Velocity and Acceleration

Acceleration

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}$$

acceleration vector

Average

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}.$$

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

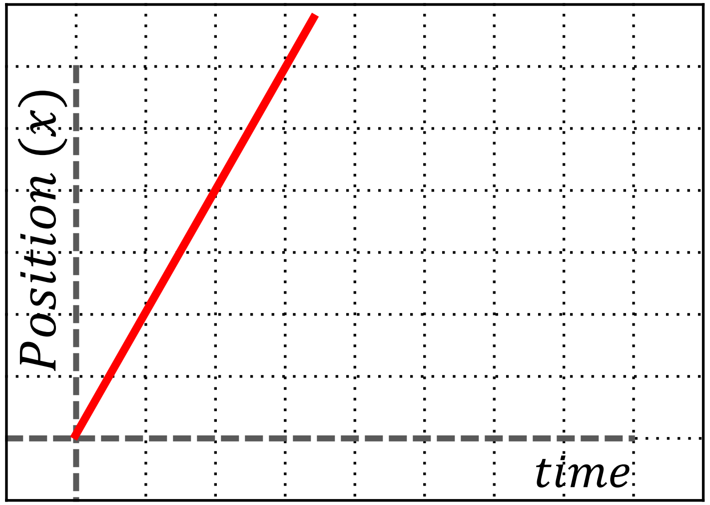
Instantaneous acceleration

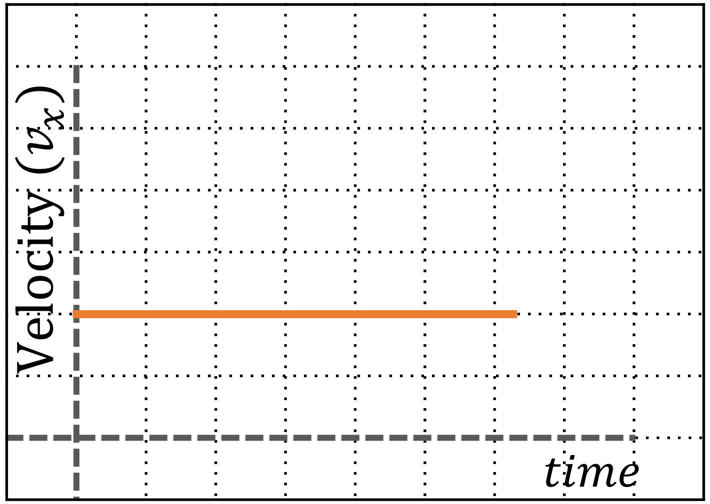
Acceleration

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}$$

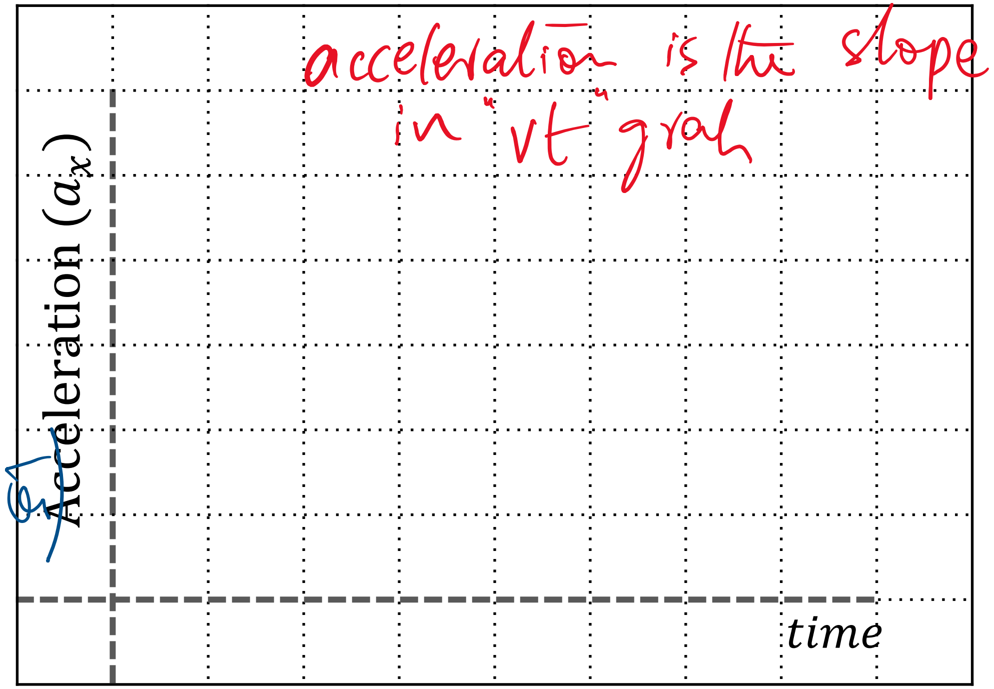
$$\vec{a}_{\mathrm{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}.$$

