therefore T_n = (F - 1)T \rightarrow T_{n,\text{total}} = 882 \text{ K}$$

$$b) \therefore (\delta_i)_{\min} = F_{\text{total}} \cdot k \cdot T \cdot B \cdot \left(\frac{S}{N}\right)_0 = 8.08 \times 10^{-14} \text{ W}$$

$$\rightarrow (\delta_i)_{\min} = -100.9 \text{ dBm}$$

$$\lambda (E_i)_{\min} = \sqrt{4 \cdot R_0 \cdot (\delta_i)_{\min}} = 4.02 \mu\text{V}$$

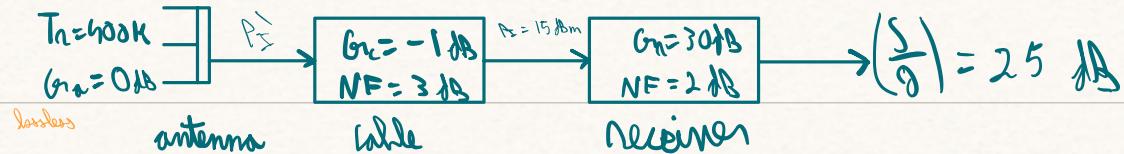
$$c) \therefore \left(\frac{S}{N}\right)_i = F \left(\frac{f}{N}\right)_0 \rightarrow \left(\frac{S}{N}\right)_i = 36.1 \text{ dB}$$

$$\lambda N_0 = F_{\text{total}} k T B \cdot G_{\text{ra}} \cdot G_{\text{rc}} \cdot b_n = 6.42 \times 10^{-15} \text{ W}$$

$$d) \therefore DR = \left(\frac{P_t}{N_0}\right)^{1/2} = \left(\frac{P_t}{G_{\text{rc}} \cdot N_0}\right)^{1/2} = 77.95$$

$$\lambda (\delta_i)_{\min} = DR \cdot N_0 = 5.04 \times 10^{-6} \text{ W}$$

12/2021:



Q1:

$$\text{a)} \quad G_{\text{total}} = 29 \text{ dB}, \quad F_{\text{tot}} = 4.46 \rightarrow NF_{\text{tot}} = 6.49 \text{ dB}$$

$$\therefore T_{n,\text{tot}} = 3.46 \cdot 290 = 1003.4 \text{ K}$$

$$\text{b)} \quad (S_i)_{\text{min}} = 5.64 \times 10^{-14} \text{ W} = -102.5 \text{ dBm}$$

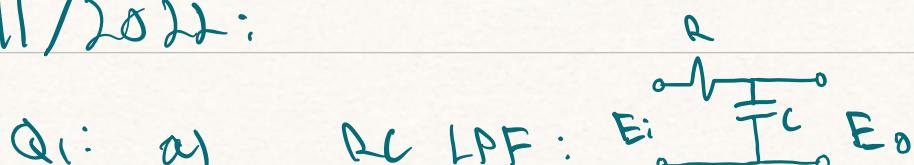
$$\therefore (E_i)_{\text{min}} = 3.36 \text{ nV}, \quad (f_N)_i = NF_{\text{tot}} + (S_N)_i = 31.49 \text{ dB}$$

$$\text{c)} \quad N_0 = 1.42 \times 10^{-13} \text{ W} = -98.5 \text{ dBm}$$

$$\text{d)} \quad DR = \frac{2}{3} [P_s - (S_i)_{\text{min}}] = 78.3 \text{ dB}$$

$$\therefore (S_i)_{\text{max}} = -24.2 \text{ dBm} = 3.83 \text{ mW}$$

11/2022:



$$\because E_o = E_i \cdot \frac{1}{j\omega RC + 1} \rightarrow H(\omega) = \frac{1}{j\omega RC + 1}$$

$$\therefore |H(\omega)|^2 = \frac{1}{1 + (\omega RC)^2}$$

$$\therefore B_n = \int_0^\infty |H(f)|^2 df = \frac{1}{\pi RC} = 250 \text{ Hz}$$

usually true, but not sure.

assuming signal bandwidth = 3-dB bandwidth

$$\rightarrow f_{3-\text{dB}} \text{ at } \frac{1}{1 + 4\pi^2 f^2 R^2 C^2} = \frac{1}{2} \rightarrow f = \frac{1}{2\pi RC} = 159 \text{ Hz}$$

$$d) \cos(2\pi f_{LO}t) \cdot \cos(2\pi f_{RF}t) = \text{Im} \cos(2\pi f_{IF}t) \xrightarrow{\text{IF}} \text{BPF}$$

$$\rightarrow \cos[2\pi(f_{LO} - f_{RF})t] = \cos(2\pi f_{IF}t) \rightarrow f_{LO} = 110.7 \text{ MHz}$$

- the local oscillator frequency could be chosen as  $f_{RF} - f_{IF}$

however it is usually taken as  $f_{RF} + f_{IF}$  to further separate the local oscillator frequency and intermediate frequency.

c) 1 - amplifiers: amplify received signals and reduces noise contributions of later stages.

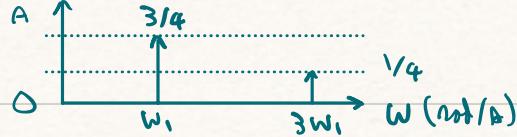
2 - oscillators: enable modulation, multiplexing, mixing, etc.

3 - filters: isolate the desired signals

$$d) \cos^3(w_i t) = \cos(w_i t) \cdot \cos^2(w_i t) = \cos(w_i t) \cdot \left[ \frac{1}{2} + \frac{1}{2} \cos(2w_i t) \right]$$

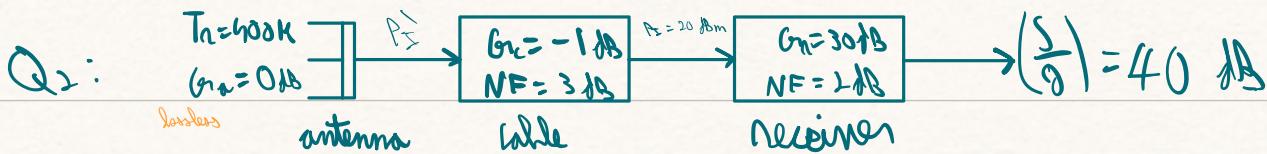
$$\rightarrow \frac{1}{2} \cos(w_i t) + \frac{1}{2} \cos(w_i t) \cdot \cos(2w_i t)$$

$$\rightarrow \frac{1}{2} \cos(w_i t) + \frac{1}{4} [\cos(3w_i t) + \cos(w_i t)] = \frac{3}{4} \cos(w_i t) + \frac{1}{4} \cos(3w_i t)$$



e) 1 - mean or mean-square, 2 - variance or standard deviation

3 - probability density function, 4 - power spectral density



a) (check previous solutions for details)  $G_{\text{total}} = 29 \text{ dB}$

$$F_{\text{total}} = 4.46 \rightarrow NF_{\text{total}} = 6.49 \text{ dB}$$

$$\rightarrow T_{n,\text{eff}} = 1003.4 \text{ K}$$

$$\text{d)} (S_i)_{\min} = 3.59 \times 10^{-12} \text{ W} = -84.5 \text{ dBm}$$

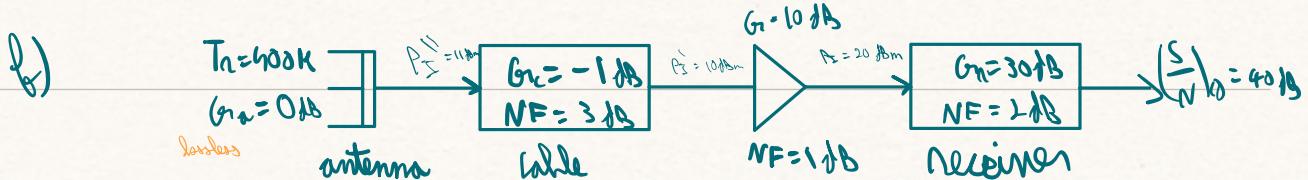
$$\lambda(E_i)_{\min} = 26.7 \text{ nV}$$

$$\text{e)} N_i = kTB = 8 \times 10^{-17} \text{ W}, N_a = (F-1)N_i = 2.768 \times 10^{-16} \text{ W}$$

$$\sim N_0 = (N_i + N_a) \cdot G_{\text{total}} = F kTB G_{\text{total}} = 2.84 \times 10^{-13} \text{ W}$$

$$\text{d)} \frac{S_i}{N_i} = 62 \text{ dB} = 47.96 \text{ dB} \quad \lambda \frac{S_0}{N_0} = \frac{S_i \cdot G_{\text{total}}}{N_0} = 41.46 \text{ dB}$$

$$\text{e)} P_i' = 21 \text{ dBm} \rightarrow DR' = 70.3 \text{ dB} \quad \lambda(S_i)_{\max} = -14.19 \text{ dBm}$$



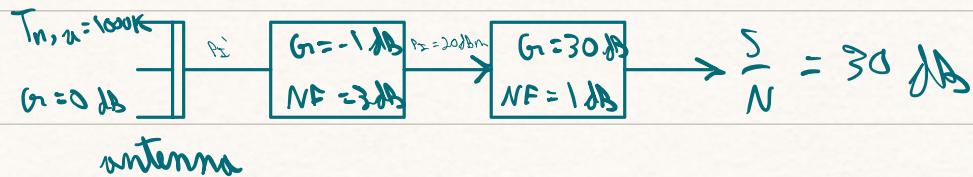
$$F_{\text{total}} = 1 + \frac{1000}{290} + 10^{0.3} - 1 + \frac{10^{0.1} - 1}{10^{-0.1}} + \frac{10^{0.2} - 1}{10^{-0.1} \cdot 10^1} = 4.12$$

$$\rightarrow N_f = 3.29 \times 10^{-12} \text{ W} = -84.8 \text{ dBm}$$

$$\rightarrow DR'' = \frac{2}{3}(11 + 84.8) = 63.9 \text{ dB}$$

10/2014:

Q1:



$$\text{a)} F_{\text{total}} = \frac{1000}{290} + 10^{0.3} + \frac{10^{0.1} - 1}{10^{-0.1}} = 5.99$$

$G_{\text{total}} = 29 \text{ dB}$

$$T_{n,\text{total}} = 4.71 \times 290 = 1383.3 \text{ K}$$

$$\text{b)} (S_i)_{\min} = F_{\text{total}} \ln T_B \cdot 10^3 = 2.31 \times 10^{-13} \text{ W} = -126.4 \text{ dBW}$$

$$\rightarrow (E_i)_{\min} = 6.79 \text{ nV}$$

$$\sim \frac{(S_i)_{\min}}{N_i} = F_{\text{total}} \cdot \left(\frac{S}{N}\right)_0 = 37.6 \text{ dB} = 5990:1$$

$$\text{c)} \because F = \frac{N_i + N_a}{N_i} \rightarrow N_a = N_i(F-1) = 1.91 \times 10^{-16} \text{ W}$$

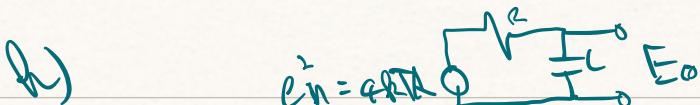
$$N_a = F N_i \cdot G_{\text{total}} = 1.83 \times 10^{-13} \text{ W}$$

$$\text{d)} DR = \frac{2}{3} [P_1' - -126.4 + 30] = 78.3 \text{ dB}$$

$$\rightarrow (S_i)_{\max} = DR + N_b = -18.1 \text{ dBm}$$

11/2013:

Q1: a) Resistor: thermal noise, diode and transistor: shot noise



$$\rightarrow H(\omega) = \frac{1}{j\omega RC + 1} \rightarrow |H(\omega)|^2 = \frac{1}{1 + (\omega RC)^2}$$

$$\rightarrow E_o^2 = E_n^2 \cdot \int_0^\infty \frac{1}{1 + \frac{\omega^2}{(\omega RC)^2}} d\omega = E_n^2 \cdot \frac{1}{\pi RC}$$

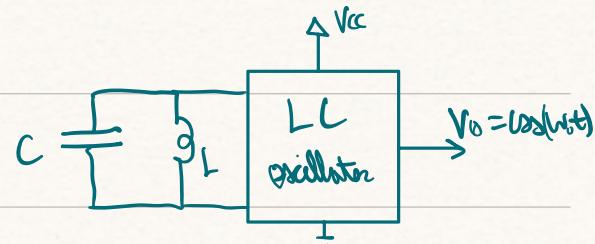
$$\therefore E_o^2 = \frac{kT}{C} \text{ W (V^2)}$$

## Chapter 7:

- amplifiers are the most important devices in communication systems.
- electronic oscillators convert DC power to periodic AC signals at specific frequency.   
 no input, only output      harmonic oscillator produce sinusoidal AC signals
- harmonic oscillators are used in communication systems to produce carriers, which can be used in modulators, demodulators, multiplexers, demultiplexers, upconverters and downconverters.   
 which allows bandwidth channels the free-space to be used      frequency multiplexing (FDM)

+ major types of oscillators :

1 - LC oscillator (conventional) :

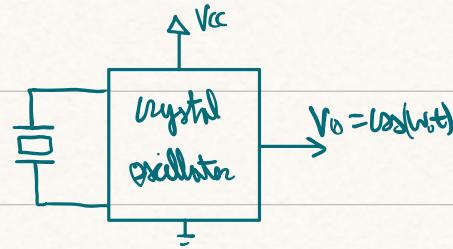


- frequency isn't precise since L and C aren't precise and may vary

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

2 - Crystal oscillator :

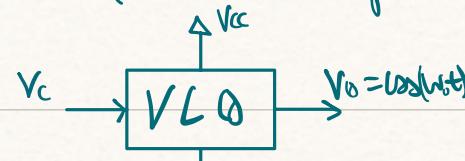
- very precise.



3 - Voltage controlled oscillator (VCO) :

- output voltage is controlled by a DC input control voltage ( $V_c$ )

$$\rightarrow f_0 = f(V_c)$$

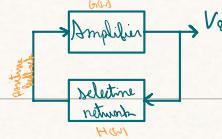


- used in FM modulators and phase-locked loops (PLL)

- oscillators are represented as unstable networks made of amplifiers with positive feedback from frequency selective networks.

- oscillators have AC outputs without an input, instead noise triggers the oscillation.

only DC power

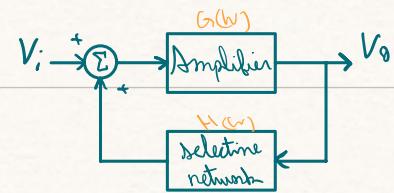


- an input is assumed to derive the oscillation conditions

$$\rightarrow V_0 = [V_i + H(w)V_0] G(w)$$

$$\therefore V_0 = \frac{G(w)}{1 - (G(w)H(w))} \cdot V_i$$

closed loop gain

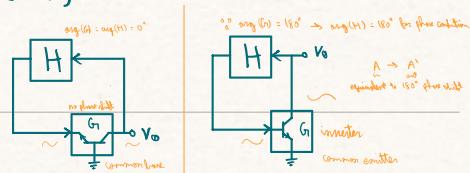


- if  $G(w)H(w) = 1$ , the closed-loop gain becomes infinite, implying that there will be an output even for  $V_i = 0$ .

- the frequency selective network  $H(w)$  will have one frequency ( $\omega_0$ ) at which the condition  $G(\omega_0)H(\omega_0) = 1$  is satisfied.

+ Therefore, there are two conditions for  $G(\omega_0)H(\omega_0) = 1$ :

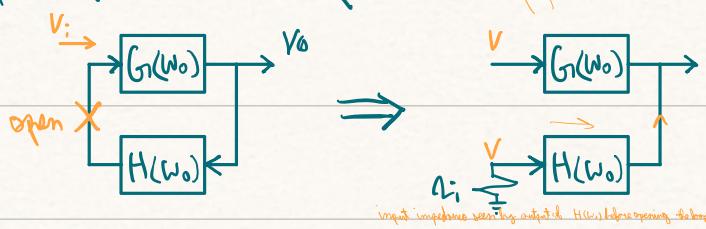
- magnitude condition:  $|G(\omega_0)H(\omega_0)| = 1$



- phase condition:  $\arg(G(\omega_0)H(\omega_0)) = 0 \text{ or } 2\pi n + \pi, n = \text{integer}$

- three different approaches to analyse oscillators will be explored.

1- open feedback loop: first approach

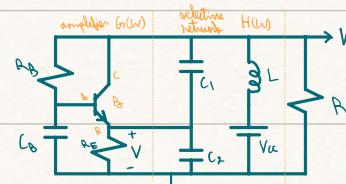


$$\text{find } G_1 = \frac{V_0}{V}, \quad H = \frac{V}{V_0}$$

for  $|G_1 H| \geq 1, \arg(G_1 H) = 0 \text{ or } 2\pi n$

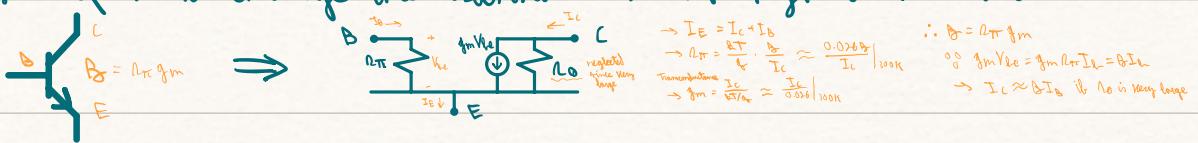
+ common base oscillator analysis:

- capacitor  $C_B$  is added to short the base and



make the amplifier common base in AC analysis, provided  $\frac{1}{\omega_0 C_L} \approx 0$

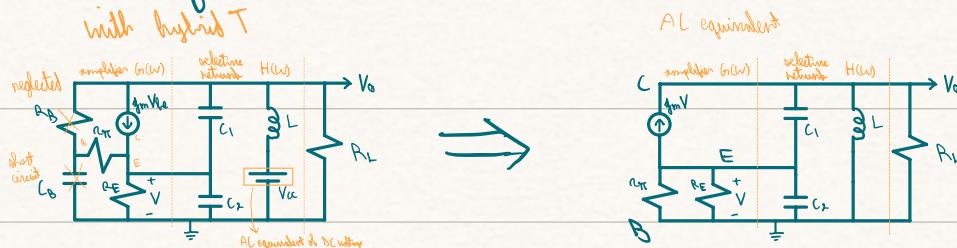
- the first step is to change the transistor to its hybrid T model.



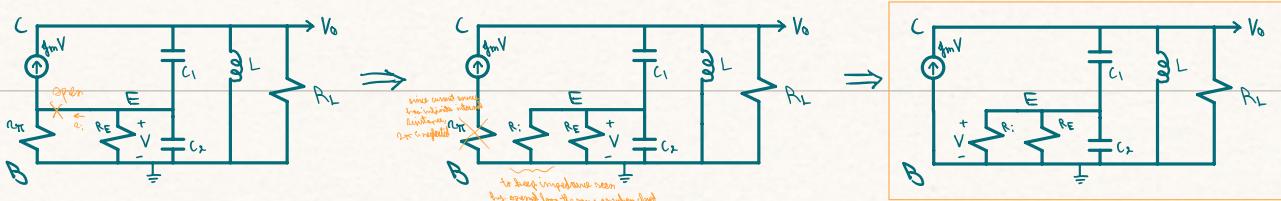
- important equations to recall:

$$g_m \approx \frac{I_C}{0.026}, \quad I_C \approx B I_B, \quad B = R_\pi g_m$$

- the AC equivalent circuit of the common-base oscillator becomes:

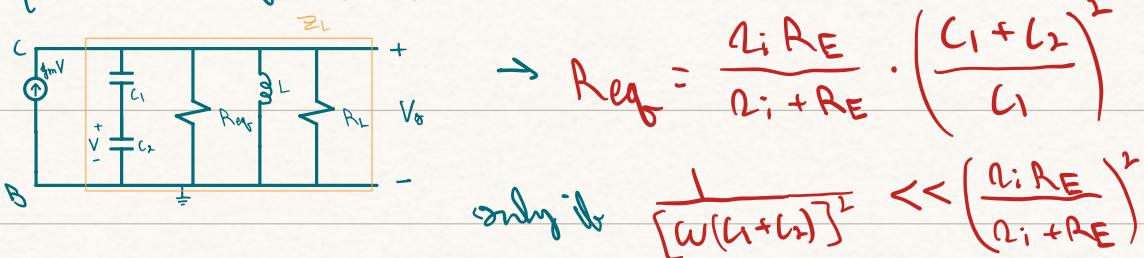


- next, the feedback loop is opened: where  $R_i = \frac{1}{g_m}$



- The analysis of the AC equivalent circuit can be simplified using

Capacitive transformation:



- finally,  $H(\omega)$  and  $G(\omega)$  are found as follows:

$$\rightarrow H(\omega) = \frac{V}{V_0} \rightarrow V = V_0 \cdot \frac{V_0 \omega C_2}{V_0 \omega C_1 + V_0 \omega C_2} \xrightarrow{\text{Voltage division across capacitor}} \frac{V}{V_0} = \frac{C_1}{C_1 + C_2}$$

$$\therefore G(\omega) = \frac{V_0}{V} \rightarrow V_0 = \frac{\text{Current}}{g_m V \cdot Z_L} \rightarrow G(\omega) = g_m Z_L$$

$$\therefore \frac{1}{Z_L} = \frac{1}{R_L} + \frac{1}{j\omega L} + \frac{1}{R_{eq}} + j\omega C_{eq} \quad \text{A.t. } C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

- now we must satisfy the oscillation conditions:

$$+ \arg(G_2 H) = 0^\circ \quad \& \quad \arg(H) = 0^\circ \rightarrow \arg(G_2) = 0^\circ$$

- for  $\arg(G_2) = 0^\circ$ ,  $G_2$  must be real  $\rightarrow \frac{1}{Z_L}$  is real

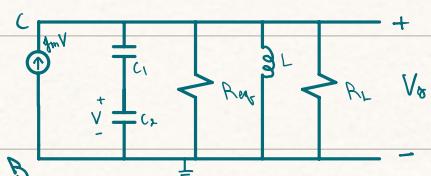
$$\therefore j\omega_0 C_{eq} = \frac{j}{\omega_0 L} \rightarrow \omega_0 = \sqrt{\frac{1}{L C_{eq}}} \rightarrow \omega_0 = \sqrt{\frac{1}{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$$

$$\rightarrow \text{at } \omega_0, Z_L = \frac{R_L R_{eq}}{R_L + R_{eq}}$$

$$+ |G_2 H| \gg 1, \text{ at resonance: } G_2(\omega_0) = g_m \frac{R_L R_{eq}}{R_L + R_{eq}}$$

$$\therefore |G_2 H| = g_m \frac{R_L R_{eq}}{R_L + R_{eq}} \cdot \frac{C_1}{C_1 + C_2} \gg 1 \quad \text{magnitude condition}$$

+ example 7.1: 20 MHz common-base oscillator with  $\beta = 100$  and  $I_C = 1 \text{ mA}$



$$R_{eq} = \frac{R_E R_L}{R_E + R_L} \cdot \left( \frac{C_1 + C_2}{C_1} \right)^2, \quad R_i = \frac{1}{g_m}$$

$$\text{only if } \frac{1}{[\omega_0(C_1 + C_2)]^2} \ll \left( \frac{R_E R_L}{R_E + R_L} \right)^2$$

$$\text{we need } 20 \text{ MHz} = \sqrt{\frac{1}{L \cdot \frac{C_1 C_2}{C_1 + C_2}}} \text{ and } g_m \frac{R_L R_{eq}}{R_L + R_{eq}} \cdot \frac{C_1}{C_1 + C_2} \gg 1$$

$$\therefore g_m = \frac{I_C}{0.026} \approx 40 \text{ mA} \quad \& \quad R_i = \frac{1}{g_m} = 25 \Omega$$

$$\& R_E \gg R_i \rightarrow R_{eq} \approx R_i \left( \frac{C_1 + C_2}{C_1} \right)^2$$

$$\rightarrow \frac{1}{R_i^2} \ll [\omega_0(C_1 + C_2)]^2$$

$$\rightarrow \text{choose } R_L \gg R_{eq} \rightarrow |G_2 H| \approx g_m R_{eq} \cdot \frac{C_1}{C_1 + C_2} = g_m \cdot R_i \cdot \frac{C_1 + C_2}{C_1}$$

$$\therefore |G_2 H| \approx \frac{C_1 + C_2}{C_1} \gg 1, \text{ if } \frac{C_1 + C_2}{C_1} \Rightarrow \rightarrow C_2 = 2C_1$$

$$\& \frac{1}{R_i^2} \ll [\omega_0(C_1 + C_2)]^2 \rightarrow 3C_1 > 1.01 \text{ nF}$$

$$\therefore C_1 > 0.336 \text{ nF} \quad \wedge \quad C_2 > 0.671 \text{ nF}$$

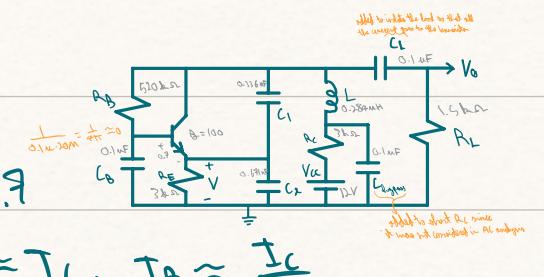
$$\therefore \omega_0 = \sqrt{\frac{1}{LC_{eq}}} \rightarrow L = \frac{1}{\omega^2 \cdot \frac{C_1 C_2}{C_1 + C_2}} = 0.283 \text{ mH}$$

- Resistors  $R_B$ ,  $R_C$ , and  $R_E$  are chosen to make  $I_C = 1 \text{ mA}$  and

to satisfy the other assumptions.

$$\rightarrow -V_{AC} + I_C R_C + I_B R_B + 0.7$$

$$+ I_E \cdot R_E = 0 \quad \wedge \quad I_E \approx I_L, \quad I_B \approx \frac{I_C}{\beta}$$

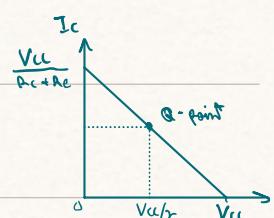


$$\rightarrow V_{AC} = I_C \left[ R_C + \frac{R_B}{\beta} + R_E \right] + 0.7$$

$$\rightarrow R_C + \frac{R_B}{\beta} + R_E = \frac{V_{AC} - 0.7}{I_C} = 11.3 \text{ k}\Omega$$

- $I_C$  must be chosen to keep the transistor at the center of the active region to have a symmetric output and avoid trimming

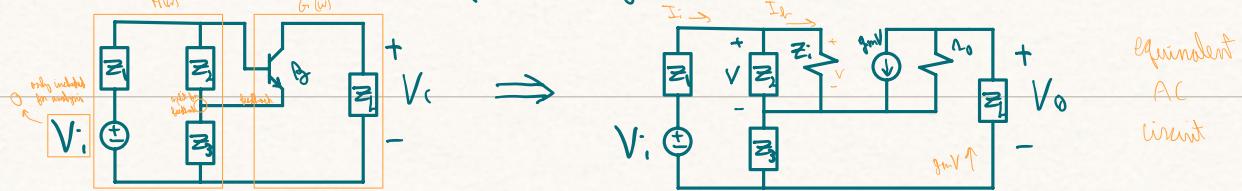
- Large  $I_C$  is desirable as it increases the transconductance  $g_m$  but a large  $I_C$  also increases power consumption, and hence there must be a tradeoff.



## 2 - Circuit analysis : second approach

- a resonant circuit must be used as the frequency selective network to determine the operating frequency of the oscillator.
- This second approach requires three impedances as the selective network and

a transistor with feedback to provide gain for the oscillator.



