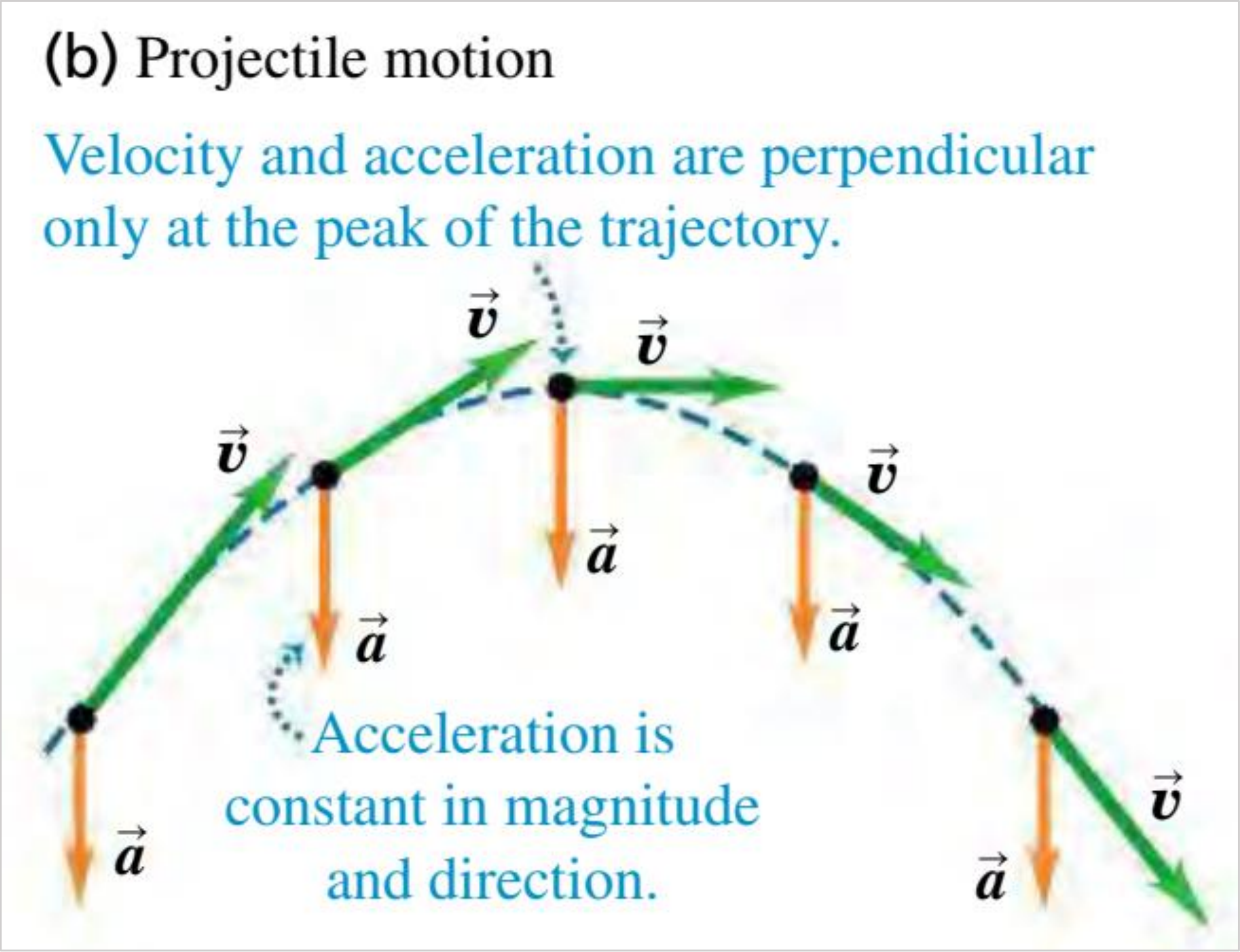
Curved paths

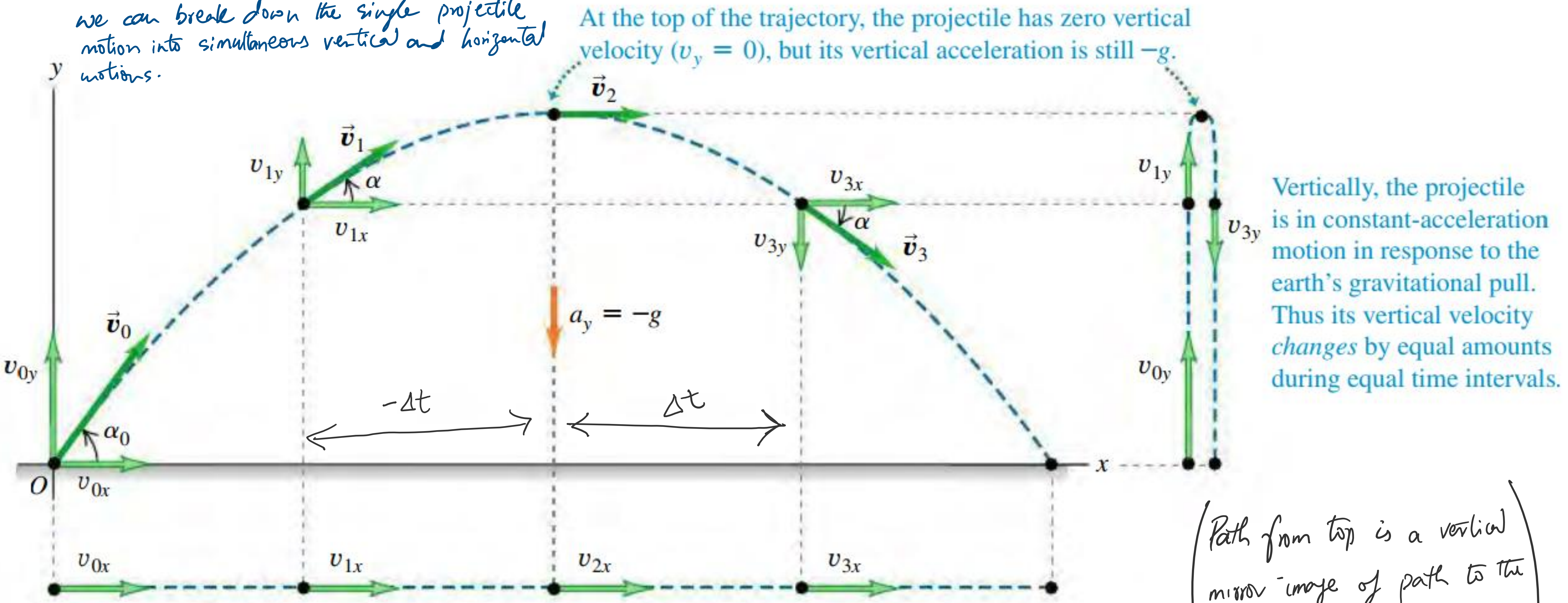
Projectile and Circular Motions



Demonstrations

Scan to watch online demonstrations

Since the axes are independent; we can break down the single projectile motion into simultaneous vertical and horizontal motions.

