Applied Physics NS (101)

LECTURE # 6&7

DATE: 6^{TH} OCTOBER , 2019

Motion in two/three dimensions
Position and Displacement

One general way of locating a particle (or particle-like object) is with a **position vector** , which is a vector that extends from a reference point (usually the origin) to the particle.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \qquad (4.1)$$

where x , y , and z are the vector components of and the coefficients x, y, and z are its scalar components.

Fig. 4-1 shows a particle with position vector

$\vec{r} = (-3\text{ m})\hat{i} + (2\text{ m})\hat{j} + (5\text{ m})\hat{k}$

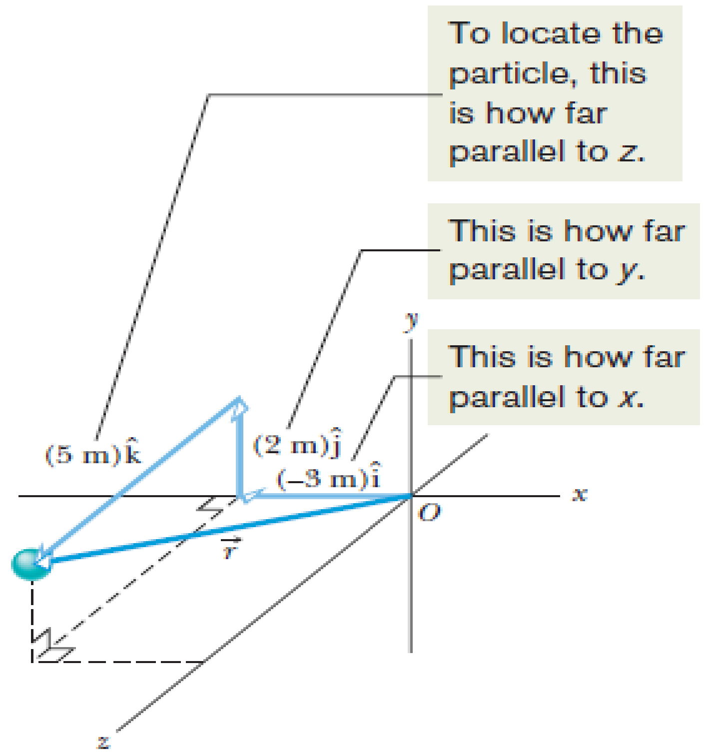


Figure 4-1 The position vector $\vec{r}$ for a particle is the vector sum of its vector components.

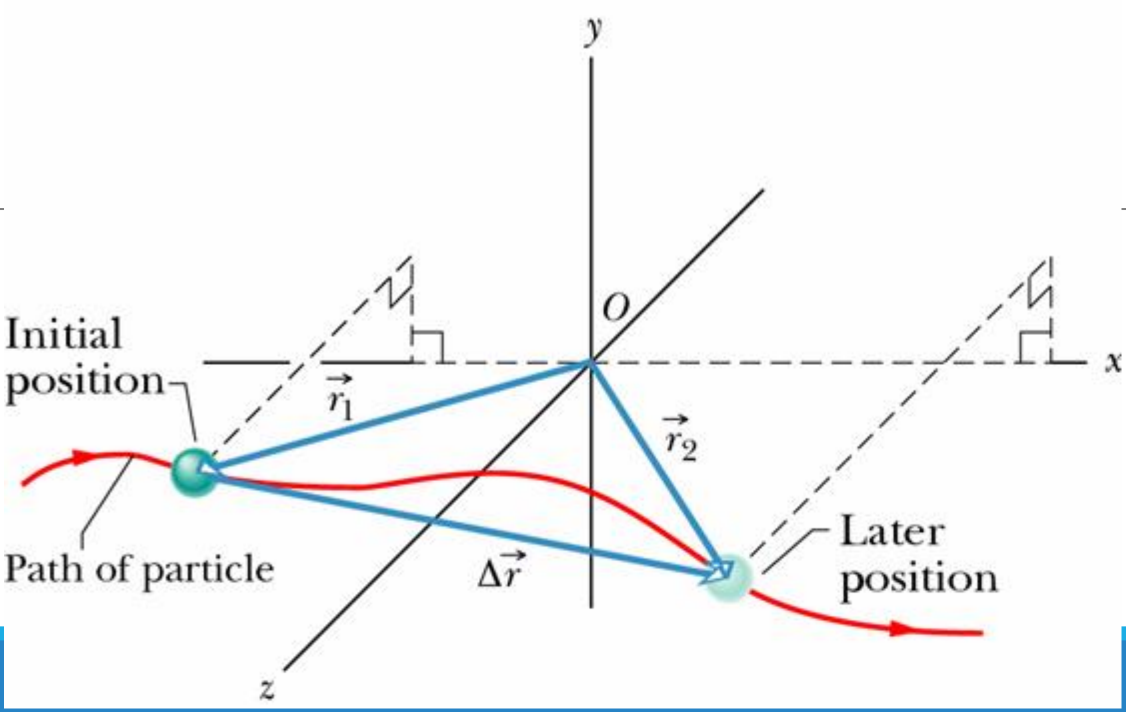
Position and Displacement

Position vector: extends from the origin of a coordinate system to the particle.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \qquad (4.1)$$

Displacement vector: represents a particle's position change during a certain time interval.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \qquad (4.2)$$



Average Velocity and Instantaneous Velocity

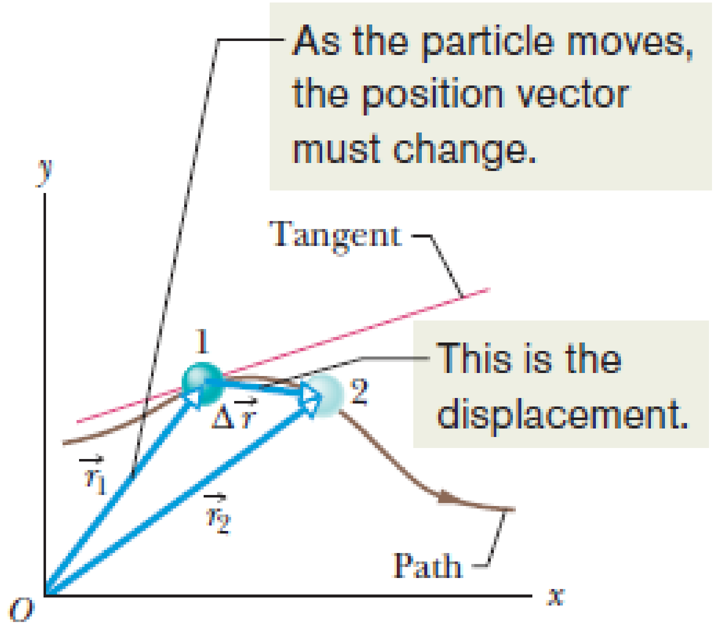
If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt, then its **average velocity** $\vec{v}_{\text{avg}}$ is

