Lecture 17

Numerical problems

Useful Formulae

$$x = x_m \cos(\omega t + \phi) \quad \text{(displacement)}$$

$$v = -\omega x_m \sin(\omega t + \phi) \quad \text{(velocity)}$$

$$a = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{(acceleration)}$$

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \text{(angular frequency).}$$

For any oscillatory object

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period).}$$

Spring mass system

$$E(t) \approx \tfrac{1}{2}k x_m^2 e^{-bt/m}.$$

For damped oscillations

$$K = \tfrac{1}{2}mv^2$$

$$U = \tfrac{1}{2}kx^2$$

for spring

1 Which of the following describe ϕ for the SHM of Fig. 15-20a:

(a) $-\pi < \phi < -\pi/2$,

(b) $\pi < \phi < 3\pi/2$,

(c) $-3\pi/2 < \phi < -\pi$?

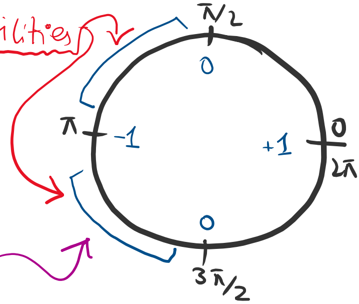
according to
$x(t) = x_m \cos(\omega t + \phi)$
at $t = 0$
$-x_m < x < 0$

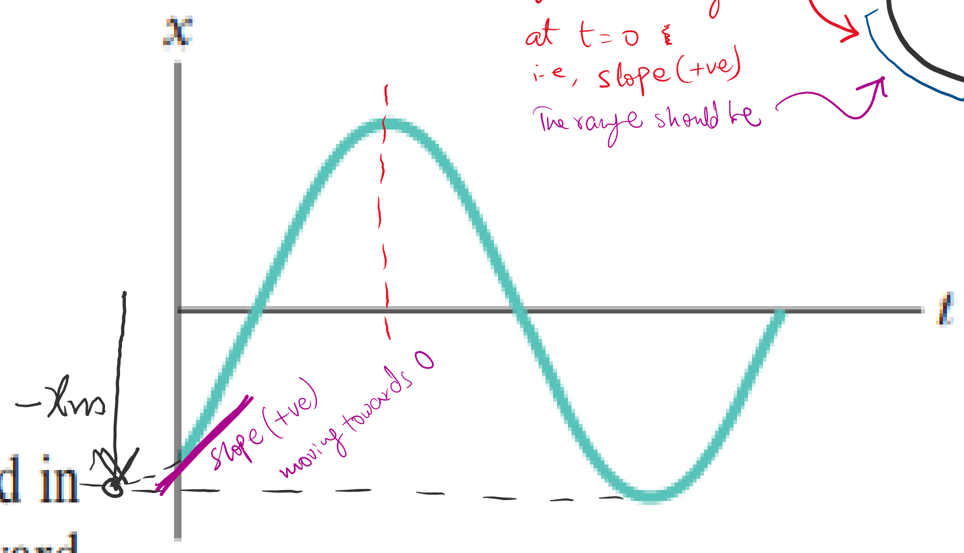
So, in the circle of $\cos\theta$ we see where this is possible

There are two possibilities

But since the value of $x(t)$ is rising at $t=0$ & i.e, slope (+ve)

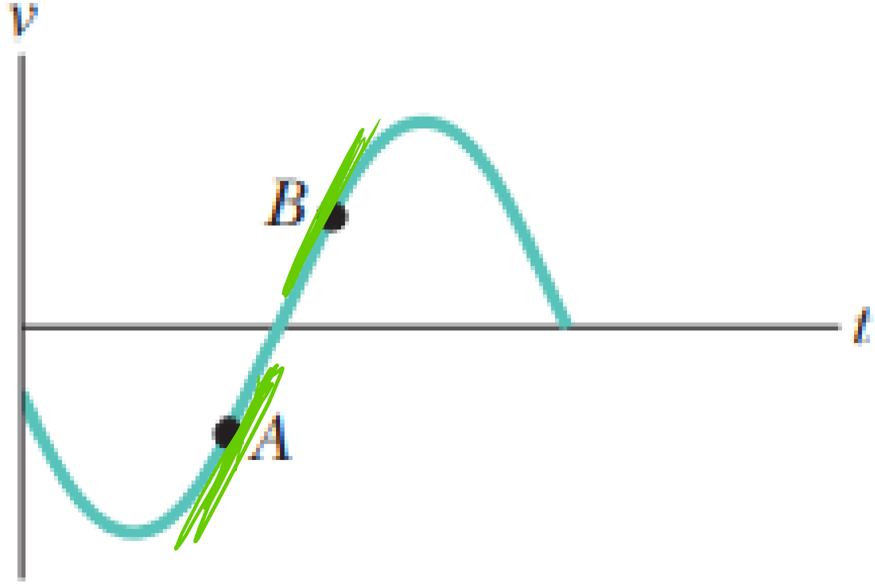
The range should be





(a)

2 The velocity $v(t)$ of a particle undergoing SHM is graphed in Fig. 15-20b. Is the particle momentarily stationary, headed toward $-x_m$, or headed toward $+x_m$ at (a) point A on the graph and (b) point B? Is the particle at $-x_m$, at $+x_m$, at 0, between $-x_m$ and 0, or between 0 and $+x_m$ when its velocity is represented by (c) point A and (d) point B? Is the speed of the particle increasing or decreasing at (e) point A and (f) point B?



