

Statistical Analysis Using R Unit-1

MOHAMMAD WASIQ

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Introduction to R language

What is R ?

R is a language and environment for statistical computing . It has following features:

- It is free and can be downloaded from internet: <http://cran.r-project.org>. It requires only 80 Mb spaces.

*It is a functional language.

- It has very powerful graphic capabilities.
- It has capabilities to interface with other important packages for data analysis and graphics.

Creating a vector in R—‘c()’

Let us define a vector x as

```
x=c(5,4,3,1,4) # define x vector
x
```

```
## [1] 5 4 3 1 4
```

```
length(x)
```

```
## [1] 5
```

Note that R commands are written in a chunk which begins with three back ticks and ends with three back ticks. The command `{r vect1}` specifies that chunk is for R language and it is labeled by ‘vect1’. All the R commands are to be written inside the chunk.

Arithmetic Operations on vectors

There are five basic operators in R, namely ‘+’, ‘-’, ‘*’, ‘/’ and ‘^’, which are meant for addition, negation, multiplication, division, exponentiation respectively. Besides these basic operators, a large number of functions are also available for arithmetic computations. Let us create a new chunk:

```
sum(x) # sum of elements
```

```
## [1] 17
```

```
sum(x^2) # sum of squares
```

```
## [1] 67
```

```
sqrt(x) # square root of x
```

```
## [1] 2.236068 2.000000 1.732051 1.000000 2.000000
```

```

log(x) # log of x with base e

## [1] 1.609438 1.386294 1.098612 0.000000 1.386294
log(x, base=10) # log of x with 10

## [1] 0.6989700 0.6020600 0.4771213 0.0000000 0.6020600
log(x, base=5) # log of x with 5

## [1] 1.0000000 0.8613531 0.6826062 0.0000000 0.8613531
1/x # inverse of x

## [1] 0.2000000 0.2500000 0.3333333 1.0000000 0.2500000
x^-1 # same as above

## [1] 0.2000000 0.2500000 0.3333333 1.0000000 0.2500000
sin(x)

## [1] -0.9589243 -0.7568025 0.1411200 0.8414710 -0.7568025
cos(x)

## [1] 0.2836622 -0.6536436 -0.9899925 0.5403023 -0.6536436
tan(x)

## [1] -3.3805150 1.1578213 -0.1425465 1.5574077 1.1578213
mean(x)

## [1] 3.4
var(x) # variance of x

## [1] 2.3
sd(x) # standard deviation of x

## [1] 1.516575
cv=function(x) sd(x)/mean(x)*100
dump("cv", file="cv.txt")
source("cv.txt")
cv(x)

## [1] 44.60515

```

Concept of recycling rule

The basic rule is that smaller vector recycles upto the length of the larger vector. Let us see the commands in the following chunk;

```

x=c(5,4,3,1,4)
x+1 # 1 will recycle up to the length of x and add

## [1] 6 5 4 2 5
2*x

## [1] 10 8 6 2 8

```

```
log(x+1)
```

```
## [1] 1.7917595 1.6094379 1.3862944 0.6931472 1.6094379
```

```
sin(2*x)
```

```
## [1] -0.5440211 0.9893582 -0.2794155 0.9092974 0.9893582
```

The function repeat—‘rep()’

```
rep(c(1,2),c(3,3)) # repeat 1 3 times and 2 also 3 times
```

```
## [1] 1 1 1 2 2 2
```

```
rep(c(1,2),each=4)
```

```
## [1] 1 1 1 1 2 2 2 2
```

```
tasbihefatimi=rep(c("SubhanAllah", "Alhamadulliah", "Allahuakbar"), c(33,33,34))  
tasbihefatimi
```

```
## [1] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [5] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [9] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [13] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [17] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [21] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [25] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [29] "SubhanAllah" "SubhanAllah" "SubhanAllah" "SubhanAllah"  
## [33] "SubhanAllah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [37] "Alhamadulliah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [41] "Alhamadulliah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [45] "Alhamadulliah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [49] "Alhamadulliah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [53] "Alhamadulliah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [57] "Alhamadulliah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [61] "Alhamadulliah" "Alhamadulliah" "Alhamadulliah" "Alhamadulliah"  
## [65] "Alhamadulliah" "Alhamadulliah" "Allahuakbar" "Allahuakbar"  
## [69] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"  
## [73] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"  
## [77] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"  
## [81] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"  
## [85] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"  
## [89] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"  
## [93] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"  
## [97] "Allahuakbar" "Allahuakbar" "Allahuakbar" "Allahuakbar"
```

```
rep(c("R1", "R2", "R3"), c(3,2,4))
```

```
## [1] "R1" "R1" "R1" "R2" "R2" "R3" "R3" "R3" "R3"
```

Logical operators—TRUE(1),FALSE(0)

>, <, <=, >=, == exactly equal to, != not equal to

```
x=c(2,5,3,1,8)
```

```
x>3
```

```
## [1] FALSE TRUE FALSE FALSE TRUE
```

```
sum(x>3)
```

```
## [1] 2
```

```
(x[x>2])
```

```
## [1] 5 3 8
```

```
mean(x[x>2])
```

```
## [1] 5.333333
```

```
mean(x[x<3])
```

```
## [1] 1.5
```

```
mean(x<3)
```

```
## [1] 0.4
```

```
#logical expression / or, &, and  
x>2 & x<4
```

```
## [1] FALSE FALSE TRUE FALSE FALSE
```

```
x[x>2 & x<4]
```

```
## [1] 3
```

Missing values— NA is.na()

```
x=c(2,5,3,NA,6)
```

```
is.na(x)
```

```
## [1] FALSE FALSE FALSE TRUE FALSE
```

```
mean(x)
```

```
## [1] NA
```

```
mean(x,na.rm=TRUE)
```

```
## [1] 4
```

```
!is.na(x)
```

```
## [1] TRUE TRUE TRUE FALSE TRUE
```

```
mean(x[!is.na(x)])
```

```
## [1] 4
```

```
## Not a number, NaN, 0/0, Inf-Inf, Inf/Inf  
x=c(0,5,3)  
x/x
```

```
## [1] NaN 1 1
```

```
is.na(x/x)
```

```
## [1] TRUE FALSE FALSE
```

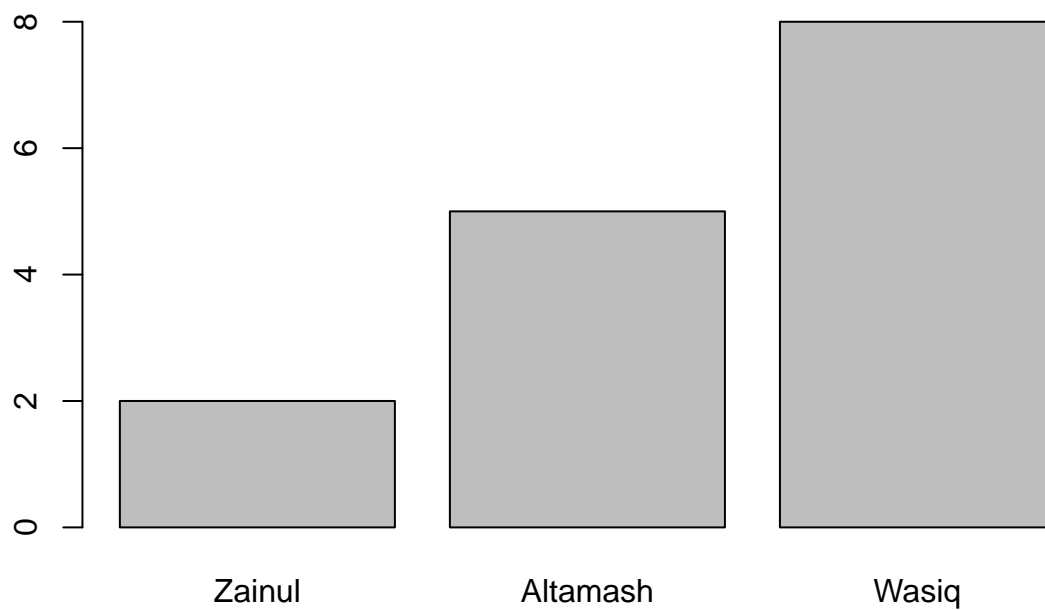
```

x=c(2,5,8)
x

## [1] 2 5 8
names(x)=c("Zainul","Altamash","Wasiq")
x

##      Zainul Altamash      Wasiq
##          2         5         8
barplot(x)

```



paste() The function 'paste()' behave like 'c()' but it always returns a character vector.

letters and **LETTERS**

```

letters

## [1] "a" "b" "c" "d" "e" "f" "g" "h" "i" "j" "k" "l" "m" "n" "o" "p" "q" "r" "s"
## [20] "t" "u" "v" "w" "x" "y" "z"

LETTERS

## [1] "A" "B" "C" "D" "E" "F" "G" "H" "I" "J" "K" "L" "M" "N" "O" "P" "Q" "R" "S"
## [20] "T" "U" "V" "W" "X" "Y" "Z"

letters[c(3,6)]

## [1] "c" "f"

```

```
LETTERS[1:6]

## [1] "A" "B" "C" "D" "E" "F"
month.abb

## [1] "Jan" "Feb" "Mar" "Apr" "May" "Jun" "Jul" "Aug" "Sep" "Oct" "Nov" "Dec"
month.abb[c(3,9)]

## [1] "Mar" "Sep"
month.abb[3:9]

## [1] "Mar" "Apr" "May" "Jun" "Jul" "Aug" "Sep"
month.name

## [1] "January" "February" "March" "April" "May" "June"
## [7] "July" "August" "September" "October" "November" "December"

substin
substr(month.name,1,4)

## [1] "Janu" "Febr" "Marc" "Apri" "May" "June" "July" "Augu" "Sept" "Octo"
## [11] "Nove" "Dece"
```

Factor vector—‘factor()’, ‘ordered()’

Factor is like the

```
grp=c("control","treatment","control","treatment")
grp

## [1] "control" "treatment" "control" "treatment"
grp=factor(grp)
as.integer(grp)

## [1] 1 2 1 2
# factor are efficient ways of storing data
as.integer(grp)

## [1] 1 2 1 2
levels(grp)

## [1] "control" "treatment"
```

Matrices and arrays— ‘matrix()’, ‘arrays()’

The function ‘matrix()’ is used to create a matrix.

```
m=matrix(1:6,nrow=2,ncol=3)
m

##      [,1] [,2] [,3]
## [1,]    1    3    5
## [2,]    2    4    6
```

```

dim(m)

## [1] 2 3

dimnames(m)=list(LETTERS[1:2],letters[1:3])
m

##   a b c
## A 1 3 5
## B 2 4 6

colnames(m)=c("Samad","Radha","Umar")
m

##   Samad Radha Umar
## A     1     3     5
## B     2     4     6

rownames(m)=c("Moradabad","Aligarh")
m

##           Samad Radha Umar
## Moradabad     1     3     5
## Aligarh       2     4     6

matrix(1:6,ncol=3,byrow=TRUE)

##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    4    5    6

# Extraction of a matrix---[]
m[1,3]

## [1] 5

m[,c(1,2)]

##           Samad Radha
## Moradabad     1     3
## Aligarh       2     4

m[c(1,2),3]

## Moradabad   Aligarh
##           5       6

## arrays()
array(1:8,c(2,2,2))

## , , 1
##
##      [,1] [,2]
## [1,]    1    3
## [2,]    2    4
##
## , , 2
##
##      [,1] [,2]
## [1,]    5    7

```

```
## [2,]    6    8
```

```
# naming arrays  
dim(array)
```

```
## NULL
```

Data frame—‘data.frame()’

These are objects, a kind of generalization of matrices. Generalization in the sense that columns can be of different types from each other. This flexibility is not with matrices. Use ‘data.frame()’ to construct a data from vector:

```
colors=c("red","yellow","blue")  
numbers=c(1,2,3)  
colors_numbers=data.frame(colors, numbers)  
colors_numbers
```

```
##   colors numbers  
## 1    red       1  
## 2 yellow      2  
## 3   blue      3
```

```
colors=c("red","yellow","blue")  
numbers=c(1,2,3)  
more_numbers=c(5,4,7)  
colors_numbers=data.frame(colors,numbers,more_numbers)  
colors_numbers
```

```
##   colors numbers more_numbers  
## 1    red       1             5  
## 2 yellow      2             4  
## 3   blue      3             7
```

```
colors=c("red","yellow","blue")  
numbers=c(1,2,3)  
more_numbers=c(5,4,7)  
colors_numbers=data.frame(Colors=colors,Numbers=numbers,MoreNUMBERS=more_numbers)  
colors_numbers
```

```
##   Colors Numbers MoreNUMBERS  
## 1    red       1             5  
## 2 yellow      2             4  
## 3   blue      3             7
```

```
# change row names  
row.names(colors_numbers)=paste0("Row",1:3)  
colors_numbers
```

```
##      Colors Numbers MoreNUMBERS  
## Row1    red       1             5  
## Row2 yellow      2             4  
## Row3   blue      3             7
```

```
# extraction from data.frame  
colors_numbers[c(1,3),]
```

```
##      Colors Numbers MoreNUMBERS  
## Row1    red       1             5
```



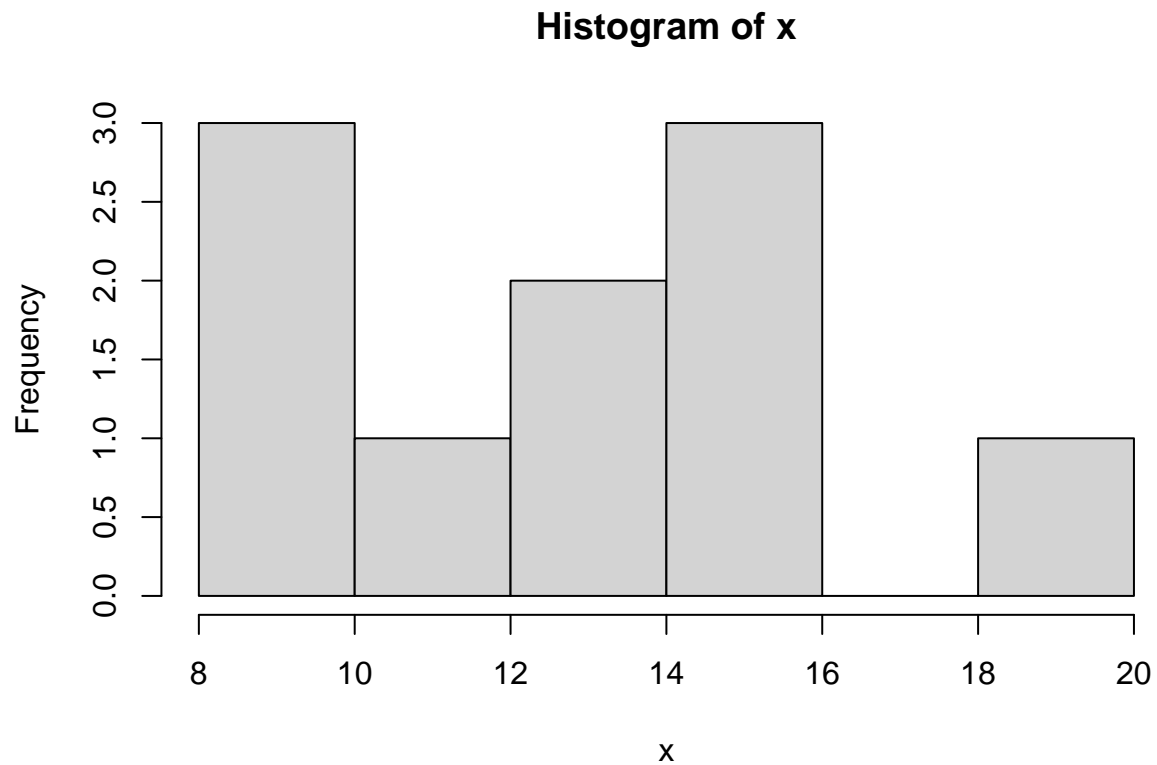
```
## Row3    blue      3      7
DF2=colors_numbers[,c(1,3)]
DF2
```

```
##      Colors MoreNUMBERS
## Row1    red           5
## Row2 yellow          4
## Row3    blue          7
```