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}},$$

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$

Average

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$



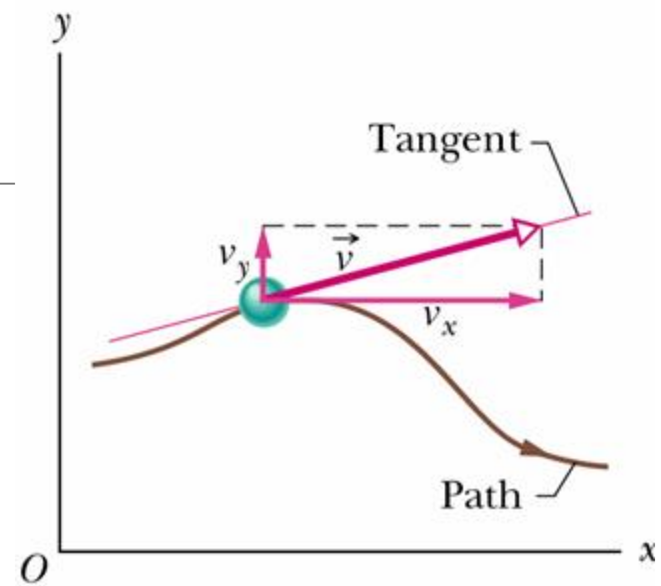
Instantaneous velocity:

When we speak of the **velocity** of a particle, we usually mean the particle's **instantaneous velocity** $\vec{v}$ at some instant. This $\vec{v}$ is the value that $\vec{v}_{avg}$ approaches in the limit as we shrink the time interval Δt to 0 about that instant. Using the language of calculus, we may write $\vec{v}$ as the derivative

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt}.$$

- The direction of the instantaneous velocity of a particle is always tangent to the particle's path at the particle's position

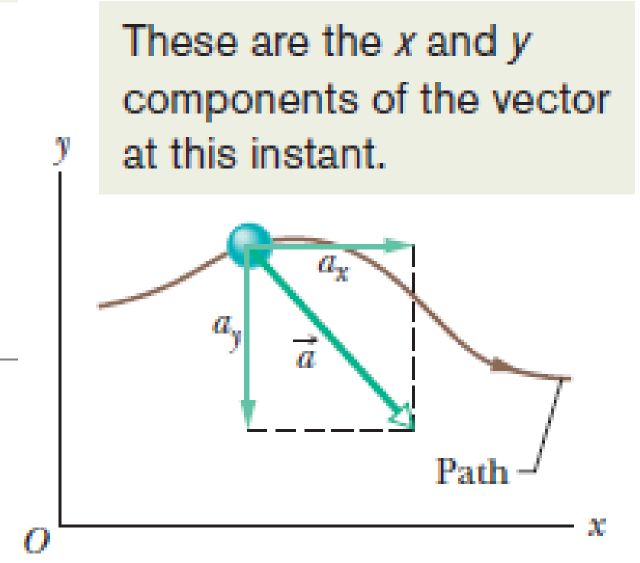


Average acceleration:

When a particle's velocity changes from $\vec{v}_1$ to $\vec{v}_2$ in a time interval Δt, its **average acceleration** $\vec{a}_{avg}$ during Δt is

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}},$$

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t} \qquad (4.5)$$



Instantaneous acceleration:

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} \qquad (4.6)$$

TWO-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

The position vector for a particle moving in the xy plane can be written

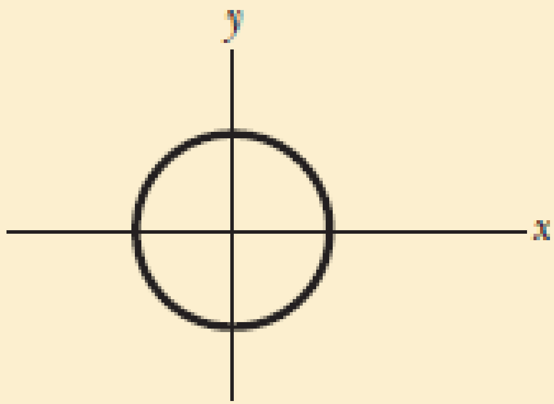
$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \qquad \begin{cases} v_{xf} = v_{xi} + a_x t \\ v_{yf} = v_{yi} + a_y t \end{cases}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \begin{cases} x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \\ y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 \end{cases}$$

Checkpoint 1



The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2\text{ m/s})\hat{\text{i}} - (2\text{ m/s})\hat{\text{j}}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.

Checkpoint 2

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$

(3) $\vec{r} = 2t^2\hat{\text{i}} - (4t + 3)\hat{\text{j}}$

(4) $\vec{r} = (4t^3 - 2t)\hat{\text{i}} + 3\hat{\text{j}}$

Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

Example

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \quad (4\text{-}5)$$

and

$$y = 0.22t^2 - 9.1t + 30. \quad (4\text{-}6)$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

For the rabbit in the preceding sample problem, find the velocity $\vec{v}$ at time $t = 15$ s.

For the rabbit in the preceding two sample problems, find the acceleration $\vec{a}$ at time $t = 15$ s.

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}. \tag{4-7}$$

(We write $\vec{r}(t)$ rather than $\vec{r}$ because the components are functions of t, and thus $\vec{r}$ is also.)

