Oscillatory Motion

Recall how we created functions for oscillatory motion

$$x(t) = x_m \cos(\omega t + \phi)$$

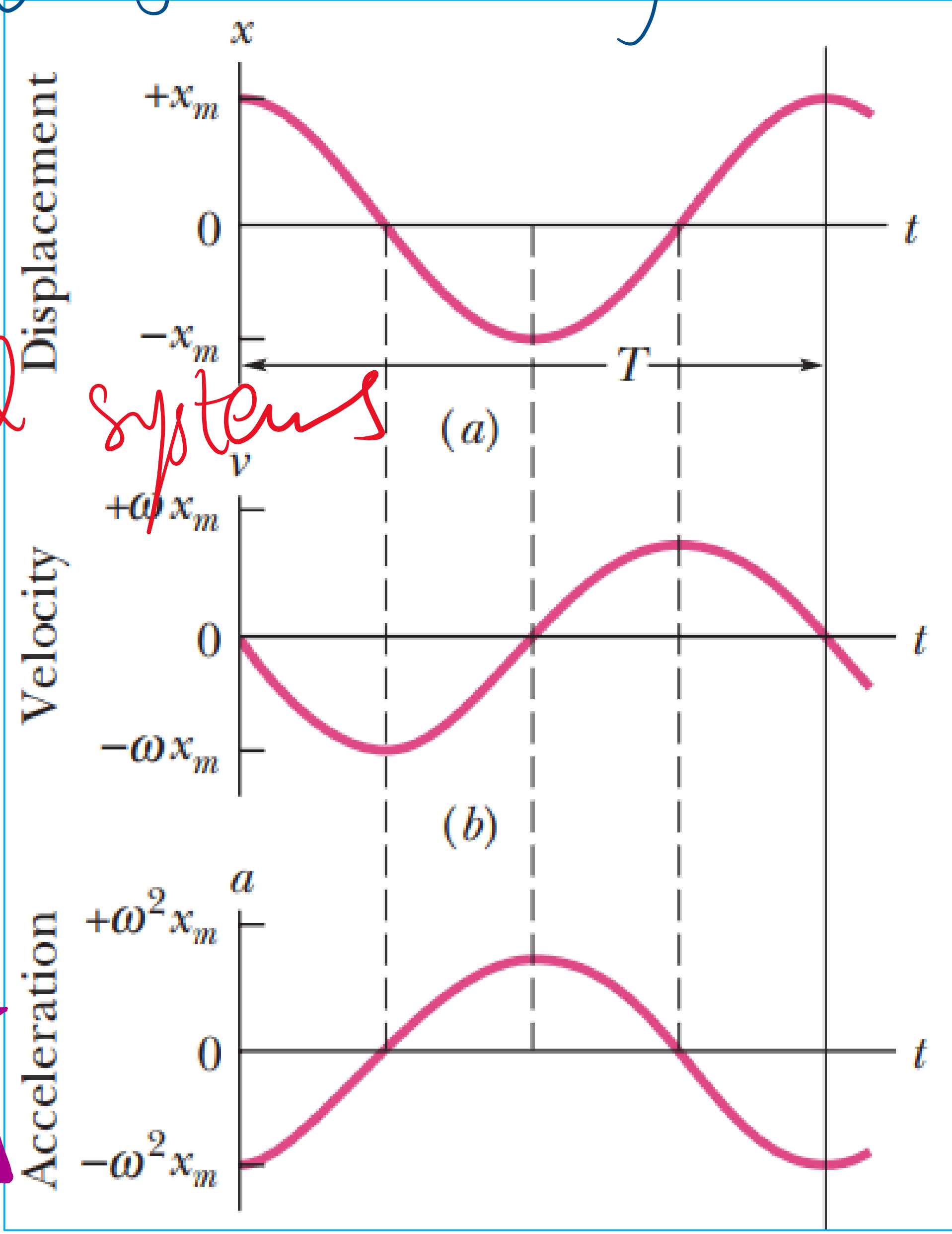
IS THIS EVEN REAL??? for physical systems

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

SHM conditions

$$\boxed{a(t) = -\omega^2 x(t)}$$



Extreme values here mean ...

zero values here and ...

extreme values here.

