

+ four modes of operation of Transistor:

1 - Saturation: Transistor functions as a short circuit between collector and emitter.  $B I_B > I_C$

2 - Cut-off: Transistor acts like an open circuit with no current flowing from collector to emitter.

3 - Forward active: The current from collector to emitter is proportional to the base current

4 - Reverse active: Current flow of the forward active mode is reversed

\* "RTL": Resistor-transistor logic \* "DTL": diode-transistor logic

\* "TTL": transistor-transistor logic \* "ECL": emitter coupled logic

- Designers should minimize the undefined input region while maximizing the noise margins as the output will be seen as either low (0) or high (1).

& RTL logic:

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assume the Transistor is operating in one of the four modes until proven otherwise.

- Forward-active:  $V_{BE\text{on}} = 0.7V$

- Cut-off:  $V_{BE} < 0.7V$

- Saturation:  $V_{BE\text{sat}} = 0.8V \wedge V_{CE\text{sat}} = 0.2V$

- Simple RTL inverter:

if  $R_B = 10k\Omega$ ,  $R_C = 1k\Omega$ ,  $\beta = 50$ ,  $V_U = 5V$

$$\therefore V_o = V_u - I_C R_C$$

$\Rightarrow I_C = 0 \rightarrow V_o = V_u \wedge I_C = 0$  in cut-off

$\therefore V_i \leq 0.7V \rightarrow V_o = V_u \rightarrow V_i = \text{low} \rightarrow V_o = \text{high}$

$$\therefore V_o = 0.2V \text{ in saturation} \rightarrow I_C = \frac{V_u - 0.2V}{R_C} = 4.8 \text{ mA}$$

$$\therefore I_C = \beta I_B \rightarrow I_B = 96 \text{ nA} \wedge -V_i + I_B R_B + V_{BE\text{sat}} = 0$$

$$\rightarrow V_i = 0.8 + 96 \text{ nA} \cdot 10k\Omega = 1.76V \therefore V_i = \text{high} \rightarrow V_o = \text{low}$$

- "VTC": Voltage transfer characteristics
  - $V_{IL}$ : maximum input voltage that will be recognized as low input logic level
  - $V_{IH}$ : the minimum input voltage that will be recognized as high input logic level
  - $V_{OH}$ : the output voltage recognized as logic high (corresponding to  $V_{IL}$  in inverter)
  - $V_{OL}$ : the output voltage recognized as logic low (corresponding to  $V_{IH}$  in inverter)
- \* noise margins: safety margin preventing erroneous outputs from noisy inputs
- If the output of one inverter is used as the input to the next:

$$\rightarrow NM_L = V_{IL} - V_{OL} = 0.7 - 0.2 = 0.5V$$

$$\rightarrow NM_H = V_{OH} - V_{IH} = 5 - 1.76 = 3.24V$$

Where NM represent the noise margin and the subscript represents the logic level

- \* fan-out: number of gate inputs driven by output of a single logic gate
- the maximum fan-out of an output measures its load driving capability
  - the driving device must be able to supply (or sink) the sum of currents needed (or provided) by all the connected inputs while maintaining the output voltage specifications (to supply or sink depends on whether the output is a logic high or low level).

- for an RTL inverter, the fan-out for a low logic level is  $\infty$ .

- for an RTL inverter, the fan-out for a high logic level is  $\neq \infty$

Ex 1: If  $V_i = \text{low} \rightarrow Q_0 \text{ off}$

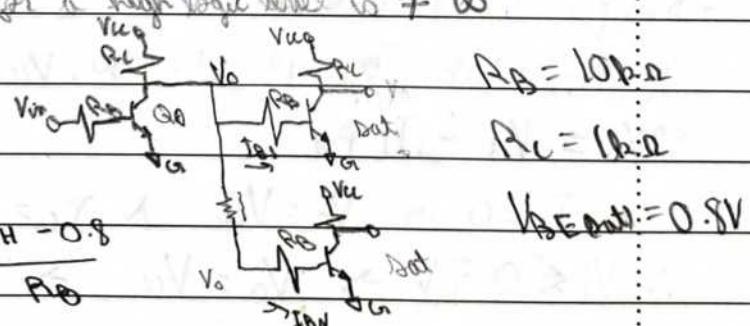
$$\rightarrow V_o = \text{high}$$

$$\therefore V_o > V_{IH} = 1.76V$$

$$\lambda I_{CO} = N \cdot I_B (1 \rightarrow N) \quad \lambda I_B = \frac{V_{IH} - 0.8}{R_B}$$

$$\lambda I_{CO} = \frac{V_o - V_{IH}}{R_C}$$

$$\therefore \frac{V_o - V_{IH}}{R_C} = N \cdot \frac{V_{IH} - 0.8}{R_B}$$



-  $B \cdot I_B > I_C$  in saturation  $\rightarrow \sigma \cdot B \cdot I_B = I_C$  where  $\sigma$ : saturation parameter.

- in Ex 1, assume transistors are at end of saturation (EOS)  $\rightarrow \sigma = 1$

$$\therefore I_{B(\text{sat})} = \frac{I_{C(\text{sat})}}{B} = \frac{V_{cc} - V_{CE(\text{sat})}}{B \cdot R_C} \quad \lambda \cdot B = 0.01$$

$$\rightarrow I_{B(\text{sat})} = 9.6 \times 10^{-5} \text{ A} \quad \lambda \cdot I_{C(0)} = N \cdot I_{B(\text{sat})} = \frac{5 - V_{IH}}{R_L}$$

$$\therefore N = \frac{3.24 \times 10^3}{9.6 \times 10^{-5}} = 33.75 \approx 33$$

\* fan-in: maximum number of inputs that a gate can have and maintain proper functionality

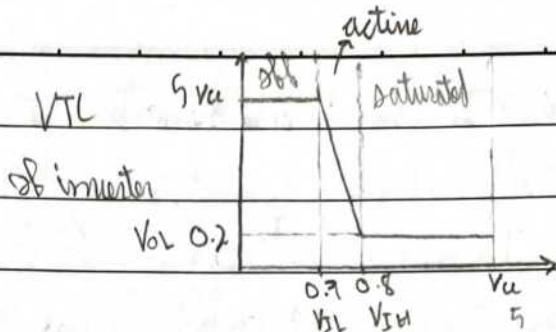
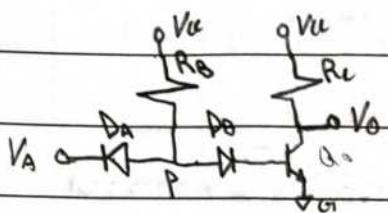
- for an RTL NAND gate, the fan-in is limited by the output voltage for low logic needs to be lower than  $V_{IL}$  ( $0.9$ )

$$\text{Thus if } V_{CE(\text{sat})} = 0.2, \text{ max fan-in} = \frac{0.9}{0.2} = 3.5 \approx 3 \text{ gates}$$

- RTL was advantageous because transistor was the most expensive component (before IC technology), hence RTL was the cheaper type of logic

- RTL dissipates high amounts of current, which causes a lot of losses to heat, hence RTL is not efficient.

## \* basic DTL inverter



assuming  $D_A$  is always on:

when  $V_A = 0V$ ,  $V_p = 0.7V$ ,  $\therefore V_{BE(on)} = 0.7V \rightarrow Q_0$  is off

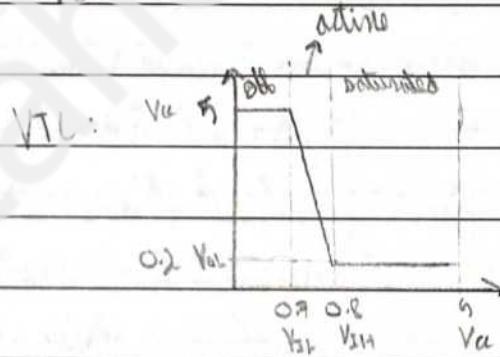
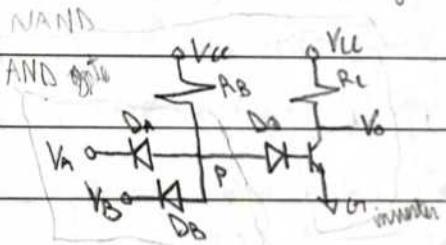
when  $V_A = 0.7V$ ,  $V_p = 1.4V \rightarrow Q_0$  is on (in active)

when  $V_A = 0.8V$ ,  $V_p = 1.5V \rightarrow V_{BE} = 0.8V \rightarrow Q_0$  is in saturation

$\therefore$  off for  $0 \leq V_A < 0.7$ , active for  $0.7 \leq V_A < 0.8$

1 saturated for  $0.8 \leq V_A \leq V_U$

## \* basic DTL NAND gate:



if  $V_A$  is grounded and  $D_A$  is assumed on:

$V_B = 0.5V$ ,  $V_p = 0.7V$  (satn)  $\rightarrow D_B$  is off and  $V_O = H$

$V_B = 1V$ ,  $V_p = 0.7$  (satn)  $\rightarrow V_O = V_U$  if  $V_B = 0$  always

if  $V_B$  is grounded and  $D_B$  is assumed on:

$V_A = 0.7$ ,  $V_p = 0.9V$ ,  $D_A$  is off,  $V_O = H = V_U$

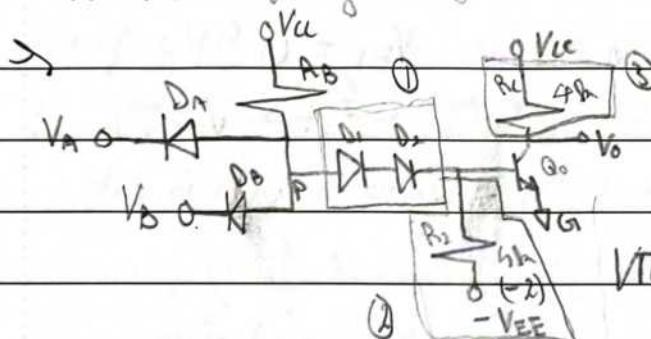
if  $V_B$  is fixed at  $0.7V$ ,  $V_p = V_A + 0.7$  if  $D_A$  is on

$\rightarrow V_B = V_A = 0.7V \rightarrow V_p = 1.4V \rightarrow Q$  is in active

if  $V_B = V_A = 0.8V \rightarrow V_p = 1.5V \rightarrow Q$  is in saturation

if  $V_B > V_A > 0.8$ ,  $V_O = 0.2V = L$

- level-shifting diodes are used to increase the noise margin by shifting  $V_{IL}$ .
- pull-up networks provide low resistance paths to  $V_{DD}$  when the transistor is off and high resistance paths when transistor is saturated.
- pull-down networks provide a path for collected charges when the transistor goes from saturation mode to cut-off.

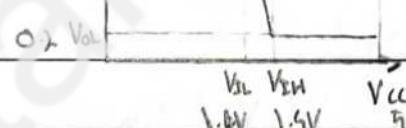


① level-shifting diode

② pull-down network

③ pull-up network

-  $-V_{EE}$  is the pull-down network (but it is given a magnitude of 0 V to speed up the drawing)



- (Reminder): if the transistor is in saturation mode, charges accumulate in the drain terminal (can be represented by a capacitor connecting the drain and source). hence, to turn off a transistor, the charges accumulated at the drain must be removed.

- the fan-out for the high output of a TTL NAND gate is effectively  $\infty$  since no current flows through the input diodes.  $N_H \rightarrow \infty$

$\Rightarrow Q_D$  must remain saturated

$$\Rightarrow I_{CO(HI)} \leq \beta I_B$$

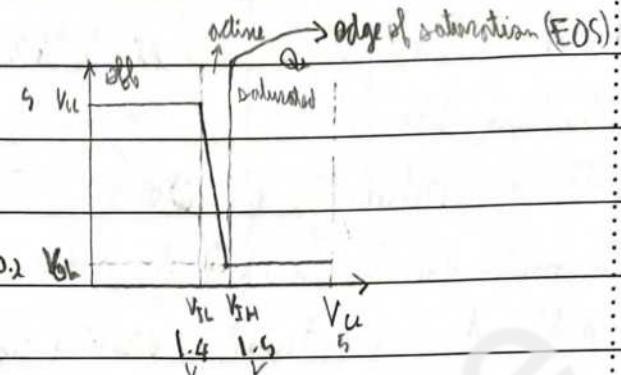
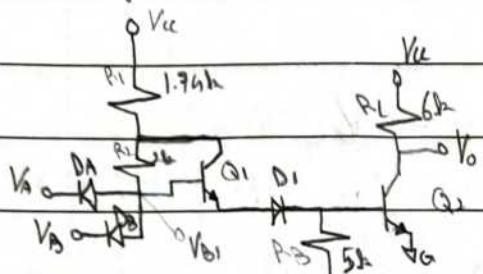
$$\lambda I_{CO(HI)} = N \cdot I_{DA} + \frac{5}{4\pi}$$

$$\Rightarrow N \cdot I_{DA} = \beta I_B \quad \lambda I_B = 0.84 \text{ mA} \quad \lambda \cdot 0.84 \text{ mA} - 1.25 \text{ mA} = 23.28 \approx 23$$

$$\lambda I_{DA} = 1.75 \text{ mA} \Rightarrow N = 1.75 \text{ mA}$$

- smaller  $\sigma$  indicates transistor is more saturated

\* modified DTL NAND:



If  $V_B$  is fixed at 5V ( $\equiv V_u$ ):

- If  $V_A = 0.2V$  (low) and  $D_A$  is on  $\Rightarrow V_{B1} = 0.9V \Rightarrow Q_1$  is on

but  $D_1$  is off  $\Rightarrow Q_1$  is off and  $Q_2$  is off  $\Rightarrow V_o = V_u$

- If  $V_A = 5V \Rightarrow Q_1$  is sat and  $D_1$  is on and  $Q_2$  is off  
 $\Rightarrow V_o = V_u$

- When  $V_A = 1.4V$  and  $D_A$  is on  $\Rightarrow Q_1$  is forward active  
 $\wedge Q_2$  is forward active

- When  $V_A = 1.5V$  and  $D_A$  is on  $\Rightarrow Q_1$  is sat and  $Q_2$  is off  
 $\Rightarrow V_o = V_u$

$$\therefore V_{IL} = -D_{A(on)} + V_{BE1(on)} + D_1(on) + V_{BE2(off)} = 1.4V$$

$$\wedge V_{IH} = -D_{A(on)} + V_{BE1(on)} + D_1(on) + V_{BE2(off)} = 1.5V$$

If  $V_A = 0.2V \Rightarrow Q_1, Q_2$  are off, assuming  $D_1$  is on

$$\Rightarrow V_{B1} = 0.9V \therefore I_A = \frac{V_u - 0.9}{3.75k} = 1.093 \text{ mA} \quad (\text{all other currents} = 0)$$

If  $V_A = V_B = V_u$  and  $D_1$  is on,  $Q_1$  is forward active  $\wedge \beta = 100$

$$\rightarrow V_{B1} = V_{BE1(on)} + V_{D1(on)} + V_{BE2(off)} = 2.2V$$

$$\therefore I_{A1} = I_{B1} + I_{U1} = I_{E1} \rightarrow I_{B1} = \frac{I_{E1}}{\beta + 1}$$

$$\rightarrow -5 + I_{E1} \cdot 1.75k + I_{E1} \cdot \frac{2k}{\beta + 1} + 2.2 = 0 \rightarrow I_{E1} = \frac{2.8}{1.75k + \frac{2k}{100}}$$

$$\rightarrow I_{E1} = 1.582 \text{ mA}$$

$$\therefore V_{BE2} = 0.8V \quad \wedge I_{E1} = I_{B2} + I_{R2}$$

$$\rightarrow I_{B2} = 1.582 \text{ mA} - \frac{0.8}{5k} = 1.422 \text{ mA}$$

$$\wedge I_{U2} = \frac{5 - 0.2}{5k} = 0.8 \text{ mA}$$

- for a high output from a modified DTL NAND, an  $\infty$  number of gates can be driven

- for a low output, the transistor  $Q_1$  must remain saturated

$$\rightarrow \beta I_{B1} \geq I_{C1}, I_{C1} = N \cdot I_{BN} + I_{AL}$$

$$\therefore I_{BN} = \frac{V_{CC} - 0.9}{3.948} = 1.093 \text{ mA}$$

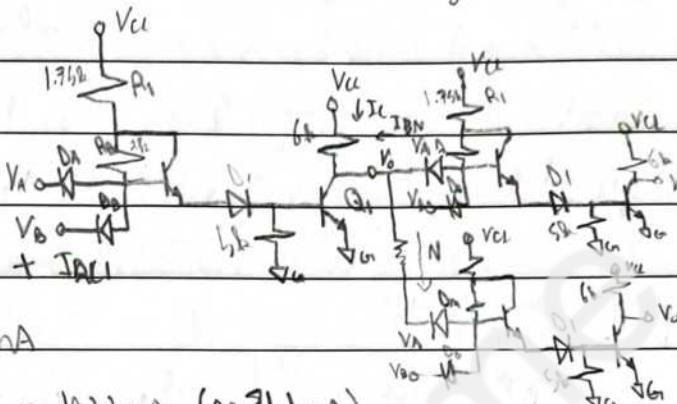
$$\lambda \beta I_{B1} = 100 \cdot 1.422 \text{ mA} = 142.2 \text{ mA (or } 141.1 \text{ mA)}$$

$$\therefore N \cdot I_{BN} + \frac{0.2}{6.2} = \beta I_{B1} \rightarrow (\beta = 100) \quad N = 144$$

$$(\beta = 50) \quad N = 64$$

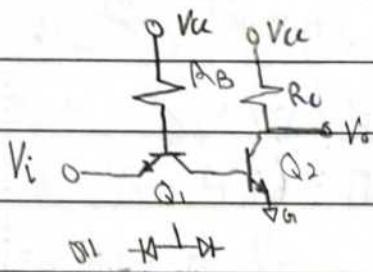
- the average power dissipation is given by :

$$P_{(avg)} = V_{CC} \cdot \frac{I_{H}(01) + I_{L}(01)}{2}$$



- In designing ICs, compactness and logic density are most important. Since resistors and capacitors are generally the largest components, reducing their count will free up space for extra transistors and save costs.
- TTL logic still requires resistors, whereas MOS gates can be made of transistors only; hence why they are more ubiquitous today.

### + Basic TTL inverter:

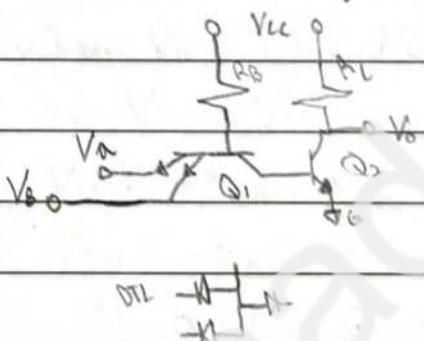


When  $V_i$  is high,  $Q_1$  is off, pushing  $Q_2$  into saturation  $\Rightarrow V_o = L$  (0.2V)

When  $V_i$  is low, the accumulated charge

at the base terminal of  $Q_1$  will form a path to the emitter of  $Q_1$ , hence turning  $Q_1$  on.

### + Basic TTL NAND gate:



$Q_1$  is a multi-emitter diode

$Q_2$ : composite current gain (normal) (emitter)

### + For a basic TTL inverter:

If  $V_i = 0.2$ , assuming  $Q_1$  is on:

$$\rightarrow V_{B1} = 0.2 + 0.8 = 1V \quad \text{so } V_{BE(\text{sat})} = 0.8V$$

$$\rightarrow I_{B1} = 1mA = I_{E1}$$

As  $I_{B2} = 0 \rightarrow Q_2$  is off  $\Rightarrow V_o = V_{CC} = 5V$

If  $V_i = 5V$ , assuming  $Q_2$  is on  $\rightarrow Q_1$  is in reverse active

$$\text{so } V_{BL(2n)} = 0.6V \quad \text{and } V_{BE(\text{sat})} = 0.8V \rightarrow V_{B1} = 1.4V$$

$$\text{so } I_u = I_{B2} = (\beta_{2n} + 1) \cdot I_{B1} \quad \text{where } \beta_{2n} \text{ is the reverse beta} = 0.1$$

$$\text{I}_{B1} = \frac{5 - 1.4V}{4k} = 0.9 \text{ mA} \rightarrow I_{C1} = (1 + 0.1) \cdot 0.9 \text{ mA} = 0.99 \text{ mA}$$

$$\text{I}_{E1} = \beta_{B1} \cdot \text{I}_{B1} = 0.09 \text{ mA}$$

$$\therefore I_{C2} = \frac{V_{cc} - V_{BE2}}{2k} \rightarrow I_{C2} = \frac{5 - 0.2}{2k} = 2.4 \text{ mA}$$

$\therefore$  for saturation,  $I_L < \beta I_B$   $\beta_2 = 10$

$$\text{I}_{C1} = 2.4 \text{ mA} < 5 \cdot 0.9 \text{ mA} = \beta_2 I_{B2} \therefore Q_2 \text{ is saturated.}$$

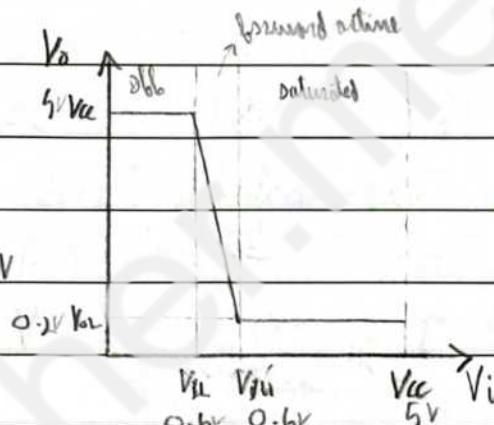
+ VTC for basic TTL inverter:  $Q_2$ :  $V_i$   $\uparrow$  forward active

$$\text{if } V_i = 0.2 \text{ V, off}$$

$Q_2$  conduction when  $V_{BE2} = 0.9 \text{ V}$

$$\rightarrow V_{B1} = 0.6 + 0.9 = 1.3 \text{ V} \rightarrow V_i = 0.5 \text{ V}$$

$$\therefore V_{CE1(\text{off})} = 0.6 \text{ V}$$



- The voltage for  $V_{CE1(\text{on})}$  is less than  $V_{BE1(\text{on})}$  because transistors are not symmetrical and the emitter is more heavily doped compared to the collector.

- immediately after switching on input of a basic TTL inverter from high (5V) to low (0.2V):

$Q_1$  turns to forward active  $\rightarrow V_{B1} = V_i + V_{BE1(\text{on})} = 0.4 \text{ V}$

momentarily,  $Q_2$  will remain saturated  $\rightarrow V_{B2} = 0.8 \text{ V} = V_{BE2(\text{sat})}$   
(due to propagation delay)

$Q_1$  immediately changes to forward active since  $V_{B2} = 0.8 \text{ V}$

$$\text{I}_{B1} = \frac{5 - 0.9}{4k} = 1.025 \text{ mA}$$

therefore, a voltage larger than the saturation voltage keeps the transistor unsaturated.

$$\therefore I_{B1} = \frac{5 - 0.9}{4k} = 1.025 \text{ mA}$$

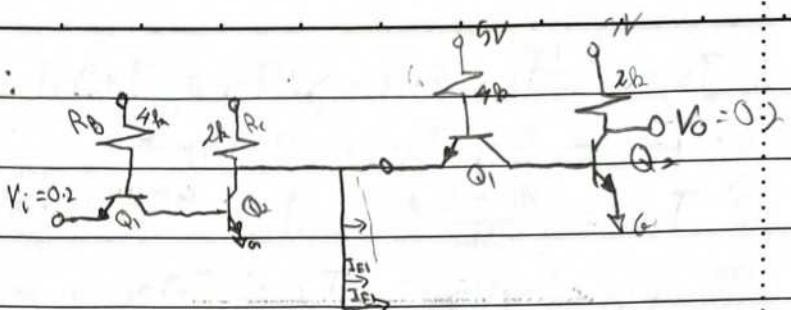
$$\rightarrow \text{initial } I_{C1} (= I_{B2}) = \beta I_{B1} = 51.25 \text{ mA}$$

fan-out for TTL inverter:

- low input case:

$\therefore Q_1$  is not (sat)

$Q_2$  should be off



$\rightarrow Q_1$  of next stage should be reverse active

$\rightarrow Q_2$  should be sat

$$\rightarrow V_{BE21} = 0.8V \quad I_{B21} = I_{E12} + I_{B12} \quad I_{E12} = \beta_1 I_{B12}$$

for  $Q_2$  sat,  $Q_1$  reverse active  $\rightarrow V_{A12} = 0.6 + 0.8 = 1.4V$

$$\rightarrow I_{B12} = \frac{5 - 1.4}{4k\Omega} = 0.9 \text{ mA} \rightarrow I_{E12} = \beta \cdot 0.9 \text{ mA}$$

$$\therefore I_{C21} = \frac{5 - V_o}{2k\Omega} = N \cdot \beta_1 \cdot (0.9) \quad \beta_1 = 0.1 \quad V_o = H \cdot S \quad V_o \geq 4.2V$$

\* Reverse Beta

$$\therefore I_{C21} = \frac{5 - 4.2}{2k\Omega} = N \cdot 0.1 \cdot 0.9 \text{ mA} = 4.444 \Rightarrow N_{max} = 4$$

- high-input case:  $V_i = 5$

$\rightarrow Q_{11}$  is reverse active,  $Q_{21}$  is sat,  $Q_{12}$  is sat,  $Q_{22}$  is off

$$\rightarrow V_{B21} = 0.8V \rightarrow V_{BH} = 0.8 + V_{BE}(\text{reverse}) = 1.4V$$

$\therefore$  for  $Q_{21}$  to remain in sat:  $\beta \cdot I_{B21} \geq I_{C21}$

$$I_{C21} = I_{A12} + N \cdot I_{E12} \quad I_{E12} = I_{B12} = \frac{5 - (0.2 + 0.8)}{4k\Omega}$$

$$\therefore I_{C21} = \frac{5 - 0.2}{2k\Omega} = 2.4 \text{ mA}$$

$$\therefore I_{B21} = I_{C21} = I_{BH} + I_{EH}, I_{BH} = \frac{5 - 1.4}{4k\Omega} = 0.9 \text{ mA}$$

$$\therefore I_{EH} = \beta_1 \cdot 0.9 \text{ mA} = 0.09 \text{ mA} \rightarrow I_{C11} = 0.99 \text{ mA}$$

$$\therefore (0.99 \text{ mA}) \cdot \beta \geq 2.4 \text{ mA} + N \cdot (1 \text{ mA})$$

$$\rightarrow N = \frac{0.99 \text{ mA} \cdot \beta - 2.4 \text{ mA}}{1 \text{ mA}} \quad 1 \cdot \beta = 50 \Rightarrow N = 47$$

\* Standard TTL NAND gate:

- $Q_4$  is included as a consequence to including  $Q_3$ .

- since the pull-up network (including  $Q_4$ )

allows  $V_o$  to go from high to low quickly.

- The pull-down network allows the output to go from low to high similarly.

- Transistor  $Q_2$  is included to control  $Q_4$ . When output is low,  $Q_3$  is saturated by the input current from  $Q_2$ , which is also saturated causing the base voltage of  $Q_4$  to be 0.2V and cutting it off.

- $Q_2$  is called a phase splitter:

a phase splitter: allows the input condition to

be produced in opposite phases;

allowing  $Q_3$  to be on while  $Q_4$  is off.

if  $V_i = \text{low}$ ,  $V_{o1} = V_u$  &  $V_{o2} = 0 \rightarrow Q_3 = \text{on}$ ,  $Q_4 = \text{off}$

if  $V_i = \text{high}$ ,  $V_{o1} = 0.9V$   $V_{o2} = V_{BE3} \rightarrow Q_3 = \text{off}$ ,  $Q_4 = \text{on}$

diode  $D_1$  is included to keep transistor  $Q_4$  off even when  $V_o = 0.9V$

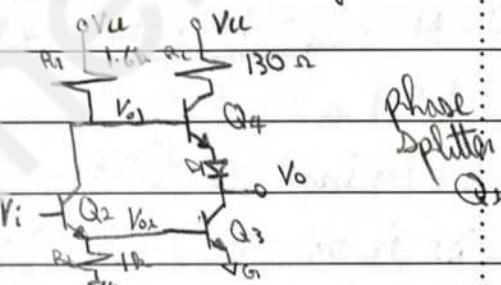
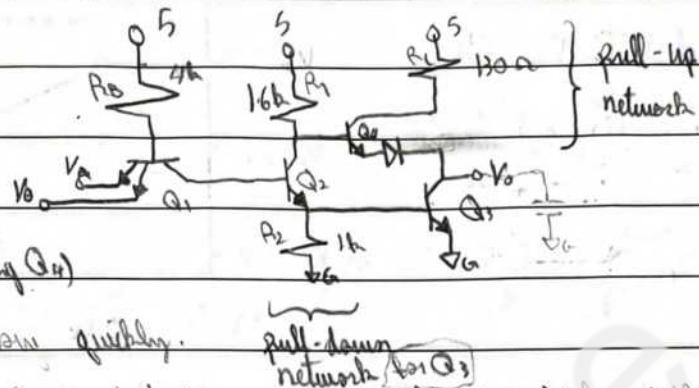
hence, 0.9V at base terminal of  $Q_4$  can be considered low

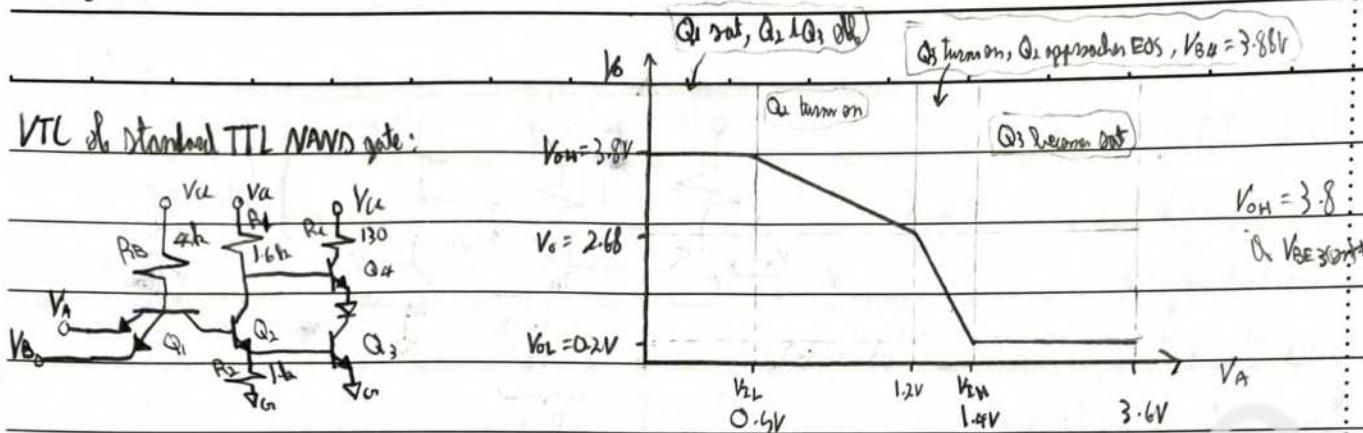
- output transistors pair,  $Q_3$  &  $Q_4$ , along with diode,  $D_1$ , are referred to as the totem pole output

- totem pole output configuration can actively source or sink current and is useful for capacitive loads.