At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

and $$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

so $$\vec{r} = (66 \text{ m})\hat{\mathrm{i}} - (57 \text{ m})\hat{\mathrm{j}}, \qquad \text{(Answer)}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2}$$
$$= 87 \text{ m}, \qquad \text{(Answer)}$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57 \text{ m}}{66 \text{ m}}\right) = -41^\circ. \qquad \text{(Answer)}$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$
$$= -0.62t + 7.2. \tag{4-13}$$

At $t = 15$ s, this gives $v_x = -2.1$ m/s. Similarly, applying the v_y part of Eq. 4-12 to Eq. 4-6, we find

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$
$$= 0.44t - 9.1. \tag{4-14}$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s. Equation 4-11 then yields

$$\vec{v} = (-2.1 \text{ m/s})\hat{\text{i}} + (-2.5 \text{ m/s})\hat{\text{j}}, \qquad \text{(Answer)}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1 \text{ m/s})^2 + (-2.5 \text{ m/s})^2}$$
$$= 3.3 \text{ m/s} \qquad \text{(Answer)}$$
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-2.5 \text{ m/s}}{-2.1 \text{ m/s}}\right)$$
$$= \tan^{-1} 1.19 = -130°. \qquad \text{(Answer)}$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 \text{ m/s}^2.$$

Similarly, applying the a_y part of Eq. 4-18 to Eq. 4-14 yields the y component as

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44 \text{ m/s}^2.$$

We see that the acceleration does not vary with time (it is a constant) because the time variable t does not appear in the expression for either acceleration component. Equation 4-17 then yields

$$\vec{a} = (-0.62 \text{ m/s}^2)\hat{\text{i}} + (0.44 \text{ m/s}^2)\hat{\text{j}}, \quad \text{(Answer)}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.62 \text{ m/s}^2)^2 + (0.44 \text{ m/s}^2)^2}$$
$$= 0.76 \text{ m/s}^2. \quad \text{(Answer)}$$

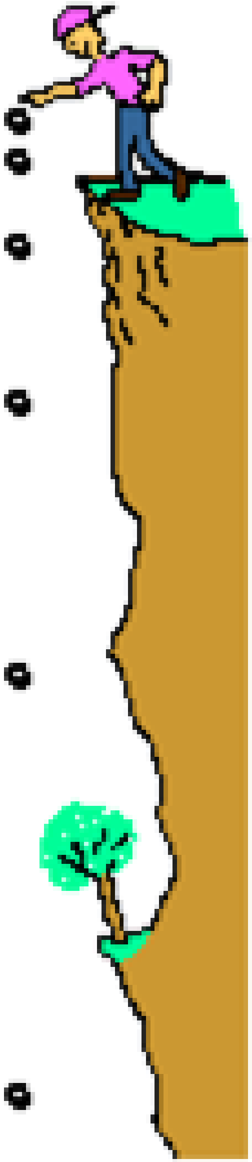
For the angle we have

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\left(\frac{0.44 \text{ m/s}^2}{-0.62 \text{ m/s}^2}\right) = -35°.$$

Free Fall and Gravitational Acceleration

A free-falling object is one which is falling under the sole influence of gravity. This definition of free fall leads to two important characteristics about a free-falling object:

- Free-falling objects do not encounter air resistance
- All free-falling objects (on Earth) accelerate downwards at a rate of -9.8 m/s^2
  - This rate is commonly referred to as *g*

