The Cluster University of Srinagar
Syllabus for 4th semester Mathematics

Semester-IV   **Subject**: Mathematics
Course title: Abstract Algebra   **End-term examination**: 56
Course code: **Internal assessment**: 30
Total credits: 06   **Attendance**: 04

Unit-I

Definition and examples of groups, examples of abelian and non-abelian groups, the group $\mathbb{Z}_n$ of integers under additional modulo n and group $U(n)$ of units under multiplication modulo n. Cyclic groups from number systems, complex roots of unity, the general linear group $GL_n(\mathbb{R})$ of $n \times n$ invertibe matrices of real numbers, groups of symmetries of (i) an isosceles triangle, (ii) an equilateral triangle, (iii) a rectangle and (iv) a square, the permutation group $\mathrm{Sym}(n)$, groups of quaternions.

Unit-II

Subgroups, examples, criteria for non-empty subset to be subgroup, cyclic subgroups, the concept of a subgroup generated by a subset and the commutator subgroup of group, examples of subgroups including the center of a group. Cosets, index of subgroup, Lagrange's theorem, order of an element, product of two subgroups, counting principle for the number of elements in HK.

Unit-III

Normal subgroups: their definition, examples and various criterion, Quotient groups, homomorphism & isomorphism, Kernel of homomorphism, normality of kernel, Fundamental theorem of homomorphism.

Unit-IV

Definition and examples of rings, examples of commutative and non-commutative rings: rings from number systems, $\mathbb{Z}_n$ the ring of integers modulo n, ring of real quaternions, rings of matrices, polynomial rings, and rings of continuous functions, subrings and ideals, integral domain and fields, examples of fields: $\mathbb{Z}_p$ $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$. Field of rational functions.

Text Books Recommended

(1) *Topics in Algebra* by I.N. Herstien (John Wiley)

(2) *Contemporary Abstract Algebra* by Joseph A Gallian (Narosa Publications)

(3) *A First Course in Abstract Algebra* by John B. Fraleigh (Pearson)