Number Systems

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Decimal Number System

2 3

2×10 + 3×1

20 + 3

23

Decimal Number System

EXAMPLE 2–2

Express the decimal number 568.23 as a sum of the values of each digit.

Solution

The whole number digit 5 has a weight of 100, which is 10^2, the digit 6 has a weight of 10, which is 10^1, the digit 8 has a weight of 1, which is 10^0, the fractional digit 2 has a weight of 0.1, which is 10^{-1}, and the fractional digit 3 has a weight of 0.01, which is 10^{-2}.

$$568.23 = (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2})$$
$$= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01)$$
$$= 500 + 60 + 8 + 0.2 + 0.03$$

Related Problem

Determine the value of each digit in 67.924.
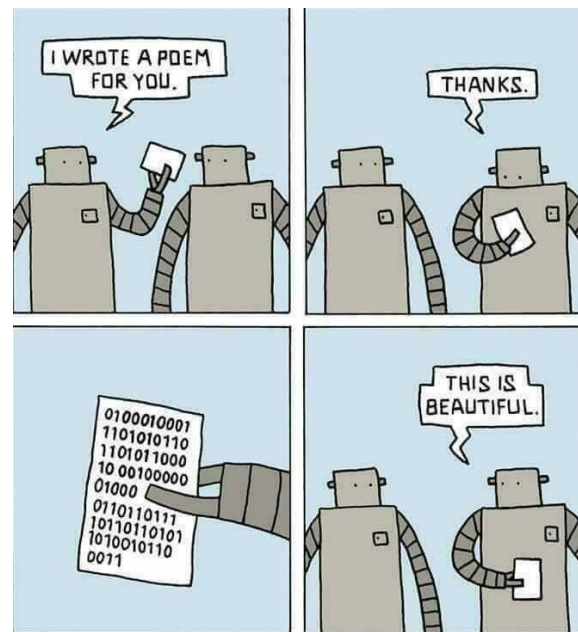
Binary Number System

There are two digits in the binary number system, 1 and 0.

Each digit is called a bit.

Binary + Digit = Bit

There are only 10
types of people
in the world:
Those who understand binary
and those who don't.

Binary Counting

With n bits you can count, 0 to 2n – 1.

Binary Counting

TABLE 2–1

Decimal Number	Binary Number			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

TABLE 2–2

Binary weights.

Positive Powers of Two (Whole Numbers)									Negative Powers of Two (Fractional Number)					
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
256	128	64	32	16	8	4	2	1	1/2 0.5	1/4 0.25	1/8 0.125	1/16 0.625	1/32 0.03125	1/64 0.015625

InfoNote

Processors use binary numbers to select memory locations. Each location is assigned a unique number called an *address*. Some microprocessors, for example, have 32 address lines which can select 2^{32} (4,294,967,296) unique locations.

Binary To Decimal Conversion

Convert the binary whole number 1101101 to decimal.

Binary To Decimal Conversion

Convert the binary whole number 1101101 to decimal.

EXAMPLE 2–3

Convert the binary whole number 1101101 to decimal.

Solution

Determine the weight of each bit that is a 1, and then find the sum of the weights to get the decimal number.

$$\text{Weight:} \quad 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$
$$\text{Binary number:} \quad 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1$$
$$1101101 = 2^6 + 2^5 + 2^3 + 2^2 + 2^0$$
$$= 64 + 32 + 8 + 4 + 1 = \mathbf{109}$$

Related Problem

Convert the binary number 10010001 to decimal.

Binary To Decimal Conversion

Convert the fractional binary number 0.1011 to decimal.

Binary To Decimal Conversion

Convert the fractional binary number 0.1011 to decimal.

EXAMPLE 2–4

Convert the fractional binary number 0.1011 to decimal.

Solution

Determine the weight of each bit that is a 1, and then sum the weights to get the decimal fraction.

$$
\begin{array}{ccccc}
\text{Weight:} & 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} \\
\text{Binary number:} \quad 0\,.\,1 & 0 & 1 & 1
\end{array}
$$

$$0.1011 = 2^{-1} + 2^{-3} + 2^{-4}$$
$$= 0.5 + 0.125 + 0.0625 = \mathbf{0.6875}$$

Related Problem

Convert the binary number 10.111 to decimal.