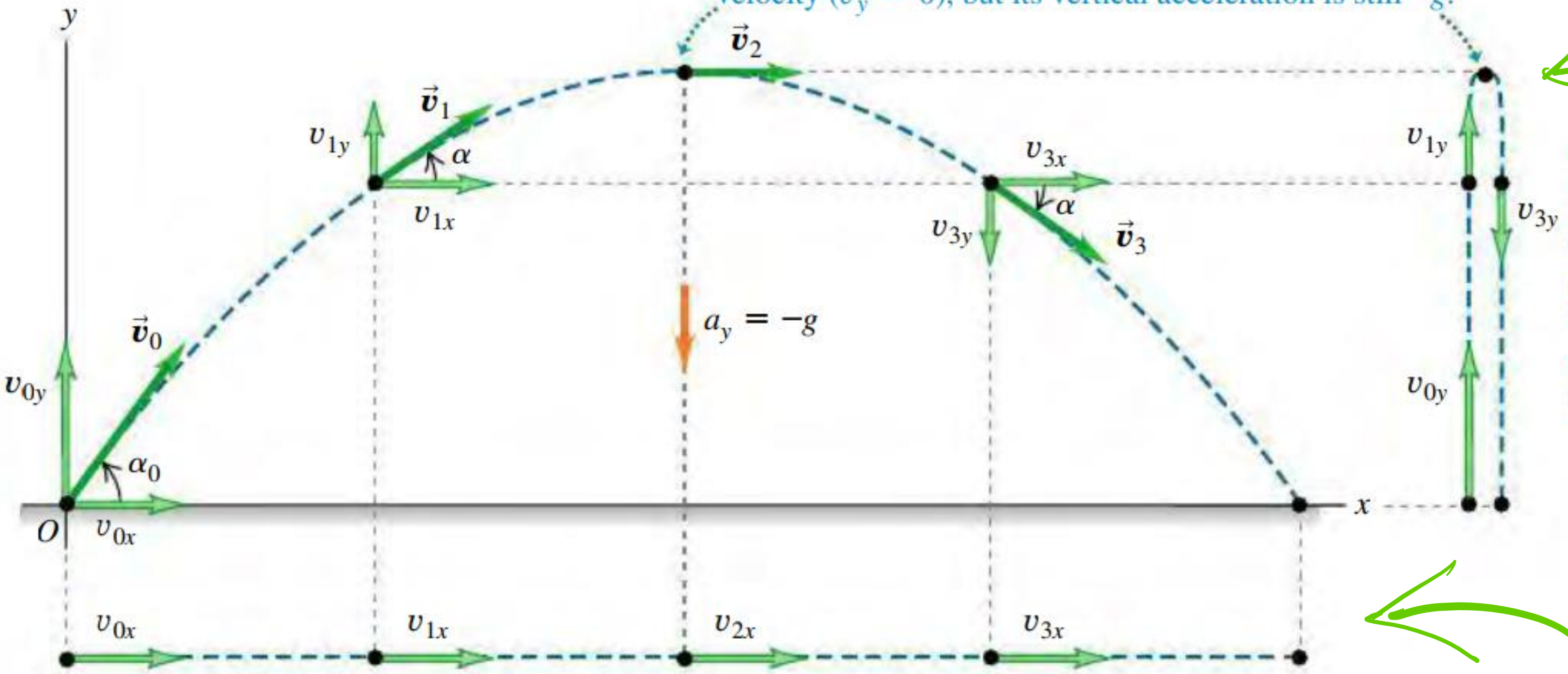


Horizontally, the projectile is in constant-velocity motion: Its horizontal acceleration is zero, so it moves equal x-distances in equal time intervals.

(Path from top is a vertical mirror-image of path to the top.)

$$\boxed{\Delta y = V_{0y} t - \frac{1}{2} g t^2}$$

At the top of the trajectory, the projectile has zero vertical velocity ($v_y = 0$), but its vertical acceleration is still $-g$.



Vertically, the projectile is in constant-acceleration motion in response to the earth's gravitational pull. Thus its vertical velocity *changes* by equal amounts during equal time intervals.

Horizontally, the projectile is in constant-velocity motion: Its horizontal acceleration is zero, so it moves equal x-distances in equal time intervals.

$$\boxed{\Delta x = V_{0x} t}$$

Coordinates at time t of a **projectile** (positive y-direction is upward, and $x = y = 0$ at $t = 0$)

$$x = (v_0 \cos \alpha_0)t$$

Speed at $t = 0$

Direction at $t = 0$

Time

$$y = (v_0 \sin \alpha_0)t - \frac{1}{2}gt^2$$

Velocity components at time t of a **projectile** (positive y-direction is upward)

$$v_x = v_0 \cos \alpha_0$$

Speed at $t = 0$

Direction at $t = 0$

Acceleration due to gravity: Note $g > 0$.

$$v_y = v_0 \sin \alpha_0 - gt$$

Time

Path of projectile

Parabolic Path

$$\boxed{y = (\tan\alpha_0)x - \frac{g}{2v_0^2\cos^2\alpha_0}x^2}$$

obtained by eleminating time variable

we can write the time t_1 when $v_y = 0$ as (the top)

initial vertical velocity

$$t_1 = \frac{v_{0y}}{g} = \frac{v_0\sin\alpha_0}{g}$$

when reaches top is when $V_y = 0$

try not to memorize these

$$\boxed{h = (v_0\sin\alpha_0)\left(\frac{v_0\sin\alpha_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0\sin\alpha_0}{g}\right)^2 = \frac{v_0^2\sin^2\alpha_0}{2g}}$$

$$\boxed{R = (v_0\cos\alpha_0)t_2 = (v_0\cos\alpha_0)\frac{2v_0\sin\alpha_0}{g} = \frac{v_0^2\sin 2\alpha_0}{g}}$$

twice the time due to mirror property

A monkey escapes from the zoo and climbs a tree. After failing to entice the monkey down, the zookeeper fires a tranquilizer dart directly at the monkey (**Fig. 3.26**). The monkey lets go at the instant the dart leaves the gun. Show that the dart will *always* hit the monkey, provided that the dart reaches the monkey before he hits the ground and runs away.

here,
we do not require to calculate any value, just need to show that the dart and the monkey can intersect.

Lecture 7

Horizontal position

$$\boxed{x_{monkey} = x_{dart}} \implies d = \frac{v_{0x}}{t} = \frac{v_0 \cos\alpha_0}{t}$$

$$\boxed{t = \frac{d}{v_0 \cos\alpha_0}}$$ time when the horizontal positions match.