- diode,  $D_1$ , increases the effective turn-on voltage of  $Q_4$ , allowing  $Q_4$  to be turned off before  $Q_3$  is fully on, preventing large surge currents from flowing in the output stage during transitions from high to low or vice versa

- the diode causes the output high voltage level to be reduced by the voltage drop across the diode





- When  $V_A$  &  $V_B$  are low,  $Q_1$  is saturated, the base current of  $Q_1$  goes to the emitters,  $Q_2$  gets no base current and is off causing  $Q_3$  to be off as well.  $V_o$  is high (There is an added voltage drop due to diode  $D_1$ ) (Q<sub>4</sub> is off)
- If either input (or both) are increased then some of the base current is directed to the collector of  $Q_1$ , base of  $Q_2$  and causes it to turn on (saturation) and direct a current to the base of  $Q_3$ , turning it on in turn.
- If either input (or both) are high,  $Q_1$  is in reverse active mode,  $Q_2$  is saturated, causing  $Q_3$  to saturate as well. hence  $Q_4$  will be off and  $V_o$  will be low ( $0.2V$ )
- Noise margins in the above circuit are required:  $M_{NH} = 0.5 - 0.2 = 0.3V$

$$\textcircled{1} \quad V_K = 9, \quad \beta = 96.6, \quad R_L = 2 \text{ k}\Omega$$

$$\frac{2 - 0.8}{R_B} = \frac{5 - 0.2}{2k\Omega \cdot 96.6} = 483 \textcircled{1}$$

$$\textcircled{2} \quad \sigma \cdot B \cdot I_B = I_{\text{const}} \quad I_{\text{const}} = \frac{5 - 0.2}{2k\Omega} = 1.2 \text{ mA}$$

$$I_B = \frac{-0.8}{5k\Omega} + \frac{5 - 2.2}{208k\Omega} = 1.1861 \text{ mA}$$

$$\rightarrow \sigma = 0.0155$$

$$\textcircled{3} \quad \sigma = 0.9 \rightarrow 0.9 \cdot 50 \cdot I_B = \frac{5 - 0.2}{1k\Omega}$$

$$\rightarrow I_B = \frac{4.8 \text{ mA}}{0.9 \cdot 50} = 0.106667 \text{ mA}$$

$$\lambda \quad V_i = 0.8 + I_B \cdot R_B = 2.199$$

$$\textcircled{4} \quad \frac{5 - V_o}{R_L} = 6 \cdot \frac{V_o - 0.8}{12.8 \text{ k}\Omega}$$

$$\rightarrow 5 - V_o = \frac{6 \cdot 1k\Omega}{12.8k\Omega} \cdot (V_o - 0.8)$$

$$\rightarrow 5 + \frac{6 \cdot 0.8}{12.8} = V_o \left( \frac{6}{12.8} + 1 \right)$$

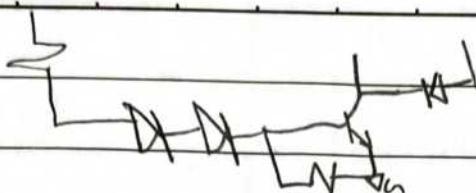
$$\rightarrow V_o = 3.699 \text{ V}$$

Subject

Date

No.

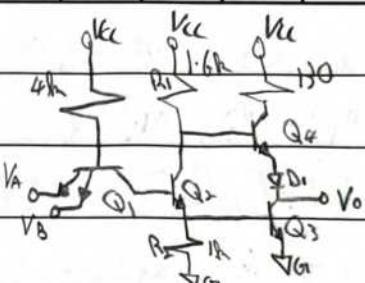
(4)



$$I_C = \frac{5 - 0.2}{0.2} + \frac{5 - 0.4}{2.42} = 1.2 +$$

\* high output:

When either (or both) inputs is low, the output will be high



\* EOC: edge of cutoff

$$0 < V_A \text{ or } V_B < 0.5$$

assume Q1 is sat,  $V_A = 0.2V \rightarrow V_{B1} = V_{BE(\text{sat})} + 0.2V = 1V$

$$\rightarrow I_{B1} = \frac{5-1}{4k} = 1\text{mA} \text{ goes to A or B (emitter)}$$

$$\therefore Q_1 \text{ is sat} \rightarrow V_{CE} = 0.2V \rightarrow V_{B2} = 0.4V$$

$$\therefore V_{B2} < V_{BE(\text{sat})} \rightarrow Q_2 \text{ is off} \quad \text{D}_{1,2}Q_4 \text{ is at EOC}$$

$$\therefore Q_2 \text{ is off} \rightarrow I_{B3} = 0 \rightarrow Q_3 \text{ is off}$$

$\therefore V_0$  is unconnected, no current flows through  $Q_4$  or  $D_1$

$$\rightarrow V_0 = V_{cc} - V_{BE(\text{sat})} - D_{1,2} = 5 - 0.7 - 0.7 = 3.6V \quad (3.8V)$$

$$I_{EB} = 0 \text{ if } V_B \text{ is high} \quad I_{EB} = I_{EA} = \frac{I_0}{2} \text{ if } V_A = V_B = \text{low}$$

\* indeterminate:  $Q_2$  is active when  $0.5V(V_A \text{ or } V_B) < 1.2V$

assuming  $Q_1$  is sat  $\rightarrow 1.3V < V_{B1} < 2V \rightarrow 0.7V < V_{B2} < 1.4V$

$$\rightarrow 0V < V_{E2} < 0.7V = V_{B3} \rightarrow V_{B3} \text{ off}$$

$\therefore V_0$  is open,  $I_{EA} = 0$ ,  $I_{B4} = 0$ ,  $D_{1,2}Q_4$  are at EOC

$$\wedge I_{C2} = I_{A1} = I_E$$

$$\rightarrow I_{E2} = \frac{V_{B2} - V_{BE(\text{on})}}{R_2} \rightarrow 0 < I_{A1} < 0.7\text{mA}$$

$$\therefore V_0 = V_{cc} - I_{B1} \cdot R_1 - V_{BE(\text{on})} - D_{1,2}$$

$$= 5 - I_{B1} \cdot 1.6\Omega - 0.6 - 0.6 \rightarrow 2.68V < V_0 < 3.8V$$

$$\therefore V_{B4} = V_{cc} - I_{A1} \cdot R_1 = 3.88V \quad \wedge V_{CE} = V_{B4} - I_{E1} \cdot R_2$$

$$\wedge I_{E1} = I_C = I_{A1} \text{ when } 0.7 < V_B < 1.4 \rightarrow V_{CE} = 3.18V$$

$\therefore Q_2$  is not in sat. but in active form.

\* indeterminate:  $Q_3$  is active,  $Q_2$  is active,  $Q_1$  is sat

when  $0.9V < V_{E2} < 0.9V$ ,  $Q_3$  will be active

$$\rightarrow 1.4V < V_{B2} < 1.6V \rightarrow 1.2V < (V_A \text{ or } V_B) < 1.4V$$

$$\therefore V_0 = V_{C1} - I_{B1} \cdot R_1 = V_{BE4(\text{sat})} - \text{Diode}$$

$$\therefore V_0 = 3.6 - I_{B1} \cdot R_1$$

$$\wedge I_{A1} = I_{C2} + I_{B4} \quad \wedge I_{E2} = I_{C2} = I_{B3} + I_{B2}$$

$$\wedge I_{B2} = \frac{V_{BE2} - 0.7}{R_2}$$

\* Low output: when both  $V_A$  &  $V_B$  are high  $\rightarrow (V_A \text{ & } V_B) > 1.4V$

$\rightarrow Q_1$  is in reverse active,  $V_{BE1(\text{rev})} = 0.6V$

$\rightarrow Q_2$  &  $Q_3$  are in sat  $\rightarrow V_{B2} = V_{CE2(\text{sat})} + V_{BE3(\text{sat})} = 1.6V$

$$\rightarrow V_{B1} = 1.6 + V_{BE1(\text{rev})} = 2.2V \rightarrow I_{B1} = \frac{5-1.6}{4k} = 0.7mA$$

$$\rightarrow I_{C1} = (1 + \text{Brewster}) \cdot I_{B1}, \text{ assuming Brewster} = 0.1$$

$$\rightarrow I_{C1} = 0.99mA = I_{B2}$$

$$\therefore Q_2 \text{ is sat} \rightarrow V_{C2} = V_{CE2(\text{sat})} = 0.2V$$

$$\wedge V_{E2} = 0.8V \rightarrow V_{C2} = V_{B4} = 1V \therefore Q_4 \text{ off (diode is off)}$$

$$\wedge I_{R1} = \frac{5-1}{R_1} = 2.5mA = I_{C2}$$

$$\therefore I_{E2} = I_{C2} + I_{B2} = 2.6 + 0.71 = 3.27mA$$

$$\wedge I_{E2} = I_{B3} + I_{B2} \quad \wedge I_{B2} = \frac{V_{E2} - 0}{R_2} = 0.8mA$$

$$\rightarrow I_{B3} = 2.49mA \quad \wedge V_0 = 0.2V (\text{low})$$

$$\therefore B7B > I_C \text{ for sat, } I_{C3} = 0 \rightarrow \text{Saturation}$$

- If  $D_1$  is not used, when  $Q_1$  is rev. active and  $Q_2$  &  $Q_3$  are sat

$$\therefore V_{B4} = 1V \rightarrow Q_4 \text{ will be sat} \rightarrow V_{C4F} = V_{CE4(\text{sat})} + V_{CE3(\text{sat})}$$

$$\rightarrow I_{C4} = I_{AC} = \frac{V_A - 0.4}{R_3} = 35.38mA \rightarrow P = 163mW$$

fence, a lot of power is being dissipated for no reason

and the large current can burn the circuit

\* fan-out for low output logic:

$$\because \beta I_B > I_C \rightarrow \sigma I_{B3} T_{B3} = I_{C3}$$

for  $\sigma = 1$   $I_{B3} = 10$ ,  $T_{B3} = 2.47 \text{ mA}$  (calculated previously)

$$\rightarrow 24.7 \text{ mA} = I_{C3} \cdot N$$

$\lambda I_{C3} = I_{AN} (\text{or } I_{AV})$ , assuming other input in next stage is high

$$I_{AV} = I_{ABN} = \frac{V_{cc} - V_{B1N}}{R_{BN}}, \quad V_{B1N} = V_0 + V_{BE1(\text{sat})} = 1V$$

$$\rightarrow I_{AV} = 1 \text{ mA}$$

$$\therefore 24.7 \text{ mA} = N \cdot 1 \text{ mA} \rightarrow N = 24$$

\* fan-out for high output logic:

$$\text{if } V_{o(\min)} = 3.3V \quad \lambda B_4 = 100$$

$$\rightarrow I_{E4} = (1+\beta) \cdot I_{B4} = N \cdot I_{EIN} \quad \lambda I_{EIN} = I_{CIN} - I_{B1N}$$

$$\lambda I_{B1N} = \frac{5 - V_{B1N}}{4k} \quad \lambda V_{B1N} = V_{B1(\text{sat})} + V_{BE1(\text{sat})} + V_{BE2(\text{sat})}$$

$$\rightarrow V_{B1N} = 2.2V \rightarrow I_{B1N} = \frac{2.2}{4k} = 0.7 \text{ mA}$$

$$\lambda I_{EIN} = I_{B1N} \cdot R_{EIN} = 0.1 \cdot 0.7 \text{ mA} = 0.07 \text{ mA}$$

$$\therefore 101 \cdot I_{B4} = N \cdot 0.07$$

$$\lambda I_{B4(\max)}: \quad V_{B4} = 3.3 + 0.9 + 0.7 = 4.9V$$

$$\rightarrow I_{B4(\max)} = \frac{5 - 4.9}{4.6k} = 0.1875 \text{ mA}$$

$$\therefore 101 \cdot (0.1875 \text{ mA}) = N \cdot 0.07$$

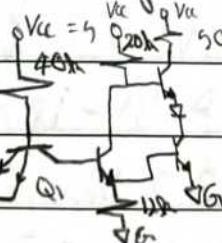
$$\rightarrow N = 270$$

\* low-power TTL: achieved by increasing the resistances as shown

$$P_{TT} = \frac{b-1}{40k} \cdot 5 = 0.5 \text{ mW}$$

$$P_L = 0.07 \cdot 5 + 0.2 \cdot 5 = 1.35 \text{ mW}$$

$$\rightarrow P_{avg} = \frac{1.35 + 0.5}{2} = 0.925 \text{ mW}$$



Example 17.8:

Assuming logic 0 = 0.1V  $\lambda V_A = V_B = 0.1V$ if  $D_A \wedge D_B$  active  $\rightarrow V_i = 0.8V$  $\therefore D_1 \wedge D_2$  off  $\wedge i_1 = 1.05mA$ ,  $Q_0$  is off  $\wedge V_o = V_{ce}$ If  $V_A = V_B = \text{Logic 1} = 5V \rightarrow D_A \wedge D_B$  off,  $D_1 \wedge D_2$  on  $\wedge Q_0$  sat

$$\rightarrow V_i = V_{BE(\text{sat})} + V_{D1\text{on}} + V_{D2\text{on}} = 2.2V$$

$$\wedge i_1 = i_2 = \frac{5-2.2}{4k} = 0.9mA$$

$$\text{Confirm sat: } B \cdot I_B > I_L, I_L = \frac{5-0.1}{4k} = 1.225mA$$

$$\wedge I_B + I_R = I_1, I_1 = \frac{0.8}{4k} \rightarrow I_B = 0.62mA$$

$$\wedge B = 29 \rightarrow B \cdot I_B = 15.5mA > 1.225mA$$

Ex 17.8:

$$P_H = V_{ce} \cdot i_{IH} + V_{ce} \cdot I_{CH} = 9.625mW$$

$$P_L = V_{ce} \cdot i_{IL} = 5.25mW$$

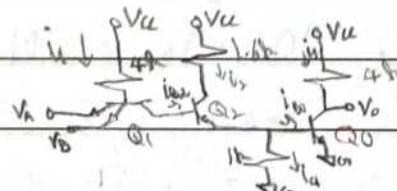
$$P_I \text{ if } V_A = V_B = 0V \rightarrow V_i = 0.9V \rightarrow i_1 = 1.075mA$$

$$\therefore P_I = 5.375mW$$

Example 17.9:

If  $V_A = V_B = 0.1V \wedge Q_1$  is off

$$\rightarrow V_{B1} = 0.9V \rightarrow i_1 = 1.025mA$$

 $\rightarrow Q_2$  is off  $\wedge Q_0$  is off  $\rightarrow V_o = V_{ce}$ If  $V_A = V_B = 5V$ ,  $Q_1$  is reverse active  $\wedge Q_2$  is on  $\wedge Q_0$  is off

$$\rightarrow V_{B2} = V_{BE(\text{off})} + V_{BE(\text{on})} = 1.6V \rightarrow V_{B1} = 1.6 + V_{ce(\text{reverse active})}$$

$$\therefore V_{B1} = 2.3V \rightarrow i_1 = \frac{2.3}{4k} = 0.675mA$$

$$\rightarrow I_{B2} = (1 + B_{\text{rev}}) \cdot i_1 = 0.9425mA$$

$$\because Q_2$$
 is on  $\wedge V_{B2} = 0.8V \rightarrow V_{C2} = 1V \rightarrow i_2 = \frac{1}{1.6k}$

$$\therefore i_{E2} = i_2 + i_{B2} = 3.2425mA = i_4 + i_{B0} \wedge i_{E2} = \frac{0.8}{1.6k}$$

$$\rightarrow i_{B0} = 2.4425mA$$

$$\text{Confirm sat: } B \cdot i_{B0} > i_{C0}$$

$$25 \cdot 2.4425 > \frac{4.8}{4k}$$

Ex 17.9:

a)  $\text{Q}_1 = \text{Q}_2 = 0.1V$ ,  $\text{Q}_1$  is sat,  $i_1 = \frac{5-0.1}{12k} = 0.34167 \text{ mA}$

$\text{Q}_2$  AQA we cutoff  $\rightarrow i_2 = i_3 = i_{E2} = 0 \quad \wedge \quad V_o = V_{AC}$

b)  $V_x = V_y = 5V$ ,  $\text{Q}_1$  is reverse active,  $\text{Q}_2$  AQA we sat

$$\rightarrow V_{B1} = 2.3V \rightarrow i_1 = \frac{2.3}{12k} = 0.225 \text{ mA} \rightarrow i_{B2} = 0.2475 \text{ mA}$$

$\text{Q}_2$  is sat  $\rightarrow V_{C2} = 0.8 + 0.1 \rightarrow i_2 = \frac{4.1}{6k} = 1.025 \text{ mA}$

$$\therefore i_{E2} = i_{B2} + i_2 = 1.2725 \text{ mA} = i_{B2} + i_4 \quad \wedge \quad i_4 = \frac{0.8}{2k}$$

$$\rightarrow i_{B2} = 0.8925 \text{ mA} \quad \wedge \quad i_3 = \frac{4.1}{6k} = 0.8167 \text{ mA}$$

confirm sat:  $B \cdot i_B > i_C \rightarrow 25 \cdot 0.8925 > 0.8167$

example 17.10:

$\therefore B \cdot I_B > I_C \rightarrow 25 \cdot 2.4025 = N \cdot i_1 + 1.025$

$$\therefore i_1 = 1.025 \text{ mA} \rightarrow N = 58$$

Ex 17.10:

when  $V_x = V_y = 3.6V \rightarrow \text{Q}_1$  is rev active,  $\text{Q}_2$  AQA we sat

$$\rightarrow V_{B1} = 2.3V \rightarrow i_{B1} = 0.225 \text{ mA} \rightarrow I_{C1} = 0.27 \text{ mA}$$

$\text{Q}_2$  is sat  $\rightarrow V_{B2} = 0.9V \rightarrow \text{Q}_3$  is cutoff  $\wedge i_{C2} = 1.025 \text{ mA}$

$$\rightarrow i_{E2} = 1.025 + 0.27 = 1.295 \text{ mA} = i_{ABO} + i_{B2}$$

$$\rightarrow i_{B2} = 0.895 \rightarrow V_o = 0.1$$

To keep  $i_1$ :  $25 \cdot 0.895 = N \cdot \frac{5-0.1}{12k} \rightarrow N = 65$

To keep  $I_C = 1.2 \text{ mA} \rightarrow 1.2 \text{ mA} = N \cdot \frac{5-0.1}{12k} \rightarrow N = 35$

TYU 17.9:

a)  $i_1 = \frac{4.2}{15k} = 0.28 \text{ mA}, i_2 = 0 = i_R = i_{AC} = 0$

(1)  $i_1 = i_2, \text{Q}_2$  is sat  $\rightarrow V_B = 0.8 \rightarrow V_i = 2.1V \wedge i_F = 0.18667 \text{ mA}$

$$i_B = i_2 - i_A = 1.33 \times 10^{-4} \text{ A} \wedge i_R = 0.05333 \text{ mA}$$

$$i_{AC} = \frac{5-0.1}{6k} = 0.833 \text{ mA} \quad \wedge \quad V_o = 0.1V$$

TYU 19.6:

for low output:  $i_B = 0.1333 \text{ mA}$   $\wedge i_L = 0.8167 \text{ mA}$ if  $B \cdot i_B > i_C$  for sat

$$\rightarrow B \cdot i_B = i_C + N \cdot \frac{4.2}{150} \rightarrow N = 11 \times (N=9 \text{ in book})$$

$$(1) I_{C,\max} = 12 \text{ mA} = i_{C1} + N \cdot \frac{4.2}{150} \rightarrow N = 39 \times (N=9 \text{ in book})$$

TYU 19.7:

low output:  $i_B = 0.8167 \text{ mA}$   $\wedge i_{B0} = 0.895$ 

$$\rightarrow 25 \cdot 0.895 = 0.8167 + N \cdot 0.342 \rightarrow N = 63$$

19.19:

i) if  $V_i = 0.1 \text{ V} \rightarrow D_1$  is active,  $Q_0$  is cut off  $\rightarrow V_1 = 0.141667 \text{ mA}$ 

$$i_B = i_3 = 0 \quad \wedge V_1 = 0.8 \text{ V} \quad \wedge V_2 = V_{ce} = 2.5 \text{ V}$$

ii) if  $V_i = 2.5 \text{ V} \rightarrow D_1$  is off,  $Q_0$  is sat  $\rightarrow V_1 = 1.5 \text{ V}$ 

$$i_B = i_1 = \frac{1}{12k} = 0.0833 \text{ mA}, V_0 = 0.4 \text{ V} \rightarrow i_3 = 0.2 \text{ mA}$$

 $\therefore B \cdot i_B > i_3 \rightarrow 25 \cdot 0.0833 > 0.2 \rightarrow \text{sat}$ at  $Q_0$  EOL  $\rightarrow V_1 = 1.4 \text{ V} \rightarrow V_2 = 0.9 \text{ V}$ at  $Q_0$  EOS  $\rightarrow V_1 = 1.5 \rightarrow V_2 = 0.8 \text{ V}$ 

19.20:

i) if  $V_i = 0 \text{ V} \rightarrow V_1 = 0.9 \text{ V}, i_1 = 0.4333 \text{ mA}, i_B = 0 = i_C$ 

$$\wedge V_0 = 3.3 \text{ V}$$

ii) if  $V_i = 3.3 \text{ V} \rightarrow V_1 = 1.5 \text{ V} \rightarrow i_1 = 0.3 \text{ mA}$ 

$$i_B = i_1 - \frac{0.8}{20k} = 0.26 \text{ mA}, V_0 = 0.1 \text{ V} \wedge i_C = 0.8 \text{ mA}$$

confirm sat:  $0.26 \cdot 25 = 6.5 \text{ mA} > 0.8 \text{ mA} \checkmark$ 

19.21:

i) if  $V_x = V_y = 0.1 \rightarrow V = 0.8 \text{ V} \rightarrow i_1 = 0.525 \text{ mA}$ 

$$i_3 = i_2 = 0 \quad \wedge V_0 = 5 \text{ V}$$

$$\text{ii) } V_x = V_y = 5V \rightarrow V = 2.2V, i_1 = 0.36 \text{ mA} = i_4 + \frac{0.8}{15k} \rightarrow i_4 = 0.2967 \text{ mA}$$
$$\therefore i_3 = \frac{4.9}{27k} = 1.842 \text{ mA}$$

17.22:

$$\text{i) } i_1 = 0.3125 \text{ mA} \quad \text{and } V = 0.8V, i_4 = i_3 = 0 \quad \text{and } V_o = 9.3V$$

$$\text{ii) } i_1 = 0.1395 \text{ mA} \quad \text{and } V = 2.2V, i_4 = i_1 - \frac{0.8}{15k} = 84.169 \mu\text{A}$$
$$\therefore i_3 = 1.33 \text{ mA}$$

17.23:

$$\text{a) Q}_1 \text{ and Q}_2 \text{ sat } \rightarrow V_1 = 2.3V \rightarrow i_1 = 0.695 \text{ mA} = i_{B1}$$

$$\therefore i_2 = \frac{3.4}{28k} = 1.2 \text{ mA} \quad \therefore i_4 = 2.375 \text{ mA}$$

$$i_B = i_4 - i_5 = 2.295 \text{ mA} \quad \text{and } i_3 = \frac{4.9}{4k} = 1.225 \text{ mA}$$

$$\text{b) } \text{so } B \cdot i_{B2} = 1.225 + N \cdot \frac{4.2}{7k}$$

$$\rightarrow N = 42$$

- main ECL advantage is the very short propagation delay.

+ disadvantages include:

- high power consumption

- poor speed-power product

- low level of integration

- not compatible with TTL and CMOS

- ECL logic and interface technology is still used in very high-speed communications gear such as fiber optic transceivers, interfaces and synchronous transfer mode networks (ATM)

\* basic ECL current switch:

if  $V_1 < V_2$

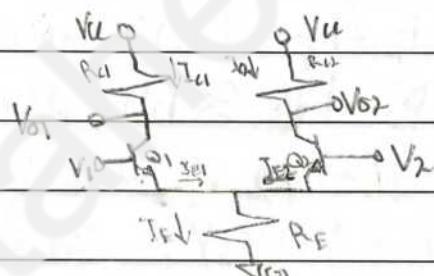
$Q_1$  is off,  $V_{O1}$  is high

$Q_2$  is forward active,  $V_{O2}$  is low

if  $V_1 > V_2$

$Q_2$  is off,  $V_{O2}$  is high

$Q_1$  is on,  $V_{O1}$  is low



hence,  $V_{O1}$  is the inverting output while  $V_{O2}$  is the non-inverting

if  $V_2$  is fixed,

assuming  $Q_1$  and  $Q_2$  are forward active:

$$\rightarrow V_{O1} = V_{CC} - I_{C1} \cdot R_{11} \quad \wedge \quad V_{O2} = V_{CC} - I_{C2} \cdot R_{22}$$

$$\wedge I_E = I_{E1} + I_{E2}, \quad \therefore \alpha = \frac{\beta}{\beta+1}, \quad \alpha_1 = \alpha_2 = \alpha$$

$$\rightarrow I_E = (I_{C1}/\alpha_1) + (I_{C2}/\alpha_2) \rightarrow \alpha I_E = I_{C1} + I_{C2}$$

recall the  $I_C - V_{BE}$  relation:  $I_C = I_S \cdot (e^{\frac{V_{BE}}{V_T}} - 1) \approx I_S \cdot e^{\frac{V_{BE}}{V_T}}$

where  $I_S$ : [reverse] saturation current,  $V_T$ : thermal voltage: 26 mV

26 mV

$$\rightarrow I_{L1} = I_{S1} e^{\frac{V_{BE1}}{V_T}} \quad \& \quad I_{L2} = I_{S2} e^{\frac{V_{BE2}}{V_T}}$$

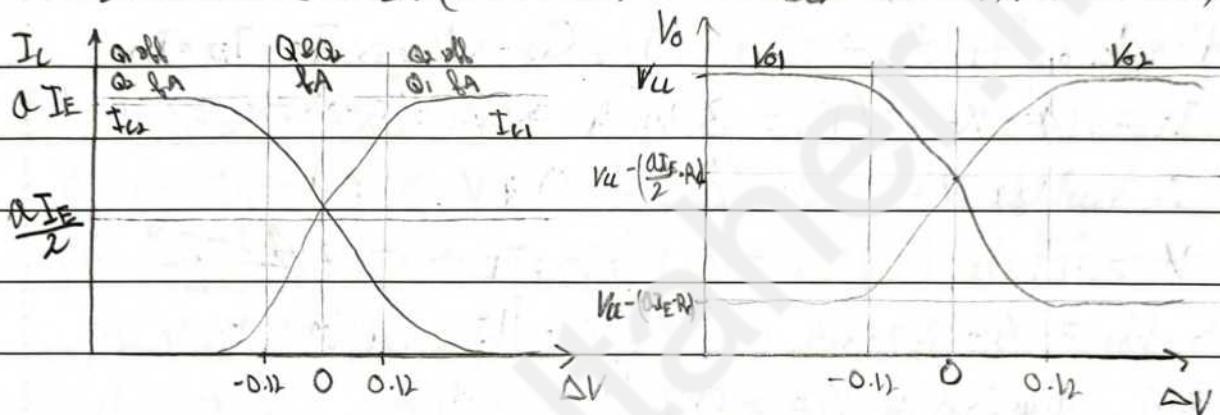
$$\therefore V_{BE1} - V_{BE2} = V_1 - V_2 = \Delta V$$

$$\therefore \frac{I_{L1}}{I_{L2}} = \frac{I_{S1}}{I_{S2}} \cdot e^{\frac{(V_{BE1} - V_{BE2})}{V_T}} \quad \text{if } I_{S1} = I_{S2}$$

$$\rightarrow \frac{I_{L1}}{I_{L2}} = e^{\frac{(V_{BE1} - V_{BE2})}{V_T}}$$

$$\therefore \alpha I_E = I_{L2} (1 + e^{\Delta V/V_T}) \quad \& \quad \alpha I_E = I_{L1} (1 + e^{-\Delta V/V_T})$$

$$\rightarrow I_{L1} = (\alpha I_E) / (1 + e^{-\Delta V/V_T}) \quad \& \quad I_{L2} = (\alpha I_E) / (1 + e^{\Delta V/V_T})$$



$$\lim_{\Delta V \rightarrow 0} (I_{L2}) = \alpha I_E, \text{ if } |\Delta V| = 0.12, e^{-\frac{0.24}{V_T}} = 1$$

- hence,  $I_E$  is constant. as  $I_{E1} \downarrow, I_{E2} \uparrow$  equally

- If the circuit shown below is to be operated as a logic element, then

$\Delta V$  should always be smaller than  $-0.12V$  or larger than  $0.12V$

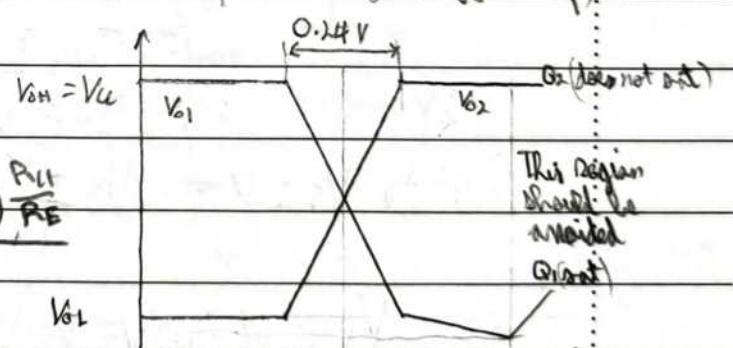
in the region where  $-0.12 < \Delta V < 0.12$  both  $Q_1$  and  $Q_2$  are on and outputs will be indeterminate. (circuit will operate as a diff amp)

- assuming  $V_o$  is fixed and  $V_1$  is  $V_2$

$V_S$ : the voltage at which  $Q_2$  saturates

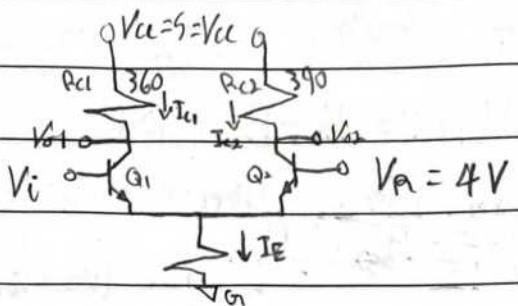
$$V_S = V_1 = \frac{V_{CE} + V_{CE1(\text{sat})} + (V_{BE1(\text{sat})} + V_{CE}) \frac{R_U}{R_E}}{1 + \frac{R_U}{R_E}}$$

$$V_{OL(\text{sat})} = V_{BE1(\text{sat})} - V_{CE(\text{sat})}$$



\* Basic ECL inverter/buffer:

- Remember the 0.24V region in which both transistors are active must be avoided.



$$\rightarrow V_{IL} < 3.88, \text{ max } V_{IL} = 3.88V \quad V_R = 0.12V$$

$$\rightarrow V_{IH} > 4.12, \text{ min } V_{IH} = 4.12V \quad V_R + 0.12V$$

- at  $V_i = L$ ;  $Q_1$  off  $\rightarrow I_{c1} = 0$ ,  $Q_2$  active  $\rightarrow I_E = I_{c2} = I_{R2}$

$$I_E = (4 - V_{BE(\text{on})}) / 1k = 3.3 \text{ mA} \rightarrow V_{c2} = 5 - 3.3 \cdot 390 = 3.913 = \boxed{L}$$

$\therefore$  buffer  $V_{c2} \approx V_{EF} \approx 0.4V \rightarrow$  not sat

$$\text{- at } V_i = H: Q_2 \text{ off } \rightarrow I_{c2} = 0, I_{c1} = I_{FE} = I_E \rightarrow I_E = \frac{4.2 - 0.9}{1k} = 3.5 \text{ mA}$$

$$\rightarrow V_{c1} = 5 - 360 \cdot 3.5 = 3.14V = \boxed{L}, V_{c1} \text{ inverter}$$

Check active/sat:  $V_{CE} = 0.24V \rightarrow$  active

- if  $V_i = 5V = V_{cc}$ :

assume  $Q_1$  is on:  $V_E = V_i - V_{BE(\text{on})} = 5 - 0.9 = 4.3V$

$$I_E = I_{c1} = \frac{4.3}{360} = 4.3 \text{ mA}, V_{c1} = 5 - 0.36 \cdot 4.3 = 3.452V = \boxed{L}$$

Check saturation:  $V_{CE} = V_{c1} - V_E = -0.848V$  Sat. must radio care.