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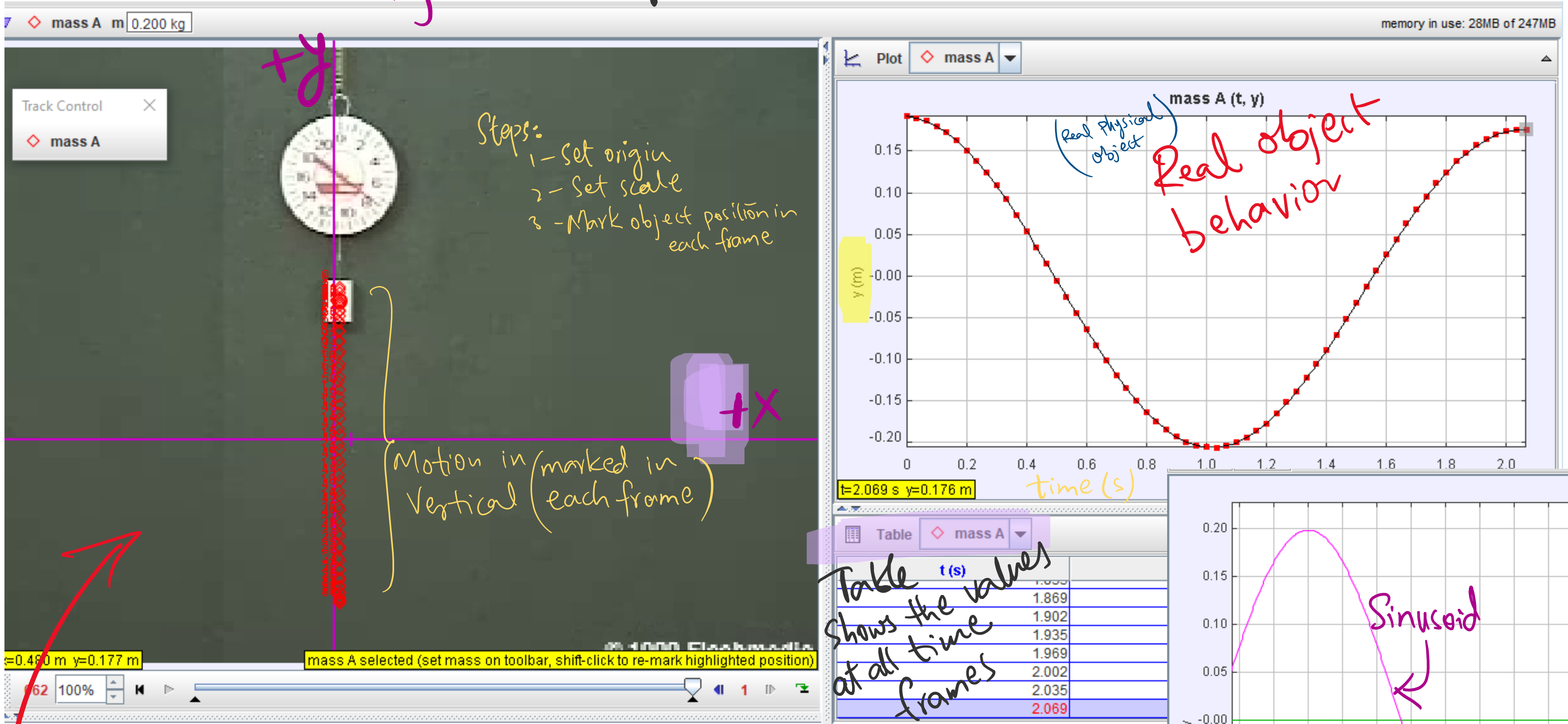
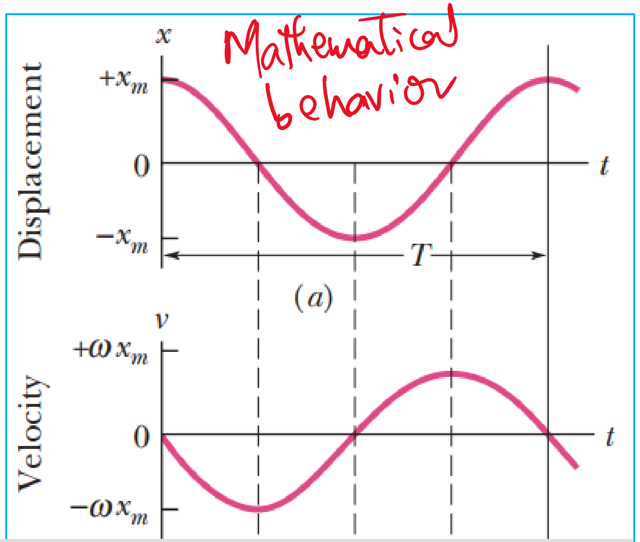
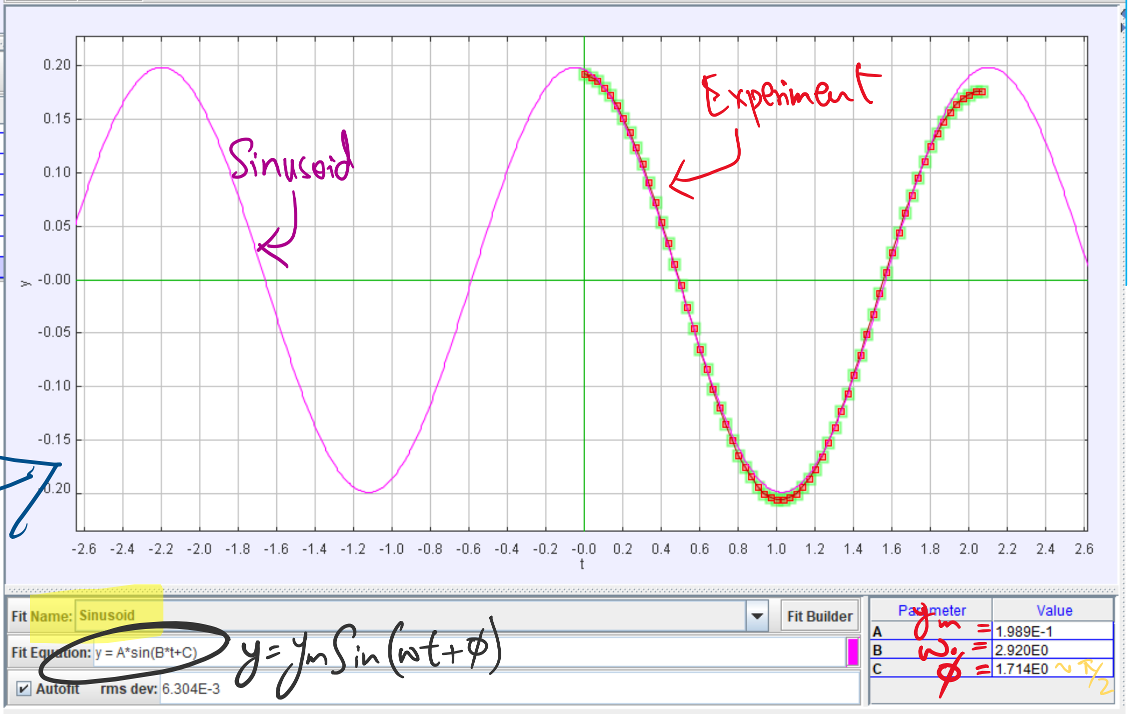
Software Tracker will be available on Google classroom







We want to prove that real systems do follow mathematical equations.

Analysis shows that there is a good fit between experiment and theory. We have concluded correct and useful mathematical fuction for position (x)

