Lecture 16

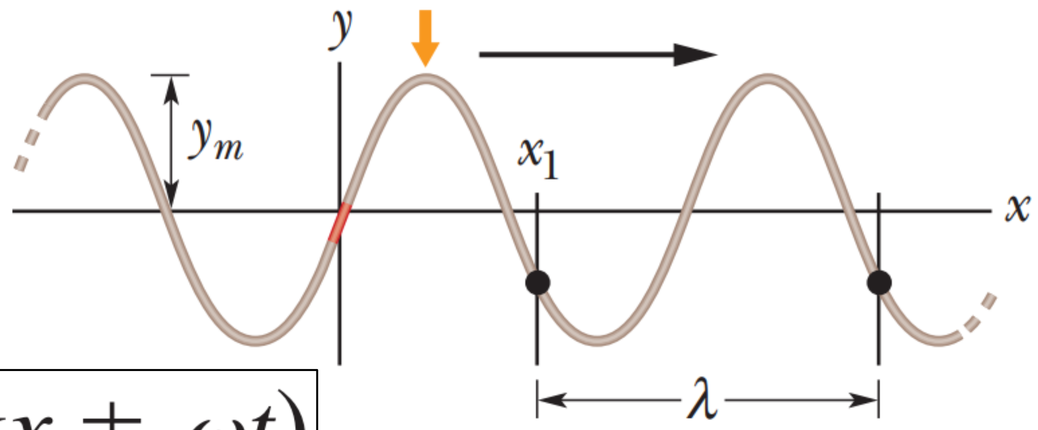
Wave Interference

And

Numerical problems

Wave

$$y = y_m \sin(kx \pm \omega t + \phi).$$



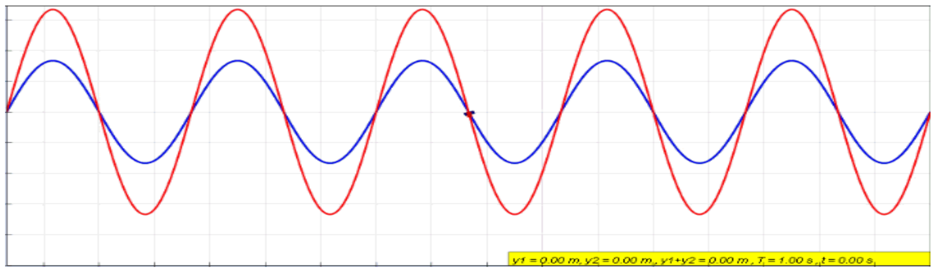
$$y(x,t) = h(kx \pm \omega t)$$

$$P_{\text{avg}} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \quad \text{(average power).}$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi).$$

$$y'(x,t) = [2y_m \sin kx] \cos \omega t.$$



Sample Problem 16.01 Determining the quantities in an equation for a transverse wave

Homework

A string has linear density $\mu = 525$ g/m and is under tension $\tau = 45$ N. We send a sinusoidal wave with frequency $f = 120$ Hz and amplitude $y_m = 8.5$ mm along the string. At what average rate does the wave transport energy?

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2 = \frac{1}{2}\left(0.525\,\frac{kg}{m}\right)\left(\sqrt{\frac{45}{0.525}}\,\frac{m}{s}\right)\left(2\pi \times 120\,\frac{1}{s}\right)^2\left(8.5\times10^{-3}\,m\right)^2$$

$$P_{avg} = 99.8 \text{ watt}$$

$y'(x,t) = [2y_m \cos \frac{\phi}{2}] \sin(kx - \omega t + \phi/2)$

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100°.

Recall the case of interference of two waves travelling in the same direction

(a) What is the amplitude y'_m of the resultant wave due to the interference, and what is the type of this interference?

The amplitude of the resultant wave is given by the relation

$$y'_m = 2y_m \cos\frac{\phi}{2} = 2(0.0098\,m)\cos(50^\circ) = 12.6\,mm$$

Since the resulting wave have large amplitude, the interference is constructive in nature.

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100°.

(b) What phase difference, in radians and wavelengths, will give the resultant wave an amplitude of 4.9 mm?

$$y'_m = 2y_m \cos\phi/2 \implies \phi = 2\left[\cos^{-1}\left(\frac{y'_m}{2y_m}\right)\right]$$

$$\phi = 2\left[\cos^{-1}\left(\frac{4.9\,mm}{2\times 9.8\,mm}\right)\right] = 151°$$

$$= 2.63 \text{ rad}$$

Since a single wavelength is $\lambda = 2\pi$ rad

$$2.63 \times \frac{2\pi}{2\pi} \text{ rad}$$

$$\frac{2.63}{2\pi} \times (2\pi \text{ rad}) = \frac{2.63}{2\pi}\lambda$$

$$\phi \text{(phase difference} = 0.418\,\lambda$$

Draw the waves to show how this phase difference occurs.