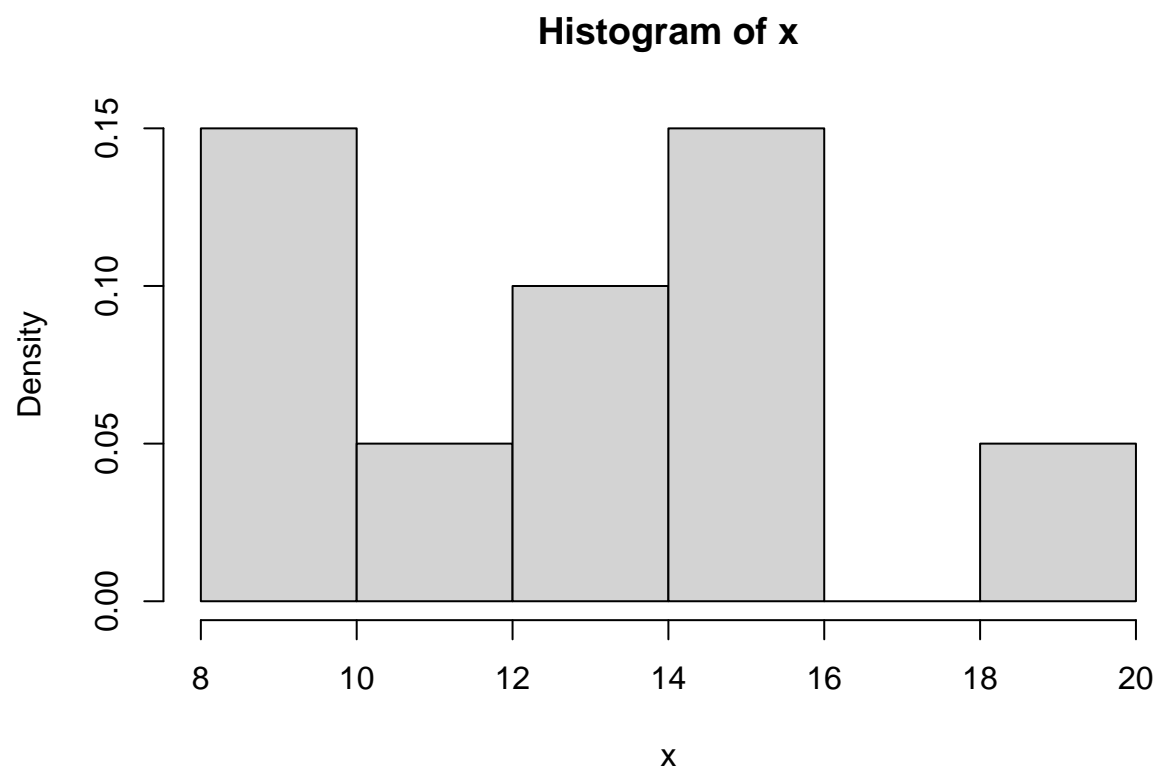
Built in graphic function—‘hist()’, ‘plot()’, ‘curve()’

**** Histogram ****

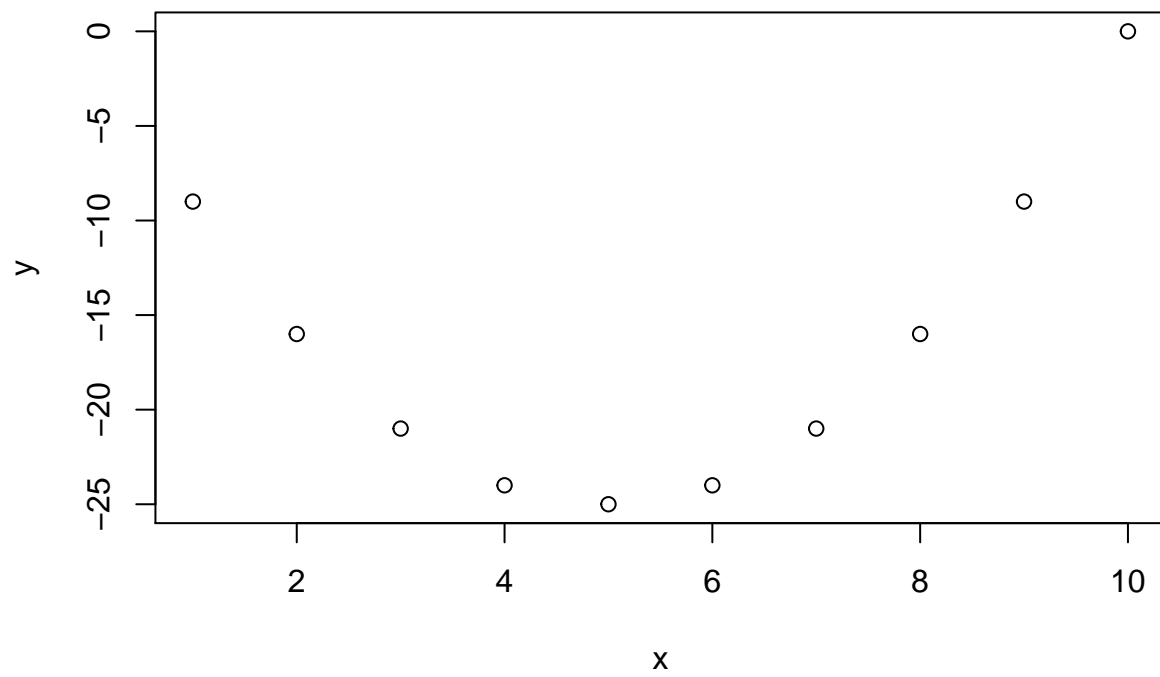
```
x=c(12,15,13,20,14,16,10,10,8,15)
hist(x)
```



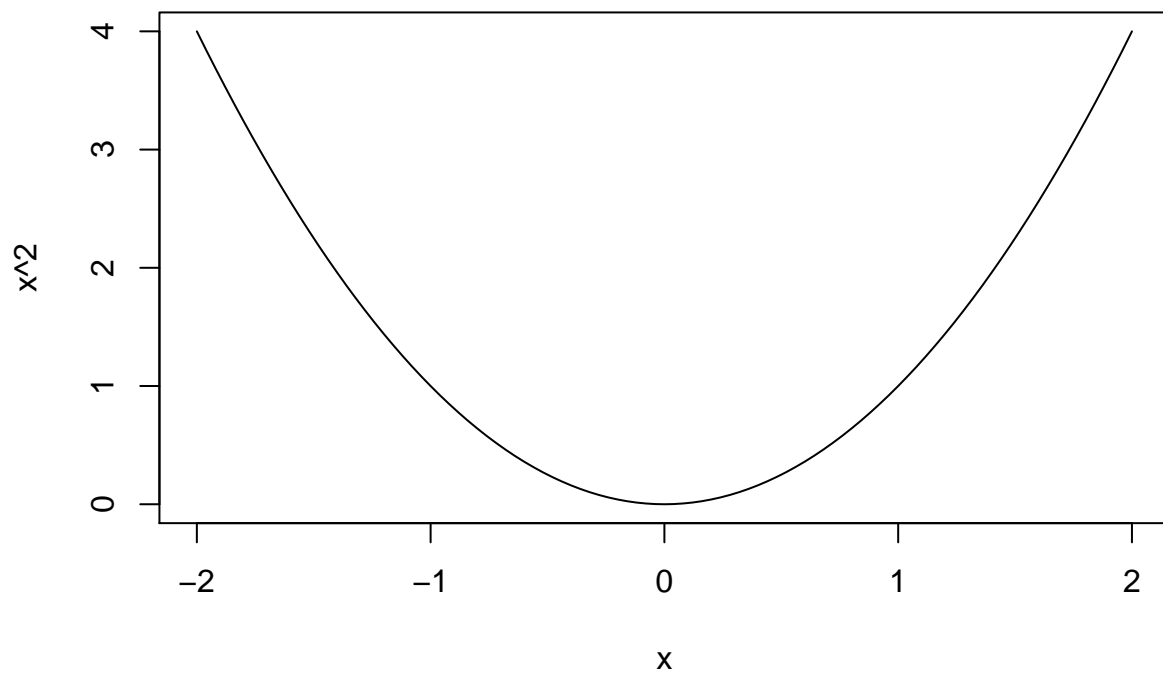
```
hist(x,prob=TRUE)
```



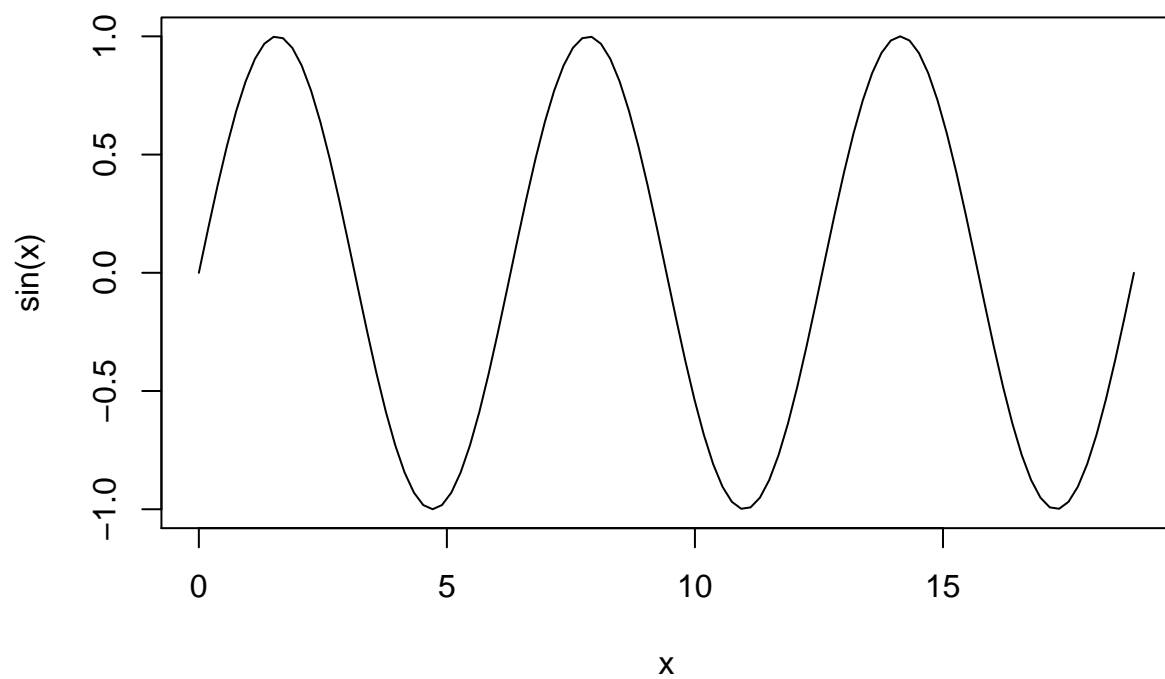
```
## Scatter plot  
x=seq(1,10)  
y=x^2-10*x  
plot(y~x)
```



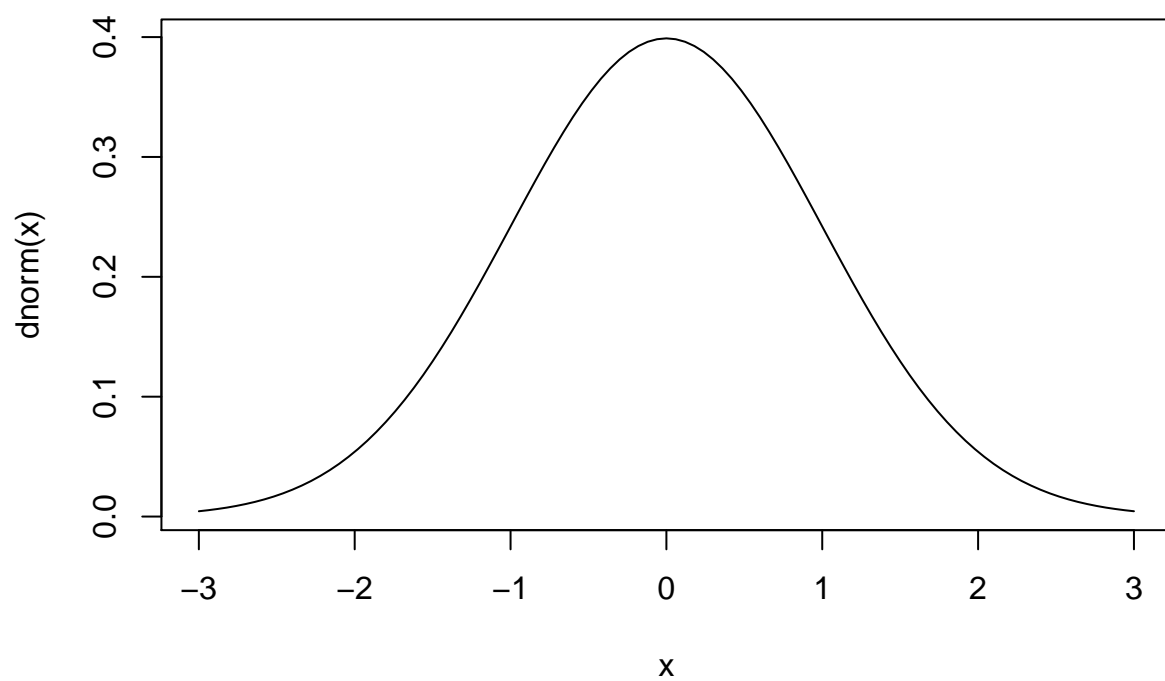
```
## Curve  
curve(x^2,from=-2,to=2)
```