Recalculate for sat:  $V_E = V_i - V_{BE(\text{sat})} = 4.2V \rightarrow I_E = 4.2 \text{ mA}$

$$\therefore V_{c1} = V_E + V_{CE(\text{sat})} = \boxed{4.4V}$$

$$\Rightarrow I_{c1} = \frac{5 - 4.4}{360} = 1.667 \text{ mA}$$

$$\therefore I_B = 2.533 \text{ mA}$$

$$\text{Calculate } V_s: V_i = V_R + V_{c1(\text{sat})} + (V_{BE(\text{sat})} + V_{EE}) \frac{R_{c1}}{R_E}$$

$$1 + \frac{R_{c1}}{R_E}$$

$$\rightarrow V_i = 4.329V = V_s$$

$$\therefore \text{max } V_{IH} = 4.329V$$

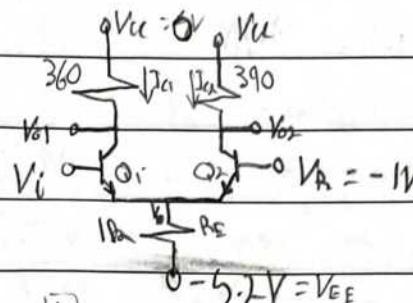
for  $V_A = -1V$  &  $V_{EE} = -5.2V$ ,  $V_U = 0V$

- if  $V_i = L$ ,  $V_i < -1.12V$ :

$Q_1$  off,  $Q_2$  on,  $I_{U1} = 0$ ,  $I_{C2} = I_E$

$$I_E = \frac{-1 - 0.9 - (-5.2)}{1k} = 3.5 \text{ mA}$$

$$V_{O2} = 0 - 3.5 \cdot 390 = -1.365V = [L]$$



Check active:  $V_{CE} = -1.365 + 1.7 = 0.335V \rightarrow \text{active}$

- if  $V_i = H$ ,  $V_i > -0.88V$ :  $V_i = 0.8V$

$Q_1$  is on,  $Q_2$  is off,  $V_E = -1.5V \rightarrow I_E = I_{U1} = \frac{3.9}{1k} = 3.9 \text{ mA}$

$$\rightarrow V_{O1} = -1.332V = [L], \text{ check active: } V_{LE} = -1.332 - (-1.5) = 0.166V$$

- if  $V_i = 0V = [H]$ :

$Q_2$  off,  $Q_1$  active  $\rightarrow V_E = -0.9V \rightarrow I_E = I_{U1} = \frac{4.5}{1k} = 4.5 \text{ mA}$

$$\rightarrow V_{O1} = -1.61V = [H]$$

Check if assumption that  $Q_1$  is active is correct:

$$V_{CE} = -1.61 - (-0.9) = -0.72V$$

Hence assumption is false and  $Q_1$  is off

$$\rightarrow V_E = 0 - V_{BE(\text{off})} = [-0.8V], I_E = \frac{4.4}{1k} = 4.4 \text{ mA}$$

$$\rightarrow V_{O1} = 0.2 - 0.8 = 0.6V \rightarrow I_{U1} = \frac{0.6}{360} = 1.667 \text{ mA}$$

$$\therefore I_{U1} = 2.933 \text{ mA}$$

\* Basic OR/NOR ECL gate:

- If  $V_A$  or  $V_B$  (or both) are

high then their respective

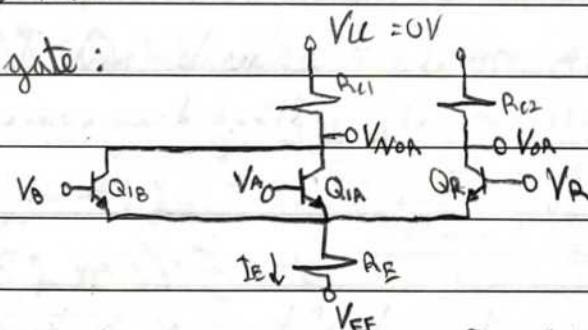
transistors will be forward

active while  $Q_A$  will be off

$$\rightarrow V_{NOR} = L \quad V_{OA} = H = V_U = 0V$$

- If both  $V_A$  &  $V_B$  are low,  $Q_{IA}$  &  $Q_{IB}$  will be off,  $Q_A$  will be forward active

$$V_{OA} = L, V_{NOR} = H = 0V = V_U$$



$Q_{IA}$  &  $Q_{IB}$  are in parallel

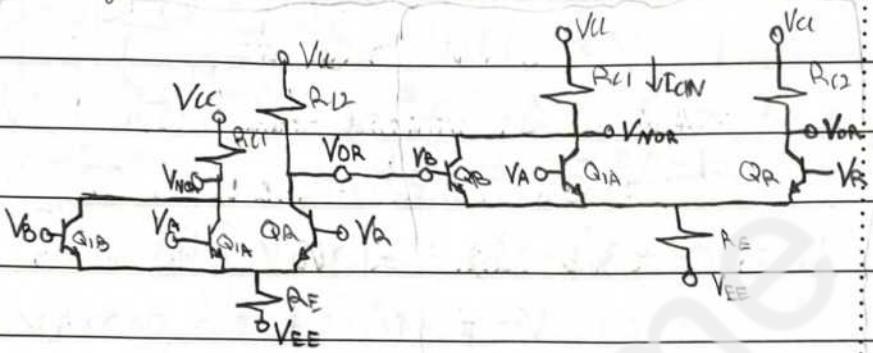
\* fan-out of ECL OR/NOR gate:  $V_{CC} = 0V$

- for an ECL OR/NOR

gate supplying the input of another

OR/NOR gate

from its NOR



Terminal:

Driver

Load

1) if  $V_B = V_A = L \rightarrow V_{OR} = L$  next stage  $V_B = L \rightarrow Q_B$  off

hence an infinite number of gates can be driven

$$N_L \rightarrow \infty$$

2) If either  $V_A$  or  $V_B$  of the driver circuit is high (or both),  $Q_A$

will be off and  $V_{OR} = V_{CC} - I_{C2} \cdot R_{C2}$

If  $V_A$  of the second stage is low  $\rightarrow Q_A$  is off,  $I_{C2}$  passes

completely through  $Q_{1B}$ , which gives the output of the emitter of the load  $Q_{1B} = I_{BB} + I_{IN}$ , where  $I_{BB}$  is the total current  $I_{C2}$  of the driver divided by the  $N$  loads.

- If  $V_A$  is high, the current  $I_{IN}$  will be divided evenly

$$\text{among } Q_{1B} \text{ and } Q_{1A}, \text{ giving } I_{EBN} = I_{BB} + \frac{I_{IN}}{2}$$

$Q_B$  must not saturate and  $I_{C2}$  must not get large enough

to cause the voltage drop across  $R_{C2}$  to reduce  $V_{OR}$  from high to low.

to be continued after quiz practice and quiz

$$P2.2: V_i = 0.1V \rightarrow V_{B1} = 0.9 \rightarrow I_B = 0.82 \text{ mA}$$

$$V_{B2} = V_{C1} = 0.3V \rightarrow Q_2 \text{ off}, Q_3 \text{ off} \rightarrow I_{C3} = 0 = I_O$$

$$V_i = 4V \rightarrow Q_1 \text{ reverse active}, Q_2 \text{ sat} \rightarrow V_{B2} = 0.8V$$

$$\rightarrow V_{B1} = 0.8 + V_{BE(\text{sat})} = 1.4V \rightarrow I_1 = 0.72 \text{ mA}$$

$$Q_3 \text{ sat as well} \rightarrow V_{B3} = 0.8V \rightarrow V_{B2} = 1.6V$$

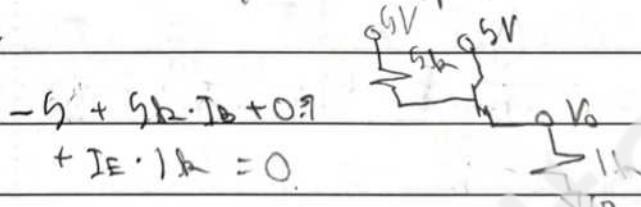
$$\rightarrow V_{B1} = 2.2V \rightarrow I_1 = 0.56 \text{ mA}$$

$$\therefore Q_2 \text{ sat} \rightarrow V_{C2} = 0.2 + 0.8 = 1V \rightarrow I_2 = 1.6 \text{ mA}$$

$$\& V_o = 0.2V \rightarrow I_3 = 2.4 \text{ mA}$$

Total power supply current = 4.56 mA

Q2:



$$I_E = (1 + \beta) \cdot I_B \quad \& \quad \beta = 50 \rightarrow I_B (5k + 51 \cdot 1k) = 4.3$$

$$\rightarrow I_B = 76.78 \text{ nA}$$

$$\rightarrow V_o = 76.78 \cdot 51 \cdot 1k = 3.9161 \text{ not sat} \\ = 3.92V$$

Q3: Q1 Rev active  $\&$  sat Q3 sat  $\rightarrow V_{B3} = 0.8V, V_{B2} = 1.6V$

$$V_{B1} = 2.1V$$

If Q3 sat  $\rightarrow V_o = 0.2V$

3 gates connected Q1-N sat  $\rightarrow V_{B1N} = 0.9V \times \boxed{IV}$

$$\rightarrow I_{EIN} = \frac{4.1}{5k} = 0.82 \text{ mA} \times \boxed{0.8 \text{ mA}}$$

$$\rightarrow I_{C1} = (0.82 \text{ mA}) \cdot 3 + \frac{5 - 0.2}{2k} = 4.86 \text{ mA}$$

$$\boxed{0.8 \text{ mA}} \cdot 3 + \boxed{4.8 \text{ mA}} = \boxed{4.8 \text{ mA}}$$

$$Q_4: V_0 = 0.2V \times$$

$$I_{L1} = \frac{5-0.2}{2k} + 2 \left( \frac{5-(0.2+0.8)}{4k} \right) \times \times$$

$Q_1$  sat  $\Rightarrow$  input low  $\rightarrow Q_2$  off  $\rightarrow V_0 = V_{CC}$

$V_0$  input to two gates

$Q_{1N}$  reverse active  $\rightarrow Q_{2N}$  sat  $\rightarrow V_{BE1} = 1.4V$

$$\rightarrow I_{BE1} = 0.9 \text{ mA} \quad \beta_{BE1} = 0.2$$

$$\rightarrow I_{BE1} = 0.18 \text{ mA}$$

$$\rightarrow I_{BE1} = I_{C1} = 0.18 \cdot 2 = 0.36$$

$$\therefore V_0 = 5 - 0.36 \cdot 2k = 4.28V$$

H  $\rightarrow$  L input

$$I_{BE1} = \frac{5-0.9}{4k} + I_{C1} = \frac{5-0.9}{4k} + 51 \cdot \frac{5-0.9}{4k} = 53.33 \text{ mA}$$

$$I_{C1} = (\beta_{BE1} + 1) I_{B1} \quad \rightarrow I_{B1} = \frac{5-0.9}{4k}$$

$$I_{B1} = V_{CC} - V_{BE1} \quad \rightarrow V_{BE1} = 0.2 + 0.9 + 0.2 = 1.3V$$

$$R_1 \rightarrow I_{B1} = 3.3 / 1.6k = 2.0625 \text{ mA}$$

$$I_{C1} = \frac{V_{CC} - V_{BE1}}{130} \quad V_C = 0.2 + 0.9 + 0.2 = 1.3V$$

$$\rightarrow I_{C1} = \frac{3.9}{130} = 0.30 \text{ mA}$$

$$\rightarrow I_{BE1} \text{ changing} = 30 + 2.0625 = 32.0625$$

$$Q_5: Q_1 \text{ low} \rightarrow V_{BE1} = 0.9 \quad I_{B1} = \frac{4.1}{4k} = 1.025 \text{ mA}$$

$$\rightarrow I_{BE1} = (\beta_{BE1} + 1) \cdot I_{B1} = 57.4 \text{ mA}$$

$$\rightarrow I_{C1} = I_{B1} = \beta \cdot I_{B1} = 56.395 \text{ mA} \text{ output low}$$

$$Q_3: V_0 = 4V \rightarrow Q_1 \text{ in rev. active}, Q_3 \text{ & } Q_2 \text{ sat}$$

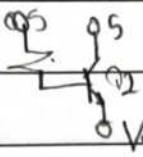
$$\rightarrow V_{BE1} = 1.6V \quad \rightarrow V_{BE1} = 2.2V \rightarrow I_{B1} = 0.56 \text{ mA}$$

$$\rightarrow I_{L1} = 0.46 \cdot (1 + 0.1) = 0.616 \text{ mA}$$

$$\text{Q2 sat} \rightarrow I_{C1} = \frac{5-1}{2k} = 1.6 \text{ mA}$$

$$\rightarrow I_{E2} = 2.216 \text{ mA}$$

$$I_{B3} = I_{E2} - 0.8112 = 1.416$$

P1:  Q<sub>1</sub> sat  $\rightarrow$  no current since  $R_L = \infty$   
 $V_{BEQ1} = 0.7V \rightarrow V_o = 5 - 0.6 = 4.4V$

P2: Q<sub>2</sub> cut off  $\rightarrow Q_{1N}$  - New active  $\rightarrow I_{BQ1} = \frac{3.6}{1k} \rightarrow I_{EIN} = 0.18mA$   
 $\therefore I_{AL} = 0.18mA \quad V_o = 4.64V$

## Quiz 2:

1) Q<sub>2</sub> sat  $\rightarrow V_o = 0.2V$

$$\textcircled{2} I_B \text{ (I)} = I_C$$

$$I_B = \left( \frac{4.8 - 1.4}{R_E} \right) (1 + 0.11) = 0.999mA$$

$$I_C = \frac{4.8}{2k} + 8 \cdot 1mA = 10.4mA$$

$$10.4 > 0.999 \cdot 42 \cdot 0 \rightarrow \boxed{0 = 0.24986}$$

2)   $V_{BE1} = 2.2 \rightarrow I_{B1} = 0.46mA$

$$V_o = 0.2mA$$

$$0.8mA \cdot 5 = 4mA \\ = 6mA$$

3) Q<sub>2</sub> & Q<sub>3</sub> cut  $\rightarrow V_o = 0.2V$

$$V_2 \cdot 1 = 12mA$$

$$\frac{5 - 1.4}{1.6k} \rightarrow$$

4)  $5 \cdot 1 = 5mA \rightarrow Q_{1P}$

$$\frac{5 - 1.4}{1.6k} = \boxed{0.4} \times 3 \quad 0.18 = \beta_{Q1N}$$

$$I_{EIN} = I_{B1N} \cdot \beta_{Q1N} = \boxed{0.162mA}$$

If we calculate  $I_{B1N}$   $\therefore I_{C2} = 0.162 \cdot 3 = 0.486mA$

power consumed

$$\therefore Q_2 P = 5 \cdot 0.486 = 2.43mA$$

for a single circuit

$$\rightarrow P_{total} = \boxed{7.43mW}$$

## &amp; modified OR/NOR ECL gate: MECL

$Q_3$  and  $Q_4$  are always forward active

If  $Q_3$  is forward active

If  $Q_3$  is forward active  $V_{B3} = V_B$

$$\rightarrow Q_A \wedge Q_B \text{ off} \rightarrow V_A = V_B = L \quad V_{NOR} = H$$

If  $Q_3$  is off,  $Q_3$  or  $Q_4$  or both

are forward active  $V_{OR} = H$

1) If  $V_A$  and  $V_B$  are low:

$$0 + 270 \cdot I_{B4} + 0.7 + I_{E4} \cdot 2h - 5.2 = 0 \quad \wedge I_{E4} = (\beta + 1)I_{B4}$$

$$\rightarrow I_{B4} [270 + (\beta + 1) \cdot 2h] = 5.2 - 0.7$$

$$\text{If } \beta = 100 \rightarrow I_{B4} = 22.25 \text{ mA} \rightarrow V_{NOR} \approx 0.7V = H$$

$$\wedge I_{B4} - I_{B3} = I_E = \frac{-1.4 - 5.2}{1.2 \cdot h} = 2.8258$$

$$N \quad 0 + I_{B4} \cdot 300 + 0.7 + I_{E4} \cdot 2h - 5.2 = 0$$

$$\rightarrow (2.8258 + I_{B4}) \cdot 300 + 0.7 - 5.2 + (\beta + 1)I_{B4} \cdot 2h = 0$$

$$\rightarrow I_{B4} [300 + (\beta + 1)2h] = 3.65326 \quad \beta = 100$$

$$\rightarrow I_{B4} = 18.0648 \text{ mA} \rightarrow V_{OR} = -1.6422V = L$$

2) If  $V_A$  and/or  $V_B$  are high:

$$V_A = V_B = H = -0.7V \rightarrow V_E = -1.4 \rightarrow I_E = I_{EA} + I_{EB} = \frac{-1.4 - 5.2}{1.2 \cdot h} = 3.0645 \text{ mA}$$

$$\rightarrow V_{NOR} = -3.0645 \text{ mA} \cdot 270 - 0.7 = -1.527 = L = V_{NOR}$$

if  $Q_3$  off &  $V_{B3}$  negligible  $\rightarrow V_{OR} = -0.7V = H$

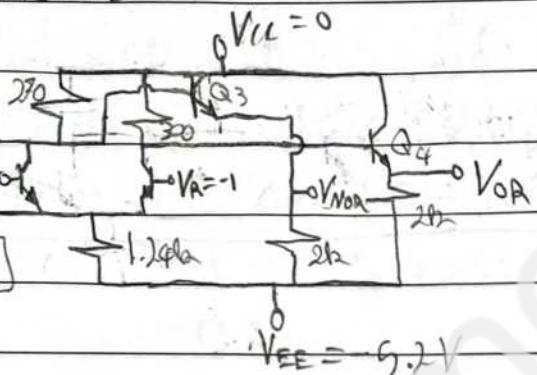
$$\therefore V_A = -1 \rightarrow V_{IH} = -0.88 \quad V_{IL} = -1.12$$

$$\wedge \text{NOR: } V_{OH} = -0.7V \quad V_{OL} = -1.628V$$

$$\text{OR: } V_{OH} = -0.7V \quad V_{OL} = -1.65V$$

$\therefore$  noise margins: [OR:  $NM_H = 0.18V$   $\wedge NM_L = 0.43V$ ]

[NOR:  $NM_H = 0.18V$   $\wedge NM_L = 0.408V$ ]



\* modified OR/NOR ECL gate: MECL fan-out

if  $V_A$  &  $V_B$  are high, OR output will be high and OR off.

$$I_{E4} = I_{RE4} + N \cdot I_{load} \quad \text{assume } Q_{out} = \text{inactive}$$

$$\text{assuming } V_{OA} = -0.75V \quad \lambda: I_{E11} = I_{IBF1} = I_{load} + I_{C1}$$

$$\lambda + 0.75 + 0.7 + 1.48 \cdot I_{ED} - 5.2 = 0$$

$$\rightarrow I_{ED} = 3.0242 \text{ mA} = (B+1) \cdot I_B$$

$$\therefore I_B = I_D \quad B = 100 \rightarrow I_D = 29.9426 \text{ mA}$$

$$\therefore I_{BE4} = (B+1) I_{B4} = I_{AE4} + N I_D$$

$$\lambda I_{AE4} = \frac{-0.75 - 5.2}{2k} = 2.225 \text{ mA} \quad \lambda I_{B4} \cdot 300 + 0.7 - 0.75 = 0 \rightarrow I_{B4} = 0.1667 \text{ mA}$$

$$\therefore (B+1) \cdot 0.1667 \text{ mA} = 2.225 \text{ mA} + N \cdot 29.9426 \text{ mA}$$

$$\rightarrow N = \frac{14.608}{0.0299} = 484 \quad \text{fan-out for high output logic}$$

\* modified 2:

$$V_B = 0.7 + 0.9 + I_{A2} \cdot 2.3k - 5.2$$

$$\lambda V_B = 0.7 + I_{ES} \cdot 2k \quad I_{ES} = B \cdot I_{BS} \quad \text{assume } I_{BS} = 0 \text{ negligible}$$

$$\rightarrow 0.7 + I_{B2} \cdot 2k - 5.2 = (B+1) I_{BS} = (B+1) (I_{A1} - I_{A2})$$

$$\lambda I_{B1} \cdot 300 + 0.7 + 0.9 + I_{A2} \cdot 2.3k - 5.2 = 0$$

$$\rightarrow I_{A1} = \frac{5.2 - 1.4 - I_{A2} \cdot 2.3k}{300}$$

$$\rightarrow -4.6 + I_{A2} \cdot 2.3k = 101 \cdot \left( \frac{3.8}{300} - I_{B2} \left( \frac{2.3k}{2.3k} + 1 \right) \right)$$

$$\rightarrow I_{A2} \cdot 2.3k = 4.6 + 1.29933 - 875.333 \cdot I_{B2}$$

$$\rightarrow I_{B2} (1300 + 875.333) = 4.6 + 1.299333$$

$$\rightarrow I_{B2} = 1.82 \text{ mA} \rightarrow I_{A1} = -$$

$$\rightarrow 0 + 300 I_{A1} + 0.7 + 0.9 + 2.3k I_{A2} = 5.2, \quad I_{A1} = I_{A2}$$

$$\rightarrow I_{A1} = 1.46 \text{ mA}$$

$$\lambda V_B = -0.4374 \quad \lambda V_B = V_B - V_{BE(on)} = -1.1374 \approx -1.14 \text{ V}$$

Quiz 1 Redone:

$$1) NM_H = 3V \Rightarrow V_{OH} - V_{IH} = 3V$$

$$NM_L = V_L - V_{OL} \quad \therefore V_{OH} = V_L = 5V$$

$$\Rightarrow V_{IH} = 5 - 3 = 2V \quad \therefore -2 + I_B R_B + 0.8 = 0$$

$$\lambda - 5 + I_C R_C + 0.2 = 0$$

$$\rightarrow I_C = \frac{4.8}{2k} = 2.4mA$$

$$\text{assume EoS: } \sigma I_B \beta = I_C, \sigma = 1 \rightarrow I_C = \beta I_B$$

$$\rightarrow I_B = 24.8mA \rightarrow A_B = 48.3$$

$$2) \sigma I_B \beta = I_C \rightarrow \sigma = \frac{I_C}{I_B \beta}, I_C = 0 \text{ if } V_0 = H$$

$$\rightarrow I_C = \frac{5 - 0.2}{4k} = 1.1mA$$

$$\lambda I_B = I_D - \frac{0.8}{5k}$$

$$\lambda I_D = I_{D1} \quad \therefore V_A \propto V_B \text{ high}$$

$$V_p = 0.8 + 0.7 + 0.7 = 2.2V \rightarrow I_{D1} = 1.34667mA$$

$$\rightarrow I_B = 1.186147846 \rightarrow \sigma = 0.015516$$

$$Q_3: \sigma = 0.9 = \frac{I_C}{I_B \beta} \quad \lambda I_C = \frac{5 - 0.2}{1k} = 4.8mA$$

$$\rightarrow I_B = 0.106667mA \quad \lambda I_B = \frac{V_0 - 0.8}{1.186147846} \rightarrow V_0 = 2.14733V$$

$$Q_4: I_C = 0 \text{ if } V_A \propto V_B \text{ high}$$

$$\rightarrow I_C = \frac{V_{CL} - V_0}{4k} + \frac{5 - 0.9}{2.4k} = \frac{4.8}{4k} + \frac{4.1}{2.4k} \quad \text{since other input bias}$$

$$Q_5: Q_1 \text{ cutoff, assume } \sigma \text{ for all not } \rightarrow$$

$$-V_L + R_L \cdot N I_B + V_0 = 0$$

$$\lambda -V_0 + I_B \cdot A_B + 0.8 = 0 \rightarrow V_0 = I_B \cdot R_B + 0.8$$

$$\rightarrow -V_L + I_B (N \cdot A_B + R_B) + 0.8 = 0$$

$$\rightarrow I_B = 53.98mA \quad 223.4mA$$

$$\rightarrow V_0 = 3.66V$$

first S17-18:

P1: If assume EOS,  $V_{IH} \rightarrow V_{OL}$

$$V_A = 0.2V \rightarrow I_{B1} = \frac{5-0.2}{1k} = 4.8mA$$

$$\text{inputs tied together} \rightarrow \frac{4.8mA}{2} = I_C = 2.4mA$$

$$\text{so EOS} \rightarrow \sigma B I_B = I_C \text{ and } \sigma = 1 \rightarrow I_B = \frac{2.4}{20} = 0.12mA$$

$$\therefore V_{IH} = 0.8 + 0.12m \cdot 15m = 2.6V$$

2) for high logic:

$$-5 + N I_B \cdot 1k + I_B \cdot 15k + 0.8 = 0$$

$$\therefore V_{O \min H} = V_{IH} \rightarrow V_o = 5 - 1k \cdot N I_B = 2.6V$$

noise margin  $\rightarrow V_{IH} < V_o < V_{OL}$

$$\rightarrow N \cdot I_B = 2.4mA$$

$$\therefore -5 + 2.4 + I_B \cdot 15k + 0.8 = 0 \rightarrow I_B = 0.12mA$$

$$\rightarrow N = 20$$

$$P2) 1) V_A = V_B = 0.1V \rightarrow V_o = 0.8V \rightarrow I_{B1} = \frac{4.7}{4k} = 1.075mA$$

$$\rightarrow I_{D1} = 0.675mA \quad \text{and} \quad I_{B2} = 0 \quad \text{so Q1 is off}$$

$$2) 0.8 \cdot B \cdot I_B = I_C, \quad V_o = 0.2V \rightarrow I_C = 1.2mA$$

$$V_{B1} = 2.3V \rightarrow I_{E1} = \frac{5-2.3}{4k} + \frac{5-1.7}{2k} = 2.325mA$$

$$\rightarrow I_{B2} = 2.325 - \frac{0.8}{1k} = 2.245mA$$

$$\therefore 0.8 \cdot 2.245mA \cdot B = 1.2mA \rightarrow B = 0.668$$

$$3) \text{ so } V_A = V_B = 5V \rightarrow P_d = 5(0.675 + 1.65 + 1.2)m = 17.625mW$$

$$4) \text{ for a low output: } I_C = N \cdot \left( \frac{5-0.9}{4k} \right) + \frac{5-0.2}{4k}$$

$$\text{EOS, } I_C = B I_B = 20 \cdot 2.245mA$$

$$\rightarrow N = 42$$

P3: 1) Q<sub>1</sub> non active, Q<sub>3</sub> = Q<sub>2</sub> = sat  $\rightarrow V_{B2} = 1.6V$

$$\therefore I_{B1} = \frac{5 - 1.6 - 0.6}{6k} = 0.46667 \text{ mA} \rightarrow I_C = I_{B2} = 0.5133 \text{ mA}$$

$$\wedge I_{B3} = I_{E2} - \frac{0.8}{1.5k} \quad \wedge I_{E2} = 0.5133 \text{ mA} + \frac{4}{2k}$$

$$\rightarrow I_{B3} = 1.97499 \text{ mA}$$

2) If  $V_L = 0.3V$ , Q<sub>1</sub> sat  $\rightarrow V_{C1} = 1.1V \rightarrow$

Q<sub>2</sub> off, Q<sub>3</sub> off, Q<sub>4</sub> forward active:

$$-5 + I_{B4} \cdot 2k + 0.1 + 0.7 + (1+B)I_{B4} \cdot 1k = 0$$

$$\rightarrow I_{B4} = 0.15667 \text{ mA} \rightarrow V_o = 3.287 \text{ V}$$

$$3) -5 + I_{B4} \cdot 2k + 1.4 + 10I_{B4} \cdot 1k = 0$$

$$\rightarrow I_{B4} = \frac{3.6}{11k} = 0.2916667 \text{ mA}$$

$$\rightarrow V_o = 2.416667 \text{ V} \rightarrow V_{E4} = 3.616667 \text{ V}$$

$$\wedge V_L = 5 - B \cdot I_B \cdot 80 = 4.79 \text{ V}$$

$$\rightarrow V_{CE} = 1.17$$

4) Low - output:  $I_{B3} \approx 2 \text{ mA}$

$$FOS \rightarrow 4 \cdot 1 \text{ mA} = I_C \quad \wedge I_C = N \cdot I_{E1}$$

$$\wedge I_{E1} = \frac{5 - 0.9}{6k} \rightarrow N = 26$$

\* ECL positive:

P1:  $V_A = 0$

$$1) V_i = 0 \rightarrow I_E = \frac{3.3}{10k} = 7.33 \text{ mA}$$

$$\rightarrow I_{E1} = I_{E2} = \frac{7.33}{2} = 3.6667 \text{ mA}$$

$$\rightarrow I_C = I_A$$

2)  $V_i = -0.12 \quad \because V_i < V_A \rightarrow Q_1 \text{ off}, V_{o1} = 2V$

$$Q_2 \text{ on} \rightarrow I_{E2} = 7.33 \text{ mA} \quad \wedge I_C = 7.26 \text{ mA}$$

mid practice:

$$2) V_A = \frac{V_{OL} + V_{OH}}{2} \quad Q_A \text{ mA}$$

$$V_A = 0 + I_{AV} \cdot 0.3k\Omega + 0.7 + I_{FO} \cdot 2k\Omega - 5 = 0$$

$$\rightarrow V_{OH} = -0.7V$$

$$\rightarrow V_{OL} \rightarrow V_x \text{ or } V_y \text{ or } \rightarrow I_C = \frac{(V_A + 0.1) - 0.7 - 5}{1.24k\Omega}$$

$$\rightarrow V_{OL} = 0.3k\Omega \cdot \frac{4.3 + V_A + 0.1}{1.24k\Omega} + 0.7 - 5$$

$$\text{or assume } V_x = 0 \rightarrow I_C = \frac{-0.7 - 5}{1.24k\Omega} = 3.468 \text{ mA}$$

$$\rightarrow V_{OL} = -1.7403 \rightarrow V_A = -1.228V$$

$$I_E = +2.903 \text{ mA} = I_C \rightarrow V_{OL} = -1.5909$$

$$\rightarrow V_B = -1.135V$$

$$4) V_{IH} = 4.79 + 0.12 = 4.81V \quad \rightarrow V_{OH} = V_{AC} = 6V$$

$$\rightarrow NM_H = 0.13$$

$$b) V_x = V_y = 1V, Q_A \text{ off} \rightarrow I_E = \frac{5 - 0.85 - 0.7}{1.24k\Omega}$$

$$\rightarrow I_E = 2.78225 \text{ mA} \rightarrow I_{Cx} = I_{Cy} = \frac{I_E}{2} = 1.39 \text{ mA}$$

$$2) Q_C \text{ is sat}, I_E = \frac{5 - 0.8}{1.24k\Omega} = 3.23077 \text{ mA}$$

$$\rightarrow V_o = 5 - 3.23077 \cdot 0.3 = 4.031V$$

$$NM_I = V_{IL} - V_{OL}$$

$$NM_H = V_{OH} - V_{IH}$$

$$2) V_{OH} = 0, V_{OL} = -1.7403 \rightarrow V_A =$$

$$1) Q_3 \text{ off}, Q_1 \text{ off} \quad -5 + I_{B2} \cdot 5\text{k}\Omega + 0.7 + (1+\beta)I_{B2} \cdot 1\text{k}\Omega = 0$$

$$\rightarrow I_{B2} = \frac{5 - 0.7}{(1+\beta)1\text{k}\Omega + 5\text{k}\Omega} = 91.6667 \text{ mA}$$

$$\rightarrow V_o = 3.94$$

$$2) V_A = V_b = H \rightarrow Q_1 \text{ sat } \wedge Q_2 \text{ sat}$$

$$\rightarrow V_{B2} = 2.3 \rightarrow I_{B2} = \frac{5 - 2.3}{38\text{k}\Omega} = \frac{2.7}{38\text{k}}$$

$$I_{B2} = I_{E1} - \frac{0.8}{2\text{k}}$$

$$I_{E1} = \frac{2.7}{38\text{k}\Omega} + \frac{3.3}{2\text{k}\Omega} \rightarrow I_{B2} = 1.32$$

$$3) V_{BEQ1} = 0.7$$

$Q_1$  off

$$5 - 0.3 \cdot I_C = V_o$$

$$I_E = I_B + I_C$$

$$I_E = (1+\beta) I_B$$

$$-5 + N I_B \cdot 0.3 + 0.7 + I_B (1+\beta) \cdot 1.3 = 0$$

$$\rightarrow I_B = \frac{4.3}{N \cdot 0.3 + (1+\beta) \cdot 1.3} = 96.11 \text{ mA}$$

$$V_o = 5 - 1.5 \cdot I_B \cdot 0.3 = 4.6675$$

4)  $Q_1$  off  $Q_3$  off:

$$Q_1 \wedge Q_3 \text{ sat} \rightarrow -V_i + I_{B1} \cdot 4\text{k}\Omega + 0.8 = 0$$

$$\wedge -V_i + I_{B2} \cdot 10\text{k}\Omega + 0.8 = 0$$

$$-5 + 5\text{k}\Omega \cdot I_{B2} + 0.7 = V_i$$

$$I_{B3} + I_{B1} = I_{B2}(1+\beta)$$

6)  $V_A = V_B = 4V \rightarrow Q_1$  is sat. not.