Practice Questions

1. What is the largest decimal number that can be represented in binary with eight bits?
2. Determine the weight of the 1 in the binary number 10000.
3. Convert the binary number 10111101.011 to decimal.

Decimal To Binary Conversion

Sum-of-Weights Method:

2^3 2^2 2^1 2^0

1   0   0   1   Binary number for decimal 9

Repeated Division Method:

Decimal to Binary

2 | 47

   2 | 23 ———————— 1

     2 | 11 ———————— 1

       2 | 5 ———————— 1

         2 | 2 ———————— 1

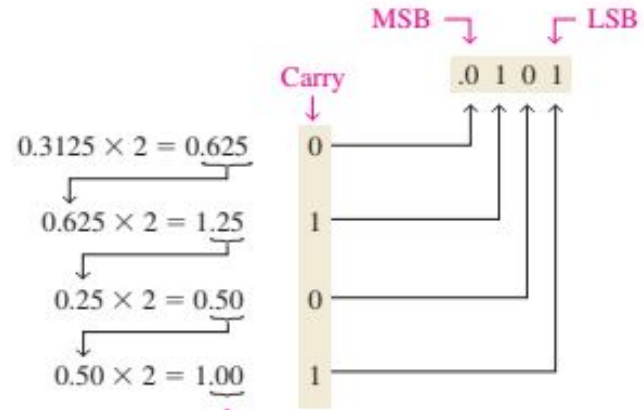   2 | 1 ——— 0

             0 ——— 1

Remainder

$(47)_{10} = (101111)_2$

Decimal Fractions To Binary Conversion

Sum-of-Weights Method:

$$0.625 = 0.5 + 0.125 = 2^{-1} + 2^{-3} = 0.101$$

Repeated Multiplication Method:

Practice Questions

1. Convert each decimal number to binary by using the sum-of-weights method:

   (a) 23 (b) 57 (c) 45.5

2. Convert each decimal number to binary by using the repeated division-by-2 method (repeated multiplication-by-2 for fractions):
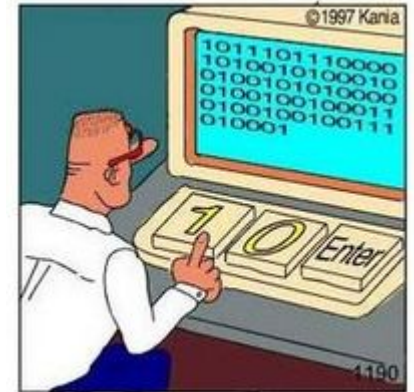
   (a) 14 (b) 21 (c) 0.375

Binary Arithmetic

Binary Addition

$$0 + 0 = 0 \quad \text{Sum of 0 with a carry of 0}$$
$$0 + 1 = 1 \quad \text{Sum of 1 with a carry of 0}$$
$$1 + 0 = 1 \quad \text{Sum of 1 with a carry of 0}$$
$$1 + 1 = 10 \quad \text{Sum of 0 with a carry of 1}$$

Add the following binary numbers:

(a) 11 + 11

(b) 100 + 10

(c) 111 + 11

(d) 110 + 100

Real programmers code in binary.

Binary Arithmetic

Add the following binary numbers:

(a) 11 + 11 (b) 100 + 10

(c) 111 + 11 (d) 110 + 100

Solution

The equivalent decimal addition is also shown for reference.

(a)
$$\begin{array}{r} 11 \\ + 11 \\ \hline \mathbf{110} \end{array} \qquad \begin{array}{r} 3 \\ + 3 \\ \hline 6 \end{array}$$

(b)
$$\begin{array}{r} 100 \\ + 10 \\ \hline \mathbf{110} \end{array} \qquad \begin{array}{r} 4 \\ + 2 \\ \hline 6 \end{array}$$

(c)
$$\begin{array}{r} 111 \\ + 11 \\ \hline \mathbf{1010} \end{array} \qquad \begin{array}{r} 7 \\ + 3 \\ \hline 10 \end{array}$$