Spring-mass system experiment video

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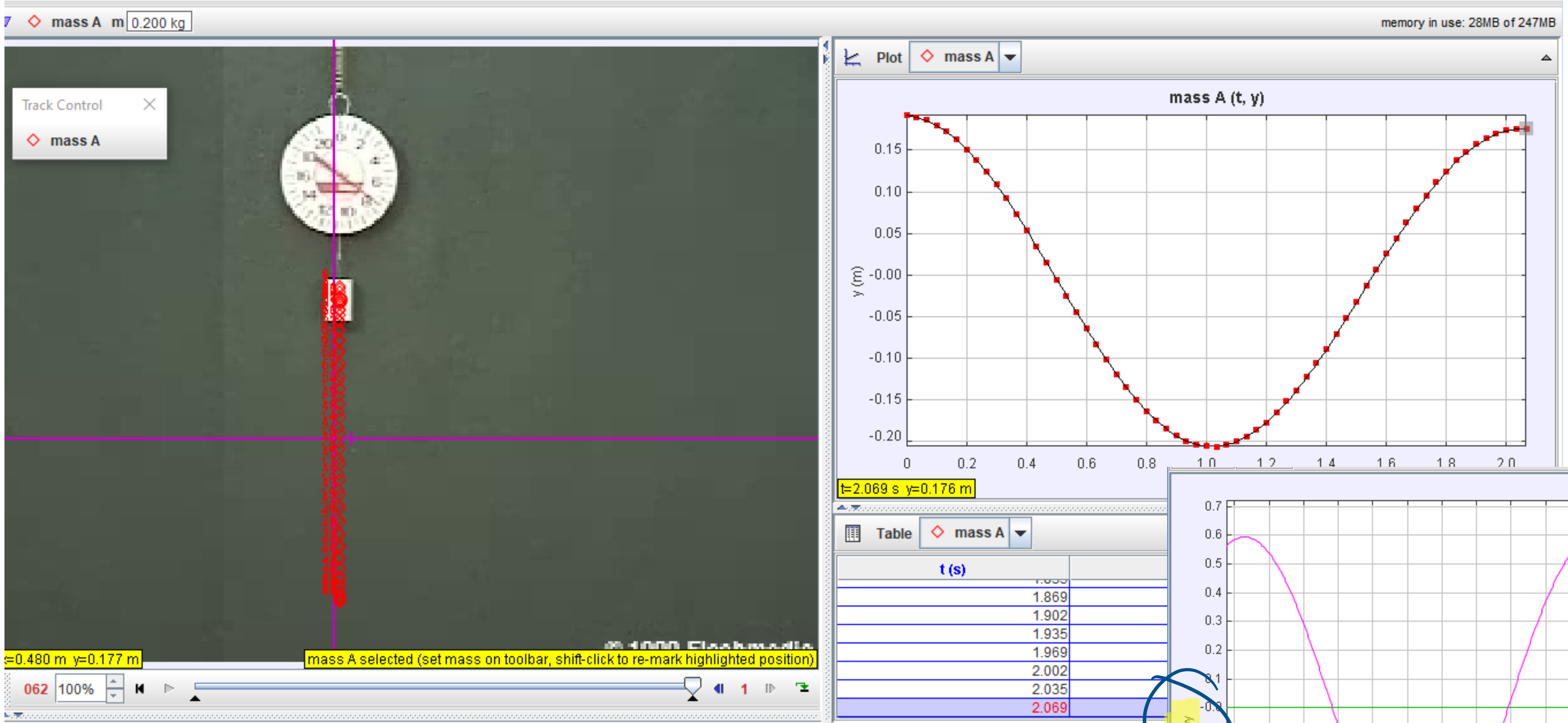
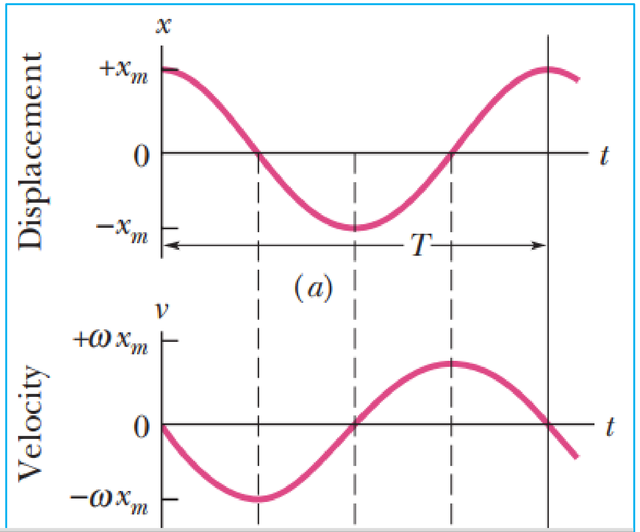
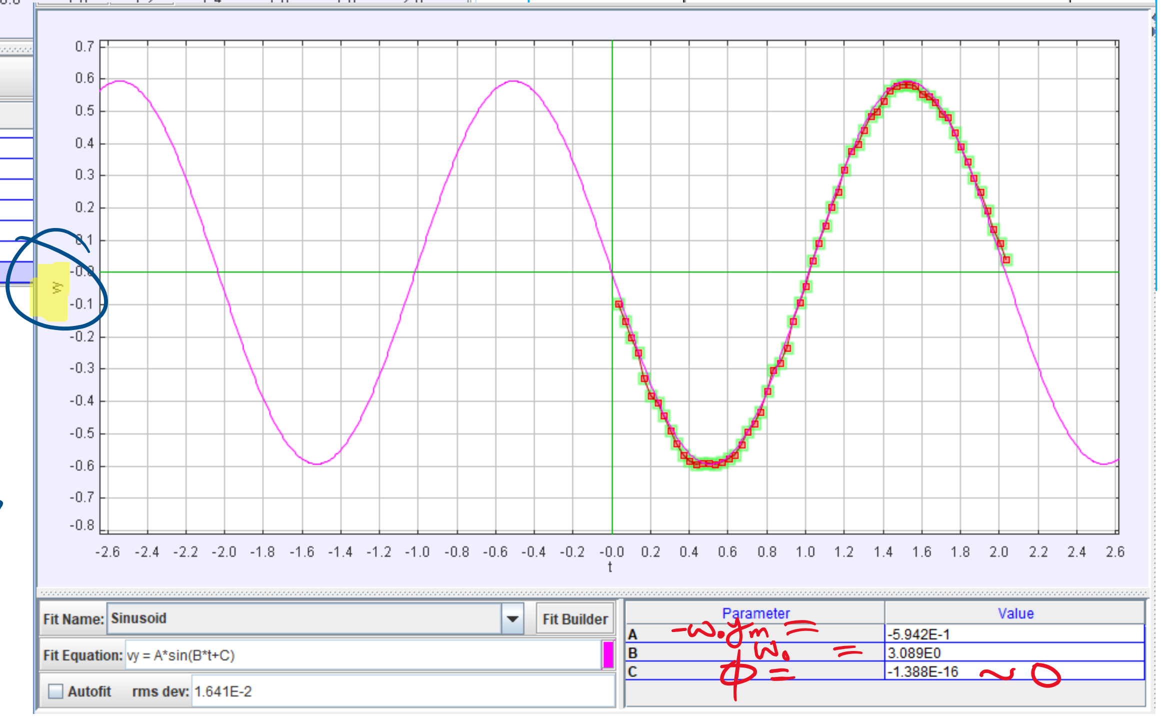