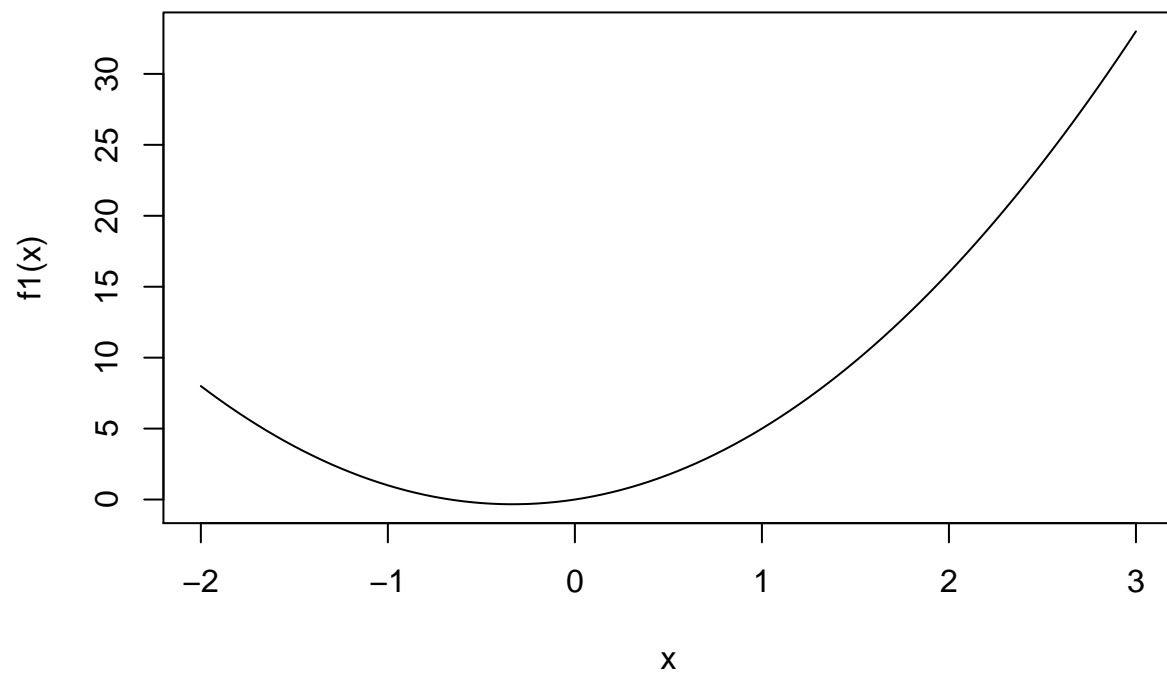
```
curve(expr=sin,from=0,to=6*pi)
```



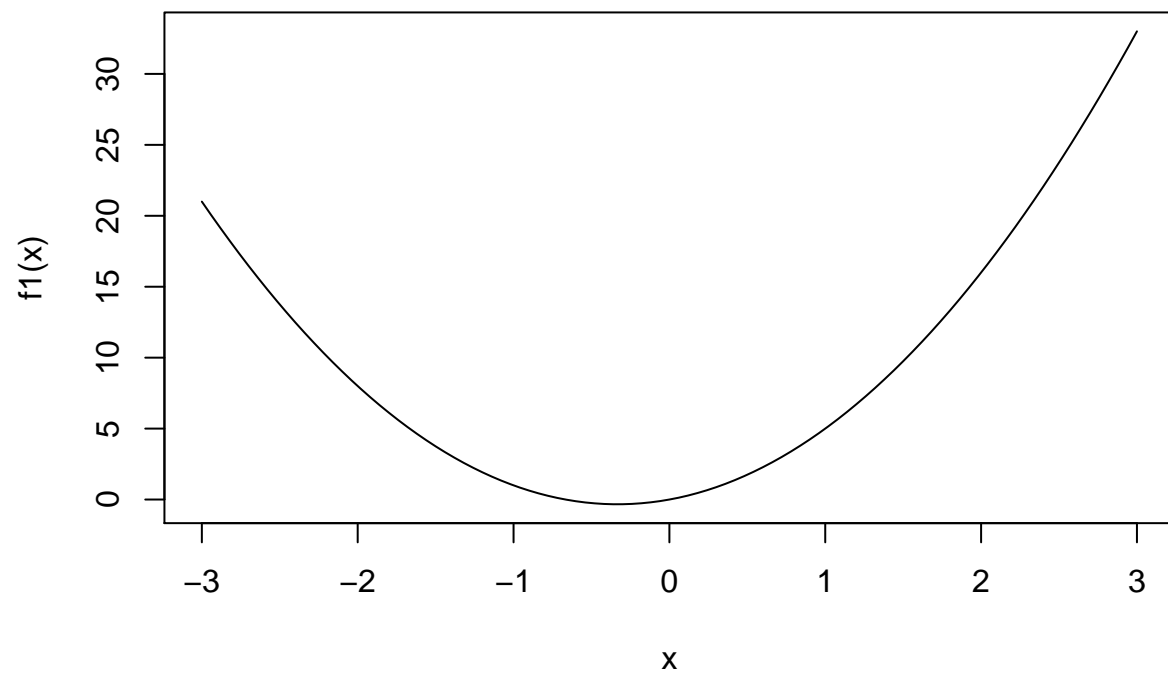
```
curve(dnorm(x),from=-3,to=3)
```



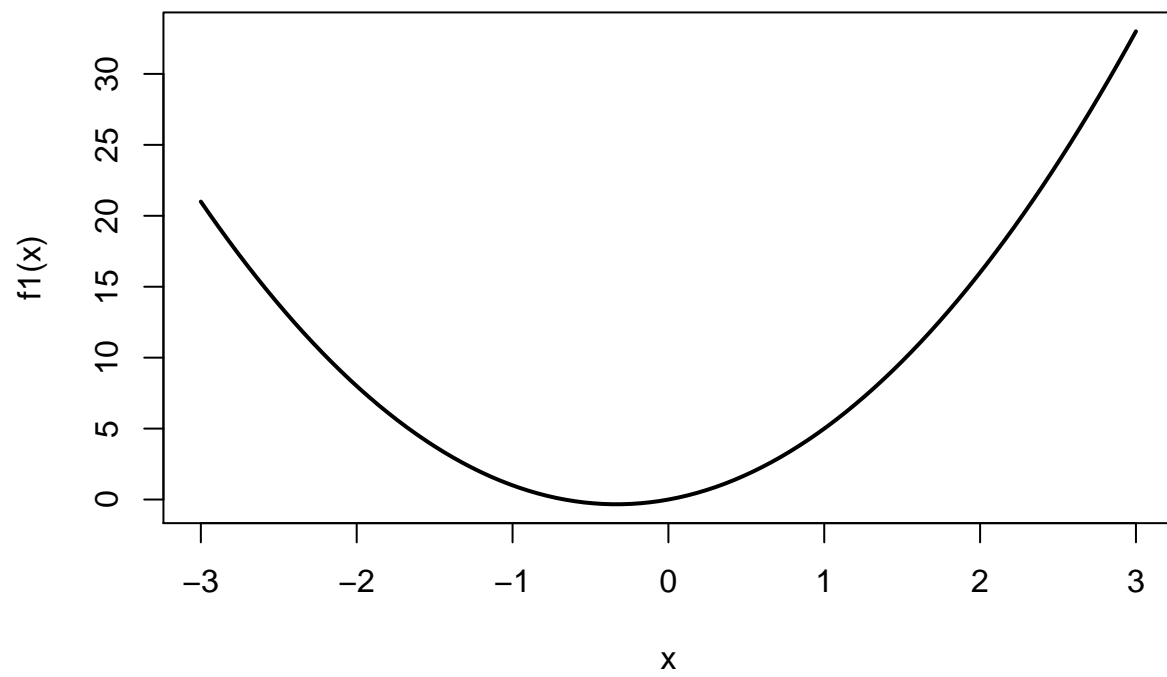
```
## Suppose you want to plot a function which is not defined in R.  
# Plot  $f_1(x)=2x+3x^2$ , -2 to 3  
f1=function(x) 2*x+3*x^2  
curve(f1,from=-2,to=3)
```



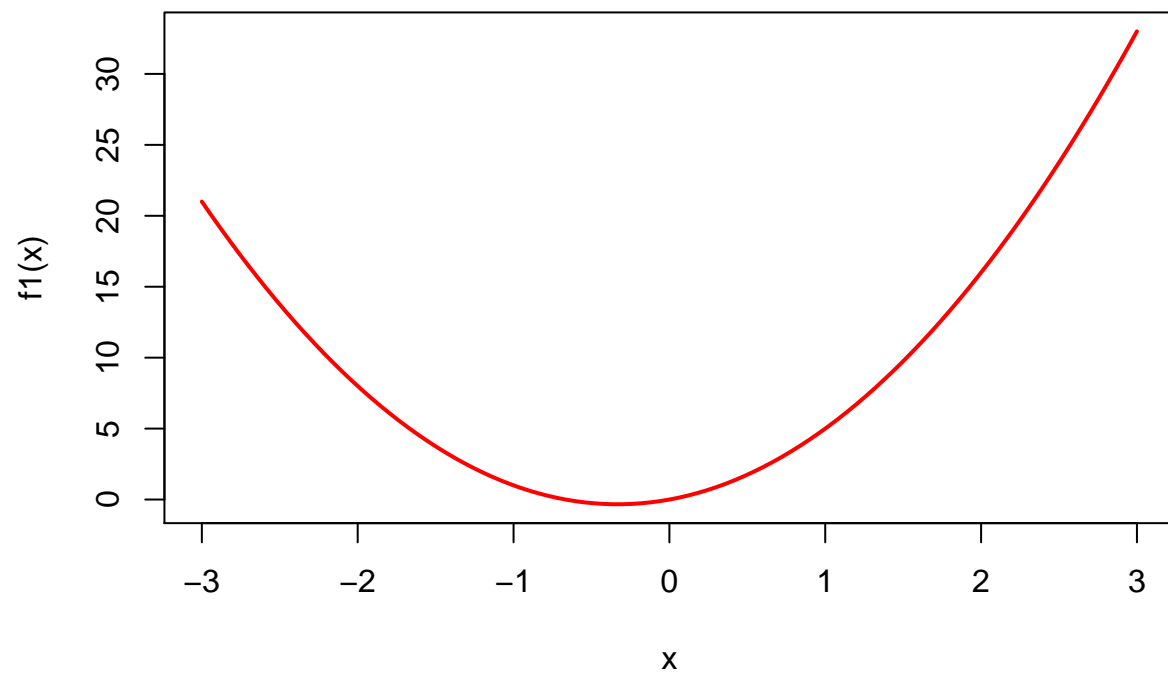
```
curve(f1,from=-3,to=3)
```



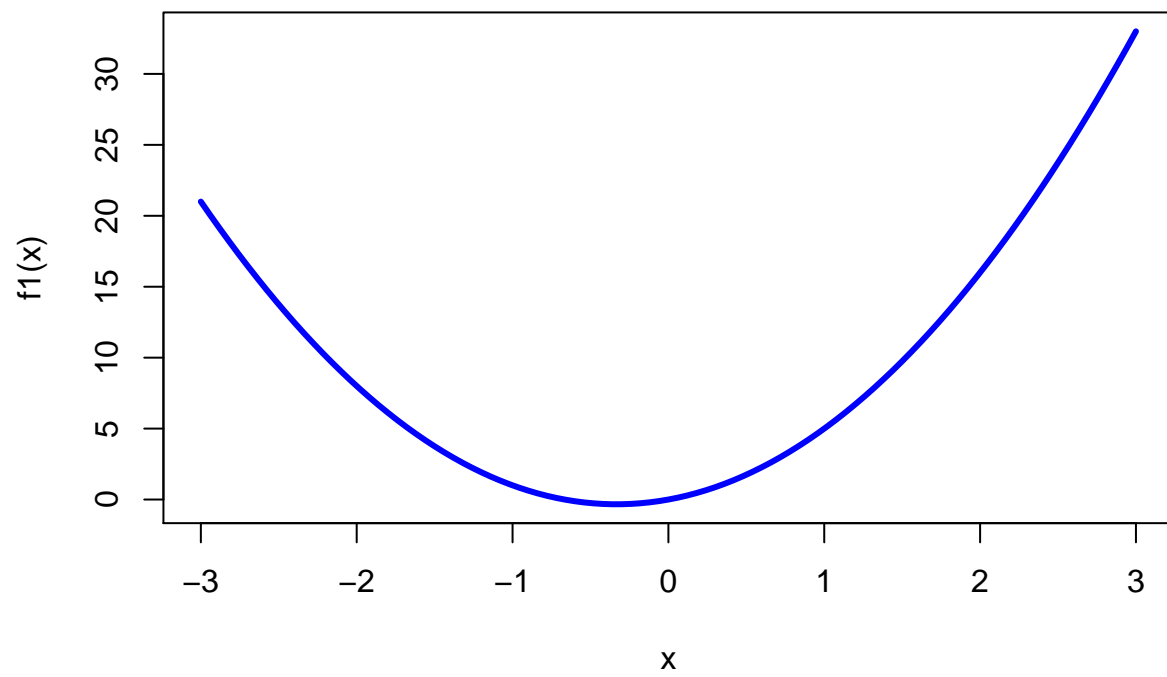
```
curve(f1,from=-3,to=3,lwd=2)
```

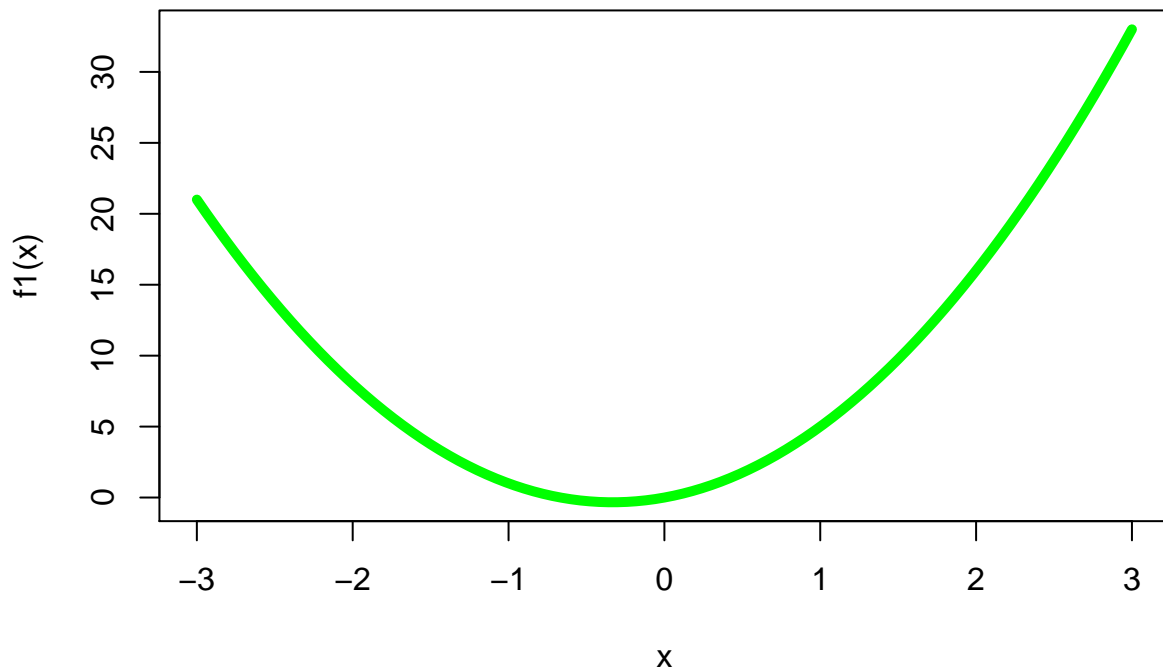
```
curve(f1,from=-3,to=3,lwd=2,col="red")
```



```
curve(f1,from=-3,to=3,lwd=3,col="blue")
```



```
curve(f1,from=-3,to=3,lwd=5,col="green")
```