(d)
$$\begin{array}{r} 110 \\ + 100 \\ \hline \mathbf{1010} \end{array} \qquad \begin{array}{r} 6 \\ + 4 \\ \hline 10 \end{array}$$

Related Problem

Add 1111 and 1100.
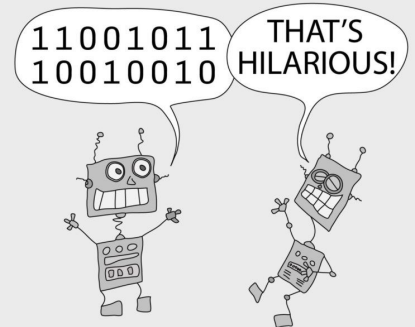
Binary Arithmetic

Binary Subtraction

$0 - 0 = 0$

$1 - 1 = 0$

$1 - 0 = 1$

$10 - 1 = 1 \qquad 0 - 1$ with a borrow of 1

Subtract 011 from 101

11001011
10010010

THAT'S HILARIOUS!

Binary Arithmetic
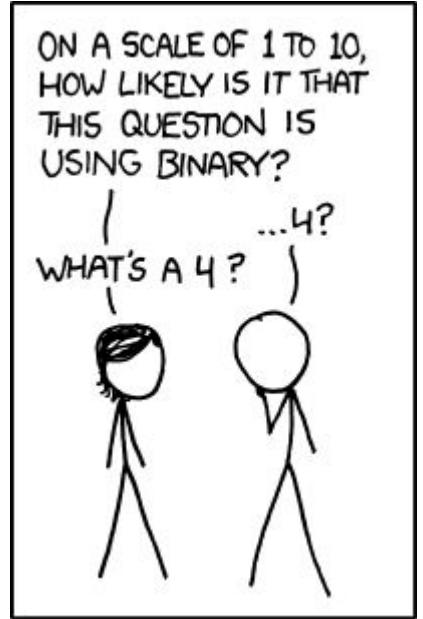
Binary Multiplication

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

Multiply 101 with 111

Binary Arithmetic

Binary Multiplication

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

Multiply 101 with 111

Binary Arithmetic

Binary Division

Divide 110 by 10

Practice Questions

1. Perform the following binary additions:

   (a) 1101 + 1010 (b) 10111 + 01101

2. Perform the following binary subtractions:
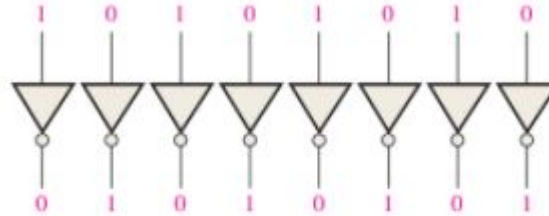
   (a) 1101 2 0100 (b) 1001 2 0111

3. Perform the indicated binary operations:

   (a) 110 * 111 (b) 1100 , 011

Complement Of Binary Number

1's Complement

2's complement = (1's complement) + 1

10110010	Binary number
01001101	1's complement
+ 1	Add 1
01001110	2's complement

Signed Number

Sign Magnitude Form

The left-most bit in a signed binary number is the sign bit, which tells you whether the number is positive or negative.

0 -> positive number

1 -> negative number

e.g:

+25=00011001

-25=10011001

EXAMPLE 2-15

Determine the decimal value of this signed binary number expressed in sign-magnitude: 10010101.

Solution

The seven magnitude bits and their powers-of-two weights are as follows:

$$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$$
$$0 \quad\; 0 \quad\; 1 \quad\; 0 \quad\; 1 \quad\; 0 \quad\; 1$$

Summing the weights where there are 1s,

$$16 + 4 + 1 = 21$$

The sign bit is 1; therefore, the decimal number is **−21**.

Related Problem

Determine the decimal value of the sign-magnitude number 01110111.