(c) A: $\boxed{\text{The velocity is negative and increasing}}$ (e) i.e, moving toward 0 from $+x_m$ (a)

B: $\boxed{\text{The velocity is positive and increasing}}$ (f) i.e, moving toward 0 from $-x_m$ (b)

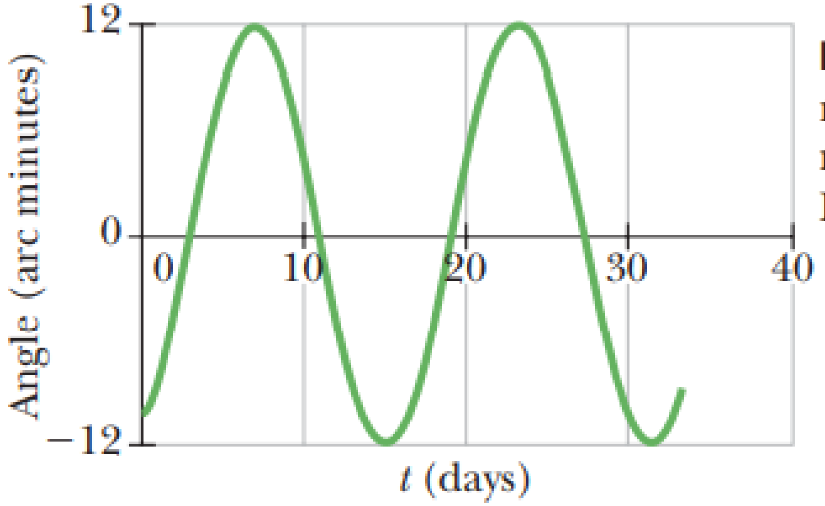
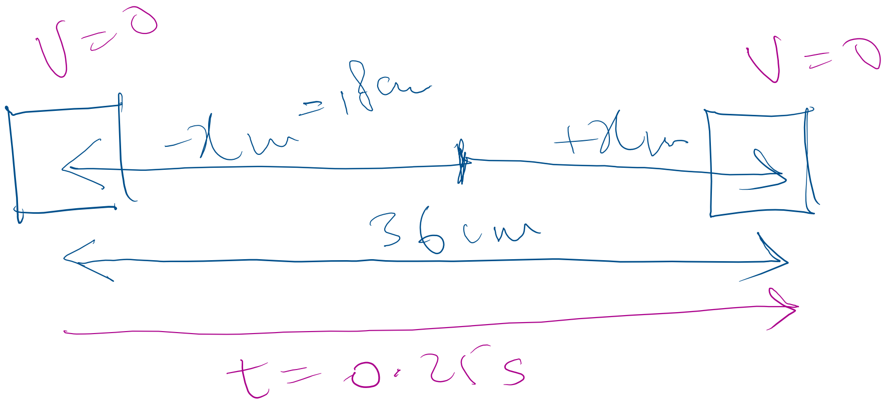


Figure 15-14 The angle between Jupiter and its moon Callisto as seen from Earth. Galileo's 1610 measurements approximate this curve, which suggests simple harmonic motion. At Jupiter's mean distance from Earth, 10 minutes of arc corresponds to about 2×10^6 km. (Based on A. P. French, *Newtonian Mechanics*, W. W. Norton & Company, New York, 1971, p. 288.)

Try and explain this graphed data using the given description.

•1 An object undergoing simple harmonic motion takes 0.25 s to travel from one point of zero velocity to the next such point. The distance between those points is 36 cm. Calculate the (a) period, (b) frequency, and (c) amplitude of the motion.

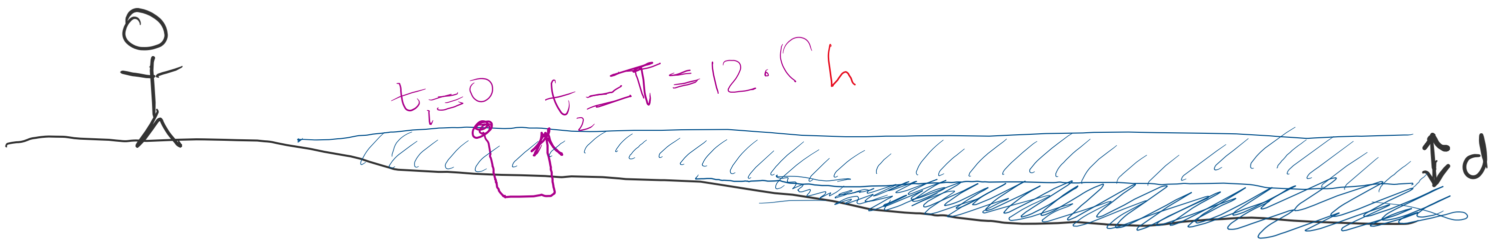


$T = 0.25\text{s} \times 2 = 0.5\text{s}$

$f = 1/0.5 = 2\text{Hz}$

$x_m = 36$ cm

••18 At a certain harbor, the tides cause the ocean surface to rise and fall a distance d (from highest level to lowest level) in simple harmonic motion, with a period of 12.5 h. How long does it take for the water to fall a distance $0.250d$ from its highest level?