$$\vec{a} = \frac{d\vec{v}}{dt}.$$



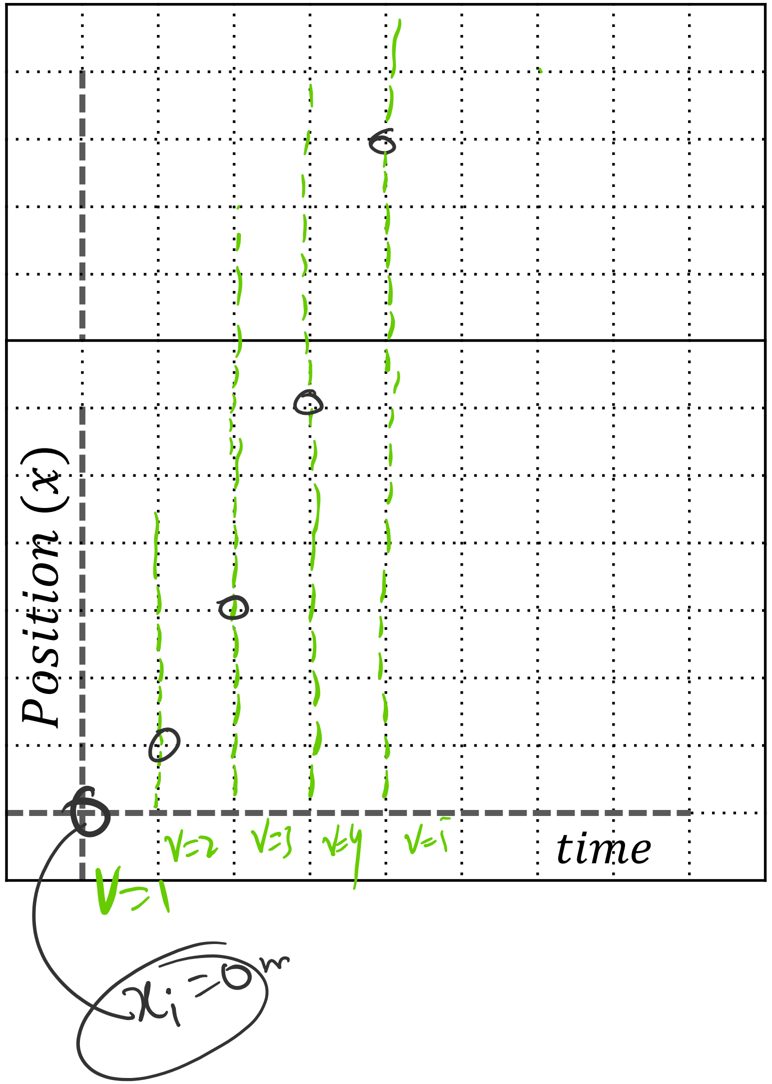


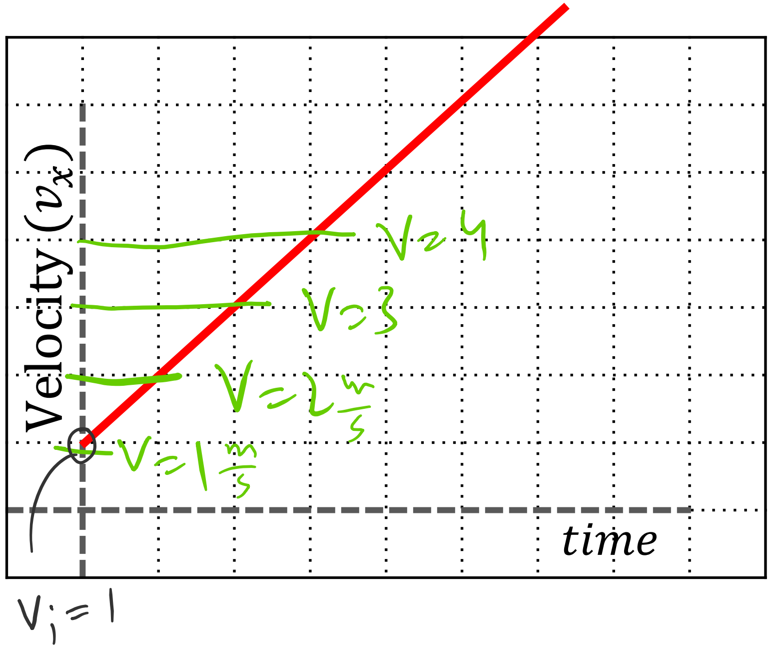
what should be the acceleration

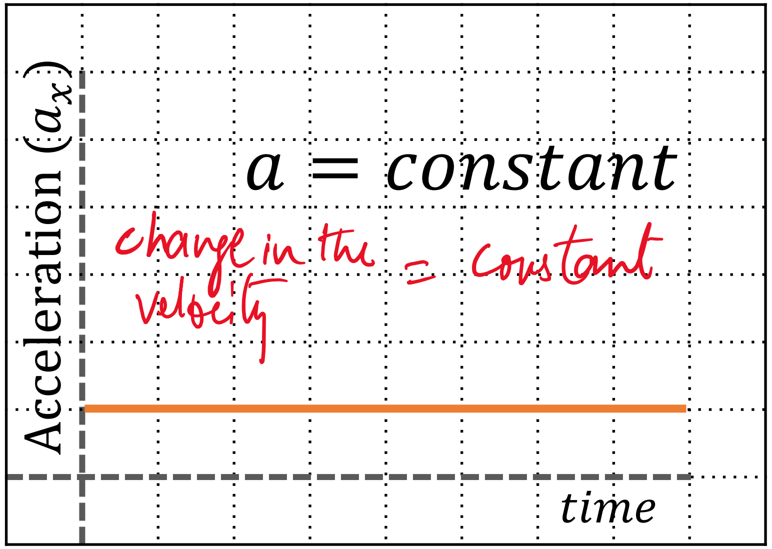


acceleration is the slope in "vt" graph

Constant Acceleration

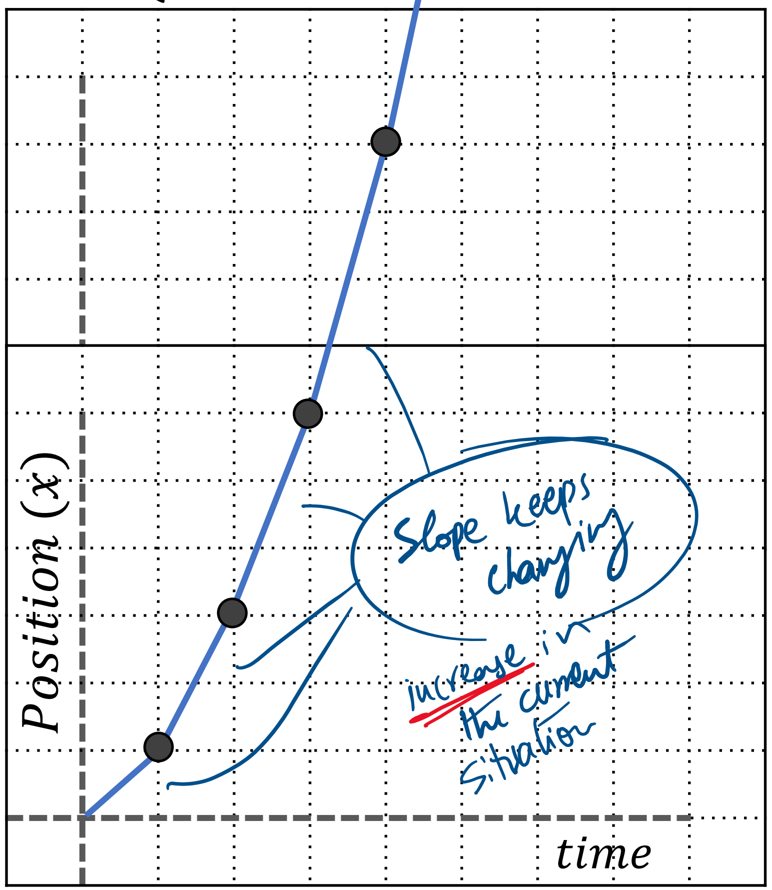




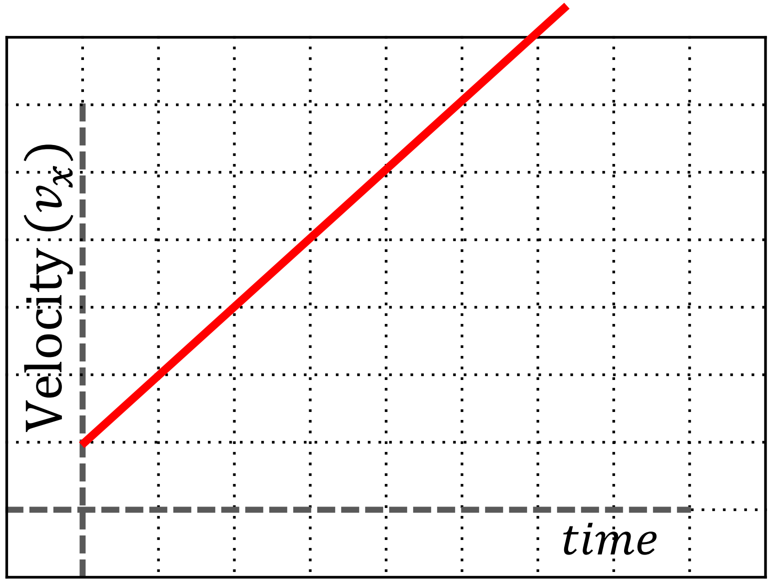


Constant Acceleration

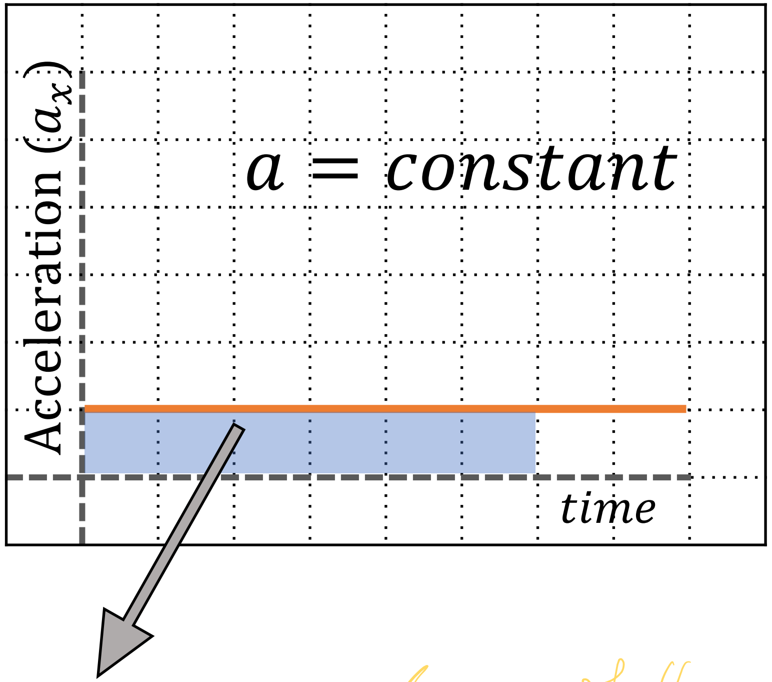
Quadratic in time



Linear in time