Signed Number

1's Complement Form

+ve number is same as sign magnitude form

-ve number is the 1's complement of the corresponding positive number

e.g:

+25=00011001

-25=11100110

EXAMPLE 2–16

Determine the decimal values of the signed binary numbers expressed in 1's complement:

(a) 00010111   (b) 11101000

Solution

(a) The bits and their powers-of-two weights for the positive number are as follows:

$$-2^7 \quad 2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$$
$$0 \quad \ \ 0 \quad \ \ 0 \quad \ \ 1 \quad \ \ 0 \quad \ \ 1 \quad \ \ 1 \quad \ \ 1$$

Summing the weights where there are 1s,

$$16 + 4 + 2 + 1 = +23$$

Signed Number

2's Complement Form

+ve number is same as sign magnitude form

-ve number is the 2's complement of the corresponding positive number

e.g:

+25=00011001

-25=11100111

EXAMPLE 2–17

Determine the decimal values of the signed binary numbers expressed in 2's complement:

(a) 01010110   (b) 10101010

Solution

(a) The bits and their powers-of-two weights for the positive number are as follows:

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	0	1	0	1	1	0

Summing the weights where there are 1s,

$$64 + 16 + 4 + 2 = +86$$

(b) The bits and their powers-of-two weights for the negative number are as follows. Notice that the negative sign bit has a weight of $-2^7 = -128$.

-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	1	0	1	0

Summing the weights where there are 1s,

$$-128 + 32 + 8 + 2 = -86$$

Related Problem

Determine the decimal value of the 2's complement number 11010111.
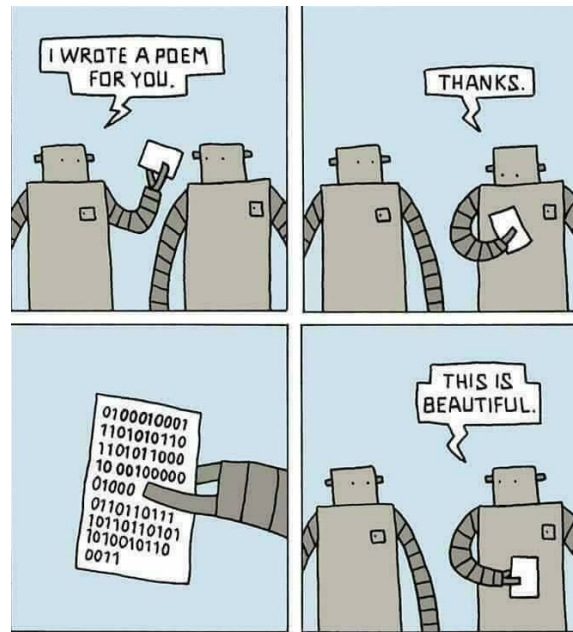
Practice Questions

1. Express the decimal number +9 as an 8-bit binary number in the sign-magnitude system.
2. Express the decimal number 233 as an 8-bit binary number in the 1's complement system.
3. Express the decimal number 246 as an 8-bit binary number in the 2's complement system.

Binary Counting For Signed Numbers

With n bits you can count,

-2^(n-1) to 2^(n-1) - 1

Addition With Signed Numbers

1. Both numbers positive
2. Positive number with magnitude larger than negative number
3. Negative number with magnitude larger than positive number
4. Both numbers negative

Addition With Signed Numbers

1. **Both numbers positive:**

$$
\begin{array}{r}
00000111 \\
+\ 00000100 \\
\hline
00001011
\end{array}
\qquad
\begin{array}{r}
7 \\
+\ 4 \\
\hline
11
\end{array}
$$

The sum is positive and is therefore in true (uncomplemented) binary.

2. **Positive number with magnitude larger than negative number:**

$$
\begin{array}{r}
00001111 \\
+\ 11111010 \\
\hline
\text{Discard carry} \longrightarrow 1 \quad 00001001
\end{array}
\qquad
\begin{array}{r}
15 \\
+\ -6 \\
\hline
9
\end{array}
$$

The final carry bit is discarded. The sum is positive and therefore in true (uncomplemented) binary.

Addition With Signed Numbers

3. **Negative number with magnitude larger than positive number:**

$$
\begin{array}{rr}
00010000 & 16 \\
+\ 11101000 & +\ -24 \\
\hline
11111000 & -8
\end{array}
$$

The sum is negative and therefore in 2's complement form.