Studying the Standing Waves

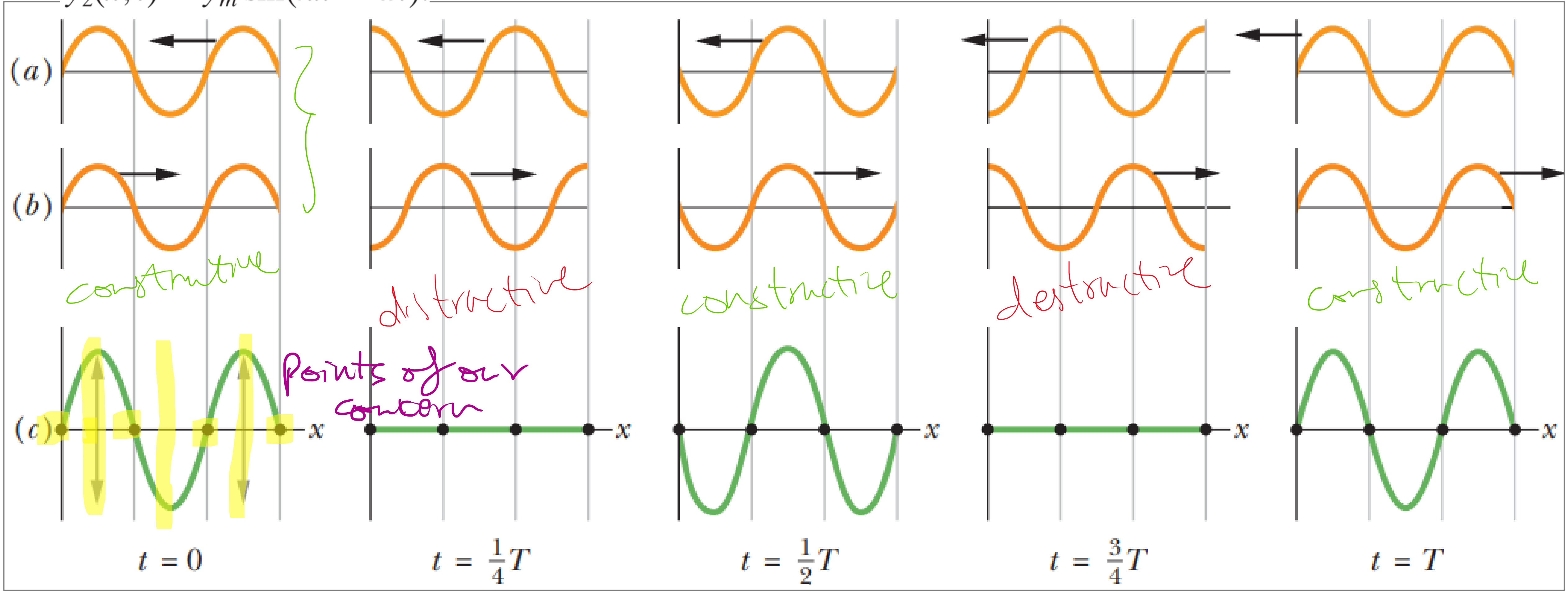
Wave Interference

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx + \omega t).$$

The resulting wave pattern is not travelling But it is oscillating

$$\boxed{y'(x, t) = [2y_m \sin kx] \cos \omega t.}$$

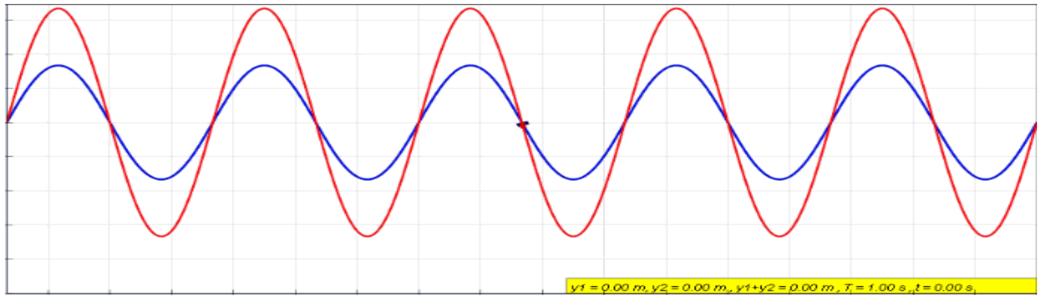


Nodes

where the sinusoidal part of amplitude is minimum

$\sin kx = 0$

Points of Zero Amplitude



$$kx = n\pi, \qquad \text{for } n = 0, 1, 2, \ldots .$$

$\boxed{k = \frac{2\pi}{\lambda}}$

$$x = n\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, 3, \ldots \qquad \text{(nodes)},$$