Histogram, Frequency Curve, Polygon, Ogive for a data

```
# weight of 30 students
weight=c(68,53,69.5,55,71,63,76.5,65.5,69,75,76,57,70.5,71.5,56,81.5,69,59,67.5,61,68,59.5,56.5,73,61,70)
dump("weight","weight.dat")
length(weight)

## [1] 30

# Create Histogram
h1=hist(weight)
h1

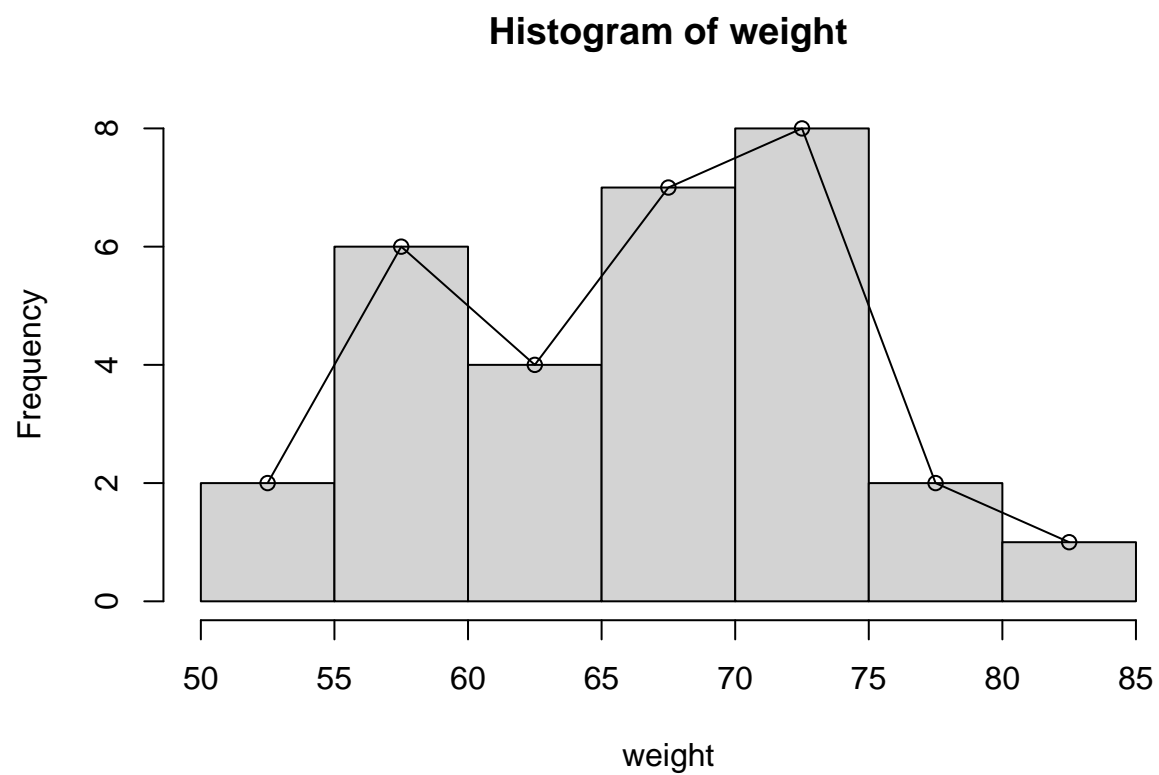
## $breaks
## [1] 50 55 60 65 70 75 80 85
## 
## $counts
## [1] 2 6 4 7 8 2 1
## 
## $density
## [1] 0.01333333 0.04000000 0.02666667 0.04666667 0.05333333 0.01333333
## [7] 0.00666667
## 
## $mids
## [1] 52.5 57.5 62.5 67.5 72.5 77.5 82.5
## 
## $xname
```

```
## [1] "weight"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
```

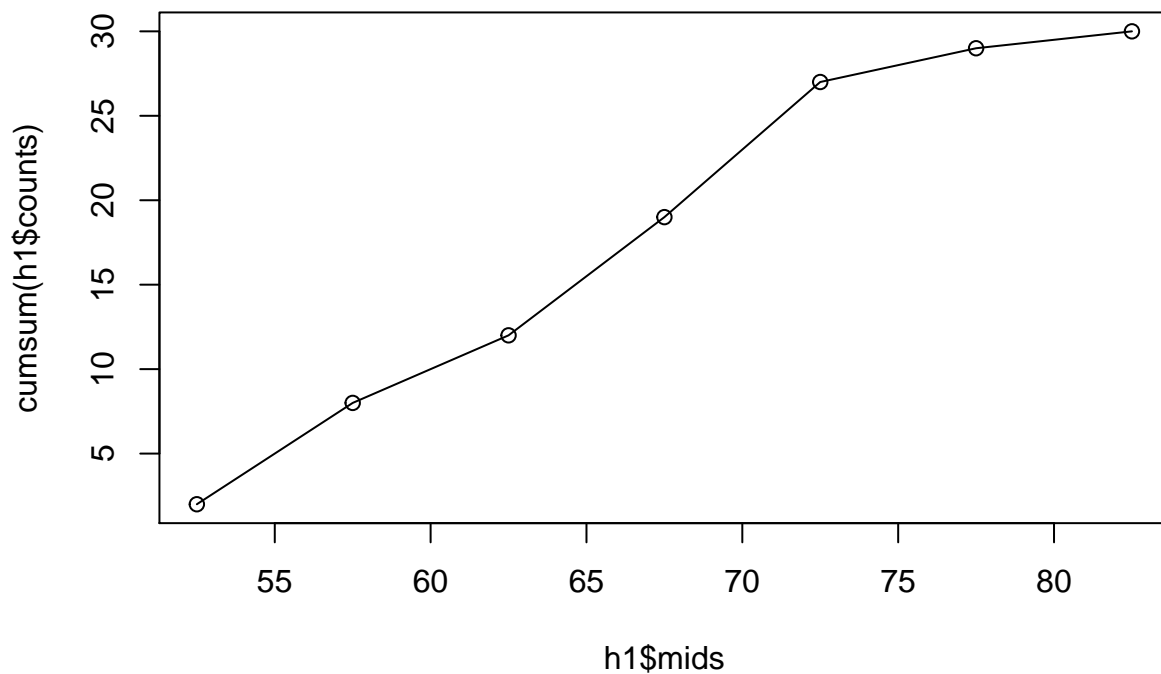
```
h1$breaks
```

```
## [1] 50 55 60 65 70 75 80 85
```

```
# Frequency polygon on histogram itself
lines(h1$mids,h1$counts,type="o")
```



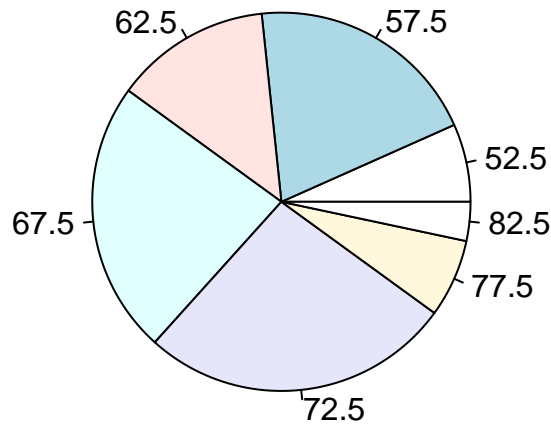
```
plot(h1$mids,cumsum(h1$counts),type="o")
```



```
stem(weight)
```

```
##
## The decimal point is 1 digit(s) to the right of the |
##
## 5 | 3
## 5 | 56779
## 6 | 001133
## 6 | 688899
## 7 | 0112233
## 7 | 5567
## 8 | 2
```

```
# Create a pie chart for the same data
pie(h1$count, label=h1$mids)
```



##Graphics with R There are two type of graphic functions:

1. High level graphic functions which always create a new plot. Examples are; plot(),hist(), barplot(), dotchart(), pie(), curve().
2. Low level graphic functions: They add on existing plot. For example, lines(), points(), mtext(), main(), sub().

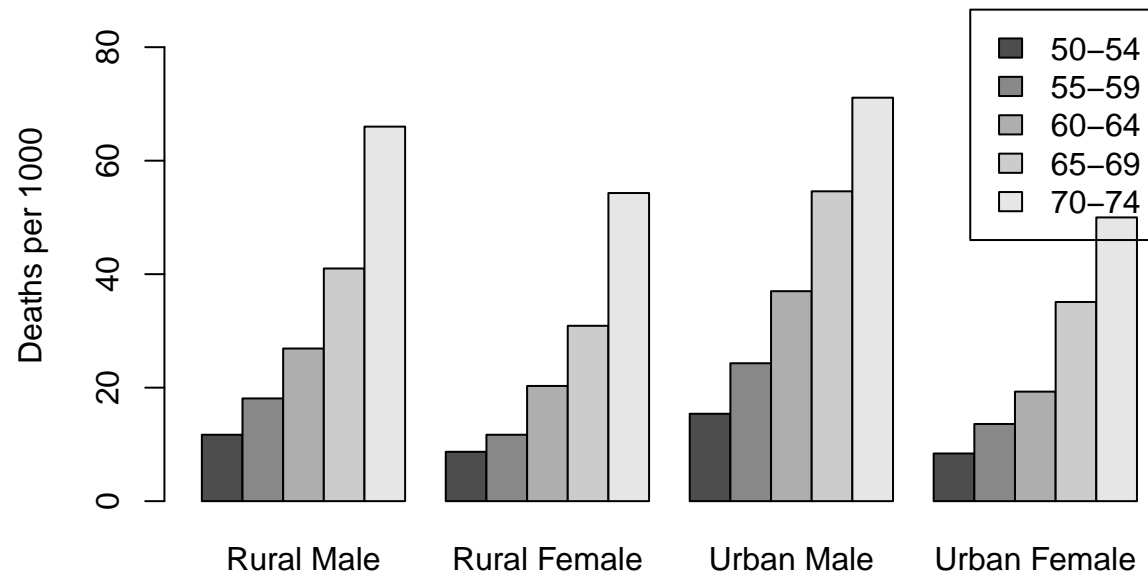
Example: Bar charts and dot charts The data VADeaths a data set in R contains death rates (number of deaths per 1000 population per year) in various subpopulations within state of Viginia in 1940. This may be plotted as barchart.

VADeaths

```
##      Rural Male Rural Female Urban Male Urban Female
## 50-54      11.7       8.7      15.4       8.4
## 55-59      18.1      11.7      24.3      13.6
## 60-64      26.9      20.3      37.0      19.3
## 65-69      41.0      30.9      54.6      35.1
## 70-74      66.0      54.3      71.1      50.0
```

```
barplot(VADeaths, beside = TRUE, legend=TRUE,ylim=c(0,90),
ylab = "Deaths per 1000", main="Death rates in Virginia")
```

Death rates in Virginia

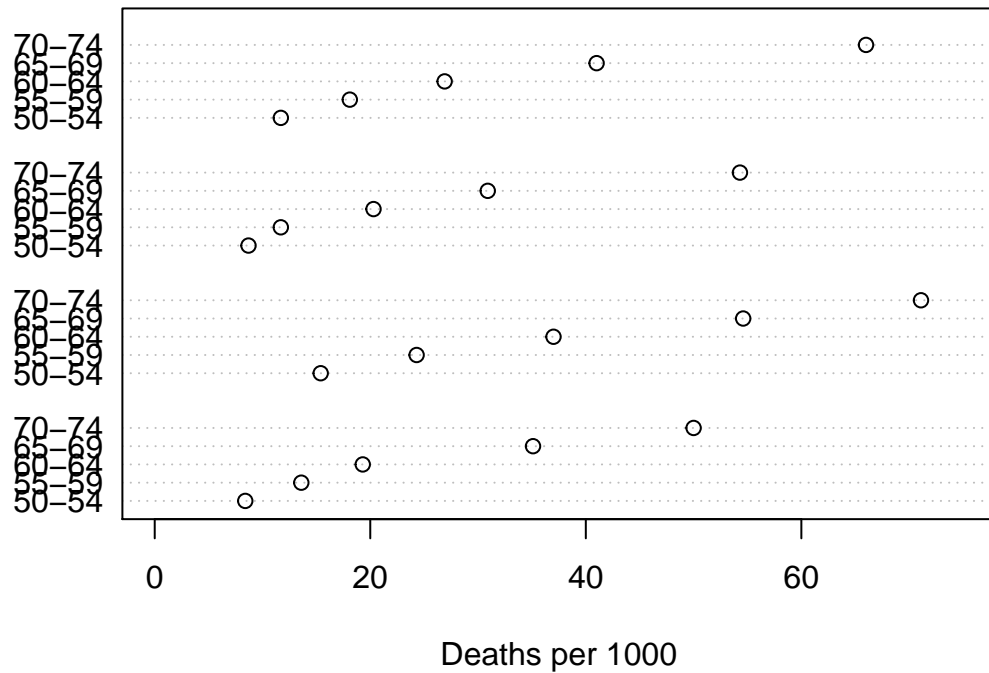


Dot Chart—'dotchart()'

Position of dot of corresponding number.

```
dotchart(VADeaths, xlim=c(0,75), xlab="Deaths per 1000", main="Death rate in Virginia")
```

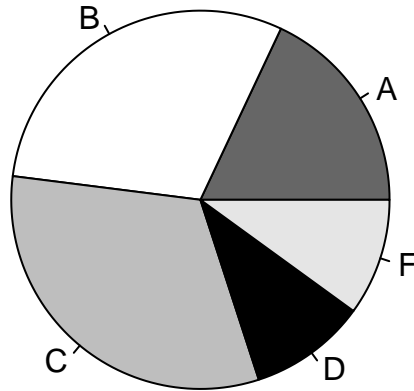

Death rate in Virginia



Pie chart

Pie chart displays a vector of numbers by breaking up a circular disk into pieces whose angle (and hence area) is proportional to each number. For example, the letter grades assigned to each class might arise in the proportions shown in the following figure:

```
groupsizes=c(18,30,32,10,10)
labels=c("A","B","C","D","F")
pie(groupsizes, labels, col=c("grey40", "white","grey","black", "grey90"))
```



Histogram—hist()