4. **Both numbers negative:**

$$
\begin{array}{rr}
11111011 & -5 \\
+\ 11110111 & +\ -9 \\
\hline
\text{Discard carry} \longrightarrow 1\quad 11110010 & -14
\end{array}
$$

The final carry bit is discarded. The sum is negative and therefore in 2's complement form.

Overflow Condition

$$
\begin{array}{rr}
01111101 & 125 \\
+\ 00111010 & +\ 58 \\
\hline
10110111 & 183 \\
\end{array}
$$

Sign incorrect
Magnitude incorrect

SUbtraction With Signed Numbers

To subtract two signed numbers, take the 2's complement of the subtrahend and add. Discard any final carry bit.

(a) 00001000 – 00000011 (b) 00001100 – 11110111

SUbtraction With Signed Numbers

EXAMPLE 2–20

Perform each of the following subtractions of the signed numbers:

(a) $00001000 - 00000011$ (b) $00001100 - 11110111$

(c) $11100111 - 00010011$ (d) $10001000 - 11100010$

Solution

Like in other examples, the equivalent decimal subtractions are given for reference.

(a) In this case, $8 - 3 = 8 + (-3) = 5$.

$$
\begin{array}{ll}
00001000 & \text{Minuend } (+8) \\
+\ 11111101 & \text{2's complement of subtrahend } (-3) \\
\hline
\text{Discard carry} \longrightarrow \mathbf{1\ 00000101} & \text{Difference } (+5)
\end{array}
$$

(b) In this case, $12 - (-9) = 12 + 9 = 21$.

$$
\begin{array}{ll}
00001100 & \text{Minuend } (+12) \\
+\ 00001001 & \text{2's complement of subtrahend } (+9) \\
\hline
00010101 & \text{Difference } (+21)
\end{array}
$$

Multiplication With Signed Numbers

- **If the signs are the same, the product is positive.**

- **If the signs are different, the product is negative.**

1. Determine if the signs of the multiplicand and multiplier are the same or different. This determines what the sign of the product will be.
2. Change any negative number to true (uncomplemented) form.
3. Multiply both numbers.
4. If the sign bit that was determined in step 1 is negative, take the 2's complement of the product. If positive, leave the product in true form. Attach the sign bit to the product.

Multiplication With Signed Numbers

Multiply the signed binary numbers: 01010011 (multiplicand) and 11000101 (multiplier).

Multiplication With Signed Numbers

Multiply the signed binary numbers: 01010011 (multiplicand) and 11000101 (multiplier).

Attach the sign bit ⟶

1 0110011011111

Division With Signed Numbers

- **If the signs are the same, the result is positive.**

- **If the signs are different, the result is negative.**

1. Determine if the signs of the divisor and dividend are the same or different. This determines what the sign of the result will be.
2. Change any negative number to true (uncomplemented) form.
3. Divide both numbers.
4. If the sign bit that was determined in step 1 is negative, take the 2's complement of the product. If positive, leave the product in true form. Attach the sign bit to the product.

Division With Signed Numbers

Divide the signed binary numbers:

00001001 / 11111101

Practice Questions

1. Add the signed numbers 00100001 and 10111100.

2. Subtract the signed numbers 00110010 from 01110111.

3. Multiply 01111111 by 00000101.

4. Divide 00110000 by 00001100.

Hexadecimal Number System

Counting in Hexadecimal

TABLE 2–3

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Binary to HexaDecimal

Convert the following binary numbers to hexadecimal:

(a) 1100101001010111 (b) 111111000101101001

Binary to HexaDecimal

EXAMPLE 2–24

Convert the following binary numbers to hexadecimal:

(a) 1100101001010111   (b) 111111000101101001

Solution

(a) $\underbrace{1100}\underbrace{1010}\underbrace{0101}\underbrace{0111}$

      C   A   5   7 $= CA57_{16}$

(b) $\underbrace{0011}\underbrace{1111}\underbrace{0001}\underbrace{0110}\underbrace{1001}$

      3   F   1   6   9 $= 3F169_{16}$

Two zeros have been added in part (b) to complete a 4-bit group at the left.

Related Problem

Convert the binary number 1001111011110011100 to hexadecimal.

