✓ Recursion

- ✓ Sum of n natural numbers
- ✓ Fibonacci
- ✓ Power of 4

Number → 123

Digits → 1, 2, 3

All digits → (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Sum of digits

$n = 123$

$sum = 1+2+3 = 6$

Problem

Iterative

Recursive

$n = 123$

$123 \% 10 = 3$

$$\boxed{\text{last digit of } n = n \% 10}$$

$87 \% 10 = 7$

$43 \% 10 = 3$

$$3 + 2 + 1 = 6$$

$$123 \xrightarrow{123\%10} 3$$

$\div 10$

$$12 \xrightarrow{12\%10} 2$$

$\div 10$

$$1 \longrightarrow 1$$

Divide by 10 $\rightarrow$ last digit removed

$$123/10 = 12$$

$$472/10 = 47$$

$$275/10 = 27$$

1. $N \% 10$ → gives me last digit
2. $N / 10$ → removes the last digit

$$N = 123 \xrightarrow{\%10} \underline{3}$$

$$\downarrow$$

$$N = 12 \xrightarrow{\%10} \underline{2}$$

$$\downarrow$$

$$N = 1 \xrightarrow{\%10} \underline{1}$$

$$\downarrow$$

$$0$$

$$\boxed{\text{digit} = N \% 10}$$

$7638 \rightarrow \%10 = 8$

$\div 10$

$763 \rightarrow \%10 = 3$

$\div 10$

$76 \rightarrow \%10 = 6$

$\div 10$

$7 \rightarrow \%10 = 7$

$\div 10$

0

$8 + 3 + 6 + 7$

24

123

digit = N % 10

ans = digit

3 +

= 6

SumofDigits (123) = 3 + SumofDigits (12) = 3+3

= 6

= 2 + SumofDigits (1) = 2+1 = 3

Base case

= 1 + 0 n = 0 = 1

0

SOD → sum of digits

SOD 4728 $\leftarrow$ 21

8 + SOD(472) $\leftarrow$ 13

digit

2 + SOD(47) $\leftarrow$ 11

7 + SOD(4) $\leftarrow$ 4

4 + 0

$\boxed{\text{ans} = 21}$

4+7+2+8 = 21

$$SOD(1002) = 3$$

$$SOD(1002)$$

$$\downarrow$$

$$\underline{2} + SOD(100)$$

$$\downarrow$$

$$\underline{0} + SOD(10)$$

$$\downarrow$$

$$\underline{0} + SOD(1)$$

$$\downarrow$$

$$\underline{1} + \underline{0}$$

1

1

1

3

Time Complexity

4738

↓

473

↓

47

↓

4

↓

0

$$\underline{\underline{O(\ln(N))}}$$

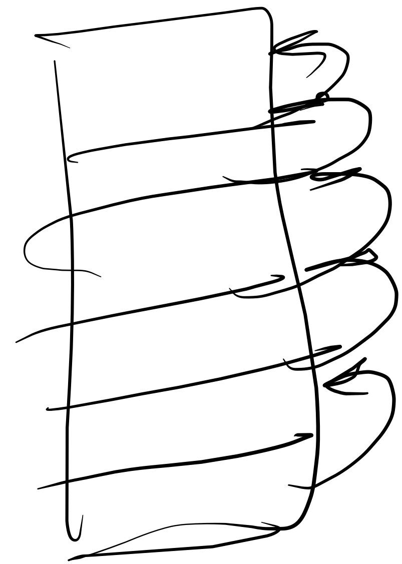
$$O(\log_{10} N)$$

$$3487 \xrightarrow{\div 10} 348 \xrightarrow{\div 10} 34 \xrightarrow{\div 10} 3 \xrightarrow{\div 10} 0$$

