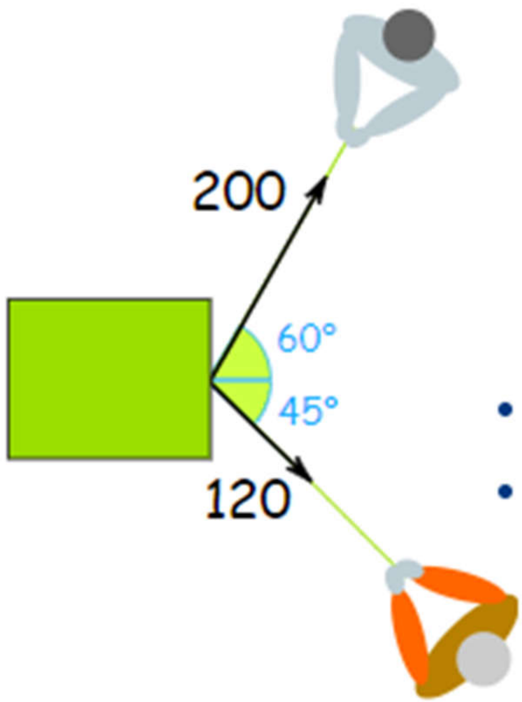
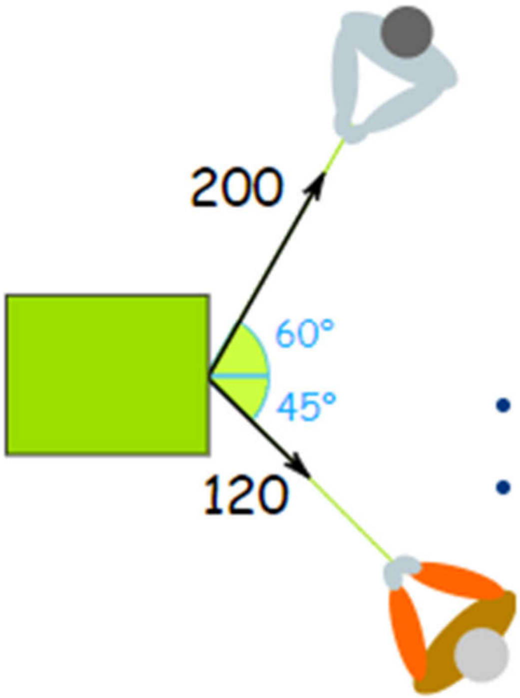


An Example

Sam and Alex are pulling a box.

- Sam pulls with 200 Newtons of force at 60°
- Alex pulls with 120 Newtons of force at 45° as shown

What is the combined force, and its direction?



An Example

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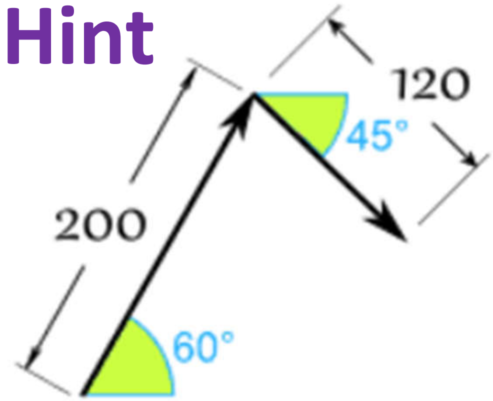
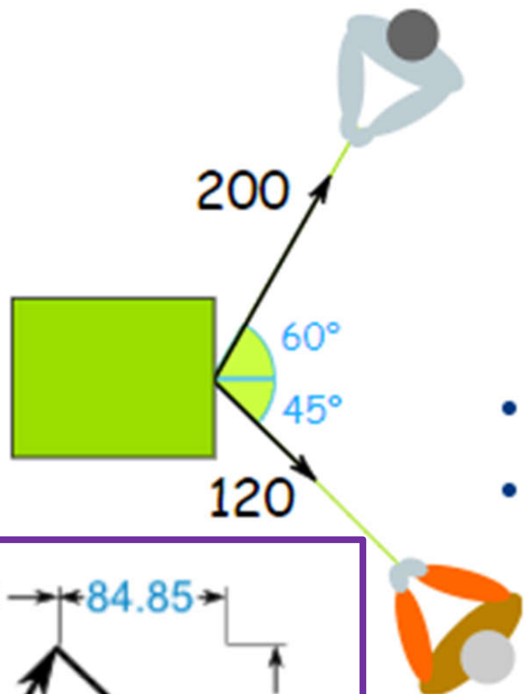
Hint