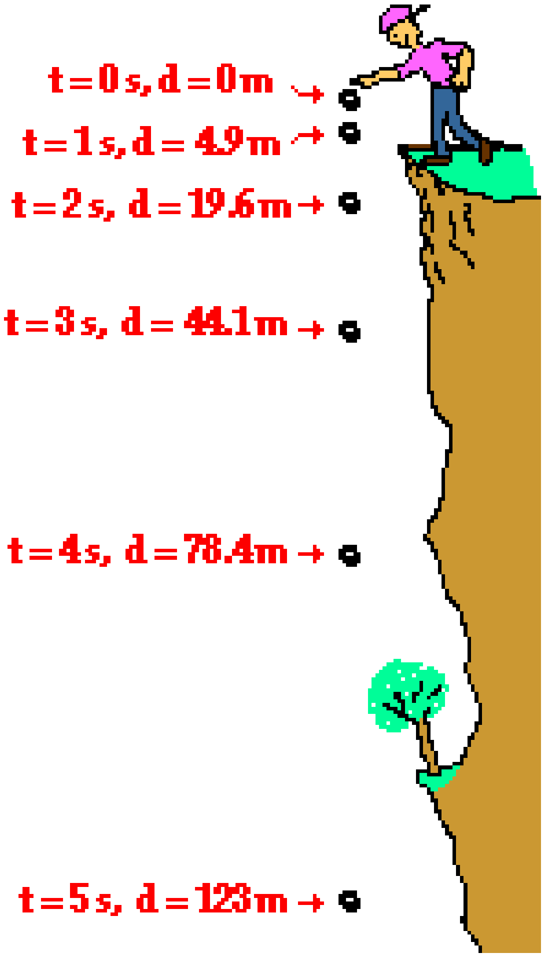


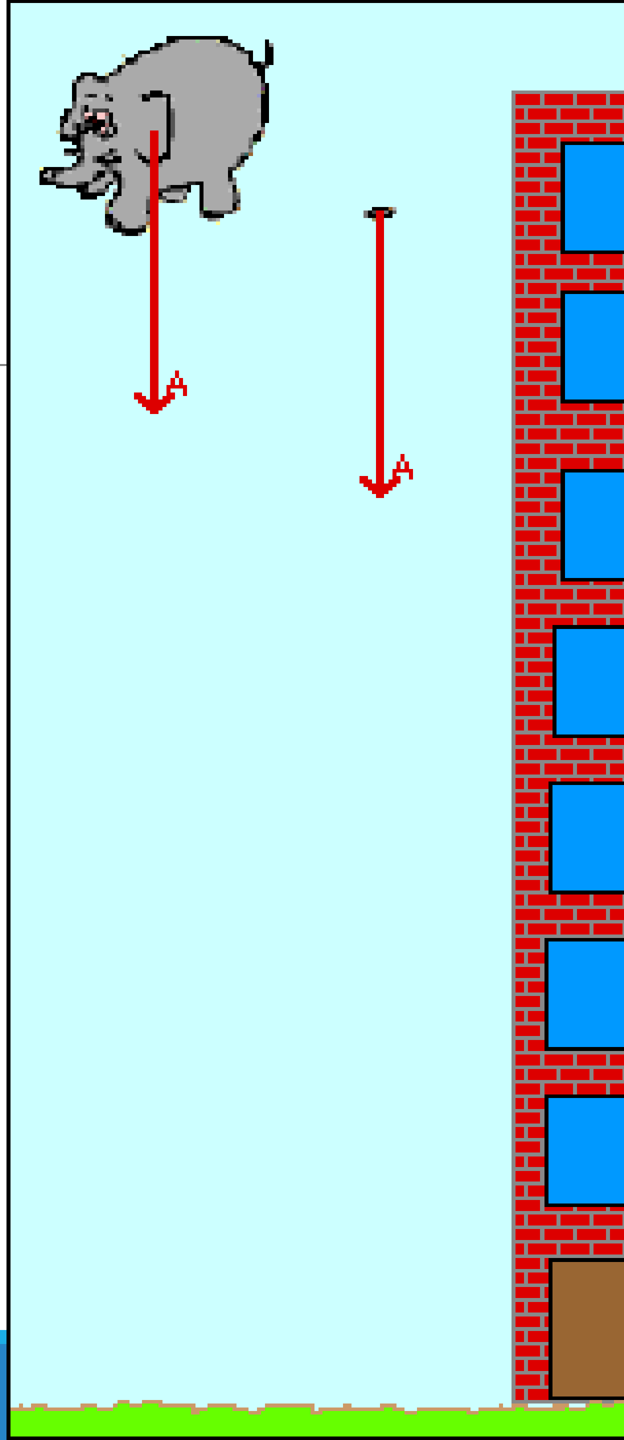
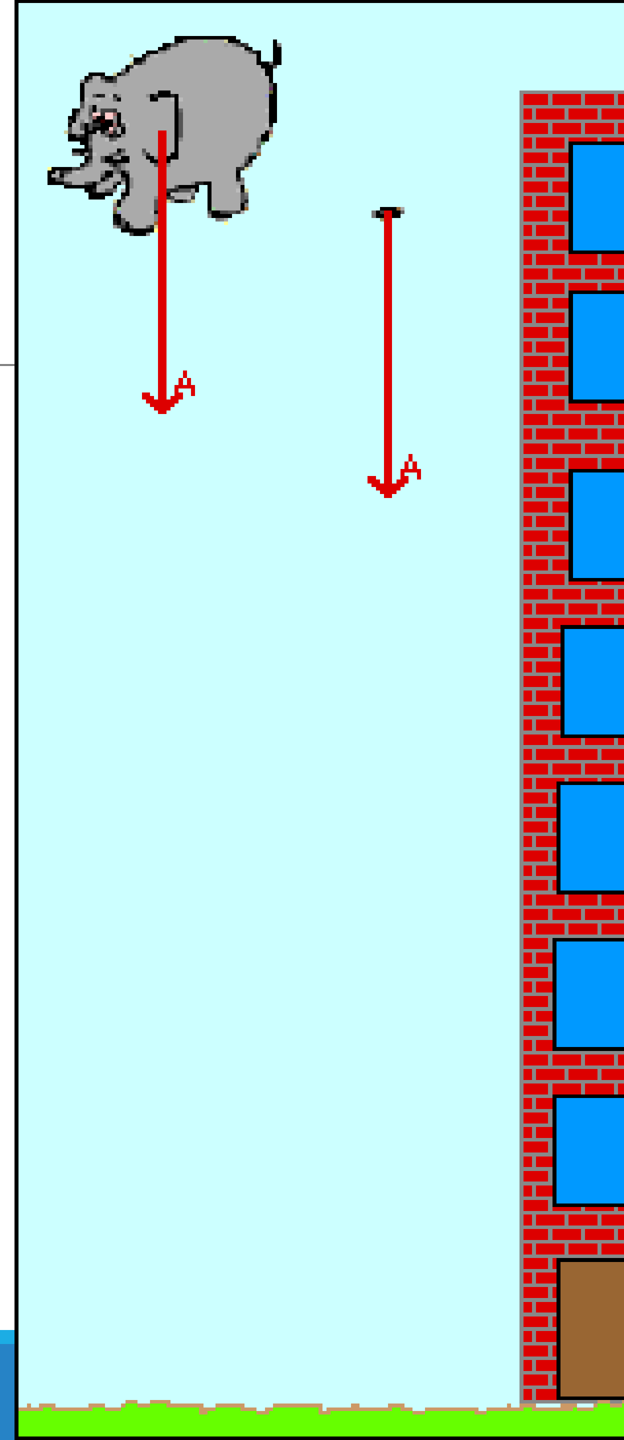
Free Fall and Gravitational Acceleration

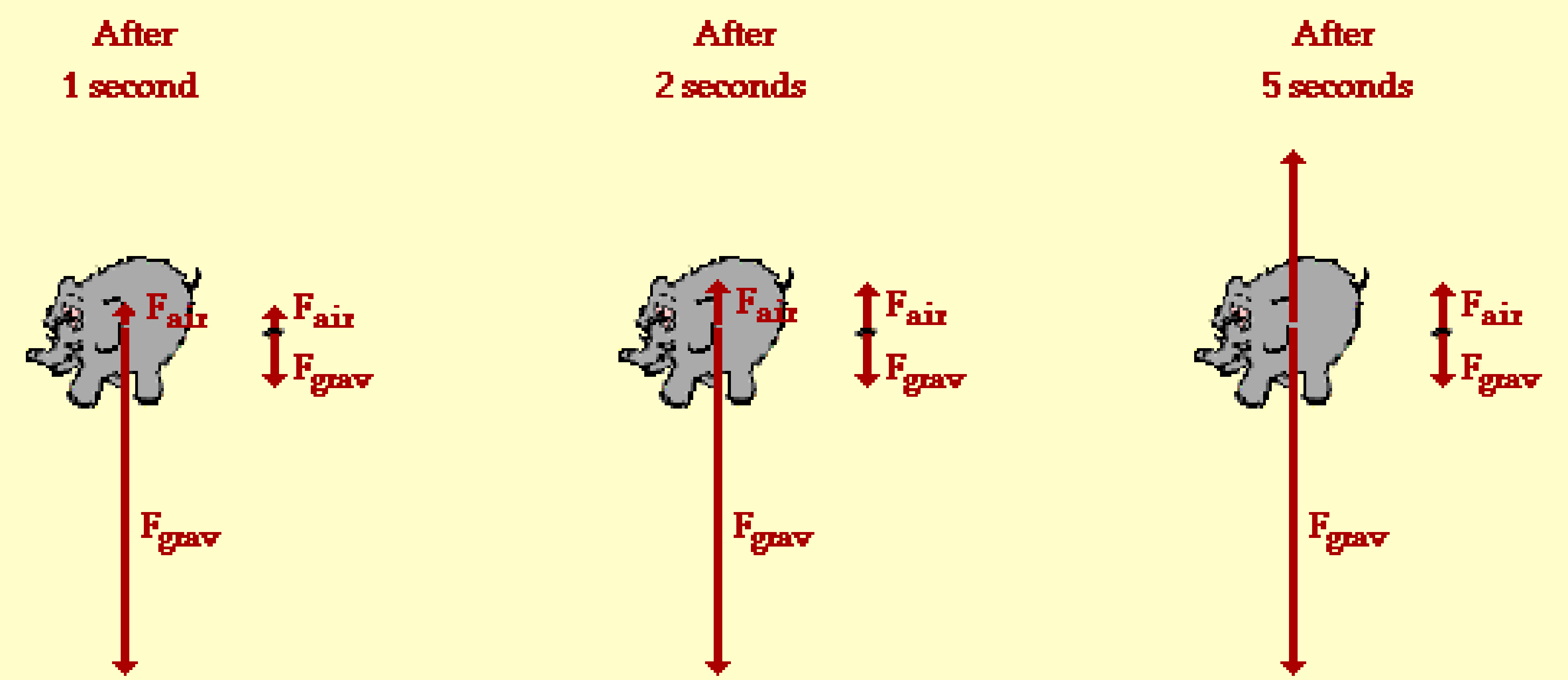
All of the equations that work for HORIZONTAL (x-direction motion ALSO work for VERTICAL (y-direction) motion.