Constant in time

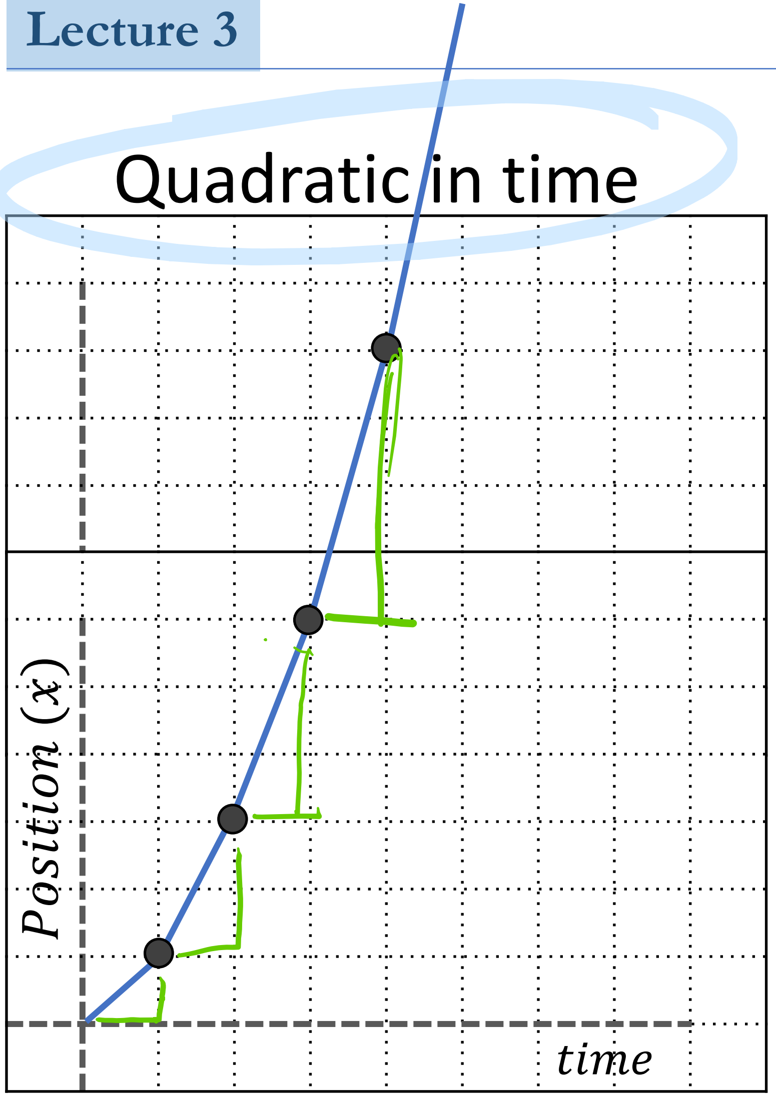


$$\Delta v = \underbrace{at}_{\text{unit} = m/s}$$

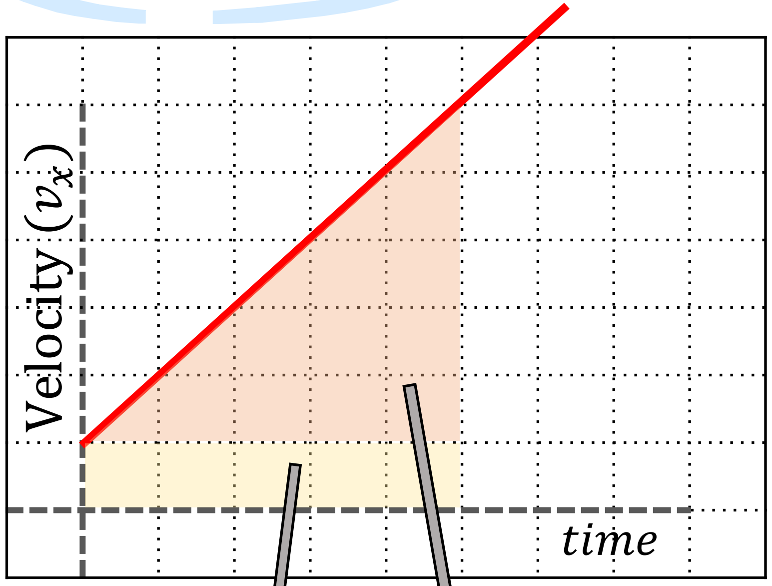
(area of the rectangle)

Constant Acceleration

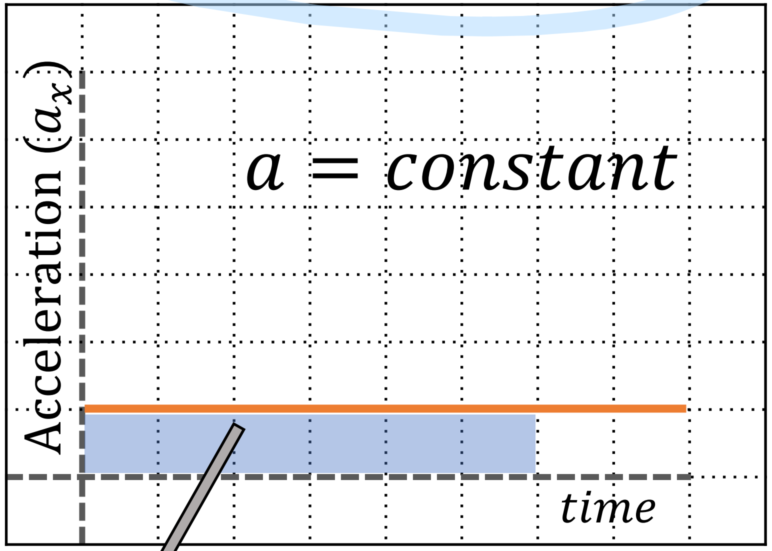
Quadratic in time



Linear in time



Constant in time



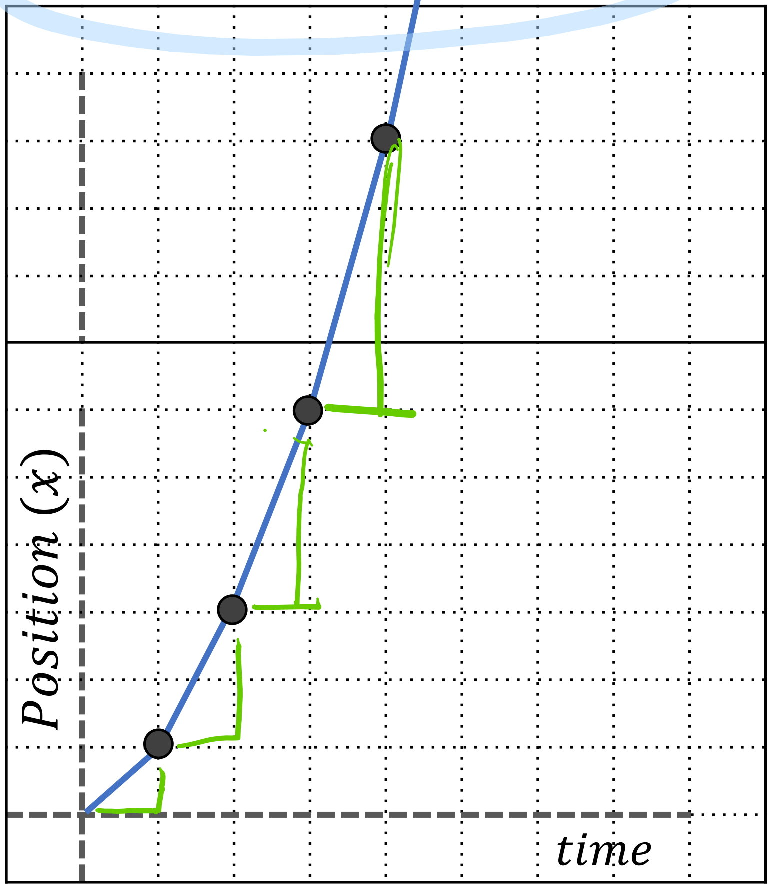
$$\Delta x = \underbrace{v_1 t}_{\text{unit} = m} + \underbrace{\frac{1}{2}\Delta v t}_{\text{unit} = m}$$

