

Integration Formulas

$$\int u^n du = \frac{u^{n+1}}{n+1} \quad \int \frac{1}{u} du = \ln|u|$$

$$\int e^u du = e^u \quad \int b^u du = \frac{b^u}{\ln b}$$

$$\int \sin u du = -\cos u + C \quad \int \cot x dx = \ln|\sin x| + C$$

$$\int \cos u du = \sin u + C$$

$$\int \tan u du = -\ln|\cos u| + C \quad \int \operatorname{cosec} x dx = \ln|\tan \frac{x}{2}| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \operatorname{cosec}^2 u du = -\cot u + C$$

$$\int \operatorname{cosec} u \cot u du = -\operatorname{cosec} u + C$$

$$\int \cot u du = \ln|\sin u| + C$$

Hyperbolic Functions

$$\int \sinh u du = \cosh u + C$$

$$\int \operatorname{sech}^2 u du = \tanh u + C$$

$$\int \cosh u du = \sinh u + C$$

$$\int \operatorname{cosech}^2 u du = -\coth u + C$$

$$\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$\int \operatorname{cosech} u \coth u du = -\operatorname{cosech} u + C$$

Algebraic Functions

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$$

Integration Formulas

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

$$\int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$\int \frac{du}{u \sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

Trigonometric Substitution:

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin x \quad ; \quad x = a \cos x$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec x$$

$$\sqrt{a^2 + x^2} \Rightarrow x = a \tan x$$

$$\Rightarrow \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\Rightarrow \cos 2\theta = 2 \cos^2 \theta - 1 \quad \text{OR} \quad \cos 2\theta = 1 - 2 \sin^2 \theta$$

Magic substitution:-

$$\tan \frac{x}{2} = u$$

$$dx = \frac{2}{1+u^2} du$$

$$\sin x = \frac{2u}{1+u^2}$$

$$\cos x = \frac{1-u^2}{1+u^2}$$

Riemann's Sum:-

Left case:-

$$x_k = a + (k-1) \Delta x$$

Right case:-

$$x_k = a + \Delta x k$$

$$\Delta x = \frac{b-a}{n}$$

Centre case:-

$$x_k = a + \left(k - \frac{1}{2}\right) \Delta x$$

Reduction Formulas:-

$$\begin{aligned} \int \cos^n x \, dx &= \int \cos^{n-1} x \cos x \, dx \\ \hookrightarrow &= \frac{\sin x \cos^{n-1} x}{n} + (n-1) \int \frac{\cos^{n-2} x}{n} \, dx \end{aligned}$$

$$\begin{aligned} \int \sin^n x \, dx &= \int \sin^{n-1} x \sin x \, dx \\ \hookrightarrow &= -\frac{\sin^{n-1} x \cos x}{n} + \frac{(n-1)}{n} \int \sin^{n-2} x \, dx \end{aligned}$$

$$\begin{aligned} \int \tan^n x \, dx &= \int \tan^{n-2} x \tan^2 x \, dx \\ \hookrightarrow &= \frac{\tan^{n-1} x}{(n-1)} - \int \tan^{n-2} x \, dx \end{aligned}$$

$$\begin{aligned} \int \sec^n x \, dx &= \int \sec^{n-2} x \sec^2 x \, dx \\ \hookrightarrow &= \frac{\sec^{n-2} x \tan x}{n-1} + \frac{(n-2)}{(n-1)} \int \sec^{n-2} x \, dx \end{aligned}$$

By-Parts:- LIATE OR ILPET

$$\int u \cdot v \, dx = u \int v \, dx - \int \left[\frac{du}{dx} \int v \, dx \right] dx$$

Hyperbolic Functions:-

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\sinh x \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\cosh^2 \theta - \sinh^2 \theta = 1$$

Area Under the Curve $\sum k$:-

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2} \right]^2$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$