$\log_{10} N$

$TC = \underline{\log_{10} N}$

$\text{Aux space} = O(\log N)$



Auxiliary space → extra space used to solve the problem

Total space complexity = Input space + Aux Space

Auxiliary space → extra space used to solve the problem

Total space Complexity = Input space + Aux Space

GCD → Greatest Common Divisor

HCF → Highest Common Factor

GCD(16 , 24)

<u>common</u>

16 → 1, 2, 4, 8, 16

24 → 1, 2, 3, 4, 6, 8, 12, 24

GCD(10, 20)

10 → 10 ✓

20 → 10 ✓

$$GCD(24, 16)$$

$$GCD(8, 16)$$

$$GCD(16-8, 8)$$

$$GCD(8, 8)$$

$$GCD(8-8, 8)$$

$$GCD(0, 8) = 8$$

$$\mathrm{GCD}(15, 20)$$
$$\mathrm{GCD}(5, 15)$$
$$\mathrm{GCD}(10, 5)$$
$$\mathrm{GCD}(5, 5)$$
$$\mathrm{GCD}(0, 5) = \underline{5}$$

Base case $\boxed{GCD(N, 0) = N}$

$$GCD(N, 1) = 1$$

GCD(100, 5)

$\downarrow$

GCD(95, 5)

$\downarrow$

GCD(90, 5)

$\downarrow$

GCD(85, 5)

⋮

GCD (a, b)

if (b == 0)

return a

29, 7.

$$29 \xrightarrow{-7} 22 \xrightarrow{-7} 15 \xrightarrow{-7} 8 \xrightarrow{-7} 1$$

$29 \% 7 = 1$

$GCD(19, 17) = 1$

$GCD(19 \% 17, 17)$

$GCD(2, 17)$

$GCD(17 \% 2, 2)$

$GCD(1, 2)$

$GCD(\underline{2 \% 1}, 1) = GCD(0, 1) = \underline{1}$

$$GCD(4000, 800)$$

$$GCD(4000 \% 800, 800)$$

$$GCD(0, 800) = \underline{\underline{800}}$$

if (b == 0)

return a

$GCD(21, 13)$

$\downarrow$

$GCD(21 \% 13, 13)$

$\downarrow$

$GCD(8, 13)$

$\downarrow$

$GCD(13 \% 8, 8)$

$\downarrow$

$GCD(5, 8)$

$\rightarrow$

$GCD(8 \% 5, 5)$

$\downarrow$

$GCD(3, 5)$

$\downarrow$

$GCD(2, 3)$

$\downarrow$

$GCD(1, 2)$

$\downarrow$

$GCD(0, 1)$

$\rightarrow$

1

21 ,	13
13 ,	8
8	5
5	3
	2
0	

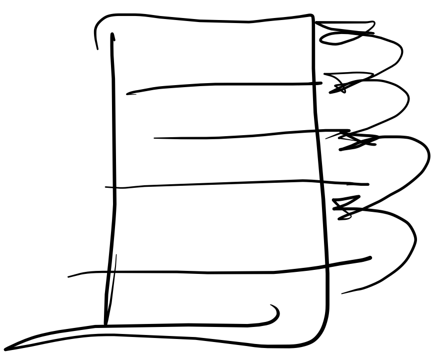
$$32 \to 16 \to 8 \to 4 \to 2 \to 1$$

5

$$\log_2 32 = 5$$

$$TC = O\left[\log_2\left(\min(A,B)\right)\right]$$

$$SC = O\left(\log_2\left(\min(A,B)\right)\right)$$



$$TC = O\left[\log_2\left(\min(A,B)\right)\right]$$

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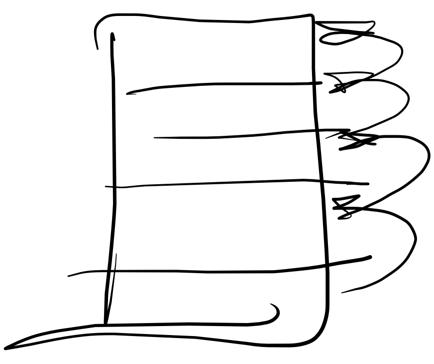

