National University of Computer & Emerging Sciences, Karachi

GRAPH THEORY- Assignment#02

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Total Marks: 35

Question 1.[3 marks] Prove the following:

(i) The total degree of a tree on n vertices is $2n - 2$.

(ii) Let G be a graph with n vertices, then $\kappa(G) = \delta(G)$ if $\delta(G) \geq n - 2$.

(iii) Find the relation between diameter of (C_n) and its complement $\overline{C}_n$. Is it true that $diam(C_n) \geq diam(\overline{C}_n)$? Justify your answer.

Question 2.[2 marks] Let G be a connected graph.

(a) Prove that if a graph G has a bridge then G has a cut-vertex.

(b) Is the converse true? Prove or give a counterexample.

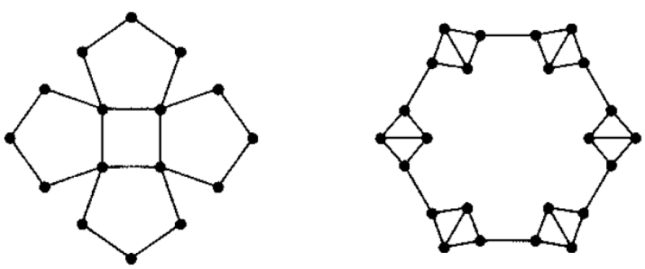
Question 3.[2 marks] There are five cities in a network. The cost of building a road directly between i and j is the entry a_{ij} in the matrix below. An infinite entry indicates that there is a mountain in the way and the road cannot be built. Determine the least cost of making all the cities reachable from each other.

$$\begin{bmatrix} 0 & 3 & 5 & 11 & 9 \\ 3 & 0 & 3 & 9 & 8 \\ 5 & 3 & 0 & \infty & 10 \\ 11 & 9 & \infty & 0 & 7 \\ 9 & 8 & 10 & 7 & 0 \end{bmatrix}$$

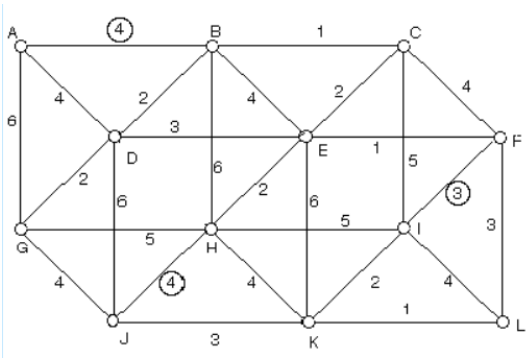
Question 4.[4 marks] There are five cities in a network. The travel time for traveling directly from i to j is the entry a_{ij} in the matrix below. The matrix is not symmetric (use directed graphs), and $a_{ij} = \infty$ indicates that there is no direct route. Determine the least travel time and quickest route from i to j for each pair i, j.

$$\begin{bmatrix} 0 & 10 & 20 & \infty & 17 \\ 7 & 0 & 5 & 22 & 33 \\ 14 & 13 & 0 & 15 & 27 \\ 30 & \infty & 17 & 0 & 10 \\ \infty & 15 & 12 & 8 & 0 \end{bmatrix}$$

Question 5.[2 marks] The graph on the left below was the logo of the 9th Quadrennial Internatio Conference in Graph Theory, held in Kalamazoo in 2000. Count its spanning trees.



Question 6.[3 marks] Find a minimum spanning tree of the given graph below.



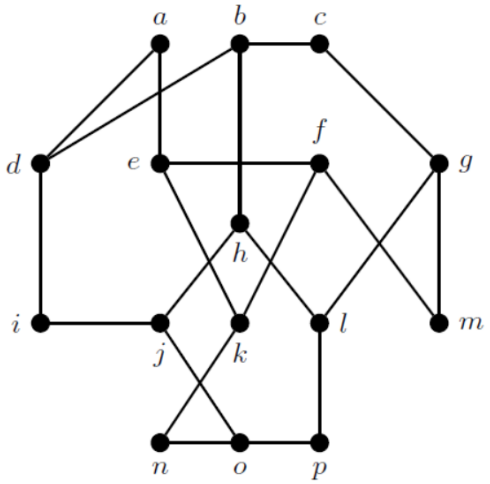
(a) Use Kruskal's algorithm to draw the resulting spanning tree and list the edges in the order they are picked by Kruskal's algorithm.