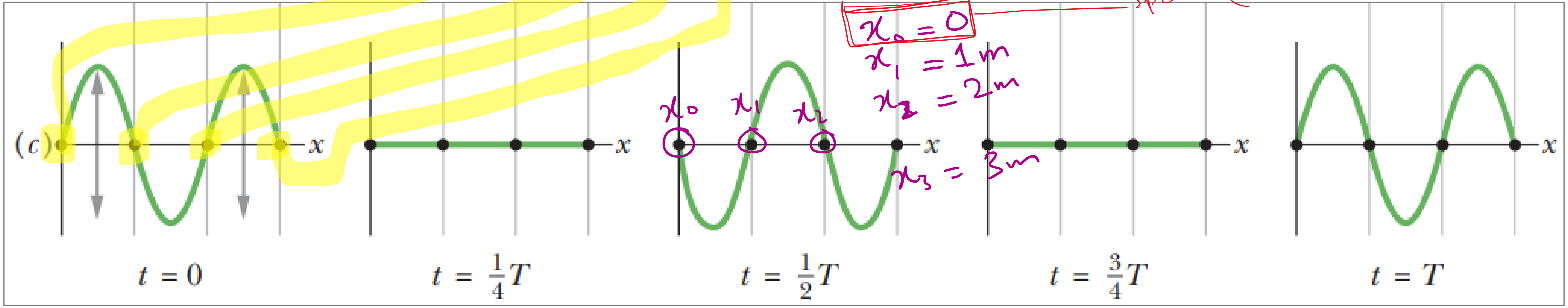
If $\lambda = 2m$ then

$\boxed{x_0 = 0}$ special (minimum) condition

$x_1 = 1m$

$x_2 = 2m$

$x_3 = 3m$

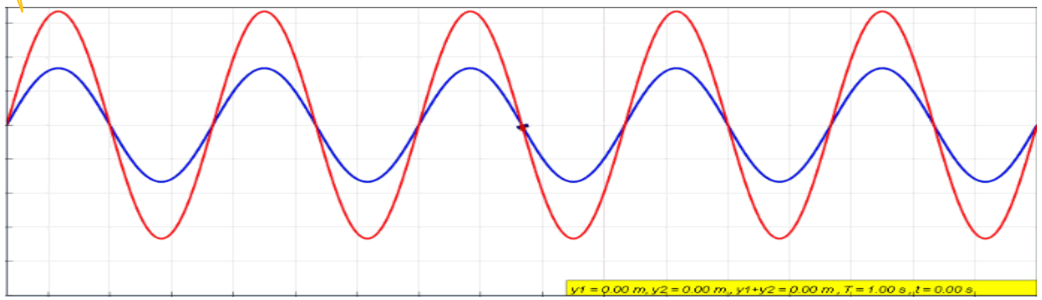


Antinodes

where the sinusoidal part of standing wave amplitude is 1