Vertical position

$$\boxed{y_{monkey} = y_{dart}}$$

$$y_{dart} = v_{0y}t - \frac{1}{2}gt^2 + y_{0,dart}$$

initial position of dart

$$y_{monkey} = \cancel{v_{0,m}}^{\,0}t - \frac{1}{2}gt^2 + y_{0,m}$$

initial position of monkey

equating the two forms

$$v_{0y}t - \cancel{\frac{1}{2}gt^2} + 0 = 0 - \cancel{\frac{1}{2}gt^2} + d\tan\alpha$$

$$v_{0y}\left(\frac{d}{v_0\cos\alpha}\right) = d\tan\alpha$$

$$\left(\frac{v_0\sin\alpha}{v_0\cos\alpha}\right)d = d\tan\alpha$$

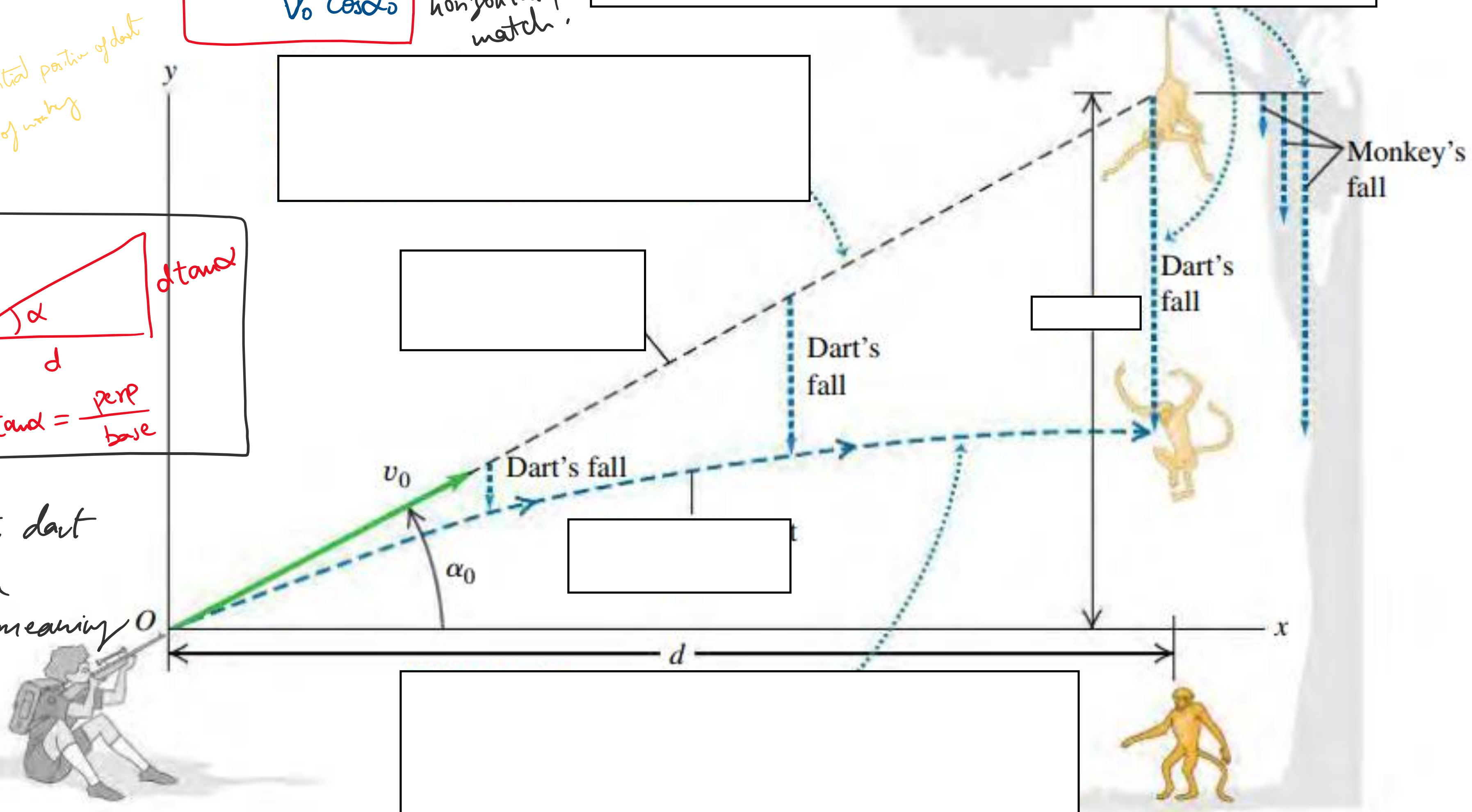
d tan α

α

d

tan α = perp / base

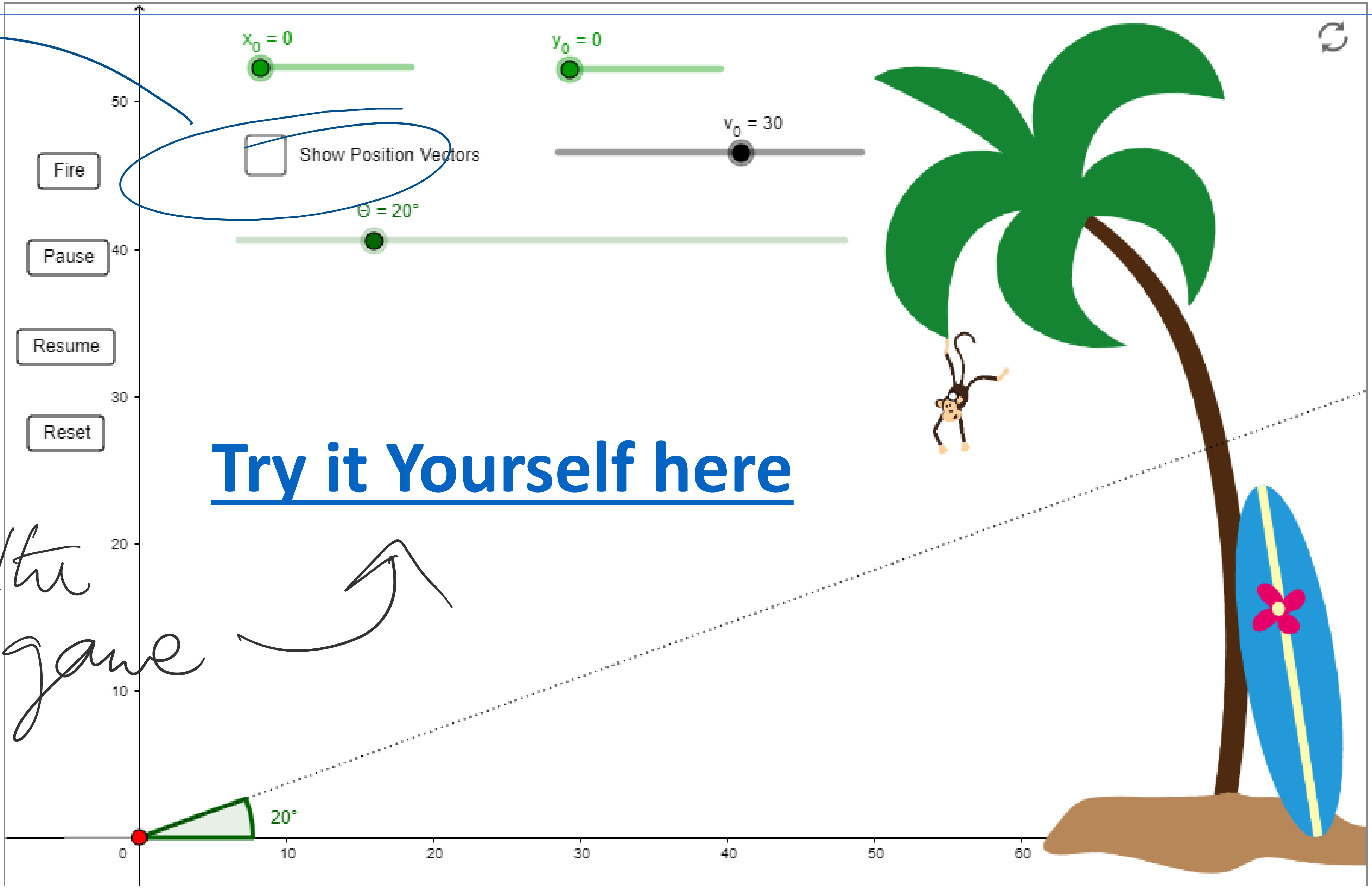
Monkey and the dart share common coordinates, meaning that they will always intersect if we keep the $\boxed{t = \frac{d}{v_0\cos\alpha}}$