$Q_2$  not  $Q_3$  not

$$V_c = L$$

on

7) ~~DAH~~  $V_{DH} = 3.6$

$$V_{IH} \Rightarrow -V_{IH} + I_B \cdot R_B + 0.8 = 0$$

$$V_{IH} = I_B \cdot R_B - 0.8$$

$$EOS = I_B \cdot B = I_C \quad \text{and} \quad I_C = \frac{1}{2} \frac{3.6 - 0.2}{R_H}$$

$$\rightarrow I_B = 73 \text{ mA} \rightarrow V_{IH} = 0.8489$$

8)  $V_x = 1V$  active

$$I_E = -0.76 - 0.7 + 5 / 1.049 \Omega = 2.855 \text{ mA}$$

$$\wedge I_{EY} = \frac{1}{2} I_E = 1.427$$

9)  $Q_1$  der aktive

$$I_C = \frac{V_a - 0.2}{2k} + 7 \cdot 1 \text{ mA} = 1.4 \text{ mA}$$

10)  $V_A = V_B = 1V$

$$V_P = 2.2V \rightarrow I_{D1} = 1.34 \text{ mA}$$

$$\rightarrow I_B = 1.18 \text{ mA}$$

$$\sigma = 59.1 \cdot 1.18 = I_C, I_C = \frac{5 - 0.2}{0.5} = 1.2 \text{ mA}$$

$$\rightarrow \sigma = 0.01721$$

$$11) \frac{P_H + P_L}{2}, P_H = 0$$

$$P_L = 3.6 \cdot \frac{3.4}{4.49} = 26.78 \text{ mA}$$

11) Put ab base

$$0.8V \quad I_{B1} =$$

$$13) 0.9 \cdot \frac{5-0.4}{1k} \cdot 53 = 43.46 \text{ mA out}$$

$$14) \sigma = \frac{I_C}{\beta \cdot I_B}$$

$$-4.1 + I_{B1} \cdot R_B + 0.8 + 0.2 = 0 \rightarrow I_{B1} = 0.2641 \text{ mA}$$

$$V_o = 0.4 \rightarrow I_{C1} = \frac{5-0.4}{1k} = 4.6 \text{ mA}$$

$$\rightarrow \sigma = 0.362$$

$$15) V_A \text{ high} \rightarrow V_o = L \quad I_C = \frac{4}{5k} \cdot 6 = 4.8 \text{ mA}$$

$$\lambda I_{B3} = I_{E2} - \frac{0.8}{1k} \quad \lambda I_{E2} = \frac{4}{2.5k} + (1+\lambda \cdot 0.9) 0.96$$

$$\rightarrow I_{E2} = 2.216 \text{ mA} \rightarrow I_{B3} = 1.416 \text{ mA}$$

$$\rightarrow \sigma =$$

$$I_L = 4.8 + \frac{5-0.2}{2k} = 7.2 \text{ mA}$$

$$V_o = L = 0.2 \rightarrow I_C = \frac{5-0.2}{2k} + 6 \cdot \frac{4}{5} = 7.2 \text{ mA}$$

$$\lambda I_B = I_{E2} - 0.8 \text{ mA} \quad I_{E2} = \frac{4}{2.5} + \frac{2.8}{5} \cdot 1.1$$

$$\rightarrow I_{E2} = 2.216 \rightarrow I_{B3} = 1.416 \text{ mA}$$

$$\sigma =$$

1.394

$$5 \cdot 0.2 \cdot \frac{4}{5k} \cdot 5 + \frac{5-0.2}{2k} = 6.4 \text{ mA}$$

$$I_{B2} = 1.1 \cdot \frac{5-2.2}{2.5k} = 0.616$$

$$0.616 + \frac{5-1}{2.5k} = 2.216 \text{ mA} \rightarrow I_{B2} = 1.416$$

$$\rightarrow \sigma = \frac{6.4}{50 \cdot 1.416}$$

\* MOSFET: N-channel or P-channel

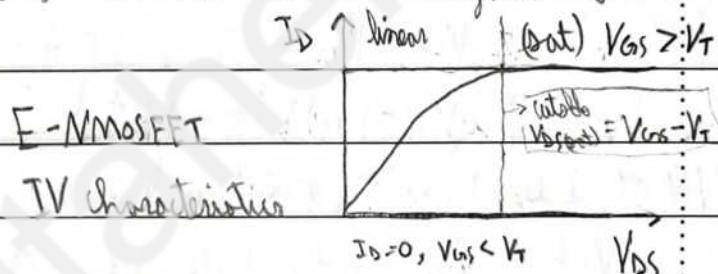
- Subcategory: enhancement or depletion (simplified to single letter)

$\rightarrow$  NMOS: E-NMOS & D-NMOS | PMOS: E-PMOS & D-PMOS

- depletion type (D-NMOSFET) is normally on, whereas enhancement mode (E-NMOSFET) is normally off.

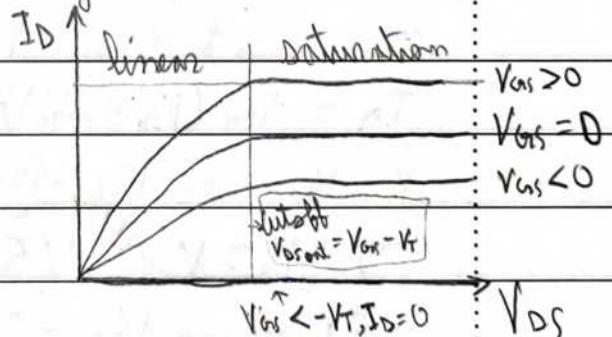
- depletion type has a built-in channel whereas enhancement type must create it

- the  $V_{GS}$  required to create the channel is called the threshold voltage ( $V_T$ ) and is positive for E-NMOS and negative for D-NMOS.



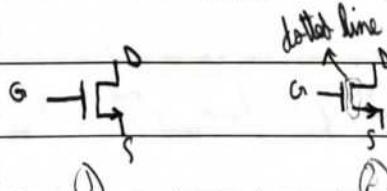
- for D-NMOS, the required  $V_{GS}$  to deplete the channel is the threshold voltage ( $V_T$ ) and is negative

- an oxide separates the gate from the p-substrate (which separates the source and drain). The thickness of this oxide layer is ( $t_{ox}$ ).

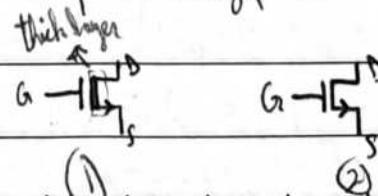


\* Symbols for enhancement-type NMOSFETs, and corresponding symbols for depletion-type

enhancement type:



depletion type



either (1) or (2) will be consistently used.

\* three modes of operation of the NMOS:

1- cutoff:

$$V_{GS} < V_{TN}, I_D = 0 \quad \text{for enhancement: } V_{GS} > 0$$

2- saturation:

$$V_{GS} > V_{TN}$$

$V_{TN}$ : threshold voltage

$$V_{DS} > V_{GS} - V_{TN}$$

$k_m$ : channel conduction parameter,  $k_m = \frac{W}{L} \cdot \frac{\mu_n C_{ox}}{2}$

$$I_D = k_m (V_{GS} - V_{TN})^2$$

$W$  &  $L$ : channel length and width |  $C_{ox}$ : capacitance of oxide per unit area

3- ohmic (linear):

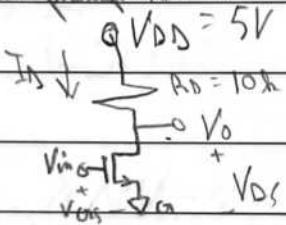
$$V_{GS} > V_{TN}$$

The minimum parameters of a MOSFET are  $V_T$  &  $k_m$

$$V_{DS} < V_{GS} - V_{TN}$$

$$I_D = k_m [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$R_o$  is the pulling network



\* NMOS inverter with resistive load:

$$V_{GS} = V_{in} \quad \& \quad V_0 = V_{DD} - I_D \cdot R_o$$

$$+ \text{if } V_{GS} < V_{TN}: I_D = 0 \quad \& \quad V_0 = V_{DD} = 5V$$

$$+ \text{if } V_{in} > V_{TN}:$$

- assume set then clear assumption:

$$V_{TN} = 1V, k_m = 100mA/V^2$$

$$R_o \cdot k_m = 1V$$

$$I_D = k_m (V_{in} - V_{TN})^2$$

$$V_0 = V_{DD} = V_{DD} - [k_m (V_{in} - V_{TN})^2] \cdot R_o > V_{in} - V_{TN} \quad (\text{forward})$$

$$\rightarrow V_0 = V_{DS} = 5 - (V_{in} - 1)^2 > V_{in} - 1$$

$$\text{solving } V_{DD} - k_m R_o [V_{GS} - V_{TN}]^2 = V_{GS} - V_{TN}$$

$$\text{for } V_{GS} = V_{in} = V_s$$

$\rightarrow$

$$V_s = V_{TN} - \frac{1}{2k_m R_o} + \sqrt{\left(\frac{1}{2k_m R_o}\right)^2 + \frac{V_{DD}}{k_m R_o}}$$

$$\rightarrow V_s = 2.99V$$

- assume linear:

$$I_D = \frac{1}{2} R_{DS} [2(V_{GS} - V_{TN})V_{GS} - V_{DS}^2]$$

$$\rightarrow V_o = V_{GS} - R_{DS} \ln [2(V_{GS} - V_{TN})V_{GS} - V_{DS}^2]$$

$$\therefore V_o = 5 - [2(V_{GS} - 1)V_{GS} - V_{DS}^2]$$

- point A:  $V_{in} = V_{TN}$  and transistor turns

cutoff (output voltage is high)

- point B:  $\frac{dV_o}{dV_{in}} = -1$  (gain), here the input

voltage is maximum low

(any voltage level below it),  $V_{o1}$

- point C: the device leaves sat and

enters linear mode

- point D:  $\frac{dV_o}{dV_{in}} = -1$ , voltages larger than  $V_{in}$  at this point will be high

- point E: the output at this point is low,  $V_{inpt} = V_{oD} = V_{IH}$

+ to calculate input voltage low ( $V_{IL}$ ):

$$\frac{dV_o}{dV_{in}} = -1 = -2R_{DS} \ln (V_{in} - V_T)$$

$$\rightarrow V_{in} = \frac{1}{2R_{DS} \ln} + V_{TN}$$

$$1 \quad V_o = V_{DD} - \frac{1}{4R_{DS} \ln}$$

+ to calculate input voltage high ( $V_{IH}$ ):

$$V_o \text{ at } D: V_o = \sqrt{\frac{V_{DD}}{3R_{DS} \ln}}$$

$$\rightarrow V_{IH} = \frac{2}{\sqrt{3}} \sqrt{V_{DD} \cdot R_{DS} \ln + V_{TN} \cdot R_{DS} \ln - \frac{1}{2}}$$

+ to calculate output voltage low ( $V_{o1}$ ):

$$V_{o1} = \left[ (V_{DD} - V_{TN}) + \frac{1}{2R_{DS} \ln} \right] \pm \sqrt{\left( (V_{DD} - V_{TN}) + \frac{1}{2R_{DS} \ln} \right)^2 - \frac{V_{DD}}{R_{DS} \ln}}$$

∴ for a resistive load inverter:

$$V_{OH} = V_{DD}$$

$$V_{IL} = V_{IN} + \frac{1}{2R_s I_{DN}}$$

$$V_{IH} = V_{IN} + 2 \left[ \frac{V_{DD}}{3R_s I_{DN}} - \frac{1}{2R_s I_{DN}} \right]$$

$$V_{OL} = \left[ V_{DD} - V_{IN} + \frac{1}{2R_s I_{DN}} \right] \pm \sqrt{\left[ V_{DD} - V_{IN} + \frac{1}{2R_s I_{DN}} \right]^2 - \frac{V_{DD}}{R_s I_{DN}}}$$

- the sharpness of the transition region increases with increasing  $R_s$   
(as  $R_s \cdot I_{DN}$  increases)

- the minimum output voltage (the logic zero level) for which input decreases with increasing  $R_s$ .  
( $V_{OL}$  decreases as  $R_s \cdot I_{DN}$  increases)

- hence the inverter approaches ideal as  $R_s$  increases.  
(limit  $R_s \cdot I_{DN} \rightarrow \infty$ )

- power dissipation :  $P_d = \frac{V_{DD}}{2} [I_D(V_{in}=V_{OL}) + I_D(V_{in}=V_{OH})]$

for inverter with resistive load :  $P_d = \frac{V_{DD}}{2} \cdot \frac{V_{DD} - V_{OL}}{R_s}$

\* Steps to finding:

1- check  $V_{DS}$ , if  $V_{DS} > V_T$  then saturation else off

2- check  $V_{DS}$ . if  $V_{DS} > V_{GS} - V_T$ , then saturated  
else  $V_{DS} < V_{GS} - V_T$ , then linear regime

3- If  $V_{DS}$  can't be checked first, assume sat then check if assumption is correct.

design example:  $I_{DN} = 150 \mu A/V^2$ ,  $V_{TN} = 0.5 V$

a)  $R_D$  for  $V_o = 0.1 V$  when  $V_i = 3 V$

$$\text{of linear } \rightarrow V_o = V_{DD} - R_D \cdot I_{DN} [2(3 - 0.5) \cdot 0.1 - 0.1^2]$$

$$\rightarrow 0.1 = 3 - R_D \cdot 150 \mu [0.49] \rightarrow R_D = \frac{2.9}{7.5 \times 10^{-6}} = 39.58 \Omega$$

b) transition point:  $V_o = V_s - V_{TN}$  stat

$$\rightarrow V_{DD} - 39.58 \cdot 150 \mu [V_s - 0.5]^2 = V_s - \frac{1}{2}$$

$$\text{define: } x = V_s - \frac{1}{2} \rightarrow 39.58 \cdot 150 \mu \cdot x^2 + x - V_{DD} = 0$$

$$\rightarrow x = 0.63219 \text{ or } -0.5809$$

$$\rightarrow V_s = 1.13219 V$$

\* NMOS MOS with resistive load:

- transistors do not saturate

A	B	$V_o$	$M_1$	$M_2$	$V_{GS1}$	$A = 1$	$m_1, m_2, b$	$M_1 \downarrow I_{D1}$	$M_2 \downarrow I_{D2}$	$V_o$
0	0	$V_{DD}$	off	off	1					$V_o = V_{DD} - I_{D1} R_D$
0	1	$V_{GS1}$	off	linear	0					$V_o = V_{GS1} = V_{IX1}$
1	0	$V_{GS2}$	linear	off	0					
1	1	$V_{DD}$	linear	linear	0					

$$\text{of } I_{D1} = I_{DN} V_{GS1} [2(V_{GS1} - V_{T1}) - V_{DS1}] \text{ and } I_{RD} = \frac{V_{DD} - V_o}{R_D} = I_{D1} + I_{D2}$$

for  $A=B$ ,  $I_{D1}=I_{D2} \rightarrow I_{RD} = m \cdot I_{D1}$ ,  $m$ : number of transistors (here  $m=2$ )

$$\rightarrow \frac{V_{DD} - V_o}{R_D} = m I_{DN} V_o [2(A - V_{T1}) - V_o], A = \text{Voltage input at A}$$

\* NMOS NAND with resistive load:

A	B	$V_o$ (Logic)	$V_{DD}$	$M_1$	$M_2$	
0	0	1	$V_{DD}$	off	off	
0	1	1	$V_{DD}$	off	linear	
1	0	1	$V_{DD}$	linear	off	
1	1	0	$V_{DD} + V_{DS1}$	linear	linear	$V_o = V_{DD} - I_{D1} \cdot R_D = V_{DS2} + V_{DS1}$

$$\text{if } V_{GS1} = A \quad \text{and } V_{GS2} = B - V_{DS1}$$

$$\therefore I_{D1} = f_{m1} V_{DS1} [2(V_{GS1} - V_T) - V_{DS1}] \quad \text{and } I_{RD} = \frac{V_{DD} - V_o}{R_D} = I_{D1} - I_{D2}$$

$$I_{D2} = f_{m2} V_{DS2} [2(V_{GS2} - V_T) - V_{DS2}]$$

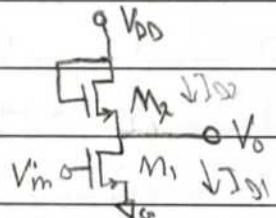
$$\frac{V_{DD} - (V_{DS1} + V_{DS2})}{R_D} = f_{m1} V_{DS1} [2(A - V_T) - V_{DS1}]$$

$$= f_{m2} V_{DS2} [2(B - V_{DS1} - V_T) - V_{DS2}]$$

\* NMOS inverter with active load

$$\text{if } V_{GS2} = V_{GS1} \rightarrow V_{GS2} - V_T < V_{DS2}$$

$\rightarrow M_2$  is always saturated regardless of  $M_1$



Cases:

1-  $V_m < V_T \rightarrow M_1$  is off  $\rightarrow I_{D1} = 0$ ;

$$I_{D1} = I_{D2} = 0 \rightarrow f_{m2} (V_{GS2} - V_{TM2})^2 = 0 \rightarrow V_{GS2} = V_{TM2}$$

$$\therefore V_o = V_{DD} - V_{GS2} = V_{DD} - V_{TM2}$$

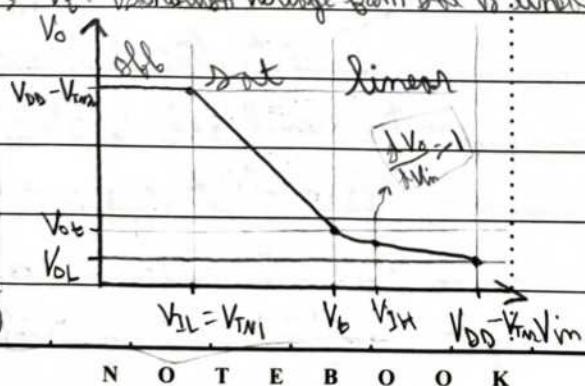
2-  $V_{TM1} < V_m < V_T \rightarrow M_1$  is sat,  $V_t$ : transition voltage from sat to linear

$$I_{D1} = f_{m1} (V_{GS1} - V_{TM1})^2 = f_{m1} (V_m - V_{TM1})^2$$

$$V_{GS1} = V_{DD} - V_{DS1} = V_{DD} - V_o$$

$$I_{D2} = f_{m2} [V_{GS2} - V_{TM2}]^2 = f_{m2} [V_{DD} - V_o - V_{DS2}]^2$$

$$\therefore I_{D1} = I_{D2} \rightarrow f_{m1} (V_{m} - V_{TM1})^2 = f_{m2} (V_{DD} - V_o - V_{DS2})^2$$



$$\therefore V_o = V_{DD} - V_{TN2} - \sqrt{\frac{k_m}{k_n}} \cdot (V_{in} - V_{TH}) \rightarrow V_{IL} = V_{TN1}$$

3-  $V_{in} = V_t \rightarrow M_1$  saturated Ishemic (edge)

$$V_{DS1} = V_{GS1} - V_{TM1} \rightarrow V_{ot} = V_t - V_T \wedge I_{D1} = I_{D2}$$

$$k_m (V_{GS1} - V_{TH})^2 = k_n (V_{GS2} - V_{TM2})^2$$

$$\rightarrow k_m (V_t - V_{T1})^2 = k_n (V_{DD} - V_{ot} - V_{T2})^2 = k_n (V_{DD} - V_t + V_{T1} - V_{T2})^2$$

4-  $V_{in} > V_t \rightarrow M_1$  Ishemic:

$$I_{D1} = k_m V_{DS1} [2(V_{GS1} - V_T) - V_{DS1}] , V_o = V_{DS1} , V_{GS1} = V_{in}$$

$$\wedge V_{GS2} = V_{DD} - V_o$$

$$\therefore I_{D1} = I_{D2} \wedge I_{D2} = k_n (V_{GS2} - V_{T2})^2$$

$$\rightarrow k_m V_{DS1} [2(V_{GS1} - V_T) - V_{DS1}] = k_n (V_{GS2} - V_{T2})^2$$

- in this region,  $V_{TH}$  can be found by taking the derivative of the above equation:  $\frac{dV_h}{dV_{in}} = -1$

- taking the second case and solving for  $V_t$ :

$$V_t = \frac{\sqrt{\frac{k_m}{k_n}} \cdot V_T + V_{DD}}{1 + \sqrt{\frac{k_m}{k_n}}} , \text{ define } k_m = \frac{k_m}{k_n}$$

$$\rightarrow V_t = \frac{\sqrt{k_m} \cdot V_T + V_{DD}}{1 + \sqrt{k_m}} , k_m \text{ should be large}$$

$$\therefore k_m = \frac{W_1}{L_1} \frac{Un Cox}{2} \wedge k_n = \frac{W_2}{L_2} \frac{Un Cox}{2} , \frac{Un Cox}{2} \text{ is usually constant}$$

$$\therefore k_m = \left( \frac{W_1}{L_1} \right) : \text{ratio of the aspect ratios of the transistors}$$

$\rightarrow M_1$  should be wide and  $M_2$  narrow

example:  $V_t = 1V , k_m = 10^3 A/V^2 , k_n = 0.34 \times 10^3 A/V^2 , V_{DD} = 5V$

$$1) V_t = \frac{\sqrt{k_m} \cdot V_T + V_{DD}}{1 + \sqrt{k_m}} \wedge \sqrt{k_m} = 1.715 \rightarrow V_t = 2.493V$$

$$V_{ot} = V_t - V_T = 1.493V$$

$$2) \text{ for } V_{in} = V_{DD} - V_T = 4V , \text{ shemic} \rightarrow V_o [2(3) - V_o] = \frac{k_n}{k_m} (5 - V_o)^2$$

$$\rightarrow \frac{k_m + k_n}{k_m} V_o^2 - \frac{6k_m + 8k_n}{k_m} \cdot V_o + \frac{k_n}{k_m} 16 = 0$$

$$\rightarrow V_o = 0.6989V$$

- Note on previous example: Maximum output voltage of the circuit is  $V_{DD} - V_T$ , hence the high input corresponding to  $V_{OL}$  is  $V_{in} = V_{DD} - V_T$

### \* NMOS NOR gate:

A	B	C	$V_{out}$
0	0	0	1
0	0	1	0
0	1	0	0
1	1	1	0

example:  $\text{g}_m = \text{g}_{m2} = \text{g}_{m3} = \text{g}_m = 2 \times 10^{-3} \text{ A/V}^2$ ,  $V_{T1} = V_{T2} = V_{T3} = 1 \text{ V}$

$$\lambda I_D = 1 \text{ mA}, V_{DD} = 5 \text{ V}$$

$$1) V_{G2} = V_{DS3} + V_{DS2} + V_{GS1}$$

$$\therefore V_{GS1} = 5 - 1 = 4 \text{ V}$$

$$\text{DYNAMIC} \rightarrow I_D = \text{g}_m \cdot V_{GS1} [2(V_{GS1} - V_T) - V_{DS1}] = 1 \text{ mA}$$

$$\therefore 0.5 = 6V_{DS1} - V_{DS1}^2 \rightarrow V_{DS1} = 84.52 \text{ mV}$$

$$\therefore V_D = V_{DD} - V_T \rightarrow V_{GS2} = V_D - V_{GS1} = 3.9155 \text{ V}$$

$$\lambda I_D = \text{g}_m V_{DS2} [2(V_{GS2} - V_T) - V_{DS2}] \rightarrow -V_{DS2}^2 + 5.83 V_{DS2} - 0.5 = 0$$

$$\therefore V_{DS2} = 89.05 \text{ mV}$$

$$\therefore V_c = V_D - V_T \rightarrow V_{GS3} = V_c - V_{GS1} - V_{DS2} = 3.8284 \text{ V}$$

$$\rightarrow -V_{DS3}^2 + 5.65686 V_{DS3} - 0.5 = 0 \rightarrow V_{DS3} = 89.81 \text{ mV}$$

$$\therefore V_G = 89.81 + 89.05 + 84.52 = \boxed{0.2614 \text{ V}}$$

$$2) \therefore V_{GS1} \text{ always saturated: } I_D = \text{g}_{m1} (V_{GS1} - V_T)^2$$

$$V_{GS1} = V_{DD} - V_T - V_{G2} = 3.739 \text{ V} \rightarrow \text{g}_{m1} = 71.55 \text{ mA/V}^2$$

$\alpha_2, \beta_2, \alpha_1, \beta_1$

$1001$

$\alpha_3, \beta_3, \alpha_1, \beta_1$

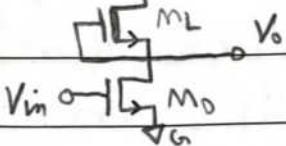
$0011$

$1100$

$V_{GSL} = 0 \rightarrow M_L$  is always on, since gate and source are connected to  $V_{DD}$

$V_{DSL} < V_{GSL} - V_{TL} \rightarrow M_L$  is linear

$V_{DSL} > V_{GSL} - V_{TL} \rightarrow M_L$  is saturated



- since the depletion NMOS has a negative threshold voltage, the device is normally on

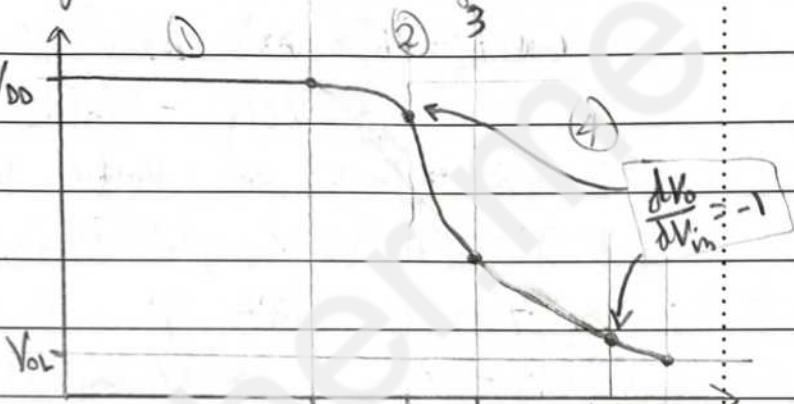
①  $V_{in} < V_{TD}, M_D$  is off,  $I_D = 0 : V_{DH} = V_{DD}$

-  $M_L$  could be sat or linear

assuming sat:

$$I_D = \frac{g_m}{2} (V_{in} - V_T)^2 \neq 0$$

hence not sat  $\rightarrow$  linear



$$\therefore I_D = \frac{g_m}{2} V_{DSL} [2(V_{DSL} - V_T) - V_{DSL}] = 0 \quad V_{TL} \quad V_{DL} \quad V_{SS} \quad V_{HH} \quad V_{DD}$$

$$\rightarrow V_{DSL} = 0 \quad \text{or} \quad V_{DSL} = -2V_{TL}, \text{ which is larger than } V_{DSL} - V_{TL}$$

$$\therefore V_O = V_{DD}$$

hence break the condition for sat

②  $V_{in} > V_{TD} \rightarrow M_D$  becomes saturated,  $M_L$  remains linear:

$$\frac{g_m}{2} (V_{DSL} - V_{TD})^2 = \frac{g_m}{2} V_{DSL} [2(V_{DSL} - V_{TL}) - V_{DSL}]$$

$$\therefore \frac{g_m}{2} (V_{in} - V_{TD})^2 = \frac{g_m}{2} (V_{DD} - V_O) [-2V_{TL} - (V_{DD} - V_O)]$$

then find  $V_{DL}$

③ at  $V_{in} = V_{SS}$ ,  $M_D$  will stay in sat, whereas  $M_L$  will enter sat

$$\text{EOS: } V_{DSL} = V_{GSL} - V_{TL} = V_{SS} - V_{TL} = V_{SS}$$

$$\therefore \frac{g_m}{2} (V_{DSL} - V_{TD})^2 = \frac{g_m}{2} (V_{DSL} - V_{TL})^2 \rightarrow V_{GDSL} = 0 \quad V_{DSL} = V_{SS}$$

$$\therefore V_{SS} = V_{TD} \pm \sqrt{\frac{g_m}{2} \cdot V_{TL}}$$

$$\rightarrow V_{SS}^2 = \frac{g_m}{2} \cdot V_{TL}^2$$

	①	②	③	④
$M_L$	linear	linear	sat	sat
$M_D$	off	linear	sat	linear

④  $V_{in} > V_{ss}$ ,  $M_D$  is linear,  $M_L$  remains saturated

$$\text{for } V_{os1} [2(V_{GSL} - V_T) - V_{os1}] = \text{for } (V_{GSL} - V_T)^2$$

$$\rightarrow \text{for } [2(V_i - V_T) - V_o] = \text{for } (0 - V_T)^2$$

then find  $V_{ZH}$ , or  $V_{OL}$  if  $V_i = V_{DD}$

-  $V_{o1}$  is the output voltage corresponding to an input equal to  $V_{DD}$

- Larger  $\frac{\text{for}}{\text{for}}$  makes the curve steeper and closer to ideal

example:  $V_{T1} = 1V$ ,  $V_{TL} = -1V$ ,  $\text{for} = 1 \times 10^3 \text{ A/V}^2$

$$\text{for} = 0.2 \times 10^{-3} \text{ A/V}^2, V_{DD} = 5V. \text{ find } V_{ss}, V_{os}, \text{ and } V_{o1}$$

$$\therefore V_{ss} = V_{T1} \pm \sqrt{\frac{\text{for}}{\text{for}}} \cdot V_{TL} \rightarrow V_{ss} = 0.553V \text{ or } 1.449V$$

$0.553 \rightarrow V_{GSL} = V_{in} - V_{T1} \rightarrow$  negative  $\rightarrow M_L$  cutoff

$$\therefore V_{ss} = 1.4492V$$

$$\therefore V_{os} = \pm V_{TL} \sqrt{\frac{\text{for}}{\text{for}}} \rightarrow V_{os} = 0.449V$$

$$\text{for } V_{o1} \quad \text{for } V_o [2(V_{DD} - V_{T1}) - V_o] = \text{for } (-V_{TL})$$

$$\therefore 2V_o \cdot (4) - 16^2 = 0.2 \rightarrow V_o = 25 \text{ mV}$$

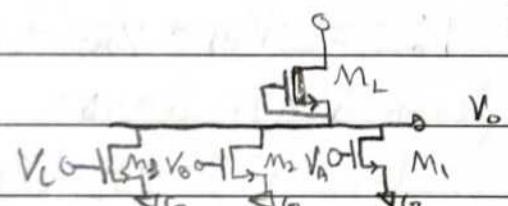
\* N-MOS NOR gate:

A B C  $V_o$

0 0 0 1  $V_{DD}$

0 0 1 0  $V_{os1}$

1 1 1 0  $V_{os2}$



load is sat and others active  $\rightarrow I_1 = \text{for}_L (V_{GSL} - V_{TL})^2$

$\lambda I_L = m I_D$ ,  $m$ : number of transistors, for input

$M_L \rightarrow$  are linear:  $I_D = \text{for}_D [2(V_{GSD} - V_{TD}) - V_{osD}]$

$$\therefore m I_D = m \text{for}_D \cdot V_{osD} [2(V_{GSD} - V_{TD}) - V_{osD}] = \text{for}_D (V_{osD} - V_{TD})^2$$

- worst case is when  $m=1$   $\lambda V_{o2} < V_{T1}$

example on NMOS NOR

$$\textcircled{1} \quad A=B=C=5V \rightarrow m=3$$

$$\rightarrow 3 \cdot 10^{-3} \cdot V_o [2(V_{DD} - V_T) - V_o] = 0.34 \times 10^{-3} (-V_{T1})^2$$

$$\rightarrow 8V_o - V_o^2 = \frac{19}{150} \rightarrow V_o = 14.19 \text{ mV}$$

$$\textcircled{2} \quad \text{if } A=5 \wedge B=C=0V \rightarrow m=1$$

$$\therefore 8V_o - V_o^2 = 0.34 \rightarrow V_o = 42.93 \text{ mV}$$

\* NMOS NAND gate:

- very low input gives  $I_L=0 \wedge V_o=V_{DD}$

- if all inputs are high ( $A=B=C=V_{DD}$ ), then all MOSFETs are in linear mode

$$\rightarrow V_{GSI} = A = V_{DD}$$

$$V_{GS2} = B - V_{DS1} = V_{DD} - V_{DS1}$$

$$V_{GS3} = V_{DD} - V_{DS1} - V_{DS2}$$

$$V_o = V_{DS1} + V_{DS2} + V_{DS3}$$

example on NMOS NAND:

$$\textcircled{1} \quad V_o = V_{DS1} + V_{DS2} + V_{DS3} \quad \wedge \quad I_D = 1 \text{ mA} = J_{DS1}(-V_{T1})$$

$$= 2 \mu_A \cdot V_{DS1} \cdot [2(V_{GS2} - V_{T1})^2 - V_{DS1}]$$

$$\therefore -V_{DS1}^2 + 2(V_{GS2} - 1) \cdot V_{DS1} - 0.5 = 0$$

$$\rightarrow V_{DS1} = 62.99 \text{ mV}, V_{DS2} = 64.02 \text{ mV}, V_{DS3} = 65.09 \text{ mV}$$

$$\therefore V_o = 0.1921 \text{ V}$$

$$\textcircled{2} \quad \text{so } 1 \text{ mA} = J_{DS1}$$

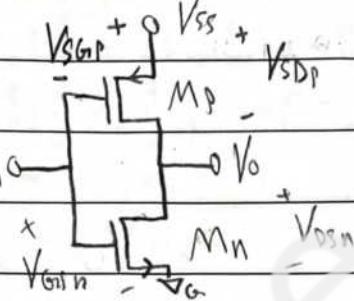
a CMOS : complementary metal oxide semiconductors

& CMOS inverter: consists of NMOS and PMOS

①  $0 < V_{in} < V_{TN}$ ,  $M_n$  off,  $M_p$  ohmic:

$$\Rightarrow V_{SDP} [2(V_{SGP} - |V_{TP}|) - V_{SDP}] = 0 \Rightarrow V_{SDP} = 0$$

$$\Rightarrow V_o = V_{DD}$$



②  $V_{TN} < V_{in} < V_{SS}$ ,  $M_n$  sat,  $M_p$  ohmic:

$$\Rightarrow f_{DN}(V_{GSN} - V_{TN})^2 = f_{DP}(V_{SDP} [2(V_{SGP} - |V_{TP}|) - V_{SDP}])$$

$$\therefore f_{DN}(V_{in} - V_{TN})^2 = f_{DP}(V_{DD} - V_i) [2(V_{DD} - V_{in} - |V_{TP}|) - (V_{DD} - V_o)]$$