Also for velocity



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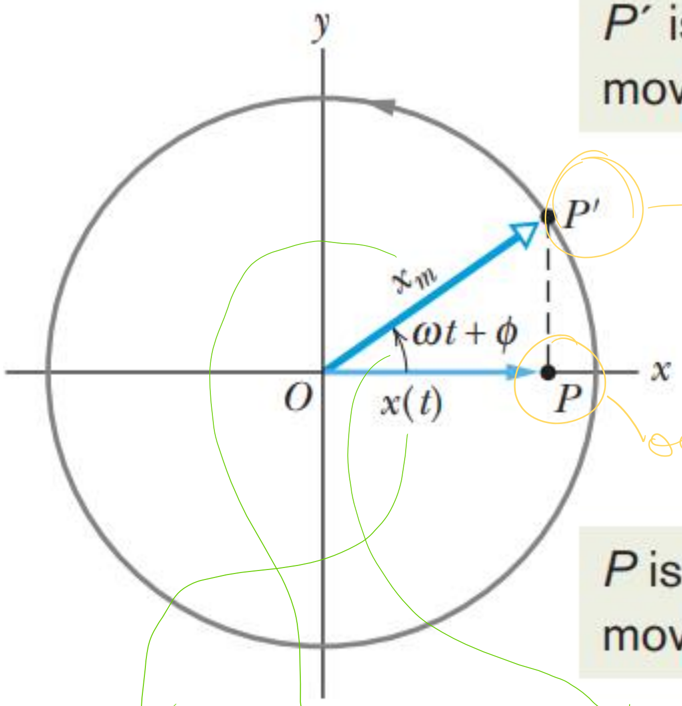
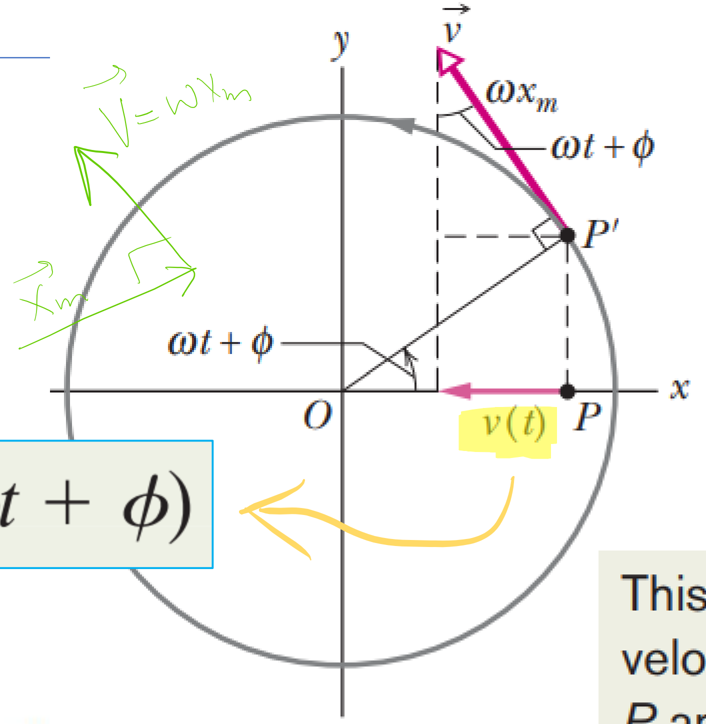
Projection of uniform circular Motion is also Oscillating Behavior



P′ is a particle moving in a circle.

P is a projection moving in SHM.

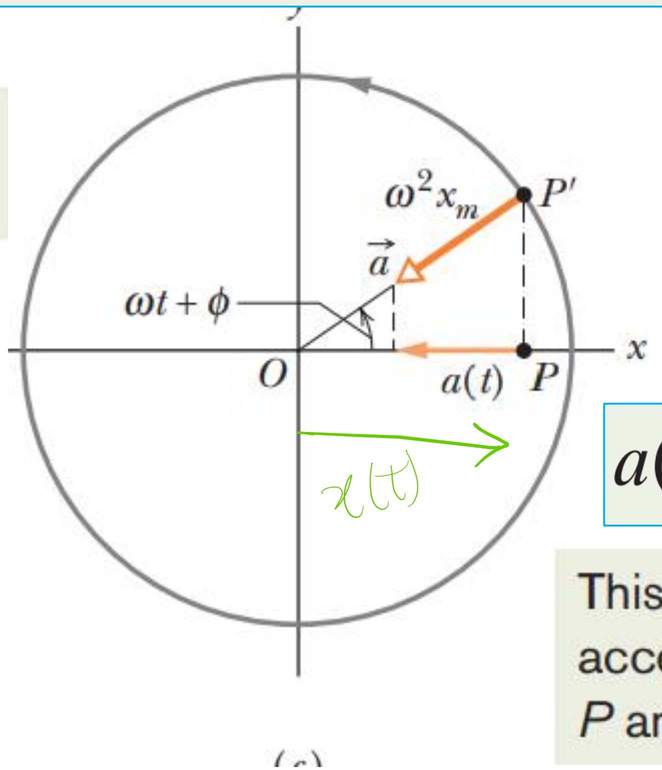
$$x(t) = x_m \cos(\omega t + \phi)$$



$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

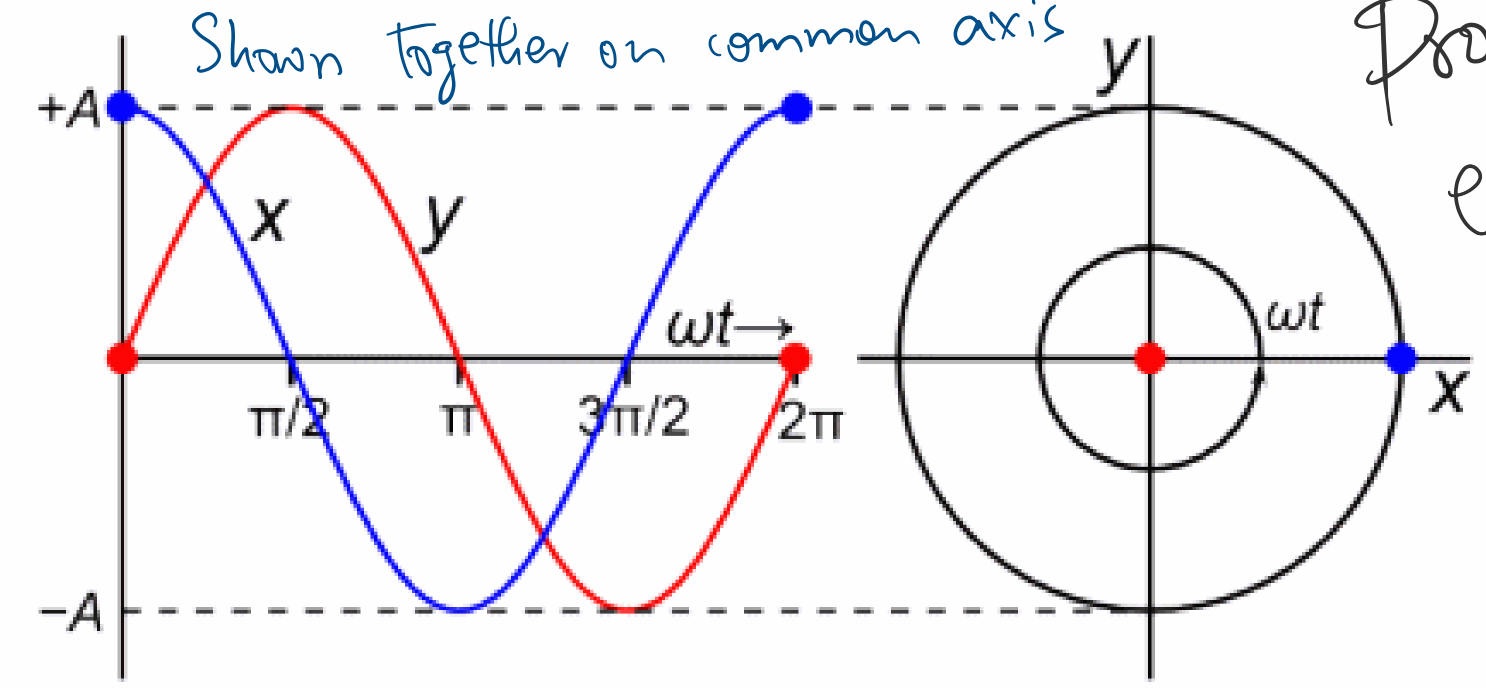
Velocity is $\frac{\pi}{2}$ ahead of position

This relates the velocities of *P* and *P′*.