$$x_f - x_i = vt + \frac{1}{2}at^2$$

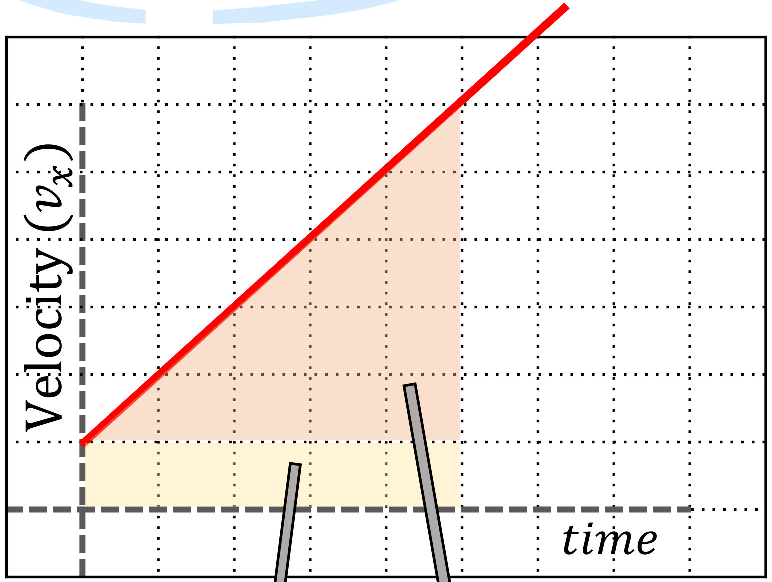
$$\Delta v = at$$

Constant Acceleration

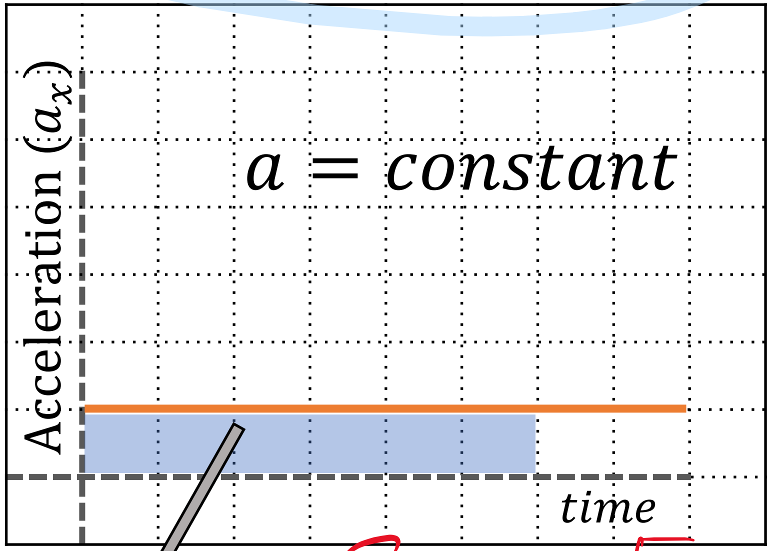
Quadratic in time



Linear in time



Constant in time



$$\Delta x = v_1 t + \frac{1}{2}\Delta v t$$

$$\Delta v = at$$

$x_f - x_i = vt + \frac{1}{2}at^2$

These equations of motion are only valid for constant accelerations.

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

Remember the rabbit!

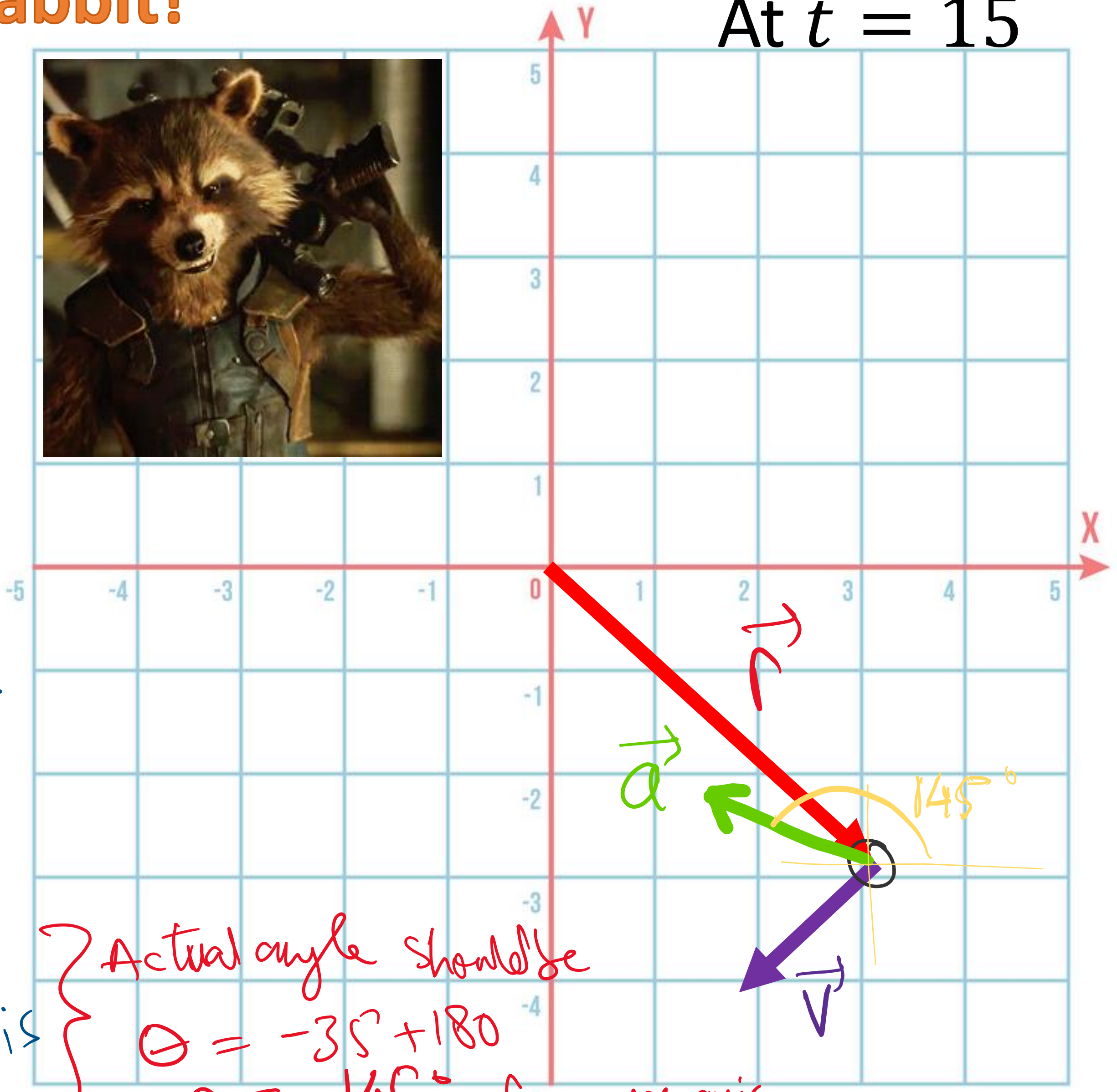
$$a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt}$$

$$a_x = -0.62 \qquad a_y = 0.44$$

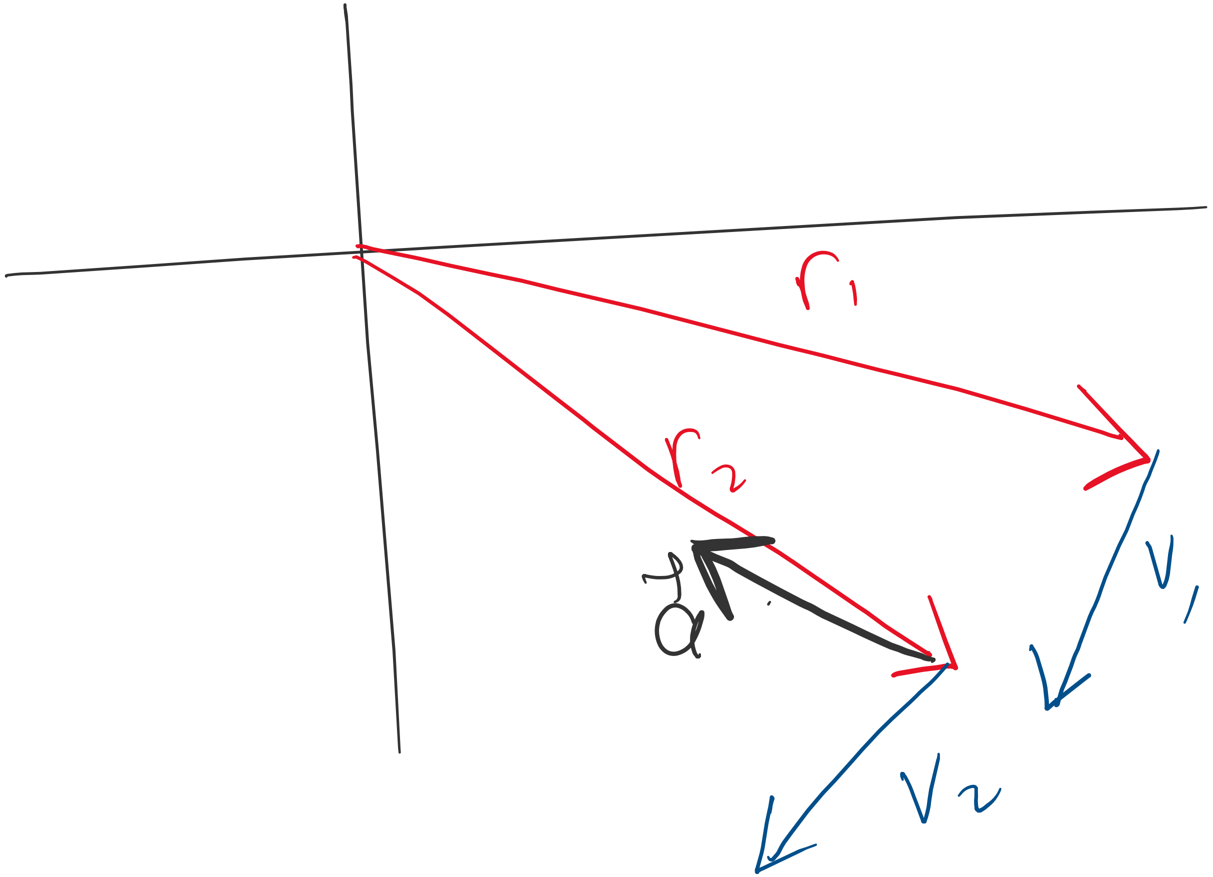
$$\boxed{\vec{a} = -0.62\,\hat{i} + 0.44\,\hat{j}}$$