③  $V_{in} = V_{SS}$ ,  $M_n$  sat,  $M_p$  off:

$$\Rightarrow f_{DN}(V_{GSN} - V_{TN})^2 = f_{DP}(V_{SDP} - |V_{TP}|^2)$$

$$\therefore f_{DN}(V_i - V_{TN})^2 = f_{DP}(V_{DD} - V_i - |V_{TP}|)^2$$

④  $V_{SS} < V_{in} < V_{DD} - V_{TN}$ ,  $M_n$  ohmic and  $M_p$  sat:

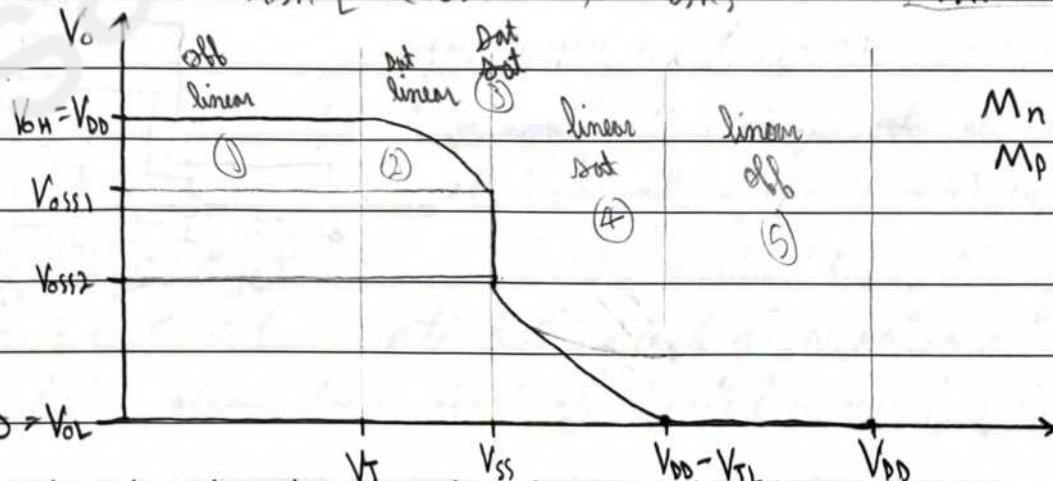
$$\Rightarrow f_{DN} V_{GSN} [2(V_{GSN} - V_{TN}) - V_{GSN}] = 0$$

$$\Rightarrow f_{DP} (V_{GSN} - V_{TN})^2$$

$$\therefore f_{DN} V_o [2(V_i - V_{TN}) - V_o] = f_{DP} (V_{DD} - V_i - |V_{TP}|)^2$$

⑤  $V_{DD} - V_T < V_{in} < V_{DD}$ ,  $M_n$  is ohmic,  $M_p$  is off:

$$\Rightarrow f_{DN} V_{GSN} [2(V_{GSN} - V_{TN}) - V_{GSN}] = 0 \Rightarrow V_{GSN} = 0V = V_o$$



### \* advantages of CMOS logic:

- low power dissipation
  - better packaging design
  - high noise immunity
  - if  $I_{Dn} = I_{Dp}$ ,  $I_{Dn}(V_{GSn} - V_{Tn})^2 = I_{Dp}(V_{GSp} - |V_{Tp}|)^2$
- $$\Rightarrow V_{SS} = \frac{V_{Tn} + \sqrt{\frac{I_{Dp}}{I_{Dn}} [V_{DD} - |V_{Tp}|]}}{1 + \sqrt{\frac{I_{Dp}}{I_{Dn}}}}$$
- if  $I_{Dn} = I_{Dp}$  &  $V_{Tn} = |V_{Tp}|$
- $$\Rightarrow V_{SS} = \frac{V_{DD}}{2}$$

- at  $V_{SS}$ ,  $M_n$  switches from sat to linear:

$$EOS \rightarrow V_{GSn} = V_{GSn} - V_{Tn} = V_{SS} - V_{Tn} = V_{oss_1}$$

- at  $V_{SS}$ ,  $M_p$  switches from linear to sat:

$$\Rightarrow V_{SDp} = V_{GSp} - |V_{Tp}| = V_{DD} - V_{SS} - |V_{Tp}|$$

$$\lambda V_{SDp} = V_{DD} - V_0 \rightarrow V_{DD} - V_{SS} - |V_{Tp}| = V_{DD} - V_0$$

$$\therefore V_0 = V_{SS} + |V_{Tp}| = V_{oss_2}$$

example on inverter:

$$\textcircled{1} \quad \text{if } I_{Dn} = I_{Dp} \quad \lambda V_{Tn} = |V_{Tp}| \rightarrow V_{SS} = \frac{V_{DD}}{2} = 2.5V$$

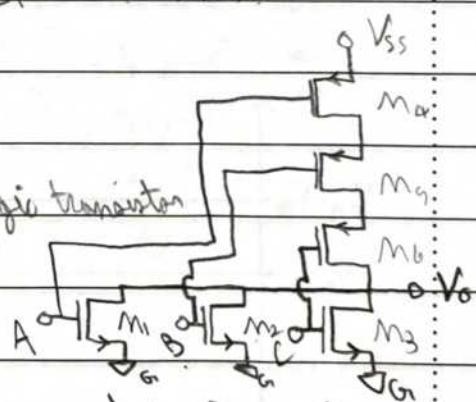
$$\textcircled{2} \quad \text{if } V_{in} = V_{SS} \rightarrow \text{both sat}$$

$$I_p = I_{Dp} (V_{GSp} - |V_{Tp}|)^2 \quad \lambda V_{SDp} = V_{DD} - V_{in}$$

$$\rightarrow I_p = I_{Dp} (2.5 - 1)^2 \rightarrow I_p = 4.5 \text{ mA}$$

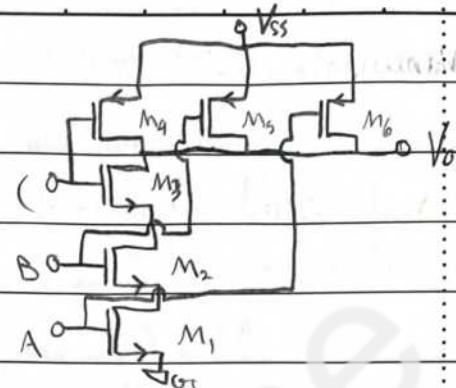
### \* NOR CMOS:

- two transistors are required for every input
- every input transistor requires a complementary logic transistor
- if one input (at least) is high, then switch the logic transistor (at least) will be off
- if a transistor is in linear but its current is zero then the voltage difference between its drain and source will be zero



## &amp; NAND CMOS:

- if one of the input transistors has low voltage, it will be cutoff, causing the other transistors' currents to equal zero hence the output voltage will be high.



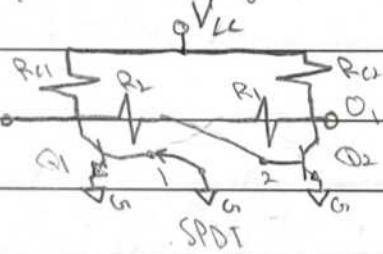
\* Multivibrators: sequential logic circuits that operate continuously between two distinct states of high and low

### + Types of multivibrators:

- Astable: free-running multivibrator that has no stable states but switches continuously between two states, producing a train of square-wave pulses at a fixed frequency
- Monostable: a one-shot multivibrator that has only one stable state and is triggered externally and returns back to its first stable state
- Bistable: a flip-flop that has two stable states, which produce a single pulse (either positive or negative)

### \* Bistable multivibrator:

- When SPDT is at position 1),  $Q_1$  will be off and output  $Q_1$  will be high.
- $Q_2$  will turn on and  $Q_2$  will be low



- This state will remain unless the switch is flipped (externally)

$$V_{ce} = V_{BE2\text{sat}}$$

$$\rightarrow I_{B1} = I_{B2} = \frac{V_{ce} - V_{BE1\text{sat}}}{R_{B1} + R_2}$$

$$\therefore V_{o1} = V_{ce} - I_{B1} \cdot R_{B1} \quad \text{and} \quad V_{o2} = V_{BE2\text{sat}} = 0.1V$$

- If the switch connects to position 2,  $Q_2$  will cutoff.  $Q_2$  will be high, and soon.

- Linear and Reverse active modes are ignored in multivibrators

Given  $C = 50 \text{ fF} = 50 \times 10^{-15} \text{ F}$ ,  $V_{DD} = 0.6V$ ,  $V_{DH} = 5V$

$$V_T = 1 \quad \lambda \quad k_n = 100 \mu\text{A/V}^2$$

$$R_D = 10k$$

- If the input is low, output should switch to high

$$\therefore i = C \frac{dV_o}{dt} = \frac{V_{DD} - V_o}{R_D}$$



- If the input switches from low to high, output should switch back to low :  $I_D = I_{D0} + I_{L0}$ ,  $I_{L0} = -\frac{dV_o}{dt} \cdot C$

$$\text{If } V_o > 4: M \rightarrow \text{sat} \rightarrow I_D = k_n [V_{GS} - V_{Tl}]^2 = \frac{V_{DD} - V_o}{R_D} - C \frac{dV_o}{dt}$$

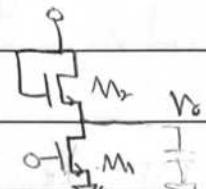
$$\text{If } V_o < 4: M \rightarrow \text{ohmic} \rightarrow I_D = k_n \cdot R_D [8V_o - V_o^2] = \frac{V_{DD} - V_o}{R_D} - C \frac{dV_o}{dt}$$

8 NMOS with saturated ENMOS as load:

input going from high to low:

$$\therefore M_2 \text{ sat} \rightarrow I_{D2} = k_n (V_{DD} - V_o - V_T)^2$$

$$\lambda \quad i_L = C \frac{dV_o}{dt} = k_n (2 - V_o)^2$$

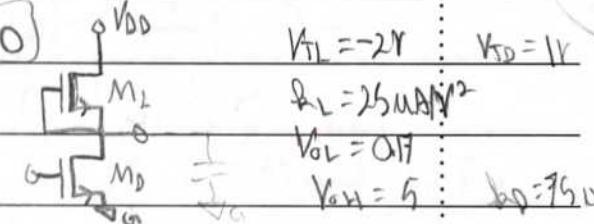


8 NMOS inverter with DMOS as load :  $V_{GS0}=0$

for  $V_o < 3$ :  $V_{DS1} > V_{GS1} - V_{Tl} \rightarrow \text{sat}$

$$\therefore I_D = C \frac{dV_o}{dt} = k_n (V_{GS1} - V_{Tl})^2 \\ = k_n (-V_{Tl})^2$$

$$\therefore V_o(t) = V_{oL} + \frac{k_n}{2} |V_{Tl}|^2 t$$



} for high to low input

for  $V_o > 3$ :  $V_{DS1} < -V_{Tl} \rightarrow \text{ohmic}$

$$I_D = C \frac{dV_o}{dt} = k_n R_D (2(V_{GS1} - V_{Tl}) \cdot V_{oL} - V_{DS1}^2)$$

$$\rightarrow C \frac{dV_o}{dt} = k_n \cdot R_D (4(V_{DD} - V_o) - (V_{DD} - V_o)^2)$$

- NMOS with DMOS load as input goes from low to high

If  $V_o > 4$ :  $M_1$  ohmic,  $M_2$  sat  $\therefore V_{DS0} > V_{GS0} - V_{Tl}$

$$\rightarrow I_D = k_n \cdot (V_{GS0} - V_{Tl})^2 = 1.2 \text{ mA}$$

$$\text{If } V_o > 3 \rightarrow M_1: I_D = k_n (4(V_{DD} - V_o) - (V_{DD} - V_o)^2)$$

\* for  $3 < V_o < 4$ ,  $M_D \wedge M_L$  shunt

$$M_D: I_D = I_{D0} (8V_o - V_o^2) = I_{D0} (4(V_{DD} - V_o) - (V_{DD} - V_o)^2)$$

\* for  $V_o < 3$ ,  $M_L$  is sat &  $M_D$  is linear

- hence, an ENMOS driver connected to a DMNOS load will pass through three stages ( $M_L$  linear &  $M_D$  sat, both linear, and  $M_L$  sat &  $M_D$  linear) while switching from low input to a high input (or high output to low)

\* CMOS inverter:

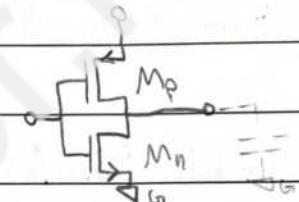
- high to low input:

$$\text{for } V_o < |V_{TP}|: I_D = I_{D0} (V_{DD} - V_i - |V_{TP}|)^2$$

$$= I_{D0} (V_{DD} - V_{TP})^2 = C \frac{dV_o}{dt}$$

$$\therefore V_{SS} - V_o > V_{SS} - V_i - |V_{TP}| \quad [V_{DD} > V_{SS} - V_{TP}] \text{ for pass}$$

- if  $V_o > |V_{TP}| \rightarrow M_D$  goes to shunt mode



$$\rightarrow C \cdot \frac{dV_o}{dt} = I_{D0} (2(V_{DD} - |V_{TP}|)(V_{DD} - V_o) - V_{DD} - V_o^2)$$

- for low to high input:

- initially  $M_N$  will be sat,  $M_D$  will be cutoff

$$\rightarrow C \frac{dV_o}{dt} = -I_{D0} (V_{DD} - V_{TN})^2$$

- next the driver will enter linear, whereas  $M_D$  will remain off.

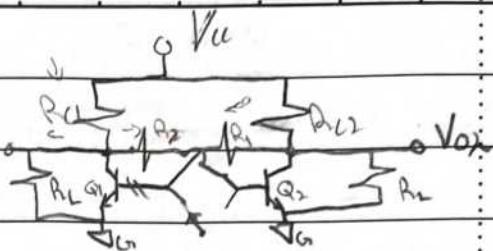
$$C \frac{dV_o}{dt} = -I_{D0} (2(V_{DD} - V_{TN})V_o - V_o^2)$$

\* Loaded bistable MV:

-  $Q_1$  is off since its base is grounded

-  $Q_2$  is saturated

$$I_{L2} = \frac{V_{CE2\text{ sat}}}{R_L} = \frac{0.1}{R_L}$$



$$I_{L1} = I_{R_{Q1}} - I_{B2} \rightarrow \frac{V_{bb} - V_{BE2\text{ sat}}}{R_L} = \frac{V_{cc} - V_{bb} - V_{bb}}{R_L} = \frac{V_{cc} - V_{bb}}{R_L}$$

- For  $Q_2$  to be saturated,  $I_{L2} \geq B_2 \cdot I_{B2}$

$$I_{L2} = \frac{V_{cc} - V_{bb}}{R_{L2}} - \frac{V_{bb}}{R_L}$$

$$\therefore B_2 \left[ \frac{V_{cc} - V_{bb}}{R_L} - \frac{V_{bb}}{R_L} \right] \leq \left[ \frac{V_{cc} - 0.1}{R_L} - \frac{0.1}{R_L} \right]$$

\* RS flip-flop using bistable:

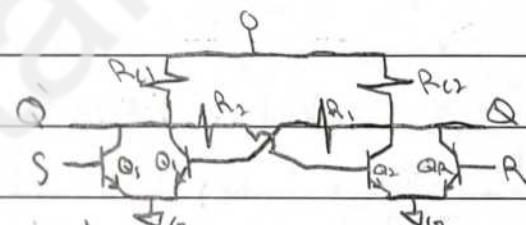
- If the S input is low

while the R input is high

$Q_S$  will be off,  $Q_R$  will be on  $\rightarrow$  output  $Q$  will be low

$\because Q$  output is low and connected to the base of  $Q_1$ , then

the output  $Q'$  will be high since  $Q_1$  will be off



R	S	Q
0	0	$Q(t-1)$
0	1	Set
1	0	Reset
1	1	Indeterminate

holds previous

$Q_S$  off  
 $Q_R$  off

set

$Q_S$  on  
 $Q_R$  off

Reset

$Q_S$  off  
 $Q_R$  on

indeterminate

## &amp; monostable multivibrator:

- before triggering,  $TR_2$  will be saturated, giving  $TR_1$  off.

The voltage difference across

$$LT = 5 - 0.8$$

$$V_{ic} - V_{BE(\text{sat})2}, V_{out} = 0.1V$$

- the voltage on both terminals of the capacitor must drop by the same amount, hence  $LT$  after the trigger :  $0.1 \xrightarrow{LT} 0.8 - (V_u - 0.1)$

Since  $TR_1$  is saturated as  $V_{BE1} = 0.8V \rightarrow TR_2$  goes off  
 $\therefore$  during trigger:  $V_{out} = V_u - I_{D2} \cdot R_2 \quad \text{and} \quad TR_2 = \frac{V_u - V_{BE(\text{sat})1}}{R_2 + R_3}$   
 $\Rightarrow \frac{5 - 0.8}{20k} = 0.2 \text{ mA} \Rightarrow V_{out} = 2.6V$

otherwise,  $V_{out} = V_{BE2(\text{sat})} = 0.1V$

$$(T \text{ charging}) \quad V_{B2}(t) = V_{B2(0)} - [V_{B2(0)} - V_{B2(0)}] e^{-t/T}$$

$$V_{B2(0)} = V_u + V_{B2(0)} = 0.8 - (V_u - 0.1)$$

$$\therefore V_{B2(0)} = V_u - [2V_u - 0.1] e^{-t/T} \\ = 5 - 9.1 e^{-t/T} \quad T = R_2 \cdot LT$$

- to find the pulse duration, set  $V_{B2(t)} = V_{B2(T)}$  equal to the stable value of  $V_{BE(\text{sat})} = 0.8$

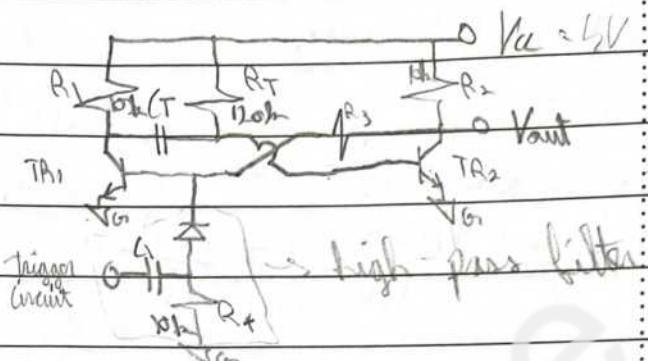
$$\therefore V_{B2(T)} = 0.8 = 5 - 9.1 e^{-T/T} = V_u - [2V_u - 0.1] e^{-T/T}$$

$$\Rightarrow e^{-T/T} = \frac{V_u - 0.8}{2V_u - 0.1} \approx \frac{1}{2} \quad \text{assuming } V_u \gg 1$$

$$\Rightarrow T = T \cdot \ln(2) = R_2 \cdot LT \cdot \ln(2)$$

- monostable circuits are called "one-shot" as they give a high output pulse which returns to the stable steady-state low output

- the duration between pulses from the triggering circuit should be  $5 \cdot R_1 \cdot LT$



NMOS :

$$V_{GS} < V_{IN}$$

off

$$V_{GS} > V_{IN}$$

$$V_{DS} \geq V_{GS} - V_{IN}$$

$$V_{DS} \leq V_{GS} - V_{IN}$$

$$\text{Sat: } I_D = k_m (V_{GS} - V_{IN})^2 \quad \text{Zener: } I_D = k_m [2(V_{GS} - V_{IN})]$$

example :

$$a) V_{IN} = V_{GS} = 3V \rightarrow V_o = 0.1V$$

$$V_o = V_{DS} \text{ if } V_o = 0.1 \rightarrow V_{DS} \leq V_{GS} - V_{IN} \quad V_{IN} \leftarrow V_{GS} \quad V_{DS}$$

$$\rightarrow \text{Zener: } \therefore I_D = k_m [2(V_{GS} - V_{IN})] V_{DS} - V_{DS}^2$$

$$I_D = \frac{3 - 0.1}{R_D}$$



$$\rightarrow \frac{2.9}{R_D} = 150 \mu [2(2.5)0.1 - (0.1)^2]$$

$$\rightarrow R_D = 39.4 \Omega, \text{ Zener}$$

b) substituting in equation

$$V_S = V_{IN} - \frac{1}{2R_D k_m} \pm \sqrt{\left(\frac{1}{2R_D k_m}\right)^2 + \frac{V_{DS}}{R_D k_m}}$$

$$\rightarrow V_S = 0.5 - 0.084495 \pm 0.9190148$$

$$\rightarrow V_S = 1.13252$$

Example :  $V_t \rightarrow$  transition point from Sat to linear $N_1$  Sat  $\rightarrow$  load  $N_2$  linear

$$1) k_m (V_{GS1} - V_{IN1})^2 = k_m [2(V_{GS2} - V_{IN2})] V_{DS2} - V_{DS2}^2$$

$$k_m = \frac{1}{0.34} \quad \& \quad V_{GS1} = V_{IN} = V_t \quad \& \quad V_{T1} = V_2 = V_T$$

$$\rightarrow \frac{1}{0.34} (V_t - 1)^2 = 2(V_{DD} - V_o - 1)(V_{DD} - V_o) - (V_{DD} - V_o)^2$$

$$V_o = V_{ot} = V_t - V_{IN} \rightarrow \frac{1}{0.34} (V_t - 1)^2 = 2(8 - V_t)(6 - V_t) - (6 - V_t)^2$$

$$\rightarrow \frac{1}{0.34} [V_t^2 - 2V_t + 1] = 2[30 - 11V_t + V_t^2] - (36 - 12V_t + V_t^2)$$

$$\rightarrow \frac{1 - 0.34}{0.34} V_t^2 - \frac{2 + 3.4}{0.34} V_t - 21.0588 = 0$$

$$\rightarrow V_t = 2.39965 \text{ or } -2.12 \quad \rightarrow V_{ot} = 1.39965$$

1) load is always sat:

$$V_{OL} = V_T - V_T$$

$$I_{m1} (V_T - V_T)^2 = I_{m2} (V_{DD} - V_0 - V_T)^2$$

$$\text{Simplifying} \rightarrow I_{m2} = I_{m1} \rightarrow V_T^2 - 2V_T + 1 = 0.34 (5^2 - 10V_T + V_T^2)$$

$$\rightarrow V_T = 2.49330 \quad \& \quad V_{OL} = 1.49330$$

2)  $V_{in} = 5 - 1 = 4V$

$M_1$  is ohmic

$$V_{GS1} = 4$$

$$\rightarrow I_{m2} [V_{DD} - V_0 - 1]^2 = I_{m1} [2(V_{GS1} - V_T) V_{GS1} - V_{DS1}^2]$$

$$\rightarrow 0.34 [4 - V_0]^2 = 6V_0 - V_0^2 \quad V_0 = V_{DS1}$$

$$\rightarrow 0.34 [16 - 8V_0 + V_0^2] = 6V_0 - V_0^2 \rightarrow V_0 = 0.69892$$

example:

$$1) I_{m2} (V_{GS1} - V_T)^2 = m I_{m1} V_{GS1} [2(V_{GS1} - V_T) - V_{DS1}]$$

$$\rightarrow 0.34 (5 - V_0 - 1)^2 = 3 \cdot [2V_0 (3) - V_0^2]$$

$$\rightarrow 0.34 (4 - V_0)^2 = 18V_0 - 3V_0^2$$

$$\rightarrow V_{OL} = 0.294913$$

$$2) 0.34 [16 - 8V_0 + V_0^2] = 6V_0 - V_0^2$$

$$\rightarrow V_{OL} = 0.6989$$

example:  $V_{in} \rightarrow$  all inputs high,  $I_D = 1mA$ , all ohmic

$$\rightarrow M_1: \text{input high} = \text{high output} = V_{DD} - V_{TL}$$

$$1mA = 2 \times 10^{-3} [2(V_{DD} - V_{TL} - V_T) V_{GS1} - V_{DS1}^2]$$

$$\rightarrow 0.5 = 2(3)V_{GS1} - V_{DS1}^2$$

$$\rightarrow V_{GS1} = 0.084524$$

$$M_2: 1mA = 2 \times 10^{-3} [2(V_{DD} - V_{TL} - V_{GS1} - V_T) V_{GS2} - V_{DS2}^2]$$

$$\rightarrow 0.5 = 5.830952 V_{GS2} - V_{DS2}^2$$

$$\rightarrow V_{GS2} = 0.08704882$$

$$M_3: V_{GS3} = V_{DD} - V_{TL} - V_{DS1} - V_{DS2} = 3.87842918$$

$$\therefore 0.5 = 6.6568544 V_{DS3} - V_{DS3}^2$$

$$\rightarrow V_{DS3} = 0.0818143$$

$$\rightarrow V_{OL} = V_{DS1} + V_{DS2} + V_{DS3} = 0.261389V$$

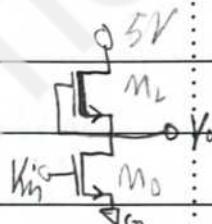
$$\therefore I_D = \frac{1}{2} [V_{GS1} - V_{TL}]^2$$

$$\rightarrow \frac{1m}{I_D} = \frac{1}{(4.738613 - 1)^2}$$

$$\rightarrow \frac{1m}{I_D} = (4.738613 - 1)^2 \rightarrow k_D = 91.543 \text{ mA/V}^2$$

example:

$$V_{SS} = V_{TL} \pm \sqrt{\frac{k_D}{m_1}} V_{TL}$$



$$= 1 \pm \sqrt{0.2 \cdot (-1)} = 0.952786 \text{ or } 1.447214V$$

$$\rightarrow V_{SS} = 1.447214V$$

$$\wedge V_{DS}^2 = \frac{k_D}{m_1} \cdot V_T^2 \Rightarrow V_{DS} = \pm \sqrt{\frac{k_D}{m_1}} \cdot V_T$$

$$\rightarrow V_{DS} = \pm \sqrt{0.2} = 0.4472V$$

to bind  $V_{OL}$ :  $M_1$  is sat  $V_{GS1} > V_T \wedge V_{DS1} > V_{DS1}$

$$\rightarrow I_{OL} = \frac{1}{2} [V_{GS1} - V_T]^2 = \frac{1}{2} [1]^2$$

$$\wedge I_{DS} = I_{OL} = \frac{1}{2} [2(V_{in} - V_{TN})V_o - V_o^2]$$

$$\rightarrow 0.2 = 8V_o - V_o^2 \rightarrow V_{OL} = 0.025V$$

example:

$$\frac{1}{2} (V_{GS1} - V_{TL})^2 = m \frac{1}{2} m [2(V_{GS1} - V_{TN})V_o - V_o^2]$$

$$\rightarrow 0.34 = m [8V_o - V_o^2]$$

$$(a) m=3 \rightarrow V_{OL} = 0.014192V$$

$$(b) m=1 \rightarrow V_{OL} = 0.04293V$$

example:  $I_D = 1 \text{ mA}$

$$\text{c) } M_1: 0.5 = 8V_{DS1} - V_{DS1}^2 \rightarrow V_{DS1} = 0.06716 \text{ V}$$

$$M_2: 0.5 = 2(4 - V_{DS1})V_{DS2} - V_{DS2}^2 = 0.0640 \text{ V}$$

$$M_3: 0.5 = 2(4 - V_{DS1} - V_{DS2})V_{DS3} - V_{DS3}^2 = 0.06509 \text{ V}$$

$$\rightarrow V_{DS3} = 0.1911 \text{ V}$$

$$\text{d) } I_D = 1 \text{ mA} = k_m(V_{DS1} - (-1))^2 \rightarrow k_m = 1 \text{ mA/V}^2$$

example

$$\text{a) } k_m = k_p \rightarrow V_m = |V_{TP}| \rightarrow V_{SS} = \frac{|V_{DD}|}{2}$$

$$\rightarrow V_{SS} = 2.5 \text{ V}$$

$$\text{b) both sat} \rightarrow I_D = k_m (V_{SD} - |V_{TP}|)^2$$

$$= k_m (V_{DS1} - V_{TP})^2$$

$$= 2 \times 10^{-3} \cdot (2.5 - 1)^2 = 4.5 \text{ mA}$$

past

mid exam practice:

$$1) \quad k_m = \frac{k_m 1}{k_m 2}$$

$$I_D = k_m (-(-1))^2 =$$

$$V_{SS} = V_{TL} \pm \sqrt{\frac{k_m 1}{k_m 2}} \cdot V_{RL}$$

$$\rightarrow 1.6 = 1 \pm 1.2 \cdot \sqrt{\frac{1}{4}} \rightarrow 0.5 = \frac{1}{2}$$

$$\rightarrow k_m 2 = 4$$

2)  $V_{SS} \rightarrow$  both sat

$$\rightarrow I_{DD} = I_{DL} \rightarrow k_m 1 \cdot (V_m - V_T)^2 = k_m 1 (-V_{TL})^2$$

$$\rightarrow k_m 2 = \frac{(1.2)^2}{(1.6 - 1)^2} \rightarrow k_m 2 = 4$$

$$3) \quad NM = V_{TL} - V_{RL}, \quad V_{RL} =$$

$$b) \quad V_{RL} = 0.2 \quad V_{TL} = 1.2 \rightarrow NM = 1$$

$$7) NM_{II} = V_{OH} - V_{IH} \quad V_{OH} = 160$$

$$V_{IH} = V_{TN} \rightarrow NM_{II} = 4$$

$$V_{IL} = V_{TN} + \frac{1}{2R_D} I_{DN}$$

$$\lim_{R_D \rightarrow \infty} = V_{TN}$$

$$8) \text{ if } V_I = 4.2V \rightarrow \text{high}$$

$$V_{SG} = 0.8V(5-4.2)$$

for on  $V_{SG} > |V_{TP}|$   $\times \rightarrow$  load is off

$$\rightarrow V_{DL} = 0 \rightarrow V_{DSL} = -5$$

$$9) P_{MOS} = \frac{P_{DH} + P_{SL}}{2} \quad P_{DL} = 0$$

$$P_{DH} = \frac{(5-0.5)^2}{1k\Omega} = 20.25 \text{ mW}$$

$$\frac{1}{2}[V_I] = \frac{5}{2} \cdot \frac{4.2}{1k\Omega} = 11.25 \text{ mW}$$

$$10) \text{ if } V_{IN} = 0.3 \rightarrow M_1 \text{ off}$$

$$V_o = 5 \rightarrow V_{DS1} = V_{DD} - V_o = 0$$

$$11) \text{ if } V_{IN} = 1.2 \rightarrow V_{DS1} - V_{TN} = 0.2$$

$$V_o = 0.2$$

$$M_1: h - V_o = 1.2$$

$$h \quad V_o > 0.2 \rightarrow$$

linear sat

$$\text{if } V_o < 0.2 \rightarrow h - V_o > 1.2$$

$$V_o = 4.6 \rightarrow V_{DS1} = 0.4V < V_{DS1} - V_{IL}$$

$\rightarrow$  Dsat is linear

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$$V_{DS1} = 4.6 \rightarrow 4.6 > V_{DS1}^{\text{on}} - V_{TN} \rightarrow \text{Dsat}$$

$$B) I_{DA} \cdot (V_{in} - 0.8)^2 = (V_{DD} - V_0 - 1.2)^2$$

$$\rightarrow I = 5 - V_{DD} - 1.2$$

$$V_{DD} = 2.8V$$

$$A) V_{GSN} = 1.5 \rightarrow V_{GSN} ? 0.5$$

$$V_{GDP} = 3.9$$

$$V_{DDP} > V_{GDP} - V_{TP} \text{ for sat}$$

$$V_{DDP} = V_{SS} - V_0$$

$$\text{oo } I_{DA} + I_{DDP} \rightarrow V_{SS} = \frac{V_{TN} + \frac{I_{DDP}}{I_{DA}} [V_{DD} - V_{TP}]}{1 + \frac{I_{DDP}}{I_{DA}} \frac{1}{V_{TN}}} \rightarrow V_{SS} = 1 + 0.5 [V_{SS} - 1]$$

$$\rightarrow V_{SS} = \frac{2}{3} + \frac{1}{3} V_{SS} - \frac{1}{3}$$

$$\rightarrow \frac{2}{3} V_{SS} = \frac{1}{3} \rightarrow V_{SS} = 0.5V$$

i.e.  $M_P$  saturation

$$V_{SS} = \frac{V_{TN} + \frac{I_{DDP}}{I_{DA}} [V_{SS} - (V_{TP})]}{1 + \frac{I_{DDP}}{I_{DA}} \frac{1}{V_{TN}}}$$

$$\rightarrow V_{SS} = \frac{1 + 0.5 [V_{SS} - 1]}{1 + 0.5}$$

$$\rightarrow 1.5 V_{SS} - 0.5 V_{SS} = 0.5 \rightarrow V_{SS} = 0.8$$

$V_{SS}$  saturation  $\neq V_{SS}$  source

$$\rightarrow 1.5 V_{SS,source} - 0.5 V_{SS} = 0.5$$

$$\therefore V_{SS} = \frac{0.5 + 2.5}{1.5} = 2V$$

oo  $V_{TN} < V_{in} < V_{SS} \rightarrow M_N$  sat &  $M_P$  ohmni

$$I_D = 200 \mu A [1.5 - 1]^2 = 50 \mu A$$

Q3)  $V_{GSL} = V_{DD} - V_o$

$$V_{DSL} = V_{DD} - V_o \rightarrow V_{DSL} = V_{GSL}$$

$\therefore$  Sat  $\rightarrow V_{DSL} \geq V_{GSL} - 1.2$

$$V_o = V_{DD} - V_{GSL} = 5 - 1.2 = V_o$$

$$V_{DSL} = 5 - V_o = 1.2 V$$

Q4)  $I_{DD} = I_{DL} \rightarrow (V_{GSD} - V_{IN})^2 = (V_{GSL} - V_{TL})$

$$V_{GSL} = 5 - V_o \quad \wedge \quad V_{GSD} = V_{IN} = 2.4$$

$$\rightarrow 2.4 - 0.8 = 3.8 - V_o \rightarrow V_o = 1.2 V$$

Q5) more power dissipation in CMOS DC/DCs when both transistors are sat

Q6)  $V_{DSL} = 4.5 > V_{IN} - V_{TN} \rightarrow$  sat M<sub>1</sub>

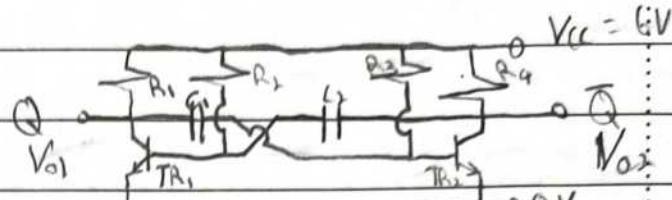
$\therefore M_2: V_{DS2} = 0.5 \geq |-1.2| \rightarrow$  linear

## Astable multivibrator:

1 - if  $TA_1$  is off,  $TA_2$  is on,

$$V_{B1} = V_{CC}, V_{B2} = 0.1 \text{ V}$$

$$V_{CC} - \frac{C_1}{R_1} \approx V_{CC} - \frac{0.8V}{R_1} \approx 0.1$$



$$C_1 = C_2 \quad R_2 = R_1$$

2 -  $C_2$  will start charging through  $R_2$  until  $V_{B1} = 0.8V$ when  $TA_1$  turns on and  $TA_2$  turns off  $\rightarrow V_{B1} = 0.1V \wedge V_{B2} = V_{CC}$ 

$$0.1 \quad | \quad 0.8 - V_{CC} - 0.1$$

$$0.8 \quad | \quad V_{CC}$$

3 -  $C_1$  will then start to charge through  $R_1$  until  $V_{B2} = 0.8V$   
which is when  $TA_2$  becomes on and  $TA_1$  goes off

4 - then 2 repeat and so on.