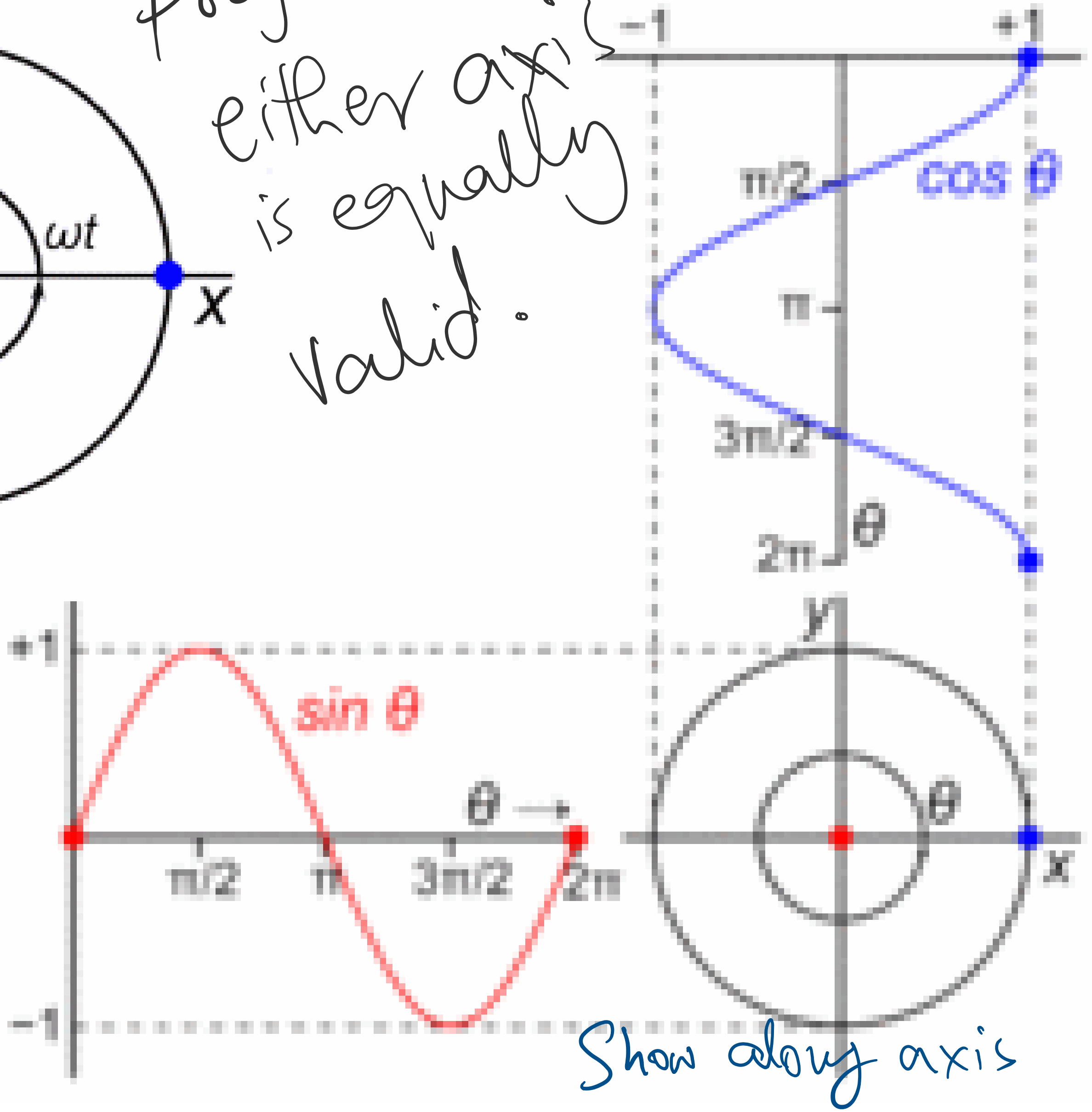


$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

This relates the accelerations of *P* and *P′*.



Projection on either axis is equally valid.



Sine and Cosine Functions were essentially made to represent the points on circle (and triangle)

Path of a Pendulum also contains the part of circle (we'll deal with it in next lecture)

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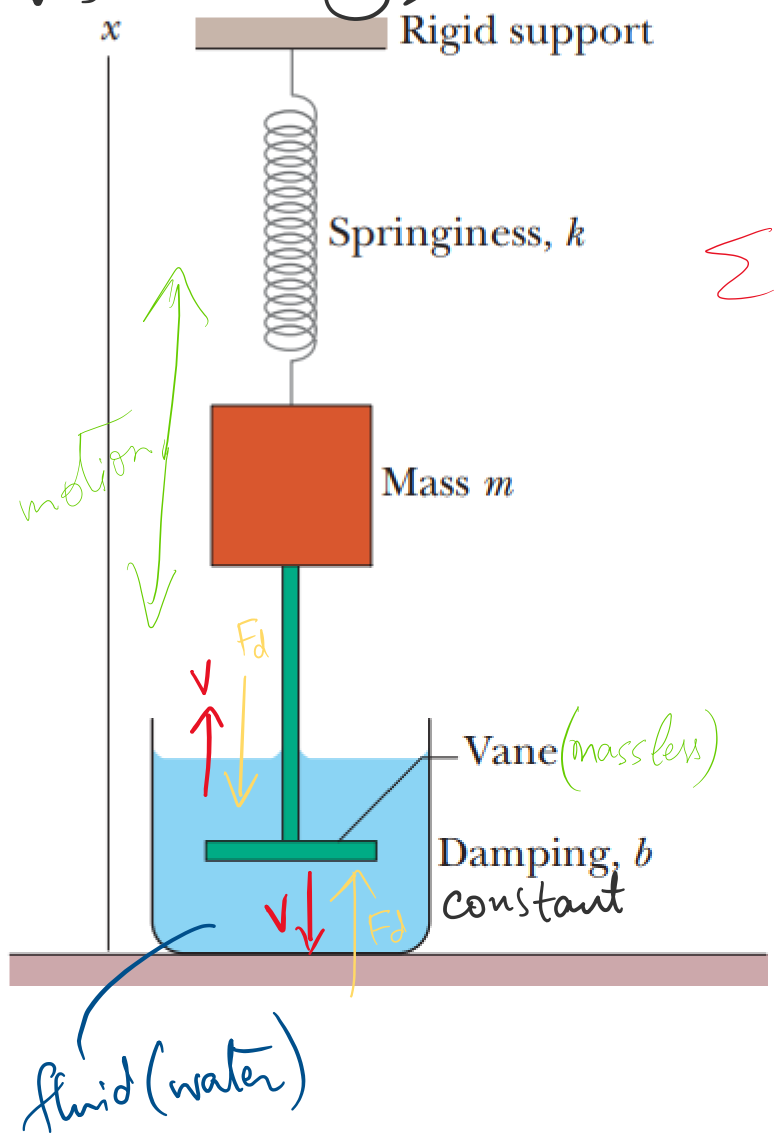
DAMPED OSCILLATIONS



damping force

damping constant (depends on fluid and vane properties)

velocity

$$F_d = -bv$$

damping force always opposes the velocity

$$\Sigma F = \underbrace{-bv}_{F_d} \underbrace{-kx}_{F_s} = ma.$$

using Newton's second law

rearranging

$$m \boxed{\frac{d^2x}{dt^2}}_{a} + b \boxed{\frac{dx}{dt}}_{v} + kx = 0.$$

Solution to this differential equation

Because solving this differential equation is not part of our course.