- assuming  $A_{\text{v}}$  is very large (s.c.) : KVL

$$1) -V_i + I_i Z_1 + V + Z_3 (I_i + \text{gm}V) = 0$$

$$2) V = I_i \cdot (Z_2 / |Z_1|) = I_i \frac{Z_1 Z_2}{Z_1 + Z_2} \rightarrow I_i Z_2 - V \frac{Z_1 + Z_2}{Z_1} = 0$$

- for oscillation,  $I_i$  and  $I_h$  must be non-zero even when  $V_i$  is zero, which is only possible if the determinant is zero

$$\text{from (1)}: I_i (Z_1 + Z_3) + V (1 + \text{gm}Z_3) = 0$$

$$\text{from (2)}: I_i Z_2 - V \left( \frac{Z_1 + Z_2}{Z_1} \right) = 0$$

$$\rightarrow \begin{vmatrix} Z_1 + Z_3 & 1 + \text{gm}Z_3 \\ Z_2 & -\frac{Z_1 + Z_2}{Z_1} \end{vmatrix} = 0$$

$$\text{or } \text{so } I_i = V \cdot \frac{Z_1 + Z_2}{Z_1 Z_2}$$

$$\rightarrow \left[ (Z_1 + Z_3) \frac{Z_1 + Z_2}{Z_1 Z_2} + 1 + \text{gm}Z_3 \right] \cdot V = 0 \quad \text{not zero}$$

must equal zero

$$\rightarrow (Z_1 + Z_3) \cdot \left( 1 + \frac{Z_2}{Z_1} \right) + Z_2 + \text{gm}Z_2 Z_3 = 0$$

$$\rightarrow Z_1 + Z_2 + Z_3 + \frac{Z_2}{Z_1} (Z_1 + Z_3) + \text{gm}Z_2 Z_3 = 0$$

$$\text{so } R_{\pi} = Z_1 \text{ for BJT: multiply by } Z_1$$

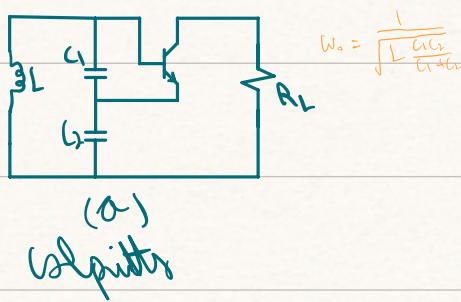
$$\rightarrow \underbrace{(Z_1 + Z_2 + Z_3) \omega_T}_{\text{imaginarily}} + \underbrace{Z_2 (Z_1 + Z_3)}_{\text{real}} + \underbrace{\beta Z_2 Z_3}_{\text{real}} = 0$$

Jm 2T

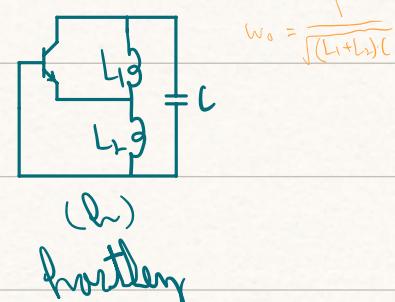
$\because \text{Imaginary} \times \text{Imaginary} = \text{Real}$

- To reduce power consumption, shunt impedances should be purely reactive (i.e., no real part)
  - For an equation involving complex numbers to equal zero, both real and imaginary parts must equal zero.
- $$\therefore Z_2 (Z_1 + Z_3) + \beta Z_2 Z_3 = 0 \rightarrow Z_1 = -Z_3(\beta + 1)$$
- $-Z_3(\beta + 1) \rightarrow Z_3(-\beta - 1) + Z_1 = 0$
- $\lambda \pi (Z_1 + Z_2 + Z_3) = 0 \rightarrow Z_2 = \beta Z_3$
- $Z_3$  must have an opposite sign to  $Z_1$   
 $\rightarrow Z_1$  inductive,  $Z_3$  capacitive  
 $Z_2$  and  $Z_3$  both must have the same sign  
 $\rightarrow Z_2$  inductive or capacitive

- Therefore, the two possible configurations following this approach are : (a) two capacitors + one inductor, (b) two inductors + one cap.



$$w_o = \frac{1}{\sqrt{C_1 C_2}}$$



$$w_o = \frac{1}{\sqrt{(L_1 + L_2)C}}$$

- Practically, the impedances have resistive components. If we reanalyze the shunt oscillator with  $Z_1 = R + j\omega L$ :

$$(R + j\omega L + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}) \omega_T + \frac{1}{j\omega C_1} (R + j\omega L + \frac{1}{j\omega L_2}) - \beta \frac{1}{\omega^2 C_1 C_2} = 0$$

$\xrightarrow{\text{real}}$   $R \omega_T + \frac{L}{C_1} - \frac{1}{\omega^2 C_1 C_2} - \frac{\beta}{\omega^2 C_1 C_2} = 0$

$\xrightarrow{\text{imaginary}}$   $\omega_T (w_L - \frac{1}{\omega L_2} - \frac{1}{\omega C_2}) - \frac{R}{\omega C_1} = 0$

$$Z_2 = \frac{1}{j\omega C_1}$$

$$Z_3 = \frac{1}{j\omega L_2}$$

$$\therefore \text{magnitude condition: } R_{\pi} \leq \frac{\beta + 1}{\omega_0^2 C_{L\pi}} - \frac{L}{C_1}$$

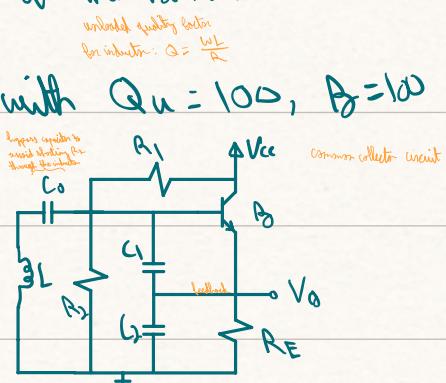
$$1 \text{ phase condition: } \omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}} \text{ s.t., } C_1 = \frac{C_2}{1 + \frac{R_{\pi}}{\beta}}$$

- hence, the internal resistance changes the resonant frequency, as observed from the phase condition. However  $C_1 \approx C_2$  if  $R_{\pi} \gg R$
- the magnitude condition indicates that the gain of the transistor has to be large enough to overcome the resistive losses.
- there is another constraint on the capacitors, as they need to be much larger than the stray/parasitic capacitance of the transistor.

example 7.2: 5MHz colpitts, 10uH inductor with  $Q_u = 100$ ,  $\beta = 100$

$$\therefore \omega_0 = 10\pi \text{ MHz}, Q_u = \frac{\omega_0 L}{R}$$

$$\rightarrow R = \frac{3.14}{\pi} \Omega$$



$$1 \text{ phase condition: } \omega_0 = \frac{1}{\sqrt{L C_{eq}}} \rightarrow C_{eq} = 1.013 \times 10^{-10} = 1.013 \text{ pF}$$

$$\text{if } C_1 = C_2 = C \rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \rightarrow C_1 = C_2 = 202.6 \text{ pF}$$

$$\rightarrow \text{magnitude condition: } R_{\pi} \leq \frac{\beta + 1}{\omega_0^2 C_{L\pi}} - \frac{L}{C_1} \rightarrow R_{\pi} \leq 0.778 \text{ M}\Omega$$

$$\therefore R_{\pi} = \frac{0.026 \beta}{I_C} \rightarrow I_C = 3.34 \text{ mA}$$

### 3 - negative resistance: (third approach)

- ideally, an excited LC circuit will oscillate forever. in reality, the inductor has a large resistance (drawn from its Q-factor) that causes the oscillations to attenuate

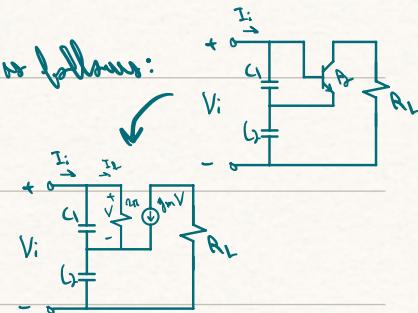


- To counter the attenuation, a "negative resistance" is inserted such that  $|R_i| > |R_L|$

- the input impedance of the AC equivalent active circuit is found as follows:

$$\rightarrow -V_i + \tilde{V} + (I_i + g_m V) X_{C_2} = 0$$

$$\rightarrow V_i = I_i (X_{C_1} + X_{C_2}) - I_d (X_{C_1} - \beta X_{C_2})$$



$$\therefore (I_d - I_i) X_{C_1} + I_d R_d = 0 \rightarrow -I_i X_{C_1} + I_d (R_d + X_{C_1}) = 0$$

$$\therefore I_d = I_i \cdot \frac{X_{C_1}}{R_d + X_{C_1}} \rightarrow V_i = I_i \left[ (X_{C_1} + X_{C_2}) - \frac{X_{C_1}}{R_d + X_{C_1}} (X_{C_1} - \beta X_{C_2}) \right]$$

$$\rightarrow Z_i = \frac{V_i}{I_i} = \frac{\alpha \pi (X_{C_1} + X_{C_2}) + (\beta + 1) X_{C_1} X_{C_2}}{\alpha \pi + X_{C_1}}$$

$\cancel{\text{if}} \quad X_{C_1} \ll R_d \quad \text{and} \quad \beta \gg 1$

$$\rightarrow Z_i = X_{C_1} + X_{C_2} + g_m X_{C_1} X_{C_2}$$

$$\therefore Z_i = \left( \frac{1}{j \omega C_1} + \frac{1}{j \omega C_2} \right) - \frac{g_m}{\omega^2 C_1 C_2}$$

real part  
in negative  
impedance  
circuit  
 $-R_i$

$$\rightarrow Z_i = -R_i + \frac{1}{j \omega C_i} \rightarrow \frac{C_i}{C_i + L}$$

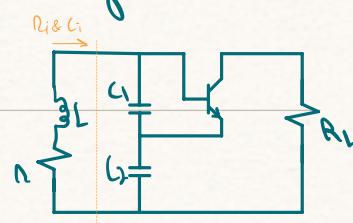
- Therefore, if a coil with resistance  $R$  is connected across the above equivalent circuit, it becomes a <sup>inductor</sup> ~~series~~ oscillator with the following oscillation conditions:

• magnitude:  $R_i > R$

$$\rightarrow \frac{g_m}{\omega^2 C_1 C_2} > R$$

equivalent to second approach:  
 $\frac{1}{R_d} R_i R_d \leq \frac{1}{R_d} \frac{(R_i + R)}{\omega^2 C_1 C_2} - \frac{1}{L} \frac{1}{R_d} \rightarrow R \leq \frac{g_m}{\omega^2 C_1 C_2}$

• phase condition:  $\omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_i}{C_i + L}}}$

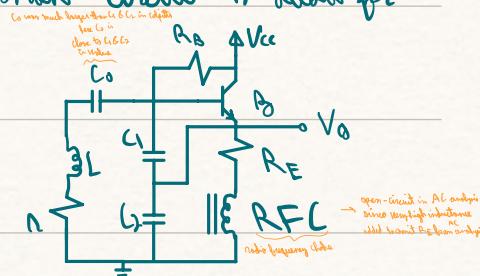


## + Chapp - gowinet oscillator:

- modified version of the Colpitts oscillator, where the bypass capacitor ( $C_0$ ) is made small enough to be included in the resonant circuit to allow for

more freedom in designing the oscillator.

$$\rightarrow \omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_0 C_i}{C_0 + C_i}}} \quad L \cdot C_i = \frac{C_0 C_i}{C_0 + C_i}$$



example 7.4:  $f_0 = 1 \text{ MHz}$ ,  $X_L = 800 \Omega$ ,  $Q_m = 200$ ,  $g_m = 6 \text{ mS}$

$$\therefore X_L = \omega_0 L \Rightarrow L = 129.3 \mu\text{H}, \quad R = \frac{\omega_0 L}{Q_m} = 4 \Omega$$

$$\therefore \text{phase condition: } \sqrt{L \cdot \frac{C_0 C_i}{C_0 + C_i}} = \omega_0, \quad C_i = \frac{C_m}{2}, \quad C_m = C_1, C_2$$

$\rightarrow$  need two equations to find  $C_m$  and  $C_0$ ,  $\therefore g_m = \frac{I_c}{0.026}$   $\xrightarrow{\text{assuming load}}$

$$\text{from magnitude condition: } \frac{g_m}{(\omega_0 C_m)^2} \geq 1 \Rightarrow C_m = 6.16 \text{ nF}$$

from phase condition

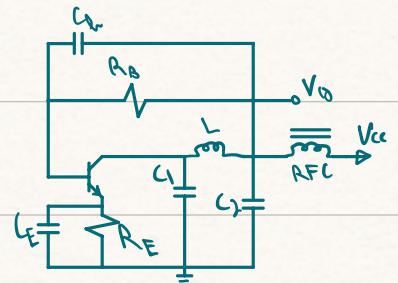
$$\rightarrow C_0 = 212.9 \text{ pF}$$

## + Pierce oscillator: common emitter

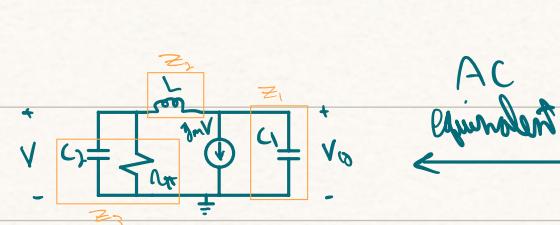
- bias resistors are shorted and do not shunt the

tuned circuit, which gives a more stable

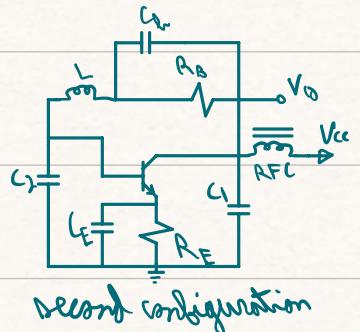
frequency.



first configuration



AC equivalent



second configuration

- find  $G(w)$  and  $H(w)$  to find oscillation conditions.

$$H(\omega) = \frac{V}{V_0} : \text{ Voltage division, } V = V_0 \cdot \frac{Z_3}{Z_1 + Z_2}$$

$$G_r(\omega) = \frac{V_0}{V} : V_0 = I z_1 \cdot Z_1, -I z_1 = g_m V \cdot \frac{Z_2 + Z_3}{Z_1 + Z_2 + Z_3}$$

$$\therefore G_r(\omega) = -g_m Z_1 \cdot \frac{Z_2 + Z_3}{Z_1 + Z_2 + Z_3}$$

$$\text{for } G_r H > 1 \rightarrow \frac{-g_m Z_1 Z_3}{Z_1 + Z_2 + Z_3} > 1$$

$$\therefore Z_1 = \frac{1}{j\omega C_1}, Z_2 = j\omega L, Z_3 = \frac{\pi}{\pi j\omega C_2 + 1}$$

$$\rightarrow G_r H = \frac{-g_m \pi \pi}{(j\omega \pi C_2 + 1)(j\omega C_1) \left\{ \frac{1}{j\omega C_1} + j\omega L + \frac{\pi}{\pi j\omega C_2 + 1} \right\}}$$

$$\rightarrow G_r H = \frac{-\beta}{1 - \omega^2 L C_1 + j\omega \pi \underbrace{\{C_1 + C_2 - \omega^2 L C_1 C_2\}}_{\text{imaginary part}}}$$

$$\therefore \text{Im}\{G_r H\} = 0 \rightarrow j\omega \pi \{C_1 + C_2 - \omega^2 L C_1 C_2\} = 0$$

$$\rightarrow \omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}} \quad (\text{phase condition})$$

$$\lambda \frac{\frac{\beta}{\omega_0^2 L C_1 - 1}}{\omega_0^2 L C_1 - 1} > 1 \quad (\text{magnitude condition})$$

## + Crystal oscillators:

- frequency stability and precision depend on the Q-factor.

higher Q-factor  $\rightarrow$  more stable and precise

- because of the inductor's resistances, the Q-factor of LC resonant circuits is

less than 1.

Q-factors  
up to 10<sup>6</sup>

- crystals, such as quartz or ceramic, are electromechanical devices that exploit

piezoelectric effect:



reverse piezoelectric effect:



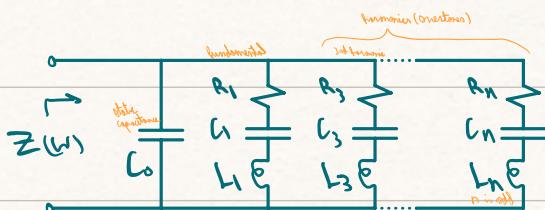
expands and contracts at a specific frequency

the reverse piezoelectric effect to generate voltage at a rate depending on  
much more precise and stable than LC resonator  
the crystal resonant frequency.

- a crystal is essentially a thin wafer between two conductive plates. since the crystal material is a dielectric, the crystal behaves as a static capacitor with capacitance:

$$\frac{\epsilon_0 \cdot \epsilon_r \cdot A}{t}$$

- the fundamental resonant frequency of a crystal is inversely proportional to its thickness.
- the equivalent electric circuit of a crystal is made up of many resonant circuits in parallel with each other, with each being an odd harmonic of the fundamental frequency.



- the crystals cannot oscillate at multiple modes simultaneously. They are often made to oscillate at one of the harmonics to achieve higher frequencies; however, the generated signal becomes weaker at higher harmonics.
- the static capacitance is typically in the pF range, whereas the n-th capacitance is in the fF range.
- The resonant frequencies at the fundamental mode are found from:

$$Z(w) = X_{C_0} / \left[ R_1 + X_{L_1} + X_{C_1} \right] = \frac{\frac{1}{jwC_0} \left[ R_1 + \frac{1}{jwL_1} + jwL_1 \right]}{R_1 + \frac{1}{jwC_0} + \frac{1}{jwL_1} + jwL_1}$$

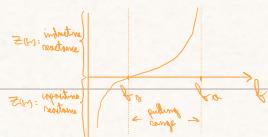
if  $R_1$  is assumed to be very small  $\rightarrow Z(w) = \frac{\frac{1}{jwC_0} \left[ \frac{1}{jwL_1} + jwL_1 \right]}{\frac{1}{jwC_0} + \frac{1}{jwL_1} + jwL_1}$

- hence resonance occurs when  $Z(w) = 0$  or  $Z(w) = \infty$

1- for  $Z(w) = 0$   $\rightarrow \frac{1}{jwC_0} \cdot \left[ \frac{1}{jwL_1} + jwL_1 \right] = 0 \rightarrow \frac{1}{w_0 L_1} = w_0 L_1$   
 $\therefore w_0 = \frac{1}{\sqrt{L_1 C_0}}$  if  $L_1 \ll C_0 \rightarrow w_0 \approx w_0$

2- for  $Z(w) = \infty \rightarrow \frac{-j}{w_0 C_0} + \frac{-j}{w_0 L_1} + jw_0 L_1 = 0 \rightarrow w_0 = \frac{1}{\sqrt{L_1 \frac{C_0 L_1}{C_0 + L_1}}}$   
 $f_a$  is generally used in defining crystals rather than  $f_0$

- a relation between  $f_a$  and  $f_0$  can be derived to give:



$$f_a = \left( 1 + \frac{L_1}{C_0} \right)^{1/2} \cdot f_0 \quad \begin{matrix} \text{binomial approximation} \\ \because \frac{L_1}{C_0} \ll 1 \text{ given } L_1 \ll C_0 \end{matrix} \quad f_a \approx f_0 \cdot \left( 1 + \frac{L_1}{2C_0} \right) \quad \begin{matrix} \text{between } f_0 \text{ and } f_a \end{matrix}$$

\* pulling range: the region in which the crystal oscillator operates

example 7.6:  $C_0 = 5.1 \mu F$ ,  $C_1 = 21 \mu F$ ,  $R_1 = 29 \Omega$  (from table)

$$\therefore Q_u = \frac{w_0 L_1}{R_1}, \quad jw_0 L_1 = \frac{j}{w_0 C_0} + \frac{j}{w_0 L_1}$$

$$\rightarrow L_1 = \frac{1}{Q_u w_0} = \frac{1}{53403.2} \times \frac{1}{2\pi \times 10^6} \quad \begin{matrix} \text{very large} \\ \rightarrow \text{must take fine or more significant figures} \end{matrix} \quad \rightarrow L_1 = 0.037278 \text{ H}$$

- crystals are used in oscillators as either parallel or series resonators

- if the parallel mode is chosen, the inductor in simple inductor

oscillators (e.g., Colpitts) is removed and replaced by the crystal.



- the magnitude oscillation conditions are the same for crystal oscillator

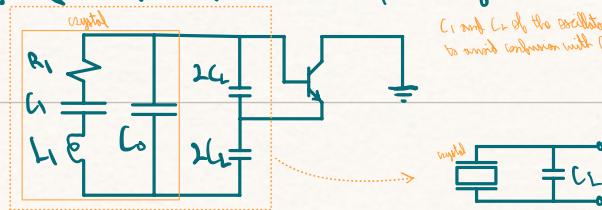
except  $R_m$  for the crystal replaces  $R_1$  in the previous induction

## as LC oscillators.

colpit's capacitances

- for a colpitts crystal oscillator, the magnitude condition is:  $\frac{gm \cdot X_{C1} \cdot X_{C2}}{R_1} > 1$

- The resonance frequency is found by replacing the crystal with its equivalent circuit:



$C_1$  and  $C_2$  of the oscillator were assumed equal  $\rightarrow C_1 = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{2} = \frac{C_2}{2}$   
to avoid confusion with  $C_1$  do the crystal,  $C_1$  and  $C_2$  were replaced by  $2L_1$  each

looked anti-resonance frequency

$$\rightarrow \omega_0 L_1 = \frac{1}{\omega_0 C_1} + \frac{1}{\omega_0 (C_0 + L_1)} \xrightarrow{\text{series resonance}} \omega_0 = \frac{1}{2\pi \sqrt{L_1 \cdot \frac{C_1 \cdot (C_0 + L_1)}{C_1 + C_0 + L_1}}} = \omega_a'$$

$$\therefore \text{if } C_1 = \infty \rightarrow \omega_0 = \frac{1}{2\pi \sqrt{L_1 C_1}} \rightarrow \omega_a' = \omega_a$$

$$\wedge \text{ if } C_1 = 0 \rightarrow \omega_0 = \frac{1}{2\pi \sqrt{L_1 \frac{C_1 C_0}{C_1 + C_0}}} \rightarrow \omega_a' = \omega_a$$

$$\omega_0 \leq \omega_a' = \omega_0 \leq \omega_a$$

- therefore, the operating frequency is always within the pulling range.

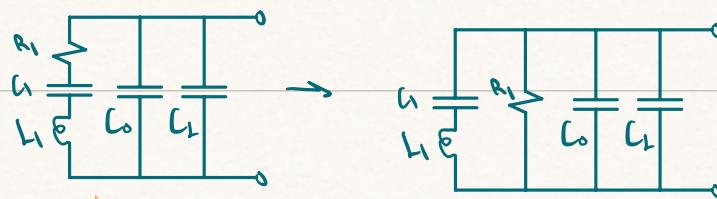
$$\omega_0 = \omega_a$$

- since  $C_1$  is in parallel, the series resonance frequency is unaffected by it.

- The capacitor in parallel with the crystal also affects the loaded Q-factor of the oscillator.

- To find the loaded quality factor,  $R_p$ , is made parallel with the

$$\text{Resonant circuit: } R_p = R_1 \left(1 + \frac{X_p^2}{R_1^2}\right), \quad X_p = \frac{1}{\omega (C_0 + C_1)} \xrightarrow{\text{loading}}$$

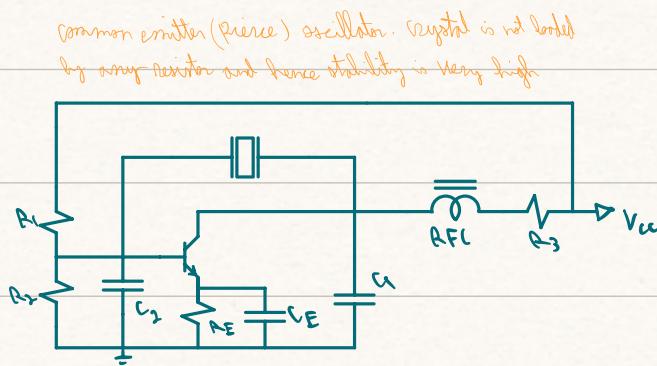
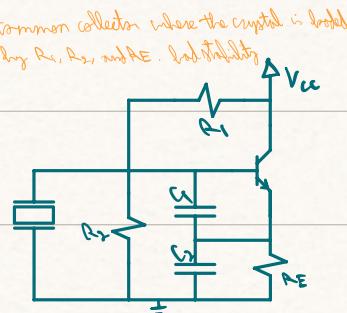
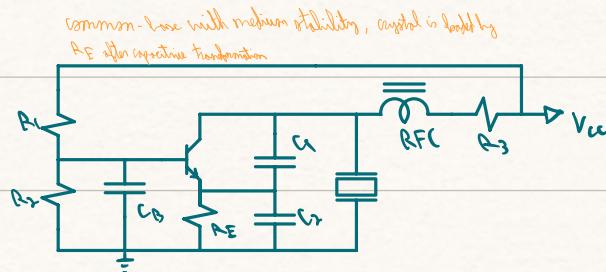


$$\rightarrow Q_L = \frac{R_p}{\omega L_1} \xrightarrow{\text{parallel}} \approx \frac{\omega L_1}{R_1}$$

increasing  $C_1$  decreases frequency stability

- hence, increasing  $C_1$  decreases  $R_p$  and, in turn, the Q-factor.

- the loaded Q factor of a crystal reduces when it is shunted by a resistor.  $Q_u = \frac{R_p}{wL}$ ,  $Q_L = \frac{R_p || R_s}{wL} < Q_u$



### + series mode crystal oscillators:

- in parallel mode, the inductor is simply replaced by the crystal whereas in series mode, all the components are left unchanged and the crystal is added to the feedback path.
- the crystal in the feedback path will act as a short circuit if the operating frequency matches the crystal's series resonance frequency.

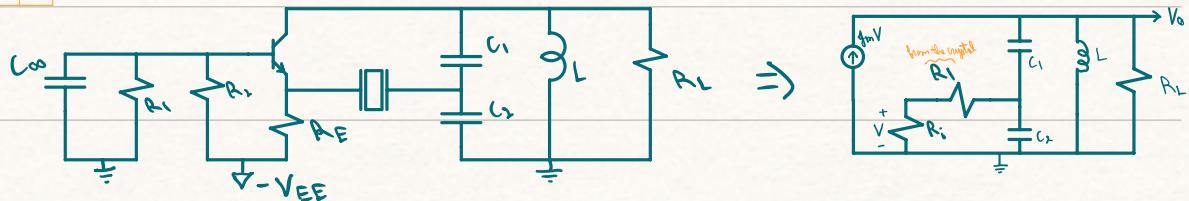
$$\rightarrow f_{\text{eff}} = f_0 \quad : \quad \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{LC_{\text{eq}}}}$$

at resonance

$$\boxed{\square} = \boxed{\frac{1}{L_1 C_1}}$$

common-base series-mode oscillator

C is coupled with the output  
with  $L_1$  as the LC circuit

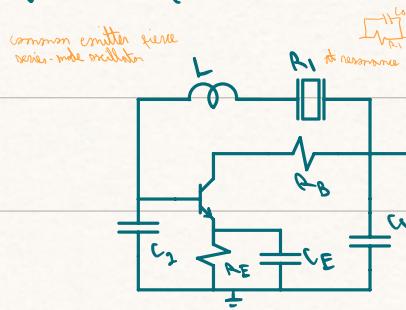


$$\rightarrow V = V_0 \cdot \frac{C_1}{L_1 + L_2} \cdot \frac{R_i}{R_i + R_f}$$

same analysis as conventional oscillators

the inductor is in the feedback path

- for a Pierce oscillator, the crystal is inserted in series with the inductor



$$\rightarrow f_{\text{eff}} = f_{\text{eff}} = \frac{1}{2\pi\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$$

- the Q factor of the above circuit is affected by the crystal's

$$\text{resistance, such that: } Q = \frac{\omega_0 \cdot L}{R + R_i} \xrightarrow{\substack{\text{inductive resistance} \\ \text{crystal resistance}}} \omega_0 = \omega'$$

- if a capacitor is placed in series with the crystal the parallel resonance does not change, but the series resonance changes when the capacitor is taken into account.

$$f'_0 = \frac{1}{2\pi\sqrt{L_1 \cdot \frac{C_1 (C_0 + C_s)}{C_1 + C_0 + C_s}}}$$

, if  $C_s = 0 \rightarrow f'_0 = f_0$   
if  $C_s = \infty \rightarrow f'_0 = f_{\text{eff}}$

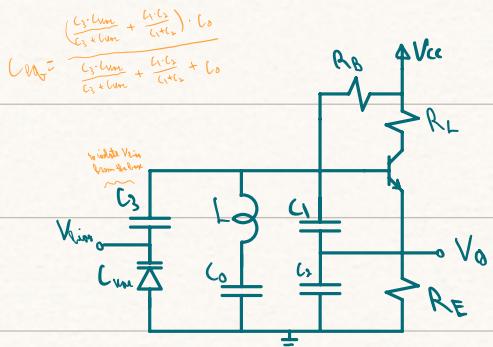
$$\boxed{\square} = \boxed{\frac{1}{L_1 C_1}}$$

## + Voltage controlled oscillators:

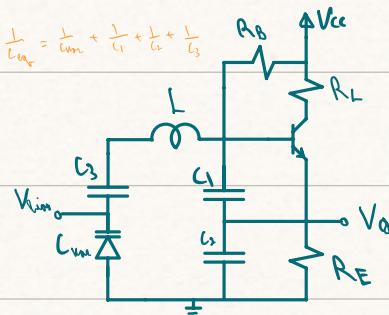
- a local oscillator is required at the receiver to demodulate signals.

- One way to vary an oscillator's frequency is to change the capacitors' values.
- a capacitor whose value depends on the voltage across it can be created by using a reverse biased pn junction.
 

Varactor or Varicap diode
- increasing the voltage across a reverse biased pn junction increases the depletion regions thickness, thereby decreasing the capacitance
- a conventional LC oscillator can be converted to a VCO by inserting a varactor diode.
- the tuning range of a VCO depends on the capacitance range of the varactor diode and its location in the LC circuit.



Deg. oscillation with varactor diode in parallel



Deg. oscillation with varactor diode in series (all capacitors in series)

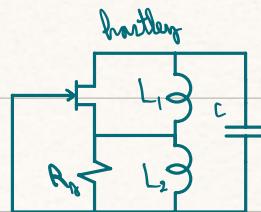
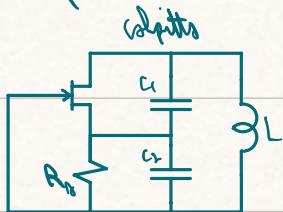
$$\rightarrow f_{o,min} = \frac{1}{2\pi\sqrt{L \cdot C_{var,max}}} , \quad f_{o,max} = \frac{1}{2\pi\sqrt{L \cdot C_{var,min}}}$$

at  $C_{var,max}$       at  $C_{var,min}$

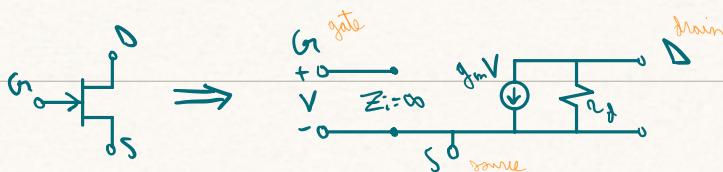
- tuning range:  $f_{o,min} \rightarrow f_{o,max}$  (large range is better)

+ field effect transistor oscillator:

- all BJTs in previous oscillators can be connected to FETs.