- given  $V_{B1}(t) = V_{B1}(0) + [V_{B1}(0) - V_{B1}(0)] e^{-t/T_1}$

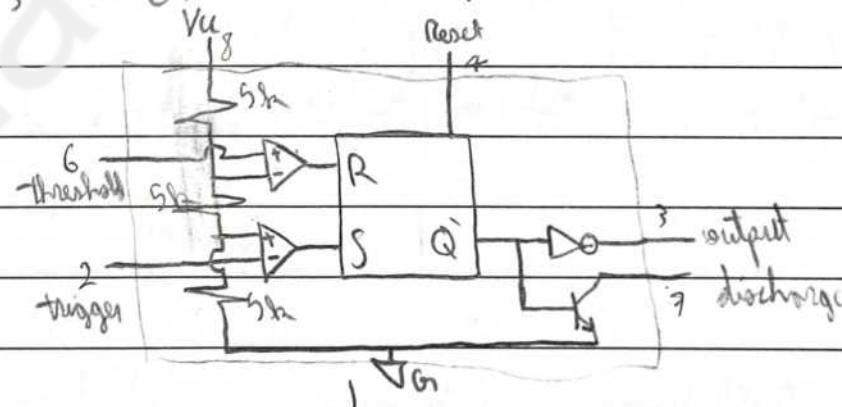
$$\rightarrow \text{time for } V_{B1} \text{ to equal } 0.8V : T_1 = R_2 C_2 \ln(2)$$

$$\text{and } T_2 = R_1 C_1 \ln(2) \text{ time for } V_{B2} \text{ to equal } 0.8V$$

$$\therefore T = R_2 C_2 \ln(2) + R_1 C_1 \ln(2)$$

$$\text{if } R_2 = R_1 \quad \wedge \quad C_1 = C_2 \quad \rightarrow \quad T = 2 R C \ln(2)$$

&amp; 555-timer:



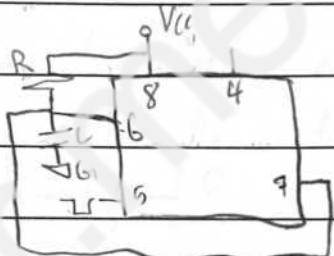
- The open-loop op-amp can be used as a comparator whose output is high if the voltage at its positive terminal is higher than that at its negative terminal else the output will be low (i.e., negative > positive > output high).

\* mono-stable MV using 555:

1 - before trigger:  $S=0, R=0, Q=0, \bar{Q}=1$ , transistor is off sat. This state is stable. (is discharged)

2 - after trigger:  $S=1, R=0, Q=1, \bar{Q}=0$ , transistor is (negative trigger) off, C starts charging

3 - Remove the trigger before  $V_C$  reaches  $\frac{2}{3} V_{cc}$ :  
 $S=0, R=0, Q=1, \bar{Q}=0$ , transistor is off, C will continue charging



4 - When  $V_C$  exceeds  $V_{cc} \cdot \frac{2}{3}$ :

$S=0, R=1, Q=0, \bar{Q}=1$ . Transistor is on sat, C will discharge.

5 - When  $V_C$  goes below  $\frac{1}{3} V_{cc}$ :

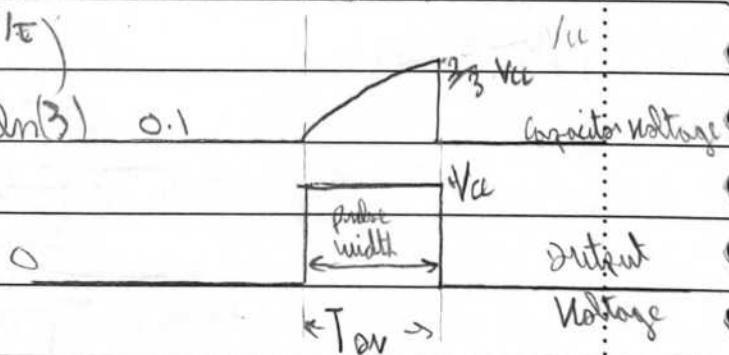
$S=0, R=0, Q=0, \bar{Q}=1$ , transistor is on sat, C will fully discharge. This is the stable state of the circuit

$$\text{so } V_C(t) = V_C(0) - [V_C(0) - V_C(\infty)] e^{-t/T} \quad V_C$$

$$\rightarrow V_C(t) = V_{cc} [1 - e^{-t/T}], T = RL$$

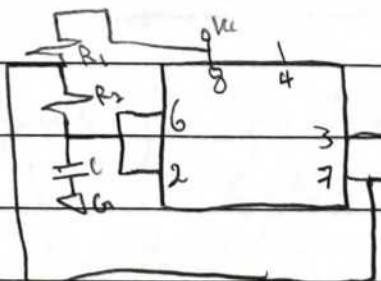
$$\therefore V_C(T) = \frac{2}{3} V_{cc} = V_{cc} (1 - e^{-T/T})$$

$$\rightarrow -\ln(3) = -\frac{T}{T} \rightarrow T = R(C \ln(3)) \approx 0.1$$



\* Astable MV using 555 timer:

1 - assuming the ff is initially Reset, then the transistor will be saturated and the capacitor will discharge to less than  $\frac{1}{3} V_{cc}$ , which will cause the ff to be set.



2- When the ff is set,  $Q=0$ , and the transistor is off hence the capacitor will charge.

at  $V_L = \frac{1}{3} V_U$ , the output of the S comparator will go low.

and at  $V_C = \frac{2}{3} V_U$ , the output of the R comparator will go high and the ff will be reset.

3- If the ff is reset, the transistor will be set and the capacitor will discharge to repeat the process.

$$\text{so } V_{Cn}(t) = V_C(\infty) - [V_C(\infty) - V_C(0)] e^{-t/T_C}$$

$$\rightarrow V_{L(t)} = V_U - V_U \left[ \frac{2}{3} \right] e^{-t/T_C} = 1 - \frac{2}{3} e^{-t/T_C}$$

$$\text{where } T_C = (R_1 + R_2) L$$

$$\lambda V_{C,d} = V_U - \left[ V_U - \frac{2}{3} V_U \right] e^{-t/T_D}$$

$$\rightarrow V_{L,d} = V_U \left[ 1 - \frac{2}{3} e^{-t/T_D} \right] \rightarrow T_D = R_2 L$$

$$T_{on} = (R_1 + R_2) L \cdot \ln(2) \quad \lambda T_{off} = R_2 L \cdot \ln(2)$$

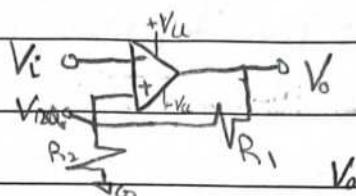
$$T = T_{on} + T_{off} \rightarrow D = \frac{T_{on}}{T} > 60\%$$

\* Multivibrator using Schmitt trigger:

+ Schmitt trigger operation (inverting)

$$V_i > V_{th} \rightarrow V_o = -V_{out}$$

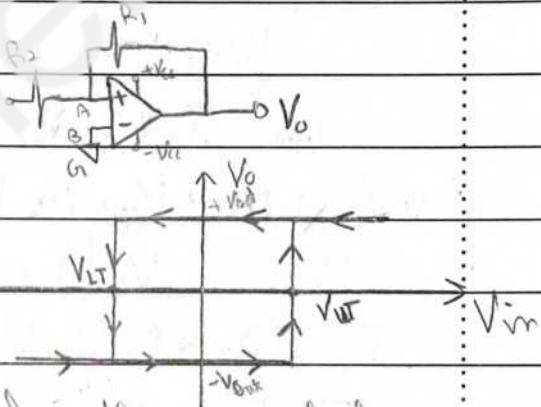
$$V_i < -V_{th} \rightarrow V_o = V_{out}$$



$$V_{th} = \frac{R_2}{R_1+R_2} \cdot V_o = \begin{cases} \frac{R_2}{R_1+R_2} \cdot V_{out} = V_{UT} \\ -\frac{R_2}{R_1+R_2} \cdot V_{out} = V_{LT} \end{cases} \quad \text{Graph: } V_o \text{ vs } V_i$$

- for a low input:  $V_o = V_{out}$

for a high input:  $V_o = -V_{out}$



+ Non-inverting Schmitt trigger:

$$V_A = \frac{R_1}{R_1+R_2} \cdot V_i + \frac{R_2}{R_1+R_2} \cdot V_o$$

$$V_B = 0$$

$$V_o = \pm V_{out}$$

- assuming  $V_o = -V_{out}$  implies that  $V_A$  is less than zero for low input, then increase the input until  $V_A = 0$

$$\rightarrow V_A = \frac{R_1}{R_1+R_2} \cdot V_i + \frac{R_2}{R_1+R_2} (-V_{out}) = 0 \rightarrow V_i = V_{UT} = \frac{R_2}{R_1} V_{out}$$

- hence  $V_{UT}$  is our high input that gives  $V_o = V_{out}$

- next, set  $V_o = V_{out}$  and decrease the input until  $V_A = 0$ .

$$V_i = V_{LT} = -\frac{R_2}{R_1} V_{out}$$

- therefore,  $V_{LT}$  input is the low input that gives  $V_o = -V_{out}$

\* Astable multivibrator:

$$V_{c, th} = V_{out} - [V_{out} - (-V_T)] e^{-t/RC} \quad \text{at } t=0$$

$$\text{at } t=t_1, V_c = V_T = \frac{R_2}{R_1+R_2} \cdot V_{out} \quad \text{at } t=0$$

$$\text{sub } ② \text{ in } ① \rightarrow V_{out} - [V_{out} + \frac{R_2}{R_1+R_2} V_{out}] e^{-t_1/RC}$$

$$= \frac{R_2}{R_1+R_2} \cdot V_{out} \quad \therefore t_1 = RC \ln \left( 1 + \frac{2R_2}{R_1} \right)$$

$$V_{C, \text{dis}} = -V_{\text{out}} - [-V_{\text{out}} - V_t] e^{-t/\tau_{AC}} = -V_t = \frac{-R_2}{R_1 + R_2} V_{\text{out}}$$

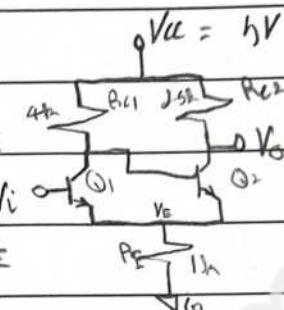
$$\therefore t_r = \tau_{AC} \ln(1 + \frac{2R_2}{R_1})$$

& Schmitt trigger Using BJT:

- for  $V_i = 0$ ,  $Q_1$  is off and  $Q_2$  must be on

$$\therefore I_F = I_B + I_C = I_{AC2} + I_{AC1} = I_{AF}$$

$$\therefore \frac{V_E}{R_E} = \frac{V_u - V_{BE, \text{sat}} - V_E}{R_{C1}} + \frac{V_u - V_{CE, \text{sat}} - V_E}{R_{C2}}$$



$$V_{CE, \text{sat}} = 0.2V$$

$$V_{BE, \text{sat}} = 0.9V$$

$$\therefore V_E = 1.8V \quad \text{and} \quad V_c = 2V$$

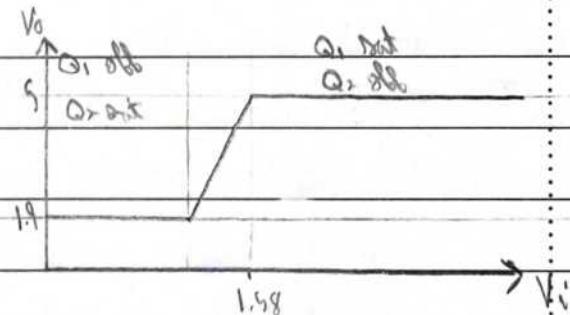
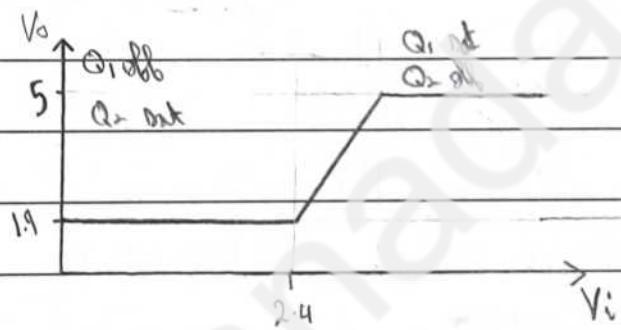
If  $Q_1$  is placed at the edge of conduction by increasing  $V_i$  until  $V_{BE1} = 0.6V$ :

for  $V_i > 2.4$ ,  $Q_1$  will be forward biased, neglecting  $I_S \rightarrow I_C = I_E \rightarrow \frac{V_E}{R_E} = \frac{V_E - 0.6 - V_E}{R_{C1}}$

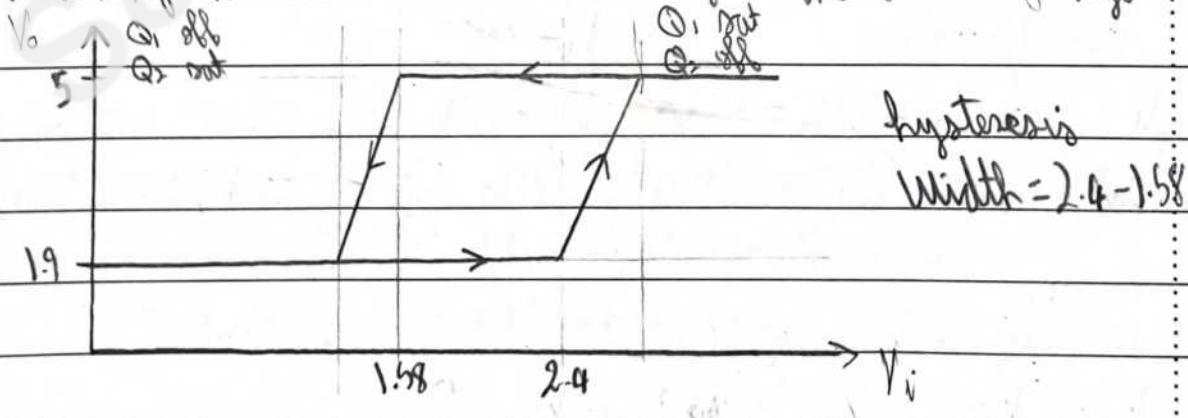
not causing  $Q_2$  to go off

$$\therefore V_E = 0.88V \quad \text{and} \quad V_i = 1.58V$$

$$V_o = V_u$$



- Combining the above two VTL will give the following hysteresis



& monostable MV using TTL NOR gates:

- if the trigger input is initially low:

Voltage across capacitor  $\frac{V_{in}}{R_T} \rightarrow V_L$  → the output from 2 will be low.

- this low input - low output state represents the circuit's stable state.
- when a positive trigger is applied, the output of the first NOR gate will go low and the capacitor will begin to charge through  $R_T$ :

$$V_{OL} \rightarrow V_L = V_{OH} - (V_{on} - V_{OL}) \quad \therefore V_L(t) = V_{OH} - [V_{OH} - V_{OL}] e^{-t/R_T C_T}, \quad T = R_T C_T$$

$$\rightarrow V_L(t) = V_{OC} - [V_{OC} - (V_{OC} - V_{on} + V_{OL})] e^{-t/R_T C_T}$$

$$\therefore V_L(t) = V_{OC} - [V_{on} - V_{OL}] e^{-t/T}$$

- define T: time for  $V_L$  to equal  $V_{IH}$   $\rightarrow T = R_T C_T \ln \frac{V_{on} - V_{OL}}{V_{OC} - V_{IH}}$

- the large jump in  $V_L$  voltage

occurs when  $V_L$  reaches  $V_{IH} - V_{on}$

hence turning the output of NOR2

low, which is input into NOR1

causing it to go high. Therefore, the sudden increase in voltage occurs from the capacitor charging through NOR1.

$$V_L \leq V_{IH}$$

$$V_{on} \rightarrow V_{IH}$$

$$V_L > V_{IH}$$

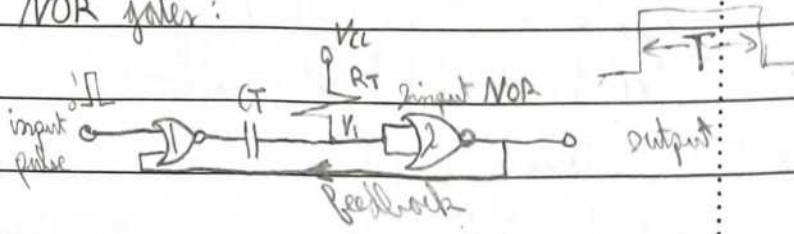
$$V_{on} \rightarrow V_{IH} + (V_{OH} - V_{IH})$$

example: assume  $V_{OH} = 3.8V$ ,  $V_{OL} = 0.1$ ,  $V_{IH} = 1.6V$ ,  $R = 15\Omega$ ,  $C = 1\text{nF}$

$$\text{pulse width: } T = 15 \times 10^6 \cdot \ln \left[ \frac{3.7}{3.4} \right] \approx 1.27 \times 10^{-6} \text{ s}$$

& Monostable MV using CMOS NOR gates:

$$\therefore V_{on} = V_{DD}, \quad V_{IH} = V_{DD}/2, \quad V_{OL} = 0$$



$$\rightarrow V_1(t) = V_{1(\infty)} - [V_{1(\infty)} - V_{1(0)}] e^{-t/\tau}$$

$$\rightarrow V_1(t) = V_{DD} - [V_{DD} - (V_{DD} - (V_{DD}-0))] e^{-t/\tau}, \quad \tau = RC$$

$$\therefore V_1(t) = V_{DD} [1 - e^{-t/\tau}]$$

$$\text{if } t = T \rightarrow V_1 = V_{IH} \rightarrow T = RC \ln \left( \frac{V_{DD}}{V_{DD} - V_{IH}} \right)$$

\* Astable MV using inverters:

+ two possible states:

$$1) \quad V_1 = L \rightarrow V_1 = H \rightarrow V_{02} = L$$

$$2) \quad V_1 = H \rightarrow V_{02} = L \rightarrow V_{02} = H$$

- Both States are unstable

- assuming  $V_{01} = H$ , (will start

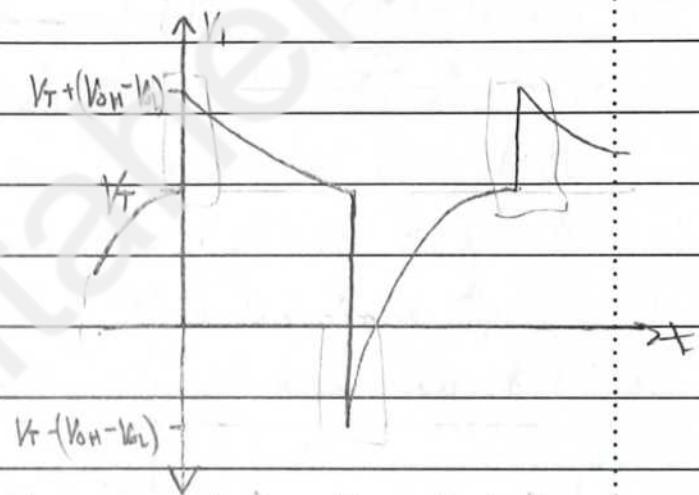
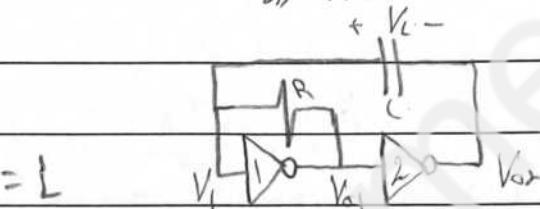
to charge through R until the time

after voltage ( $V_T$ ) of inverter 1 is

reached, which is registered as a

high input and causes  $V_{01}$  to turn

low.



- following the large jump in voltage, caused by the output of inverter 2 suddenly turning high, the capacitor will begin to discharge through R, until the voltage ( $V_1$ ) is low enough to turn the output  $V_{01}$  high. Another jump is seen here as  $V_{02}$  suddenly goes low.

$$\text{- discharging: } V_{1d}(t) = V_{1d}(\infty) - [V_{1d}(\infty) - V_{1d}(0)] e^{-t/\tau}$$

$$\rightarrow V_{1d}(t) = V_{0L} - [V_{0L} - (V_T + (V_{0H} - V_{0L}))] e^{-t/\tau}$$

$$\therefore V_{1d}(t) = V_{0L} - [2V_{0L} - V_T - V_{0H}] e^{-t/\tau}$$

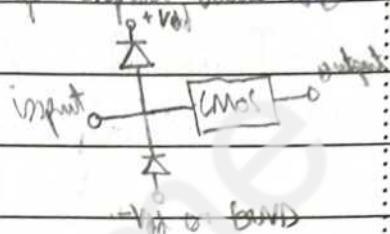
define  $T_1$ : time at which  $V_{1d} = V_T \rightarrow V_{1d}(T_1) = V_T$

$$\therefore T_1 = T \ln \left( \frac{2V_{0L} - V_{0H} - V_T}{V_{0L} - V_T} \right)$$

- charging:  $V_{IC}(t) = V_{OH} - [2V_{OH} - V_{OL} - V_T] e^{-t/T}$

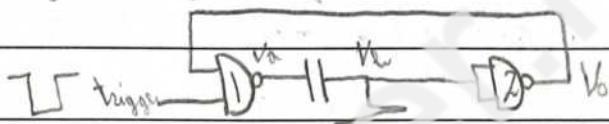
-  $T_2$ : time at which  $V_{IC}(t) = V_T \therefore T_2 = T \ln \left[ \frac{2V_{OH} - V_{OL} - V_T}{V_{OH} - V_T} \right]$

- if CMOS is used, then the maximum voltage input will be limited to  $V_{DD}$  by the protection diodes:



x Monostable using NAND gates:

- the circuit is triggered by a negative pulse and gives a negative pulse at the output.
- the circuit's stable state occurs when the output of  $V_o$  is low since the inputs at NAND 1 will be low and high respectively, which will give no change to the output when the input is triggered [(low, high) = high]
- the circuit must be started at the quasi-stable state ( $V_o = \text{high}$ ) so that will allow the circuit to vibrate?



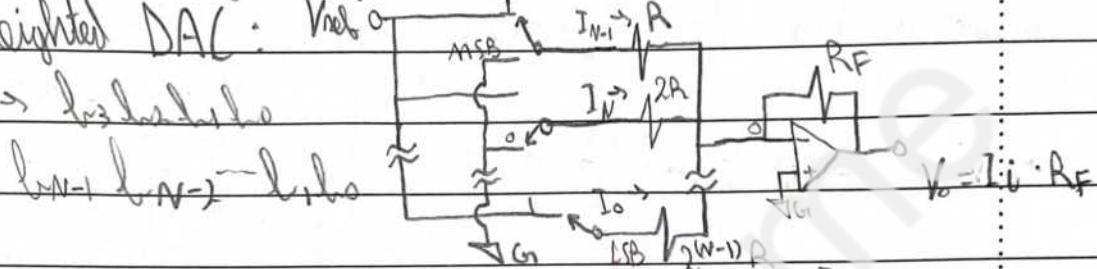
\* DAC : digital to analog converter.

\* ADC : analog to digital converter.

+ types of DACs : 1) binary weighted, 2) R-2R ladder.

1) binary weighted DAC :

- If  $N=4 \rightarrow$  4 bits



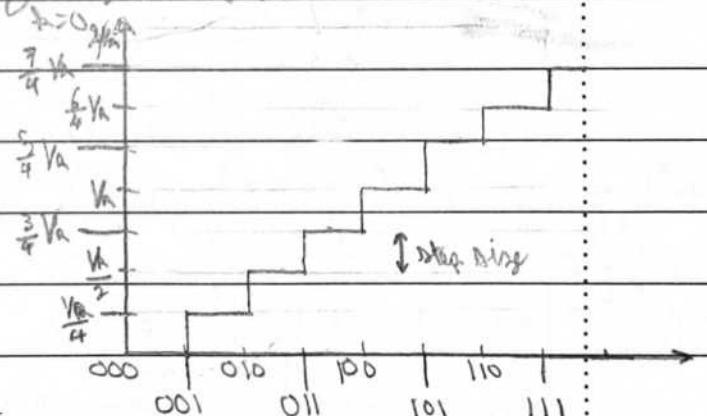
$$- V_o = - I_0 \cdot R_F = - R_F [I_0 + I_1 + \dots + I_{N-1}]$$

$$\therefore V_o = - V_{ref} \cdot \frac{R_F}{R} \cdot \frac{1}{2^{N-1}} \cdot \sum_{k=0}^{N-1} 2^k \cdot b_k = - V_{ref} \cdot \frac{R_F}{R} \cdot \frac{1}{2^{N-1}} \left[ \frac{b_0}{2^0} + \frac{b_1}{2^1} + \dots + \frac{b_{N-2}}{2^{N-2}} + \frac{b_{N-1}}{2^{N-1}} \right]$$

example:  $N=3$ ,  $R_F=R$ ,  $V_{ref}=-5V$

$$\rightarrow V_o = 5 \cdot \frac{1}{4} \cdot \sum_{k=0}^3 2^k \cdot b_k$$

$b_2$	$b_1$	$b_0$	$V_o$
0	0	0	$\frac{1}{4} V_{ref} = 0$
0	0	1	$\frac{1}{4} V_{ref}$
0	1	0	$\frac{3}{4} V_{ref}$
0	1	1	$\frac{5}{4} V_{ref}$
1	0	0	$\frac{7}{4} V_{ref}$
1	0	1	$\frac{9}{4} V_{ref}$
1	1	0	$\frac{11}{4} V_{ref}$
1	1	1	$\frac{13}{4} V_{ref}$

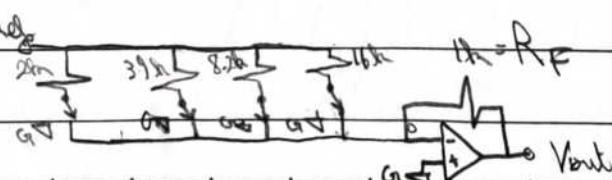


- binary weighted DACs require a wide range of precision matching resistors: if  $N=8 \rightarrow$  resistors needed:  $2R, 4R, \dots, 128R$

- resistors should also have low tolerance and low temperature variation

2. Constant reference current DAC

- another issue with binary weighted DACs is the



Large jumps in current passing, which might cause overshoot.

The circuit shown on the previous page will have a constant current passing through the resistors. Based on the switch position, the current will either flow from/to the ground or the RF.

(2) R-2R ladder DAC:

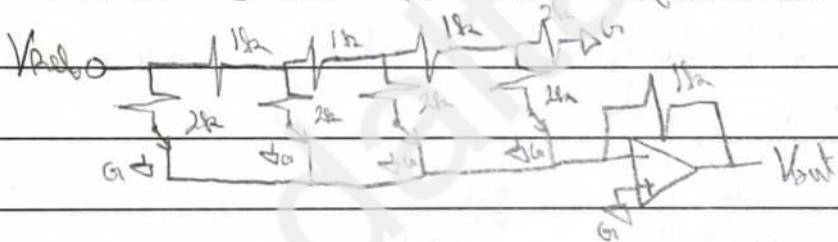
$$\rightarrow V_o = -V_{ref} \cdot \frac{RF}{R} \left[ \frac{B_0}{16} + \frac{B_1}{8} + \frac{B_2}{4} + \frac{B_3}{2} \right]$$

$$\therefore V_o = -V_{ref} \cdot \frac{RF}{R} \cdot \frac{1}{16} [2^0 B_0 + 2^1 B_1 + 2^2 B_2 + 2^3 B_3]$$

$$\rightarrow V_o = -V_{ref} \cdot \frac{RF}{R} \cdot \frac{1}{2^N} \sum_{k=0}^{N-1} 2^k B_k$$

- the same current issue still stands with the standard R-2R circuit.

\* modified R-2R circuit: (constant reference current)



+ Types of ADCs: (1) parallel or flash converter, (2) successive approximation

(1) 2-bit flash converter:

Analog input condition	Comparator output	Digital output	
$0 \leq V_{in} < \frac{V}{4}$	0 0 0	B <sub>1</sub> B <sub>0</sub>	apply mask voltage
$\frac{V}{4} \leq V_{in} < \frac{V}{2}$	1 0 0	0 1	
$\frac{V}{2} \leq V_{in} < \frac{3V}{4}$	1 1 0	1 0	
$\frac{3V}{4} \leq V_{in} \leq V$	1 1 1	1 1	

- for an N-bit output,  $2^N - 1$  comparators must be used

+ advantages of flash converter:

- fastest type of ADC since the conversion is performed simultaneously typically taking 100 ns
- easy to design and construction is simple.

+ disadvantages of flash ADC:

- not suitable for large numbers of bits as the number of comparators nearly doubles for every added bit.