HexaDecimal To Binary

Determine the binary numbers for the following hexadecimal numbers:

(a) 10A416 (b) CF8E16 (c) 974216

HexaDecimal To Binary

EXAMPLE 2–25

Determine the binary numbers for the following hexadecimal numbers:

(a) $10A4_{16}$ (b) $CF8E_{16}$ (c) 9742_{16}

Solution

(a) 1   0   A   4   (b) C   F   8   E   (c) 9   7   4   2

1000010100100   1100111110001110   1001011101000010

In part (a), the MSB is understood to have three zeros preceding it, thus forming a 4-bit group.

Related Problem

Convert the hexadecimal number 6BD3 to binary.

HexaDecimal To Decimal

Convert the following hexadecimal numbers to decimal:

(a) 1C16 (b) A8516

HexaDecimal To Decimal

EXAMPLE 2–26

Convert the following hexadecimal numbers to decimal:

(a) $1C_{16}$ (b) $A85_{16}$

Solution

Remember, convert the hexadecimal number to binary first, then to decimal.

(a)   1   C
         ↓   ↓
    $\overbrace{0001}\overbrace{1100} = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = \mathbf{28}_{10}$

(b)   A   8   5
         ↓   ↓   ↓
    $\overbrace{1010}\overbrace{1000}\overbrace{0101} = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = \mathbf{2693}_{10}$

Related Problem

Convert the hexadecimal number 6BD to decimal.

HexaDecimal To Decimal

EXAMPLE 2–27

Convert the following hexadecimal numbers to decimal:

(a) $E5_{16}$ (b) $B2F8_{16}$

Solution

Recall from Table 2–3 that letters A through F represent decimal numbers 10 through 15, respectively.

(a) $E5_{16} = (E \times 16) + (5 \times 1) = (14 \times 16) + (5 \times 1) = 224 + 5 = \mathbf{229}_{10}$

(b) $B2F8_{16} = (B \times 4096) + (2 \times 256) + (F \times 16) + (8 \times 1)$

$\qquad = (11 \times 4096) + (2 \times 256) + (15 \times 16) + (8 \times 1)$

$\qquad = \quad 45{,}056 \quad + \quad 512 \quad + \quad 240 \quad + \quad 8 \quad = \mathbf{45{,}816}_{10}$

Related Problem

Convert $60A_{16}$ to decimal.

Decimal To HexaDecimal

Convert the decimal number 650 to hexadecimal by repeated division by 16.

HexaDecimal Addition

Add the following hexadecimal numbers:

(a) $23_{16} + 16_{16}$ **(b)** $58_{16} + 22_{16}$

HexaDecimal Subtraction

Subtract the following hexadecimal numbers:

(a) $84_{16} - 2A_{16}$ **(b)** $C3_{16} - 0B_{16}$

Practice Questions

1. Convert the following binary numbers to hexadecimal:(a) 10110011 (b) 11001110100

2. Convert the following hexadecimal numbers to binary:(a) 57 (b) 3A5 (c) F80B

3. Convert 9B3016 to decimal.

4. Convert the decimal number 573 to hexadecimal.

5. Add the following hexadecimal numbers directly: (a) 18 + 34 (b) 3F + 2A

6.

7. Subtract the following hexadecimal numbers: (a) 75 – 21 (b) 94 – 5C

Octal Number System

Counting in Octal

TABLE 2–4								
Octal/binary conversion.								
Octal Digit	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

Decimal to Octal and Octal To Decimal

Convert the following decimal numbers to octal:

(a) 359

Convert the following octal numbers to decimal:

(a) 2374

Decimal to Octal and Octal To Decimal

Convert the following decimal numbers to octal:

(a) 359 ------> 547 in octal

Convert the following octal numbers to decimal:

(a) 2374 -----> 1276 in decimal

Binary to Octal and Octal To Binary

Convert the following binary numbers to octal:

(a) 110101 (b) 101111001

Convert the following octal numbers to binary:

(a) 140   (b) 7526

Binary to Octal and Octal To Binary

Convert the following binary numbers to octal:

(a) 110101 (b) 101111001

(a) $\underbrace{110}\underbrace{101}$
$\downarrow \quad \downarrow$
$6 \quad 5 = 65_8$