Example for histogram

```
# simulate 5000 observations from N(0,1)
# make a histogram for the simulated data
x=rnorm(n=1000,mean=0,sd=1)
x
```

```
##      [1]  0.1052027104 -0.3084765708 -1.9991944892 -1.2592551526  1.5793188272
##      [6]  0.6267640375  0.6245298228  0.6818180820  0.1501646573  0.2379173873
##     [11]  1.0373886134 -1.5435226793  0.0690591245 -1.4123320962 -1.3784309980
##     [16]  0.0331350277 -0.5778872452 -0.8521736068 -0.4561704177 -0.0034668910
##     [21] -1.1259995016 -0.3204263716 -0.9013836436 -1.0294233224 -0.6264214813
##     [26]  0.0352620815  0.0668443206 -1.0116676377  0.1045350823 -2.4587902297
##     [31] -3.5688303218  0.0465314697 -0.1757626827  0.4510521704 -0.8581849776
##     [36]  1.1612630471 -0.1604437898  2.2169665525 -0.1179537327  0.3420286202
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## [956]  0.0553337951  0.3599691186 -0.4893066579  1.3655698356  1.0665707930
## [961] -0.5342244420 -0.0382079474 -0.8663823251 -0.6322328415 -0.8708468134
## [966]  0.2371176074 -0.1174438250 -0.7638960137 -0.7877759840 -0.9419285099
## [971]  0.7886895826  1.0540125484 -2.3116646312  0.0417254140  1.4316062844
## [976]  0.9629673523 -1.7548757654  0.0588601866 -0.7861373398 -0.0442734031
## [981]  0.1987368535 -0.6053162548 -0.8075372192  1.4326539650  0.0540148218
## [986]  1.7939045857  0.6346471813 -1.2422044083 -0.0555730726  0.0829397507
## [991]  0.1665541147  0.8086654576 -0.1366476158 -1.6932806546 -0.9177512378
## [996] -1.2480091310  2.2352144987 -1.0876223086  1.9978945980  0.9418505676
```

```
mean(x)
```

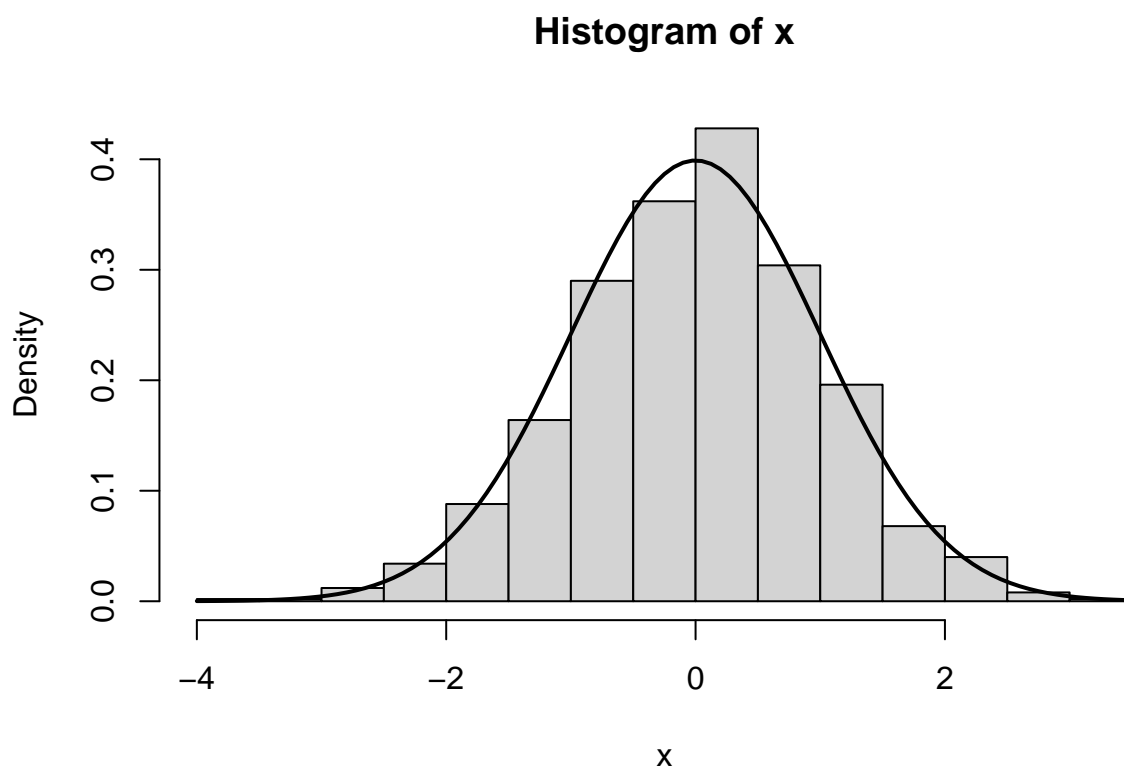
```
## [1] 0.01007682
```

```
sd(x)
```

```
## [1] 0.9975375
```

```
hist(x,prob=TRUE)
```

```
curve(dnorm(x,mean=0,sd=1),add=TRUE,lwd=2) #to add on histogram
```

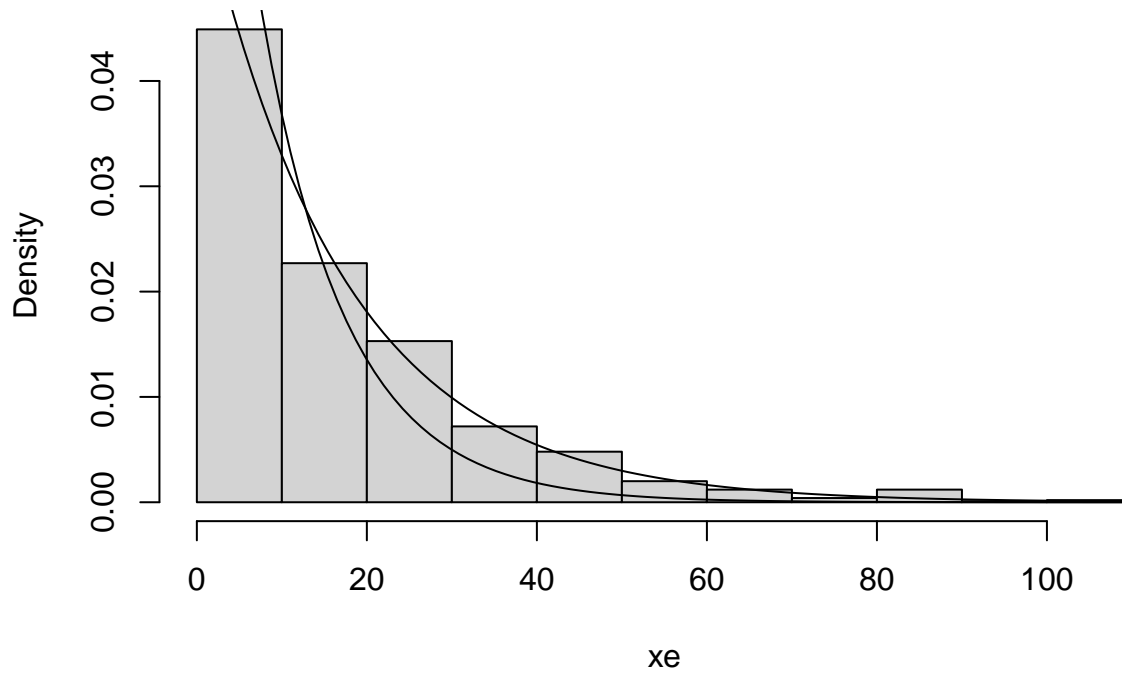


```
# An example with exponential  
xe=rexp(1000,rate=0.06)  
mean(xe)
```

```
## [1] 17.07742
```

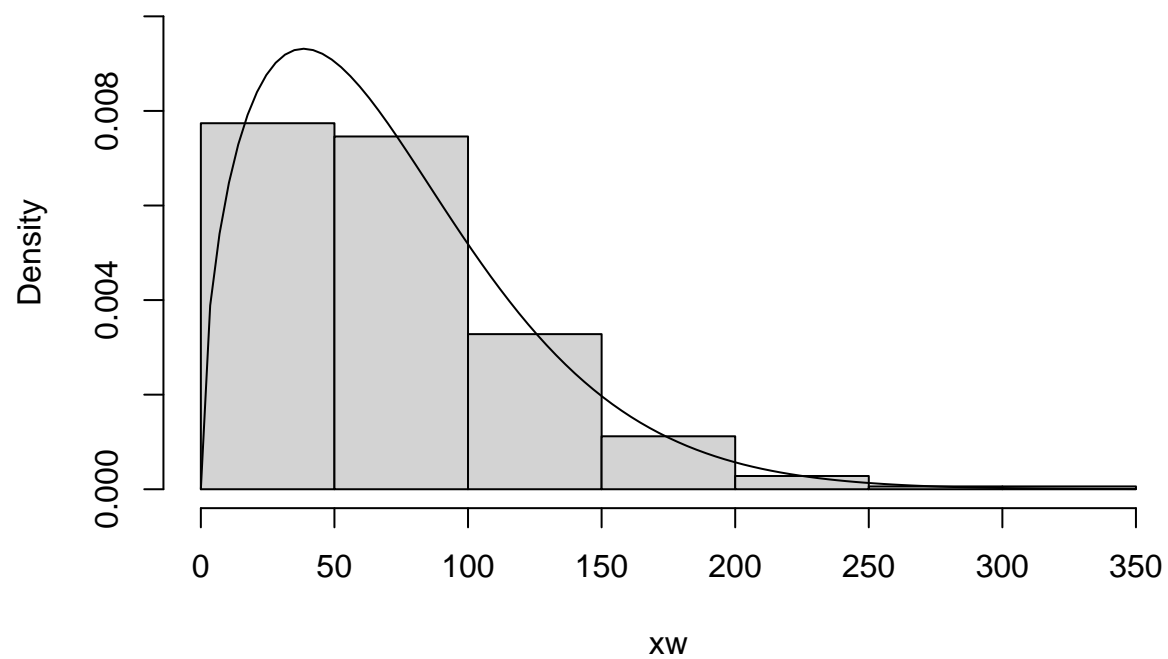
```
hist(xe, prob=TRUE)  
curve(dexp(x,rate=0.06),add=TRUE)  
curve(dexp(x,rate=0.10),add=TRUE)
```

Histogram of xe



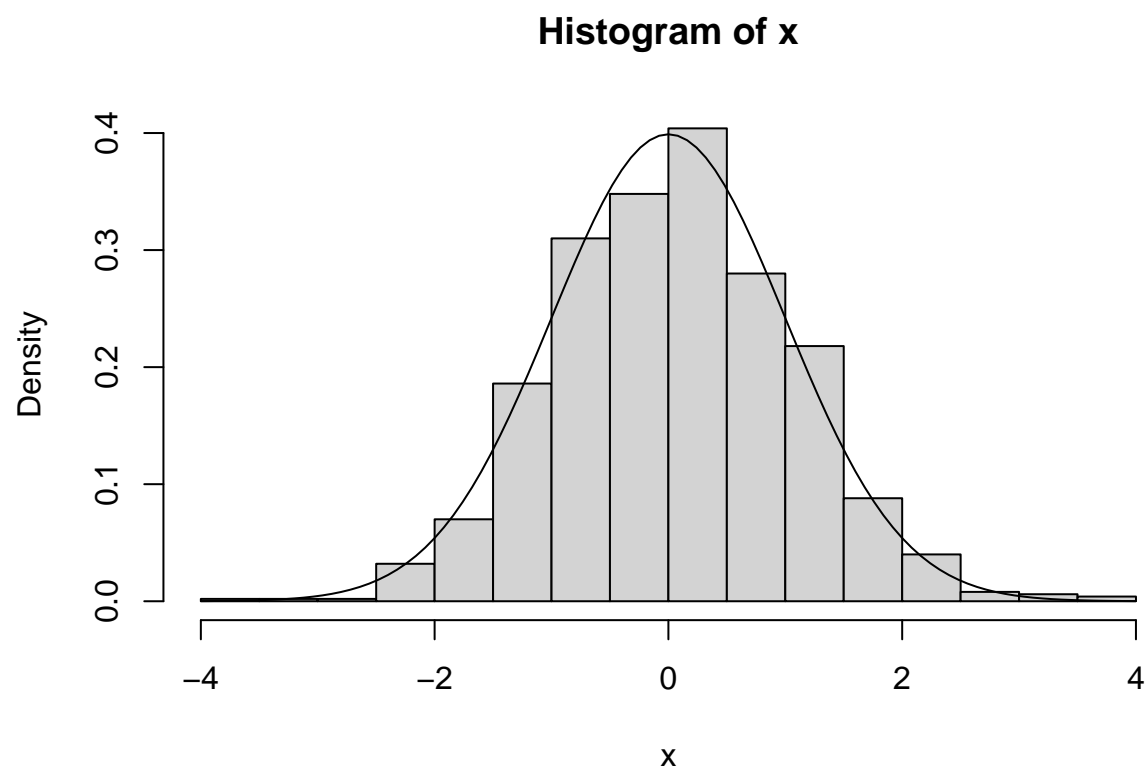
```
# Simulate 1000 random values from Weibull(shape=1.2,scale=16)
xw=rweibull(n=1000,shape=1.5, scale=80)
hist(xw,prob=TRUE,ylim=c(0,.01))
curve(dweibull(x,shape=1.5,scale=80),add=TRUE)
```


Histogram of xw



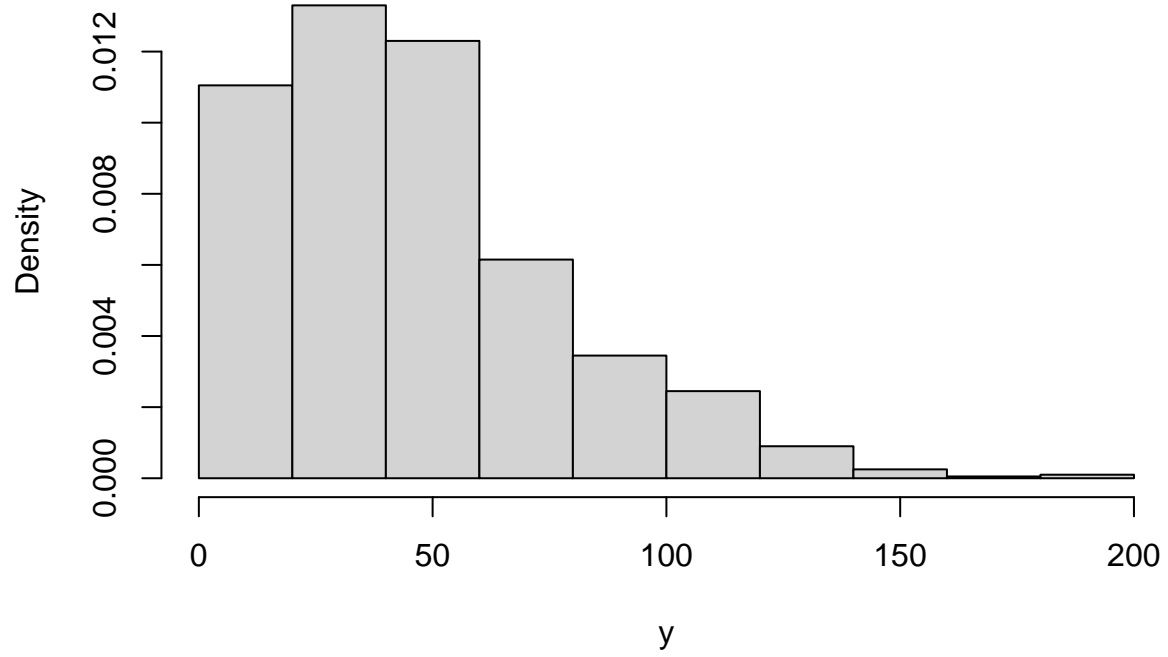
Details of histogram

```
x=rnorm(1000)
hist(x,prob=TRUE)
curve(dnorm(x),add=TRUE)
```