Simply substitute g for a and Δy for Δx

Generally, free falling objects start falling from rest, so it tends to simplify the equations by eliminating v_i



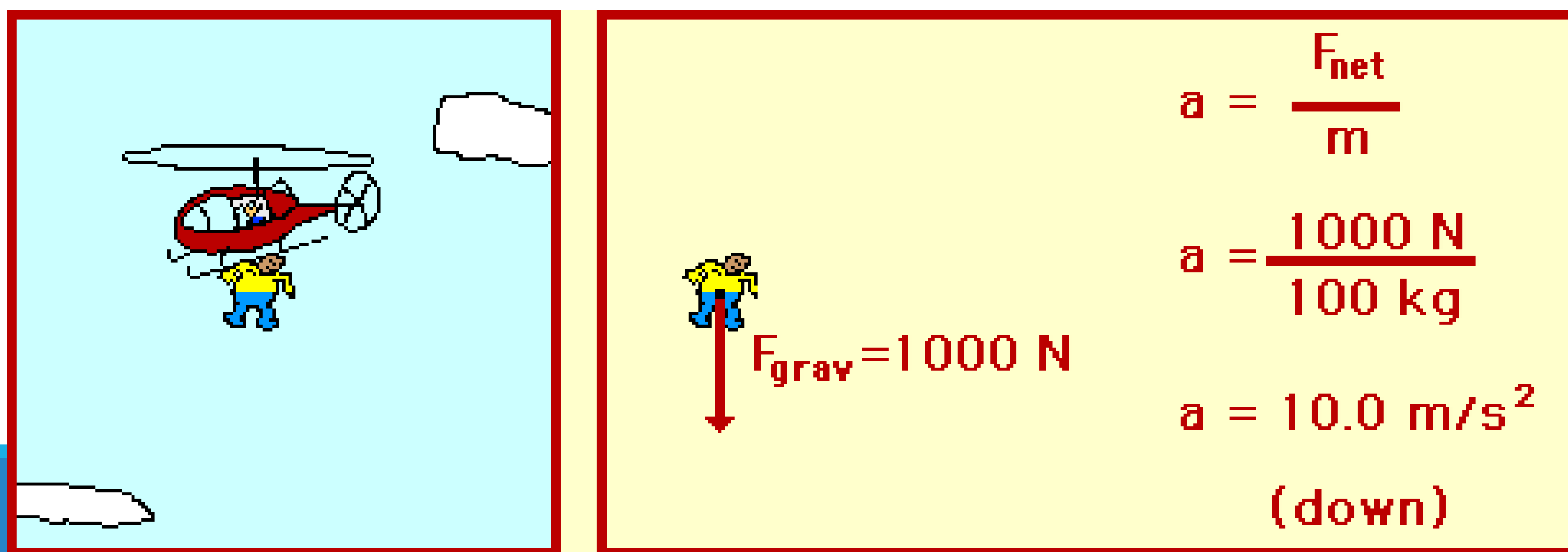





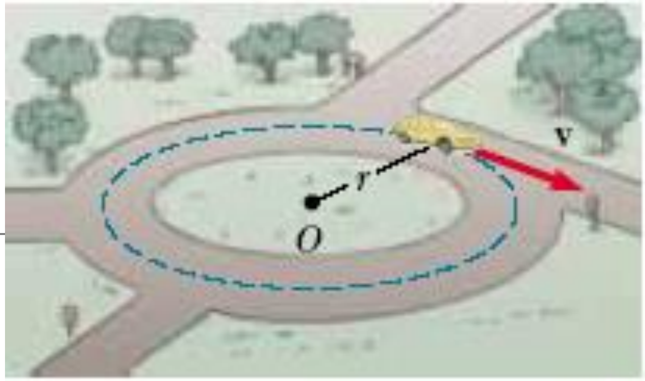
Free-body diagrams for the elephant and the feather at various times during the course of their fall reveal that the feather quickly reaches terminal velocity while the elephant continues to accelerate for the entire fall.

Terminal Velocity

Forces cause objects to accelerate (2nd Law).
When the force of gravity on a falling object equals the force of the air resistance going against gravity, the forces balance out and the object stops accelerating.
The object will travel at a constant velocity – the terminal velocity.