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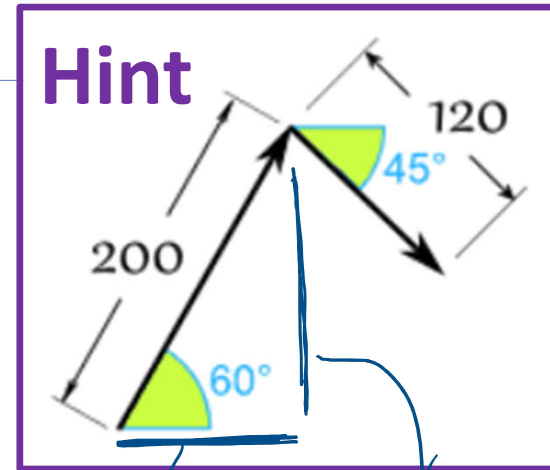
Sam and Alex are pulling a box.



- Sam pulls with 200 Newtons of force at 60°
- Alex pulls with 120 Newtons of force at 45° as shown

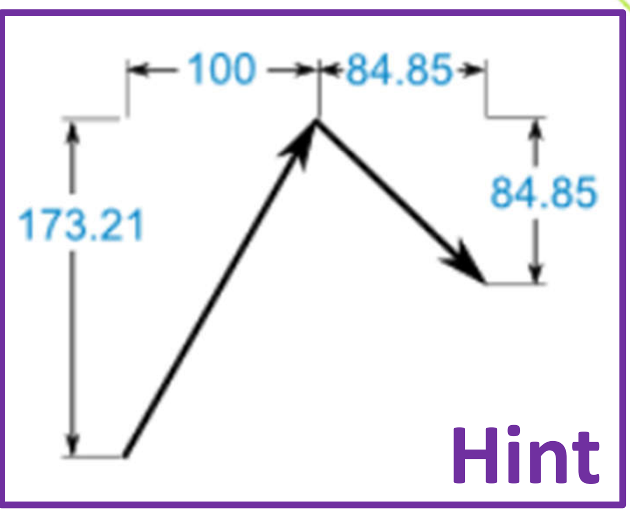
200 Sin(60°)

What is the combined force, and its direction?

Sam x = 200 Cos(60°)