(b) Use Prim's algorithm, starting at H, to draw the resulting spanning tree and list the edges in the order they are picked by Prim's algorithm.

Question 7.[2 marks] Draw the labelled tree corresponds to the given prüfer sequence.

(i) $(3, 2, 2, 3, 2, 2)$ (ii) $(5, 3, 2, 3, 3, 1)$

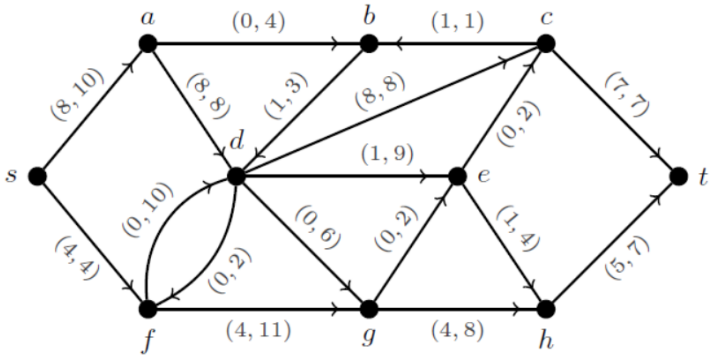
Question 8.[3 marks] Complete each of the following on the two graph shown below.



(a) Find the breadth-first search tree with root i.

(b) Find the depth-first search tree with root a.

Question 9.[4 marks] For the graph given below, use the Augmenting Flow Algorithm to maximize the flow and the Min-Cut Method to find a minimum cut.



Question 10.[4 marks]

Consider the 3×5 Latin rectangle $L = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 5\,1\,2\,3\,4 \\ 4\,5\,1\,2\,3 \end{pmatrix}$.

Define a bipartite graph G with bipartition (X, Y) associated with L as follows:

(i) $X = \{1, 2, 3, 4, 5\}$,
(ii) $Y = \{C_1, C_2, C_3, C_4, C_5\}$, where C_i is the ith column of L,
(iii) 'i' in X is adjacent to 'C_j' in Y if and only if 'i' does not appear in 'C_j'.

(a) Draw the diagram of G.
(b) What is the degree of each vertex in X?
(c) What is the degree of each vertex in Y?
(d) Is G 2-regular?
(e) Does G contain a perfect matching?
(f) Display a perfect matching in G if your answer to (e) is 'yes'.

Question 11.[2 marks]

Consider the following graph G:

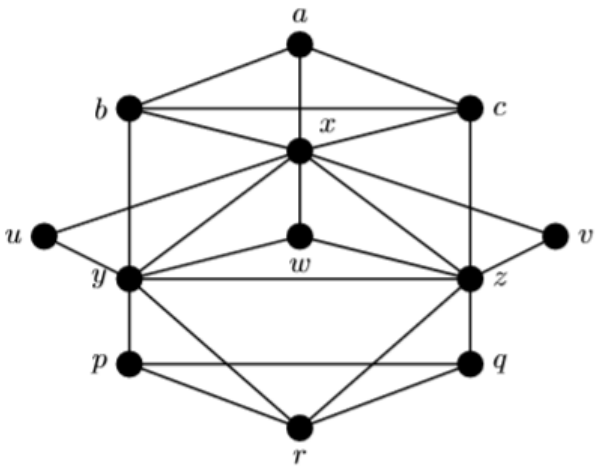


Fig. 7.5.10

(i) Does G have a perfect matching?
(ii) What is the size of a maximum matching in G?

Question 12.[4 marks]

Determine the stable matchings resulting from the Proposal Algorithm run with men proposing and with women proposing, given the preference lists below.

Men $\{u, v, w, x, y, z\}$	**Women** $\{a, b, c, d, e, f\}$
$u: a > b > d > c > f > e$	$a: z > x > y > u > v > w$
$v: a > b > c > f > e > d$	$b: y > z > w > x > v > u$
$w: c > b > d > a > f > e$	$c: v > x > w > y > u > z$
$x: c > a > d > b > e > f$	$d: w > y > u > x > z > v$
$y: c > d > a > b > f > e$	$e: u > v > x > w > y > z$
$z: d > e > f > c > b > a$	$f: u > w > x > v > z > y$

The End