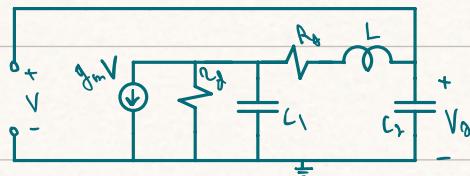
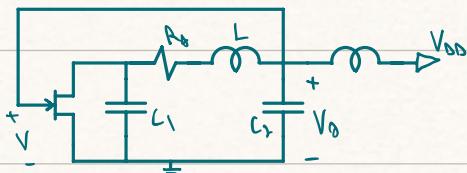


- equivalent model of FET is the same as BJT with  $Z_i = \infty$ :



$$g_m = g_{m0} \cdot \left( \frac{I_D}{I_{DSS}} \right)^{\frac{1}{2}}$$

- the FET pierce oscillator shown below is analyzed as follows:



$$\therefore V = V_0 \rightarrow H = 1, \quad \text{if } R_f \gg \frac{1}{\omega C_L} \quad \text{assume open circuit}$$

$$\rightarrow V_0 = -g_m \cdot V \cdot \frac{jC_L}{jC_L + R_S + X_L + X_{C2}} \cdot X_{C2}$$

$$\rightarrow j = \frac{-g_m \cdot jC_L \cdot X_{C2}}{jC_L + X_{C2} + X_L + R_S} = \frac{g_m \cdot \frac{1}{\omega^2 C_L C_2}}{R_S + j\omega(\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2})}$$

$$\text{for } G \cdot H = 1 : \quad \omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2} = 0$$

$$\therefore \text{phase condition: } \omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$$

$$\text{magnitude condition: } \frac{g_m}{R_S \cdot \omega_0^2 \cdot L_1 C_2} \geq 1$$

## EE 524: Homework #2

$$7.1 : \text{ if } H = \frac{V_1}{V_0} \text{ and } G_1 = \frac{V_0}{V_1}, \quad V_1 = -g_m V_0 \cdot 1k$$

$$\rightarrow H = -g_m \cdot 1k \quad \text{and} \quad V_0 = -g_m \cdot V_1 \cdot Z_L \rightarrow G_1 = -g_m \cdot Z_L$$

$$\text{d.t. } Z_L = X_L // X_C // R_L \rightarrow \frac{1}{Z_L} = \frac{1}{j\omega L} + j\omega C + \frac{1}{R_L}$$

$$\therefore G_1 = -g_m \cdot \frac{1}{\frac{1}{j\omega L} + j\omega C + \frac{1}{R_L}}$$

$$\rightarrow G_1 H = \frac{g_m \cdot 1k}{\frac{1}{j\omega L} + j\omega C + \frac{1}{R_L}} \gg 1$$

\* phase condition:  $\arg\{G_1 H\} = 0 \rightarrow \text{Im}\{G_1 H\} = 0$

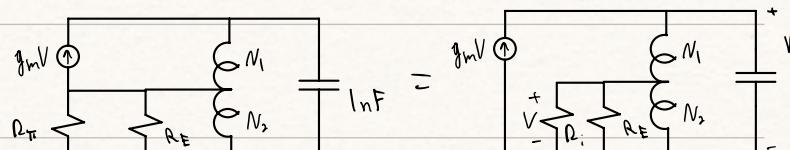
$$\rightarrow \frac{1}{j\omega L} + j\omega C = 0 \rightarrow f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

\* magnitude condition:  $\frac{g_m \cdot 1k}{1/5k} \gg 1 \rightarrow g_m^2 \cdot 5 \times 10^6 \gg 1$

from phase condition:  $L = 2.533 \mu H$  for  $10 MHz$

from magnitude condition:  $g_m \geq 0.45 \text{ mS}$

7.3: the AC equivalent circuit is:



$$\rightarrow V_0 = g_m V \cdot Z_L \rightarrow G_1 = g_m \cdot Z_L$$

$$\text{and } V = V_0 \cdot \frac{X_{L2} // R_E // R_i}{X_{L1} + X_{L2} // R_E // R_i} \rightarrow H = \frac{X_{L2} // R_E // R_i}{X_{L1} + X_{L2} // R_E // R_i}$$

$$\rightarrow \frac{G_1 H}{g_m} = (X_{L2} // R_E // R_i + X_{L1}) // X_C \gg 3$$

After calculating  $I_C$ ,  $N_i$  is found as  $12.9 \rightarrow R_E // R_i \approx N_i$

$$\rightarrow G_{TH} = \left( \frac{j\omega L_2 \cdot R_i}{j\omega L_2 + R_i} + j\omega L_1 \right) // \frac{1}{j\omega C} \cdot g_m$$

- assuming that  $N_i \gg \omega L_2$

$$\rightarrow G_{TH} = (j\omega L_1 + j\omega L_2) // \frac{1}{j\omega C} \cdot g_m$$

$$H = \frac{L_2}{L_1 + L_2}$$

$$\rightarrow G_{TH} = \frac{L_2 / L \cdot g_m}{j\omega (L_1 + L_2) + \frac{1}{j\omega C}}$$

$$G_{TH} = \frac{N_i R_E \cdot (L_1 + L_2)^2}{R_i + R_E} \text{ for } \text{Im}\{G_{TH}\} = 0 \rightarrow \omega = \frac{1}{\sqrt{(L_1 + L_2) \cdot C}}$$

inductive transformation

$$\therefore L_{\text{total}} = L_1 + L_2 = 1.013 \text{ nH for } 5 \text{ MHz}$$

$$\text{magnitude condition: } \frac{L_2 \cdot g_m}{C} \geq 3$$

$$\text{however, } \omega L_2 \ll R_i \rightarrow L_2 = 41 \text{ nH}$$

$$\therefore \frac{L_1}{L_2} = \left( \frac{N_1}{N_2} \right)^2 \rightarrow \frac{N_1}{N_2} = 4.87$$

$$7.4: \text{phase condition for a colpitts FET oscillator: } \omega = \frac{1}{\sqrt{L \frac{C_1 C_2}{C_1 + C_2}}}$$

$$\text{magnitude condition: } \frac{g_m}{R_s \omega C_1 C_2} \geq 2.5$$

$$\text{from the phase condition: } C_{eq} = 2.533 \times 10^{-10} \text{ F}$$

$$\therefore R_s = \frac{\omega \cdot L}{Q_H} = \frac{\pi}{5} \Omega \quad \times \quad Q_H \text{ is source}$$

$$\text{from mag. condition: } C_1 C_2 \leq 8.06 \times 10^{-19}$$

$$\therefore C_1 = C_2 = 0.51 \text{ nF} \text{ satisfies both conditions}$$

$$C_1 = 379.5 \text{ pF}, C_2 = 759 \text{ pF}$$

$$7.6: \text{phase condition of a Colpitts: } \omega_0 = \frac{1}{\sqrt{LC_{eq}}}$$

$$\rightarrow \text{for } 3.5 \text{ MHz: } C_{eq} = 1.38 \text{ nF}$$

$$\text{magnitude condition: } R_{IT} \leq \frac{1 + \beta_B}{\omega_0^2 C_1 C_2} - \frac{L}{C_1}$$

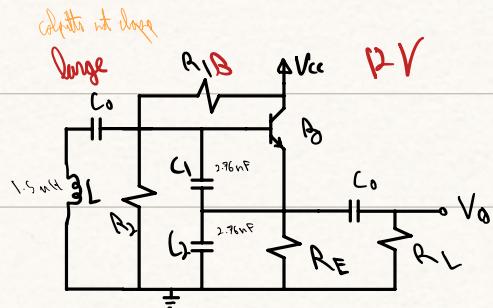
$$\text{assuming } C_1 = C_2 = 2 C_{eq} = 2.76 \text{ nF} \quad \& \quad R = \frac{\omega_0 L}{Q_H} = 0.22 \Omega$$

$$\rightarrow R_{IT} \leq 122.15 \Omega$$

$$\therefore R_{IT} = \frac{0.026 \Omega}{I_C} \rightarrow I_C \geq \frac{0.026 \Omega}{122.15 \Omega}$$

$$\therefore I_C \geq 2.13 \times 10^{-8} \text{ A}$$

$2.13 \times 10^{-5} \text{ A}$



Homework #2 redone:

$$7.1: \because L \parallel C \rightarrow f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \rightarrow L = 2.53 \mu H$$

$$\therefore H = \frac{V_o}{V_i} = -g_m \cdot 1k$$

$$\therefore G_r = \frac{V_o}{V_i} = -g_m \cdot 5k$$

$$\rightarrow G_r \cdot H > 1 \rightarrow g_m^2 \cdot 5 \times 10^6 > 1$$

$$\therefore g_m > 0.449 \text{ mS}$$

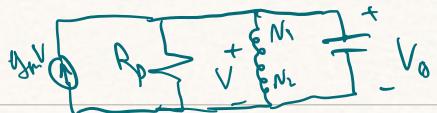
$$7.3: \text{equivalent: } \text{[circuit diagram showing a dependent current source with gain } g_m \text{ and a voltage-controlled voltage source with gain } 1 \text{ in series with the dependent source]} \quad n_i = \frac{1}{g_m} \rightarrow g_m = \frac{I_C}{0.026}$$

$$\text{to find } I_C: -1k + \frac{I_C}{8} \cdot 360k + 0.9 + I_C \cdot 2k = 0$$

$$\rightarrow I_C = 2.02 \text{ mA} \rightarrow n_i = 12.9 \Omega$$

$$\rightarrow L_1 \parallel R_E \approx R_i$$

inductive Transformation:



$$\frac{1}{L_1 + \frac{R \cdot L_2}{R+L_2}} = \frac{1}{R_p} + \frac{1}{L_1 + L_2} \rightarrow \frac{1}{R_p} = \frac{R+L_2}{L_1(R+L_2) + R \cdot L_2} - \frac{1}{L_1 + L_2}$$

$$\rightarrow \frac{1}{R_p} = \frac{(R+L_2) \cdot (L_1 + L_2) - L_1(R+L_2) - RL_2}{(L_1(R+L_2) + RL_2)(L_1 + L_2)}$$

$$\rightarrow R_p = \frac{(L_1(R+L_2) + RL_2) \cdot (L_1 + L_2)}{(R+L_2)(L_1 + L_2) - L_1(R+L_2) - RL_2}$$

$$\rightarrow R_p = \frac{(L_1R + L_1L_2 + RL_2)(L_1 + L_2)}{RL_1 + RL_2 + L_1L_2 + L_2^2 - RL_1 - L_1L_2 - RL_2}$$

$$\rightarrow R_p = \frac{L_1^2R + L_1^2L_2 + 2RL_1L_2 + L_1L_2^2 + L_2^2R}{L_2^2}$$

if  $L_1 \cdot L_2$  and  $L_1 \cdot L_2^2$  are neglected

$$\rightarrow R_p \approx R \cdot \left( \frac{L_1 + L_2}{L_2} \right)^2 \quad \text{inductive transform}$$

$$\therefore L_1 + L_2 = L_T \quad \text{and} \quad \frac{L_1}{L_2} = \left( \frac{N_1}{N_2} \right)^2 \rightarrow L_2 = L_1 \cdot \left( \frac{N_2}{N_1} \right)^2$$

$$\rightarrow R_p \approx R_i \cdot \left( \frac{L_1 \left( 1 + \left( \frac{N_2}{N_1} \right)^2 \right)}{L_1 \left( \frac{N_2}{N_1} \right)^2} \right)^2 = R_i \cdot \left( \frac{N_1^2 + N_2^2}{N_1^2} \cdot \frac{N_1^2}{N_2^2} \right)$$

$$\rightarrow R_p \approx R_i \cdot \left( 1 + \left( \frac{N_2}{N_1} \right)^2 \right)$$

$$\therefore f_{b_0} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L_T \cdot C}} \rightarrow L_T = 1.01 \text{ nH for } 5 \text{ MHz}$$

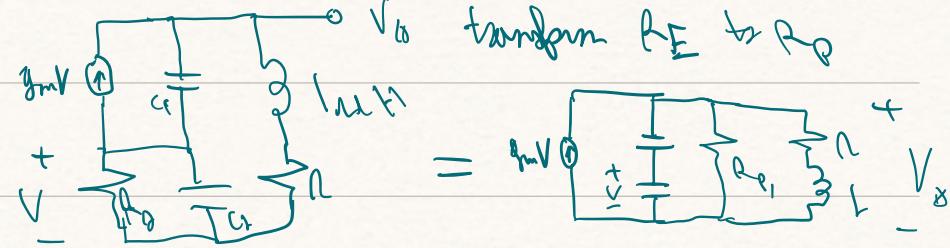
$$\therefore H = \frac{V}{V_0} = \frac{L_2}{L_1 + L_2} \quad \text{and} \quad G_r = \frac{V_0}{V} = g_m \cdot R_p$$

$$\rightarrow G_r \cdot H = g_m \cdot R_p \cdot \frac{L_1 + L_2}{L_2} = \frac{L_1 + L_2}{L_2}$$

$$\therefore G_r \cdot H > 3 \rightarrow L_2 = \frac{1}{3} L_1 \approx 0.33 \text{ nH}$$

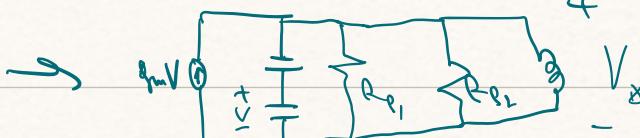
$$\rightarrow L_1 \approx 0.67 \text{ nH} \rightarrow \frac{N_1}{N_2} = \sqrt{2}$$

7.4: equivalent circuit:



$$R_{P1} = R_p \cdot \left( \frac{L_1 + L_2}{L_1} \right)^2$$

$$\therefore f = 10 \text{ MHz} = \frac{1}{2\pi} \sqrt{\frac{1}{L} \cdot \frac{C_1 C_2}{L_1 + L_2}} \rightarrow C_{eq} = 0.2533 \text{ nF}$$



$$R_{P2} = Q \cdot W L$$

$$\rightarrow R_{P2} = 6.283 \Omega$$

$$\rightarrow G_r = g_m \cdot (R_{P1} // R_{P2}) > 2.5$$

$$\text{if } R_{P2} \gg R_{P1} \rightarrow G_r = g_m \cdot R_{P1} > 2.5$$

$$\text{assuming } R_{P1} = R_p / Q_1 \cdot \left( \frac{L_1 + L_2}{L_1} \right)^2$$

$$\rightarrow G_r \approx g_m \cdot R_p \cdot \frac{L_1 + L_2}{L_1} > 2.5$$

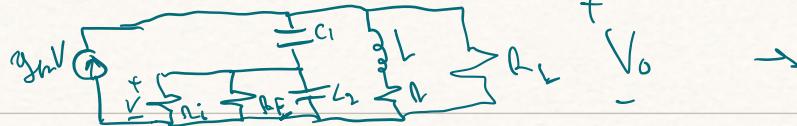
$$\rightarrow L_2 > 1.5 L_1$$

$$\rightarrow \frac{1.5 L_1^2}{2.5 L_1} = 0.2533 \text{ nF}$$

$$L_2 = 0.633 \text{ nF} \rightarrow C_1 = 0.422 \text{ nF}$$

$$\text{if } G_r H > 3 \rightarrow C_1 = 0.38 \text{ nF} \text{ and } L_2 = 0.96 \text{ nF}$$

7.6:



$$R_{P1} = \mu_i \cdot \left( \frac{C_1 + C_2}{C_1} \right)^2, \quad R_{P2} = Q \cdot W_L$$

$$\rightarrow g_r = g_m \cdot \left( R_{P1} / R_1 / (R_{P2}) \right) \quad \text{if } R_{P1} \ll R_{P2} \text{ and } R_1$$

$$\wedge \quad H = \frac{C_1}{C_1 + C_2} \rightarrow g_r H \approx \frac{C_1 + C_2}{C_1} \geq 1$$

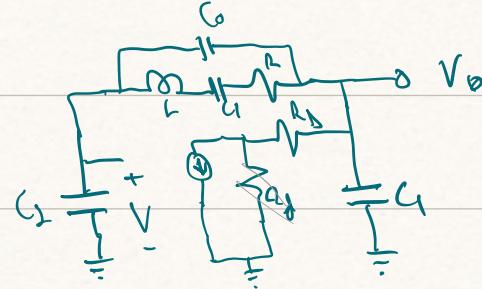
$$\wedge \quad f_n \text{ MHz} = \frac{1}{2\pi C_{eq}} \rightarrow C_{eq} = 1.38 \text{ nF}$$

$$\rightarrow I_C > 23 \times 10^{-5} \text{ A} \quad \text{if } h = h_2 = 296 \text{ nF}$$

$$\rightarrow -I_2 + R_B \cdot \frac{I_L}{100} + 0.7 + R_E \cdot I_C = 0$$

$$\rightarrow \frac{R_B}{100} + R_E = \frac{12 - 0.7}{I_L}$$

7.22: equivalent circuit:



## Second practice:

12/2020

$$Q_2: R_1 = 40 \Omega, L_1 = 3 \text{ mH}, C_0 = 6.2 \mu\text{F}$$

$$a) \quad w_s = \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_a = 2\pi \text{ MHz} = \frac{1}{2\pi} \cdot \sqrt{\frac{L_{\text{tot}}}{L_{\text{tot}} + L_0}} \rightarrow L = 12.44 \text{ mH}$$

$$\rightarrow f_8 = \frac{f_{50}}{\left(1 + \frac{c_1}{f_0}\right)^{v_2}} \rightarrow f_8 = 25.9939 \text{ Hz}$$

→ pulling range:  $f_a - f_s = 6288 \text{ Hz}$

$$\rightarrow Q_n = \frac{w_a \cdot L}{n} = 51010$$

$$d) C_p = C_0 + 32 \rho F$$

$$f_{\text{res}} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{\frac{L \cdot C_1 C_p}{C_1 + C_p}}} \approx 25.9947 \text{ MHz}$$

must be  
very accurate

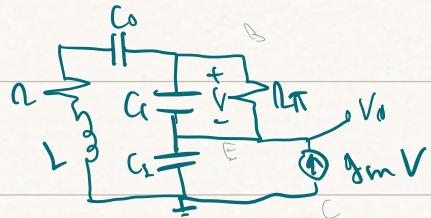
so  $f_s = \text{no change}$  since series

$$\rightarrow d_2' = f_{\text{FS}} \cdot \left(1 + \frac{c_1}{c_0}\right)^{k_2} = 25.9957 \text{ m}$$

for this is for

$$Q3: \text{ a) } -10 + I_C \cdot \frac{b_2}{100} + 0.9 + I_C \cdot 10k = 0$$

$$\rightarrow I_C = 0.5 \text{ mA}$$



$$\text{d}) \quad z_1 = \chi_{C_1} \| M_1, \quad z_2 = \chi_{C_2}, \quad z_3 = \chi_{C_3} + \chi_L$$

$$(z_1 + z_2 + z_3) R_T + z_1 (z_2 + z_3) + 8 z_2 z_3 = 0$$

$$z_1 \approx \chi_{C_1}$$

$$\rightarrow (\chi_{C_1} + \chi_{C_2} + \chi_L) R_T + R R_T + \chi_{C_1} (\chi_{C_2} + \chi_L) \\ + \chi_{C_1} \cdot R + 8 \chi_{C_1} \chi_{C_2}$$

$$\rightarrow (\chi_{C_1} + \chi_{C_2} + \chi_L) R_T + \chi_{C_1} R = 0 \quad \text{Phase}$$

$$\therefore R R_T + \chi_{C_1} (\chi_{C_2} + \chi_L) + 8 \chi_{C_1} \chi_{C_2} = 0$$

$$\therefore \frac{R + R_T + R_T}{j\omega C_1} + j\omega L R_T = 0 \quad \omega L = \frac{2}{T} \quad \rightarrow \omega = \frac{1}{\sqrt{L^2}}$$

$$\rightarrow (R + 2R_T) \cdot \frac{1}{\omega C_1} = \omega L \cdot R_T$$

$$\therefore \boxed{\frac{R + 2R_T}{2R_T} = \omega^2 C_1 L} \quad \approx R_T = \frac{0.0268}{I_C} \quad \rightarrow C_1 = 1.266 \mu \text{F}$$

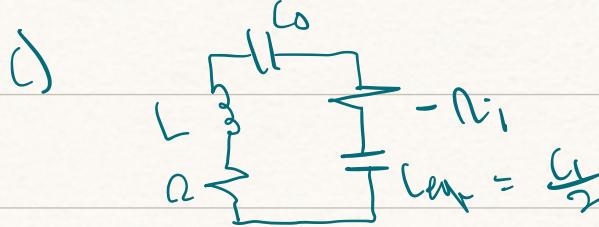
$$\therefore R R_T - \frac{1}{\omega^2 C_1 L} + \frac{L}{C_1} - \frac{R}{\omega^2 C_1 L} = 0 \quad \approx C_2$$

$$\therefore \boxed{R R_T = \frac{R+1}{\omega^2 C_1 L} - \frac{L}{C_1}}$$

$$R = \frac{\omega_0 L}{Q_R} = \frac{\pi}{25}$$

$$\rightarrow R_T \leq \frac{25}{\pi} \cdot \left[ \frac{R+1}{\omega^2 C_1^2} - \frac{L}{C_1} \right] = 31102.5$$

$$\therefore I_C \geq \frac{0.0268}{31102.5} = 83.6 \text{ mA}$$



$$\text{d) } I = \frac{\pi}{25}$$

$$\text{d) } W_0 = \sqrt{\frac{1}{L} \cdot \frac{C_{total}}{C_1 + C_2}}$$

$$\rightarrow L_{eq} = 6.3326 \times 10^{-6} \text{ F}$$

$$r - r_i = \frac{-j_m}{\omega^2 C_1^2}, \quad r_i \geq r \text{ moy}$$

$$\therefore j_m = \frac{I_c}{0.026} = 19.23 \text{ mA}$$

$$\rightarrow C_1 \leq 3.113 \text{ nF}$$

$$\text{assume } C_1 = C_2 = 3.113 \text{ nF} \rightarrow C_{eq} = 1.556 \text{ nF}$$

$$\rightarrow C_0 = 1.07 \text{ nF}$$

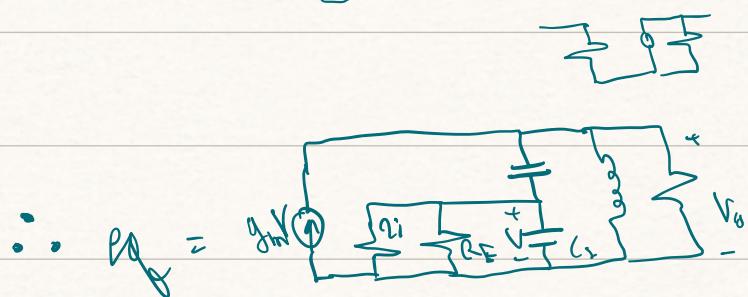
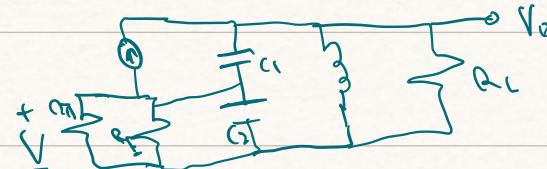
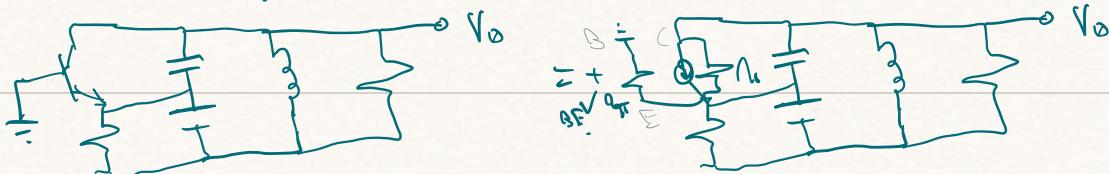
$$C_0 \cdot L_{eq} + C_{eq} C_{eq}' = C_0 C_{eq}$$

$$\rightarrow C_0 = \frac{C_{eq} C_{eq}'}{C_{eq} - C_{eq}'}$$

12/2021

Q2: Q1 C<sub>B</sub> is a bypass transistor that grounds the base

C<sub>0</sub> is used to isolate the load from the DC bias of the transistor, only AC passes to output.



$$\therefore R_B = \frac{V_B}{I_B}$$

$$\text{b) } \therefore -10 + I_c \cdot \frac{R_E}{100} + 0.7 + I_c R_E = 0$$

$$\rightarrow \frac{R_B}{100} + R_E = \frac{9.3}{I_C}$$

$$\rightarrow \frac{10}{A_E} = 1 \text{ mA} \rightarrow R_E = 10k\Omega$$

$$\therefore R_B = 860k\Omega$$

Q)  $\therefore H = \frac{A}{C_1 + C_2}$  and  $G_L = g_m \frac{R_L R_{eq}}{R_L + R_{eq}}$   $\approx g_m R_{eq}$   
 $R_E \gg R_L$

assuming  $R_L \gg R_{eq}$  or  $R_{eq} \approx R_L \cdot \left(\frac{C_1 + C_2}{C_1}\right)^2$

$$\rightarrow G_L H = \frac{C_1 + C_2}{C_1} = 2 \rightarrow C_1 = C_2$$

$$\frac{1}{(w(C_1+C_2))^2} \ll R_L^2 \rightarrow C_1 + C_2 = 2L = 0.968 \text{ nF}$$

$$\rightarrow C_1 = C_2 = 0.484 \text{ nF}$$

$$\therefore w_o = \sqrt{\frac{C_1 C_2}{C_1 + C_2}} \rightarrow L = 1.047 \text{ mH}$$

Q3: a)  $Z_3 = R + jwL$ ,  $Z_2 = \frac{1}{jwC_2}$ ,  $Z_1 = \frac{1}{jwC_1}$

$$\rightarrow (Z_1 + Z_2 + Z_3)R_T + Z_1(Z_2 + Z_3) + \beta Z_1 Z_2 = 0$$

$$\therefore \frac{R}{jwC_1} + \frac{R}{jwC_2} + jwL R_T + \frac{1}{jwC_1} = 0 \quad \lambda L = 0$$

$$\therefore \frac{2R_T + R}{wC_1} = wL R_T \rightarrow \frac{2R_T + R}{2\pi} = w^2 LC$$

$$\sim R R_T + \frac{1}{C_1} - \frac{1}{w^2 C_1^2} - \frac{\beta}{w^2 C_1^2} = 0$$

$$\therefore R R_T \leq \frac{\beta + 1}{w^2 C_1^2} - \frac{1}{C_1}$$

$$\therefore R_T \gg R \rightarrow \beta = w^2 L C_1$$

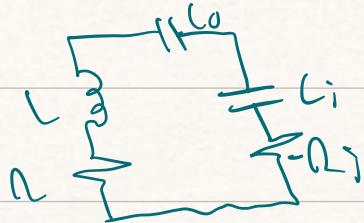
$$\therefore C_1 = C_2 = 4.05 \text{ nF}$$

$$\rightarrow R_T \leq \frac{Q_n}{w \cdot L} \cdot \left[ \frac{\beta + 1}{w^2 C_1^2} - \frac{1}{C_1} \right] = 38.93 \text{ k}\Omega$$

$$\therefore R_T = \frac{0.026\Omega}{I_C} \rightarrow I_C > 66.78 \text{ mA}$$

$$\therefore g_m > 2.57 \text{ mS}$$

b)



$$R_i > R_L = \frac{V_o}{I_C} \Omega$$

$$\approx R_i = \frac{g_m}{(w_0 C_f)} \quad \because C_1 = C_2, \quad g_m = \frac{I_C}{0.026}$$

$$\therefore C_1 = C_2 = 90.9 \text{ nF} \rightarrow C_1 = 45.45 \text{ nF}$$

$$\rightarrow w_0 = \sqrt{\frac{C_0 C_1}{C_0 + C_1}} \rightarrow C_0 = 1.939 \text{ nF}$$

12/10/2022:

$$Q1: a) Z_3 = L + j\omega L, Z_2 = Z_1 = \frac{1}{j\omega L_2} = \frac{1}{j\omega L_1}$$

$$\text{from } (Z_1 + Z_2 + Z_3) \cdot R_T + Z_1 (Z_2 + Z_3) + A_Z Z_1 Z_2 = 0$$

$$\rightarrow \text{phase condition: } \frac{L + 2\Omega_T}{\Omega_T} = \omega_0^2 \cdot L C_1 \approx 2$$

$$\therefore C_1 = C_2 = 5.07 \times 10^{-9} \text{ F}$$

$$\text{from mag. condition: } RL \leq \frac{\Delta + 1}{(\omega_0 \cdot C_1)^2} - \frac{1}{4}$$

$$\therefore R = \frac{\omega_0 L}{Q_H} = \frac{\pi}{25} \approx$$

$$\rightarrow \Omega_T \leq 7.963 \text{ rad} \therefore I_C \geq 0.335 \text{ mA}$$

b)

$$\therefore R = \frac{\pi}{25} \approx$$

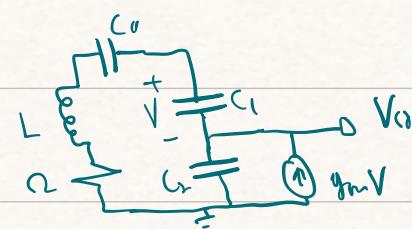
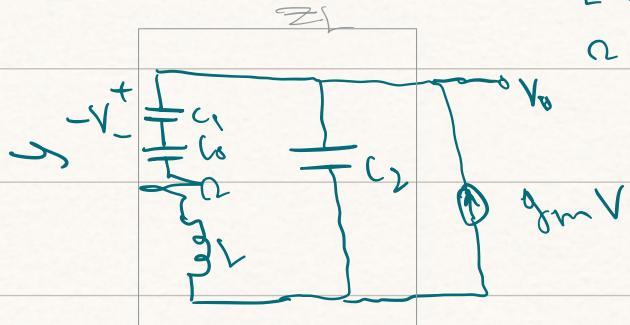
$$\text{and } R_L \geq R \quad (\text{mag. condition})$$

$$\rightarrow \frac{g_m}{(\omega_0 \cdot C_1)^2} \geq \frac{\pi}{25} \rightarrow C_1 = C_2 \leq 6.23 \text{ nF}$$

$$\text{and } f_{\text{ro}} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \cdot \frac{C_1 \cdot C_2}{C_{V_2} + C_0}}} \rightarrow C_0 = 18.56 \text{ nF}$$

$$\therefore -10 + I_C \cdot \frac{R_E}{100} + 0.9 + I_C \cdot R_E = 0 \rightarrow R_E = 860 \text{ k}\Omega$$

c) common collector:



$$C_{eq} \cdot C_0 = X_{L_{eq}} + C_0 X_L$$

$$\rightarrow C_0 = \frac{X_{L_{eq}}}{C_{eq} - X_L}$$

$$\rightarrow G_v = g_m \cdot Z_L, H = \frac{-X_{L1}}{X_{C1} + X_{L0} + X_L + X_L}$$

$$r \geq L = \frac{\chi_{L2} \cdot (\chi_{C_1} + \chi_{L0} + r + \chi_L)}{\chi_{L2} + \chi_{C_1} + \chi_{L0} + r + \chi_L}$$

$$\therefore G_H = g_m \cdot \frac{-\chi_{C_1} \cdot \chi_{C_2}}{\chi_{L2} + \chi_{C_1} + \chi_{L0} + r + \chi_L}$$

$$\hookrightarrow G_H = \frac{g_m \cdot \frac{1}{w^2 C_1 L_2}}{\frac{1}{jwL_2} + \frac{1}{jwC_1} + \frac{1}{jwC_0} + jwL + r} \gg 1$$

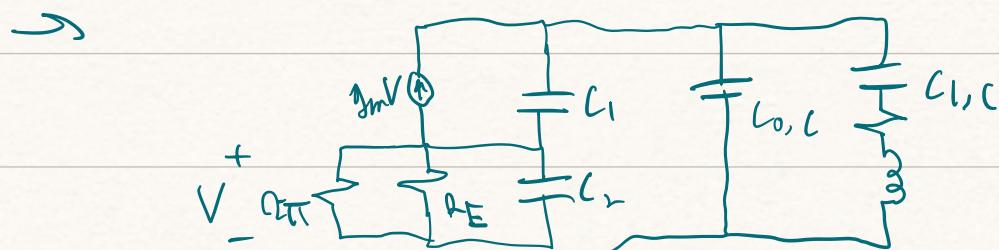
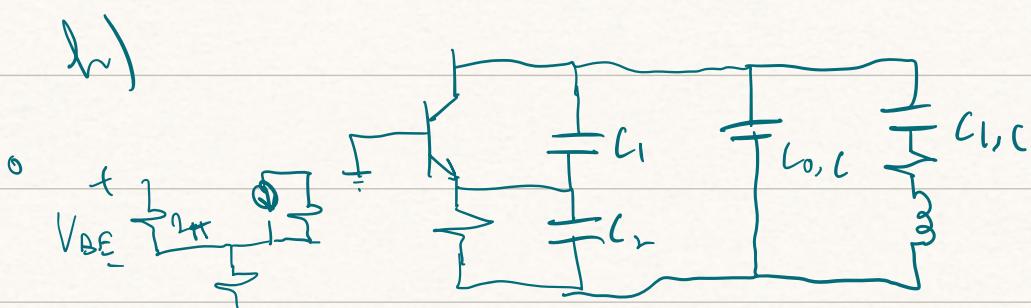
real  $\rightarrow \text{Im}\{G_H\} = 0 \rightarrow w = \sqrt{L \cdot \frac{1}{L_1 + L_2 + C_0}}$

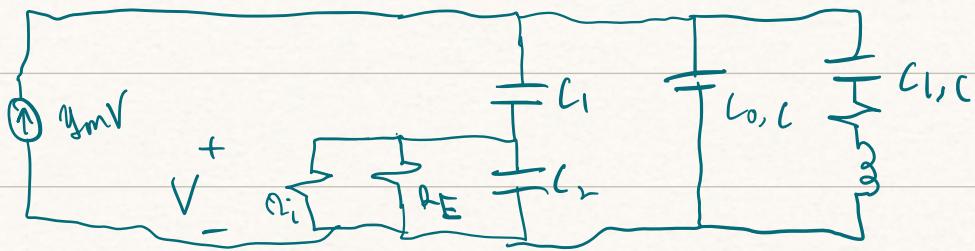
max:  $G_H = \frac{g_m}{w^2 L_1 L_2 r} \gg 1$

Q2: a)  $\circlearrowleft 26 \text{ MHz} = f_m = \frac{1}{2\pi} \cdot \sqrt{L \cdot \frac{L_1 C_0}{L_1 + L_0}}$   
 $\hookrightarrow L = 11.41 \text{ mH}$

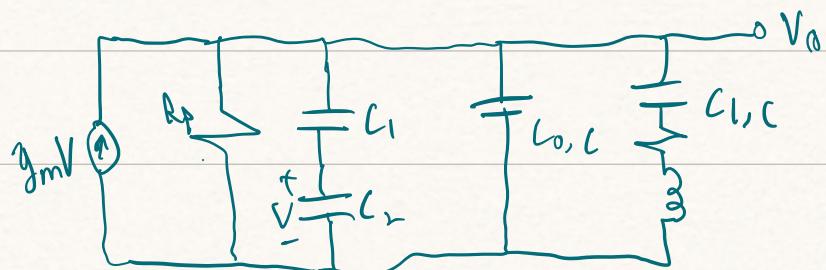
$$\hookrightarrow f_m = 25.9937 \text{ MHz}$$

$$\hookrightarrow \text{pulling range} = 6.3 \text{ kHz}$$





$$\rightarrow R_P \approx R_i \cdot \left( \frac{C_1 + C_2}{C_1} \right)^2$$



$$\rightarrow \omega = \sqrt{L \cdot \frac{(C_L + C_0) \cdot C_1}{C_L + C_0 + C_1}}$$

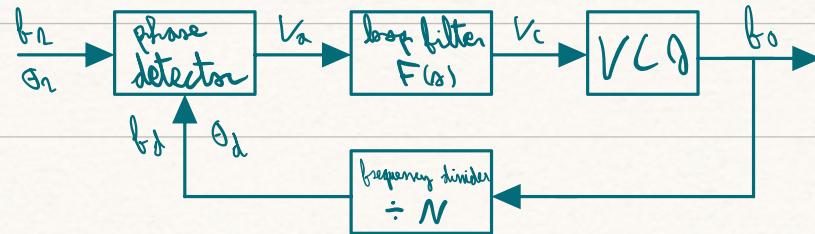
$$\stackrel{00}{\circ} f_0 = 25.4439 \text{ MHz}$$

$$\rightarrow f_2' = 25.4439 \cdot \left( 1 + \frac{C_1}{C_0 + C_1} \right)^2$$

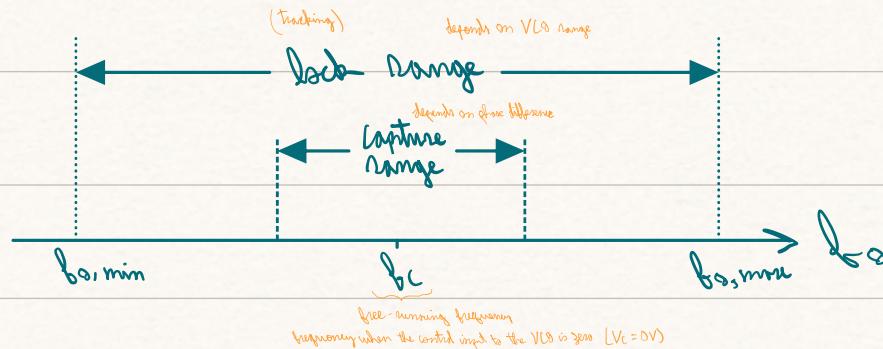
$$\rightarrow f_2' \approx 25.991$$

## Chapter 8:

- phase-locked loop (PLL) is a negative feedback system in which the output frequency and phase are locked to those at the input.