120 Cos(-45) = 12 cos(**315**)

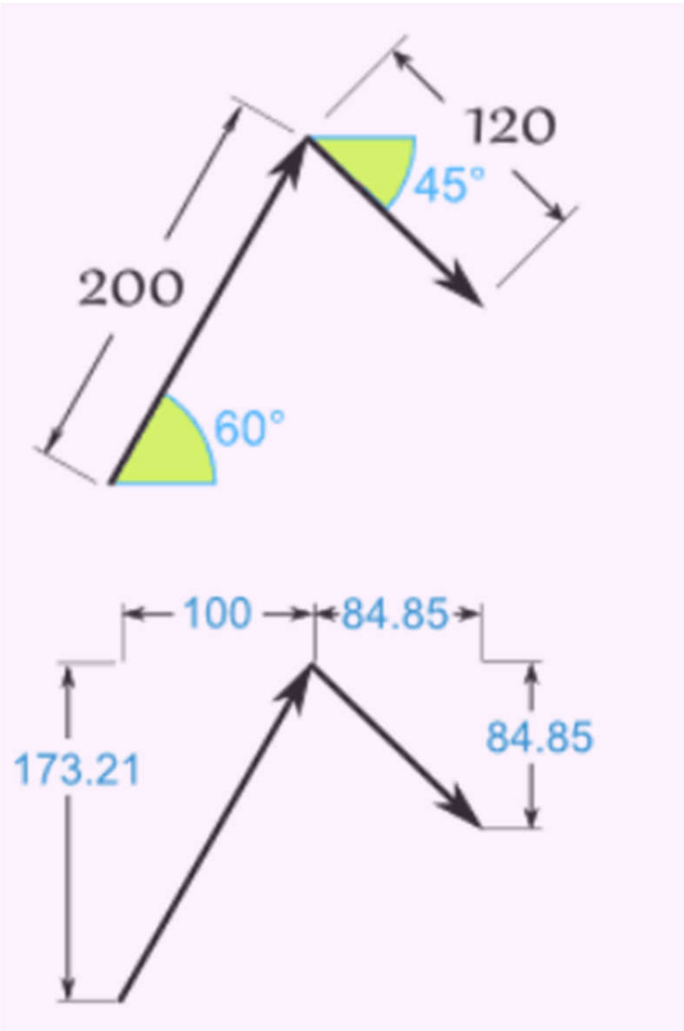
-40

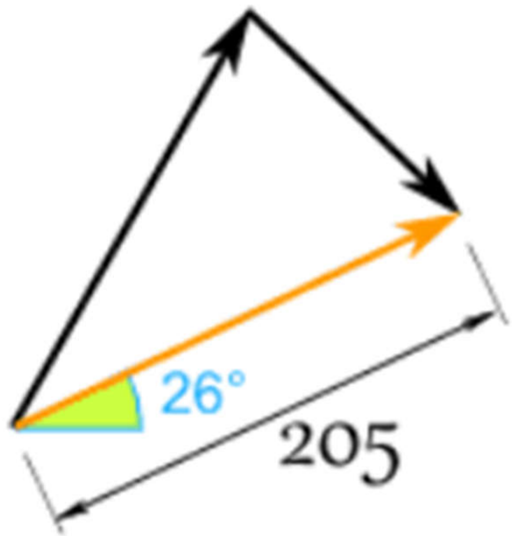


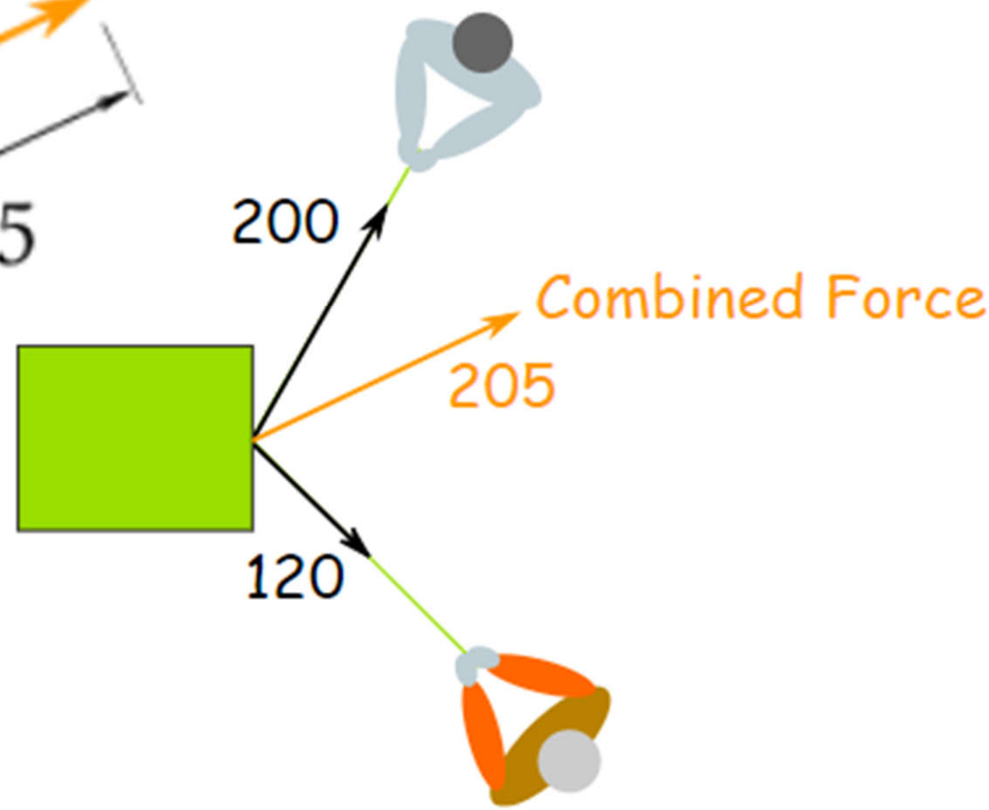
120 Sin(-40) = 120 Sin(320)

$R_x = 100 + 84.45$

$R_y = 173.21 + (-84.85)$


120
45°
200
60°
100
84.85
173.21
84.85


26°
205


200
Combined Force
205
120

Vector Multiplication

(Scalar . Vector) = Vector

Scalar Product

- The product of a scalar s and a vector $\vec{v}$ is a new vector whose magnitude is sv and whose direction is the same as that of $\vec{v}$ if s is positive, and opposite that of $\vec{v}$ if s is negative. To divide $\vec{v}$ by s, multiply $\vec{v}$ by $1/s$.