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi),$$

function of time

exponential factor

Angular frequency of damped oscillations

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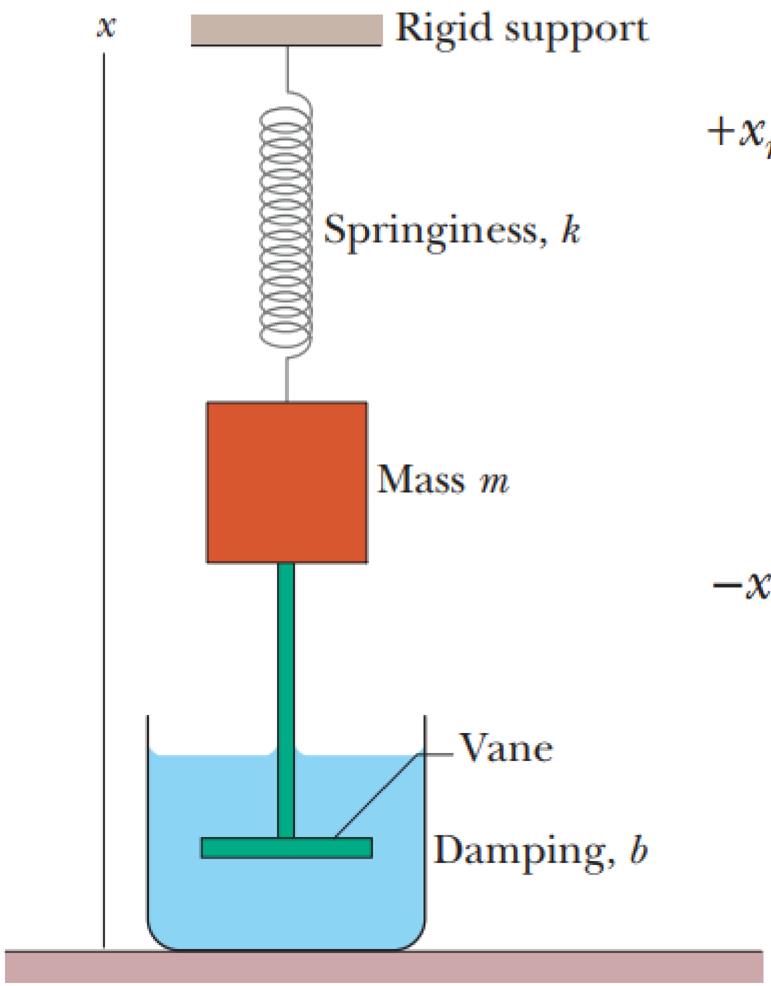
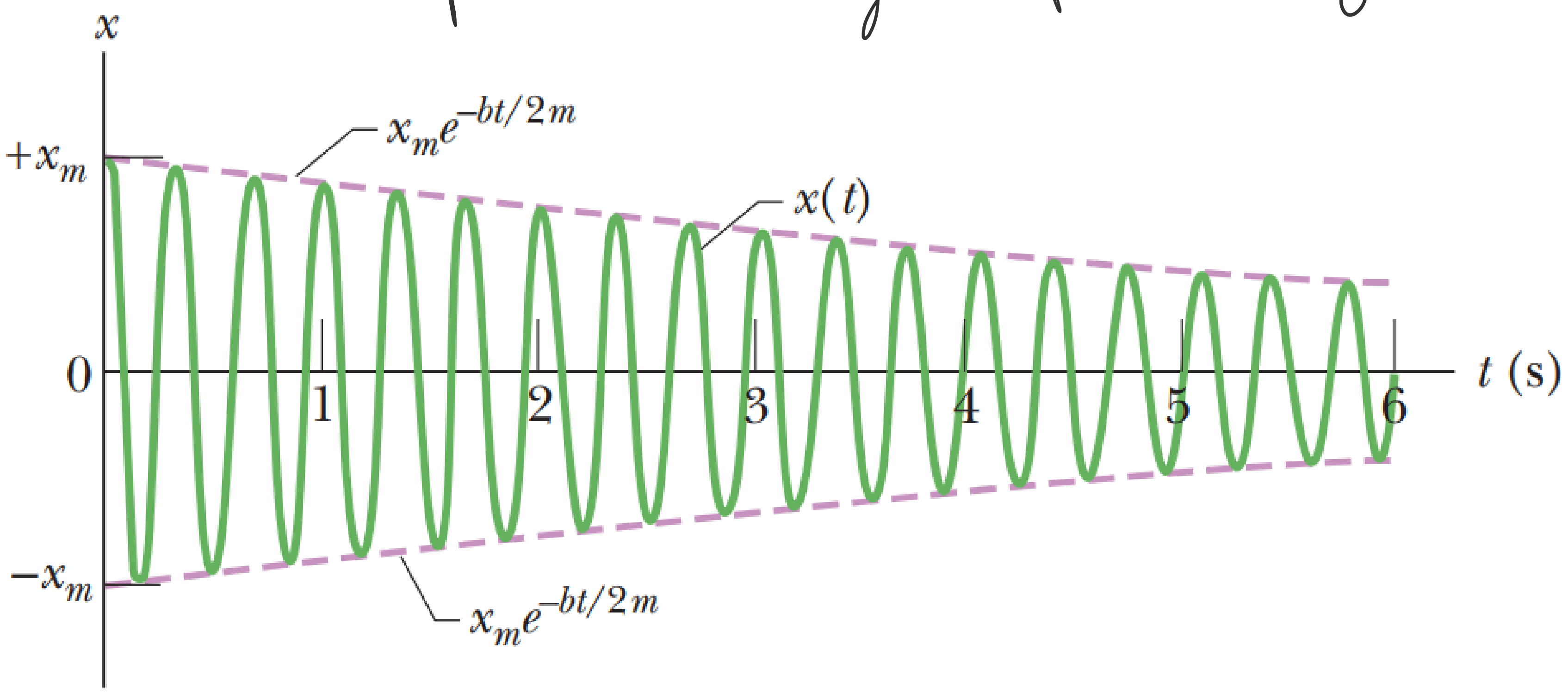
The amplitude decays exponentially