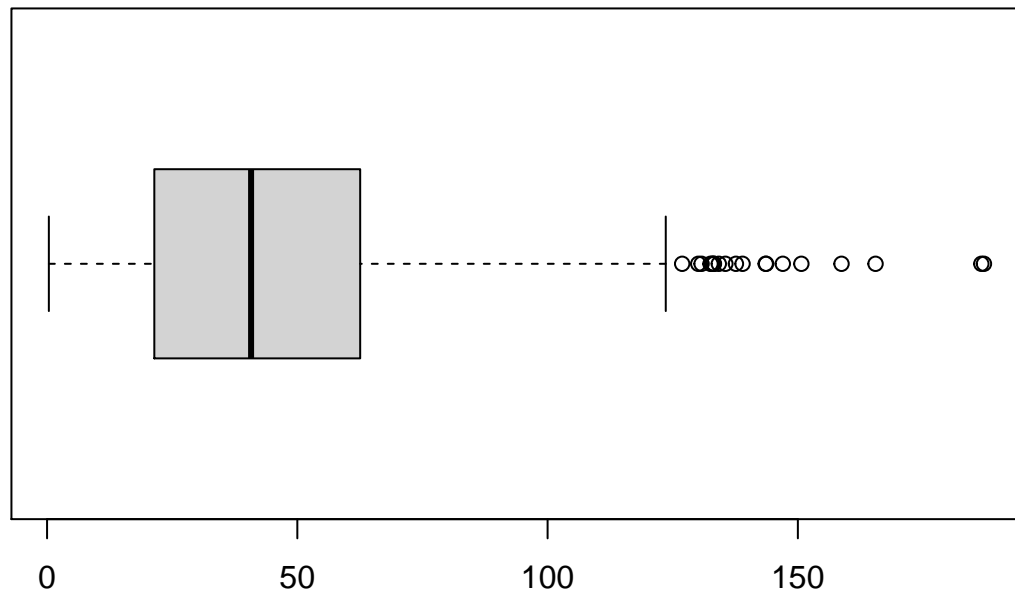


```
## Box plot—boxplot()
y=rweibull(1000,shape=1.5,scale=50)
hist(y,prob=TRUE)
```

Histogram of y



```
boxplot(y, horizontal=TRUE)
```



```
median(y)
```

```
## [1] 40.76161
```

```
quantile(y,prob=c(0.25,.50,.75))
```

```
##      25%      50%      75%
## 21.43287 40.76161 62.50444
```

```
min(y)
```

```
## [1] 0.3527543
```

```
max(y)
```

```
## [1] 187.1773
```

```
IQR(y)
```

```
## [1] 41.07156
```

```
LowerWhisker=max(min(y),20.449-1.5*IQR(y))
LowerWhisker
```

```
## [1] 0.3527543
```

```
UpperWhisker=min(max(y),60.734+1.5*IQR(y))
UpperWhisker
```

```
## [1] 122.3413
```

Box plot of 'Iris' data

```
names(iris)
```

```
## [1] "Sepal.Length" "Sepal.Width" "Petal.Length" "Petal.Width" "Species"
```

```
head(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1         5.1         3.5         1.4         0.2   setosa
## 2         4.9         3.0         1.4         0.2   setosa
## 3         4.7         3.2         1.3         0.2   setosa
## 4         4.6         3.1         1.5         0.2   setosa
## 5         5.0         3.6         1.4         0.2   setosa
## 6         5.4         3.9         1.7         0.4   setosa
```

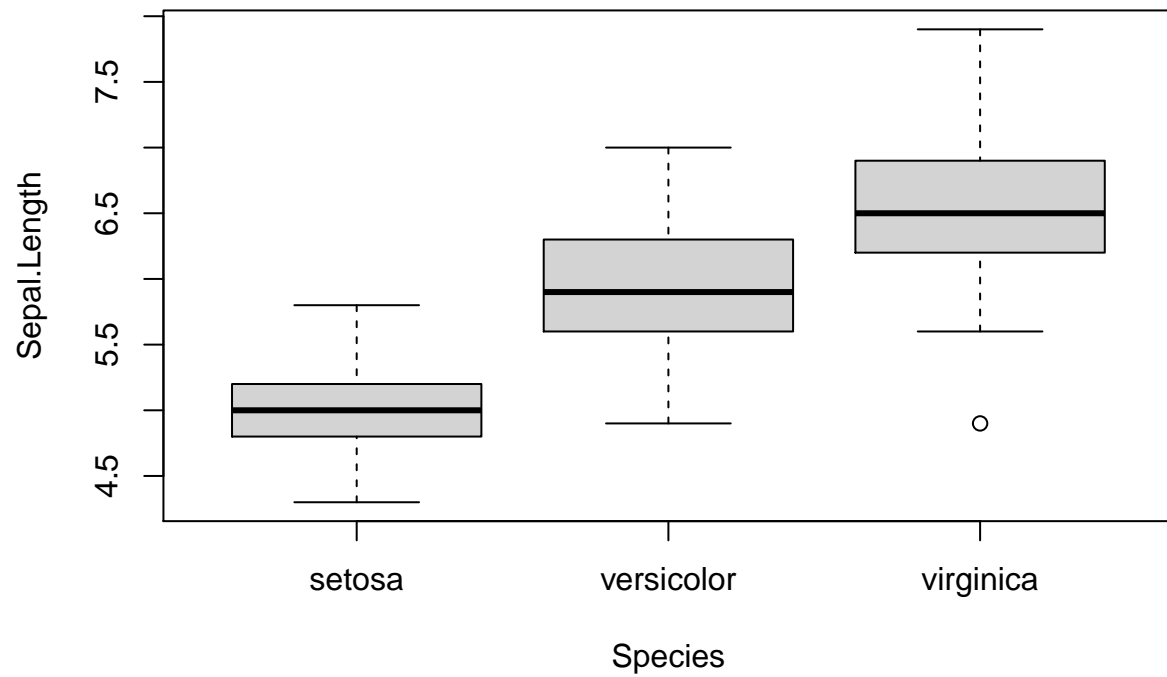
```
tail(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width  Species
## 145         6.7         3.3         5.7         2.5 virginica
## 146         6.7         3.0         5.2         2.3 virginica
## 147         6.3         2.5         5.0         1.9 virginica
## 148         6.5         3.0         5.2         2.0 virginica
## 149         6.2         3.4         5.4         2.3 virginica
## 150         5.9         3.0         5.1         1.8 virginica
```

```
str(iris)
```

```
## 'data.frame':   150 obs. of  5 variables:
##  $ Sepal.Length: num  5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
##  $ Sepal.Width : num  3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
##  $ Petal.Length: num  1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
##  $ Petal.Width : num  0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...
##  $ Species      : Factor w/ 3 levels "setosa","versicolor",...: 1 1 1 1 1 1 1 1 1 1 ...
```

```
boxplot(Sepal.Length~Species,data=iris)
```



Scatter plot—‘plot()’

```
x= rnorm(100) # assigns 100 random normal observations to x
y=rpois(100,30)
mean(y)
```

```
## [1] 29.68
```

```
mean(x)
```

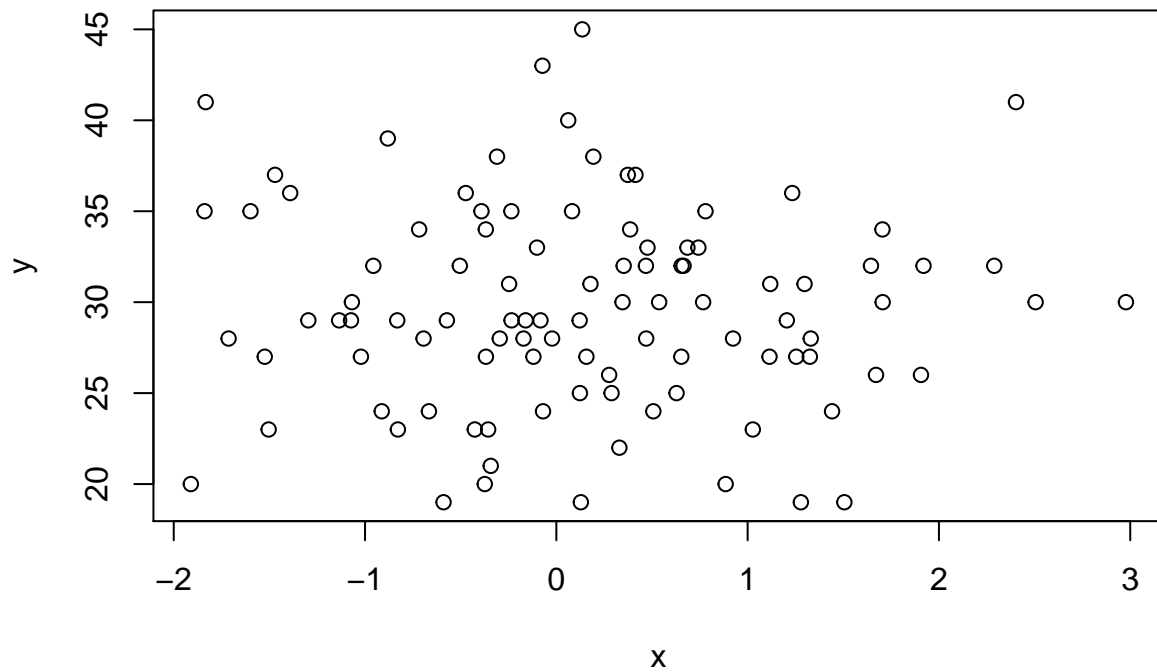
```
## [1] 0.1589246
```

```
sd(x)
```

```
## [1] 1.045671
```

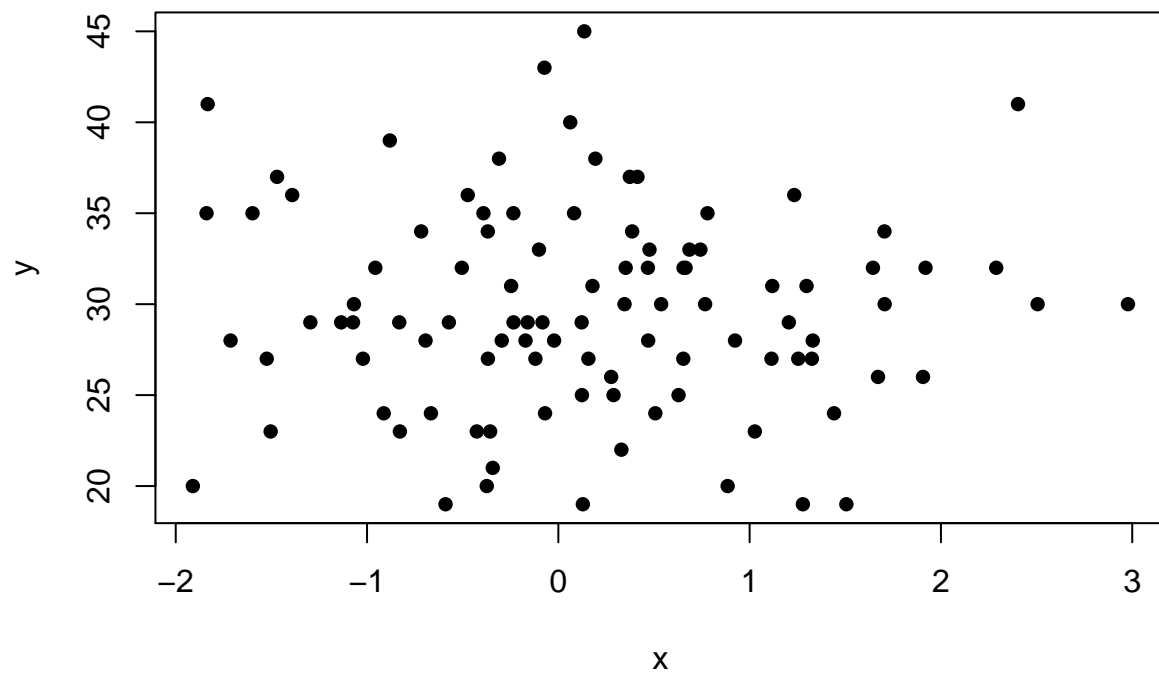
```
plot(x,y,main="Poisson versus Normal")
```

Poisson versus Normal

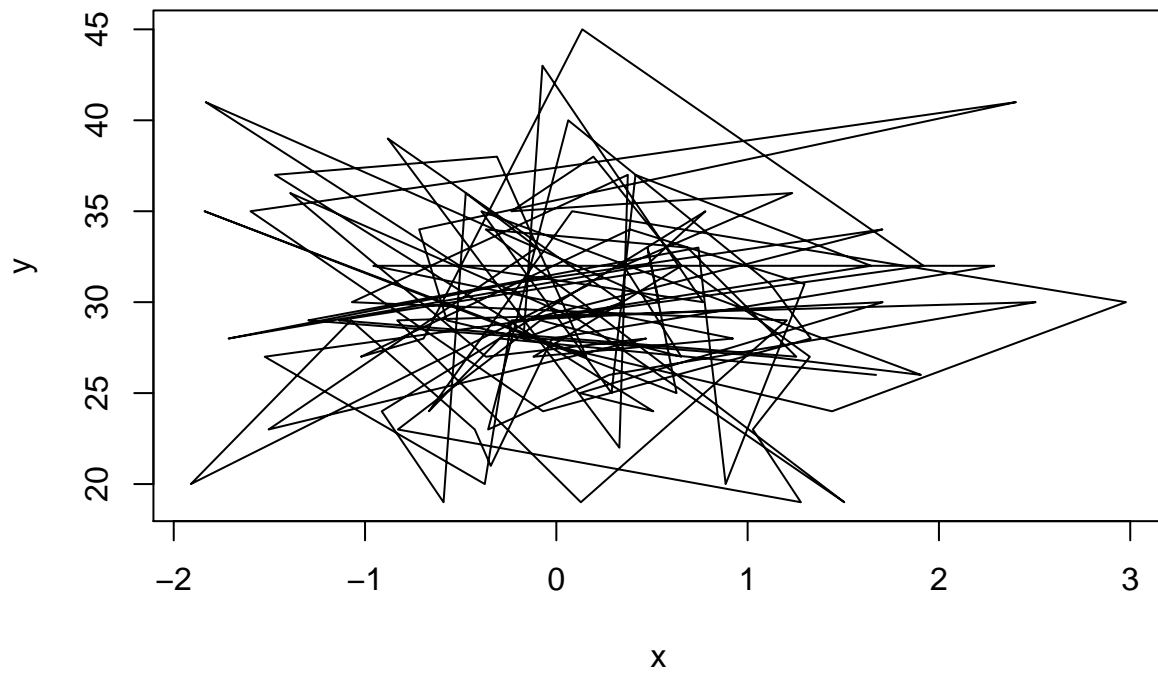


```
plot(x,y,pch=16,type="points")
```