Lecture 7

Projectile Motion: Tranquilize the Monkey

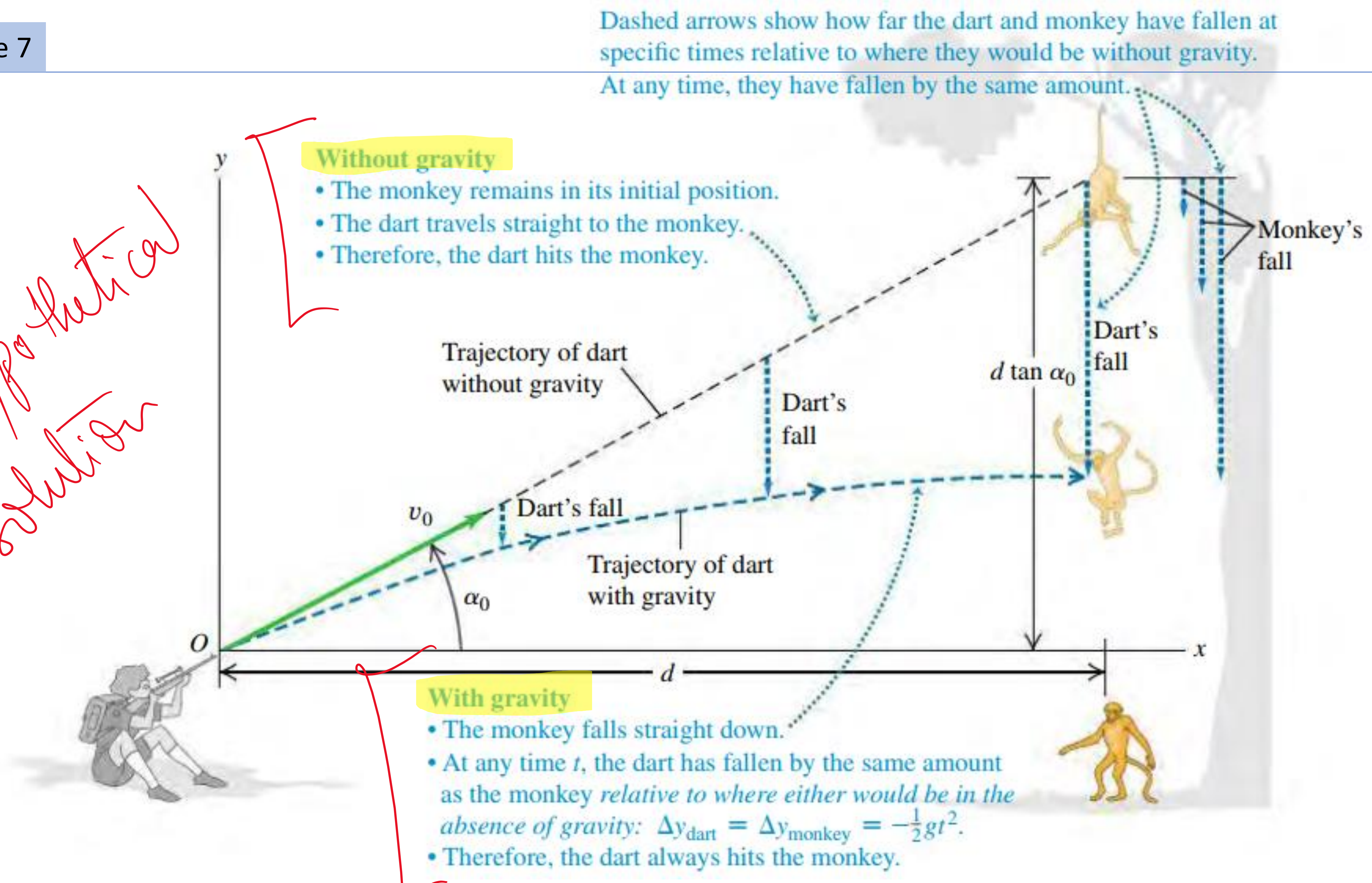
Do not forget to check this



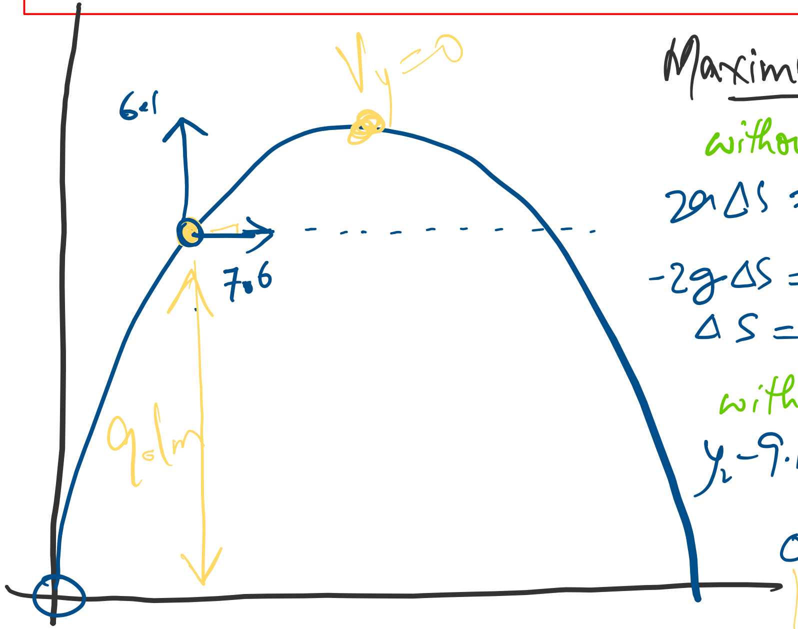
[Try it Yourself here]