scaling up $\rightarrow$ $100\vec{V} = \vec{\omega}$, direction remains the same

$|\vec{\omega}| = 100|\vec{V}|$

scaling down $\rightarrow$ $0.5\vec{V} = \vec{u}$

inverting $\rightarrow$ $-2\vec{V} = \vec{S}$

$2|\vec{V}| = |\vec{S}|$

$\theta_S = 180^\circ + \theta_V$

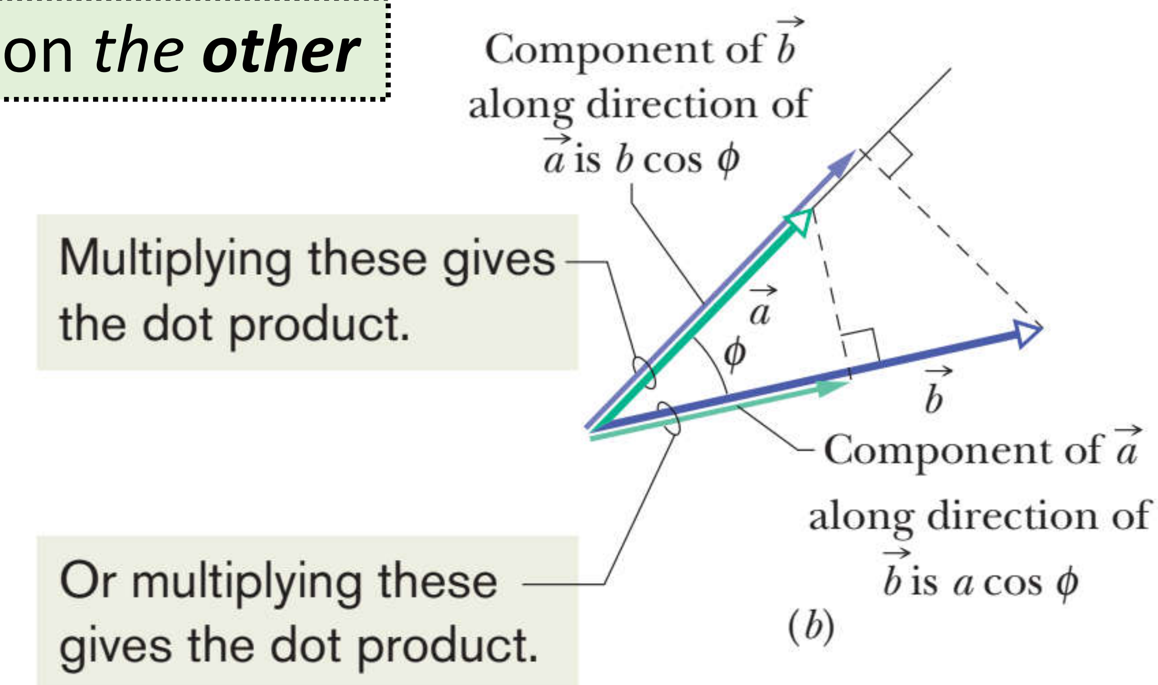
Vector Multiplication

(Vector . Vector) = Scalar

Dot Product

The Projection of one ***vector*** on *the **other***

How much does two vector point in the same direction



Vector Multiplication

(Vector . Vector) = Scalar

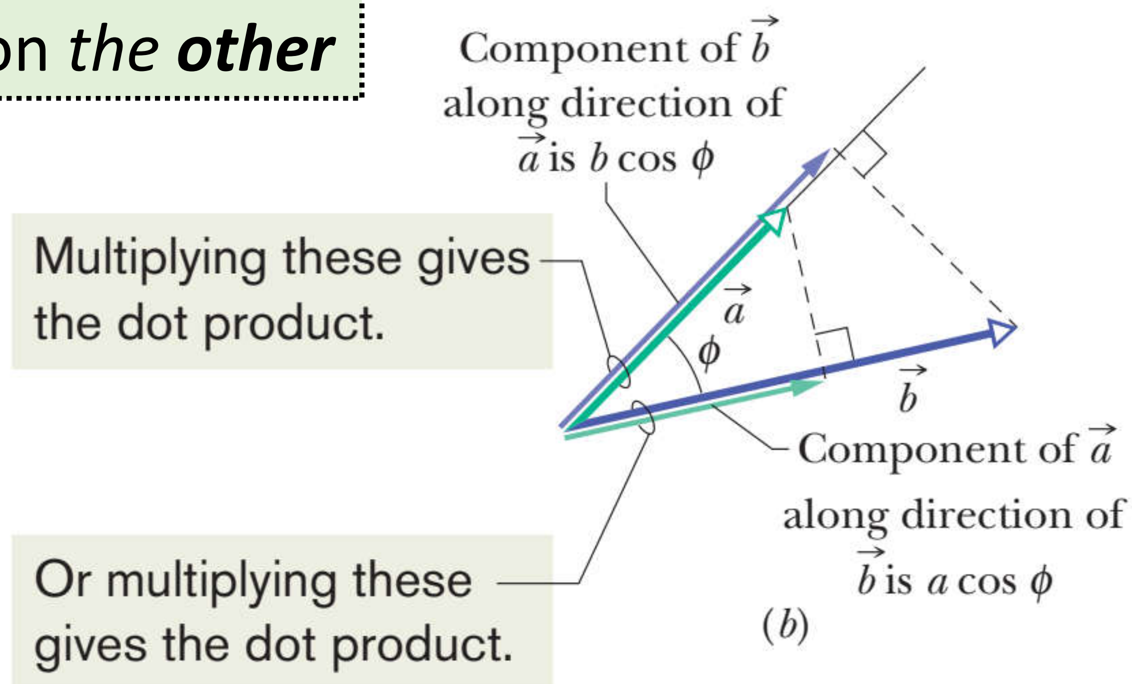
Dot Product

The Projection of one ***vector*** on *the* ***other***

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \emptyset$$

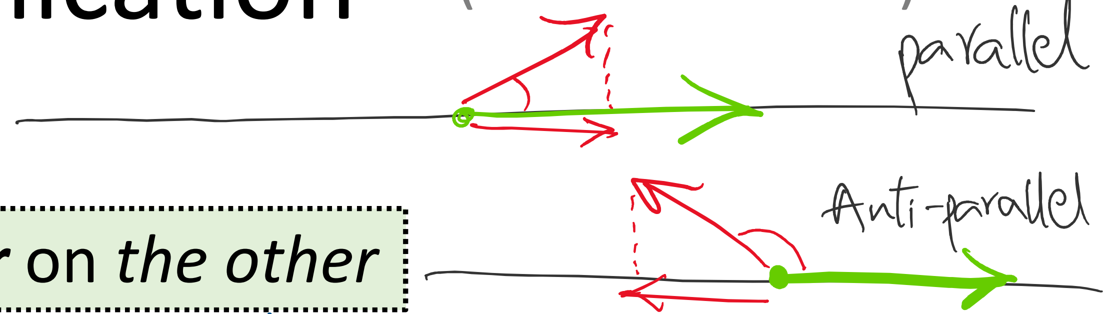
$$\vec{b} \cdot \vec{a} = |\vec{b}||\vec{a}| \cos \emptyset$$

$$\boxed{\vec{b} \cdot \vec{a} = \vec{a} \cdot \vec{b}}$$



Vector Multiplication

(Vector . Vector) = Scalar



Dot Product

The Projection of one ***vector*** on *the other*

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \emptyset$$

parallel $\longrightarrow$ Dot product is +ve

Anti parallel $\longrightarrow$ Dot product is -ve

If the angle ϕ between two vectors is 0°, the component of one vector along the other is maximum, and so also is the dot product of the vectors. If, instead, ϕ is 90°, the component of one vector along the other is zero, and so is the dot product.

$\sum_{m=0}^{4}(a_m b_m) = a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$

Vector Multiplication

(Vector . Vector) = Scalar

Dot Product

$(a_x \hat{i} + a_y \hat{j}) \cdot (b_x \hat{i} + b_y \hat{j})$

just multiply and remember

$\hat{i} \cdot \hat{i} = 1$

$\hat{j} \cdot \hat{j} = 1$

Or Sum of (**Element wise multiplication**)

summation upper limit

repeated index

$$\vec{A} \cdot \vec{B} = \sum_{u=1}^{2} (a_u b_u)$$

Summation

sum starts from

(x, y)