UNIFORM CIRCULAR MOTION

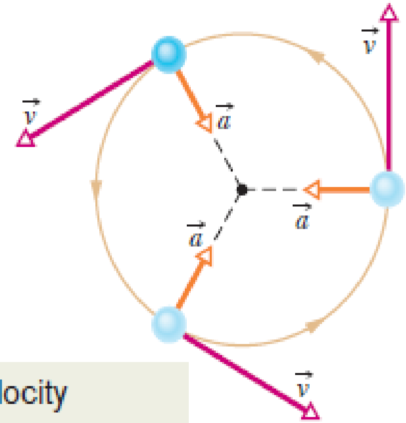


The acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An Acceleration of this nature is called a centripetal (center-seeking) acceleration, and its magnitude is

$$a = \frac{v^2}{r} \quad \text{(centripetal acceleration)}, \qquad T = \frac{2\pi r}{v} \quad \text{(period)}.$$

Consider a particle moving along a curved path where the velocity changes both in direction and in magnitude

The acceleration vector always points toward the center.

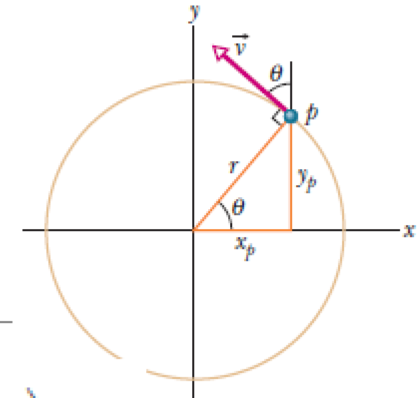


The velocity vector is always tangent to the path.

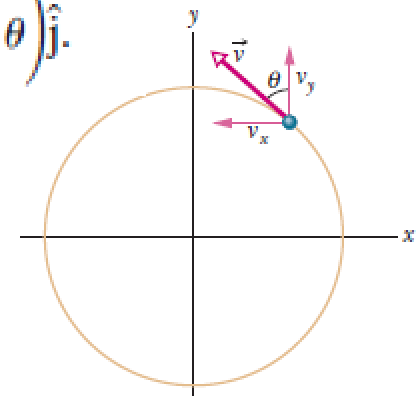
$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-v\sin\theta)\hat{\mathrm{i}} + (v\cos\theta)\hat{\mathrm{j}}.$$

replace sin with y_p/r and cos with x_p/r

$$\vec{v} = \left(-\frac{vy_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{vx_p}{r}\right)\hat{\mathrm{j}}.$$

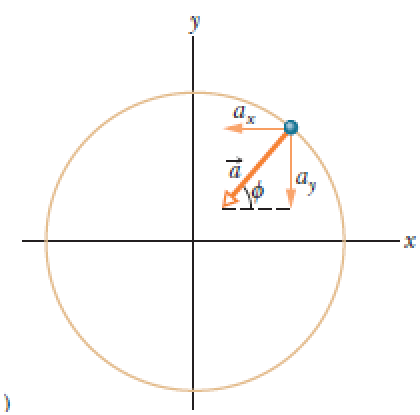


$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}. \qquad \vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}.$$

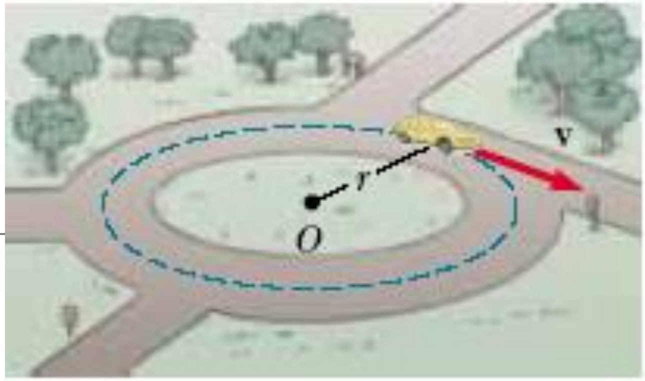


$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r},$$

$$\tan\phi = \frac{a_y}{a_x} = \frac{-(v^2/r)\sin\theta}{-(v^2/r)\cos\theta} = \tan\theta.$$



Tangential and Radial Acceleration

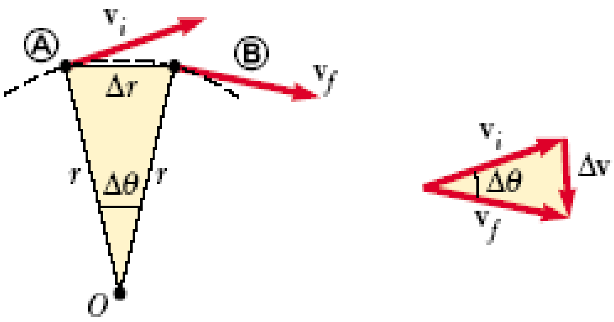


Radial acceleration

$$a_t = \frac{d|\mathbf{v}|}{dt}$$

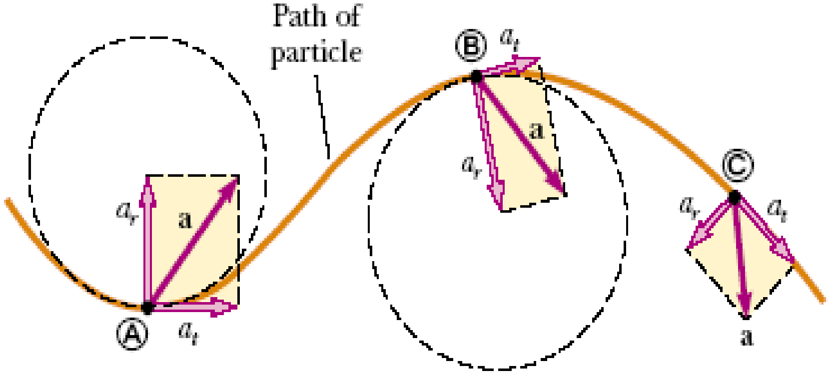
Tangential acceleration

$$a_r = \frac{v^2}{r}$$



Total acceleration

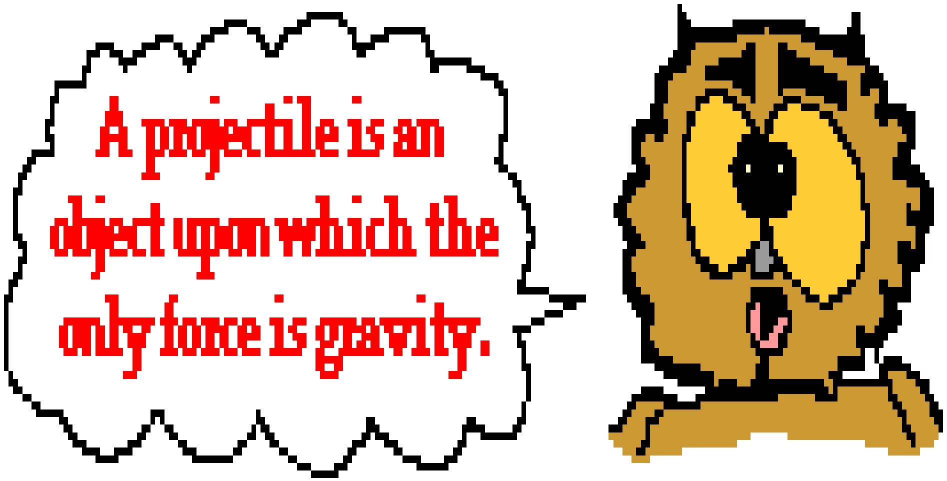
$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$$