Play the game

Hypothetical solution



••43 ILW A ball is shot from the ground into the air. At a height of 9.1 m, its velocity is $\vec{v} = (7.6\hat{i} + 6.1\hat{j})$ m/s, with $\hat{i}$ horizontal and $\hat{j}$ upward. (a) To what maximum height does the ball rise? (b) What total horizontal distance does the ball travel? What are the (c) magnitude and (d) angle (below the horizontal) of the ball's velocity just before it hits the ground?



Maximum height

without time

$2a\Delta S = V_f^2 - V_i^2$

$-2g\Delta S = 0 - (6.1)^2$

$\Delta S = 1.89$ m

with time

$y_2 - 9.1 = (6.1)t - \frac{1}{2}gt^2$

$0 = 6.1 - gt$

$\boxed{t = 0.622 \text{ sec}}$

$y_2 = 11$ m

Horizontal Range

A simple way to find complete time period:

$0 - 9.1 = (6.1)t - 4.8t^2$

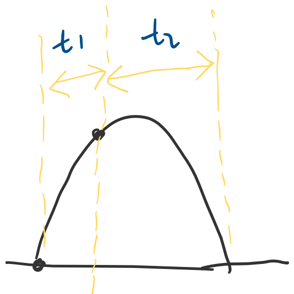
solving this Quadratic eq.

$t_2 = 2.12$ sec

$t_1 = -0.87$ sec

total time is therefore

$t = 2.99$ sec



Now, because horizontal velocity remaining constant throughout the motion we can use $V_x = 7.6$ m/s

$R = V_x t = 7.6 \times 2.99$

$\boxed{R = 22.724 \text{ m}}$

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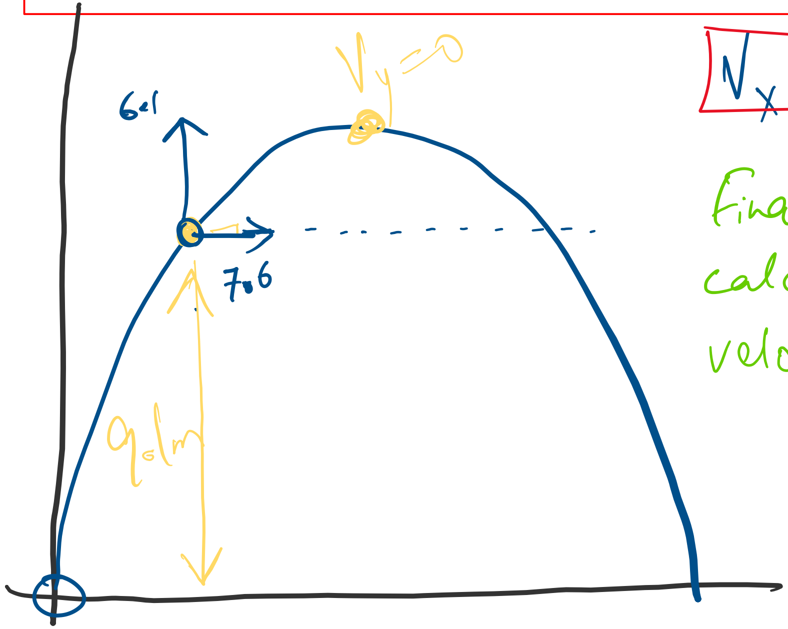


$\boxed{V_x = 7.1\,m}$

correct these results for Vx = 7.6 m/s

Final vertical velocity can be calculated via change of vertical velocity at the given point.