$(1, 2)$

$= a_1 b_1 + a_2 b_2 \Rightarrow a_x b_x + a_y b_y$

Vector Multiplication

(Vector . Vector) = Scalar

Dot Product

Or Sum of (**Element wise multiplication**)

$$\vec{A} \cdot \vec{B} = \sum_{u=1}^{2} (a_u b_u)$$

$$\vec{A} \cdot \vec{B} = a_x b_x + a_y b_y$$

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = (1)(1) \cos 0° = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\imath} \cdot \hat{k} = \hat{\jmath} \cdot \hat{k} = (1)(1) \cos 90° = 0$$

Vector Multiplication

(Vector x Vector) = Vector

Cross Product

Rotational Information

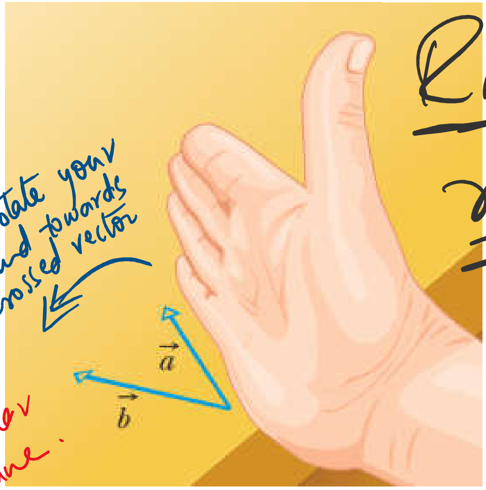
Important points

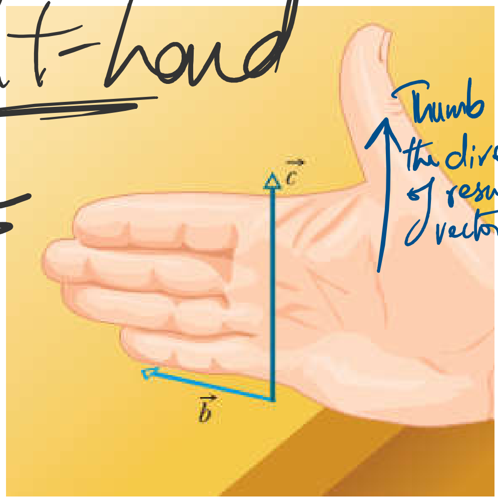
The system must be in three dimensions or more.

The resultant vector is always perpendicular to the two vectors multiplied.

Right-hand rule

rotate your hand towards crossed vector





Thumb gives the direction of resultant vector

$\vec{a} \times \vec{b}$

$\vec{a}$ and $\vec{b}$ are necessarily on a plane. and $\vec{c}$ is perpendicular to this plane.