(b) $\underbrace{101}\underbrace{111}\underbrace{001}$
$\downarrow \quad \downarrow \quad \downarrow$
$5 \quad 7 \quad 1 = 571_8$

Convert the following octal numbers to binary:

(c) 140   (d) 7526

(c) $\begin{array}{ccc} 1 & 4 & 0 \\ \downarrow & \downarrow & \downarrow \\ \underbrace{001} & \underbrace{100} & \underbrace{000} \end{array}$

(d) $\begin{array}{cccc} 7 & 5 & 2 & 6 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \underbrace{111} & \underbrace{101} & \underbrace{010} & \underbrace{110} \end{array}$

Practice Questions

1. Convert the following octal numbers to decimal:
    (a) 73 (b) 125

2. Convert the following decimal numbers to octal:
    (a) 9810 (b) 16310

3. Convert the following octal numbers to binary:
    (a) 468 (b) 7238 (c) 56248

4. Convert the following binary numbers to octal:
    (a) 110101111 (b) 1001100010 (c) 10111111001

Binary Coded Decimal

- 8421 BCD Code
- Invalid Codes

- Digital clocks
- Digital thermometers
- Digital meters

What is Binary Coded Decimal?
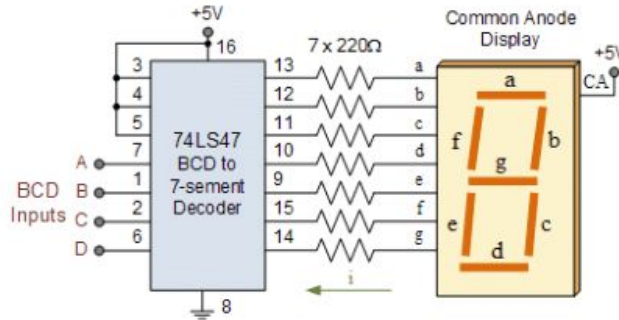
Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

E4U Electrical 4 U

Decimal to BCD and BCD to Decimal

Convert each of the following decimal numbers to BCD:

(a) 35 (b) 98 (c) 170 (d) 2469

Convert each of the following BCD codes to decimal:

(a) 10000110 (b) 001101010001 (c) 1001010001110000

Decimal to BCD and BCD to Decimal

EXAMPLE 2–33

Convert each of the following decimal numbers to BCD:

(a) 35   (b) 98   (c) 170   (d) 2469

Solution

(a) 3 5
00110101

(b) 9 8
10011000

(c) 1 7 0
000101110000

(d) 2 4 6 9
0010010001101001

Related Problem

Convert the decimal number 9673 to BCD.

Decimal to BCD and BCD to Decimal

EXAMPLE 2–34

Convert each of the following BCD codes to decimal:

(a) 10000110   (b) 001101010001   (c) 1001010001110000

Solution

(a) 10000110   (b) 001101010001   (c) 1001010001110000

     8   6   3   5   1   9   4   7   0

Related Problem

Convert the BCD code 1000001000100111 0110 to decimal.

BCD Addition

Add the following BCD numbers:

(a) 0011 + 0100 (b) 00100011 + 00010101

(c) 10000110 + 00010011 (d) 010001010000 + 010000010111

BCD Addition

EXAMPLE 2–35

Add the following BCD numbers:

(a) 0011 + 0100

(b) 00100011 + 00010101

(c) 10000110 + 00010011

(d) 010001010000 + 010000010111

Solution

The decimal number additions are shown for comparison.

(a)
```
  0011        3
+ 0100      + 4
  ────       ──
  0111        7
```

(b)
```
  0010  0011      23
+ 0001  0101    + 15
  ────  ────      ──
  0011  1000      38
```

(c)
```
  1000  0110      86
+ 0001  0011    + 13
  ────  ────      ──
  1001  1001      99
```

(d)
```
  0100  0101  0000      450
+ 0100  0001  0111    + 417
  ────  ────  ────      ───
  1000  0110  0111      867
```

Note that in each case the sum in any 4-bit column does not exceed 9, and the results are valid BCD numbers.

BCD Addition

Add the following BCD numbers:

(a) 1001 + 0100 (b) 1001 + 1001

(c) 00010110 + 00010101 (d) 01100111 + 01010011

BCD Addition

The decimal number additions are shown for comparison.