Set = $\{1, 2, 3\}$

$\{\}$ $\{1\}$ $\{2\}$ $\{3\}$ $\{1, 2\}$ $\{2, 3\}$ $\{1, 3\}$ $\{1, 2, 3\}$

$\{0, 1, 2, 3, 3, 5, 4, 6\}$

set = {1, 2}

no. of subsets = $2^n = 2^2 = 4$

{ }, {1}, {2}, {1, 2}

{ 0 1 2 3 }

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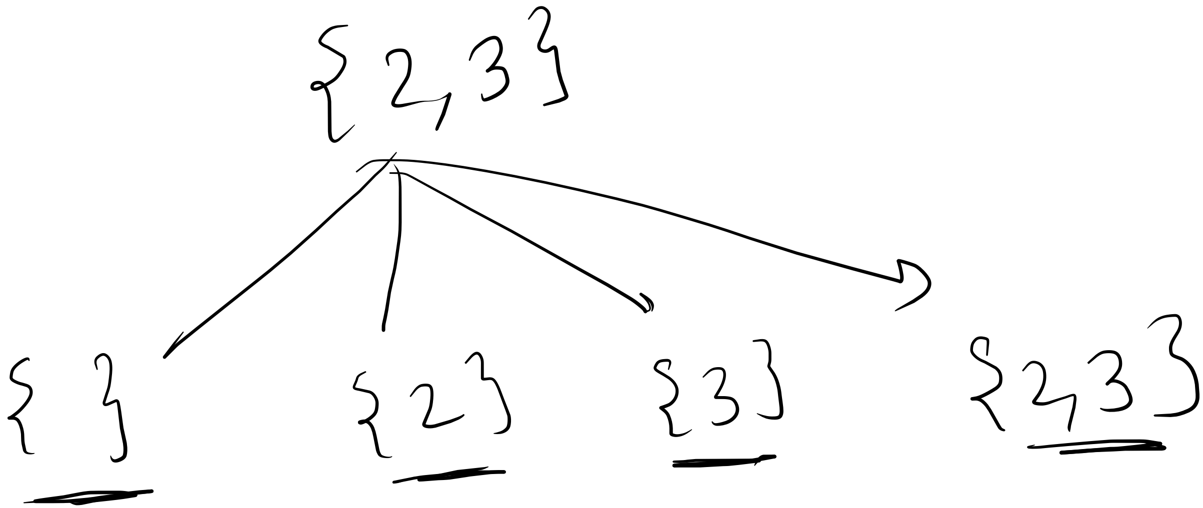
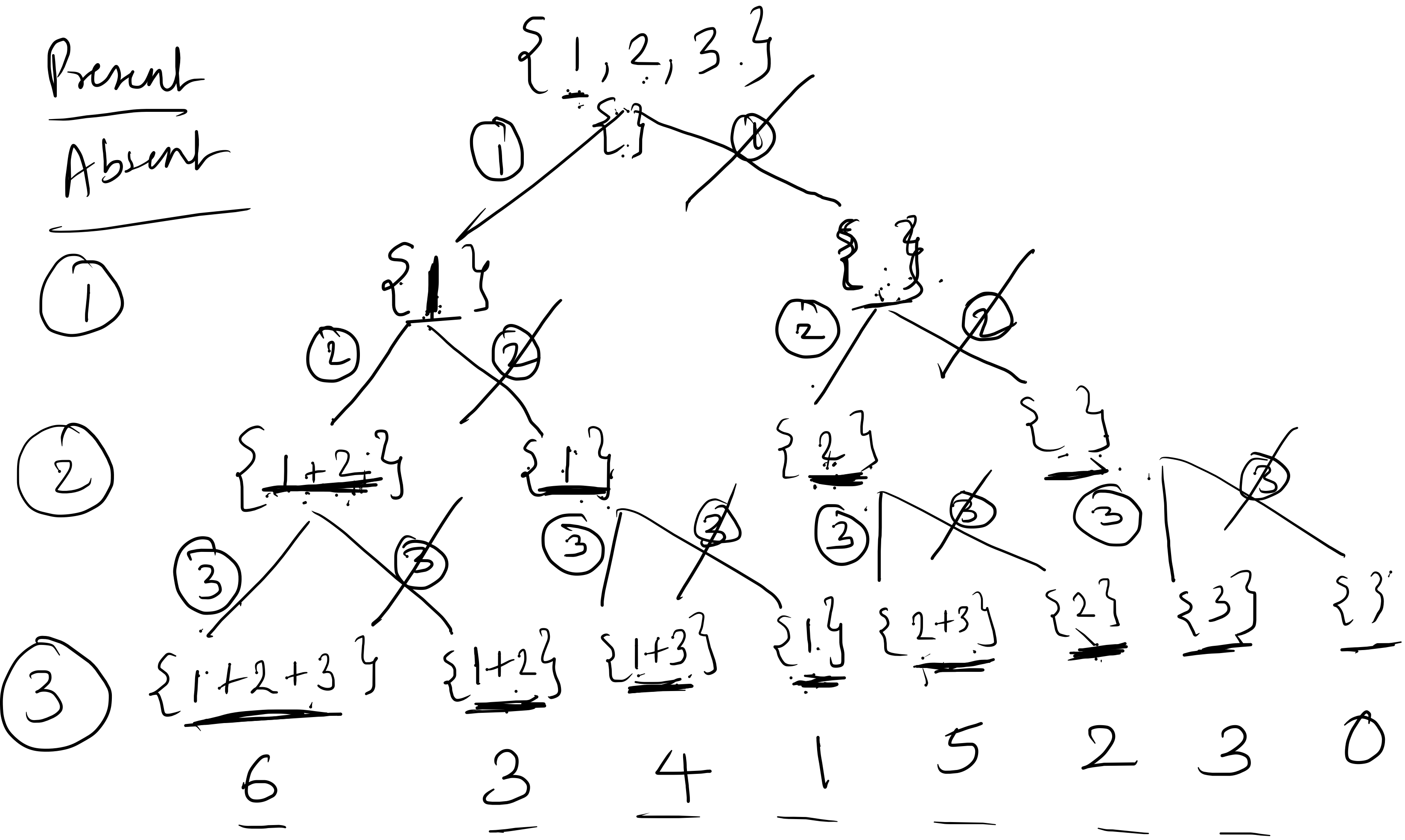