$\sin kx = 1$

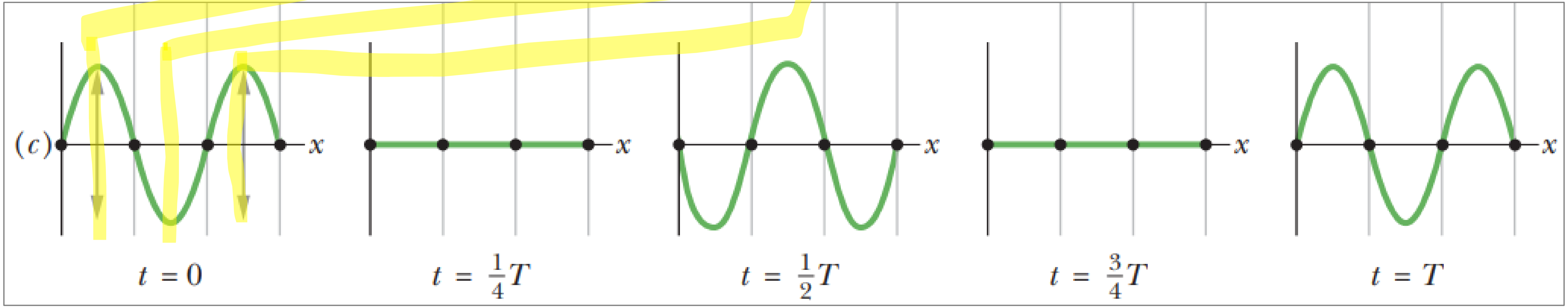
Points of Maximum Amplitude



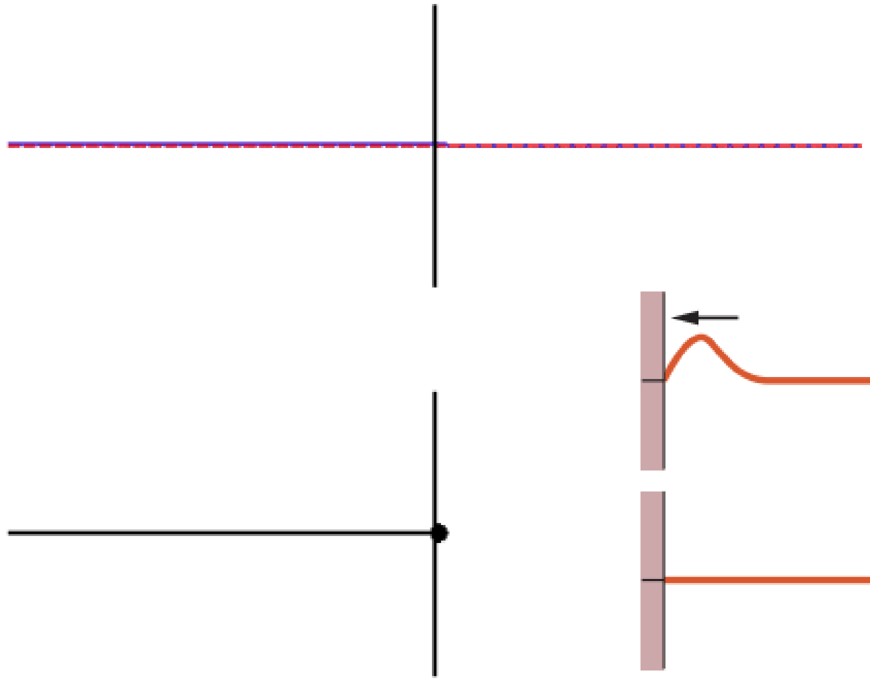
$$kx = \left(n + \tfrac{1}{2}\right)\pi, \qquad \text{for } n = 0, 1, 2, \ldots$$