(2) Successive approximation ADC:

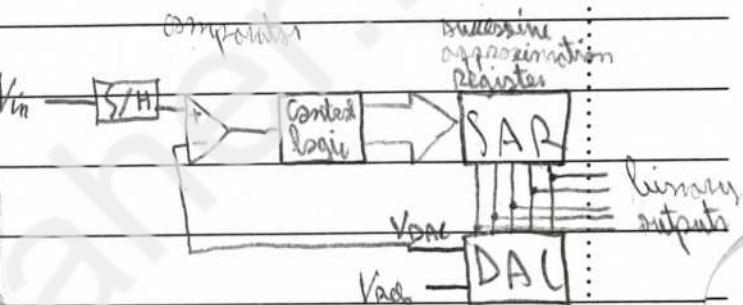
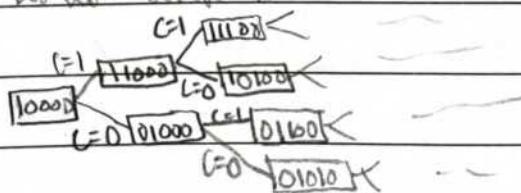
+ conversion procedure:

- 1) start of conversion (SOC) input is set to (1), then the SAR will set the MSB equal to 1 and 0s rest equal to zero as an initial guess (i.e., for 5 bits: 10000)

2) the  $V_{DAC}$  output is then compared with  $V_{in}$

- if  $V_{in} > V_{DAC}$ , the output of the comparator will be (1) and the SAR will maintain the MSB as (1) and set the next bit equal to (1) (i.e., 11000)
- if  $V_{in} < V_{DAC}$ , the output of the comparator becomes (0) and the MSB is changed to zero while the second is set to (1) as the second guess (i.e., 01000)

- 3) the cycle repeats for all other bits until they have all been tested and set.



example :  $N=4$ ,  $V_{ref} = -5V$ ,  $V_m = 3.45V$ , DAC: binary weighted,  $R=R_f$

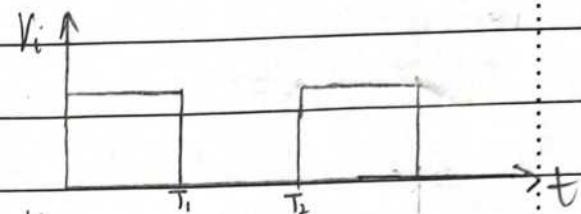
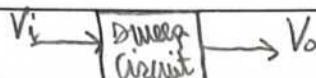
$$\therefore V_{DAC} = \frac{V_{ref}}{2^{N-1}} \cdot \frac{R_f}{R} \sum_{k=0}^{N-1} 2^k l_k = \frac{5}{8} \sum_{k=0}^{3} 2^k l_k$$

$$\therefore 1l_3l_2l_1l_0 = 1000 \rightarrow V_{DAC} = 5V \rightarrow V_m \quad \checkmark$$

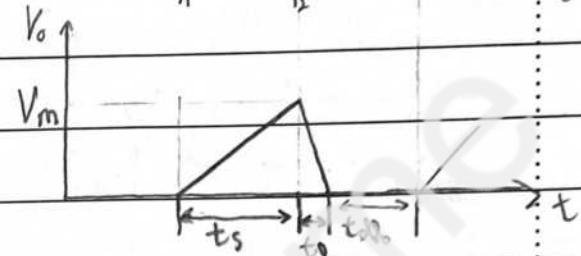
$$2) 0100 = 2.5V < V_m(\sqrt{3}) \quad 0110 = 2.5 + 1.25 > V_m \quad \times$$

$$4) 0101 < V_m \rightarrow 3.45 = 0101 \text{ (closest)} \quad \checkmark$$

## \* Sweep Circuits:



-  $V_o$  should be linear as a function of time in interval  $T_1 - T_2$

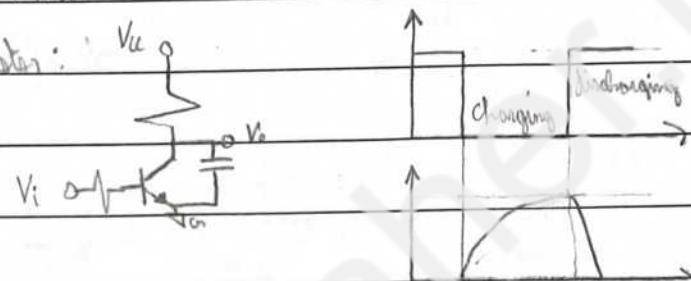


\*  $t_s$ : sweep time,  $t_f$ : flyback time

-  $t_f$  should be very small.

\* Sweep circuit with resistor:

- Since the capacitor is charging through  $V_i$  and  $\frac{1}{R}$ , the output will be exponential.



\* Constant current sweep circuit:

• When Q is off:  $I_{DC} = C \frac{dV}{dt}$

$$\Rightarrow V_o(t) = \frac{I_{DC}t}{C}$$

- If Q is saturated:  $B I_B > I_{DC} + I_{C,dis}$

• When charging:  $I_{DC} = C \frac{dV}{dt}$

$$\Rightarrow V_o(t) = \frac{I_{DC}t}{C} \Rightarrow V_m = \frac{I_{DC}(T_2 - T_1)}{C} \text{ (max)}$$

• When discharging, Q is forward active:

$$I_{DC} = B I_B + C \frac{dV}{dt} \Rightarrow I_{DC} = B I_B + C \cdot \frac{(0 - V_m)}{t_f}$$

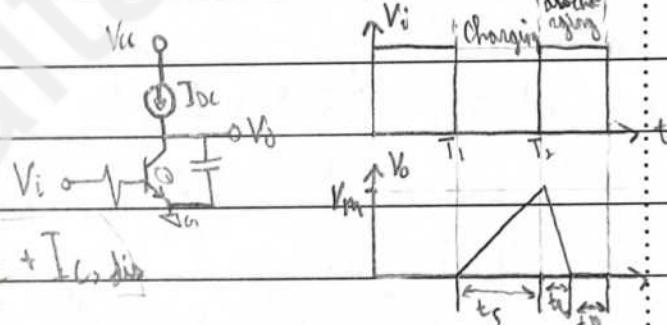
$$\therefore t_f = \frac{C V_m}{B I_B - I_{DC}}$$

Example: QP is forward active

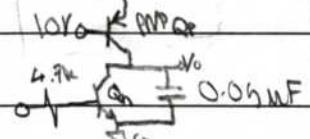
$\therefore QP$  is FA  $\rightarrow$  Voltage on collector terminal is

always 0.9V higher than that on the base.

$$\therefore I_{EP} = \frac{20 - 10.9}{22k\Omega} = 0.4293 \text{ mA}$$



$V_{CC} = 20V$   
constant current



$$\therefore I_{EP} = \frac{g+1}{g} I_{CP} \rightarrow I_{CP} = 0.4185 \text{ mA constant}$$

$$\therefore I_{CP} = C \frac{dV_o}{dt} \rightarrow V_o = \frac{I_{CP} \cdot t}{C} = 8.37 \times 10^3 t$$

- the maximum output voltage is set to be 10.9V, so the voltage drop across the PNP will become zero then and cause the sweep to plateau.

$$\rightarrow V_{om} = 10.9 > \frac{I_{CP}}{C} \cdot t_s \quad \text{and } t_s = T_2 - T_1$$

$$\therefore t_s < 1.2784 \text{ ms}$$



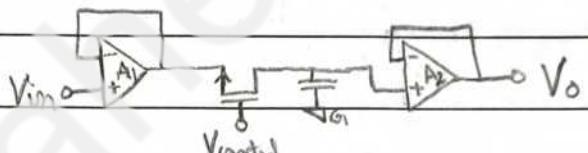
- for discharging: Qn first goes to forward active

$$I_{CP} = \beta_{on} T_{Bn} + C \frac{dV_o}{dt} \rightarrow 0.4185 \text{ mA} = 100 \cdot I_B + C \cdot \frac{-V_m}{t_f}$$

$$\therefore t_f = \frac{C V_m}{\beta_{on} T_{Bn} I_C}$$

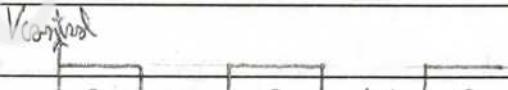
### \* Sample & Hold:

- Voltage follower op-amp are used as buffers to eliminate loading effect.



- When active, the mosfet can be

modeled as a resistor, and it will allow the capacitor to charge and sample the input.



- When the control input is low, the fet will go off and the input will not be sampled.

$$Q_1) T = t_2 + t_1, t_1 = R_1 \cdot \ln \left( 1 + \frac{2R_2}{R_1} \right) = \\ = 1\text{h} \cdot 1\text{MF} \cdot \ln(1+2) = 1.0986\text{ms}$$

$$t_2 = R_2 \cdot \ln(1+2) \rightarrow I = 0.4551 \text{ A}$$

$$Q_2) Q_s \text{ is always FA} \Rightarrow V_{IN} = -0.9V$$

$$V_{OL} = -0.9 - I_E \cdot 300 \quad I_E = \frac{V_E + 5}{R_E}, V_E = V_{BE} - 0.9$$

$$\rightarrow V_{OL} = -0.9 - \frac{300}{1240} \left[ \frac{V_{OL} - 0.9}{2} - 0.9 + 5 \right]$$

$$\rightarrow V_{OL} = -0.9 - \frac{15}{124} V_{OL} + \frac{15}{124} \cdot 0.9 + \frac{15}{62} \cdot 0.9 - \frac{15}{124} \cdot 5$$

$$\rightarrow V_{OL} \left[ 1 + \frac{15}{124} \right] = \frac{-2063}{1240} \rightarrow V_{OL} = -1.479V$$

$$\rightarrow V_{REF} =$$

$$I_E = \frac{V_E + 5}{1240} \rightarrow V_E = V_{BE} - 0.9$$

$$\rightarrow I_E = \frac{V_{REF} + 4.3}{1240}$$

$$-\frac{15}{124} V_{OL} + \frac{15}{124} \cdot 0.9 - \frac{15}{124} \cdot 4.3$$

$$Q_3) T_{on} = RL \ln(3) = 11\text{ms}$$

$$Q_4) V_{IA}, V_E = -1.6942 \rightarrow I_E = 2.763\text{mA}$$

$$\rightarrow I_{EB} = 1.381\text{mA}$$

$$Q_5) V_E = 4.5 - 0.9 \rightarrow I_E = 2.923\text{mA}$$

$$Q_6) V_{SS} = \frac{1 + \frac{\sqrt{2}}{2} [V_{AO} - 1]}{1 + \frac{\sqrt{2}}{2}} = 2.242641\text{V}$$

$$I_{DD} = 60\text{mA} = k_m (V_{in} - 1)^2 \rightarrow V_{in} = 1.9946$$

$$Q_7) 60\text{mA} = k_m (V_{DD} - V_{in} - 1)^2 \rightarrow V_{in} = 2.904495 \quad 75$$

$$\text{Q1a) } V_{ob} = -1.69V$$



$$\text{Q1b) } V_{in} < V_i < V_{ss} \rightarrow I_{ob} = g_m (V_{ob} - V_{Tn})^2 = g_m (1.5 - 1)^2 = 2.5mA$$

$$\text{Q1c) } 0.8 - (V_a - 0.1) = -6.1V$$

Solve examples:

$$2) V_i = 13 \rightarrow V_{in} < V_i < V_{ss}$$

$$\therefore V_{ss} = \frac{1 + \frac{1}{\sqrt{2}}[4]}{1 + \frac{1}{\sqrt{2}}} = -2 + 3\sqrt{2} V \approx -2.24264$$

$$\rightarrow I_{DD} = k_m (V_{DSN} - V_{Tn})^2 = 0.25 \cdot 100 \text{ mA/V}^2 = 25 \text{ mA}$$

$$3) V_{DD} - V_{Tn} = 4.4V$$

$$4) I_{DR} = \frac{k_m}{2} \Rightarrow V_{ss} = V_{Tl} + \sqrt{\frac{I_{DR}}{k_m}} V_{Tl} \rightarrow I_{DR} = 9$$

$$5) V_{OL} = 0.2, V_{OH} = 5 \rightarrow P_{OH} = 0$$

$$V_{OL} = 0.2 \rightarrow I_{OL} = \frac{5 - 0.2}{1k} = 4.8 \text{ mA}$$

$$\rightarrow P_{OL} = 24 \text{ mW} \rightarrow P_{avg} = \frac{P_{OL} + P_{OH}}{2} = 12 \text{ mW}$$

$$6) \text{ linear mode of operation: } V_{DS} > V_{Tn}$$

$$V_{DS} < V_{DSL} - V_{Tn}$$

$$\rightarrow V_{GSL} \geq -2 \wedge V_{GSL} - -2 = 0 - -2 = 2V$$

$$V_{GSL} - V_{Tn} \geq V_{DSL} \quad \therefore V_{DSL} = 5 - V_o = 1V$$

$$\rightarrow \text{linear} \quad \therefore I_{DD} = k_m \sqrt{2(V_{DS} - V_{Tn})(V_{DS} - V_{DSL})}$$

$$\rightarrow I_{DD} = 25 \mu A \cdot [2(2) \cdot 1 - \frac{1}{2}] = 75 \mu A$$

$$7) V_o = V(0) - [V(0) - V(0)] e^{-t/T}$$

$$= 5 - (5 - 0.6)e^{-2} = 4.4045V$$

$$\rightarrow I = \frac{5 - 4.4045}{10 \Omega} = 0.5955 \text{ mA}$$

$$8) V_{ss} = 5V \quad V_o = 0.5V \rightarrow V_{DS} < V_i < V_{DS} - V_{Tl} \rightarrow \text{Non Sat}$$

$$\rightarrow M_n \text{ ohmic} \wedge M_p \text{ sat} \quad V_i \neq 0 \wedge$$

$$\rightarrow I_D = k_m (V_{GDP} - V_{Tl})^2 = k_m (5 - 0 - 1.5)^2 = 0.245 \text{ mA}$$

example 8.8:  $T_1 = T \ln \left[ \frac{2V_{OL} - V_{OH} - V_T}{V_{OL} - V_T} \right]$

 $V_{OL} = 0, V_{OH} = V_{DD} \rightarrow \frac{2 \cdot 0 - V_{DD} - V_T}{0 - V_T} = \frac{V_{DD} + V_T}{V_T}$

$\therefore T_1 = R \ln \left( \frac{V_{DD} + V_T}{V_T} \right)$

$\lambda T_2 = T \ln \left[ \frac{2V_{OH} - V_{OL} - V_T}{V_{OH} - V_T} \right] \quad \lambda V_{OH} = V_{DD}$

$\lambda V_{OL} = 0 \rightarrow T \ln \left[ \frac{2V_{DD} - V_T}{V_{DD} - V_T} \right]$

$\therefore R = 10\Omega \quad \lambda C = 1\text{nF}$

$\rightarrow T = 10\text{ns} \rightarrow T_1 = 10\text{ns} \ln \left( \frac{5+1}{2} \right) = 12.53\text{ns}$

$\lambda T_2 = 10\text{ns} \ln \left( \frac{10-2}{5-2} \right) = 9.81\text{ns}$

$\therefore$  the period is  $T = T_1 + T_2 = 22.34\text{ns}$   $\lambda$  duty cycle =  $\frac{T_1}{T} = 56.1\%$

$\text{If } R = 12\Omega \rightarrow T_1 = 12\text{ns} \ln(3.5) = 15.03\text{ns}$

$T_2 = 11.99\text{ns} \rightarrow T = 26.8\text{ns}$

$\text{duty cycle} = \frac{15.03}{26.8} = 56.1\% \text{ same}$

~~1)  $T_{off} = R_2 C \ln 2$~~

~~$T_{off} \rightarrow V_{C,off} = V(0) - (V(0) - V_{OL}) e^{-t/T_D}$~~

~~$V(0) = \frac{5+2}{2} = \frac{10}{3} \quad \lambda V_{OL} = 0$~~

~~$\rightarrow V_{C,off} = \frac{10}{3} - \left[ \frac{10}{3} e^{-t/T_D} \right] \quad \lambda T_D = 20\text{ns} \cdot 2$~~

~~$V_{C,off} = 0 \rightarrow t = T$~~

~~$\rightarrow \frac{10}{3} = \frac{10}{3} e^{-T/T_D}$~~

~~$V(0) = \frac{2}{3} \cdot V_{OL} = \frac{10}{3} \quad \lambda V(0) = V_{OL} \cdot \frac{10}{3} = \frac{10}{3}$~~

~~$\text{or } V(T) = \frac{5}{3}$~~

$\rightarrow \frac{5}{3} = \frac{10}{3} - \left( \frac{10}{3} - \frac{10}{3} \right) e^{-T/T_D}$

$\rightarrow \frac{\frac{5}{3} - \frac{10}{3}}{\frac{10}{3} - \frac{10}{3}} = \frac{1}{8} = e^{-T/T_D} \rightarrow T_D = T_D \cdot \ln(8)$ 
 $= R_2 C \cdot \ln(8)$

$\lambda R_{on} = R_1/R_2 = 14.2857$

$$2) V_{out} = V_{CE2}$$

$$V_{CE2} = V_{DD} - (V_{DD} - V_{BE}) e^{-t/T}$$

$$\lambda T = 1\mu \cdot 1\mu, V_{DD} = 10$$

$$V_{DD} = V_{CE\text{sat}} = 0 \Rightarrow V_{CE2} = 10 - 10e^{-1} = 8.66V$$

$$3) V_C = \frac{R_2}{R_1 + R_2} V_{out} = 2V$$

$$4) t_1 = R_L \ln \left( 1 + \frac{2A_{v2}}{A_1} \right) = R_L \cdot \ln (1+1) = R_L \ln (2)$$

$$5) V_{LT} = \cancel{\frac{R_2}{R_1 + R_2}} V_{out} = \frac{R_2}{R_1} V_{out} = 3V$$

$$\wedge V_{LT} = -\frac{R_2}{R_1} V_{out} = -3V$$

$\rightarrow$  we are in the saturation region

$\therefore$  Voltage is decreasing  $\rightarrow V_{out} = 6V$

examples from slides:

$$1) V_{DD} = 3V, I_{DN} = 150\mu A/V^2, V_{TN} = 0.5V$$

$$\text{a)} V_o = 0.1 \text{ for } V_i = 3V$$

$$\text{assume linear: } V_o = V_{DD} - R_D I_{DN} [2(V_{GS} - V_{TN}) V_{DS} - V_{DS}^2]$$

$$V_{DS} = V_o \rightarrow 0.1 = 3 - R_D \cdot 150\mu [2 \cdot (3 - 0.5) \cdot 0.1 - 0.1^2]$$

$$\rightarrow \frac{2.9}{150\mu} = 0.1 R_D - (0.1)^2 R_D \\ \rightarrow R_D = 39.455 \mu\Omega$$

$$\text{b)} \text{ transition point: } V_{in} = V_s \quad \wedge \quad V_o = (V_{GS} - V_{TN})$$

$$V_{GS} = V_o = V_{DD} - I_D \cdot R_D \quad \wedge \quad V_{GS} = V_s = V_{in}$$

$$\rightarrow (V_s - V_{TN}) = 3 - 5.91825 (V_s - V_{TN})^2$$

$$\text{define } x = (V_s - V_{TN}) \rightarrow 5.91825 x^2 + x - 3 = 0$$

$$\rightarrow x = 0.632484 \quad \text{or} \quad -0.86145 \quad x = V_o$$

$$\rightarrow V_s = 1.132484 V$$

example ②:

$$V_6: \text{transition from Sat to linear}, V_t = \frac{V_T + \frac{V_{DD}}{I_{Tn}}}{1 + \frac{V_{DD}}{V_{Th}}}$$

$$1) \text{ or } V_t = \frac{V_T / \beta_{Tn} + V_{DD}}{1 + \frac{V_{DD}}{\beta_{Tn} V_{Th}}} \quad \beta_{Tn} = \frac{I_{DS1}}{I_{DS2}} = \frac{g_m}{g_{m2}} = \frac{1}{0.34} = 2.947330421$$

$$\rightarrow V_t = \frac{1.7149859 + 5}{1.7149859 + 5} = 2.47330421$$

$$\lambda V_{ot} = V_t - \frac{1}{\beta_{Tn}} V_{Th} = 1.4733 - 5$$

$$2) V_{DS} = 5 - 1 = 4V \rightarrow I_D = \lambda m_1 [2(4-1)V_6 - V_6^2]$$

$$\rightarrow I_D = \lambda m_1 (V_{DD} - V_6 - V_{Tl})^2$$

$$\rightarrow 0.34 \cdot (4 - V_6)^2 = 6V_6 - V_6^2$$

$$\rightarrow 0.34 \cdot (16 - 8V_6 + V_6^2) = 6V_6 - V_6^2$$

$$\rightarrow 5.44 - 2.72V_6 + 0.34V_6^2 - 6V_6 + V_6^2 = 0$$

$$\rightarrow V_6 = 0.698919 V$$

example ③:

$$\text{or } I_{DS1} (V_{DS1} - V_{Tl})^2 = 3 \cdot \lambda m_1 \cdot [2(V_{DS1} - V_{Tl})V_{DS1} - V_{DS1}^2]$$

$$1) \lambda V_{DS1} = V_6 \rightarrow 0.34 (V_{DD} - V_6 - V_{Tl})^2 = 3 [6V_6 - V_6^2]$$

$$\rightarrow 0.34 (16 - 8V_6 + V_6^2) = 18V_6 - 3V_6^2$$

$$\rightarrow 5.44 - 2.72V_6 + 0.34V_6^2 - 18V_6 + 3V_6^2 = 0$$

$$\rightarrow V_6 = 0.2947134 V$$

$$2) m=1$$

$$\rightarrow 5.44 - 2.72V_6 + 0.34V_6^2 - 6V_6 + V_6^2 = 0$$

$$\rightarrow V_6 \approx 0.9 V$$

example ④: all inputs high for low output,  $I_D = 1 \text{ mA}$

$$V_{ot} = V_{DS1} + V_{DS2} + V_{DS3}$$

assume ohmic:

$$I_{DS} = \lambda m_1 \cdot [2(V_{DS1} - V_{Tl})V_{DS1} - V_{DS1}^2] \rightarrow \frac{1}{2} = 2(V_{DS1} - 1) \cdot V_{DS1} - V_{DS1}^2$$

$$\lambda V_{DS1} = V_A = V_{DD} - V_T = 4V$$

$$\rightarrow -V_{DS1}^2 + 6V_{DS1} - 0.5 = 0 \rightarrow V_{DS1} = 0.084924V$$

$$\wedge \frac{1}{2} = \boxed{2(V_{DS2} - 1) \cdot V_{DS1} - V_{DS2}^2}$$

$$V_{DS2} = 4 - V_{DS1} = 3.915476$$

$$\rightarrow -V_{DS2}^2 + 5.830492V_{DS2} - 0.5 = 0$$

$$\therefore V_{DS2} = 0.087049V$$

$$\wedge 2(V_{DS3} - 1) \cdot V_{DS3} - V_{DS3}^2 - 0.5 = 0$$

$$\wedge V_{DS3} = 4 - (V_{DS2} + V_{DS1}) = 3.828429V$$

$$\rightarrow -V_{DS3}^2 + 5.656854V_{DS3} - 0.5 = 0$$

$$\therefore V_{DS3} = 0.089843V \rightarrow V_{D1} = 0.2614V$$

$\therefore$  look in diagram that  $\rightarrow I_{MT} = R_T \cdot (V_{GSE} - V_T)^2$

$$\wedge V_{GSE} = V_{DD} - V_0 = 4.7386V$$

$$\rightarrow I_{MT} = 71.545 \text{ mA/V}^2$$

final exam practice:

$$1) V_C = \frac{R_2}{R_1 + R_2} \cdot 6 = 2V \quad \boxed{1}$$

$$2) T = R_T \cdot \ln \left( \frac{V_{OH} - V_{OL}}{V_{OH} + V_{OL}} \right) = \ln \cdot \ln \frac{3.8 - 0}{5 - 1.6} \quad \cancel{\frac{3.8 - 0}{5 - 1.6}}$$

$$V_{I(00)} = 0 \quad V_{I(0)} = 0 + (V_{OH} - V_{OL})$$

$$\rightarrow V_{I(t)} = V_{I(00)} - [V_{I(00)} - V_{I(0)}] e^{-\frac{t}{RC}}$$

$$\rightarrow V_{I(T)} = V_{IL} = 0 - [0 - (V_{OH} - V_{OL})] e^{-T/RC}$$

$$\rightarrow V_{IL} = [V_{OH} - V_{OL}] e^{-T/RC}$$

$$\rightarrow \ln \left[ \frac{V_{IL}}{V_{OH} - V_{OL}} \right] = -\frac{T}{RC}$$

$$\rightarrow T = RC \cdot \ln \left[ \frac{V_{OH} - V_{OL}}{V_{IL}} \right] \quad \begin{array}{l} \text{NAND} \\ \text{ASTABLE} \end{array}$$

$$\rightarrow T = \ln(2.395) \text{ ms} = \boxed{1}$$

3)  $V_o = 2 \rightarrow V_{ds} = 2V$ ,  $V_{GDS} - V_{Th} = 4V$   
 $\rightarrow$  ~~linear~~  $\rightarrow I_D = f_{Dn} [2 \cdot 4 \cdot R - 2^2] = f_{Dn} \cdot 12$   
 $\rightarrow I = \boxed{C} = 1.2 \text{ mA}$

4)  $V_o = 4 \rightarrow M_n \text{ not } \propto m_p \text{ linear} \rightarrow I = f_{Dn} (V_i - V_{Th})^2$   
 $V_i = 0 \rightarrow I = 50 \text{ mA}$   
 $\wedge M_p \text{ linear} \rightarrow I_D = f_{Dp} (2(V_{GDS} - V_{Th}) \cdot V_{Dn} - V_{SD})$   
 $\wedge V_{SD} = 1 \rightarrow I_D = 20 \text{ mA} (2 \cdot 3.5 \cdot 1 - 1^2) = 100 \text{ mA}$   
 $V_{SS1} = V_{SS} - V_{Th} \quad \wedge V_{SS} = \frac{1 + \frac{f_{Dp}}{f_{Dn}} [5 - 1.5]}{1 + \frac{f_{Dp}}{f_{Dn}}} = 1.968665V$   
 $\rightarrow V_{SS1} = 0.96866V$   
 $\rightarrow I_C = 0.12 \text{ mA} = \boxed{D}$

5)  $M_L : \text{linear} \rightarrow I_D = f_{D1} [2(0 - V) V_{BSL} - V_{BSL}^2] = 0.04395 \text{ mA}$

$\rightarrow \boxed{D}$

6)  $V_o = 3V \quad \wedge V_{SS1} = V_{SS} - V_{Th} \quad \wedge V_{SS} = \frac{1 + \frac{\sqrt{2}}{f_{Dn}} [3.5]}{1 + \frac{\sqrt{2}}{f_{Dn}}}$   
 $\rightarrow V_{SS} = 1.968665 \rightarrow V_{SS2} \approx 3.5V$   
 $\rightarrow V_o < V_{SS2} \rightarrow M_p \text{ not } \propto M_n \text{ linear}$   
 $\therefore I_D = f_{Dp} (V_{Gn} - V_{Th})^2 = 20 \text{ mA} (3.5 - 1.5) =$   
 $I_D = f_{Dp} [2 (V_{Gn} - V_{Th}) V_{SD} - V_{SD}^2] = f_{Dp} [-6 - 9]$   
 $= -750 \text{ mA} = \boxed{A}$

7)  $-10 \cdot \frac{R_A}{2k} \left[ \frac{1}{8} + \frac{1}{4} + \frac{1}{2} \right] = -17.5V \quad \boxed{D}$

8)  $M_n \text{ not } \propto \rightarrow I_D = -f_{Dn} (5 - 1)^2 = -0.8 = \boxed{D}$

9)  $1000 \rightarrow 1100 \approx 1.5V \rightarrow 1010 \quad \boxed{D}$

10)  $I_D = C \frac{V}{R} \rightarrow V(00) - [V(00) - V(00)] e^{-\frac{V}{R}}$

$\rightarrow I = \frac{5 - (5 - 0.6) e^{-2}}{10k} = 59.54 \text{ mA} \approx 60 \text{ mA}$

(D)  $M_{sat} \rightarrow I_D = b_m (S-1)^2 = 1.6 \text{ mA} = \boxed{D}$

(B)  $M_{min} \wedge M_D \text{ sat} \rightarrow I_D = I_C = b_m (V_{SGD} - V_{TO})^2$   
 $\rightarrow I_C = 0.245 \quad \boxed{B}$

(4)

?

(B)

$$0 \rightarrow T^0 \quad \boxed{A}$$

(6)

$$V_{DT} = V_U - (V_U - (0.8 - V_U - 0.1)) e^{-t/0.1} \\ \approx t = 0.25T$$

$$\rightarrow 10 - [10 + 9.2] e^{-0.25} = -4.9524 \text{ V} \quad \boxed{D}$$

(7)

$$\therefore V_{UT} = \frac{10}{20} \cdot 6 = 3 \text{ V} \wedge V_{LT} = -3 \text{ V}$$

still in hysteresis  $\rightarrow \boxed{D}$

(8)

$$V_T = \frac{3}{30} = 0.1 \text{ mA} \rightarrow V_{DT} = V_T = 0.1 \text{ mA} \rightarrow V_{DT} = 4 \text{ V}$$

$$\rightarrow 4 = 6 - [6 + 4] e^{-t/0.1} \rightarrow -T/0.1 = \ln \frac{6-4}{6}$$

$$\rightarrow T = T \cdot \ln \left( \frac{10}{6} \right) = T \ln(5) \quad \boxed{A}$$

(9)

$$\therefore V_{oss} = 2.3094 \rightarrow M_2 \wedge M_D \text{ sat}$$

$$\rightarrow I = b_m (V_{oss} - V_{T1})^2 = 0.1 \text{ mA} = \boxed{D}$$

(10)

$$-2.5 - \frac{-1.25}{10} = 1.25 \quad \boxed{C}$$

(1)

assume  $V_{DT} \Rightarrow V_{DT} = 4 \quad \boxed{A}$

(3)  $M_{min} \rightarrow I_D = b_m [2(S-1) \cdot 2 - 2^2] = 1.2 \text{ mA}$

$$\wedge V \quad 1.2 \text{ mA} = \left( \frac{\partial V_0}{\partial t} + \frac{V_{DD} - V_0}{R_D} \right)$$

$$\rightarrow \left( \frac{\partial V_0}{\partial t} \right) = 1.2 \text{ mA} - 0.3 = 0.9$$

(20)

$$\frac{V_U}{10 \text{ mA}}$$

$$\boxed{1} \quad \frac{CdV_0}{dt} + \frac{V_{DD} - V_0}{R_0} = \ln(5-1)^2 \rightarrow \frac{CdV_0}{dt} = 1$$

$$\boxed{2} \quad 0.11 \rightarrow \begin{array}{c} 0.11 \\ \downarrow \\ \text{---} \\ \rightarrow I_E = \end{array} \quad \begin{array}{c} 43 \\ \nearrow \\ \text{---} \\ \downarrow \\ 83.8 \text{ mA} \end{array}$$