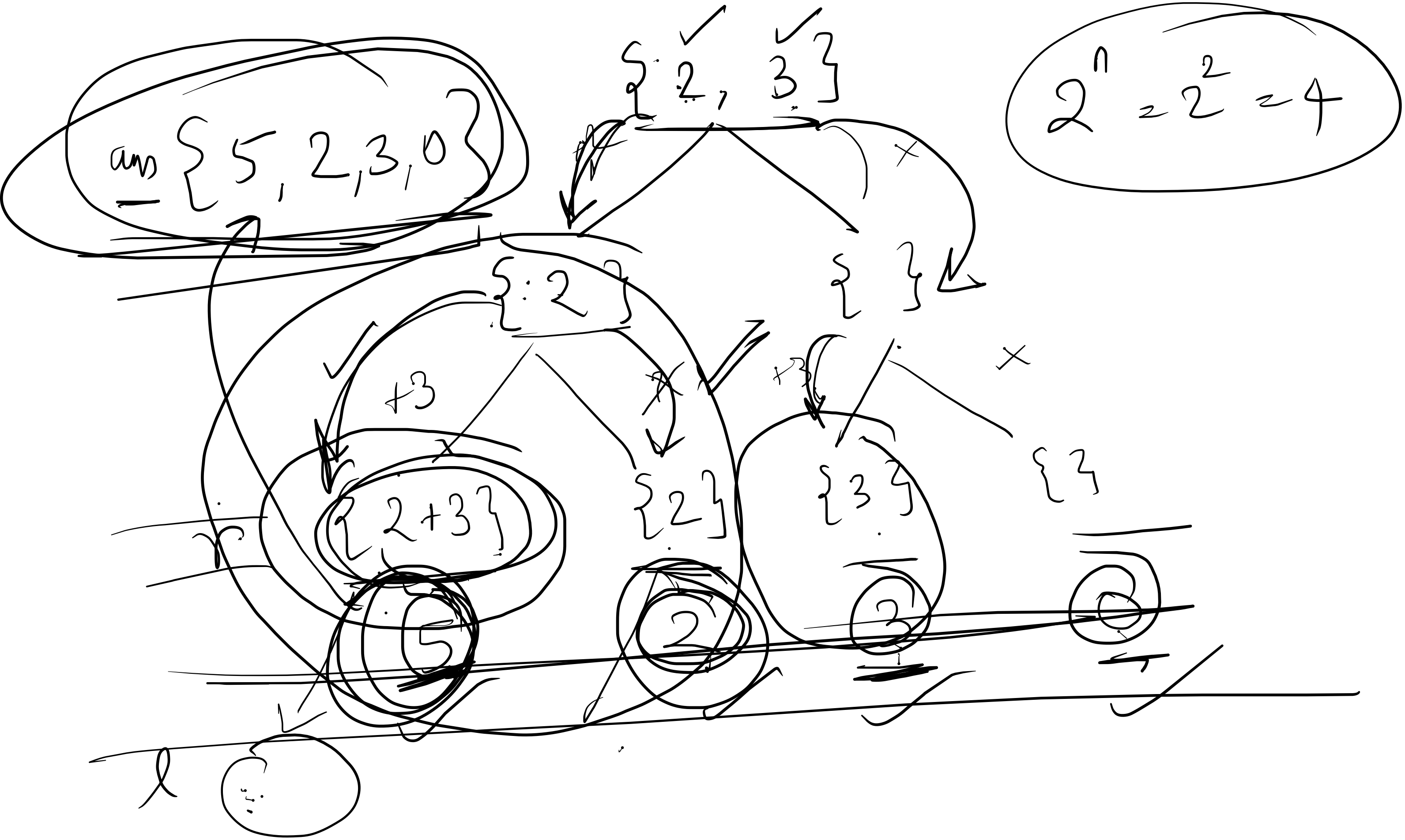
{ }, {1}, {2}, {1, 2}

{ 0 1 2 3 }



$\{ \underline{0}, \underline{2}, \underline{3}, \underline{5} \}$


Present
Absent
{1, 2, 3}
{1}
{1+2}
{1}
{2}
{1+2+3}
{1+2}
{1+3}
{1}
{2+3}
{2}
{3}
6
3
4
1
5
2
3
0



ans {5, 2, 3, 0}
{2, 3}
$2^n = 2^2 = 4$
{2}
{ }
+3
{2+3}
{2}
{3}
{ }
5
2
3
0
r
l

$\cancel{l}$ $\cancel{l}$ $\cancel{l}$ l
r

$\{1, 2, 3\}$

0 ✓ 1 ✓ 2 ✓ $\underline{3}$

$\underline{\text{if } (l > r)}$

stop

$\cancel{l}$ $\cancel{l}$ $\cancel{l}$ l

$\{ 2, 3, 5 \}$

r

if $(l > r)$

$l+1$

$\cancel{l}$ $\cancel{l}$ $\cancel{l}$ l

$\{2, 3, 5\}$

r

$\underline{\text{if}(l > r)}$

$\underline{l+1}$

T.C $= O(2^n)$

SC $= O(n)$