Projectile Motion

Two-dimensional motion of an object

- Vertical
- Horizontal



Types of Projectile Motion

Horizontal

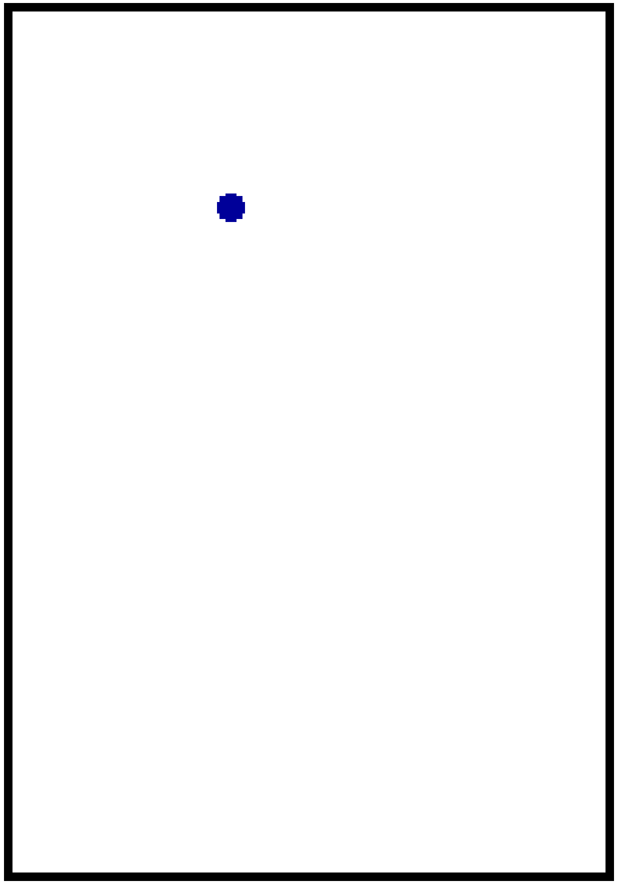
- Motion of a ball rolling freely along a level surface
- Horizontal velocity is ALWAYS constant

Vertical

- Motion of a freely falling object
- Force due to gravity
- Vertical component of velocity changes with time

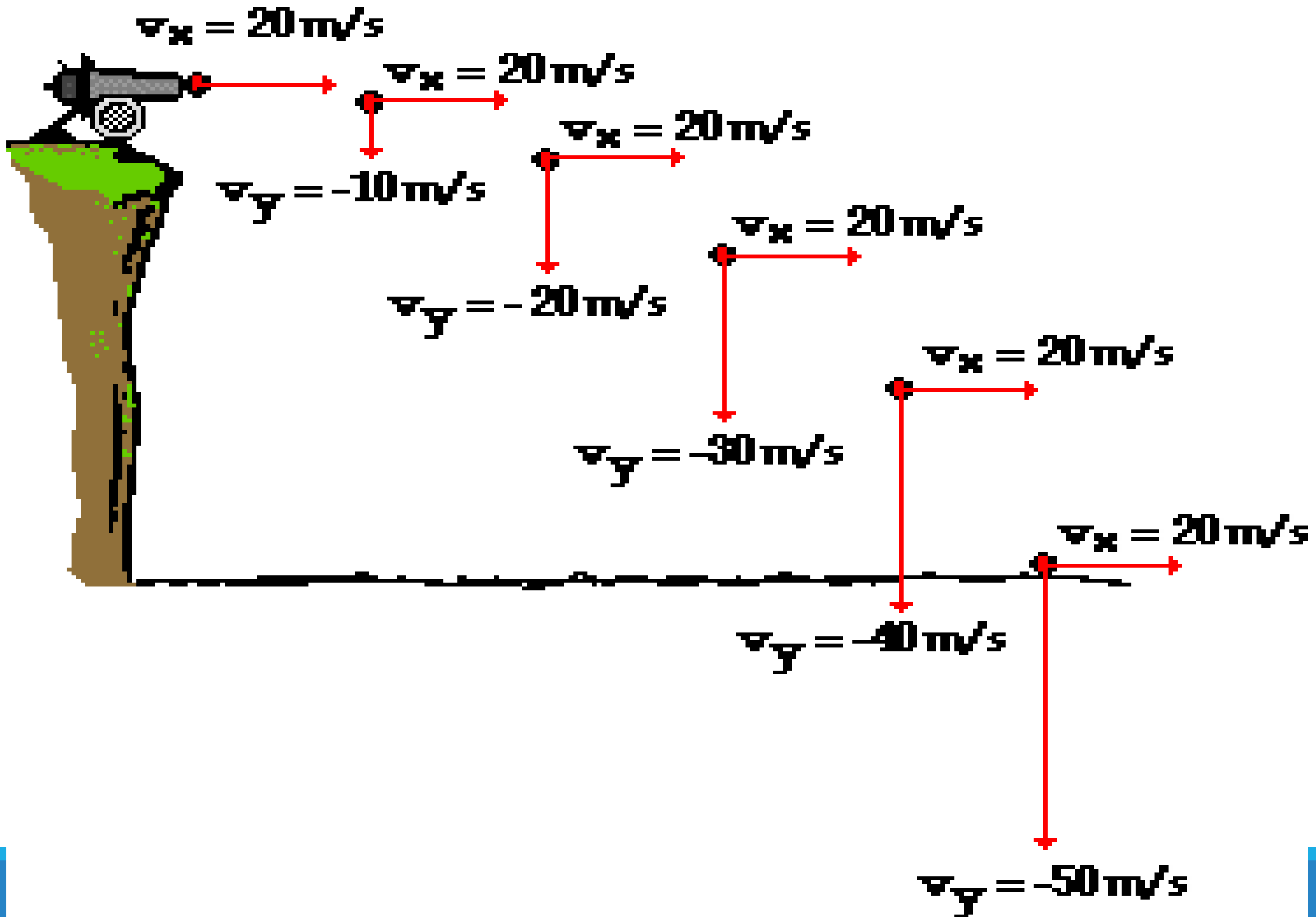
Parabolic

- Path traced by an object accelerating only in the vertical direction while moving at constant horizontal velocity