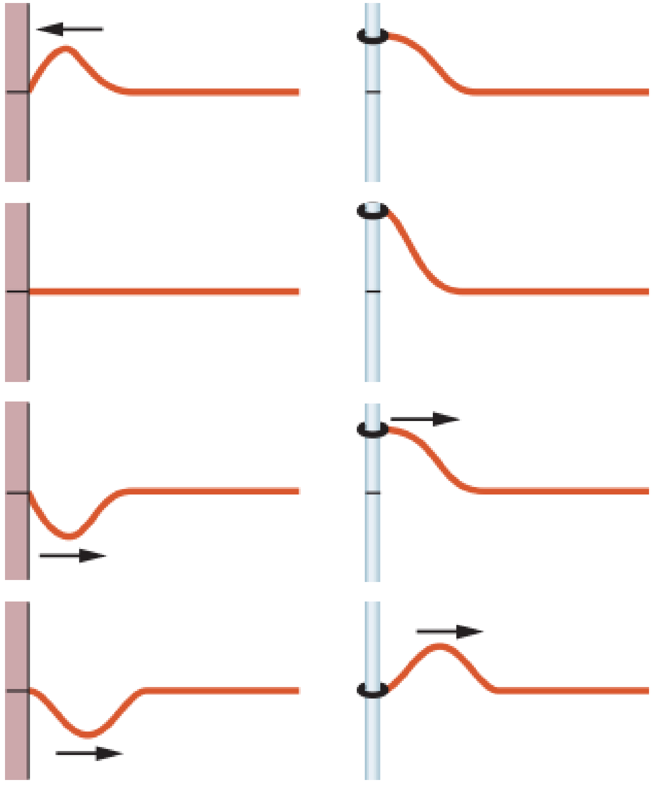
$\boxed{k = \frac{2\pi}{\lambda}}$

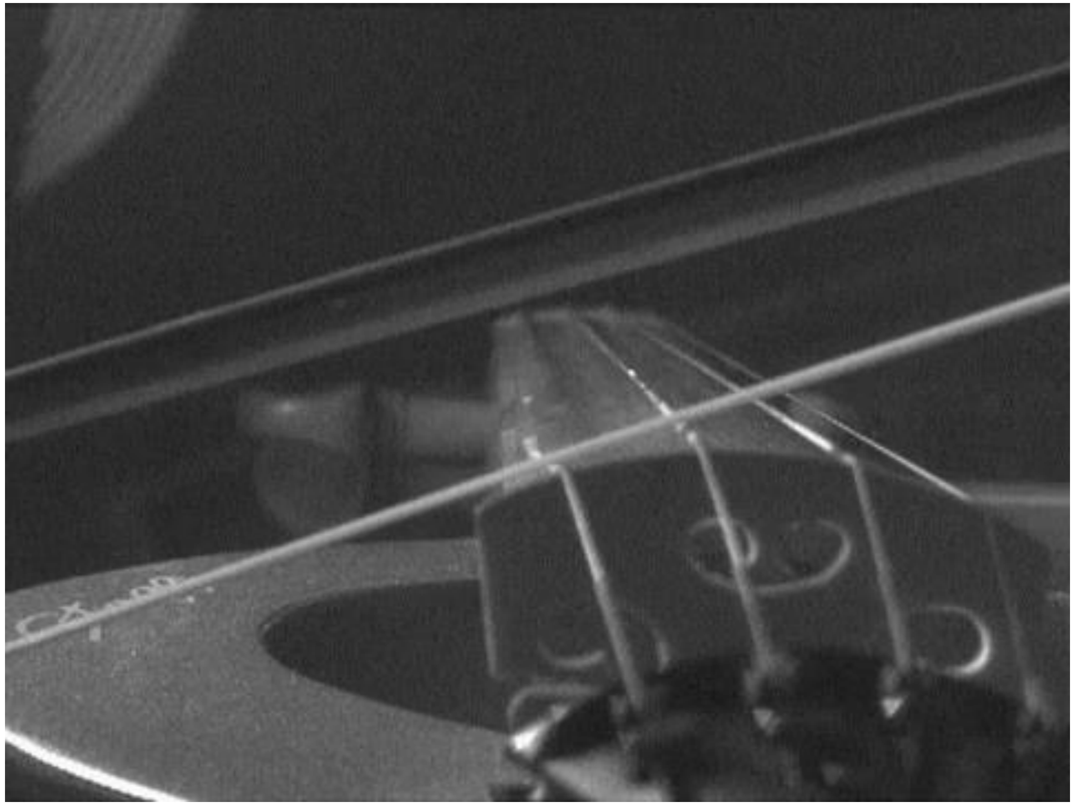
$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(antinodes)},$$