- the voltage output of the phase detector is proportional to the difference in phase ( $\Theta_n - \Theta_d$ )
- the control voltage changes the output frequency until  $f_d = f_n$ . lock condition  
If  $\Theta_d = \Theta_n$ , output of phase detector is zero
- at the locked state:  $f_d = f_n$ ,  $\Theta_d \approx \Theta_n$ ,  $f_o = N \cdot f_n$
- $\Theta_n - \Theta_d \neq 0$  is required so that the control voltage keeps the PLL locked and allows the VCO to track changes in the input.



- \* **Capture range**: the range of frequencies over which the PLL can lock.
- \* **lock range**: the range of frequencies over which the PLL can maintain its locked state. equal or less than the range of the VCO
- the output frequency of the PLL can be increased by increasing the frequency

allows PLL to be used as a frequency multiplier or synthesizer

divider ratio ( $N$ ).

$$\therefore f_o = N f_{cr} \rightarrow f_o \propto N$$

integer

- the output voltage of the phase detector is found from:

$$V_a = k_{pd} \cdot (\theta_2 - \theta_1)$$

$k_{pd}$ : phase detector gain factor in V/Ns

- the VCO's output's deviation from its free-running frequency is:

$$\Delta \omega = (\omega_0 - \omega_c) = k_{pd} V_c \rightarrow \omega_0 = \omega_c + k_{pd} V_c$$

start angular frequency  
free running angular frequency  
gain factor of VCO (Pd/V)

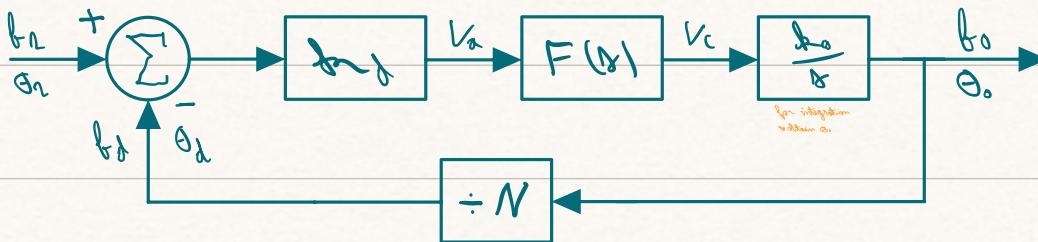
- the output phase is found from the output angular frequency as:

$$\omega_o = \frac{d\theta_o}{dt} \rightarrow \theta_o = \int \omega_o dt$$

- at the output of the frequency divider, the phase and frequency are:

$$f_{pd} = \frac{f_o}{N}, \quad \theta_1 = \frac{\theta_o}{N}$$

- therefore, the linearized PLL's block diagram becomes:



- the transfer function can be found as:

$$\frac{\theta_o(s)}{\theta_2(s)} = \frac{k_{pd} \cdot F(s) \cdot \frac{k_0}{s}}{1 + k_{pd} \cdot F(s) \cdot \frac{k_0}{s} \cdot \frac{1}{N}} = \frac{G(s)}{1 + \frac{G(s)}{N}}$$

which gives four parameters that can be controlled:  $k_{pd}$ ,  $k_0$ ,  $F(s)$ , and  $N$

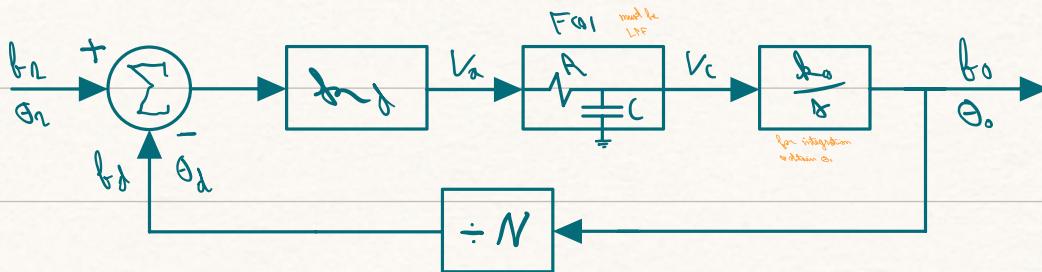
- a first order PLL has no loop filter, hence the transfer function becomes:

$$\therefore F(s) = 1 \rightarrow \frac{\theta_o(s)}{\theta_2(s)} = \frac{k_0 k_{pd}}{s + k_{pd} k_0 \cdot \frac{1}{N}} = \frac{N b_r}{s + b_r}, \quad b_r = \frac{k_0 k_{pd}}{N}$$

low pass filter  
(PLL has increased around notch frequency)

bandwidth of filter  
 $b_r = b_r \pi / \omega_0$

- LPP
- If a first order low-pass filter is used as a loop filter, then the PLL is considered second order since it behaves as a second-order LPF.



$$\rightarrow F(s) = \frac{1}{1 + \zeta \omega_n s} = \frac{\omega_L}{\omega_L + \zeta} , \quad \omega_L = \frac{1}{RC}$$

$$\rightarrow \frac{\Theta_o(s)}{\Theta_i(s)} = \frac{j\omega_L \theta_0 \omega_L}{(\omega_L + \zeta) s + j\omega_L \theta_0 \frac{\omega_L}{N}} = \frac{N \omega_L v}{(\omega_L + \zeta) \cdot \frac{1}{\omega_n} + j\omega_L v} = \frac{N}{\left(\frac{1}{\omega_n}\right)^2 + \left(\frac{2\zeta}{\omega_n}\right)s + 1}$$

$$\therefore \omega_n = \sqrt{\omega_L \theta_0} , \quad 2\zeta = \sqrt{\frac{\omega_L}{\omega_n v}} , \quad \omega_n v = \frac{\theta_0 \omega_L}{N}$$

- a Butterworth filter has a damping ratio  $\zeta = \frac{1}{\sqrt{2}}$  and its 3-dB bandwidth is equal to its natural frequency ( $\omega_h = \omega_n$ ).

- The 3-dB bandwidth  $\omega_n$  can be calculated from:

$$\omega_n = \omega_h \left[ 1 - 2 \bar{z}^2 + (2 - 4 \bar{z}^2 + 4 \bar{z}^4)^{1/2} \right]^{1/2}$$

and the rise time is found from:  $t_r \approx \frac{2.2}{\omega_n}$  (10% to 90%)

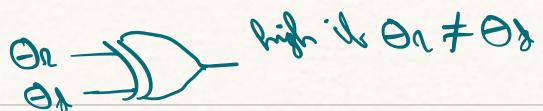
- therefore, a larger bandwidth gives a faster rise time but includes more noise.

+ There are three categories of phase detectors:

1- digital , 2- analog , 3- sampling.

+ digital phase detectors:

1- XOR phase detector :



duty cycle must be 50%, if it is less than 50% V<sub>dc</sub>-V<sub>c</sub> plot changes.

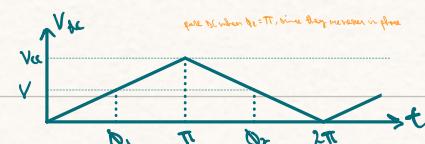
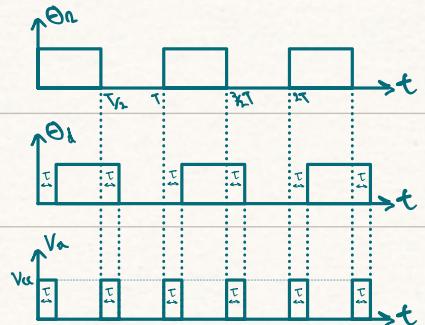
- the average DC value of V<sub>a</sub> is:  $V_{dc} = \frac{2T \cdot V_{cc}}{T}$

- the phase difference is found from:

$$\Phi_e = (\Theta_2 - \Theta_1) = \frac{T}{\tau} \cdot 2\pi \rightarrow T = \frac{\tau}{2\pi} \cdot \Phi_e$$

- when V<sub>dc</sub> is plotted as a function to  $\Phi_e$ , a phase ambiguity is noticed.  $V_{dc} = \frac{V_{cc}}{\pi} \Phi_e = \text{Indef. } \Phi_e$

Phase detector has



$\Phi_1$  and  $\Phi_2$  both produce same  $V_{dc}$

## 2 - set-reset (RS) flip-flop phase detector.

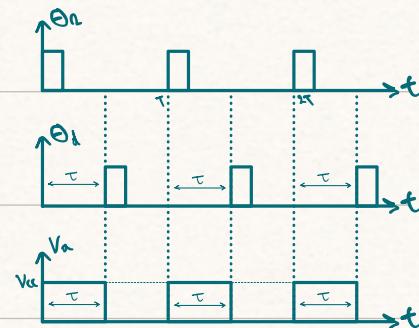
- solves phase ambiguity, but requires very narrow pulses. so that they are not high simultaneously

- the average DC voltage of V<sub>a</sub> is:  $V_{dc} = \frac{T \cdot V_{cc}}{T}$

-  $\Phi_e$  is found from the time delay  $\tau$ :

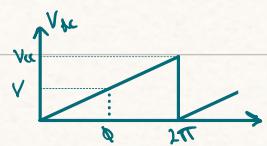
$$\Phi_e = (\Theta_2 - \Theta_1) = \frac{T}{\tau} \cdot 2\pi \rightarrow T = \frac{\tau}{2\pi} \cdot \Phi_e, \text{ equivalent to the XOR}$$

$$\rightarrow V_{dc} = \frac{V_{cc}}{2\pi} \cdot \Phi_e \rightarrow \text{Indef. } \Phi_e = \frac{V_{cc}}{2\pi}, \text{ Indef. RS} = \frac{1}{2} \cdot \text{Indef. XOR}$$



- if V<sub>dc</sub> is plotted against  $\Phi_e$ , it can be observed that there is

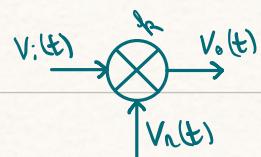
no phase ambiguity, since there is only one pulse per period.



- the RS flip-flop phase detector has the disadvantages of requiring very narrow pulses and monostable vibrator inputs.

## + Analog phase detector - mixer:

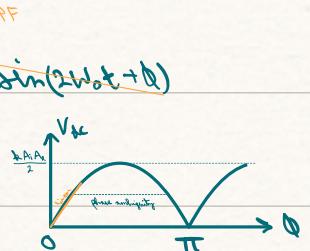
- if the inputs are sinusoidal, a mixer (multiplier) can be used as a phase detector.



$$\text{if } V_i(t) = A_i \cos(\omega_i t) \quad \& \quad V_n(t) = A_n \sin(\omega_n t + \phi)$$

$$\therefore V_o(t) = \underbrace{\text{mixer gain}}_{\approx 1} \cdot V_i(t) \cdot V_n(t) = \frac{\Im A_i A_n}{2} \sin(\phi) + \frac{\Re A_i A_n}{2} \sin(2\omega_n t + \phi)$$

$$\therefore V_{dc} = V_o(t) |_{LPF} = \frac{\Re A_i A_n}{2} \sin(\phi)$$

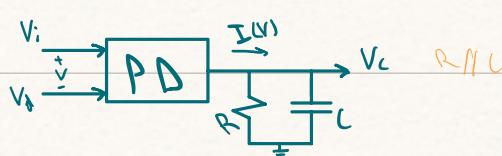


- this phase detector suffers from phase ambiguity, non-linearity for larger phase shifts, and  $V_{dc}$  depending on  $A_i$  (in addition to  $\sin(\phi)$ ), which is visible.
- for small values of  $\phi$ , the mixer phase detector is linearized as:

$$V_{dc} = \frac{\Re A_i A_n}{2} \phi \quad \text{for } \phi \ll 1$$

- the performance of a PLL depends on the characteristics of the VCO.
- the VCO's tuning range determines the tracking range of the PLL.
- the VCO's modulation sensitivity  $\beta_{vo} = \frac{\omega_{v, \max} - \omega_{v, \min}}{V_{v, \max} - V_{v, \min}}$  or  $\beta_{vo} = \frac{f_{v, \max} - f_{v, \min}}{V_{v, \max} - V_{v, \min}}$
- the VCO's gainfactor ( $\beta_{vo}$ ) shall be high.
- the VCO must have high frequency stability, fast response time, high linearity, and high spectral purity.
- the loop filter of the PLL also removes harmonics produced by the phase detector and gives the order of the PLL.

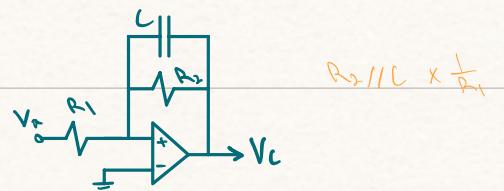
• passive charge pump:



$$V_c = I(V) \cdot \frac{R}{\sigma R C + 1}$$

$$\text{cutoff: } \omega_L = \frac{1}{RC}$$

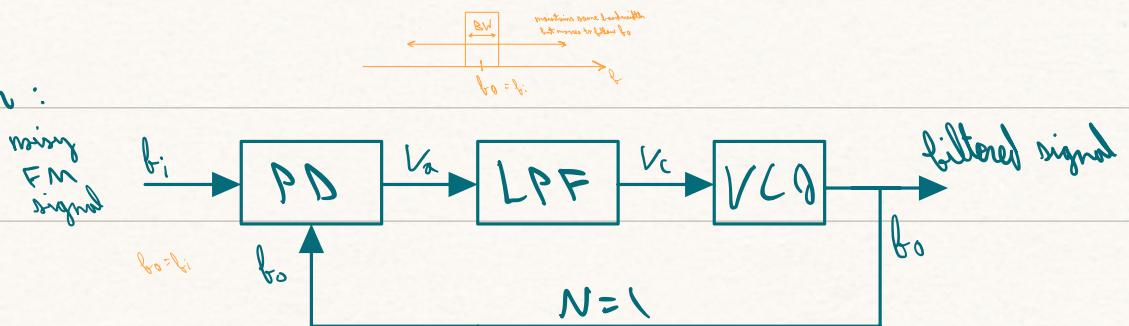
• active charge pump:



$$V_c = -\frac{R_2}{R_1} \cdot \frac{1}{\sigma R_1 C + 1} \cdot V_a$$

## + PLL applications:

### 1- Tracking filter:

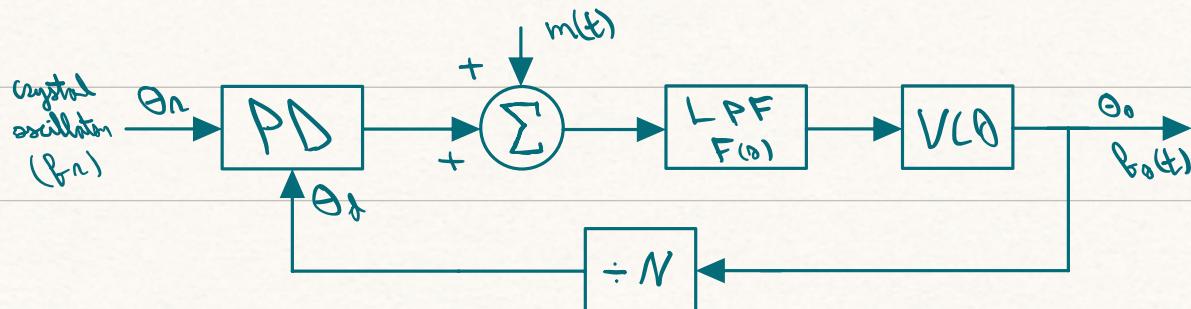


- functions as a filter that remains centered at  $f_i$ , despite variations.

### 2- Phase and frequency modulation:

- using a VCO alone for FM modulation may not be reliable since its frequency stability depends on an LC circuit.

- instead, a PLL with a crystal oscillator input can be used as a stable frequency or phase modulator.



- the output is found from:

$$\Theta_o(\delta) = \frac{\Theta_n(\delta) \cdot [k_o \cdot \delta + \frac{F(\delta)}{\delta}]}{1 + k_o \cdot \delta + F(\delta)/N\delta} + \frac{N(\delta) \cdot [\frac{k_o F(\delta)}{\delta}]}{1 + k_o \cdot \delta + F(\delta)/N\delta}$$

assuming  $k_o \cdot \delta + \frac{F(\delta)}{N\delta} \gg 1$  and  $\Theta_n(N) = 2\pi f_n$

$$\rightarrow \Theta_o(\delta) = N \cdot \Theta_n(\delta) + \frac{N N(\delta)}{k_o \delta}$$

$$\rightarrow f_o = \frac{1}{2\pi} \frac{d\theta_o(t)}{dt} = \underbrace{Nf_{cR}}_{\text{carrier frequency}} + \frac{N}{2\pi\Delta\omega} \cdot \frac{dm(t)}{dt} \quad (\text{phase modulation})$$

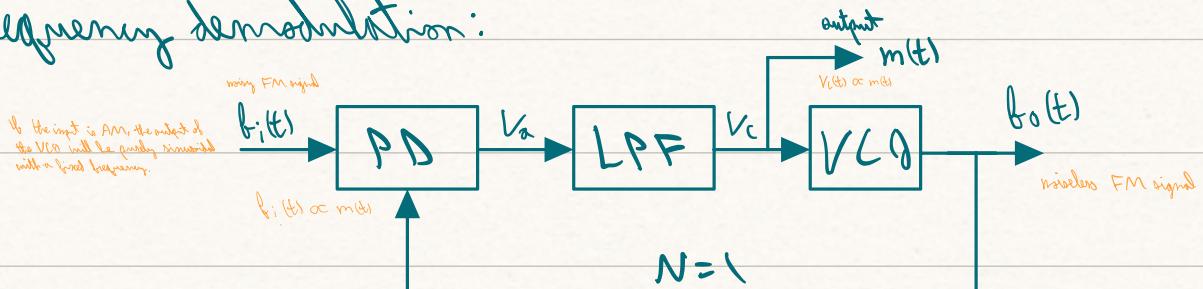
- Phase modulation: phase is linearly proportional to  $m(t)$ .
- Frequency modulation: frequency is linearly proportional to  $m(t)$ .
  - a phase modulation circuit can be transformed to a frequency modulation circuit by inserting  $m(t)$  into an integrator at first. (and vice versa)

If  $m(t) = Am \sin(\omega_m t)$ :

$$f_o(t) = f_c + \frac{NAm\omega_m}{2\pi\Delta\omega} \cdot \cos(\omega_m t)$$

$$\therefore f_o(t) = f_c + \Delta f \cos(\omega_m t) \quad \text{s.t., } 2\Delta f \leq \text{lock range}$$

### 3- Frequency demodulation:



- since the output of the VCO is the frequency modulated signal, then

its input must be the message signal

$$\therefore N=1 \rightarrow f_o(t) = f_i(t) \wedge f_o(t) \propto V_c(t) \therefore V_i(t) \propto m(t)$$

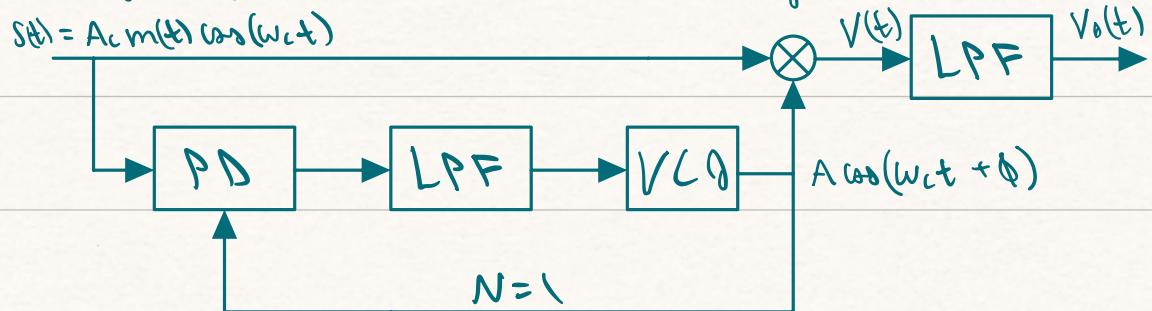
- the tracking range of the PLL as a demodulator must be greater than the frequency deviation of the input FM signal.

$$TR = 2(\Delta\omega)_{\text{max}} = 2 \frac{\Delta\omega}{\text{LRF gain}} V_{c,\text{max}}$$

## 4- Carrier Recovery (synchronization):

- in coherent demodulation, the local oscillator must have the same

frequency and phase of the received signal.



$$\therefore V(t) = \frac{AA_c}{2} m(t) \cos(\phi) + \frac{AA_c}{2} m(t) \cos(2\omega_c t + \phi)$$

- when the PLL is locked  $\phi \approx 0$ , therefore:

$$V_o(t) = \frac{AA_c}{2} \cdot m(t)$$

- integrated circuits (ICs) reduce system size, power consumption, noise, delay, etc. as compared to discrete components.

- MC14046B is a CMOS PLL IC whose pin diagram is below:

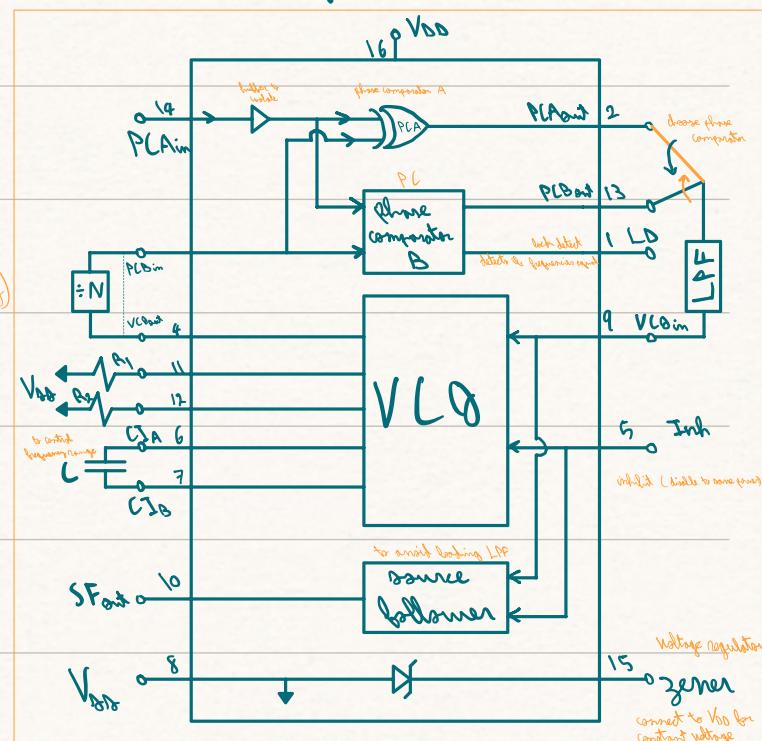
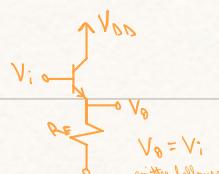
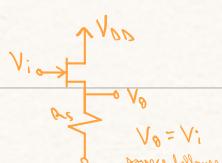
$$\rightarrow f_{min} = \frac{1}{R_2(C + 32\text{pF})}$$

$$\rightarrow f_{max} = f_{min} + \frac{1}{R_1(C + 32\text{pF})}$$

$$\rightarrow f_{2VCO} = f_{2o} = \frac{2\pi \Delta f_{VCO}}{V_{DD} - 2} \quad (\text{from } f_{max} - f_{min})$$

$$\wedge 10k\Omega < R_1, R_2 < 1M\Omega$$

$$\wedge 100\text{pF} < C < 0.01\text{nF}$$



example 8.1:

$$\text{1st order} \rightarrow TF = \frac{N_{\text{bv}}}{s + b_{\text{bv}}} , b_{\text{bv}} = \frac{f_{\text{o}} - f_{\text{c}}}{N}$$

$$\therefore b_0 = 1 \text{ MHz} \rightarrow N = \frac{1M}{2\pi b_0} = 40$$

$$r b_{\text{bv}} = \frac{200\pi \cdot 2}{40} = 10\pi \therefore TF = \frac{400\pi}{s + b\pi}$$

$$\text{BW: at } 0 \text{ rad/s } TF = 40, \text{ at } \omega_n \text{ TF} = \frac{40}{2} \rightarrow \frac{400\pi}{\omega_n + 10\pi} = 20$$

$$\therefore \text{BW} = 10\pi \text{ rad/s}$$

example 8.1:

$$\therefore Z = \frac{1}{\sqrt{2}} = \frac{1}{2} \cdot \sqrt{\frac{W_L}{b_{\text{bv}}}} \rightarrow W_L = 2b_{\text{bv}} = 20\pi \text{ rad/s}$$

$$\therefore \text{Butterworth} \rightarrow \omega_n = \sqrt{W_L \cdot b_{\text{bv}}} \rightarrow 10\pi \cdot \sqrt{2} = 14.14\pi \text{ rad/s}$$

$$r t_n = \frac{2 \cdot 2}{14.14\pi} = 49.5 \text{ ms}$$

example 8.6:

$$\therefore b_0 = 2.4 \text{ kHz}, b_0 = 100 \text{ kHz} \rightarrow N = 40$$

$$\text{1 second order PLL Butterworth filter} \rightarrow Z = \frac{1}{\sqrt{2}}$$

$$\therefore 2Z = \sqrt{\frac{W_L}{b_{\text{bv}}}}, W_L = \frac{1}{\omega C}, b_{\text{bv}} = \frac{f_{\text{o}} - f_{\text{c}}}{N}$$

assuming  $f_{\text{min}} = 90 \text{ kHz}$  and  $f_{\text{max}} = 180 \text{ kHz}$ ,  $C = 1 \text{ nF}$

$\rightarrow 10\text{kHz limit}$

$\therefore f_{\text{max}} = 2f_{\text{min}}$

$$\rightarrow R_2 = 10.77 \text{ k}\Omega \quad r R_1 = 10.77 \text{ k}\Omega = R_2$$

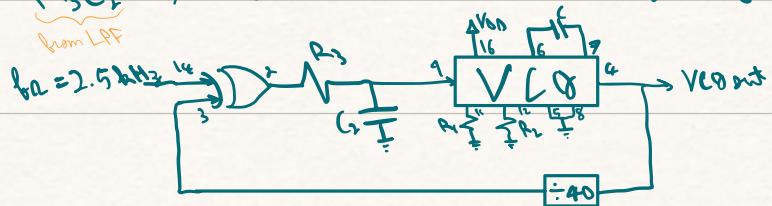
$$r b_{\text{vco}} = b_0 = \frac{2\pi \cdot \Delta f_{\text{VCO}}}{V_{\text{DD}} - 2} = 80.783 \times 10^3 \text{ rad/s/V}$$

$$\text{for VOR: } k_V = \frac{V_u}{\pi} = \frac{V_{\text{DD}}}{\pi} = \frac{9}{\pi} = 2.865 \text{ V/rad}$$

$$\rightarrow \omega_0 = 5.986 \times 10^3 \text{ rad/s} \rightarrow \omega_L = 11592 \text{ rad/s}$$

$$\rightarrow \omega_n = 8182 \cdot 6 \text{ rad/s} = 2604 \cdot 6\pi \text{ rad/s} = \omega_n = BW$$

$\therefore \omega_L = \frac{1}{R_3 C_2}$ , if  $C_2 = 1 \mu F \rightarrow R_3 = 86.42 \text{ k}\Omega$



$f_{LPF} = 2.5 \text{ kHz}$  <sup>from LPF</sup>

## EE-524: homework #3

$$8.1: \because f_R = 50 \text{ kHz} \quad \wedge \quad f_o = 1 \text{ MHz} \rightarrow N = 20$$

$$\therefore f_T = 2 \text{ V/rot}, \quad b_0 = 100 \text{ Hz/V} = 20\pi \text{ rad/s/V}$$

$$\rightarrow \omega_r = \frac{400\pi \text{ Hz}}{20} = 20\pi \text{ Hz}$$

$$\therefore \text{transfer function} = \frac{400\pi}{s + 20\pi}$$

$$\therefore t_R \approx \frac{2.2}{\omega_r} \quad \wedge \quad \omega_r = 20\pi \rightarrow t_R = 35 \text{ ms}$$

$$\text{If } f_{R_s} = 1.2 \text{ MHz} \rightarrow N = 24 \rightarrow \omega_r = 52.36 \text{ Hz}$$

$$\rightarrow t_R = 42 \text{ ms}$$

$$\because f_R = 50 \text{ kHz} \quad \wedge \quad \text{frequency range} = 400 \text{ kHz}$$

$$\rightarrow \frac{400}{50} = 8 \quad \boxed{8} \text{ different frequencies possible.}$$

$$8.2: \text{ a) } \omega_L = \omega_n \wedge \omega_n = \sqrt{\frac{1}{L} + \omega_L^2} \wedge 2\pi = \sqrt{\frac{\omega_L}{L}} \rightarrow \omega_L = 2\pi\sqrt{L}$$

$$\therefore \omega_n = 28.28 \pi \text{ rad/s} = \omega_p \rightarrow \boxed{\text{BW} = 14.14 \text{ Hz}}$$

$$\text{b) } \omega_n = 52.36 \text{ Hz} \rightarrow \omega_n = \omega_{in} = 74.05 \text{ rad/s}$$

$$\rightarrow \boxed{\text{BW} = 11.79 \text{ Hz}}$$

$$8.14: \text{ } \Delta f_{VCO} = 2\pi \text{ Hz}, \text{ assuming } V_{DD} = 9V \rightarrow f_{osc} = 1995.2 \text{ rad/s}$$

$$\text{ } \therefore \text{ demodulator} \rightarrow \text{TR} = 2\Delta \omega_{max} = 2\pi \omega_{osc} V_{C, max} \wedge \text{ XOR} \rightarrow f_{out} = \frac{V_{DD}}{\pi}$$

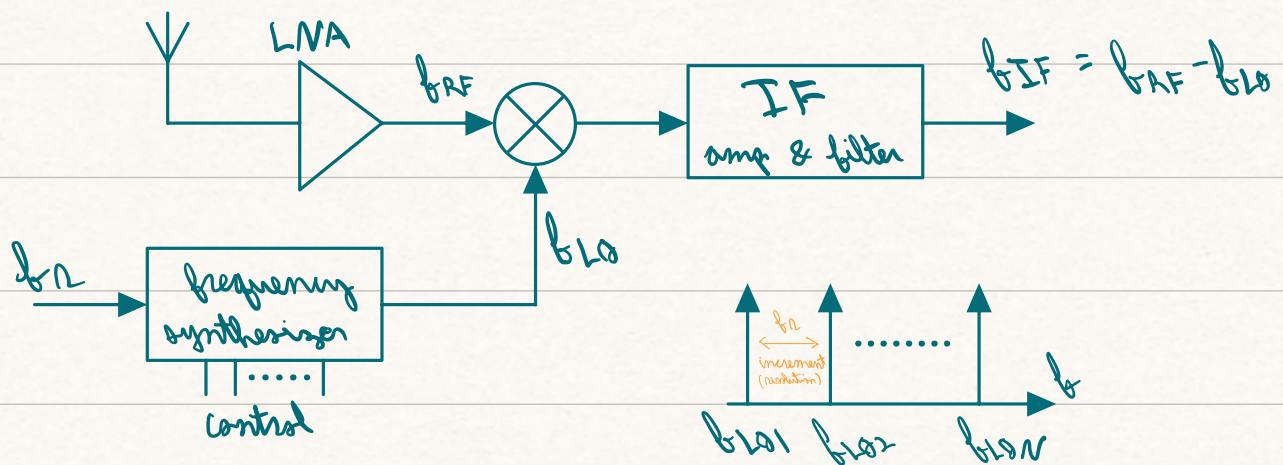
$$\text{assuming } C = 0.5 \text{ nF} \rightarrow \boxed{R_2 = 19.0 \text{ k}\Omega} \wedge \boxed{R_1 = 18.6 \text{ k}\Omega}$$

$$\text{if } \text{BW} = 2\pi \text{ Hz} \text{ and } \beta = \frac{1}{\sqrt{2}} \rightarrow \boxed{R_{LPF} = 48.6 \text{ k}\Omega} \text{ if } C_{LPF} = 2 \text{ nF}$$

# Chapter 10:

\* frequency synthesizer: device that generates many precise frequencies from one or more reference frequencies.