Examples of Projectile Motion

Launching a Cannon ball



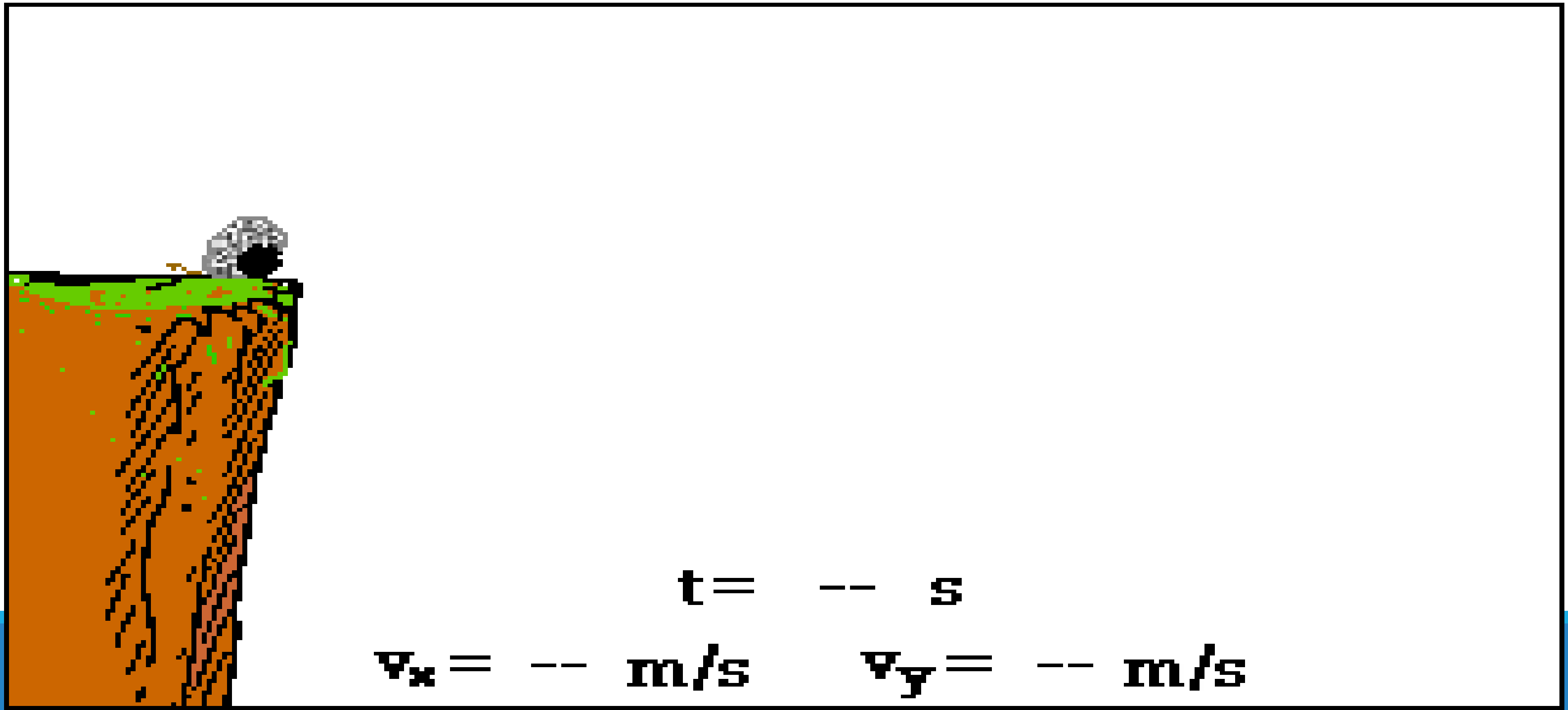
Final Horizontal and Vertical component of velocity:

$$v x_f = v\ Cos\theta$$
$$v y_f = v\ Sin\theta\ - gt$$

The projectile motion is the superposition of two motions:

(1) constant velocity motion in the horizontal direction and

(2) free-fall motion in the vertical direction.



Equations

X- Component

$$x_f = x_i + v_{xi}t$$

Y- Component

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

Note: g= 9.8 m/s^2

$$v_{yf}{}^2 = v_{yi}{}^2 - 2g\Delta y$$

$$v_{yf} = v_{yi} - gt$$

Vectors

$$v_{xi} = v_i \cos(\theta)$$

$$v_{yi} = v_i \sin(\theta)$$

Checkpoint 3

At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{\text{i}} - 4.9\hat{\text{j}}$ (the x axis is horizontal, the y axis is upward, and $\vec{v}$ is in meters per second). Has the ball passed its highest point?

The Horizontal Motion

There is *no acceleration* in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion

a = 0

the initial x and y components of velocity are

$$v_{xi} = v_i \cos\theta_i \qquad v_{yi} = v_i \sin\theta_i$$

$$x - x_0 = v_{0x}t.$$

Because $v_{0x} = v_0 \cos\theta_0$, this becomes

$$x - x_0 = (v_0 \cos\theta_0)t.$$

The Vertical Motion

In vertical the acceleration is constant i.e

a = $-g$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$= (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2,$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

The Equation Path

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \qquad \text{(trajectory).}$$

The path of a projectile, which we call its *trajectory, is always a* parabola

The Horizontal Range

The *horizontal range R* of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). To find range *R*, let us put $x - x_0 = R$ and $y - y_0 = 0$

$$R = (v_0 \cos \theta_0)t$$

and

$$0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2.$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$

Using the identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$ (see Appendix E), we obtain

$$R = \frac{v_0^2}{g} \sin 2\theta_0.$$

The horizontal range R is maximum for a launch angle of 45°.

Maximum Height

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g}$$