Wave Reflections and Harmonics

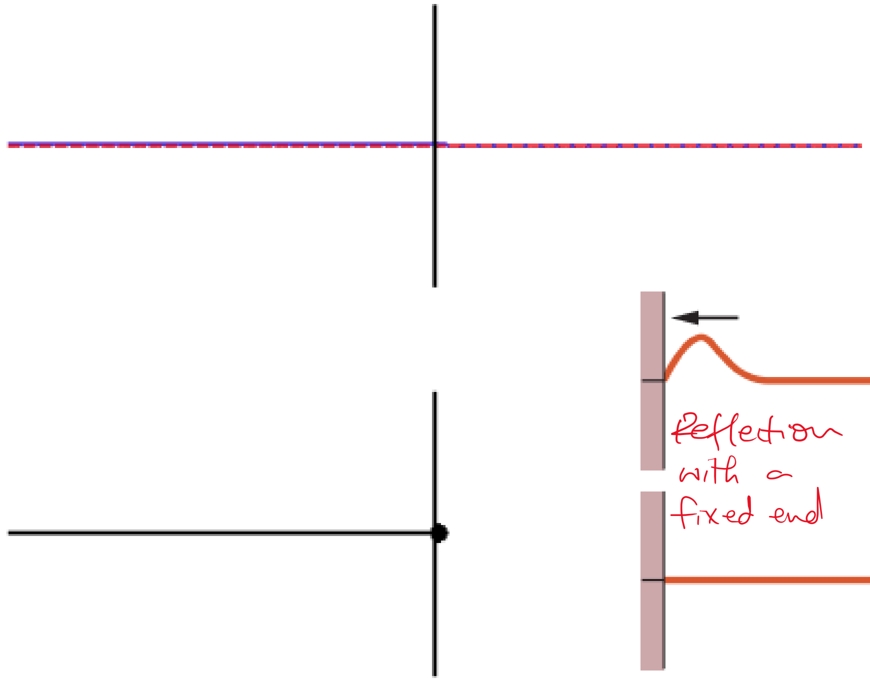


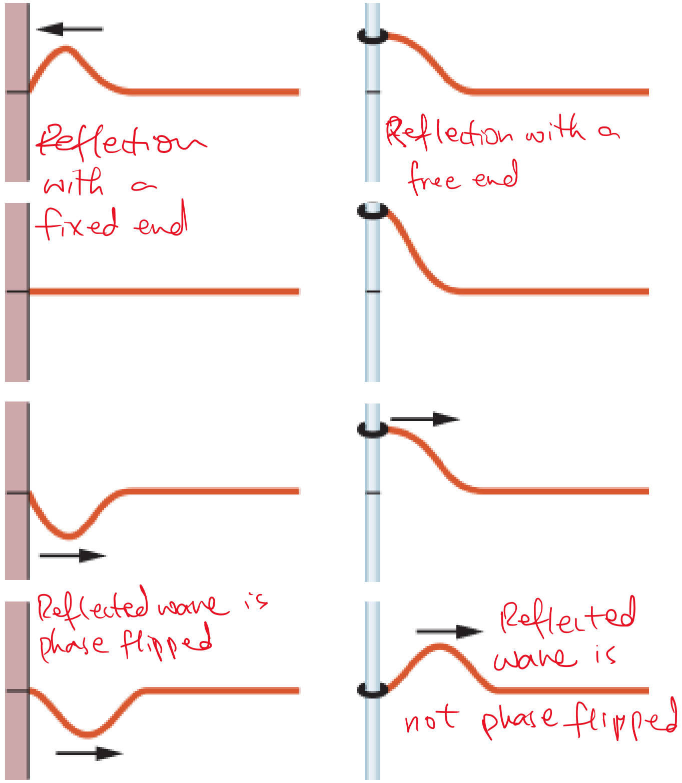


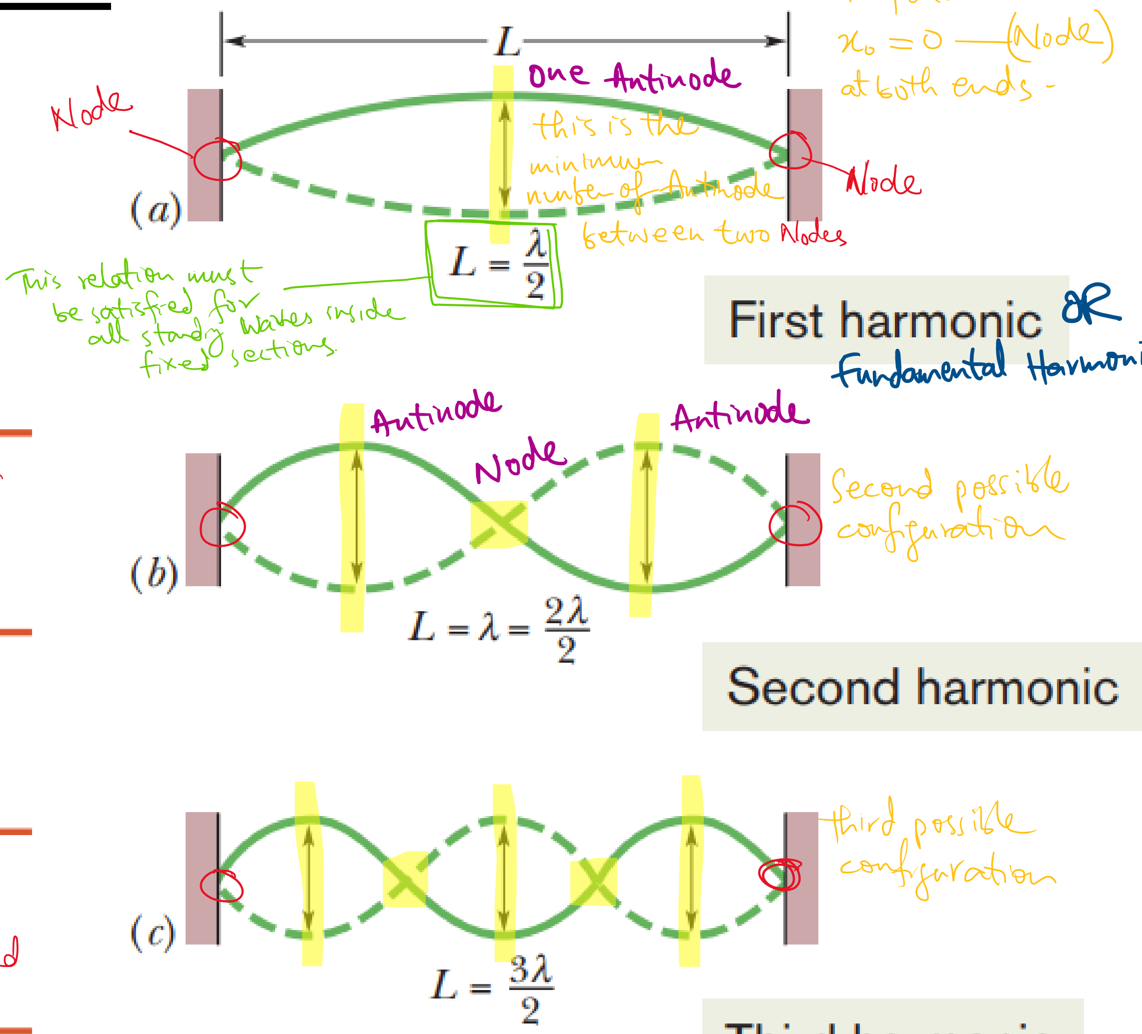


Wave Reflections and Harmonics

If we trap a travelling transverse wave between two fixed ends then the wave will interfere with itself in opposite directions resulting in a standing wave







First harmonic OR Fundamental Harmonic

Second harmonic

Third harmonic

Wave Reflections and Harmonics

Conditions to produce Harmonics:

$L = n\left(\frac{\lambda}{2}\right)$

$$\boxed{\lambda = \frac{2L}{n}}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