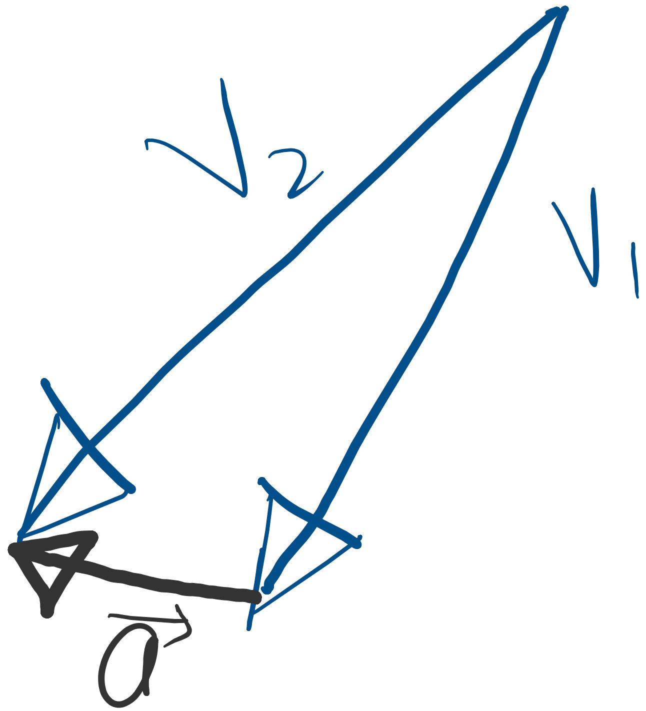
The two components indicate the acceleration vector to be in 2nd quadrant

However, $\theta = \tan^{-1}\left(\frac{0.44}{-0.62}\right) = -35°$ from $+x$-axis

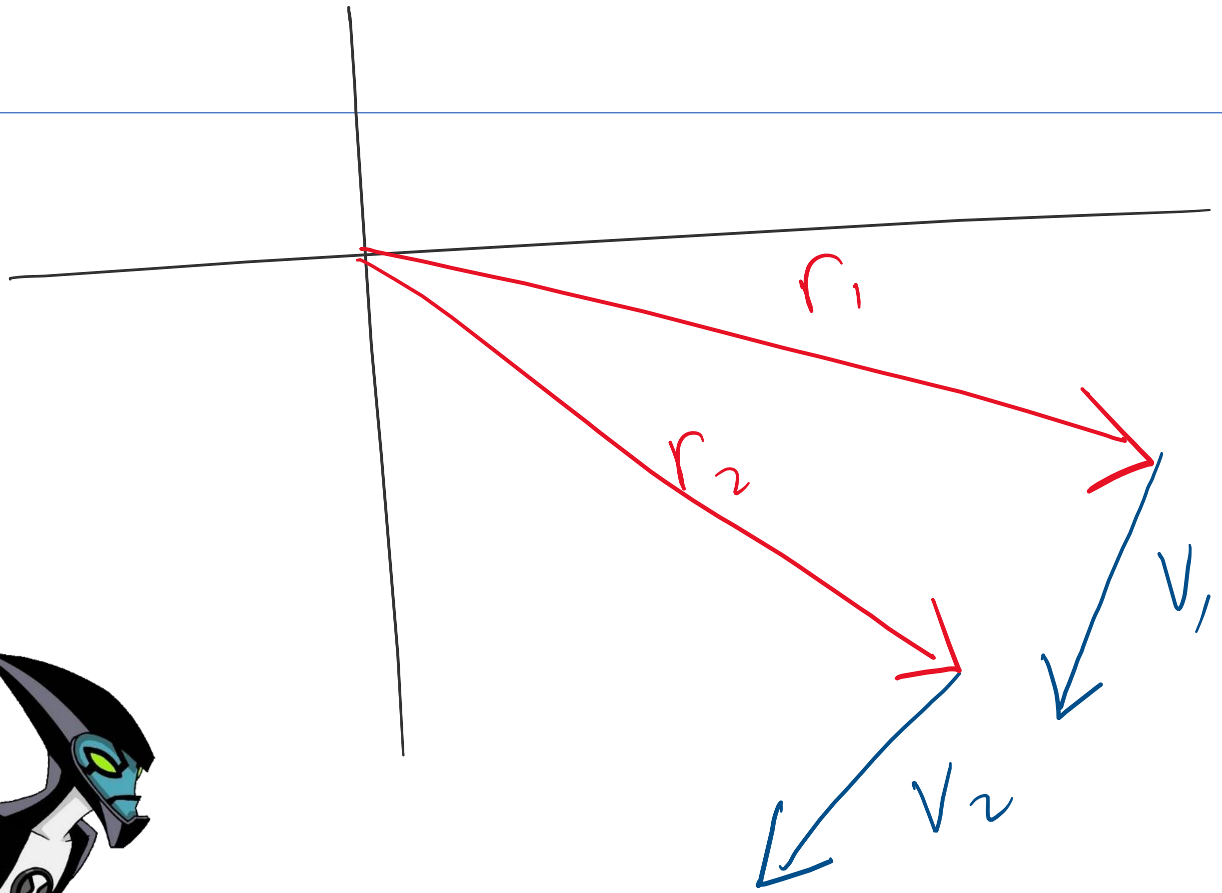


Actual angle should be
$\theta = -35 + 180$
$\theta = 145°$ from $+x$-axis

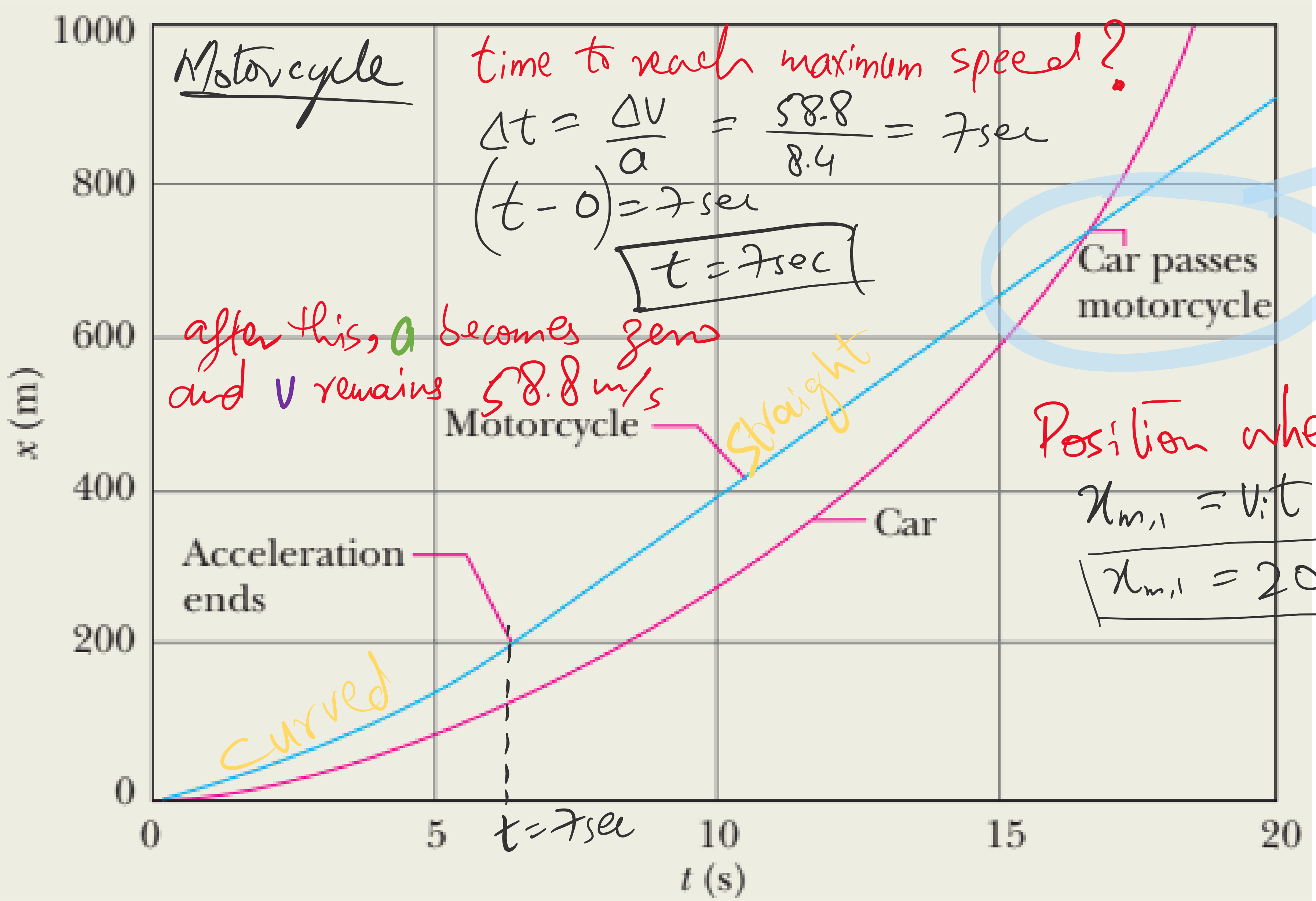




acceleration vector lives between velocity vectors.