Vector Multiplication

(Vector x Vector) = Vector

Cross Product

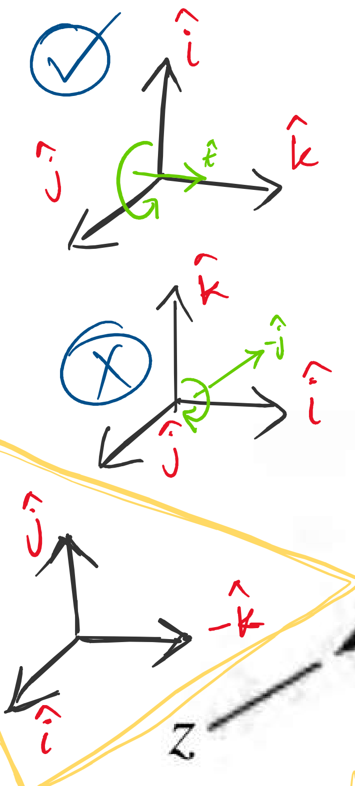
Rotational Information

using the right-hand rule

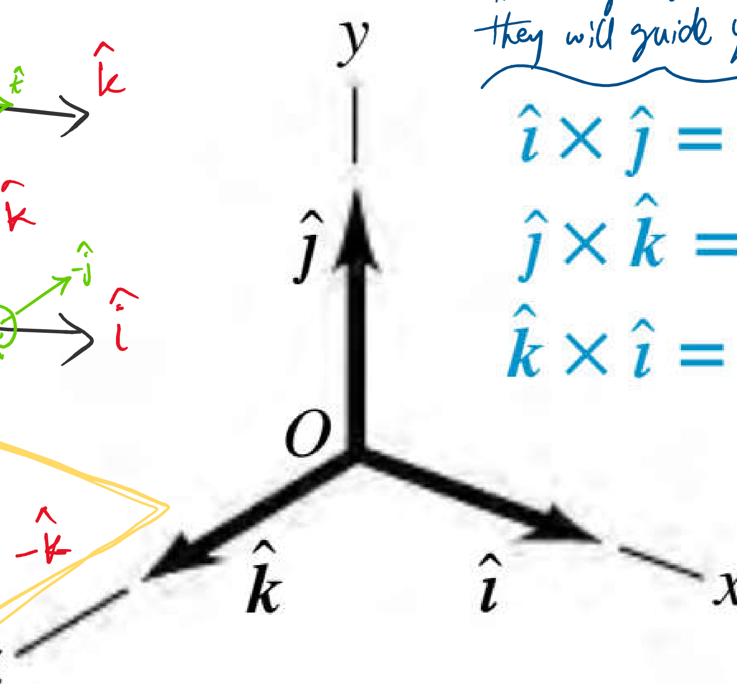
$$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$$

$$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$$



What do you think, is this correct?



Hold tight to these
they will guide you

$$\hat{\imath} \times \hat{\jmath} = \hat{k}$$

$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath}$$

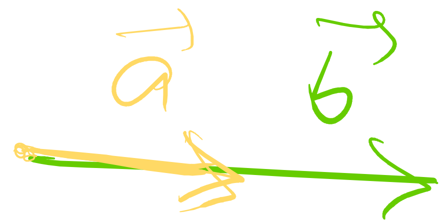
Vector Multiplication

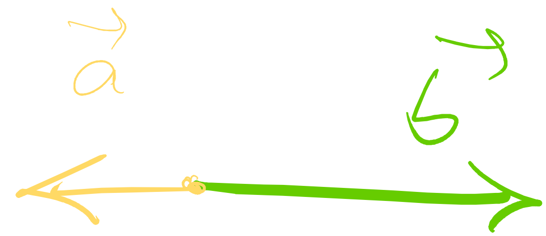
(Vector x Vector) = Vector

Cross Product

Rotational Information

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \emptyset$$

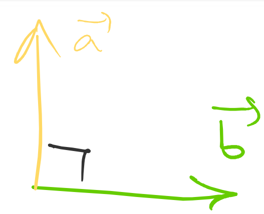




parallel
(No room for rotation)

anti-parallel
(will be understood by next explanation)

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.



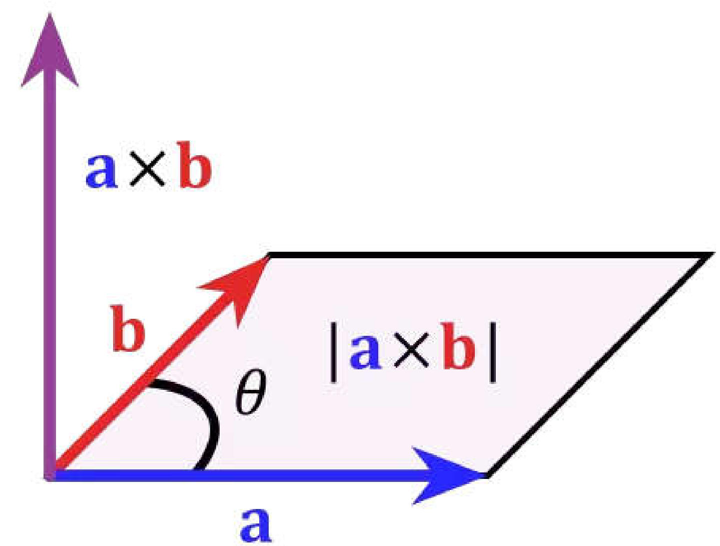
Vector Multiplication

(Vector x Vector) = Vector

Cross Product

Determinant (because determinants show how area is stretched and rotated)

$$\vec{a} \times \vec{b} = det\left(\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}\right)$$



Vector Multiplication

(Vector x Vector) = Vector

Cross Product

for parallel and antiparallel vectors, the area of parallelogram will remain zero.

Determinant (because determinants show how area is stretched and rotated)

- Length of $\vec{a} \times \vec{b}$ is the same as *area of parallelogram.*

- $\vec{a} \times \vec{b}$ is perpendicular to the $\vec{a}$ and $\vec{b}$