this gives all the allowed wavelengths inside a section of length L

for a section of length 0.42 m

Allowed waves with corresponding frequencies

$\lambda = \frac{2(0.42)}{1} = 0.84\,m$ — wave of wavelength 0.84m can exist as first Harmonic

$\lambda = \frac{2(0.42)}{2} = 0.42\,m$ — wave of wavelength 0.42m can exist as second Harmonic

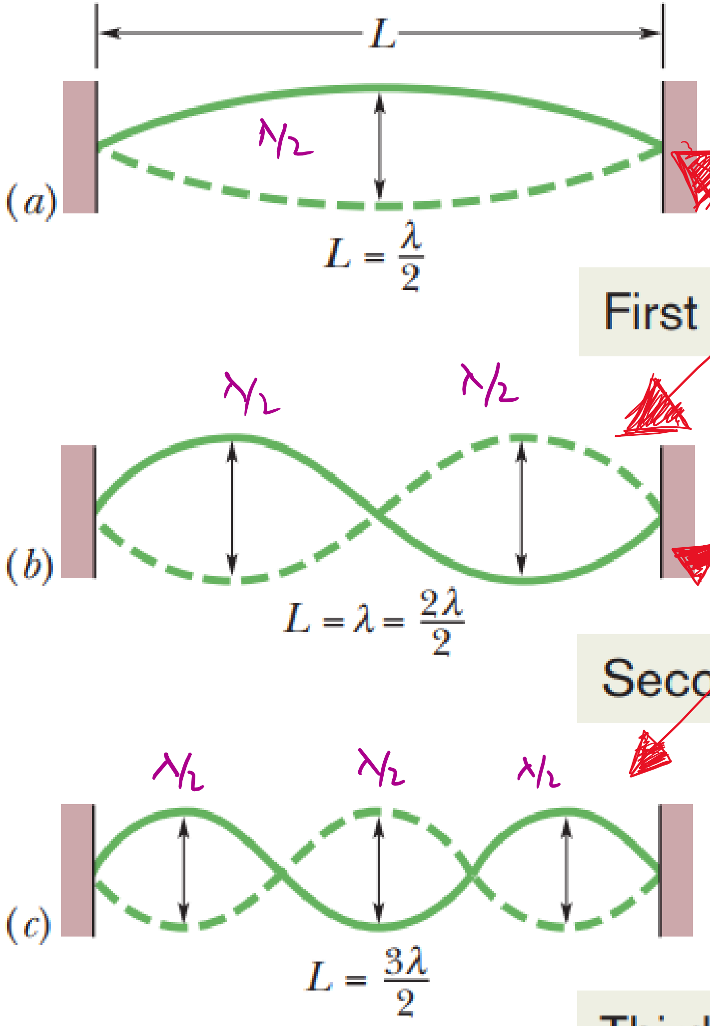
$\lambda = \frac{2(0.42)}{3} = 0.28\,m$ — wave of wavelength 0.28m can exist as third Harmonic

$\vdots$

wave velocity

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

allowed frequencies inside a section.