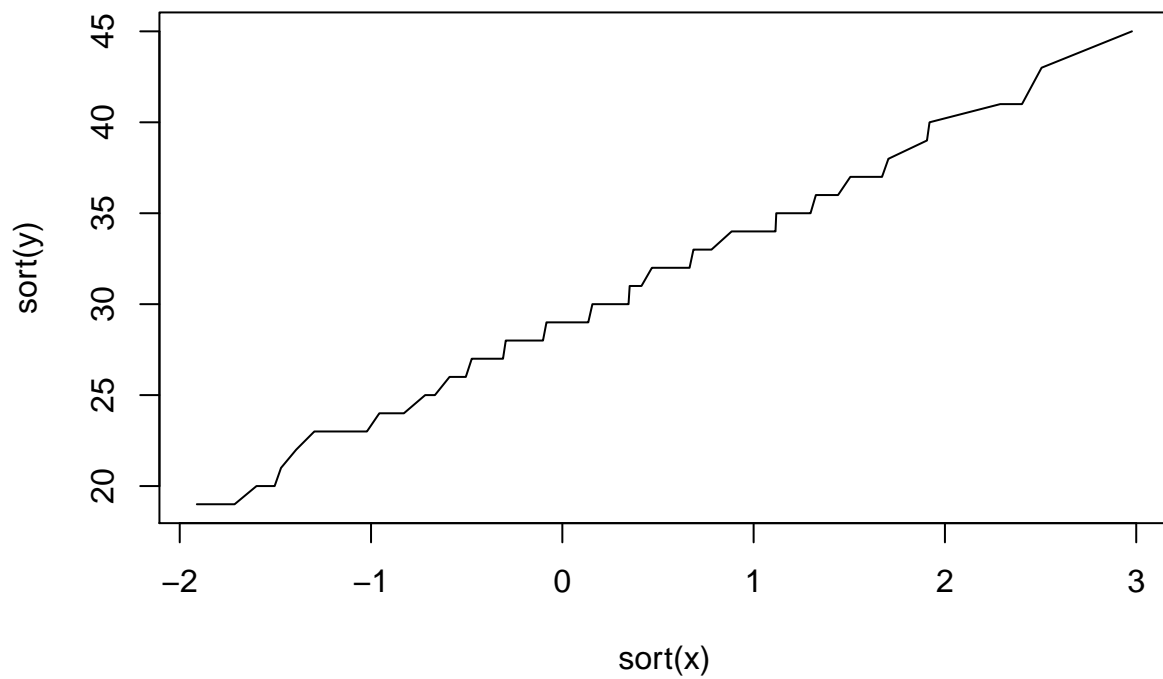
```
## Warning in plot.xy(xy, type, ...): plot type 'points' will be truncated to first  
## character
```



```
plot(x,y,type="l") #lines
```

```
plot(sort(x),sort(y),type="l")
```



Plotting From a data frame

```
data(Orange)
names(Orange)
```

```
## [1] "Tree"      "age"       "circumference"
```

```
head(Orange)
```

```
##   Tree age circumference
## 1    1 118             30
## 2    1 484             58
## 3    1 664             87
## 4    1 1004            115
## 5    1 1231            120
## 6    1 1372            142
```

```
tail(Orange)
```

```
##   Tree age circumference
## 30    5 484             49
## 31    5 664             81
## 32    5 1004            125
## 33    5 1231            142
## 34    5 1372            174
## 35    5 1582            177
```

```
str(Orange)
```

```
## Classes 'nfnGroupedData', 'nfGroupedData', 'groupedData' and 'data.frame': 35 obs. of 3 variables
## $ Tree : Ord.factor w/ 5 levels "3"<"1"<"5"<"2"<...: 2 2 2 2 2 2 2 2 4 4 4 ...
## $ age : num 118 484 664 1004 1231 ...
## $ circumference: num 30 58 87 115 120 142 145 33 69 111 ...
## - attr(*, "formula")=Class 'formula' language circumference ~ age | Tree
## ..- attr(*, ".Environment")=<environment: R_EmptyEnv>
## - attr(*, "labels")=List of 2
## ..$ x: chr "Time since December 31, 1968"
## ..$ y: chr "Trunk circumference"
## - attr(*, "units")=List of 2
## ..$ x: chr "(days)"
## ..$ y: chr "(mm)"
```

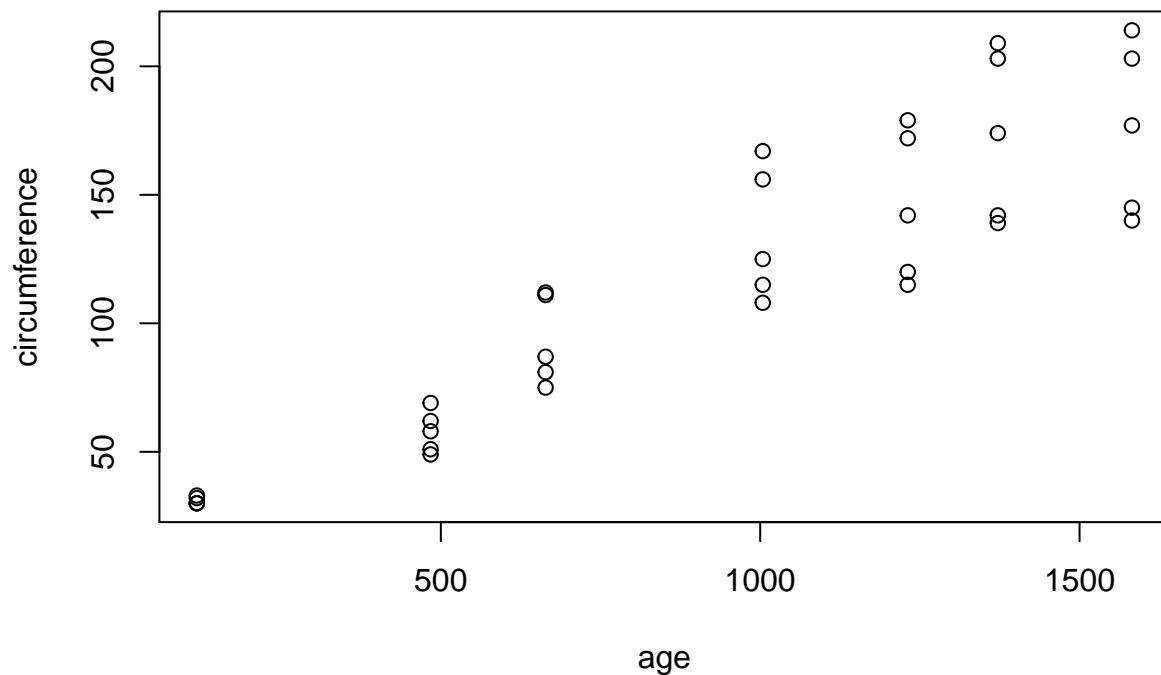
```
Orange$Tree #extract column of Tree
```

```
## [1] 1 1 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 3 3 4 4 4 4 4 4 4 5 5 5 5 5 5
## Levels: 3 < 1 < 5 < 2 < 4
```

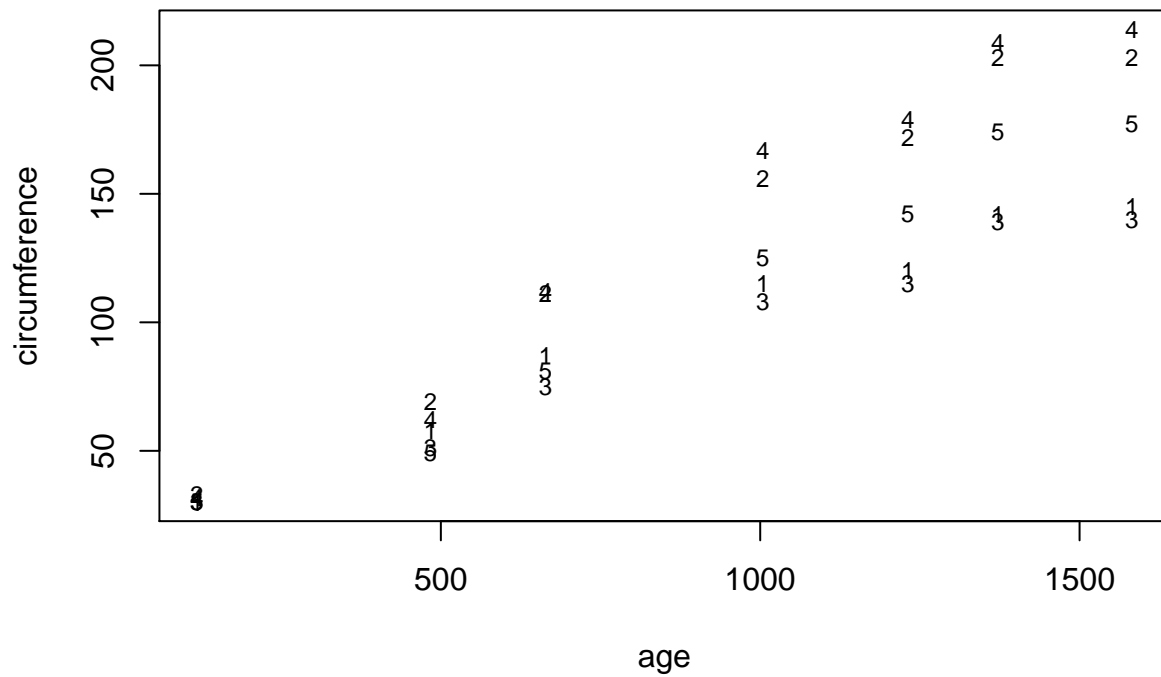
```
unique(as.character(Orange$Tree))
```

```
## [1] "1" "2" "3" "4" "5"
```

```
plot(circumference~age, data = Orange)
```



```
plot(circumference~age, data = Orange, pch = as.character(Tree),
cex = 0.75)
```



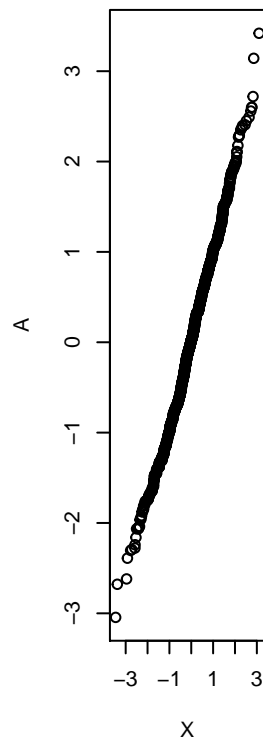
QQ plot

```
z<-rnorm(10000,mean=0,sd=1)
quantile(z,prob=c(0.25,.50,.75))
```

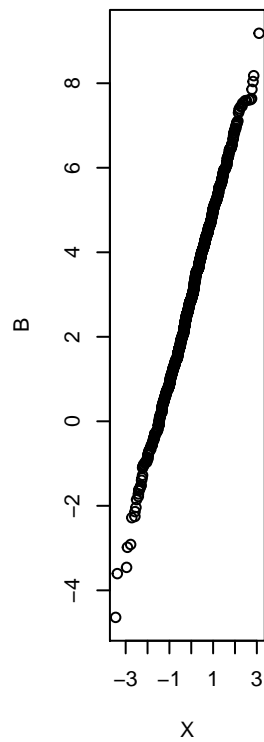
```
##          25%          50%          75%
## -0.683965522  0.008889652  0.676895665
```

```
par(mfrow = c(1,4))
X=rnorm(1000)
A=rnorm(1000)
qqplot(X,A,main="A and X are the same")
B=rnorm(1000,mean=3,sd=2)
qqplot(X,B,main="B is rescaled X")
C=rt(1000,df=2)
qqplot(X,C,main="C has heavier tails")
par(mfrow=c(1,1))
```

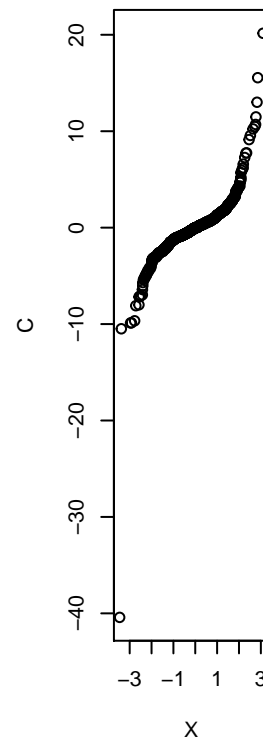
A and X are the same



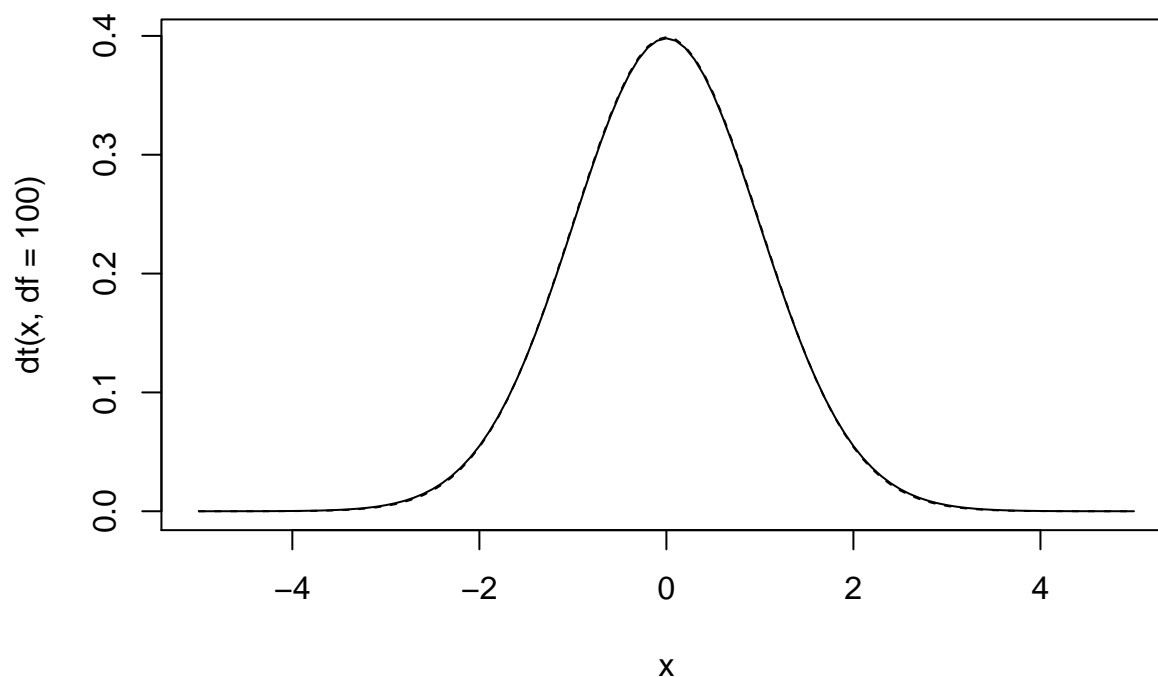
B is rescaled X



C has heavier tails



```
curve(dt(x,df=100),from=-5,to=5)
curve(dnorm(x),add=TRUE,lty=2)
```