r1
r2
V1
V2

A popular web video shows a jet airplane, a car, and a motorcycle racing from rest along a runway (Fig. 2-10). Initially the motorcycle takes the lead, but then the jet takes the lead, and finally the car blows past the motorcycle. Here let's focus on the car and motorcycle and assign some reasonable values to the motion. The motorcycle first takes the lead because its (constant) acceleration $a_m = 8.40\ \mathrm{m/s^2}$ is greater than the car's (constant) acceleration $a_c = 5.60\ \mathrm{m/s^2}$, but it soon loses to the car because it reaches its greatest speed $v_m = 58.8$ m/s before the car reaches its greatest speed $v_c = 106$ m/s. How long does the car take to reach the motorcycle?



$x_m = x_c$

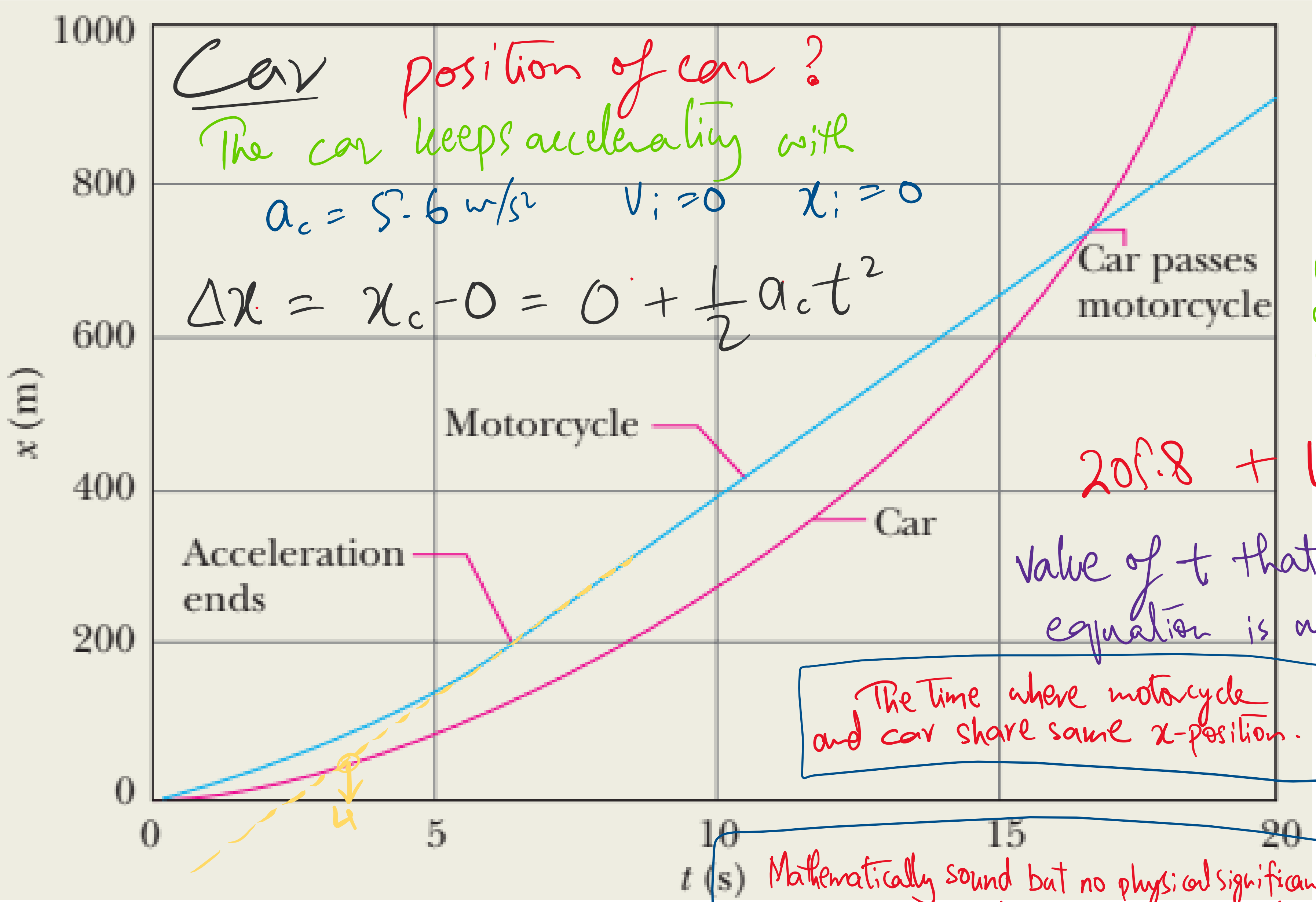
Position when acceleration end?

$x_{m,1} = v_i t + \frac{1}{2} a t^2$

$\boxed{x_{m,1} = 205.8 m}$

$v_i = 0$

$t = 7 sec$



Car

position of car ?

The car keeps accelerating with

$a_c = 5.6\ m/s^2 \qquad v_i = 0 \qquad x_i = 0$

$$\Delta x = x_c - 0 = 0 + \frac{1}{2} a_c t^2$$

According to the condition

$$x_m = x_c$$

$$\underbrace{x_{m,1}}_{\text{with acceleration}} + \underbrace{x_{m,2}}_{\text{without acceleration}} = x_c$$

$$205.8 + vt = \frac{1}{2} a_c t^2$$

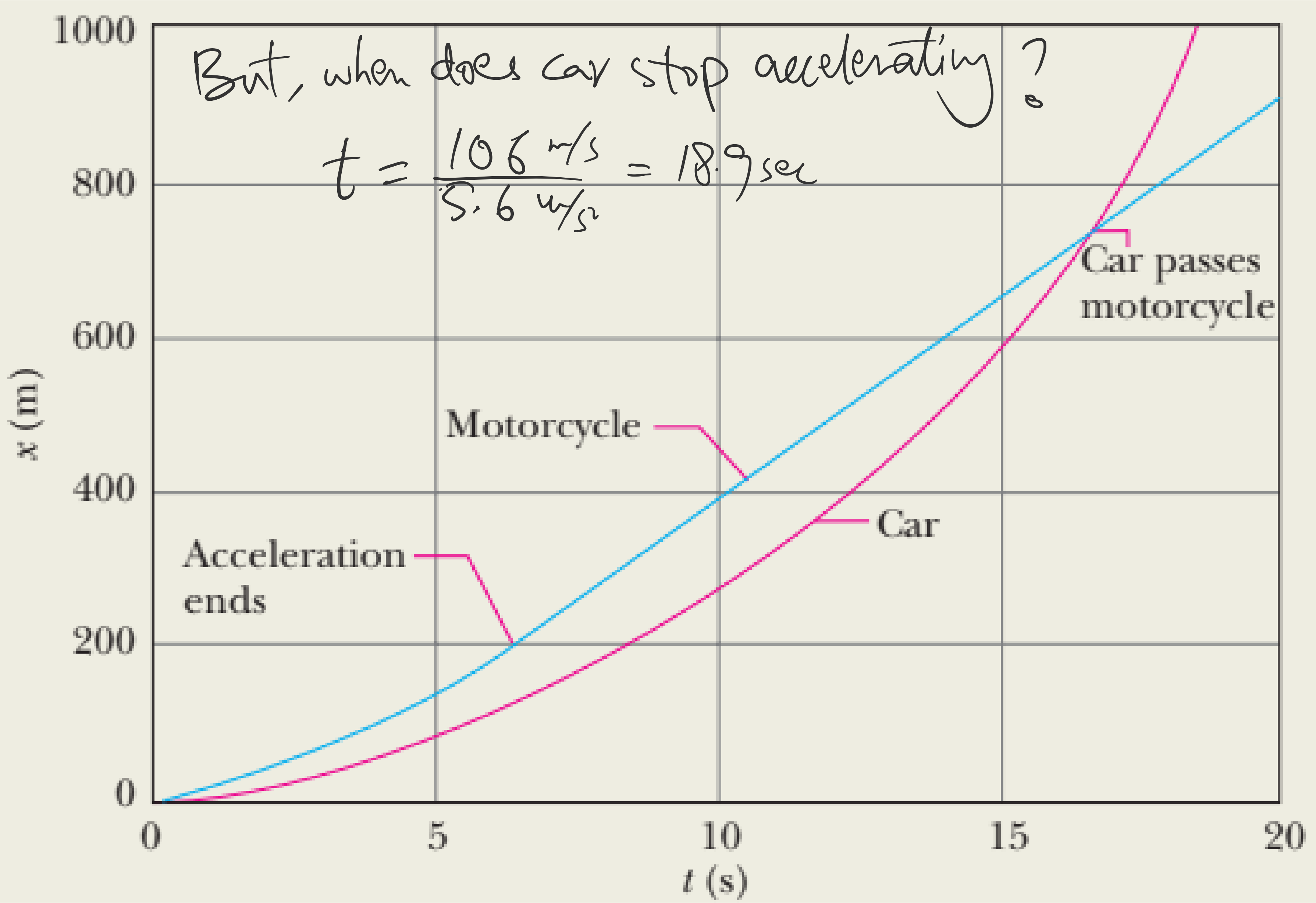
Value of t that satisfies this quadratic equation is when $x_m = x_c$