First harmonic

Second harmonic

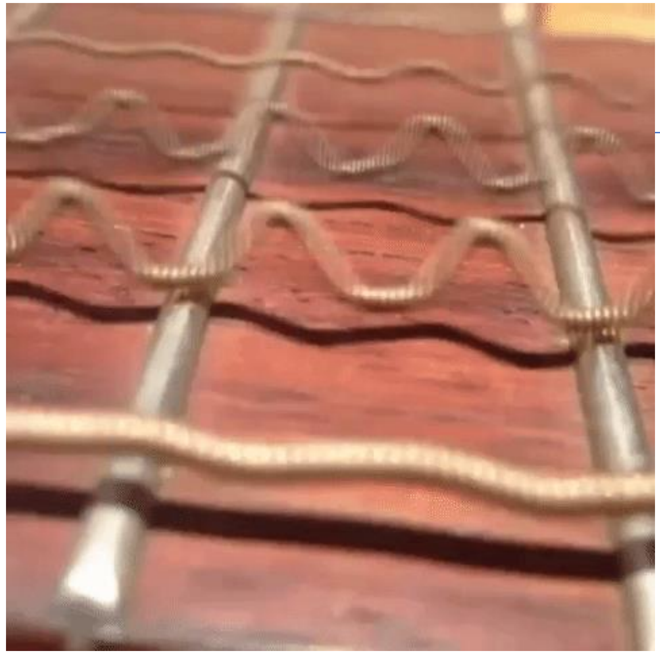
Third harmonic

No configuration in between is possible

No configuration in between is possible

this is analogous to a staircase where you can step on either step but not in between the steps

Wave Reflections and Harmonics



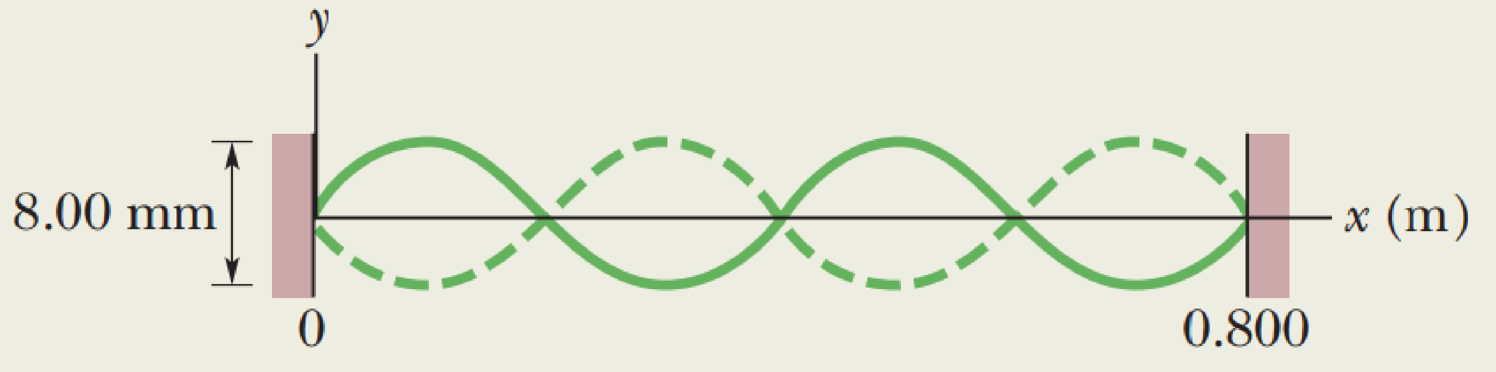
Conditions to produce Harmonics:

$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3$$

Figure 16-23 shows resonant oscillation of a string of mass $m = 2.500$ g and length $L = 0.800$ m and that is under tension $\tau = 325.0$ N. What is the wavelength λ of the transverse waves producing the standing wave pattern, and what is the harmonic number n? What is the frequency f of the transverse waves and of the oscillations of the moving string elements? What is the maximum magnitude of the transverse velocity u_m of the element oscillating at coordinate $x = 0.180$ m? At what point during the element's oscillation is the transverse velocity maximum?

Read solution of Sample Problem 16.06



Lecture 16