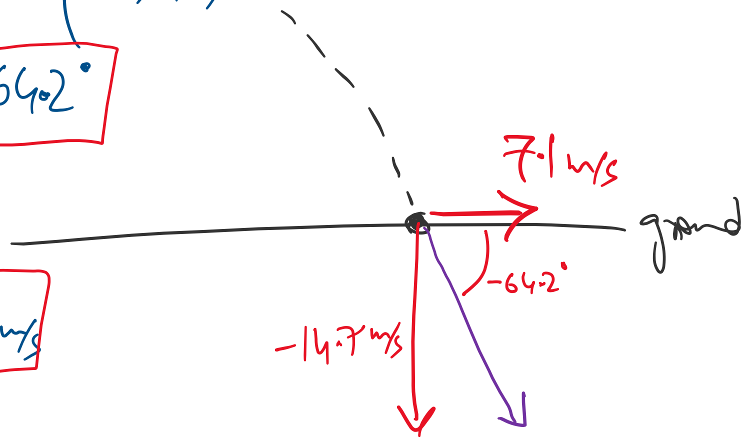
$V_y = 6.1 - 9.8(2.12)$

$\boxed{V_y = -14.676\ m/s}$

$\theta = \tan^{-1}\left(\frac{-14.7}{7.1}\right)$

$\boxed{\theta = -64.2^\circ}$

$\boxed{|V| = 16.3\ m/s}$



•61 When a large star becomes a *supernova*, its core may be compressed so tightly that it becomes a *neutron star*, with a radius of about 20 km (about the size of the San Francisco area). If a neutron star rotates once every second, (a) what is the speed of a particle on the star's equator and (b) what is the magnitude of the particle's centripetal acceleration? (c) If the neutron star rotates faster, do the answers to (a) and (b) increase, decrease, or remain the same?

time period $= T = 1$ sec

$$V = \frac{2\pi R}{T} = \frac{2\pi (20\,km)}{T}$$

$$\boxed{V = 125.6\ km/s}$$

$$a_c = V^2/R = \boxed{7901.3\ km/s^2 = a_c}$$

Faster rotation means smaller timeperiod, this will lead to higher speeds and eventually to higher centripital acceleration. However, this is the case when the radius remains constant.

•60 A centripetal-acceleration addict rides in uniform circular motion with radius $r = 3.00$ m. At one instant his acceleration is $\vec{a} = (6.00\ \text{m/s}^2)\hat{\text{i}} + (-4.00\ \text{m/s}^2)\hat{\text{j}}$. At that instant, what are the values of (a) $\vec{v}\cdot\vec{a}$ and (b) $\vec{r}\times\vec{a}$?

(a) recall that for a uniform circular motion the $\vec{a}$ and $\vec{v}$ vectors are always perpendicular to each other, therefore $\vec{v}\cdot\vec{a} = 0$

(b) In case of a circle $\vec{r}$ makes angle of 180° from acceleration vector, refore

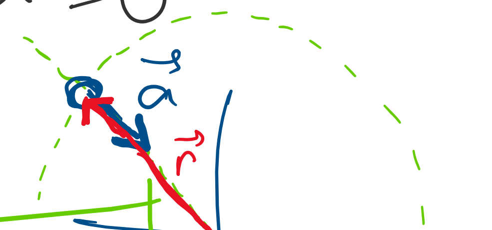
$$|\vec{r}\times\vec{a}| = |r||a|\sin(180^\circ) = |r||a|(-1) = (3)(7.21)(-1)$$

$$|\vec{r}\times\vec{a}| = 21.6\ m^2/s^2$$

Since the $|r|$ and $|a|$ remains constant throughout the motion, the $|\vec{r}\times\vec{a}|$ does not change.

•60 A centripetal-acceleration addict rides in uniform circular motion with radius $r = 3.00$ m. At one instant his acceleration is $\vec{a} = (6.00 \text{ m/s}^2)\hat{\mathrm{i}} + (-4.00 \text{ m/s}^2)\hat{\mathrm{j}}$. At that instant, what are the values of (a) $\vec{v} \cdot \vec{a}$ and (b) $\vec{r} \times \vec{a}$?

(a) recall that for a uniform circular motion the $\vec{a}$ and $\vec{v}$ vectors are always perpendicular to each other, therefore $\vec{v} \cdot \vec{a} = 0$



(b) $\theta = \tan^{-1}\left(\frac{-4}{6}\right) = -33.6^\circ = +147^\circ$

$\vec{r} = -2.51\,\hat{i} + 1.64\,\hat{j}$

$$\vec{r} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -4 & 0 \\ -2.5 & 1.64 & 0 \end{vmatrix}$$

$= (0)\hat{i} - \hat{j}(0) + \hat{k}(9.84 - 10)$

$\boxed{\vec{r} \times \vec{a} = -0.16\hat{k}}$

$\vec{a}$ is rotating but so is $\vec{r}$. They remain antiparallel. And the vector $\vec{r} \times \vec{a}$ also remains constant.

Practice problems:

Problems from **Fundamentals of Physics**
-Jearl Walker

Chapter 4 : Motion in Two and Three dimension

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17, 35, 42, 48, 67

Additional Problems

Additional Problems 90, 92, 85