- frequency synthesizers are often used as variable local oscillators for transmitters and receivers to select a specific station, or otherwise.



- frequency synthesizers are characterized by their respective frequency range and frequency resolution. ( $f_{LON} - f_{LO1}$ ) ( $f_{LO(i+1)} - f_{LOi}$ ) (increment on step)

+ main types of frequency synthesizers:

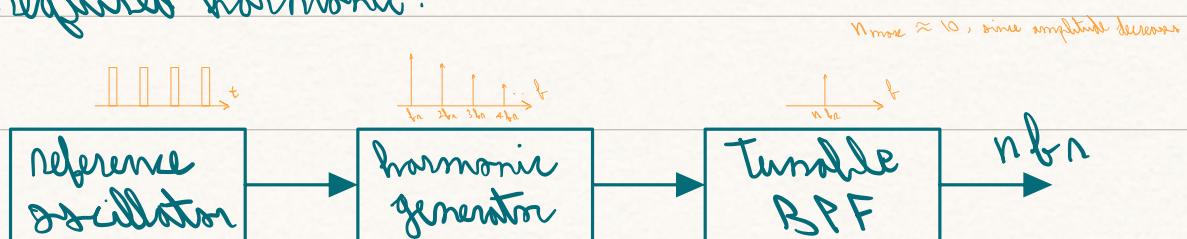
1- direct frequency synthesizer.

2- phase-locked loop frequency synthesizer.

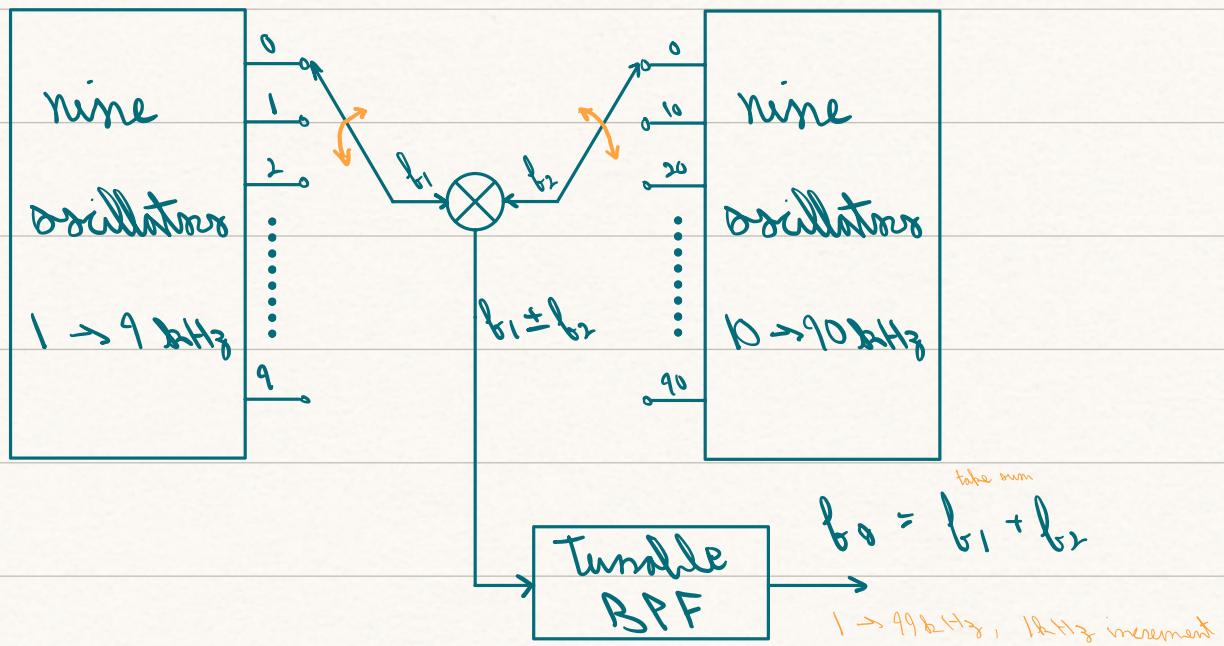
3- direct digital frequency synthesizer. will not be discussed

# 1 - Direct frequency synthesis:

- direct frequency synthesis is an old method that uses harmonic generators, filters, dividers, and mixers.
- in this technique, a reference oscillator with narrow pulses excites a harmonic generator, then a tunable BPF selects the required harmonic.



- the two decade direct frequency synthesizer shown below can generate 99 frequencies from 18 reference oscillators.



- designing a tunable BPF with a narrow passband to select

one of the 99 frequencies is challenging.

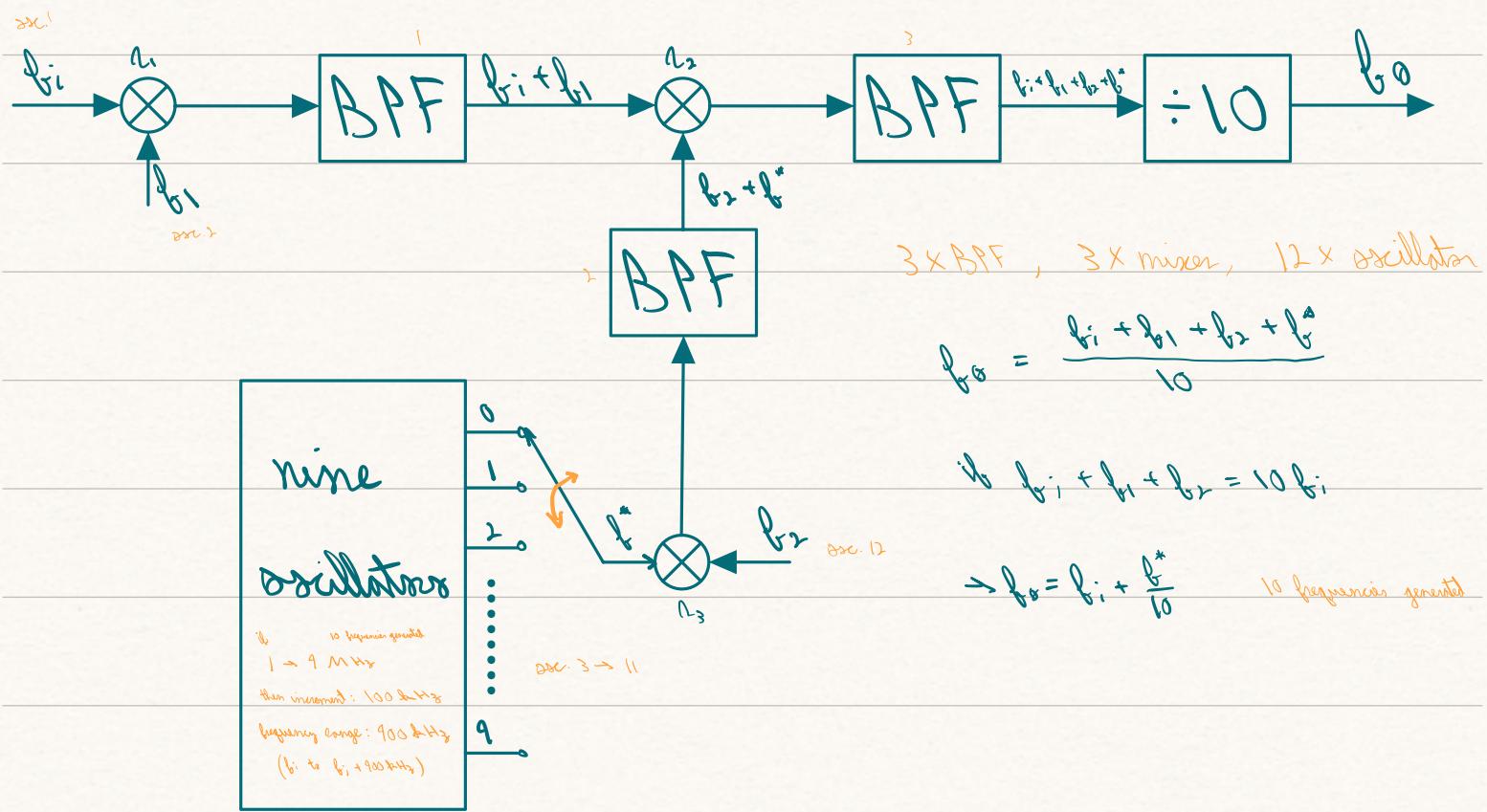
- the complexity of a BPF depends on the mixing ratio:

$$R = \frac{f_1}{f_2} \quad \begin{array}{c} \text{larger} \\ \text{smaller} \end{array} \quad R \rightarrow \text{complexity} \uparrow$$

- a small mixing ratio is desirable because the spacing between frequencies will be large, which allows easy design of the filter.

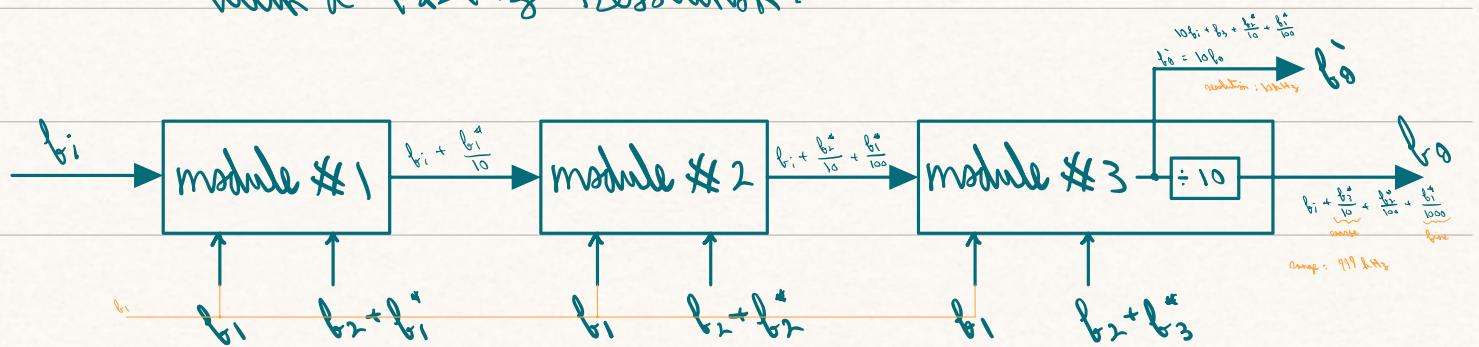
+ double-mix-divide direct frequency synthesis:

- offset frequencies are used to reduce filter complexity, as below:



-  $f_i$ ,  $f_{b1}$ , and  $f_{b2}$  are chosen such that the mixing ratios of the two mixers are close to one.

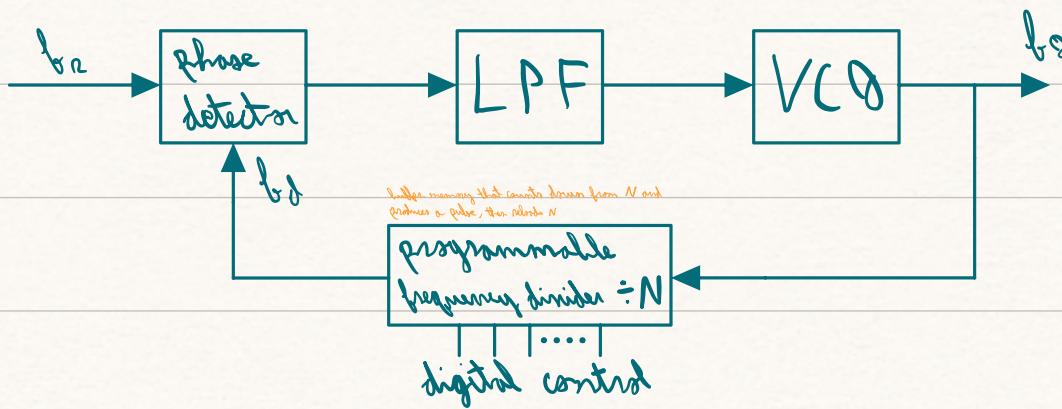
- two cascaded double-mix divide modules can generate 100 frequencies with a 10 kHz resolution. Three cascaded modules can generate 1000 frequencies with a 1 kHz increment.
- The three cascaded double-mix-divide modules shown below only need 12 reference oscillators to generate 1000 frequencies with a 1 kHz resolution.



## 2- PLL frequency synthesizers:

- PLL frequency synthesizers are low cost and only require a single reference frequency to implement any resolution or range.

### 1- simple PLL synthesizer:



- when locked:  $f_{\text{lo}} = f_{\text{rf}} = \frac{f_{\text{ref}}}{N} \rightarrow f_{\text{lo}} = N f_{\text{ref}}$

- the resolution is  $f_{\text{ref}}$  and the range is  $f_{\text{lo,min}}$  to  $f_{\text{lo,max}}$ :

$$N_{\text{min}} = \frac{f_{\text{lo,min}}}{f_{\text{ref}}} \quad \wedge \quad N_{\text{max}} = \frac{f_{\text{lo,max}}}{f_{\text{ref}}}$$

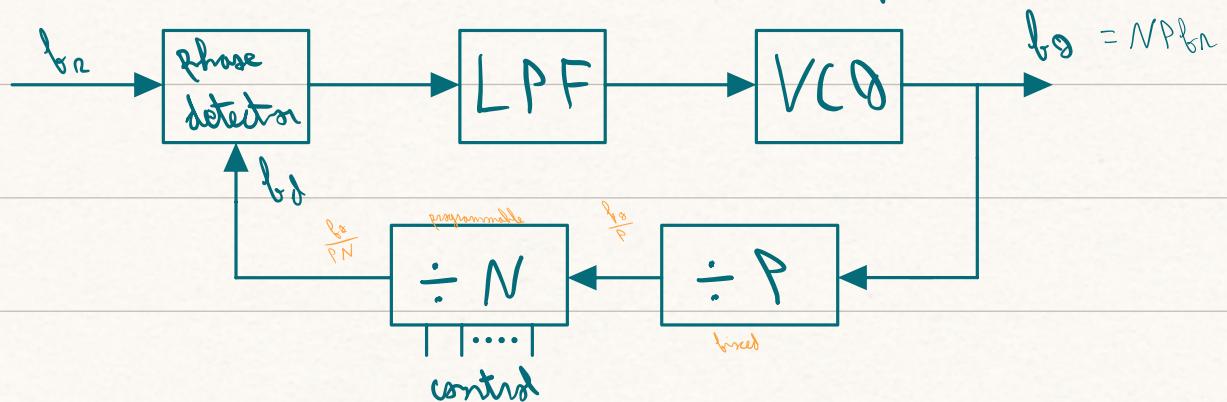
+ This simple PLL synthesizer has two main problems:

1- programmable frequency dividers are slow devices.

2- the switching time ( $t_s$ ) of the PLL is inversely proportional to the reference frequency ( $f_{\text{ref}}$ ):  $t_s \propto \frac{1}{f_{\text{ref}}}$

$t_s \approx 25 t_n$

2- PLL with fixed-modulus divider (prescaler):



- this solves the first problem since the prescaler drops the frequency to be manageable by the programmable divider.

$$\therefore f_{\text{lo,n}} = N P f_{\text{ref}}, f_{\text{lo,n+1}} = (N+1) P f_{\text{ref}}$$

→ increment:  $P f_{\text{ref}}$        $\frac{f_{\text{lo,max}}}{P} \leq 5 \text{ MHz}$  to be compatible with programmable divider

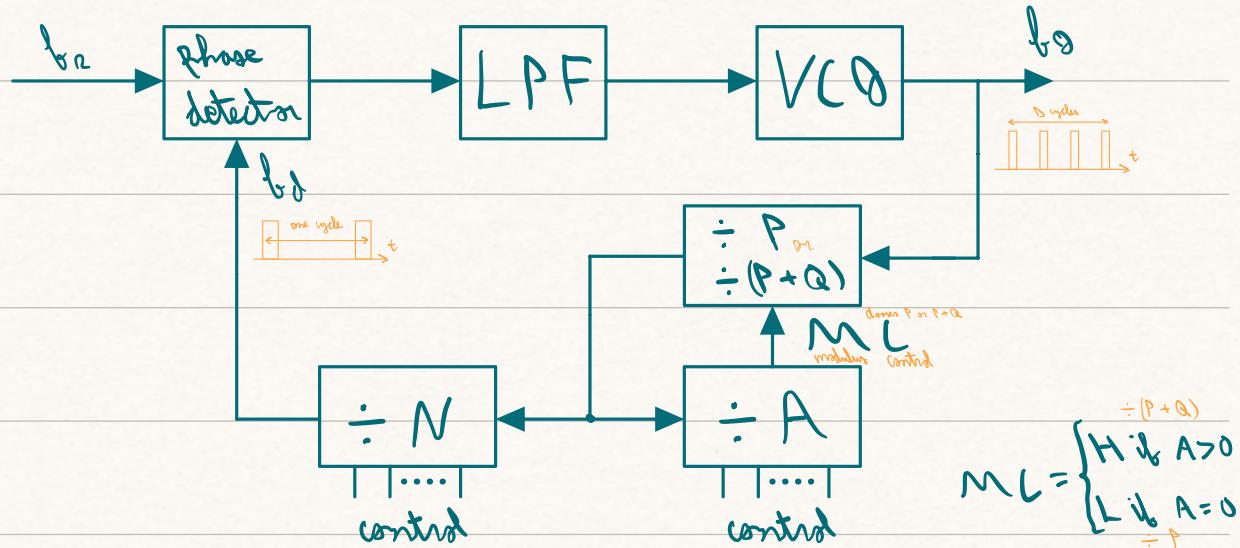
$$\therefore N_{\text{min}} = \frac{f_{\text{lo,min}}}{P f_{\text{ref}}}, \quad N_{\text{max}} = \frac{f_{\text{lo,max}}}{P f_{\text{ref}}}$$

- this type of PLL synthesizer solves the first problem,

but worsens the second since the switching time

to achieve same resolution  $f_{\text{cr}}$  for this PLL synthesizer must be smaller than the previous one ( $f_{\text{cr,new}} = f_{\text{cr,old}} \rightarrow f_{\text{cr,new}} = \frac{f_{\text{cr,old}}}{P}$ ). Therefore,  $t_{\text{cr,new}} = P \cdot t_{\text{cr,old}}$  becomes larger.

3- PLL with dual-modulus prescaler synthesizer:



- the prescaler here makes the resolution  $f_{\text{cr}}$  instead of  $P f_{\text{cr}}$

- the modulus control is high for  $A$  cycles and low for

$N-A$  cycles.

- the output frequency is found as:

$$f_O = D f_r = [(N-A)P + A(P+Q)] \cdot f_r$$

$$\rightarrow f_O = PN f_r + QA f_r$$

- if  $Q < P$ , then the resolution is  $Q f_r$ .  $Q$  is usually

chosen as 1:

$A < N$ ,  $A < P$  or signals will overlap

$$f_O = PN f_r + A f_r = P(N + \frac{A}{P}) \cdot f_r$$

- the ranges of  $N$  and  $A$  are found as:

$$D_{\min} = P N_{\min} + A_{\min} = \frac{f_{o,\min}}{f_n}$$

$$D_{\max} = P N_{\max} + A_{\max} = \frac{f_{o,\max}}{f_n}$$

$P=64, Q=1$

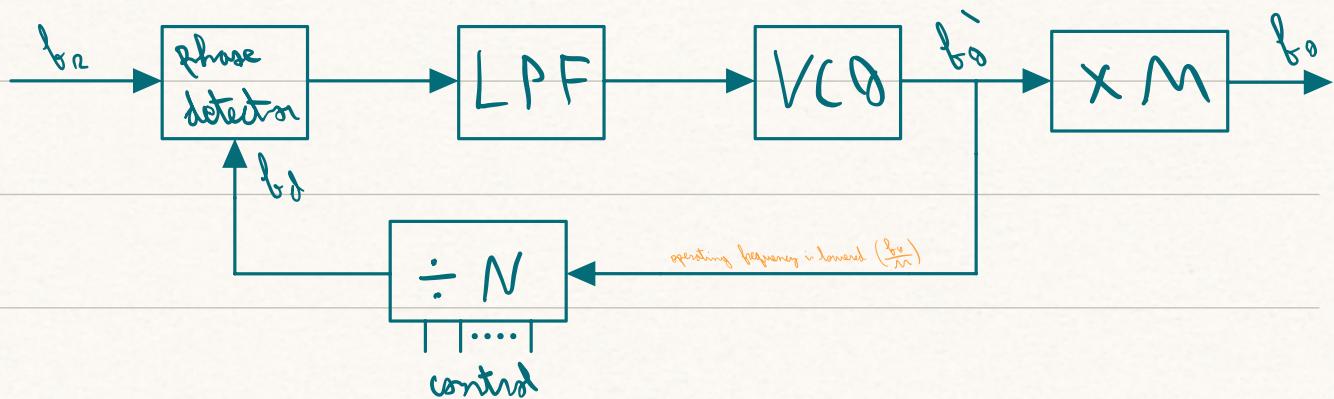
- the prescalers are written in the form  $P/(P+Q)$  (e.g.,  $64/65$ ).

- many PLL FS ILs exist, some have frequency hopping

and bring hops to a different carrier

spread spectrum capabilities, which help against jamming

#### 4 - PLL with post-multiplication:



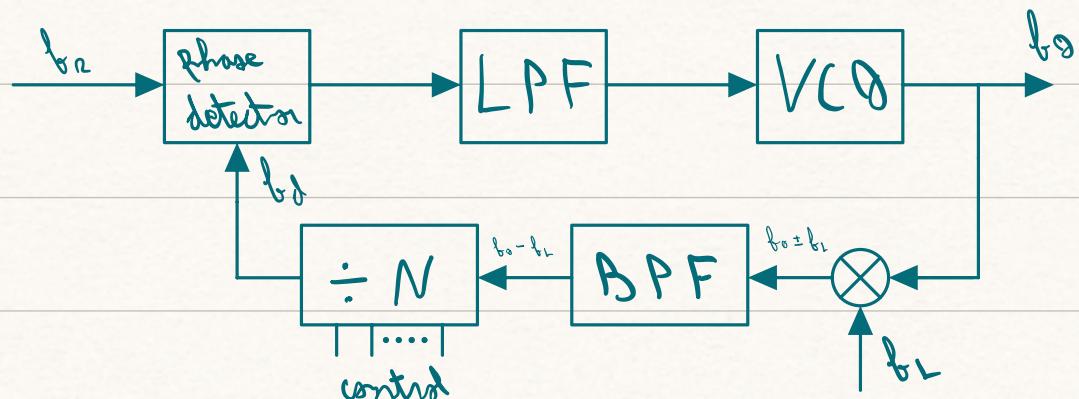
$$\text{at lock: } f_n = f_d = \frac{f'_o}{N} \rightarrow f'_o = N f_n$$

$$\therefore f_o = N M f_n$$

N is variable

→ resolution:  $M f_n$

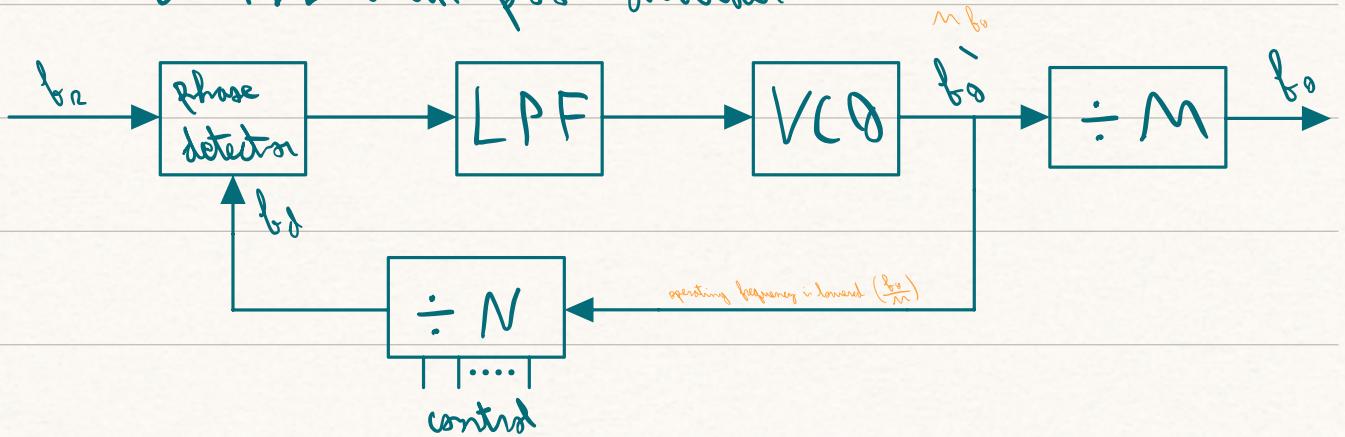
#### 5 - PLL with down conversion:



$$\therefore f_{\text{fr}} = f_d = \frac{f_o - f_L}{N} \rightarrow f_o = f_L + N f_{\text{fr}}$$

- Therefore, the resolution is  $f_{\text{fr}}$  and the programmable divider operates at  $f_o - f_L$

## 6- PLL with post-divider



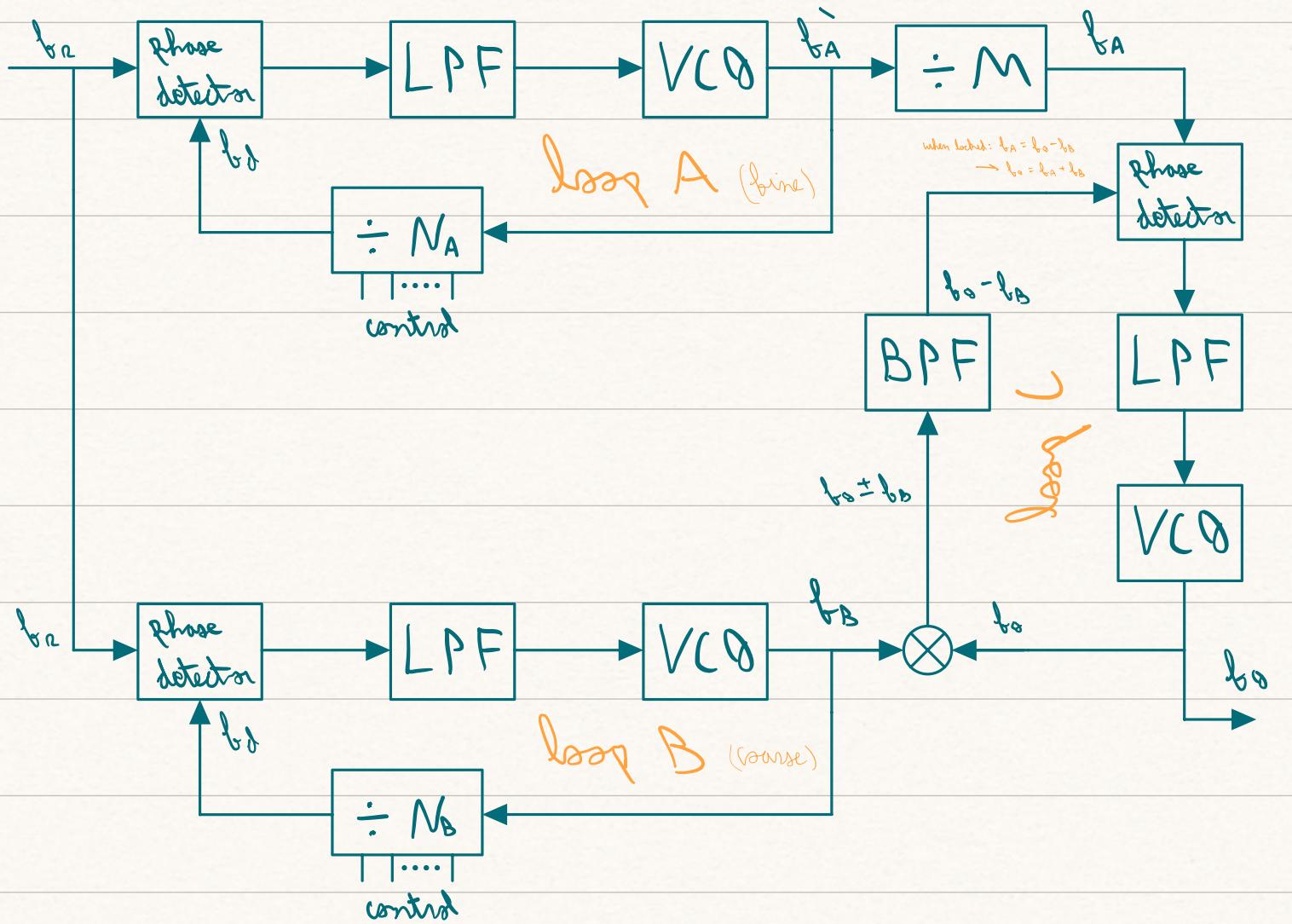
$$\therefore f_{\text{fr}} = f_d = \frac{f'_o}{N}, \quad f'_o = N f_{\text{fr}}, \quad f'_o = M f_o$$

$$\therefore f_o = \frac{N f_{\text{fr}}}{M} \rightarrow \text{resolution: } \frac{f_o}{M}$$

- the resolution is low, which implies high switching speed, but the operating frequency of the divider is high.

## 7- multiple-loop frequency synthesizers:

- a compromise between high switching speed and low operating frequency of programmable dividers can be achieved using this type of PLL frequency synthesizers.