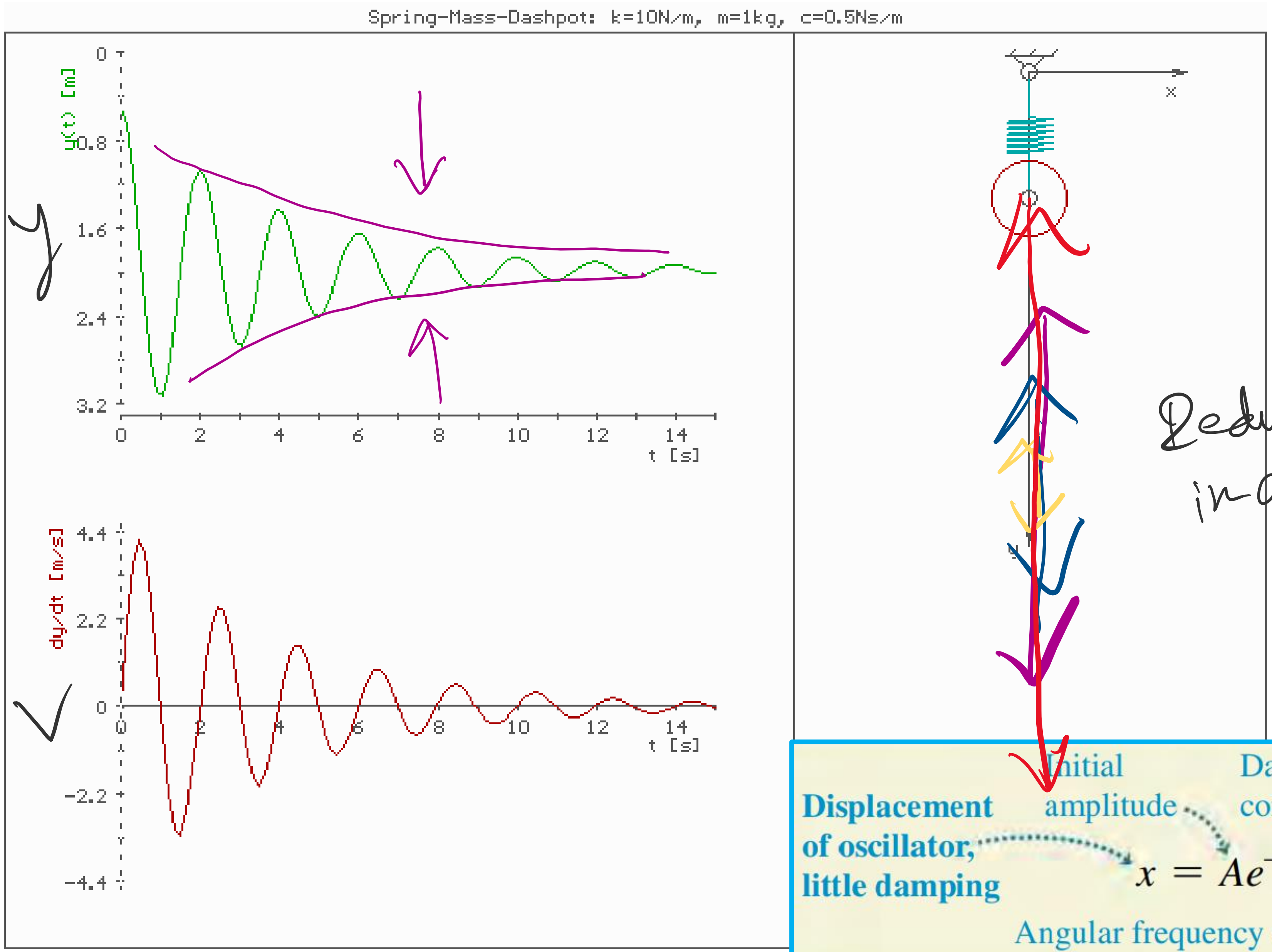
Solution to this differential equation

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi),$$

Amplitude of Damped oscillations

$\boxed{x_m e^{-bt/2m}}$

Amplitude decreases exponentially



Reduction in amplitude

Displacement of oscillator, little damping

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$$

Initial amplitude

Damping constant

Mass

Time

Angular frequency of damped oscillations

Phase angle

Damping conditions also change the angular frequency:

undamped (natural frequency) $\omega = \sqrt{k/m}$ {for spring-mass system

Angular frequency of oscillator, little damping $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$

Force constant of restoring force (k); Damping constant (b); Mass (m)

$\omega' < \omega$ by this factor

$b \ll \sqrt{km}$), then $\omega' \approx \omega$.