Low level graphic functions

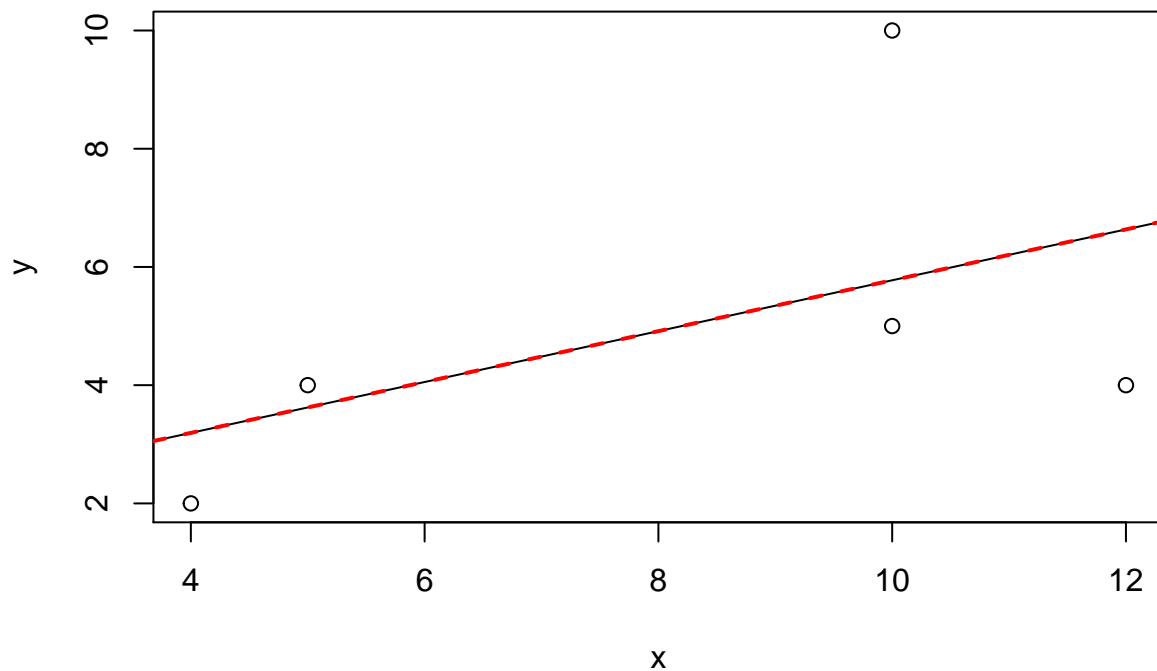
Adding to plots

- `points(x, y, ...)` # adds point
- `lines(x, y, ...)` # adds line segments
- `text(x, y, labels, ...)` # adds text into the graph
- `abline(a, b, ...)` # adds the line $y = a + bx$
- `abline(h = y, ...)` # adds a horizontal line
- `abline(v = x, ...)` # adds a vertical line
- `polygon(x, y, ...)` # adds a closed and possibly filled polygon
- `segments(x0, y0, x1, y1, ...)` # draws line segments
- `arrows(x0, y0, x1, y1, ...)` # draws arrows
- `symbols(x, y, ...)` # draws circles, squares, thermometers, etc. `legend(x, y, legend, ...)` # draws a *legend

Linear Models— ‘lm()’

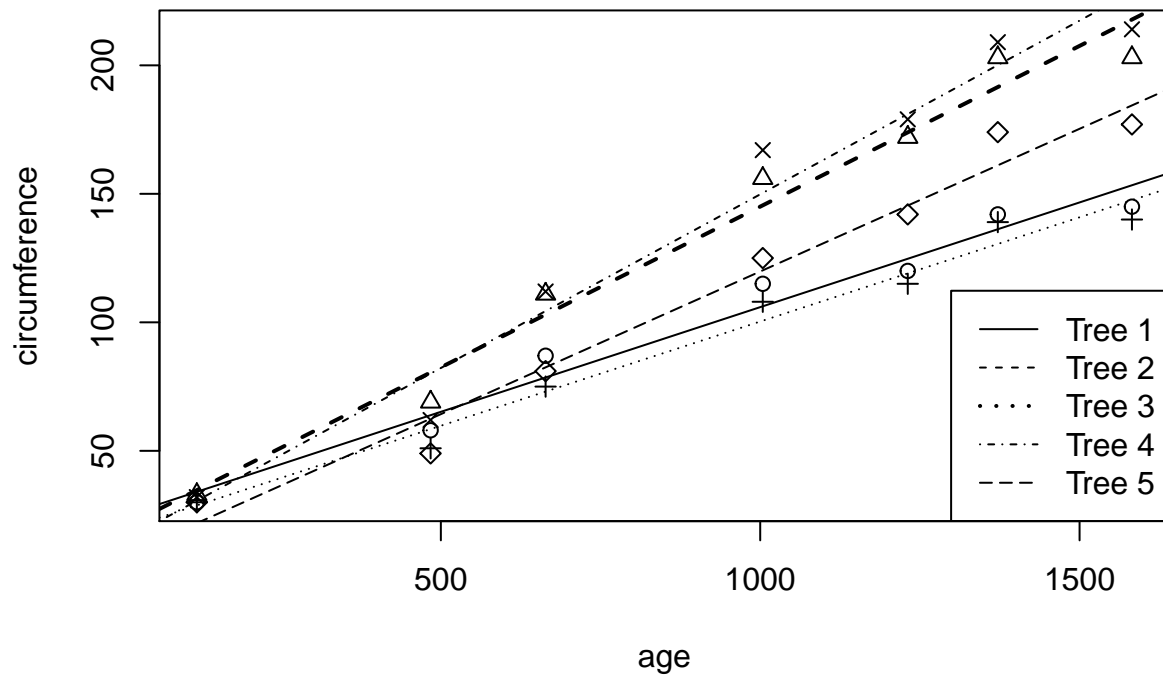
```
y=c(2,4,5,4,10)
x=c(4,5,10,12,10)
plot(y~x)
# y=beta0+beta1*x+error
M1=lm(y~x)
coef(M1)
```

```
## (Intercept)          x
##  1.4713115  0.4303279
abline(lm(y~x))
abline(M1,lwd=2,lty=2,col="red")
```



Example Orange data

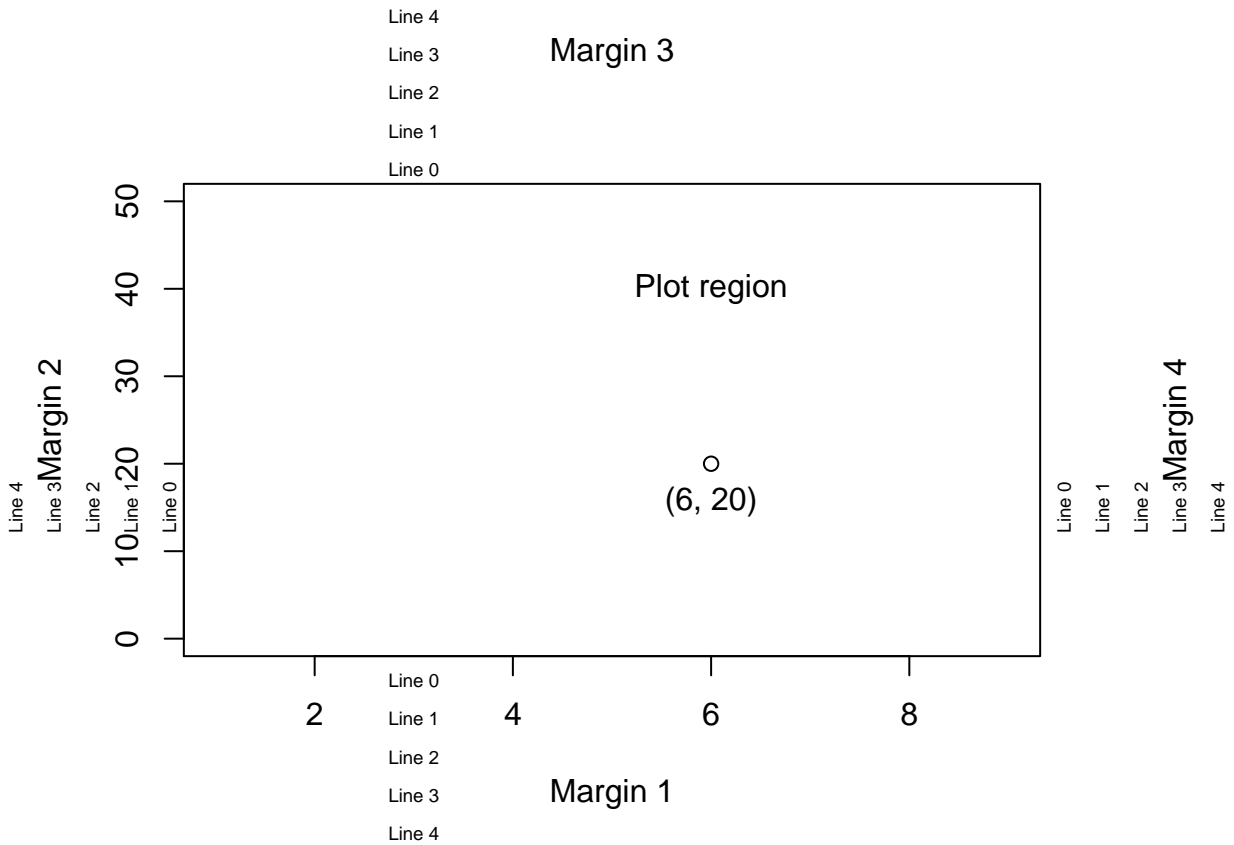
```
plot(circumference~age, pch=as.numeric(as.character(Tree)),data=Orange)
abline(lm(circumference~age,data=Orange,subset=Tree=="1"))
abline(lm(circumference~age,data=Orange,subset=Tree=="2"),lty=2,lwd=2)
abline(lm(circumference~age,data=Orange,subset=Tree=="3"),lty=3)
abline(lm(circumference~age,data=Orange,subset=Tree=="4"),lty=4)
abline(lm(circumference~age,data=Orange,subset=Tree=="5"),lty=5)
legend("bottomright",legend = paste("Tree",1:5),lty=1:5,lwd=c(1,1,2,1,1))
```



```

par(mar=c(5,5,5,5)+0.1)
plot(c(1,9),c(0,50),type='n' ,xlab="", ylab="")
text(6, 40, "Plot region")
points(6, 20)
text(6, 20, "(6, 20)", adj = c(0.5, 2))
mtext(paste("Margin", 1:4), side = 1:4, line = 3)
mtext(paste("Line", 0:4), side = 1, line = 0:4, at = 3, cex = 0.6)
mtext(paste("Line", 0:4), side = 2, line = 0:4, at = 15, cex = 0.6)
mtext(paste("Line", 0:4), side = 3, line = 0:4, at = 3, cex = 0.6)
mtext(paste("Line", 0:4), side = 4, line = 0:4, at = 15, cex = 0.6)

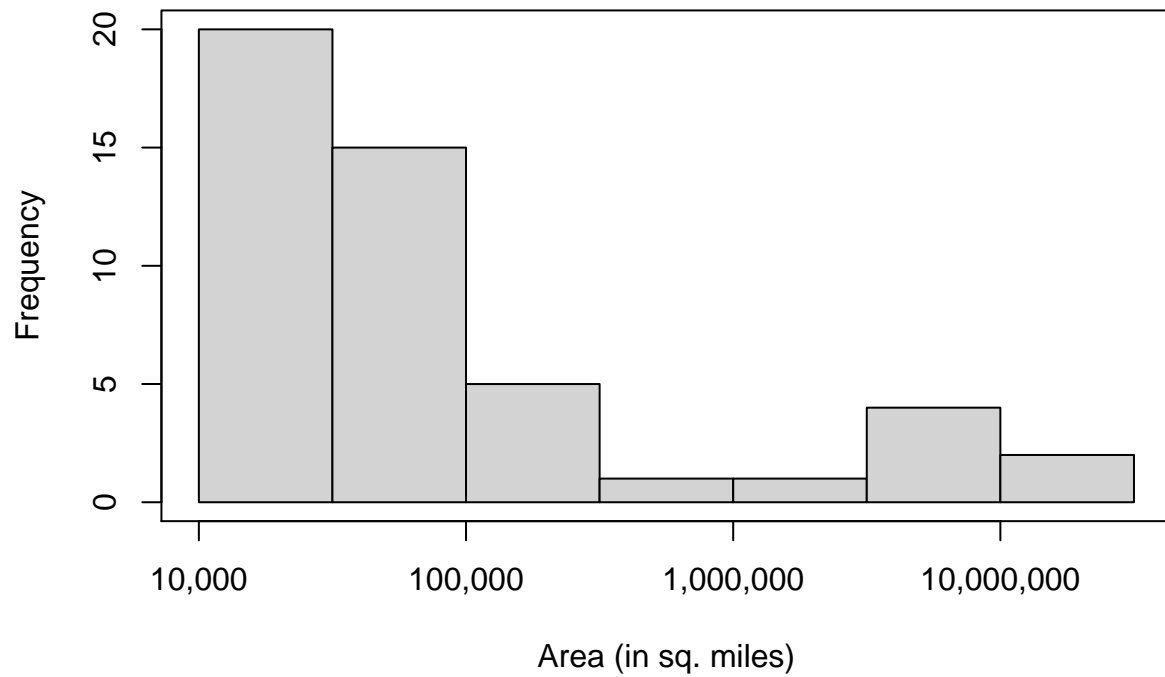
```

Axis and Tricks

```
hist(log(1000*islands, 10), axes = FALSE, xlab = "Area (in sq. miles)",
main = "Areas of the World's Largest Islands")
box()
axis(side = 1, at = 4:7, labels = c("10,000", "100,000", "1,000,000",
"10,000,000"))
axis(side = 2)
```

Areas of the World's Largest Islands



Boxplot with some additional features

motor ## Brand 1 Brand 2 Brand 3 Brand 4 Brand 5 ## 1 13.1 16.3 13.7 15.7 13.5 ## 2 15.0 15.7 13.9 13.7 13.4 ## 3 14.0 17.2 12.4 14.4 13.2 ## 4 14.4 14.9 13.8 16.0 12.7 ## 5 14.0 14.4 14.9 13.9 13.4 ## 6 11.6 17.2 13.3 14.7 12.3