- in this configuration, the PLL in Loop L is used instead of a mixer and BPF because the mixing ratio will make a BPF complex when  $f_B$  is fine.

example 10.2:

$10 \rightarrow 19.94 \text{ MHz}$ ,  $10 \text{ kHz}$  resolution implied 1000 frequencies

therefore, 3 cascaded double-mix dividers required

$$f_0' = 10 f_{b1} + f_{b3}^* + \frac{f_{b2}^*}{10} + \frac{f_{b1}^*}{100}, f_{0,\min} = 10 f_{b1} \rightarrow f_{b1} = 1 \text{ MHz}$$

$$\text{but } 10 f_{b1} = f_{b1} + f_{b1} + f_{b2} \rightarrow f_{b1} + f_{b2} = 9 \text{ MHz}$$

maximize mixing ratios: choose  $f_{b1} = 3 \text{ MHz}$ ,  $f_{b2} = 6 \text{ MHz}$

$$\rightarrow R_1 = \frac{f_{b1}}{f_{b1}} = 3 \quad \text{and} \quad R_2 = \frac{f_{b2} + f_{b3,\max}^{9 \text{ MHz}}}{f_{b1} + f_{b1}} = \frac{15}{4} = 3.75$$

if  $f_{b0}$  is 15.34 MHz  $\rightarrow f_{b3}^* = 5 \text{ MHz}$ ,

$$f_{b2}^* = 3 \text{ MHz}, f_{b1}^* = 4 \text{ MHz}$$

example 10.3: must find  $N_{\min}$ ,  $N_{\max}$ ,  $A_{\min}$ ,  $A_{\max}$

105  $\rightarrow$  127 in 1 MHz increments: 23 frequencies

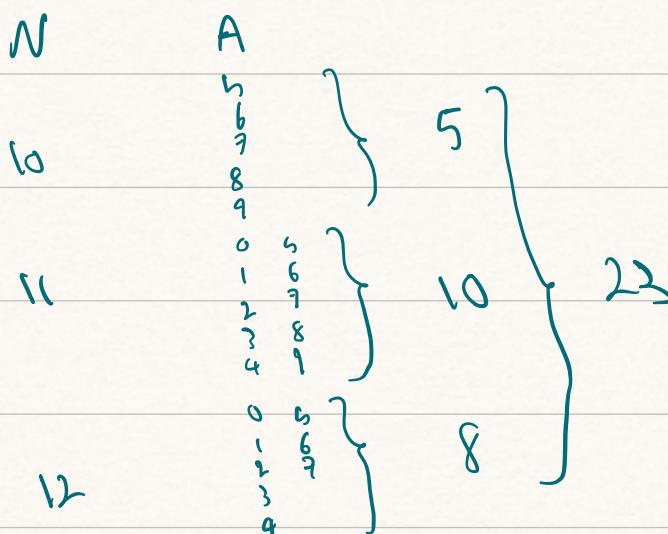
$$P = 10, P + Q = 11 \rightarrow Q = 1$$

$$D_{\max} = \frac{f_{0,\max}}{f_0} = \frac{127 \text{ MHz}}{1 \text{ MHz}} = 127 = P N_{\max} + A_{\max}$$

$$\therefore 127 = 10 N_{\max} + A_{\max} \rightarrow N_{\max} = 12, A_{\max} = 7$$

$$\text{and } D_{\min} = \frac{f_{0,\min}}{f_0} = \frac{105}{1} = 105 \rightarrow N_{\min} = 10, A_{\min} = 5$$

$\therefore$



example 10.4: find R, f<sub>r</sub>, number hops, N<sub>min</sub>, N<sub>max</sub>, A<sub>min</sub>, A<sub>max</sub>, hopping freq.

f<sub>0,min</sub> = 180.4, f<sub>0,max</sub> = 185.6, frequencies  $\geq 40$ , f<sub>r</sub>  $\geq 20$  Hz

$$P = 64, Q = 1, \text{ROM} = 2^7 \times 16 \rightarrow 2^7 = 128 \text{ hops}$$

$$\text{and } N + A = 16$$

$$\text{oscillation} = 2 \text{ MHz}, f_r = \frac{\text{oscillation}}{R} \geq 20 \text{ kHz}$$

$$\rightarrow R \leq 100 \rightarrow RA_2 RA_1 RA_0 = 001 \rightarrow R = 64$$

$$\therefore f_r = \frac{2 \text{ MHz}}{64} = 31.25 \text{ kHz}$$

with such a resolution, the number of hops is:  $\frac{(185.6 - 180.4) \text{ MHz}}{31.25 \text{ kHz}}$

$$\rightarrow 166.4 = 166 \text{ hops} > \text{required 50}$$

$$N \quad \text{A} \quad \text{Amax} \leq P-1 \neq A$$

$$\therefore D_{\min} = \frac{180.4 \text{ m}}{31.25 \text{ s}} = 5772.8 = P N_{\min} + A_{\min}$$

$$90 \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} \quad \therefore 167 \text{ combinations}$$

$$\rightarrow 5773 = 64 \cdot N_{\min} + A_{\min} \rightarrow N_{\min} = 90, A_{\min} = 13$$

$$91 \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} \quad \therefore 168 \text{ combinations}$$

$$\therefore D_{\max} = \frac{185.6 \text{ m}}{31.25 \text{ s}} = [5939.2] = 64 \cdot N_{\max} + A_{\max}$$

$$92 \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array} \quad \therefore 169 \text{ combinations}$$

$$\rightarrow N_{\max} = 92, A_{\max} = 51$$

The hopping frequency gives how quickly the output frequency can change. This is greater than or equal to the PLL's switching time

$$\rightarrow t_s \approx \frac{25}{f_r} = 8 \times 10^{-4} \text{ s} \rightarrow t_H \geq 8 \times 10^{-4} \text{ s}$$

$$\therefore f_H \leq 1250 \text{ Hz}$$

example 10.5:

$$f_{\text{A}} = 100 \text{ kHz}, \text{ resolution} = 1 \text{ kHz}$$

$$\therefore \text{resolution} = \frac{f_{\text{A}}}{M} \rightarrow M = 100$$

$$\text{Required operating frequency of divider: } f'_{\text{D}, \text{min}} = 1010 \text{ MHz}$$

example 10.6:

$$f_{\text{A}, \text{min}} = 34.4 \text{ MHz}, f_{\text{B}, \text{max}} = 40 \text{ MHz}, \text{increment: } 1 \text{ kHz}$$

$$f_{\text{D}} = 100 \text{ kHz}, \text{ loop A: fine, loop B: coarse}$$

$f_{\text{A}}$  controls the response time of loop C, hence if  $f_{\text{A}}$  is higher

response time of loop C will be lower. Therefore, offset  $f_{\text{A}}$  by 300 kHz

$$\rightarrow f_{\text{A}} = [300 \text{ kHz}, 399 \text{ kHz}]$$

loop A is responsible for the 1 kHz and 10 kHz increments

and loop B is responsible for 0.1 MHz and 1 MHz increments

$$\therefore f'_A = M f_A - f_A = N_B f_B$$

$$\therefore \text{resolution} = \frac{f_A}{M} \wedge \text{resolution} = 1 \text{ kHz} \rightarrow M = 100$$

$$\rightarrow f'_A = [30 \text{ MHz}, 39.9 \text{ MHz}]$$

$$\rightarrow N_{\text{A}, \text{min}} = \frac{30 \text{ MHz}}{100 \text{ kHz}} = 300, N_{\text{A}, \text{max}} = 399$$

$$\therefore 300 \leq N_A \leq 399$$

for loop B: since  $f_A$  was offset by + 300 kHz, loop B

Should be offset by  $-300 \text{ kHz}$

$$\rightarrow f_B = [35.1 \text{ MHz}, 39.7 \text{ MHz}]$$

$$\therefore N_{B,\min} = 341, \quad N_{B,\max} = 397$$

$$\sim f_B = f_A + f_B$$

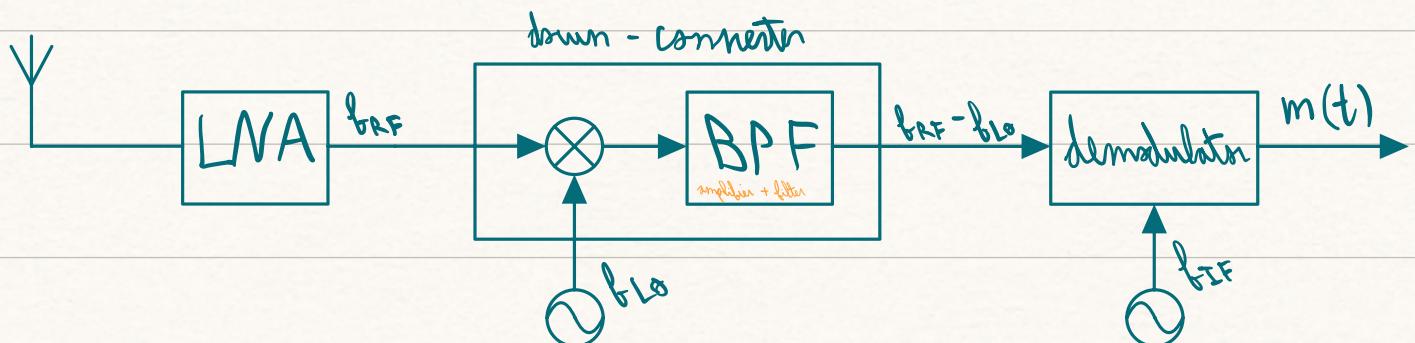
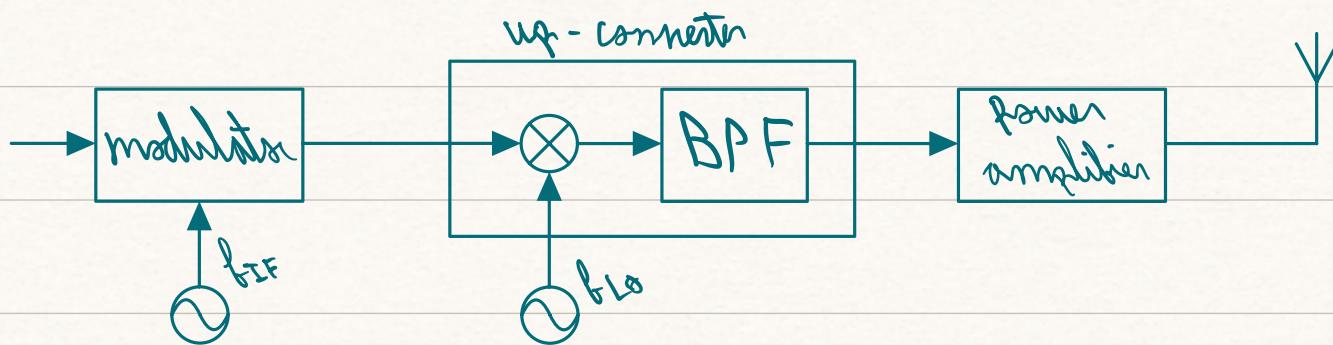
$$- \text{ If } f_B = 37.568 \text{ MHz} = (f_A + 300 \text{ kHz}) + f_B^{\text{offset}}$$

$$\rightarrow f_B = 37.2 \text{ MHz} \rightarrow N_B = 372$$

$$\sim f_A = 368 \text{ kHz} \rightarrow N_A = 368 \quad \therefore f_A' = 36.8 \text{ MHz}$$

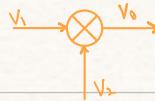
## Chapter 12:

- modulators, demodulators, and frequency converters (mixers) are fundamental devices in communication systems.



## + frequency mixers :

- most used for frequency conversion, modulation, and demodulation
- mixers are four-quadrant multipliers that multiply two input signals.



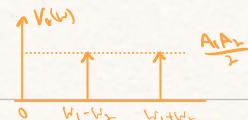
- if the two input signals are sinusoidal, then:

$$V_1(t) = A_1 \sin(\omega_1 t), \quad V_2(t) = A_2 \sin(\omega_2 t)$$

$$\rightarrow V_o(t) = V_1(t) V_2(t) = A_1 A_2 \sin(\omega_1 t) \sin(\omega_2 t)$$

$$\therefore V_o(t) = \frac{A_1 A_2}{2} [\cos((\omega_1 - \omega_2)t) - \cos((\omega_1 + \omega_2)t)]$$

this gives two frequency components:

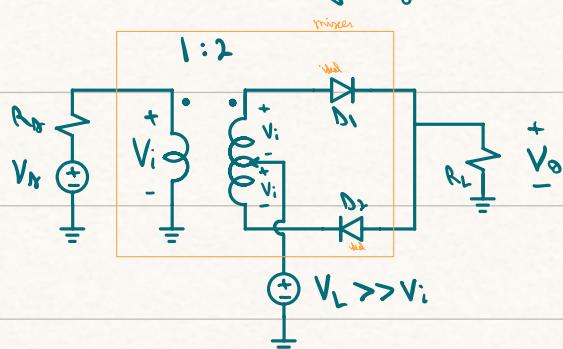


## + mixers are classified as:

1- active or passive.

2- switching-type or nonlinear.

## 1- simple two-diode switching-type mixer:



$$\text{if } V_L > 0 : D_1 \text{ on, } D_2 \text{ off, } -V_L - V_i + V_o = 0$$

$$\rightarrow V_o = V_L + V_i$$

$V_L < 0$ :  $D_1$  off,  $D_2$  on,  $-V_L + V_i + V_o = 0$

$$\rightarrow V_o = V_L - V_i \quad \text{negative}$$

hence,  $V_o = V_L + P(t) V_i$ , such that  $P(t) = \begin{cases} +1, & V_L > 0 \\ -1, & V_L < 0 \end{cases}$

- given that  $P(t)$  is a square wave, it can be expanded using the Fourier series:

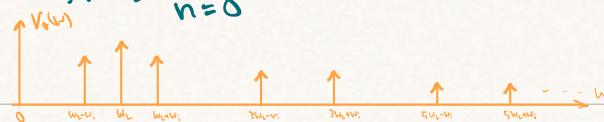
$$P(t) = \frac{4}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin[(2n+1)w_L t]}{2n+1}$$

then  $V_o(t) =$

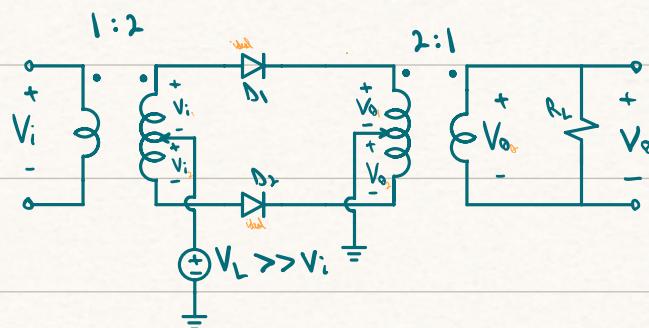
$$V_L \sin(w_L t) + V_i \sin(w_L t) \cdot \frac{4}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin[(2n+1)w_L t]}{2n+1}$$

$\rightarrow V_o(t) =$

$$V_L \sin(w_L t) + \frac{2V_i}{\pi} \sum_{n=0}^{\infty} \frac{\cos[(2n+1)w_L t - w_i t] - \cos[(2n+1)w_L t + w_i t]}{2n+1}$$



## 2- Simple two-diode switching-type mixer:



$$\begin{aligned} \text{assume } V_{o1} + V_{o2} = V_{ot} = 2V_{ot} \\ \because V_L > 0 : \\ -V_L - V_{o1} + V_{o2} = 0 \rightarrow V_{o1} = V_{o2} + V_L \\ \therefore -V_L + V_{o2} - V_{ot} = 0 \rightarrow V_{o2} = V_{ot} - V_L \\ \therefore V_{ot} = V_{o1} + V_{o2} + V_L - V_L = 2V_i \rightarrow V_{ot} = V_i \end{aligned}$$

if  $V_L > 0$ : both  $D_1$  and  $D_2$  on,  $V_o = V_i$

if  $V_L < 0$ : both  $D_1$  and  $D_2$  off,  $V_o = 0$

$$\rightarrow V_o = P(t) V_i, \text{ such that } P(t) = \begin{cases} 1, & V_L > 0 \\ 0, & V_L < 0 \end{cases}$$

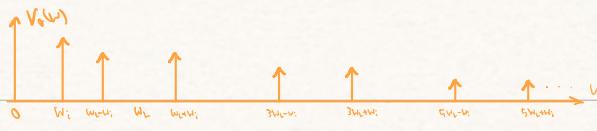
- P(t) there is also a square wave that can be expanded

$$\text{as: } P(t) = \frac{1}{2} + \frac{2}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin((2n+1)\omega_L t)}{2n+1}$$

$$\rightarrow V_o = \frac{V_i}{2} \sin(\omega_i t) + \frac{2V_i \sin(\omega_i t)}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin((2n+1)\omega_L t)}{2n+1}$$

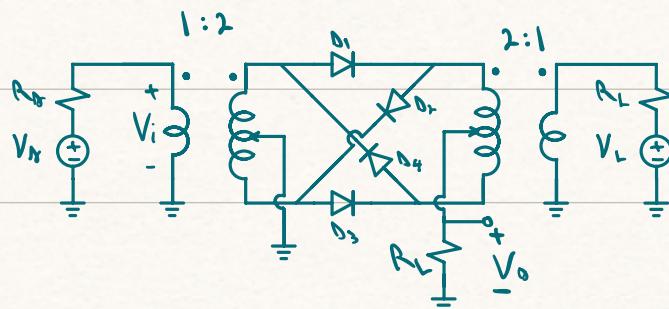
$$\therefore V_o(t) =$$

$$\frac{V_i}{2} \sin(\omega_i t) + \frac{V_i}{\pi} \cdot \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) + \cos((2n+1)\omega_L t + \omega_i t)}{2n+1}$$



### 3-four-diode switching-type mixer:

- This is a double-balanced mixer in which neither the local oscillator signal nor the input signal appear at the output.

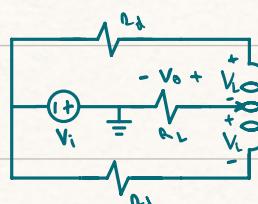


- If  $V_L > 0$ :  $D_2$  and  $D_3$  on,  $D_1$  and  $D_4$  off

- if  $V_L < 0$ :  $D_1$  and  $D_4$  on,  $D_2$  and  $D_3$  off

- for nonlinear diodes, the following is the equivalent

circuit:



$\therefore R_L \parallel R_f \rightarrow V_o$  found as: (Voltage division)

for  $V_L > 0$ :

$$V_o = -\frac{R_L}{R_L + \frac{R_f}{2}} \cdot V_i$$

for  $V_L < 0$ :

$$V_o = \frac{R_L}{R_L + \frac{R_f}{2}} \cdot V_i$$

- the output voltage can be expressed as:

$$V_o(t) = \frac{R_L}{R_L + \frac{R_f}{2}},$$

$$\text{such that: } P(t) = \begin{cases} -1, & V_L > 0 \\ +1, & V_L < 0 \end{cases}$$

again,  $P(t)$  can be expanded as:

$$P(t) = \frac{4}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\sin[(2n+1)\omega_L t]}{2n+1}$$

$$\rightarrow V_o(t) =$$

$$\frac{R_L}{R_L + \frac{R_f}{2}} \cdot \frac{2V_i}{\pi} \cdot \sum_{n=0}^{\infty} \frac{\cos[(2n+1)\omega_L t - \omega_i t] - \cos[(2n+1)\omega_L t + \omega_i t]}{2n+1}$$

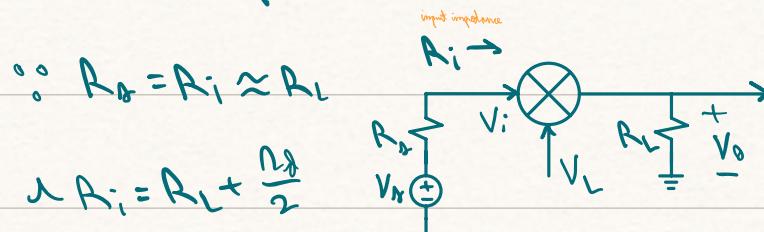


+ Conversion loss of mixer:

\* conversion loss: Ratio of output power in one sideband

to input power of the signal.

- maximum power transfer occurs when matched:



$$R_o = R_i \approx R_L$$

$$\therefore V_i = \frac{V_o}{2} \rightarrow P_i = \frac{V_o^2}{4R_i} = \frac{V_i^2}{R_L}$$

four-diode switching-type (ring) mixer

- for the double-balanced ring mixer:

$$V_o|_{W_1 \pm W_2} = \frac{2V_i}{\pi} = \frac{V_o}{\pi}$$

$$\therefore P_o = \frac{V_o^2}{\pi^2 R_L}$$

- the conversion gain of the mixer is found as:

$$G_r = \frac{P_o}{P_i} = \frac{4}{\pi^2} \rightarrow G_r(\text{dB}) = -3.92 \text{ dB}$$

- hence, the conversion loss is:

$$L = \frac{1}{G_r} = \frac{P_i}{P_o} \rightarrow L(\text{dB}) = 3.92 \text{ dB}$$

- for the single-balanced two-diode (opposite) mixer:

$$V_o|_{W_1 \pm W_2} = \frac{2V_i}{\pi}$$

$\rightarrow G_r = -3.92 \text{ dB}$ , same as ring.

- for the single-balanced two-diode (same direction) mixer:

$$V_o|_{W_1 \pm W_2} = \frac{V_i}{\pi} = \frac{V_o}{2\pi}$$

$$\therefore P_o = \frac{V_o^2}{4\pi^2 R_L}$$

$$\rightarrow L = \frac{P_i}{P_o} = \pi^2 \rightarrow L = 9.94 \text{ dB}$$

+ intermodulation distortion in mixers:

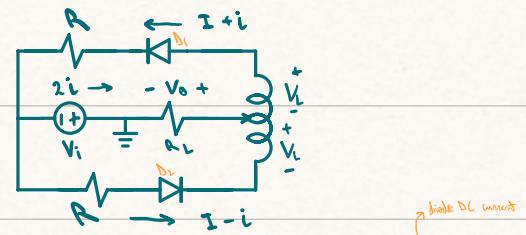
- as mentioned previously, intermodulation distortion arises from nonlinear receivers.

- for diode mixers, the current flowing in the diode is a nonlinear function of the voltage drop across it, such that:

$$i_D = I_{D0} e^{\frac{V_D}{V_T}}, \quad V_T = \frac{kT}{q} \stackrel{\text{at room temperature}}{\approx} 0.026V$$

- after adding linearizing resistors, the following

is the equivalent circuit of a double-balanced ring mixer:



$$\therefore V_i = i (2R_L + R) + \frac{V_T}{2} \ln \left( \frac{I+i}{I-i} \right)$$

if  $i \ll I$ , the  $\ln$  term is expanded to give:

$$V_i \approx \left( 2R_L + R + \frac{V_T}{I} \right) \cdot i + \frac{1}{3} \left( \frac{i}{I} \right)^3 + \dots$$

which can be inverted to find  $i$  as:

$$i \approx \frac{V_i}{2R_L + R} - \frac{V_T}{3} \cdot \frac{V_i^3}{(2R_L + R)^4 I^3}$$

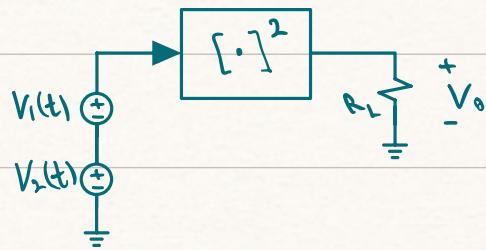
- therefore,  $V_o = 2iR_L$  is found to be:

$$V_o \approx \frac{2R_L}{2R_L + R} \cdot V_i - \frac{V_T}{3} \cdot \frac{2R_L}{(2R_L + R)^4 I^3} \cdot V_i^3$$

$$\text{or } V_o \approx k_1 V_i - k_3 V_i^3$$

- hence, the third-order intermodulation distortion is proportional to  $k_3$ . Then, the goal is to minimize  $k_3$ .
- it can be seen that  $k_3$  is inversely related to the linearizing resistors and the diode DC current.
- therefore, increasing the linearizing resistors' values will decrease IMD.

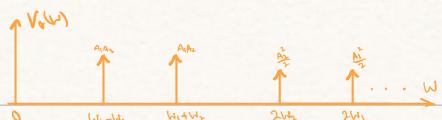
+ <sup>nonlinear</sup> Square-law mixer:



- diodes and transistors can be made to operate nonlinearly in order to be used as square-law mixers.
- the two signals are added then squared:

$$V_o(t) = [V_1(t) + V_2(t)]^2 = [A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)]^2$$

$$\Rightarrow V_o(t) = \frac{A_1^2}{2} [1 - \cos(2\omega_1 t)] + \frac{A_2^2}{2} [1 - \cos(2\omega_2 t)] + A_1 A_2 [\cos(\omega_1 t - \omega_2 t) - \cos(\omega_1 t + \omega_2 t)]$$



$$A_1 A_2 [\cos(\omega_1 t - \omega_2 t) - \cos(\omega_1 t + \omega_2 t)]$$

- diodes may be used as square-law mixers, but they introduce large conversion losses.

- for the diode to operate in its square-law region,

$$V_i \ll V_{dc}$$

$$\rightarrow V_o(t) = V_{dc} + k_{x1} V_i + k_{x2} V_i^2 + \dots$$

$$\text{if } V_i = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$

$$\rightarrow V_o(t) = V_{dc} + k_{x1} \cdot [A_1 \sin(\omega_1 t) + \dots$$

$$A_2 \sin(\omega_2 t)] + k_{x2} \cdot [ \dots$$

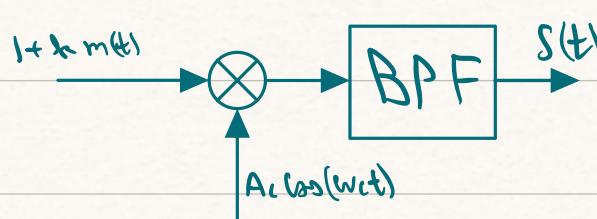
$$A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)]^2$$

+ Amplitude modulation and demodulation:

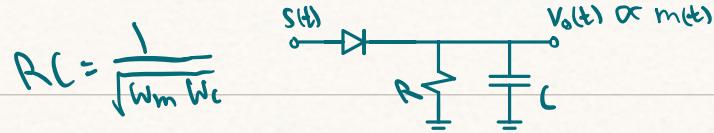
1- <sup>full-wave</sup> DSB-LL amplitude modulation:

$$S(t) = A_c [1 + k_m m(t)] \cos(\omega_c t) \rightarrow |k_m m(t)|_{max} \leq 1$$

- the modulator is a mixer followed by a BPF:



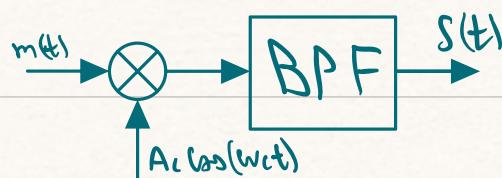
- the demodulator is an envelope detector:



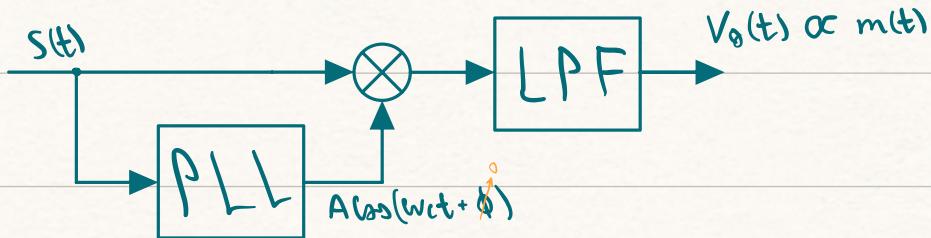
2- DSB-SC AM:

- similarly, the modulator is a mixer and BPF:

$$S(t) = A_c m(t) \cos(\omega_c t)$$



- coherent detector
- the demodulator consists of a product modulator with a PLL producing a synchronized local oscillator.



### 3- SSB-SC AM:

$$S(t) = A_c m(t) \cos(\omega_c t) \pm A_c \hat{m}(t) \sin(\omega_c t)$$

coherent transformer

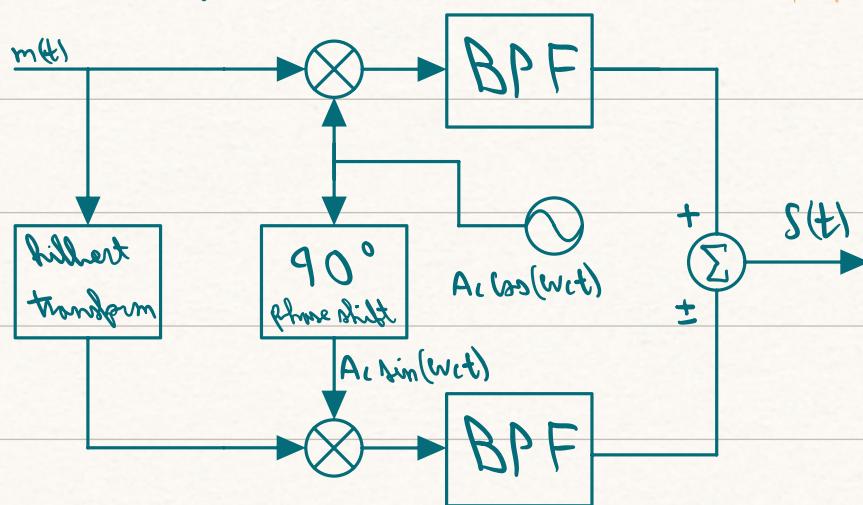
- the modulator requires two mixers, Hilbert transformer, BPFs,

widler, and phase shifter. whereas the demodulator is exactly

coherent demodulator

the same as for DSB-SC.

SSB modulator



ASK

### 4- Amplitude Shift Keying:

- this is the digital version of AM. hence, standard AM

product modulator

modulators can be used, but the message signal must be

a unipolar digital signal

- an envelope detector can be used as a demodulator.

+ phase and frequency modulation and demodulation:

- in angle modulation, the modulated signal is represented as:

$$S(t) = A_c \cos[\omega_0 t + \Theta(t)]$$

frequency modulation  $\rightarrow f(t) \propto m(t)$  i.e.,  $\frac{d\theta(t)}{dt} \propto m(t)$

phase modulation  $\rightarrow \Theta(t) \propto m(t)$

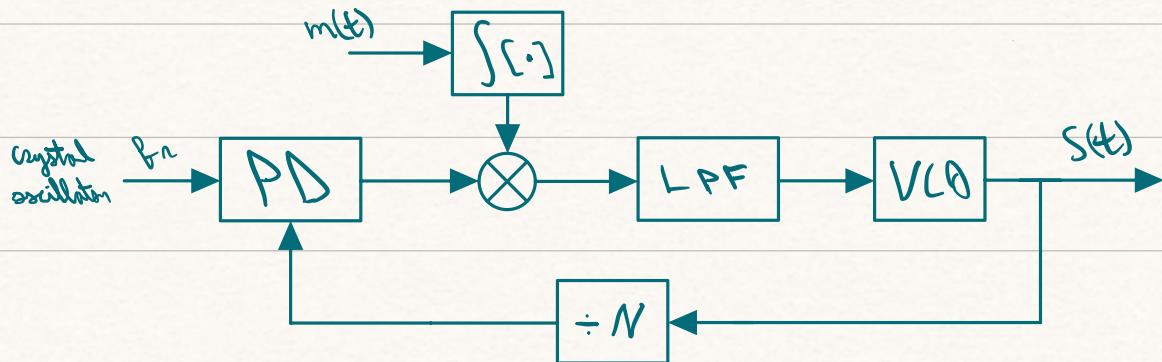
- since  $f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$ , then a frequency modulator can be converted to a phase modulator by differentiating  $m(t)$  first.

I - Analog frequency modulation:

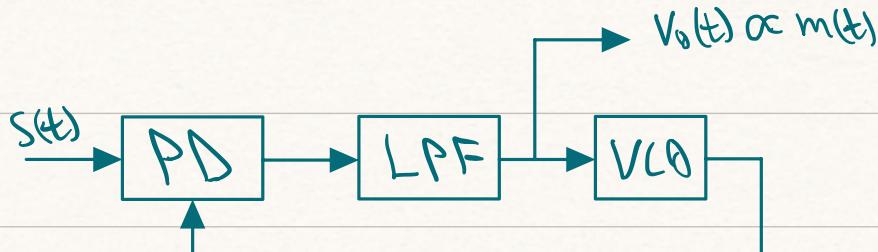
the VCO produces a signal whose frequency changes with the voltage of the input message signal

- a signal can be modulated by simply inputting it to a

VCO or a PLL for higher frequency stability.



- the demodulator can be a PLL or a balanced frequency discriminator.



## 2 - Analog phase modulation:

- the same modulator and demodulator as FM can be used here, but without the integrator

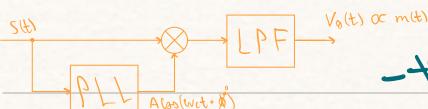
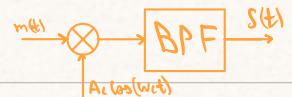
## 3 - frequency shift keying:

- same modulator and demodulator can be used here,  
*unipolar or polar*
- but the message signal has to be in digital form.