(a)
```
        1001                                      9
     +  0100                                    + 4
        1101        Invalid BCD number (>9)      13
     +  0110        Add 6
 0001   0011        Valid BCD number
   ↓      ↓
   1      3
```

(b)
```
        1001                                      9
     +  1001                                    + 9
   1    0010        Invalid because of carry     18
     +  0110        Add 6
 0001   1000        Valid BCD number
   ↓      ↓
   1      8
```

BCD Addition

(c)

```
   0001    0110                                              16
 + 0001    0101                                            + 15
   0010    1011   Right group is invalid (>9),               31
                    left group is valid.
         + 0110    Add 6 to invalid code. Add
                    carry, 0001, to next group.
   ‾‾‾‾‾‾‾‾‾‾‾‾
   0011    0001   Valid BCD number
    ↓       ↓
    3       1
```

(d)

```
          0110    0111                                       67
        + 0101    0011                                     + 53
          1011    1010   Both groups are invalid (>9)        120
        + 0110  + 0110   Add 6 to both groups
   0001   0010    0000   Valid BCD number
    ↓      ↓       ↓
    1      2       0
```

Practice Questions

1. Convert the following decimal numbers to BCD:
   (a) 6 (b) 15 (c) 273 (d) 849

2. What decimal numbers are represented by each BCD code?
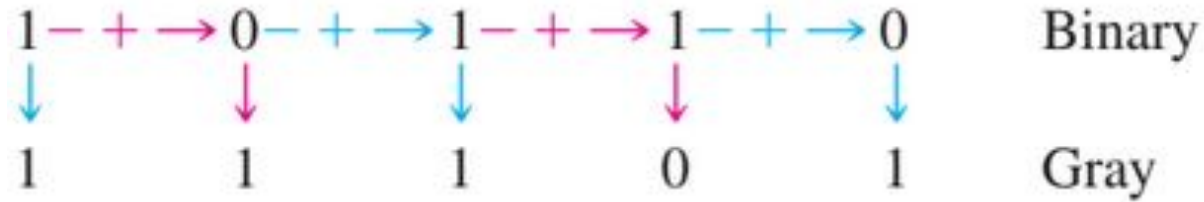   (a) 10001001 (b) 001001111000
   (c) 000101010111
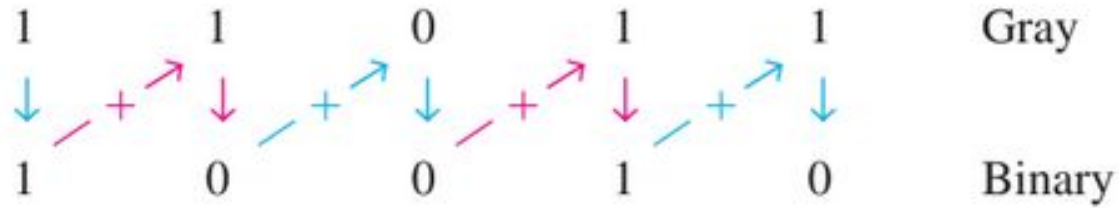
GRAY CODE

TABLE 2–6

Four-bit Gray code.

Decimal	Binary	Gray Code	Decimal	Binary	Gray Code
0	0000	0000	8	1000	1100
1	0001	0001	9	1001	1101
2	0010	0011	10	1010	1111
3	0011	0010	11	1011	1110
4	0100	0110	12	1100	1010
5	0101	0111	13	1101	1011
6	0110	0101	14	1110	1001
7	0111	0100	15	1111	1000

Binary To Gray Code

$$1 \xrightarrow{\;-\;+\;} 0 \xrightarrow{\;-\;+\;} 1 \xrightarrow{\;-\;+\;} 1 \xrightarrow{\;-\;+\;} 0 \quad \text{Binary}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$1 \qquad 1 \qquad 1 \qquad 0 \qquad 1 \quad \text{Gray}$$

(a) Convert the binary number 11000110 to Gray code.

Gray To Binary Code

1	1	0	1	1	Gray
1	0	0	1	0	Binary

(b) Convert the Gray code 10101111 to binary.