Lab exam practice:

$$\boxed{1} \quad t_1 = t_2 \rightarrow f = \frac{1}{2t_1} \wedge t_1 = RL \ln(1+2) \\ \wedge RL = 0.2 \text{ m} \rightarrow t_1 = 0.2199 \text{ ms} \\ \rightarrow \boxed{2} \quad f = 2295.59$$

$$\boxed{2} \quad \text{pulse width} = RL \ln(3) \rightarrow R = R + R_{B1} \\ \text{at } 1 \text{ ms: } R = 1820 \Omega$$

$$\text{at } 1.8 \text{ ms: } R = 32.96 \Omega \rightarrow R_B [0, 1.46 \Omega]$$

$$\boxed{3} \quad V_{in} = 6V \rightarrow \text{out}$$

$$\boxed{4} \quad Q_1 \text{ sat: } V_{D1} = 5.0 \text{ V} = 4.3 \rightarrow I_{D1} = 83.8 \text{ mA} \\ \rightarrow I_{D1} = R_0 I_{D1} = 4.19 \text{ mA} = I_{B2}$$

$$\boxed{5} \quad \text{duty cycle} = T_{on} / T \quad V_{UT} = \frac{b}{2} \cdot V_{DD} = 6V \\ \rightarrow V_U = -6V \rightarrow$$

$$2.4 \cdot 0.7 \text{ ms} \pm 8.0 \mu\text{s}$$

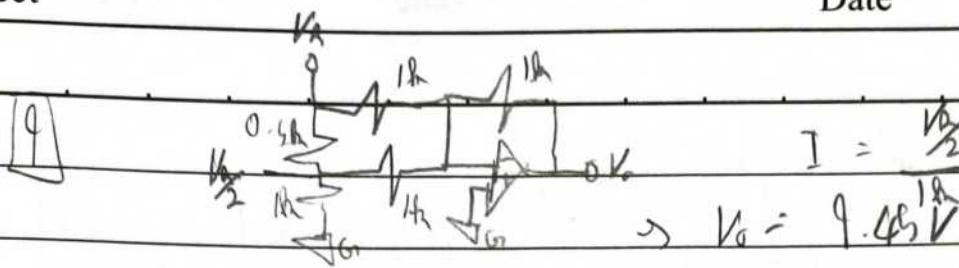
$$\boxed{6} \quad V(t) = [V_{DD} - V_{D1}] e^{-t/\tau} \quad V_{D1} = 0, V_{DD} = 10V$$

$$\rightarrow 10 - 10 e^{-0.5/T} = 5.4$$

$$\rightarrow T = 6.4389 \times 10^{-4}$$

$$T = R \times L$$

$$\rightarrow T = 1.58896 \text{ ms} \rightarrow R_L =$$



$$I = \frac{V_2}{1k} + \frac{V_3}{1k}$$

$$\rightarrow V_3 = 9.45V$$

$$\boxed{10} \quad f = 158\text{ Hz} \rightarrow T_{on} + T_{off} = 66.667 \text{ ms}$$

$$\rightarrow T_{off} (10 + R_B) C \ln(2) + R_B C \ln(2) = 66.667 \text{ ms}$$

$$\rightarrow \ln(2) \cdot C [10 + 2R_B] = 66.667 \text{ ms}$$

$$\rightarrow 10k + 2R_B = 79.2359 \text{ k} \rightarrow R_B = 4600 \Omega$$

$$\boxed{11} \quad \ln(2) \cdot C \cdot [R_1 + 2R_2] = \frac{1}{gk}$$

$$\rightarrow R_1 + 2R_2 = 18.0337 \text{ k} \Omega$$

$$\rightarrow R_1 + R_2 = 10k \rightarrow R_1 = 1966.3 \text{ m} \Omega = 1966.3 \text{ } \mu\Omega = 8033 \text{ } \mu\Omega$$

$$\rightarrow T_{on} = (R_1 + R_2) \cdot C \ln(2) = 6.9315 \times 10^{-5} \text{ s}$$

$$\rightarrow \frac{T_{on}}{T} = 8k \cdot T_{on} = 0.5545$$

$$\boxed{12} \quad R_T = R + 50\Omega$$

$$\rightarrow T_{max} =$$

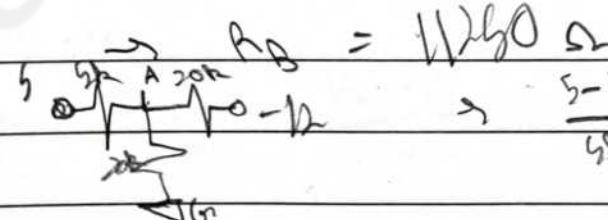
$$\boxed{13} \quad T_{on} = 0.64 \cdot T$$

$$\rightarrow T_{on} = 20k \cdot 0.05 \mu\text{s} \cdot \ln(2)$$

$$\rightarrow T = T_{on} + R_B \cdot 0.05 \mu\text{s} \cdot \ln(2)$$

$$\rightarrow \left(\frac{1}{0.64} - 1\right) T_{on} = R_B \cdot 0.05 \mu\text{s} \cdot \ln(2)$$

$$\rightarrow T_{on} = 6.93147 \times 10^4 \text{ s}$$



$$\rightarrow \frac{5-V_A}{4k} = \frac{V_A}{20k} + \frac{V_A+12}{20k}$$

$$\rightarrow 20 \cdot 4V_A = V_A + V_A + 12$$

$$\rightarrow 6V_A = 8 \rightarrow V_A = \frac{4}{3}V$$

(19)  $V_{in} = 0 \rightarrow V_o = \text{high}$



$$I_{RL} = n \cdot \frac{3.2294 - 0.8}{10k} = \frac{5 - 3.2294}{10k}$$

$$\rightarrow n = 7$$

examples continued:

example 10:  $V_{ss} = V_{T1} + \sqrt{\frac{R_1}{R_2}} \cdot V_{TL}$

$$\lambda V_{ss}^2 = \frac{R_1}{R_2} V_{TL}^2$$

$$\rightarrow V_{ss} = 1 + \sqrt{\frac{0.2}{1}} \cdot (-1) = 1 \mp \frac{\sqrt{5}}{5}$$

$$\rightarrow V_{ss} = 0.553 \times 01 \boxed{1.4492 \text{ V}}$$

$$\lambda V_{ss} = \pm \sqrt{0.2} (-1)^2 = 0.449 \text{ V}$$

$$I_D = \lambda n_1 (V_{ss} - V_{T1})^2 \approx 2.2 \text{ mA}$$

for  $V_{os}$ :  $2(V_i - V_{T1}) \cdot V_{os} - V_{os}^2 = 0.2 \cdot (V_{os} - V_{T1})^2$

$$V_i = V_{os} \quad \lambda V_{os} = 0$$

$$\rightarrow 2(4) \cdot V_{os} - V_{os}^2 = 0.2$$

$$\therefore V_{os} = 0.025074 \text{ V}$$

$$12) C \cdot 10k \cdot \ln(2) \approx 8\mu s =$$

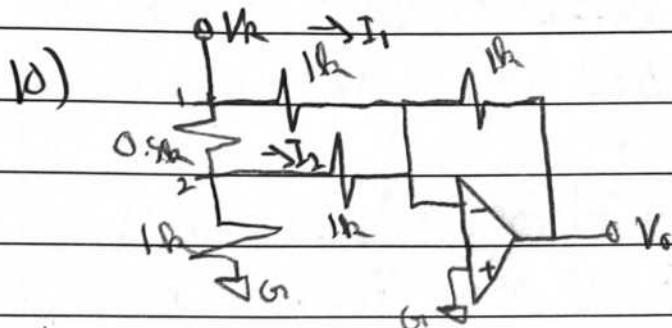
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$$12) T_{on} = (R_1 + R_2) C \cdot \ln 2 \quad \lambda R_1 + R_2 = 10k$$

$$T_{total} = \frac{1}{8\mu s}, \text{ duty cycle} = \frac{T_{on}}{T_{total}} = T_{on} \cdot 8\mu s \\ \rightarrow \text{duty cycle} = 0.59452$$



$$I_1 = \frac{V_A}{10k}$$

$$I_2 = \frac{V_2}{0.5k} \quad \lambda V_2$$

$$5) V_{VT} = 4V \quad \lambda V_{LT} = -4V$$

$$0 \rightarrow \frac{-4}{10}$$

$$0 \rightarrow 2592 \times 10^{-3} \rightarrow t_{off} = 5.144 \text{ ms}$$

$$T = \frac{1}{f} \rightarrow T_{on} = 2.6 - 5.144 = 3.1116 \mu s$$

$$\rightarrow D =$$

$$t - V_{UT} = \sin(\frac{\pi}{160}) / 160 \rightarrow t_{on} =$$

$$16) V_x > V_{in}$$

$$\frac{5 - V_{ref}}{5k} = \frac{V_{ref} + V_{out} - V_{in}}{20k}$$

$$\frac{3}{10} + \frac{1}{2} = \frac{1}{10}$$

$$\rightarrow 20 - 4/V_{ref} = 2V_{ref} - V_{in}$$

$$18) \text{ When } V_A = V_{in} \quad \lambda V_o = -V_{out} \rightarrow 2V_{ref} = 20 + V_{out}$$

$$V_A = V_{in}$$

$$\therefore V_{ref} = \frac{20 + V_{out}}{6}$$

$$V_{in} = \frac{V_x - V_{in}}{5k} - \frac{V_x - V_o}{20k}$$

$$\text{If } V_{out} = 12$$

$$V_{ref} = \frac{32}{6} =$$

$$V_{in} = 4V_x - 4V_{in} - V_x + V_{out} \rightarrow V_{in} =$$

$$V_{ref} = 4V_x - 4V_{ref} - V_x - V_o$$

$$\rightarrow V_{ref} = \frac{5V_x - V_o}{5}$$

example:

$$1) \quad f_{L2}(V_{GSS1} - V_T)^2 = m_{F_{L2}} V_{DS1} [2(V_{GSS1} - V_T) - V_{DS1}]$$

$$\rightarrow 0.34 = 3 [2(4) \cdot V_{OL} - V_{OL}^2]$$

$$\rightarrow 3V_{OL}^2 - 24V_{OL} + 0.34 \Rightarrow V_{OL} = 0.0142 \text{ V}$$

$$2) \quad V_{OL}^2 - 8V_{OL} + 0.34 = 0 \Rightarrow V_{OL} = 0.04273 \text{ V}$$

example

$$1) \quad I_D = 1 \text{ mA}$$

$$\rightarrow 1 \text{ mA} = 2 \times 10^{-3} [2(V_{GSS1} - V_T) V_{DS1} - V_{DS1}^2]$$

$$\rightarrow 0.5 = 2(5-1) \cdot V_{DS1} - V_{DS1}^2$$

$$\therefore V_{DS1} = 0.063 \text{ V}$$

$$1 \text{ mA} \rightarrow 0.5 = [2(5-V_{DS1}-1) V_{DS2} - V_{DS2}^2]$$

$$\rightarrow V_{DS2} = 0.063 \text{ V}$$

$$\rightarrow 0.5 = 2(5-V_{DS1}-V_{DS2}-1) V_{DS3} - V_{DS3}^2$$

$$\rightarrow V_{DS3} = 0.063 \text{ V}$$

$$\therefore V_{OL} = 0.1921 \text{ V}$$

$$\therefore M_1 \text{ forward} \rightarrow I_{mA} = f_{L2} \cdot V_{DS1} [2(0+1) - V_{DS1}]$$

$$\therefore I_m = f_{L2} [2(4.8079) - 4.8079^2]$$

$$\rightarrow f_{L2} = \text{negative } \times$$

$$\therefore V_{DS1} > V_{DS2} - V_{TL} \rightarrow M_2 \text{ short}$$

$$\therefore I_{mA} = f_{L2} (V_{GSS1} - V_T) \rightarrow f_{L2} = 1 \text{ mA}$$

Quiz 3:

$$1) \quad \text{if } V_{OL} \rightarrow M_1 \text{ short } \rightarrow M_2 \text{ negative}$$

$$I_D = f_{L2} (V_{GSS1} - V_T)^2 - f_{L2} V_{OL} [2(V_{GSS1} - V_T) - V_{DS1}]$$

$$\rightarrow I = 3.2 [8V_{OL} - V_{OL}^2]$$

$$\rightarrow V_{OL} = 0.0342 \text{ V}$$

$$5) \because V_{LT} = \frac{10k}{10k+20k} \cdot V_{out} \rightarrow V_{LT} = 4V$$

$$\therefore V_{LT} = \frac{1}{3}(-V_{out}) \rightarrow V_{LT} = -4V$$

assume starting at  $t=0$

$V_{in}$  reaches  $V_{LT}$  then returns to 0

$$t_{on}: V_{LT} = V_{in} = 10 \sin(160t) \rightarrow t = \frac{\arcsin(\frac{4}{10})}{160}$$

$$\rightarrow t_{LT} \approx 2.572 \text{ ms}$$

$$\therefore t_{off} = 2t_{LT}$$

$$t_{LT}: V_{LT} = -4 = 10 \sin(160t) \Rightarrow t_{LT} = 2.572 \text{ ms}$$

$$t_{off} = 2t_{LT}$$

$$T = t_{off} + t_{on} \quad \text{duty cycle} = \frac{t_{on}}{T}$$

$$\therefore T = 2t_{on} \rightarrow D = 50\%$$

$$1) V_{in} = 5V \quad M_1 \text{ is linear and } M_2 \text{ is sat}$$

$$\rightarrow I_{D1} \cdot V_{D2} [2(V_{in}) - V_{D2}] = I_{D2} (V_{in})^2$$

$$\rightarrow \frac{I_{D1}}{I_{D2}} = 8V_{D2} - V_{D2}^2$$

$$2) V_{D1} = V_{in} \quad \therefore V_{D1} = V_{D2} \rightarrow NM_H = 3.4V$$

$$3) \because V_{SS} = \frac{1 + \frac{1}{2} \cdot \frac{30}{270} [V_{DD} - 12]}{1 + \frac{1}{3}} = \frac{1 + \frac{1}{3}(3.8)}{\frac{4}{3}} = \frac{3 + 3.8}{4} = 7V$$

$$\therefore V_{in} < V_{D2} < V_{SS} \rightarrow M_1 \text{ sat and } M_2 \text{ linear}$$

$$\therefore I_D = I_{D1} (V_{in} - V_{D2})^2 = 47.628 \text{ mA}$$

$$\therefore P_{dissipated} = 238.41 \text{ mW}$$

A) M<sub>0</sub> always sat

$$\therefore k_m (V_{DS} - V_0 - 1.2)^2 = k_m (V_m - 0.8)^2$$

$$\therefore \frac{1}{k_m} \frac{1}{(V_{DS} - V_0)} = 1.96 \rightarrow$$

$$\text{or } V_0 = V_{DS} - V_T - \sqrt{1.96 \cdot (V_m - 0.8)}$$

$$\rightarrow V_0 = 4.2 - \frac{1}{2} [V_m - 0.8] \quad \text{and } V_0 < V_m < V_{DS} + V_m$$

$$\therefore V_T =$$

$$\therefore 2.45 < V_0 < 4.2$$

$$\text{otherwise } V_m = V_{DS} + V_T \rightarrow V_0 = 4.2 - \frac{1}{2} [V_m]$$

$$\rightarrow V_0 = 2.45$$

$$V_{DSL} = 5 - V_0, \quad V_{DS0} = V_0, \quad V_{DS0} = V_m$$

$$\therefore M_0 \text{ sat} \quad \therefore 0.8 < V_m < V_T$$

$$\text{find } V_T : \frac{V_T}{k_m} = \frac{(V_T - 0.8)^2}{k_m} = \frac{(V_{DS} - V_T - V_0)^2}{k_m}$$

$$\therefore \boxed{k_m = \frac{\text{driven}}{\text{load}}} \rightarrow 1.96 (V_T - 0.8)^2 = (V_{DS} - V_T - 0.8 - 1.2)^2$$

$$\rightarrow \sqrt{1.96} (V_T - 0.8) = (V_{DS} - V_T - 0.4)$$

$$\rightarrow 2.4 V_T - 1.12 = 5 - 0.4$$

$$\rightarrow V_T = 2.3833 \dots V$$

$$\therefore V_m = V_T \rightarrow V_0 = 1.9833$$

$$\therefore V_0 - 1.12 = 4.6 - V_T$$

$$\rightarrow 2.4 V_T = 6.72$$

④ M<sub>1</sub> always sat & M<sub>2</sub> sat

$$\rightarrow V_{TD} < V_{in} < V_t \quad \text{check page 22 of prob. slides}$$

$$\wedge V_o = V_{DD} - V_{TL} - \sqrt{k_R(V_{in} - V_{TD})} \quad (\text{ch-7})$$

$$\therefore k_R(V_t - V_{TD})^2 = k_R(V_{DSL} - V_{TL})^2$$

$$\wedge V_{DSL} = V_{DD} - V_{ot} \quad \wedge V_{ot} = V_t - V_{IN}$$

$$\rightarrow k_R(V_t - 0.8)^2 = (V_{DD} - V_t + 0.8 - 1.2)^2$$

$$\rightarrow \frac{1}{2}(V_t - 0.8) = 4.6 - V_t$$

$$\therefore V_t = 2.38333 \text{ V}$$

$$\text{at } V_{in} = V_{TD}, \quad V_o = V_{DD} - V_{TL} = 3.8 \text{ V}$$

$$\text{at } V_{in} = V_t, \quad V_o = 1.58334 \text{ V}$$

$$\therefore 1.58334 \text{ V} < V_o < 3.8 \text{ V}$$

⑤  $\therefore V_{SS} = V_{TD} \pm \sqrt{\frac{k_L}{k_R} \cdot V_{TL}}$

$$\rightarrow 1.6 = 1 \pm \sqrt{\frac{1}{4} \cdot (-1.2)}$$

$$\rightarrow \sqrt{k_R} = 2 \rightarrow k_R = 4$$

⑥  $V_{in} < V_{in} \rightarrow NM_L = V_{IN} - V_{ot} = V_{IN}$

⑦  $2.9 = V_{IN} \wedge 1.2 = V_{ot}$

$$NM_L = 1.2 - 0.2 = 1$$

⑧  $V_{DD} - V_{IN} = 4 \text{ V}$

$$V_{SS} = \frac{V_{IN} + \frac{1}{2} [5 - 1]}{1 + \frac{1}{2}} = 2.2426 \text{ V}$$

$$\rightarrow V_{DD} - V_t < V_i < V_{DD} \rightarrow M_2 \text{ off} \quad \wedge V_o = 0$$

$$\rightarrow V_{DSL} = -V_{SDL} = -5$$

Q1  $P_{OH} = 0$        $P_{OL} = \frac{4.5}{R_{th}} \cdot 5 = 22.5 \text{ mW}$   
 $\rightarrow P_{Oavg} = \frac{P_{OH} + P_{OL}}{2} = 11.25 \text{ mW}$

Q2  $V_{in} = 0.3$        $M_1$  sat       $\rightarrow V_o = V_{DD}$   
 $\rightarrow V_{DSL} = 0$

Q3  $V_o < V_{in}$        $\therefore V_{SS} = V_{T1} \pm \sqrt{R_{th} V_{TL}}$   
 $\rightarrow V_{SS}$        $\therefore V_{GSI} > V_{in} \text{ and } V_{DSI} = V_o > V_{GSI} - 1$

$4.6 > 0.2 \rightarrow M_1$  sat

$\therefore V_{DSI} = 0.4 < 0 - V_{TL} \rightarrow M_L$  is linear

Q4  $V_{in} = 1.8$ ,  $\sqrt{R_{th}} (V_t - 0.8) = (V_{DD} - 0.4 - V_t)$   
 $\rightarrow \sqrt{2} V_t = 5.4 \text{ V} \rightarrow V_t = 2.7 \text{ V}$

$\therefore V_{T1} < V_{in} < V_t \rightarrow M_1$  is sat

No need to calculate  $V_t$  since we know  
it is larger than  $V_{in}$  as both transistors  
are sat

$\rightarrow V_o = V_{DD} - V_{TL} - \sqrt{R_{th}} (V_{in} - V_{T1})$

$\rightarrow V_o = 2.8 \text{ V}$

Q5  $\therefore V_{SS} = \frac{V_{in} + \frac{1}{2}[4]}{1.5} = 2$

$\rightarrow V_{TD} < V_{in} < V_{SS} \rightarrow M_n$  sat,  $M_P$  linear

$\rightarrow I_o = \frac{I_{in}}{2} (V_{in} - V_{in})^2 = \frac{I_{in}}{4} = 50 \text{ mA}$

Q6  $V_{DD} - V_{TL} = 5 - 0.8 = 4.2 \text{ V}$

Q7  $\frac{V_o - V_{ref}}{R_1} = \frac{V_{ref} - V_{in}}{R_2} \rightarrow V_o - V_{ref} = 2V_{ref} - 2V_{in}$

$\rightarrow V_o + 2V_{in} = 3V_{ref} \rightarrow V_{ref} = 4$

$$\boxed{2} \quad V_o(t) = V_o(0) - [V_o(0) - V_o(\infty)] e^{-t/T}$$

$T$ : time until  $V_o = V_{IL} = 1.6$

$$\rightarrow 1.6 = 0 - [0 - 3.8] e^{-t/RC}$$

$$\rightarrow \frac{1.6}{3.8} = e^{-t/RC} \rightarrow \ln\left(\frac{1.6}{3.8}\right) = \frac{-t}{RC}$$

$$\therefore T = \text{ms } \ln(2.375)$$

$$\boxed{3} \quad \text{Given } V_{GS} = V_i = 5V \quad \text{and} \quad V_{DS} = V_o = 2V$$

$$\rightarrow 2 < 5 - V_{IN} \rightarrow \text{linear}$$

$$\therefore I_D = \text{Fan } V_o(2(V_{GS} - V_{IN})/I_0)$$

$$\rightarrow I_D = 1.2 \text{ mA} = \left( \frac{0.2}{R_s} + \frac{V_{DD} - V_o}{R_D} \right)$$

$$\rightarrow \left( \frac{M_{D2}}{A_D} \right) = 0.9 \text{ mA}$$

$$\boxed{4} \quad \text{Given } V_{SS} = 1 + \frac{\frac{2}{5}(5 - 1.9)}{1 + \frac{2}{5}} = 1.96856 \text{ V}$$

$$\rightarrow V_{GS2} = 3.03144 \text{ V}$$

$\therefore V_{GS2} < V_o \rightarrow M_D \text{ off } \& M_P \text{ linear}$

$$\therefore I_D = I_L = \text{Fan } V_o(2(V_{GS} - V_{IN}) - V_o)$$

$$\cancel{I_D = }$$

$$I_D = \text{Fan } V_{GS}[2(V_{GS} - V_{IN}) - V_{DD}]$$

$$\& V_{GS} = 5V \quad \& V_{DD} = 1$$

$$\rightarrow I_D = 2.0 \text{ mA}$$

$M_D$  linear  $M_P$  off  $\&$   $M_P$  sat  $\&$   $M_D$  linear

$$I_{DS} = \text{Fan } (V_{GS} - V_{IN})^2 = 75 \text{ mA}$$

$$\& T_{DS} = \text{Fan } (4 \cdot 0.5 - 0.5^2) = 0.048 \text{ s}$$

Check!  $V_{GS}$  can  $M_D$

$$4.5 > -V_{IN}$$

$$\text{Given } V_{GS2} = 0.5 < 0 - -2 \rightarrow \text{linear}$$

6)  $V_{SD} = 1.96V \rightarrow V_o < V_{OSA}$

$M_n$  linear  $\wedge M_p$  sat  $\wedge M_p$  off since  $V_{SG} < V_{TP}$

$$I_C = I_{DN} - I_{DP}$$

$$\wedge I_{DN} = I_D \cdot (8 \cdot 3 - 3^2) = 750 \text{ mA}$$

$$\wedge (I_C = I_{DN}) = -750 \text{ mA}$$

7)  $V_{ref} = 10V \rightarrow V_o = -10 \cdot \frac{R_f}{2R} \left( \frac{1}{8} + \frac{1}{4} + \frac{1}{2} \right)$   
 $\rightarrow V_o = -17.5V$

8)  $M_p$  off,  $M_n$  sat

$$\rightarrow I_D = I_D \cdot (5 - 1)^2 = -0.8 \text{ mA}$$

9) 1000:  $-V_{ref} \cdot \frac{R_f}{R} \cdot [1] = 5$

$\circlearrowleft 6.5 > 5 \rightarrow \text{end of first cycle} = 1000$

next 1100:  $-V_{ref} \cdot 1.5 = 7.5$

$\circlearrowleft 6.5 < 7.5 \rightarrow \text{computer output} = 0$

$\rightarrow \text{end of second cycle: } 1010$

10)  $V_C = \frac{V_{DD}}{3} = 1.667V$

11)  $V_o(\text{off}) = V_o(0) - [V_o(0) - V_o(0)] e^{-t}$

$$\rightarrow 5 - (5 - 0.6) e^{-t} \rightarrow V_o(t) = 4.404621$$

$$\rightarrow I_L = 59.54 \text{ mA} \approx 0.06 \text{ mA}$$

12)  $I_D = I_C + \frac{V_{DD} - V_o}{R_o}$

$$\circlearrowleft V_{DS} = 4.5 > V_{GS} - 1 \rightarrow \text{sat}$$

$$\rightarrow I_D = I_D \cdot (4)^2 = 1.6 \text{ mA}$$

$$\rightarrow I_f = 1.55 \text{ mA}$$

13)  $M_n$  off  $\wedge M_p$  sat  $\circlearrowleft V_o < V_{TP}$

$$\rightarrow I_C = I_{DD} = I_D \cdot (V_{SGD} - V_{TP})^2$$

$$= 20 \mu \cdot (5 - 1.5)^2 = 0.205 \text{ mA}$$

~~16~~  $V_o(t) = V_o(0) - [V_{o(0)} - V_{o(0)}] e^{-\frac{t}{R_C}}$

~~$\Rightarrow V_o(0.25T) = 10 - [10 - 10] e^{-0.25}$~~

assuming trigger at  $t=0^+$

~~$V_o(0) =$~~

at  $t=0 \rightarrow Q_2$  switches from off to on

$\rightarrow t=0^- \rightarrow Q_1$  Sat

$\therefore V_{o(0)} = 0.8 - 10 = -9.2 \text{ V}$

$V_{BE\text{sat}} = (V_{CE\text{sat}} - V_o) \rightarrow (V_{CE\text{sat}} - V_{o(0)})$

$\therefore V_o(0.25T) = 10 - [10 + 9.2] e^{-0.25}$ 
 $= -4.45 \text{ V}$

~~17~~  $\frac{V_{o\text{off}} - V_{o\text{on}}}{R_C} = \frac{V_o - V_{o\text{off}}}{R_C} \rightarrow 2V_{o\text{off}} - 2V_{o\text{on}} = V_o - V_{o\text{off}}$ 
 $\rightarrow 3V_{o\text{off}} = 6 + 6 \rightarrow V_{o\text{off}} = 4$ 
 $\rightarrow t_{on} \Rightarrow V_C = V_{o\text{off}} - V_{o\text{on}} = (V_{o\text{off}} + V_T) e^{-\frac{t_{on}}{RC}}$ 
 $\rightarrow \frac{12 - 4}{12 + 4} = e^{-\frac{t_{on}}{RC}} \rightarrow t_{on} = RC \ln \left( \frac{16}{8} \right)$

~~18~~ assume no off  $\rightarrow I_C = I_L$

$\therefore V_{DSL} = 3V > |V_{TL}| \rightarrow M_2 \text{ sat}$

$\rightarrow I_C = I_L = \frac{I_{BL}}{(0+2)^2} = 100 \text{ mA}$

~~18~~  $V_{o\text{on}} = V_{o(0)} - [V_{o(0)} - V_{o(0)}] e^{-\frac{t}{RC}}$

$\rightarrow V(t_{on}) = V_{o\text{on}} \rightarrow \frac{V_{o\text{on}} - 3}{10k} = \frac{6 - V_{o\text{off}}}{10k}$

$\rightarrow 2V_{o\text{off}} - 6 = 6 - V_{o\text{off}} \rightarrow V_{o\text{off}} = 4 \text{ V}$

$\rightarrow V_{o\text{on}}(0) = 6 - [6 + 4] e^{-\frac{t_{on}}{RC}}$

$\rightarrow \ln \left( \frac{6 - 4}{6 + 4} \right) = \frac{-t_{on}}{RC} \rightarrow t_{on} = RC \ln \left( \frac{10}{2} \right)$

$$Q_1) \frac{V_{rel} - V_x}{R_2} = \frac{V_o - V_{rel}}{R_1}$$

$$\rightarrow \frac{V_{rel} - 4.5}{10.9} = \frac{9 - V_{rel}}{20.9}$$

$$\rightarrow 2.9158 V_{rel} = 9 + 4.5 \cdot \frac{20.9}{10.9}$$

$$\rightarrow V_{rel} = 6.04$$

$$Q_2) V_{os} < V_{ns} - I \rightarrow \text{linear}$$

$$\therefore I_D = I_C + I_L \quad I_M \text{ is sat}$$

$$\rightarrow I_L = I_{os} (0 + 2)^2 = 0.1168 \text{ mA}$$

$$\wedge I_D = I_{os} (8 \cdot 1.8 - 1.8^2)$$

$$\rightarrow I_D = 8.91686 \times 10^{-4}$$

$$\rightarrow I_C = 774.88 \text{ mA}$$

$$Q_3) V_R = 0 \text{ V} \rightarrow t_{on} = R C \ln \left( 1 + \frac{2 R_2}{R_1} \right)$$

$$\wedge t_{off} = t_{on}$$

$$\rightarrow t = 2.1.43 \text{ ms}$$

$$\rightarrow t = 1.9660 \text{ ms}$$

$$\rightarrow f =$$

$$Q_4) 1.76 \text{ V}$$

$$-V_{rel} \cdot \frac{R_L}{R} \cdot \left( \frac{1}{8} + \frac{1}{4} + \frac{1}{2} \right) = 5.6 \text{ V}$$

$$Q_5) I = \text{?} \quad I_{opamp} =$$

$$Q_6) Q_2(t=0) = 0 \text{ at}, \quad Q_2(t=0) = 1 \text{ at}$$



$$\rightarrow V_{o2}(t) = V(0) - [V(0) - V(0)] e^{-t/RL}$$

$$V(0) = 0.8 \rightarrow V_{o2}(2.1T) = 0.8 - (0.8 - 10) e^{-2.1}$$

$$\rightarrow V_{o2} =$$

$$V_o = 0.8 - V_{BE} \quad V_o$$

$$\text{A) } V_o(2.1\text{V}) = 10 - (10 + 10 - 0.8) e^{-2.1}$$

$$\rightarrow V_o = 8 - (8 + 7.2) e^{-2.1} = 6.1387$$

(Q2)  $M_1$  off  $\Rightarrow V_{DS1} = 3.2 \rightarrow M_1$  is sat

$$I_D = I_C = k_s (0+2)^2 = 0.1148 \text{ mA}$$

$$\text{Q4) } t=2T \rightarrow V_C = V_{D1} - (V_{DS} - V_{DD}) e^{-2}$$

$$\text{so } V_{DS} = \cancel{0.48} \text{ V}$$

$$\rightarrow V_C = 1.06597$$

$$\text{so Min current } \rightarrow I_D = k_m (8 V_o - V_i)$$

$$\rightarrow I_D = 0.939 \text{ mA}$$

$$I_D = I_C + I_A \rightarrow I_A = \frac{5 - V_o}{10.7 \text{ k}}$$

low  $\rightarrow$  high

$$V_C(0) = V_{D1} \quad V(0) - (V(0) - V_{D1}) e^{-2}$$

$$\rightarrow V(2T) =$$

$$4.384 \text{ V} \rightarrow$$

$$\text{Q10) } V_{DS}(0) = 0.48 \text{ V}$$

$$\rightarrow 0.48 - (0.48 - 0) e^{-T/10}$$

$$\rightarrow 0.48 e^{-T/10} = 0.48$$

$$\rightarrow T =$$

$$0.48 V_{DD}$$

$$0.48 V_{DD} = 0 - (0 - V_{DD}) e^{-T/10}$$

$$\rightarrow e^{-T/10} = 0.48$$

$$\rightarrow T = T \ln \frac{1}{0.48} = 0.73$$

Q11)  $M_D$  is sat  $\rightarrow I_D = I_{DS} (4)^{\frac{1}{2}} = 1.312 \text{ mA}$

$$I_L = I_{DS} (2 \cdot 0.2 - 0.2^2) = 10.368 \text{ mA}$$

Q12)  $I_C = -I_{DS} (V_{DD} - V_{TN})^r$

Q13)  $V_{B2}(0) = 8$

$$8 = (8 + 10) e^{-0.4}$$

$$0.8 = 0.8 + 10$$

$$V_{B2}(8) = 0 \quad \& \quad V_{B2}(0) = 10$$

Q14)  $I_D = I_C + V_{DS}$

Q15)  $T_{ON} = R_1 + R_2 (1 - e^{-t/T})$

$$\begin{aligned} & \xrightarrow{R_1 + R_2 \gg 10\%} \frac{(R_1 + R_2) \cdot t}{(R_1 + R_2 + R_3)} \\ & \xrightarrow{\frac{R_1 + R_2}{R_1 + R_2 + R_3} \gg 10\%} \end{aligned}$$

$$19.6 \text{ m}\Omega \rightarrow R_1 + R_2 \gg 0.5(R_1 + R_2)$$

$$\rightarrow 0.5 R_F \gg$$

$$\frac{R_1}{R_1 + R_2} = 3f_B$$

Q16)  $V(0) = \frac{V_u}{3}$

$$\frac{V_u}{3} = b - \frac{V_{AC}}{3} e^{-t/T}$$

$$\rightarrow \frac{V_u}{3} = V_u - V_u (1 - \frac{1}{3}) e^{-t/T}$$

$$\rightarrow \frac{1}{3} = 1 - \frac{2}{3} e^{-t/T}$$

$$T = \frac{1}{e^{-t/T}} = 1 - e^{-t/T} \rightarrow T =$$

$$\frac{2}{3} = 1 - (1 - \frac{1}{3}) e^{-\frac{T}{T_0}} \rightarrow \frac{1}{3} = \frac{-2}{3} e^{-\frac{T}{T_0}}$$

Q18) at  $V_{SS}$ :  $V_{SS} = \frac{1 + \frac{\sqrt{63}}{228} (5 - 15)}{1 + \frac{\sqrt{63}}{228}} = 1.861V$

$$\rightarrow I_D = f_m (V_{SS} - V_T)^2 = 169 \text{ mA}$$

Q19)

Q20)  $\rightarrow$  v. 5. (1)  $\rightarrow$  end do first: 1000  
end do second  $\rightarrow$

Q21)  $\frac{10.7}{I_{CP}} \cdot C \rightarrow T_2 - T_1$

$$\frac{10.7}{I_{CP}} C \rightarrow 1.5 \text{ ms}$$

$$I_{CP} =$$

$$I_{CP} = 10.7 \rightarrow 0.23 \times 10^{-4}$$

$$25818 \cdot C \rightarrow 1.5 \text{ ms}$$

Q23) Mp linear  $\rightarrow$ 

$$I_n =$$

Q24)  $V_{omax} = \frac{I_{CP} \cdot t_s}{C}, t_s = 10.7 \cdot \frac{C}{I_{CP}}$

$$I_{CP} = 0.1144 \text{ mA} \rightarrow t_s = 1.80725 \text{ ms}$$

$$\rightarrow V_{omax} =$$

$$C = 0.5$$

Q25) Mp sat  $\rightarrow I_C = f_m (V_{SDS} - V_{TP})^2 =$