## 4 - phase shift keying:

- the modulator here is the same as DSB-SC, but the

*message signal must be in polar form.*



*V\_o(t) or m(t)*

*A cos(w\_c t)*

*PLL*

*S(t)*

*m(t)*

*A cos(w\_c t)*

*X*

*BPF*

*S(t)*

*m(t)*

*A cos(w\_c t)*

*X*

final practice:

Chapter 8 quiz:

$$f_{\min} = 200 \text{ kHz}, f_{\max} = 300 \text{ kHz}$$

$$\text{increment} = 10 \text{ kHz}, \text{LPF: } R_o = 10 \text{ k}\Omega, C_o = 1 \text{ nF}$$

$$V_{cc} = 10 \text{ V} \quad \text{XOR FD}$$

$$\text{a) } \therefore f_{\min} = 200 \text{ kHz} = \frac{1}{R_2(C + 32 \text{ pF})}$$

$$\sim 100 \text{ pF} < C < 0.01 \text{ nF}, 10 \text{ k}\Omega < R_1, R_2 < 1 \text{ M}\Omega$$

$$\text{try } C = 1 \text{ nF} \rightarrow R_2 = 4845 \text{ }\Omega \text{ invalid}$$

$$\text{try } C = 240 \text{ pF} \rightarrow R_2 = 17.73 \text{ k}\Omega \text{ valid}$$

$$\therefore f_{\max} = 300 \text{ kHz} = f_{\min} + \frac{1}{R_1(C + 32 \text{ pF})}$$

$$\rightarrow R_1 = 35.46 \text{ k}\Omega \text{ valid}$$

$$\text{b) } \therefore \text{XOR freq} = \frac{V_{cc}}{\pi} = \frac{10}{\pi}, f_o = \frac{2\pi \Delta f}{V_{cc}-2}$$

$$\rightarrow f_o = 24000 \text{ }\overset{\text{MHz}}{\cancel{\pi}}$$

$$\therefore f_o = 240 \text{ kHz} \rightarrow N = \frac{240 \text{ k}}{10 \text{ k}} = 24$$

$$\therefore f_V = 10 \text{ kHz}$$

$$\therefore W_L = \frac{1}{R_C} = 100 \text{ kHz}$$

$$\rightarrow z = \frac{1}{2} \cdot \sqrt{\frac{C_L}{L_V}} = 1.5811$$

$$\rightarrow w_n = 31622.7 \text{ rad/s}$$

$$\omega_{\text{Wh}} = \omega_n \left[ 1 - 2z^2 + (2 - 4z^2 + 4z^4)^{\frac{1}{2}} \right]^{\frac{1}{2}}$$

$$\rightarrow \omega_n = 11094.69 \text{ rad/s} = 1766 \text{ Hz}$$

Chapter 8 homework:

8.1:  $\text{f}_d$ : phase detector gain,  $\text{f}_{\text{av}}$ : VCO gain

$$\therefore f_d = 50 \text{ kHz}, f_{\text{av}} = 1 \text{ MHz}, k_f = 2 \text{ V/rad}$$

$$\omega_n = \frac{200\pi \text{ rad/s/V}}{100 \text{ Hz/V}}, t_n = \frac{2\pi}{\omega_n}, N = 20$$

$$\therefore \omega_{\text{Wh}} = \omega_n = \frac{\ln \omega_n}{N} = 20 \text{ rad/s} = \omega_{\text{Wh}}$$

$$\rightarrow t_n \approx 3 \text{ ms}$$

$$\text{for } f_{\text{av}} = 1.2 \text{ MHz} \rightarrow N = 24 \rightarrow \omega_n = 52.36 \text{ rad/s}$$

$$\rightarrow t_n = 42.02 \text{ ms}$$

$$\therefore f_{\text{freerunning}} = 1 \text{ MHz} \rightarrow f_{\text{min}} = 800 \text{ kHz}$$

$$\sim f_{\text{max}} = 1.2 \text{ MHz} \rightarrow N_{\text{min}} = 16 \quad N_{\text{min}} = 24$$

$$\therefore \text{Range} = 8 \text{ frequencies or } \frac{200 \text{ kHz} \times 2}{50 \text{ kHz}} = 8$$

8.2: a)  $\therefore f_0 = 1 \text{ MHz} \rightarrow N = 20 \rightarrow \omega_n = 20\pi$

$$\therefore z = \frac{1}{\sqrt{2}} = \frac{1}{2} \sqrt{\frac{\omega_L}{\omega_n}} \rightarrow \omega_L = 40\pi \text{ rad/s}$$

b)  $\omega_L = 10\pi \cdot 72 \text{ rad/s}$

8.14:  $\therefore T_A = 4 \text{ kHz} = 2 \omega_n k \sqrt{C_{\text{L, max}}}$

Final 12/2018:

Q1: w)  $f_{\min} = 21 \text{ MHz}$ ,  $f_{\max} = 26 \text{ MHz}$ , res:  $1 \text{ kHz}$

$\therefore \Delta f = 5 \text{ MHz}$  & number of freq. = 5000

$$\text{Output} = f'_o = 10f_i + f'_q + \frac{f'_3}{10} + \frac{f'_2}{100} + \frac{f'_1}{1000}$$

four modules required.

$$\rightarrow f_i = 2 \text{ MHz} \quad \text{or} \quad 10f_i = f_i + f_1 + f_2$$

$$\rightarrow 18 \text{ MHz} = f_1 + f_2 \quad \text{take } f_1 = 6 \text{ MHz}, f_2 = 12 \text{ MHz}$$

$$\rightarrow R_1 = \frac{6 \text{ MHz}}{2 \text{ MHz}} = 3, \quad R_2 = \frac{12 + f'_2}{6 + 2}$$

$$\rightarrow R_2 = \frac{12 + q}{6 + 2} = 2.625$$

$$\text{if } f_o = 23.456 \rightarrow f_i = 2 \text{ MHz}, f'_4 = 3 \text{ MHz}$$

$$f'_3 = 4 \text{ MHz}, f'_2 = 5 \text{ MHz}, f'_1 = 6 \text{ MHz}$$

w)  $f_{\min} = 934 \text{ MHz}$ ,  $f_{\max} = 960 \text{ MHz}$ , res =  $200 \text{ kHz}$

$$P = 64, Q = 1, \text{ crystal} = 10.24 \text{ MHz}$$

$\therefore \times 5$  post multiplier  $\rightarrow$  inc  $\approx 200 \text{ kHz} = 40 \text{ kHz}$

$$\therefore \frac{\text{crystal}}{40 \text{ kHz}} \leq R \rightarrow R = 256 \quad (0 \ 11)$$

$$f_{\min} = 187 \text{ MHz}, f'_{\max} = 192 \text{ MHz}$$

$$\Delta_{\min} = P N_{\min} + A_{\min} = \frac{f'_{\min}}{f'_i} = 4675$$

$$\rightarrow N_{\min} = 73, A_{\min} = 3$$

$$(A_{\max}) = P - 1 = 63$$

$$\text{N} D_{\text{max}} = P N_{\text{max}} + A_{\text{max}} = \frac{f_{\text{max}}}{f_{\text{ba}}} = 4800$$

$$\rightarrow N_{\text{max}} = 75, A_{\text{max}} = 0$$

N      A

$$73 \quad 3 - 63 \rightarrow 61$$

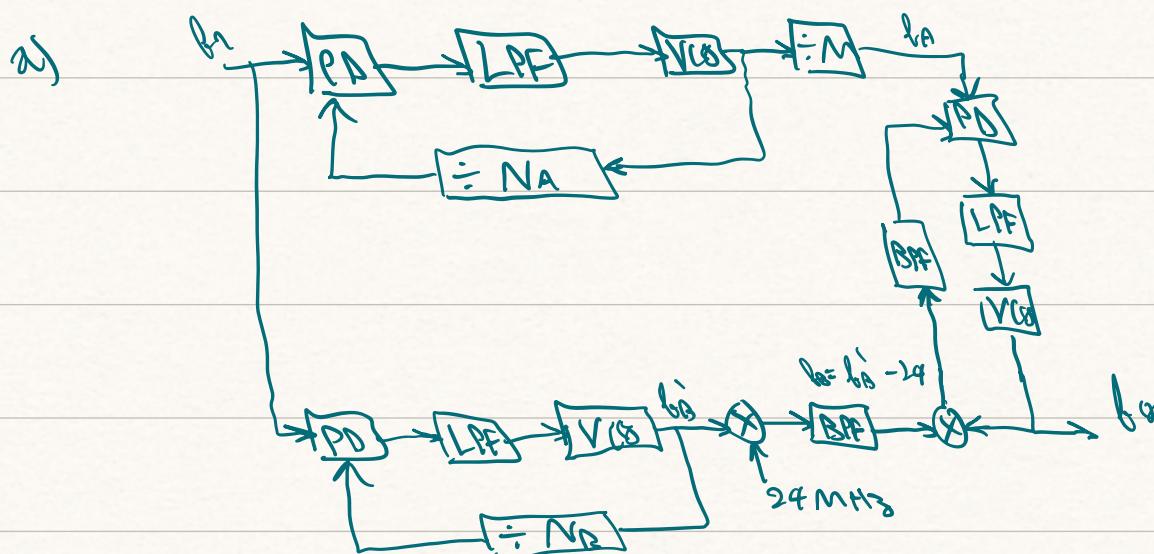
$$74 \quad 0 - 63 \rightarrow 64$$

$$75 \quad 0 - 0 \rightarrow 1$$

$$\text{formal dividers} = \frac{f_{\text{max}}}{P} = \frac{192 \text{ MHz}}{64} = 3 \text{ MHz}$$

$$\text{Q2: } f_{\text{min}} = 24 \text{ MHz}, f_{\text{max}} = 29 \text{ MHz}, f_{\text{res}} = 1 \text{ kHz}$$

$$f_{\text{r2}} = 100 \text{ kHz}$$



$$\text{for } f_{\text{res}} = 1 \text{ kHz from } f_{\text{r1}} = 100 \text{ kHz} \rightarrow M = 100$$

$$f_A = 0 \text{ kHz} \rightarrow 99 \text{ kHz offset 10 kHz}$$

$$\rightarrow f_A = 500 \text{ kHz} \rightarrow 549 \text{ kHz}$$

$$f_B = 24.5 \text{ MHz} \rightarrow 28.4 \text{ MHz}$$

$$\rightarrow f_A' = 50 \text{ MHz} \rightarrow 59.9 \text{ MHz}$$

max input to programmable divider

$$\sim f_B' = 48.5 \text{ MHz} \rightarrow 52.5 \text{ MHz}$$

max input to prog. divider

$$\therefore N_{A,\min} = \frac{50 \text{ m}}{100 \text{ m}} = 500, N_{A,\max} = 599$$

$$\sim N_{B,\min} = 485, N_{B,\max} = 525$$

$$\text{for } f_A = 26.5 \text{ kHz} \rightarrow f_B = 26.0 \text{ MHz}$$

$$\rightarrow N_B = \frac{f_B + 26 \text{ mV}_2}{f_B} = 500$$

$$\sim f_A = 543 \text{ kHz} \rightarrow f_A' = 54.3 \text{ MHz}$$

$$\rightarrow N_A = 543$$

Q3: 1) for two opposite dividers:

$$L = 4 \text{ dB}, A = \frac{2V_i}{\pi} = \frac{2}{5\pi}$$

for two same direction dividers:

$$L = 10 \text{ dB}, A = \frac{1}{5\pi}$$

for four dividers:

$$L = 4 \text{ dB}, A \approx \frac{2}{5\pi}$$

- the two opposite dividers can be used for DSB-SC

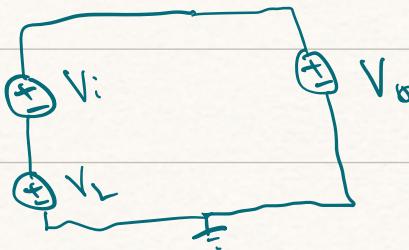
- both same direction dividers and four dividers can be used

for DSB-SC

-  $V_L$  is chosen much larger than  $V_i$  so that it contains

the switching

b) if  $D_2$  is damaged, the equivalent circuit is:



$$\rightarrow V_o = V_L + V_i \text{ when } V_i > 0$$

if  $V_L < 0$ ,  $D_1$  is off,  $V_o = 0$

$$\therefore V_o = \begin{cases} V_L + V_i, & V_L > 0 \\ 0, & V_L \leq 0 \end{cases}$$

$$\rightarrow V_o = P(t) \cdot [V_L + V_i]$$

$$P(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{\sin((2n+1)\omega_L t)}{2n+1}$$

$$\rightarrow V_o = \frac{1}{2} [V_L \sin(\omega_L t) + V_i \sin(\omega_i t)] + \frac{2}{\pi} V_L \sin(\omega_L t) \sum - \\ + \frac{2}{\pi} V_i \sin(\omega_i t) \cdot \sum - -$$

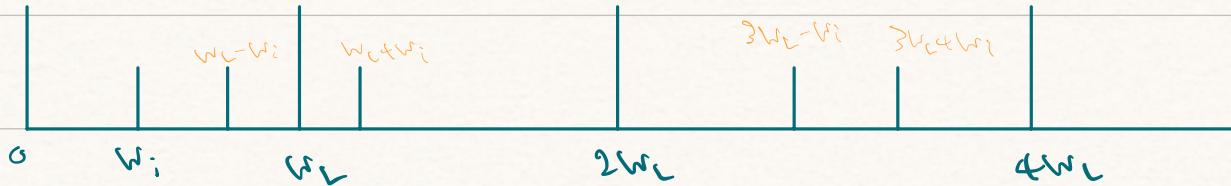
$$\rightarrow V_o = \frac{V_L}{2} \sin(\omega_L t) + \frac{V_i}{2} \sin(\omega_i t) + \frac{V_L}{\pi} \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) - \cos((2n+1)\omega_i t + \omega_L t)}{2n+1} - - - \\ + \frac{V_i}{\pi} \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) - \cos((2n+1)\omega_i t + \omega_L t)}{2n+1}$$

$$\rightarrow \text{at } \omega_L: \frac{V_L}{2}, \text{ at } \omega_i: \frac{V_i}{2}, \text{ at } 0: \frac{V_L}{\pi}$$

$$\text{at } 2\omega_L: \frac{V_L}{\pi}, \text{ at } 2\omega_i: \frac{V_i}{3\pi}, \text{ at } 4\omega_L: \frac{V_L}{5\pi}$$

$$\text{at } \omega_L - \omega_i: \frac{V_i}{\pi}, \text{ at } \omega_L + \omega_i: \frac{V_i}{\pi}, \text{ at } 3\omega_L - \omega_i: \frac{V_i}{7\pi}$$

$$\text{at } 3\omega_L + \omega_i: \frac{V_i}{3\pi}$$



Q2: a)  $I_{ds} = \frac{I_{DSSS} V_i V_L}{V_p^2} = \frac{0.1}{25} = 4 \times 10^{-3} \text{ A}$

conversion gain:  $\left(\frac{I_{ds} \cdot R_L}{V_i}\right)^2 = 0 \text{ dB}$

$\therefore R_L = 500 \rightarrow V_o = 2V$

final 1/2020:

Q1: a) output res = 200 kHz  $\rightarrow$  before LK: 40 kHz

$\therefore$  crystal: 10.24 MHz  $\rightarrow Q = \frac{10.24 \text{ M}}{40 \text{ K}} = 256$

$\rightarrow$  Lhe = 0.11

$\therefore f_{\min} = \frac{935 \text{ m}}{5} = 187 \text{ MHz}$

$\rightarrow D_{\min} = P N_{\min} + A_{\min} = \frac{187 \text{ m}}{40 \text{ k}} = 4.69 \text{ h}$

$\therefore N_{\min} = 73, A_m = 3$

$\therefore f_{\max} = \frac{960 \text{ m}}{5} = 192 \text{ MHz}$

$\rightarrow D_{\max} = P N_{\max} + A_{\max} = 4800 \text{ p-1}$

$\rightarrow N_{\max} = 75, A_{\max} = 0 \quad (A_{\max}) = 63$

N 73 74 75

A  $3 \rightarrow 63$

$0 \rightarrow 63$

$0 \rightarrow 126$

from at input is  $\frac{f_{\text{max}}}{P} = 3 \text{ MHz}$

b) 21 to 25 MHz  $\rightarrow 4 \text{ MHz} \rightarrow 400 \text{ frequencies}$

$$\therefore 3 \text{ modules required} \rightarrow f_s = 10f_1 + f_2^* + \frac{f_2^*}{10} + \frac{f_1^*}{100}$$

$$f_i = 2 \text{ MHz} \quad \text{and} \quad 10f_i = f_1 + f_2 + f_3$$

$$\rightarrow 18 \text{ MHz} = f_1 + f_2 + f_3, \text{ take } f_1 = 6 \text{ MHz}$$

$$\text{and } f_2 = 12 \text{ MHz}$$

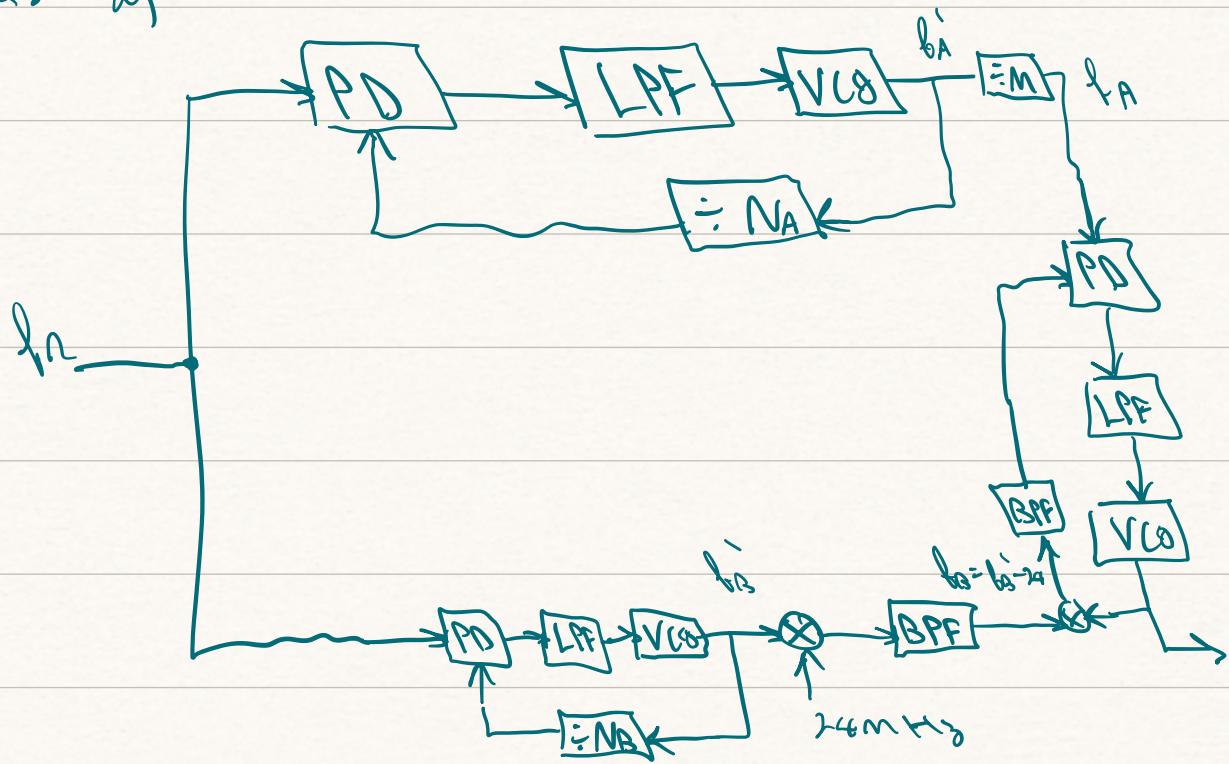
$$\therefore R_1 = \frac{f_1}{f_i} = 3, \quad R_2 = \frac{f_2 + f_3}{f_1 + f_3}$$

$$\therefore R_2 = \frac{12 + 4}{8} = 2.625$$

$$\text{if } f_s = 23.45 \rightarrow 3.45 = f_3^* + \frac{f_2^*}{10} + \frac{f_1^*}{100}$$

$$\rightarrow f_3^* = 3 \text{ MHz}, \quad f_2^* = 4 \text{ MHz}, \quad f_1^* = 5 \text{ MHz}$$

Q2: a)



for resolution = 1 kHz from 100 kHz  $f_2 \rightarrow N = 100$

to keep A/D converter operating at high speed, offset by 500 kHz

$$\rightarrow f_{BA} = 100 \text{ kHz} \rightarrow 599 \text{ kHz}$$

$$\therefore f_A' = 50 \text{ MHz} \rightarrow 59.9 \text{ MHz}$$

$$\therefore N_A = 100 \rightarrow 599$$

$$\wedge f_{BB} = 24.5 \text{ MHz} \rightarrow 28.5 \text{ MHz}$$

$$\rightarrow f_B' = 48.5 \text{ MHz} \rightarrow 52.5 \text{ MHz}$$

$$\therefore N_B = 48 \rightarrow 515$$

b) at divider A,  $f_{max} = 59.9 \text{ MHz}$

at divider B,  $f_{max} = 52.5 \text{ MHz}$

$$\wedge f_A = 27.643 \text{ MHz} \rightarrow f_{BA} = 543 \text{ kHz}$$

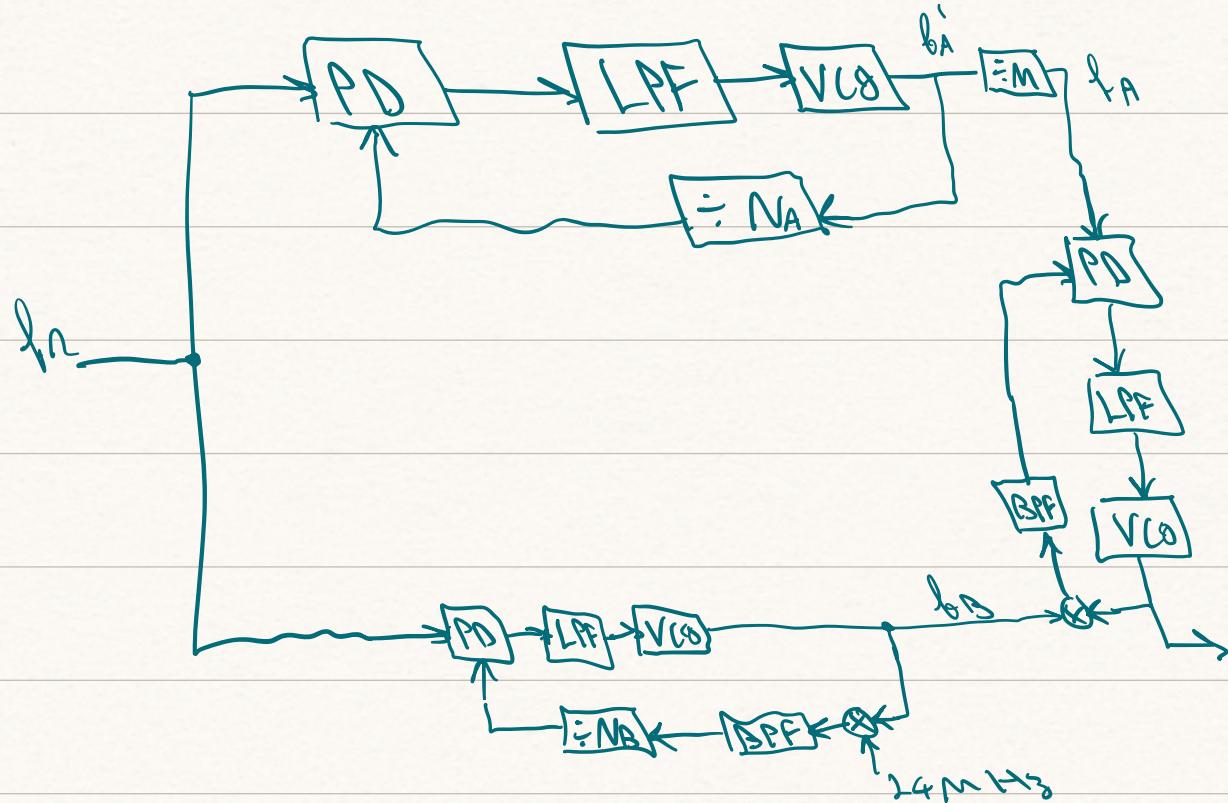
$$\wedge f_{BB} = 29.1 \text{ MHz}$$

$$\therefore N_A = 543 \quad \wedge N_B = 511$$

downconverter in the loop!

$$\rightarrow N_B = 5 \rightarrow 45 \quad \text{if } f_A \text{ offset by } 500 \text{ kHz}$$

$$\text{for } f_B = 29.643 \rightarrow N_B = 31$$



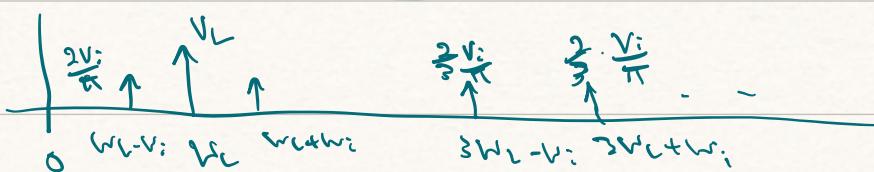
if  $f_A$  offset by  $300 \text{ kHz}$   $\rightarrow N_A = 300 \rightarrow 399$

$$\therefore N_B = 7 \rightarrow 49$$

$$\text{for } f_0 = 24.603 \text{ MHz} \rightarrow N_A = 343, N_B = 33$$

Q3: a) 1 - two-frequencies in opposite directions:

$$V_o(t) = V_L \sin(\omega_L t) + \frac{2V_i}{\pi} \sum_{n=0}^{\infty} \frac{\cos((2n+1)(\omega_L - \omega_i)t)}{2n+1}$$

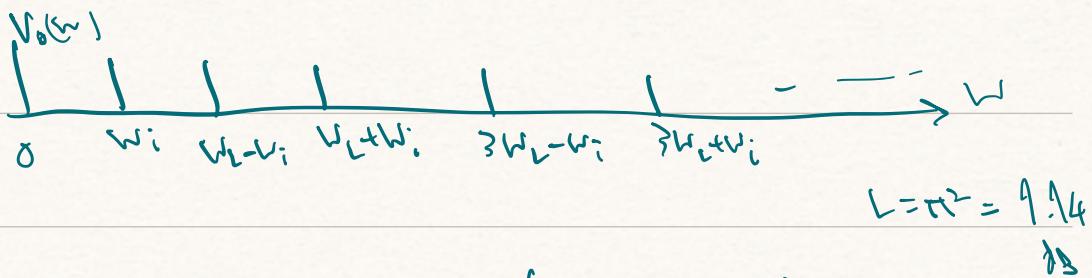


$$L = \left( \frac{0.1}{0.4/\pi} \right)^2 = \left( \frac{1}{4} \right)^2 \cdot \frac{\pi}{0.4} = \frac{\pi}{16}$$

$$\text{amplitude of } \omega_L - \omega_i \text{ component: } \frac{2V_i}{\pi} = \frac{0.4}{\pi}$$

2 - two-frequencies in same direction:

$$V_o(t) = \frac{V_i}{2} \sin(\omega_i t) + \frac{V_i}{\pi} \sum_{n=0}^{\infty} \frac{\cos((2n+1)(\omega_L - \omega_i)t) - (\cos(2n\omega_i)\omega_i)}{2n+1}$$



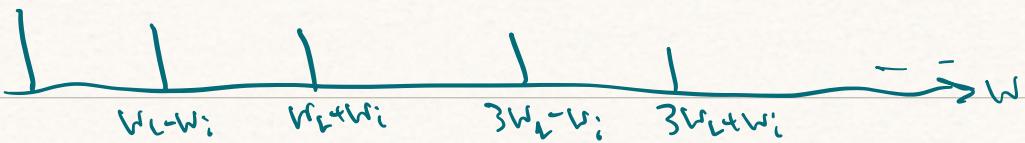
$$w_L - w_i \text{ component : } \frac{V_i}{\pi} = \frac{0.2}{\pi}$$

3-four-diodes:

$$V_o(t) = \frac{R_L}{R_L + \cancel{\frac{R_L}{2\pi}}}. \frac{2V_i}{\pi} \cdot \sum_{n=0}^{\infty} (\cos[(2n+1)w_L - w_i]t - \cos[2n\pi])$$

$$V_o(w)$$

$$L = 3.9 \approx 4.13$$



$$\text{at } w_L - w_i: A = \frac{R_L}{R_L + \cancel{\frac{R_L}{2\pi}}} \cdot \frac{0.4}{\pi} \approx \frac{0.4}{\pi}$$

- for DSB-LC, we need a component at  $w_L$ , hence only two opposite diodes can be used

- for DSB-SC, no component should be at  $w_L$ , hence both two-diodes in same direction and four diodes work.