The Time where motorcycle and car share same x-position.

$\boxed{t = 16.6 \text{ sec}}$

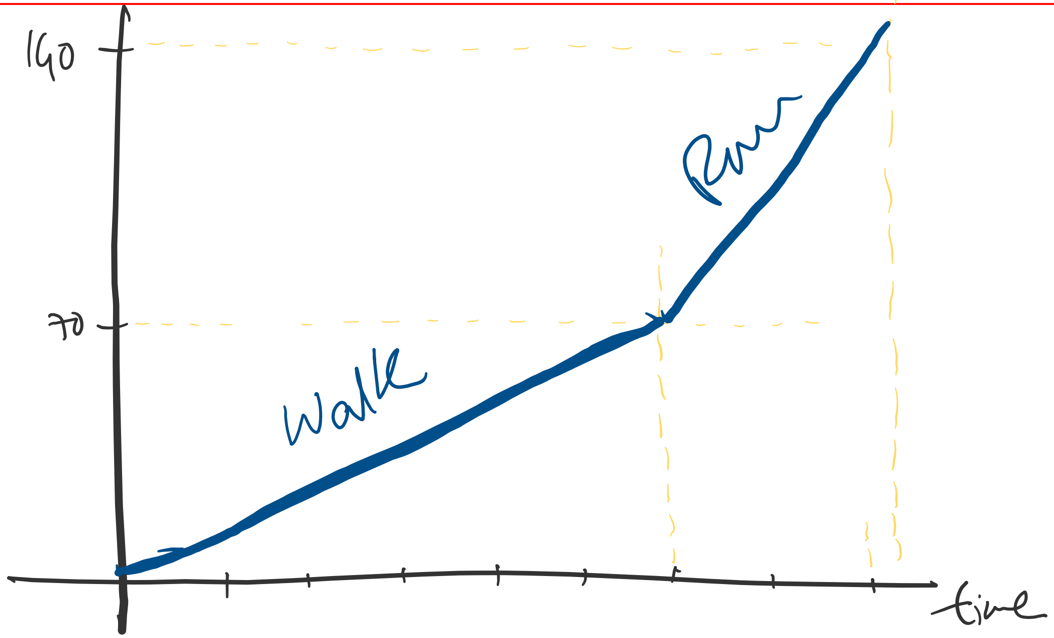
Mathematically sound but no physical significance. First point where quadratic curve cut straight line.

$\boxed{t = 4.4 \text{ sec}}$


But, when does car stop accelerating?
$t = \frac{106 \text{ m/s}}{5.6 \text{ m/s}^2} = 18.9 \text{ sec}$
1000
800
600
400
200
0
x (m)
0
5
10
15
20
t (s)
Car passes motorcycle
Motorcycle
Car
Acceleration ends

•2 Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph *x* versus *t* for both cases and indicate how the average velocity is found on the graph.

a



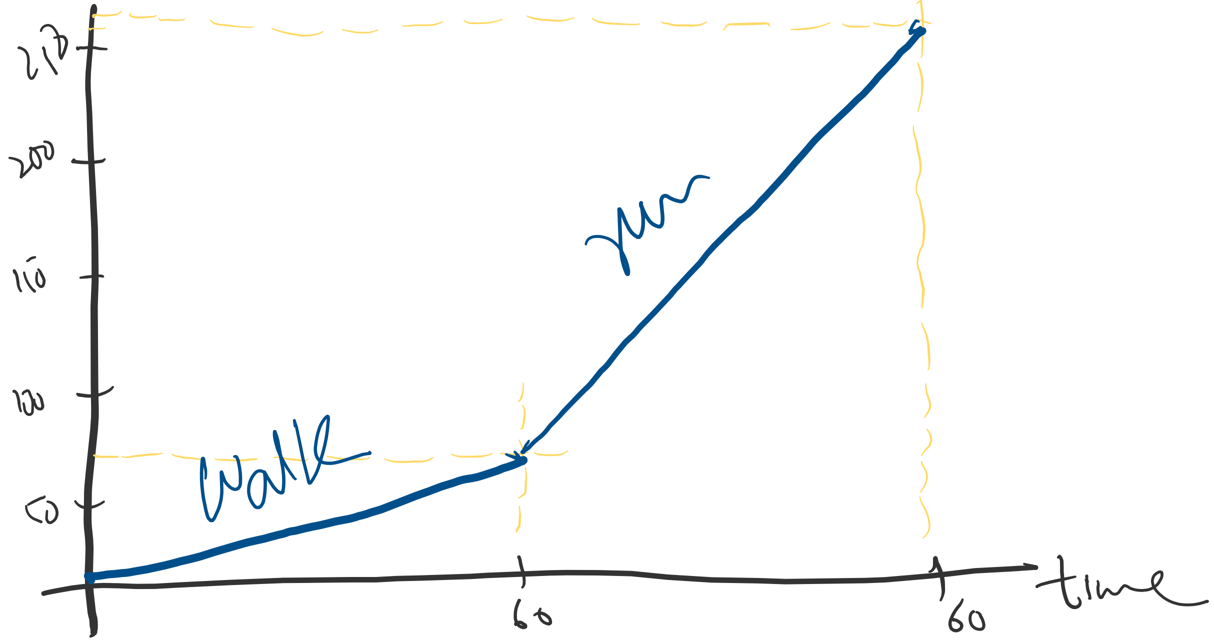
$$t_{walk} = \frac{73.2-0}{1.22\,m/s} = 60\text{ sec}$$

$$t_{run} = \frac{73.2\,m}{3.05\,m/s} = 24\text{ sec}$$

$$V_{avg} = \frac{146.4}{84} = 1.74\ m/s$$

•2 Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph x versus t for both cases and indicate how the average velocity is found on the graph.