alternate diagram

an object is oscillating vertically with period of $T = 12.5\,h$ and amplitude $= d/2$

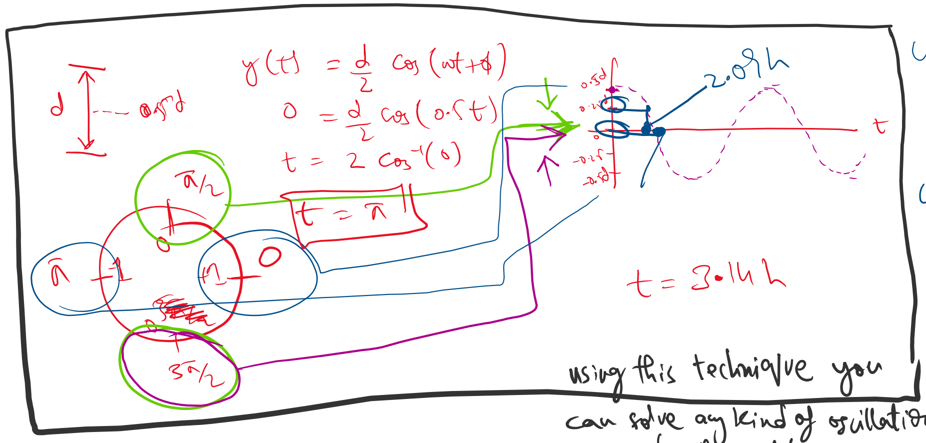
$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{12.5\,h}$$

$$\omega = 0.5\,rad/h$$

$$y(t) = y_m \cos(\omega t + \phi)$$

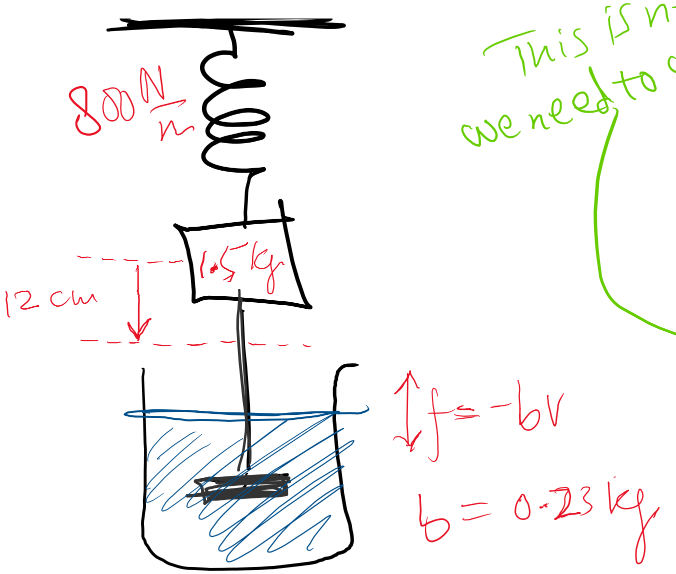
$$0.25\cancel{d} = \frac{\cancel{d}}{2}\cos(0.5\,rad/h\; t + 0)$$

$$t = \frac{1}{0.5\,rad/h}\cos^{-1}(0.5) = 2.09\,h$$



using this technique you can solve any kind of oscillation problem extremely thoroughly.

•59 SSM WWW For the damped oscillator system shown in Fig. 15-16, the block has a mass of 1.50 kg and the spring constant is 8.00 N/m. The damping force is given by $-b(dx/dt)$, where $b = 230$ g/s. The block is pulled down 12.0 cm and released. (a) Calculate the time required for the amplitude of the resulting oscillations to fall to one-third of its initial value.



initial amplitude —— 12 cm

final amplitude —— 4 cm

This is new amplitude we need to calculate the time for

$$y(t) = \underbrace{y_m e^{-bt/2m}} \cos(\omega' t + \phi)$$

$$y_m' = 4\,\text{cm} = y_m e^{-bt/2m} = 12\,\text{cm}\; e^{\left(-\frac{0.23\,\text{kg/s}\; t}{2\times 1.5\,\text{kg}}\right)}$$

$$\frac{1}{3} = \exp(-0.076\,t) \Rightarrow \ln\left(\frac{1}{3}\right) = -0.076\,t$$

$$t = 14.45\ \text{sec}$$

Practice problems:

Problems from **Fundamentals of Physics**
-Jearl Walker

Chapter 15 : SHM

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Problems; 3,5,13,23,32