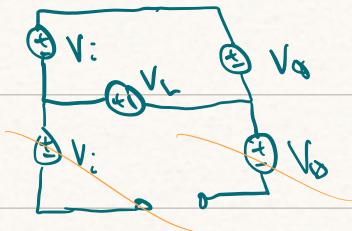
-  $V_L$  should be much larger than  $V_i$  so that it controls the switching  $\rightarrow$  switching speed will be that of  $V_L$

Q) the equivalent circuit will be:

$$\therefore V_o = V_L + V_i \text{ for } V_L > 0$$

$$\text{or } V_o = 0 \text{ for } V_L < 0$$

$$\therefore V_o(\omega) = P(\omega) \cdot (V_L + V_i) \quad \text{d.t., } P(\omega) = \begin{cases} 1, & V_L > 0 \\ 0, & V_L \leq 0 \end{cases}$$

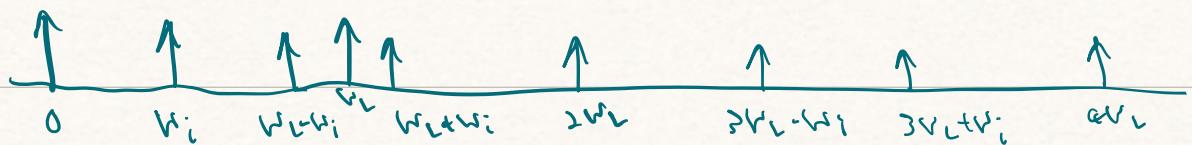


$$\rightarrow P(x) = \frac{1}{2} + \frac{1}{\pi} \sum \frac{\sin((2n+1)w_i t)}{2n+1}$$

$$\rightarrow V_o(t) = \frac{V_L}{2} \sin(w_L t) + \frac{V_i}{2} \sin(w_i t) +$$

$$+ \frac{V_L}{\pi} \cdot \sum \frac{\cos((2n+1)(w_L - w_i)t) - \cos((2n+1)(w_L + w_i)t)}{2n+1}$$

$$+ \frac{V_i}{\pi} \cdot \sum \frac{\cos((2n+1)(w_L - w_i)t) - \cos((2n+1)(w_L + w_i)t)}{2n+1}$$



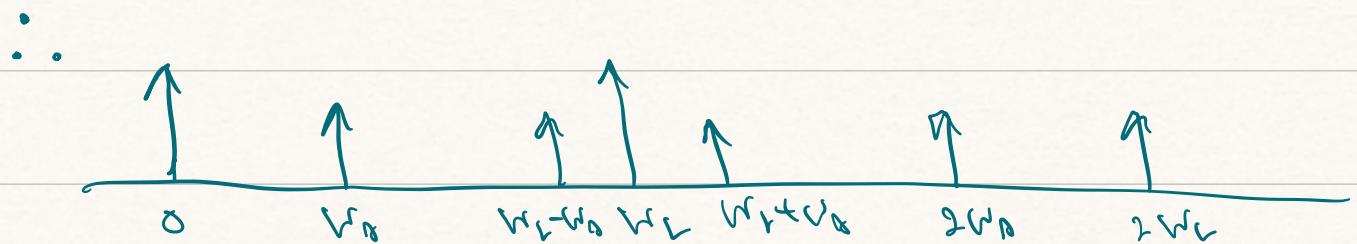
Q2: n)  $V_i = 2 \sin(w_i t) + 3 \sin(w_L t)$

$$\rightarrow V_o(t) = 1.8 \sin(w_0 t) + 2.7 \sin(w_L t) + \frac{2 \cdot 2 \cdot 3 \cdot 0.1}{2} \cdot \left[ \cos((w_L - w_0)t) - \cos((w_L + w_0)t) \right] + \left[ \frac{2 \sin(w_0 t)}{10} \right]^2 + 0.1 \left[ \frac{3 \sin(w_L t)}{10} \right]^2$$

$$\rightarrow V_o(t) = 1.8 \sin(w_0 t) + 2.7 \sin(w_L t) +$$

$$0.6 \cos((w_L - w_0)t) - 0.6 \cos((w_L + w_0)t)$$

$$+ 0.1 \left[ 1 - \cos(2w_0 t) \right] + 0.045 \left[ 1 - \cos(2w_L t) \right]$$



b) for a phase modulator PLL:  $\theta_o(\omega) = \frac{\int_{-\infty}^{\infty} V_o(t)}{\omega + k_o k_d F_o / N}$

$$\rightarrow \Theta_0(t) = N\Theta_0(0) + \frac{N m(t)}{\Delta t}$$

$$\curvearrowleft f_0(t) = \frac{1}{2\pi} \frac{d\Theta_0(t)}{dt} = Nf_0 + \frac{N}{2\pi\Delta t} \cdot \frac{dm(t)}{dt}$$

$$\curvearrowleft S(t) = A \cos[\omega t + \Theta(t)]$$

$$\therefore \Theta_0(t) = 2\pi \int f_0 dt$$

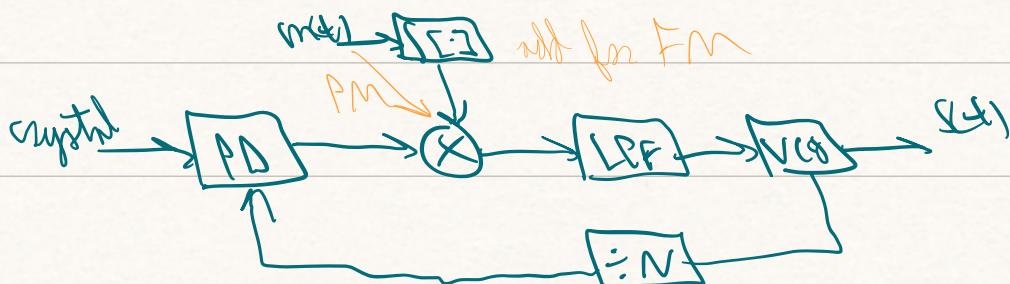
$$\rightarrow \Theta_0(t) = 50 \cdot 4\pi t \times 10^6 + 50 \cdot \sin(\omega_0 t)$$

$$\rightarrow f_0(t) = 100 \times 10^6 + \frac{50}{4\pi} \cdot 4\pi \times 10^3 \cdot \cos(2\pi \times 10^3 t)$$

$$\curvearrowleft f_0(t) = 100 \times 10^6 + \underbrace{50 \times 10^3}_{\Delta f} \cos(2\pi \times 10^3 t)$$

$$\curvearrowleft S(t) = A \cos[2\pi \times 10^3 t + 50 \sin(2\pi \times 10^3 t)]$$

$$\rightarrow f_c = 100 \text{ MHz}, \text{TR} \geq 2 \times 50 \times 10^3 = 100 \mu\text{s}$$



- with integration to go from PM to FM



Final 1/2021:

Q1: a)  $f_{\min} = 2f_i + k = \frac{1}{R_2(C+32\mu)} \rightarrow \text{take } C = 250 \mu F$

$\rightarrow R_2 = 19.184 \text{ k}\Omega$

$f_{\max} = f_{\min} + \frac{1}{R_1(C+32\mu)} \rightarrow R_1 = 11.82 \text{ k}\Omega$

b)  $\circlearrowleft f_o = 300 \text{ kHz} \rightarrow N = 30, \text{ from } \chi \otimes \Delta f = \frac{V_{DD}}{f}$

$\sim \Delta f_o = \frac{2\pi \Delta f}{V_{DD}-2} \rightarrow \Delta f = \frac{600 \text{ Hz} \cdot 10/8}{30} = 25 \text{ Hz}$

$\circlearrowleft Z = \frac{1}{\sqrt{2}} \sim 2Z = \sqrt{\frac{W_L}{\Delta f}} \rightarrow W_L = 60 \text{ rad/s}$

$\sim W_L = \frac{1}{R_3 L_2}, \text{ take } L_2 = 1 \text{ nH} \rightarrow R_3 = 20 \text{ k}\Omega$

loop bandwidth when  $Z = \frac{1}{\sqrt{2}} \rightarrow W_n = W_L$

$\sim W_n = \sqrt{\Delta f \cdot W_L} = 35.35 \text{ band/s}$

Q2: 4 modules required  $\circlearrowleft 9000 \text{ frequencies}$

$$f'_o = 10f_i + f_{\text{cr}} + \frac{f_{\text{cr}}}{10} + \frac{f_{\text{cr}}}{100} + \frac{f_{\text{cr}}}{100}$$

$f_{\min} = 10f_i = 21 \text{ MHz} \rightarrow f_i = 2.1 \text{ MHz}$

$\circlearrowleft 10f_i = f_i + f_1 + f_2 \rightarrow 18.9 \text{ MHz} = f_1 + f_2$

$\rightarrow \text{take } f_1 = 6.3 \text{ MHz}, f_2 = 12.6 \text{ MHz}$

$$\Omega_1 = \frac{f_1}{f_i} = 3, \quad \Omega_2 = \frac{f_2 + f_1}{f_i + f_1} = \frac{12.6 + 6.3}{6.3 + 21} = 2.67$$

Q3: before  $\times 5 \rightarrow f_{\max} = 400 \text{ kHz} \rightarrow R = \frac{200k}{400} = 500 \text{ (100)}$

$\circlearrowleft f_{\min} = \frac{925}{5} = 185 \text{ MHz}, P = 60\%, Q = 1$

$$\rightarrow D_{\min} = \frac{185m}{40k} = PN_{\min} + A_{\min} = 4625$$

$$\rightarrow N_{\min} = 72, A_{\min} = 17 \quad (A_{\max}) = P-1=63$$

$$f_{\text{max}} = \frac{460}{5} = 192 \text{ MHz} \rightarrow D_{\max} = \frac{192m}{40k} = PN_{\max} + A_{\max}$$

$$\rightarrow N_{\max} = 75, A_{\max} = 0$$

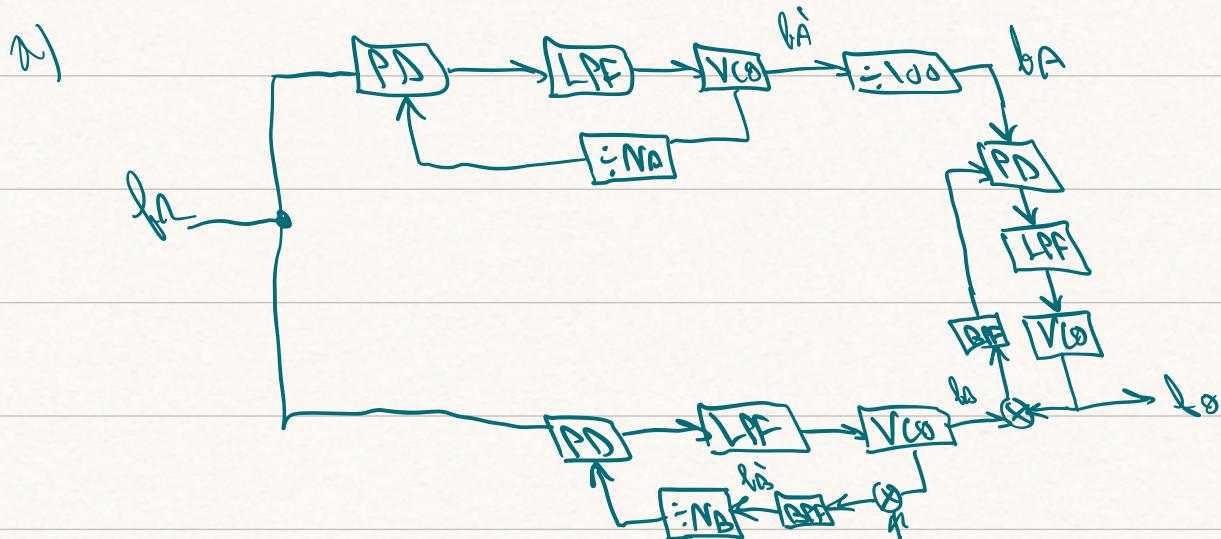
$N$       72      73      74      75

$A$        $17 \rightarrow 63$        $0 \rightarrow 63$        $0 \rightarrow 63$        $0$       *number of bins*

97      64      64      1      176

$$f_{\text{max at input}} = 192 \text{ MHz} / 64 = 3 \text{ MHz}$$

Q4:  $\because f_B = 100 \text{ kHz}, f_{\text{max}} = 18 \text{ Hz} \rightarrow N = 100$



$$\text{Offset } f_B \text{ by } 500 \text{ kHz} \rightarrow f_B = 500 \rightarrow 499 \text{ kHz}$$

$$f_B = 21.5 \text{ MHz} \rightarrow 25.5 \text{ MHz}$$

$$\therefore f_A = 50 \text{ MHz} \rightarrow 49.9 \text{ MHz} \rightarrow N_A = 500 \rightarrow 499$$

$$N_B = 0.5 \text{ MHz} \rightarrow 4.5 \text{ MHz} \rightarrow N_B = 5 \rightarrow 45$$

$$b) \text{max } B = 4.4 \text{ MHz} \rightarrow \text{max } A = 59.4 \text{ MHz}$$

$$\text{for } f_B = 24.563 \text{ MHz} \rightarrow f_A = 563 \text{ kHz} \rightarrow N_A = 563$$

$$f_B = 24 \rightarrow f_B' = 3 \text{ MHz} \rightarrow N_B = 30$$

Q5: a)

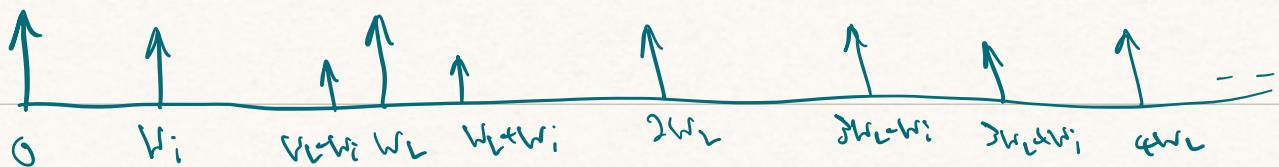


$$V_o = V_L + V_i \text{ for } V_L > 0$$

$$\hookrightarrow V_o = 0 \text{ for } V_L < 0$$

$$\rightarrow P(\omega) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{\sin((2n+1)\omega_L t)}{2n+1}$$

$$\therefore V_o(t) = \frac{V_L}{2} \sin(\omega_L t) + \frac{V_i}{2} \sin(\omega_i t) + \frac{2k}{\pi} \cdot \left[ \frac{\cos((2n+1)\omega_L t - \omega_i t)}{2n+1} \right] \\ + \frac{2V_i}{\pi} \cdot \left[ \frac{\cos((2n+1)\omega_L - \omega_i)t - \cos((2n+1)\omega_L + \omega_i)t}{2n+1} \right]$$



$- V_L \gg V_i$  so that  $V_L$  controls switching and switching speed will be  $\omega_L$

$$b) \Theta_o(t) = N\Theta_n(t) + \frac{N}{dt} m(t)$$

$$\hookrightarrow \Theta_n(t) = 2\pi \int f_B dt$$

$$\hookrightarrow \Theta_o(t) = 200\pi \cdot 10^6 t + 250 \sin(2\pi \times 10^3 t)$$

$$\hookrightarrow f_B(t) = \frac{1}{2\pi} \frac{d\Theta(t)}{dt}$$

$$\rightarrow f_o(t) = \overbrace{100 \cdot 10^6}^{f_c} + \overbrace{240 \times 10^3}^{\Delta f} \cos(2\pi \times 10^3 t)$$

$$\rightarrow S(t) = A_c \cos[\theta_o(t)]$$

$$= A_c \cos[200\pi \cdot 10^6 t + 240 \sin(2\pi \times 10^3 t)]$$

$$\therefore f_c = 100 \text{ MHz}, \Delta f > 2\Delta f = 480 \text{ kHz}$$

- integrate the message signal to go from PM to FM

Final 1/2023:

Q1: a)  $f_{\min} = 200 \text{ kHz}$ ,  $f_{\max} = 600 \text{ kHz}$ ,  $f_2 = 20 \mu\text{Hz}$   
at  $f_0 = 820 \text{ kHz}$   $\rightarrow N = \frac{f_0}{f_2} = 20$

conditions for  $VCO$ :  $100\text{pF} < L < 0.01 \text{ nF}$

$$100\text{pF} < R_1, R_2 < 1 \text{ M}\Omega$$

for  $f_{\min} = 200 \text{ kHz} = \frac{1}{R_2(L+3L_{\text{PF}})}$ , try  $[C = 160\text{pF}]$

$$\rightarrow R_2 = 27.5 \text{ k}\Omega$$

for  $f_{\max} = 600 \text{ kHz} = f_{\min} + \frac{1}{R_2((L+3L_{\text{PF}}))} \rightarrow R_1 = 13.7 \text{ k}\Omega$

to find  $R_3$  and  $L_2$ , find  $w_L \text{ so } w_L = \frac{1}{R_3 L_2}$

$$\sim 2\zeta = \sqrt{\frac{w_L}{k_V}} \rightarrow w_L = 2 \text{ kHz} \text{ so } \zeta = \frac{1}{\sqrt{2}}$$

$$\Delta f = \frac{f_{\max} - f_{\min}}{N}, N=20, \Delta f = \frac{2\pi \Delta f}{V_{DD} - 2} = 100 \text{ kHz}$$

$$\text{for } X_{GR} = \frac{V_{DD}}{\pi} \rightarrow k_V = \frac{\pi \times 10^6}{20} = 50 \text{ kT}_i$$

$$\rightarrow w_L = 100 \text{ kHz} \text{ so } L_2 = 200 \text{ pF}$$

$$\rightarrow R_3 = 15.9 \text{ k}\Omega$$

$$w_h = w_L \text{ for } \zeta = \frac{1}{\sqrt{2}} = \sqrt{k_V w_L} = 7071/\pi \text{ rad/s}$$

$$\text{or } B_H = 35.36 \text{ kHz}$$

b)  $\Theta_\alpha(t) = N \Theta_N(t) + \frac{NM(t)}{k_V}$

$$\rightarrow \Theta_\alpha(t) = N \Theta_N(t) + \frac{N}{k_V} m(t)$$

$$\therefore \theta_1(t) = 2\pi \int f_1 dt$$

$$\hookrightarrow \theta_1(t) = 200\pi \times 10^6 t + 75 \sin(2\pi \times 10^3 t)$$

$$\therefore f_0(t) = \frac{1}{2\pi} \frac{d\theta_1(t)}{dt}$$

$$\hookrightarrow f_0(t) = 100 \times 10^6 + 75 \times 10^3 \cos(2\pi \times 10^3 t)$$

$$\therefore f_0 = 100 \text{ MHz}, \Delta f = 75 \times 10^3 \text{ Hz}$$

$$TR \geq 2\Delta f = 150 \text{ kHz}$$

Q2: a) taken before decade divider  $\rightarrow f'_0$

$\because$  number of frequencies  $> 10^4$  but  $< 10^5 \rightarrow$  four module

$$\therefore f'_0 = 10f_i + f'_4 + \frac{f'_3}{10} + \frac{f'_1}{100} + \frac{f'_1}{1000}$$

$$f_i = \frac{f_{0,\min}}{10} = 3 \text{ MHz}$$

$$\hookrightarrow 10f_i = f_i + f_{01} + f_{02} \rightarrow f_{01} + f_{02} = 27 \text{ MHz}$$

$$\text{take } f_{01} = 9 \text{ MHz}, f_{02} = 18 \text{ MHz}$$

$$\rightarrow R_1 = \frac{f_1}{f_i} = 3, \quad R_2 = \frac{f_2 + f'_{1,\max}}{f_1 + f_i} = \frac{18+9}{9} = 3$$

$$\text{if } f_0 = f'_0 = 34.567 \text{ MHz}$$

$$\rightarrow f_i = 3 \text{ MHz}, f'_4 = 4 \text{ MHz}, f'_3 = 4 \text{ MHz}, f'_2 = 6 \text{ MHz}$$

$$\hookrightarrow f'_1 = 7 \text{ MHz}$$

at least 9 (4x2)+1 at least 9

only 12 oscillators, 12 DPF, and 12 mixers

$$\text{b) } f_0 = f_0 \cdot N \cdot P \rightarrow \text{resolution} = P f_0$$

$$\therefore 200 \text{ kHz} = 25 \text{ fs} \rightarrow f_2 = 8 \text{ kHz}$$

$$f_0 \approx \frac{25}{fs} = 3.125 \text{ ms}$$

$$\therefore f_{0,\text{max}} = N_{\text{max}} \cdot P_{fs} \rightarrow N_{\text{max}} = 550$$

$$\curvearrowleft f_{0,\text{min}} = N_{\text{min}} \cdot P_{fs} \rightarrow N_{\text{min}} = 450$$

max frequency at programmable divider: 4.4 MHz

$$Q3: \text{given } f_{0,\text{after } \times 4} = 200 \text{ kHz} \rightarrow f_2 = 50 \text{ kHz}$$

$$P = \frac{\text{crystal}}{f_2} = 128 \quad (010)$$

$$f'_0 = \frac{f_0}{4} \rightarrow f'_{0,\text{min}} = 231.25 \text{ MHz}$$

$$\rightarrow D_{\text{min}} = P N_{\text{min}} + A_{\text{min}} = \frac{f'_{0,\text{min}}}{f_2}, P=64, Q=1$$

$$\rightarrow N_{\text{min}} = 72, A_{\text{min}} = 17$$

$$\curvearrowleft f'_{0,\text{max}} = 231.4 \text{ MHz} \rightarrow D_{\text{max}} = 4740$$

$$\rightarrow N_{\text{max}} = 74, A_{\text{max}} = 14, (A_{\text{max}}) = P-1$$

$$N \quad 72 \quad 73 \quad 74$$

$$A \quad 17 \rightarrow 63 \quad 0 \rightarrow 63 \quad 0 \rightarrow 14$$

$$47 \quad 64 \quad 15 \quad 126 \quad \text{frequencies}$$

$$\text{frequency range of VCO} = f'_{0,\text{min}} \rightarrow f'_{0,\text{max}}$$

$$= 231.25 \text{ MHz} \rightarrow 237.5 \text{ MHz}$$

$$f_{\text{max}} \text{ at programmable divider: } \frac{f'_{0,\text{max}}}{P} = 3.71 \text{ MHz}$$

$$\text{if } f_0 = 930 \text{ MHz} \rightarrow f'_0 = 232.5 \text{ MHz}$$

$$\rightarrow D = 4650 \rightarrow N = 72, A = 41$$

Q4: a) 1- two opposite diodes:

$$V_o(t) = 0.4 \sin(\omega_i t) + \frac{0.4}{\pi} \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) - \cos((2n+1)\omega_L t + \omega_i t)}{2n+1}$$



$$A|_{w_i} = 0$$

$$A|_{w_L} = 4$$

$$A|_{w_L - w_i} = \frac{0.4}{\pi}$$

$$A|_{w_L + w_i} = \frac{0.4}{\pi}$$

2- two same direction diodes:

$$V_o(t) = 0.1 \sin(\omega_i t) + \frac{0.2}{\pi} \cdot \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) - \cos((2n+1)\omega_L t + \omega_i t)}{2n+1}$$

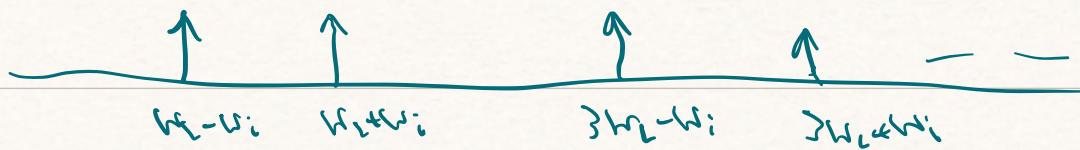


$$A|_{w_i} = 0.1, A|_{w_L} = 0$$

$$A|_{w_L - w_i} = \frac{0.2}{\pi}, A|_{w_L + w_i} = \frac{0.2}{\pi}$$

3- four-diode ring mixer:  $\frac{R_L}{R_L + \frac{Z_0}{2}} \approx 1$

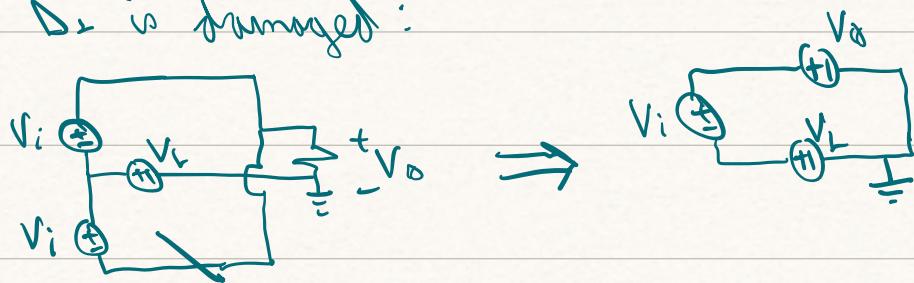
$$V_o(t) = \frac{0.4}{\pi} \cdot \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) - \cos((2n+1)\omega_L t + \omega_i t)}{2n+1}$$



$$A|_{w_i} = 0, \quad A|_{w_i} = 0$$

$$A|_{w_L - w_i} = \frac{0.4}{\pi}, \quad A|_{w_L + w_i} = \frac{0.4}{\pi}$$

b) when  $D_2$  is damaged:



$$\text{when } V_L > 0 \rightarrow V_o = V_L + V_i$$

$$\text{when } V_L < 0 \rightarrow V_o = 0$$

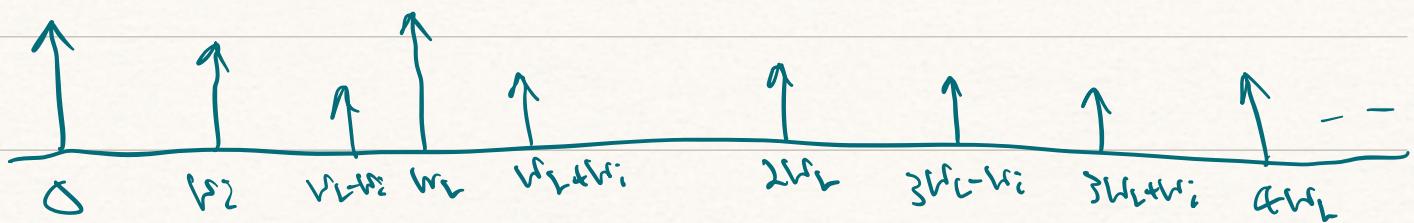
$$V_o(t) = P(t)[V_L + V_i] \quad \text{a.t., } P(t) = \begin{cases} 1, & V_L > 0 \\ 0, & V_L < 0 \end{cases}$$

$$\rightarrow P(t) = \frac{1}{2} + \frac{1}{\pi} \cdot \sum \frac{\sin((2n+1)\omega_L t)}{2n+1}$$

$$\therefore V_o(t) = \frac{V_L}{2} \sin(\omega_L t) + \frac{V_i}{2} \sin(\omega_i t) + \dots$$

$$\frac{V_L}{\pi} \cdot \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) - \cos((2n+1)\omega_L t + \omega_i t)}{2n+1} + \dots$$

$$\frac{V_i}{\pi} \cdot \sum \frac{\cos((2n+1)\omega_L t - \omega_i t) - \cos((2n+1)\omega_L t + \omega_i t)}{2n+1}$$



final 6/2021:

$$Q1: \text{At } f_0 = 200 \text{ kHz} \Rightarrow \frac{1}{R_2(C + 32\text{pF})} ; C = 200\text{pF}$$

$$\rightarrow R_2 = 21.55 \text{ k}\Omega$$

$$\approx f_{\text{max}} = f_{\text{min}} + \frac{1}{R_1(C + 32\text{pF})} \rightarrow R_1 = 10.98 \text{ k}\Omega$$

$$2) \approx f_0 = 400 \text{ kHz} \quad \approx f_1 = 10 \text{ kHz} \rightarrow N = 40$$

$$\text{for } 2\pi f = \frac{V_{DD}}{\pi}, f_{\text{osc}} = \frac{2\pi \Delta f}{V_{DD}-2} = \frac{800\pi}{8}$$

$$\rightarrow f_{\text{osc}} = \frac{f_0 - f_1}{N} = 20 \text{ kHz}$$

$$\approx Z = \frac{1}{\sqrt{2}} \approx 2Z = \sqrt{\frac{W_L}{2\pi f_{\text{osc}}}} \rightarrow W_L = 40 \text{ mW}$$

$$\approx W_L = \frac{1}{R_3 G_2} \rightarrow G_2 = 1 \text{ nF} \rightarrow R_3 = 24 \text{ k}\Omega$$

turn VCO off to consume energy when no input

o Zener: voltage regulation, inhibit: disable

LD: shows it can and be locked

source follower: isolates input and reduces loading

$$Q2: \text{At } f_0 = 10f_i + f_1 + \frac{f_3}{10} + \frac{f_2}{100} + \frac{f_1}{1000}$$

$$\rightarrow f_i = 2.2 \text{ MHz}, 19.8 \text{ MHz} = f_1 + f_2$$

$$\text{tubes } f_1 = 6.6 \text{ MHz}, f_2 = 13.2 \text{ MHz}$$

$$D_1 = \frac{f_1}{f_i} = 3, D_2 = \frac{f_2 + f_{1,\text{max}}}{f_i + f_1} = 2.423$$

2) 12 oscillators, at least 9 mixers  
four mixers and  
mixer connected to  
oscillation

$$Q3: f_R = 90 \text{ kHz} \rightarrow R = \frac{10.24 \text{ m}}{40 \text{ k}} = 256 \text{ (011)}$$

$$\therefore f_{\text{res}, \min} = 185 \text{ MHz}, P = 64, Q = 1$$

$$\rightarrow D_{\min} = PN_{\min} + QA_{\min} = \frac{185 \text{ M}}{40 \text{ k}}$$

$$\rightarrow N_{\min} = 72, A_{\min} = 17 \quad |A_{\text{more}}| = 1 - 1 = 63$$

$$\therefore f_{\text{res}, \max} = 190 \text{ MHz} \rightarrow D_{\max} = 4750$$

$$\rightarrow N_{\max} = 74, A_{\max} = 14$$

$N$       72      73      74

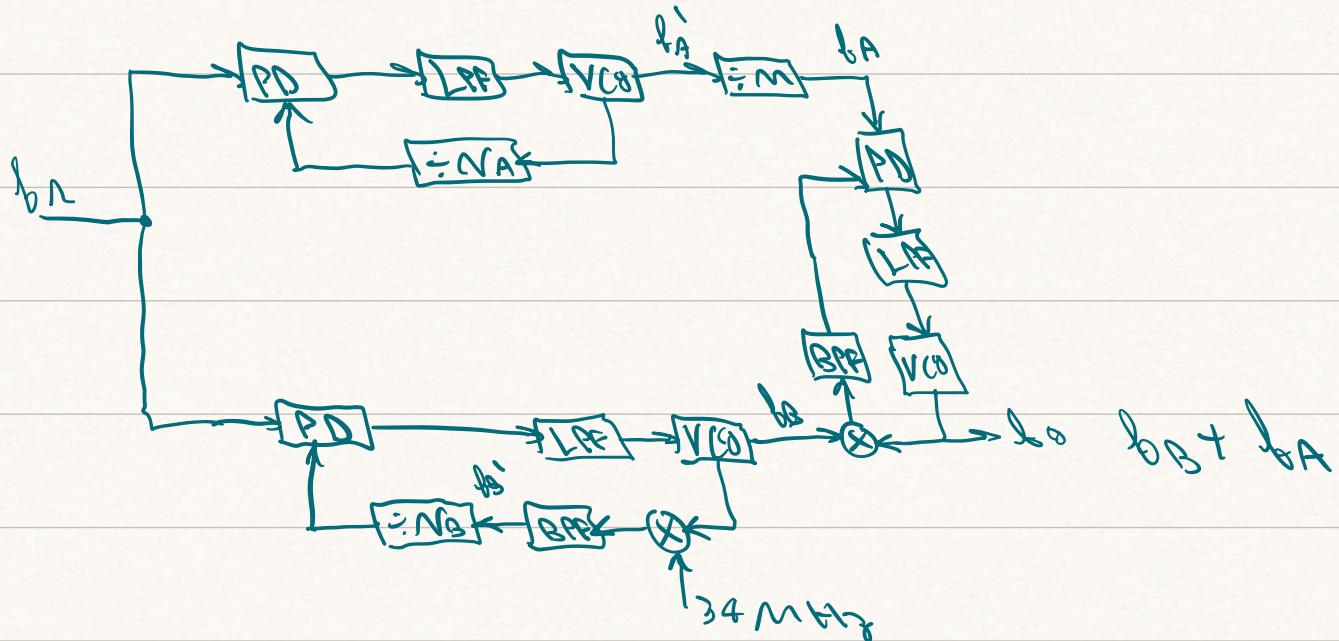
$A$        $17 \rightarrow 63$        $0 \rightarrow 63$        $0 \rightarrow 14$

47            64            15            126

tuning range of VCO:  $185 \rightarrow 190 \text{ MHz}$

from its programmable dividers:  $2 \cdot 969 \text{ MHz}$

$$Q4: f_R = 100 \text{ kHz} \quad \therefore f_{\text{res}} = 1 \text{ kHz} \rightarrow n = 100$$



a) Offset to bring 500 kHz to mid channel  
 down other loop  $\rightarrow f_A = 500 \text{ kHz} \rightarrow 599 \text{ kHz}$   
 $N_A = 500 \rightarrow 544 \quad (500 \rightarrow 344)$   
 $\rightarrow f_B' = 50 \text{ MHz} \rightarrow 59.9 \text{ MHz}$

$$\rightarrow f_B = 34.4 \text{ MHz} \rightarrow 39.5 \text{ MHz} - 1 \text{ kHz}$$

$$\therefore f_B = 500 \text{ kHz} \rightarrow 3.4 \text{ MHz} - 1 \text{ kHz}$$
 $\rightarrow N_B = 5 \rightarrow 35 - 1 (24)$

b)  $f_{\text{max},A} = 59.9 \text{ MHz}$ ,  $f_{\text{max},B} = 3.4 \text{ MHz}$

if  $f_B = 36.478 \rightarrow f_B = 36 \rightarrow N_B = 20$

$$N_A = 578$$

Q5: a)  $V_o(t) = P(t)[V_i(t) + V_L(t)]$  s.t.,  $P(t) = \begin{cases} 1, & V_L > 0 \\ 0, & V_L \leq 0 \end{cases}$

$$b) f_o(t) = N f_B + \frac{1}{2\pi} \frac{f_m(t)}{f_t} \cdot \frac{N}{k_f}$$

$$= 80 \text{ MHz} + 2.667 \times 10^5 \cos(2\pi \times 10^3 t)$$

$$\rightarrow f_c = 80 \text{ MHz} , \Delta f = 26.7 \text{ kHz}$$

$$TR \geq 2\Delta f = 53.3 \text{ kHz}$$

c) 10 dB for same direction, 4 dB for opposite and ring