with increasing damping constant (b) the angular frequency (ω) decreases

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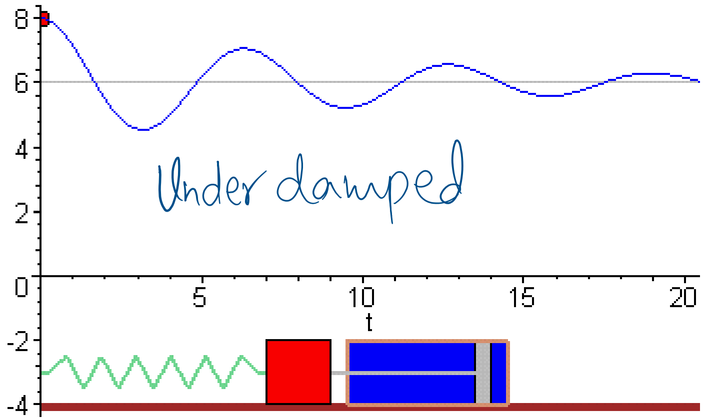
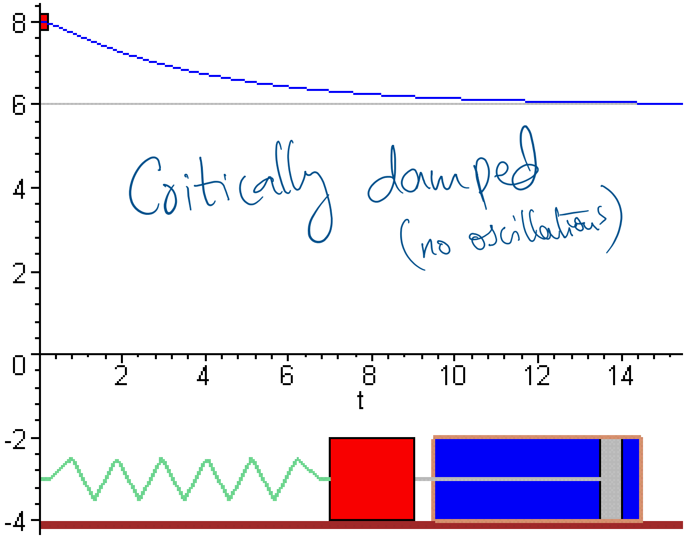
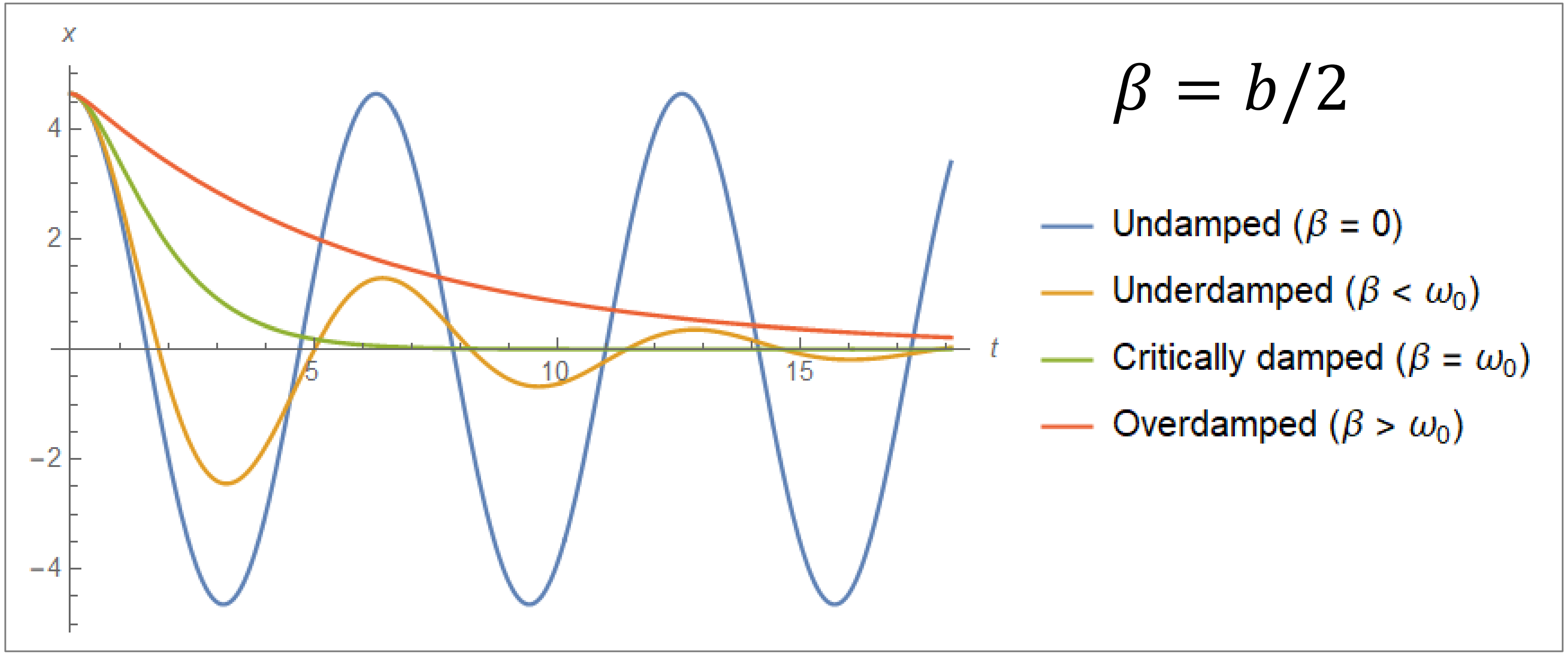
and when ω decreases down to zero (means, no oscillation. object only slowly moves to its mean position and stops)

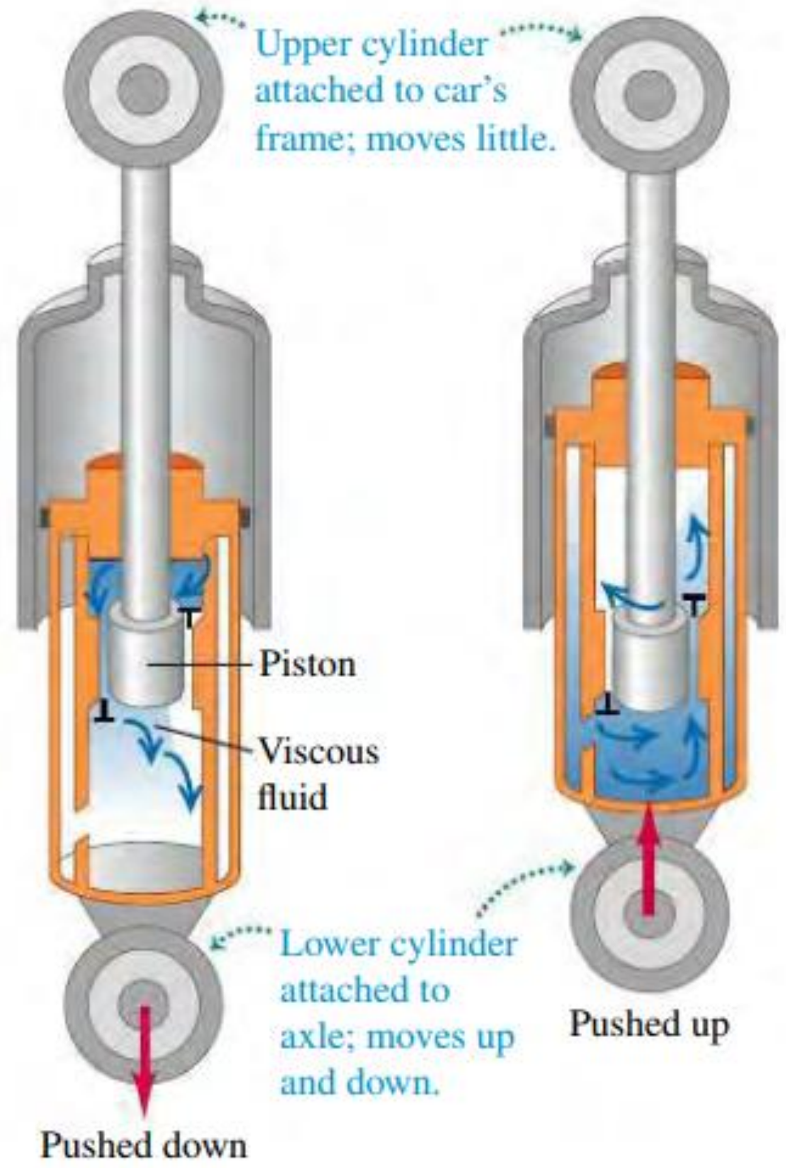
$$\frac{k}{m} - \frac{b^2}{4m^2} = 0 = \omega'^2 \quad \text{or} \quad b = 2\sqrt{km}$$

the condition is called $\boxed{\textbf{critical damping.}}$









The damping phenomenon is useful in many engineering applications.

Undamped Total Mechanical Energy

$$E = U + K = \frac{1}{2}kx_m^2.$$

Damped Total Mechanical Energy

$$E(t) \approx \frac{1}{2}kx_m^2 e^{-bt/m}$$

Softness and hardness of the shock absorbers is adjusted via damping constants.