(b)



$$x_{walk} = vt = (1.22)(60) = 73.2\,m$$

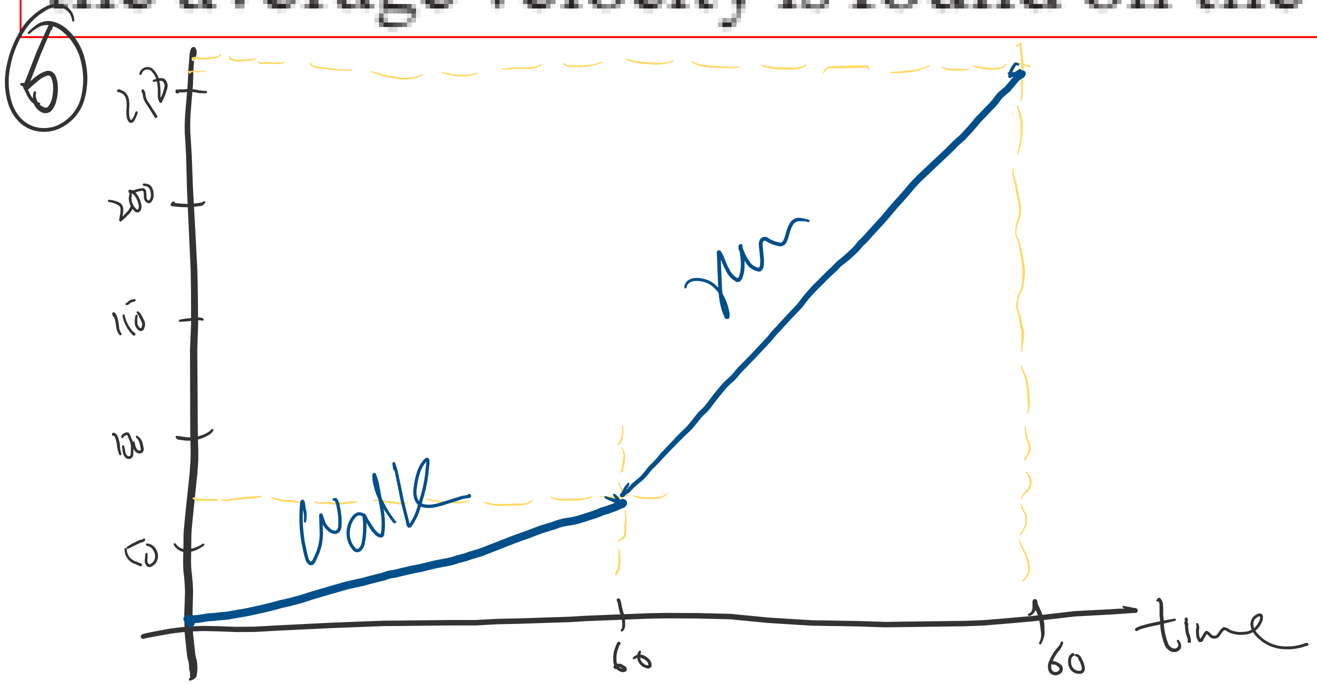
$$x_{run} = (3.05)(60) = 183\,m$$

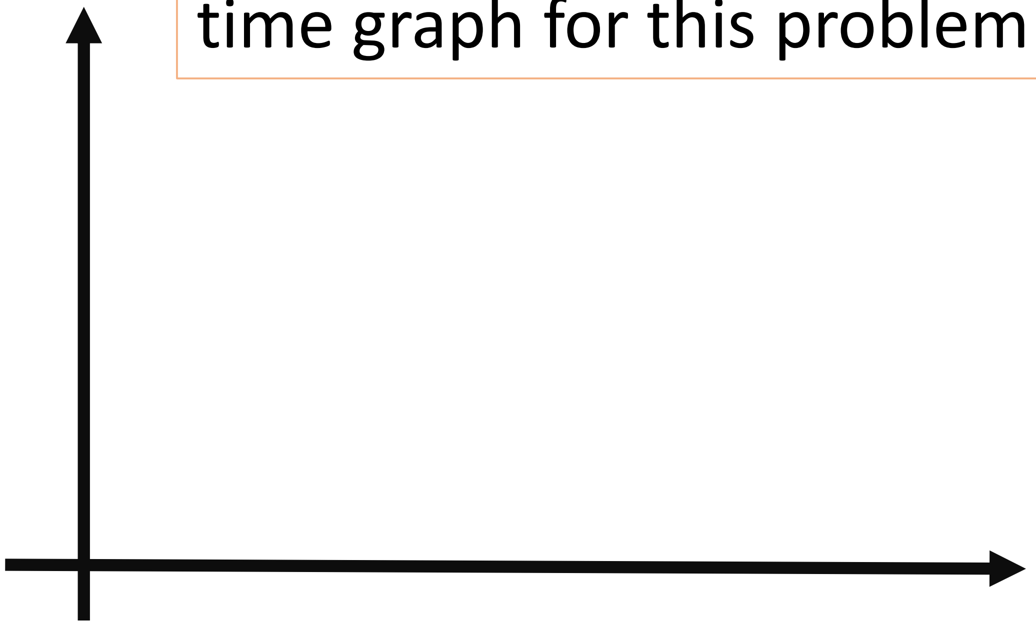
$$V_{avg} = \frac{256.2}{120} = 2.135\,m/s$$

Lecture 3

•2 Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph *x* versus *t* for both cases and indicate how the average velocity is found on the graph.

Draw the acceleration vs time graph for this problem





Acceleration - Integral form

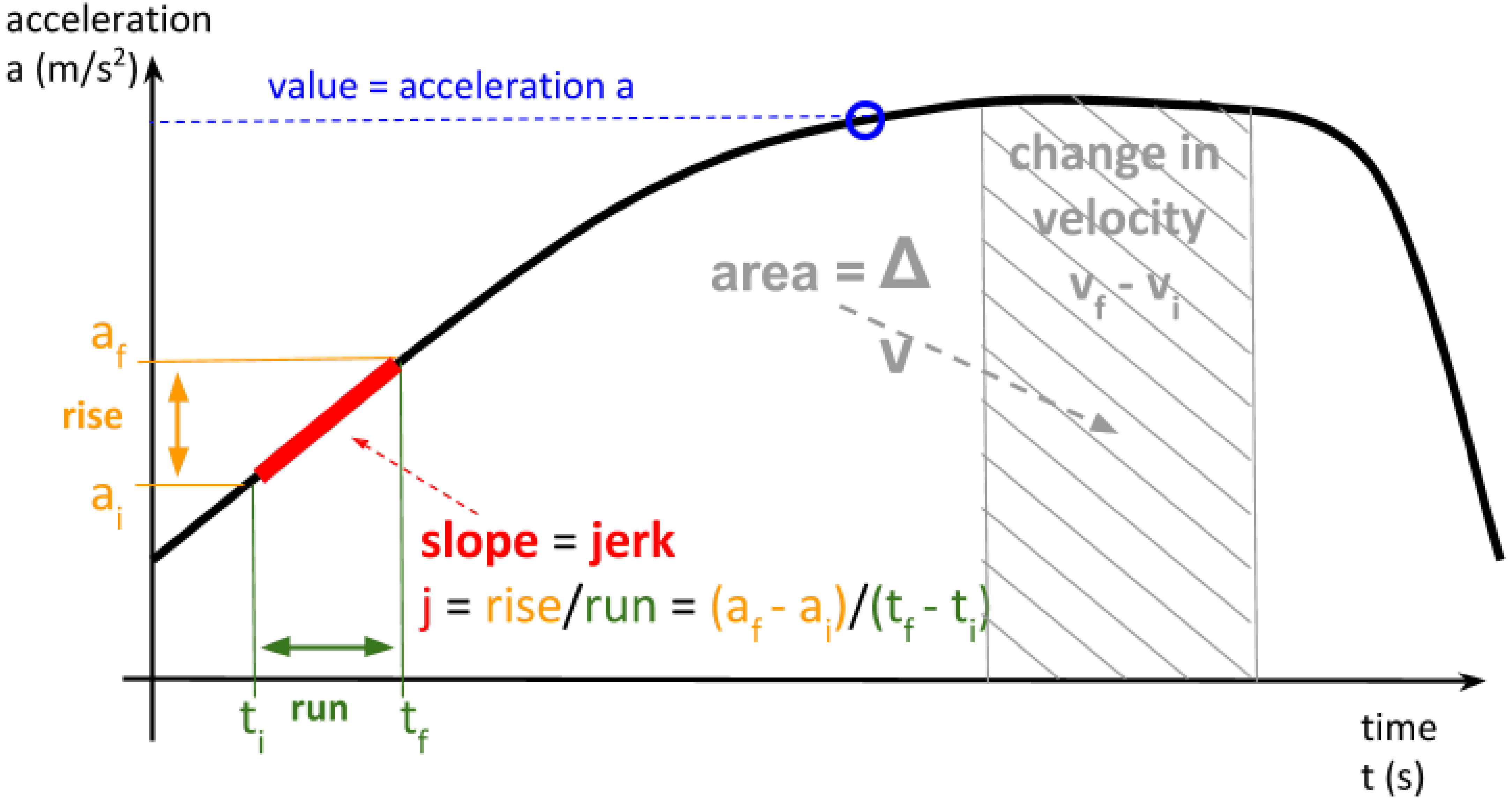
- On a graph of acceleration a versus time t, the change in the velocity is given by

$$v_1 - v_0 = \int_{t_0}^{t_1} a\,dt.$$

The integral amounts to finding an area on the graph:

$$\int_{t_0}^{t_1} a\,dt = \left(\begin{array}{c}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{array}\right).$$

Acceleration - Integral form



Practice problems:

Problems from **Fundamentals of Physics**
-Jearl Walker

Chapter 2 : Motion in One dimension

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Sample problems 2.02,03,06