1.)

$x_k = A x_{k-1} + m_q + q_{k-1}$ $\quad q_k \sim N(0, Q)$

$y_k = H x_k + m_r + r_k$. $\quad r_k \sim N(0, R)$

$p(x_k | x_{k-1}) = N(x_k | A x_{k-1}, m_q, Q)$

$p(y_k | x_k) = N(y_k | H x_k, m_r, R)$

Step 1 :-

$x_{k-1} \sim N(m_{k-1}, m_q, P_{k-1})$

$p(x_{k-1}) = p(x_{k-1} | m_{k-1}, m_q, P_{k-1})$

$p(x_k | x_{k-1}) = N(x_k | A x_{k-1}, m_q, Q)$

$$p\left(\begin{matrix} x_{k-1} \\ x_k \end{matrix} \middle| y_{1:k-1}\right) = N\left(\begin{pmatrix} x_{k-1} \\ x_k \end{pmatrix}, \begin{pmatrix} m_{k-1}, \\ A m_{k-1} + m_q \end{pmatrix}, \begin{pmatrix} P_{k-1} & P_{k-1} A^T \\ A P_{k-1} & A P_{k-1} A^T + Q \end{pmatrix}\right)$$

Step 2

$p(x_k | y_{1:k-1}) = N(x_k | A m_{k-1} + m_q, A P A^T + Q)$

$= N(x_k | m_k^-, P_k^-)$

$$\boxed{m_k^- = A m_{k-1} + m_q}$$

Prediction step

Step 3 :-

$$P(x_k) = N(x_k \mid m_k^-, P_k^-)$$

$$P(y_k \mid x_k) = N(y_k \mid H x_k + m_2, R)$$

$$P\left(\begin{matrix} x_k \\ y_k \end{matrix} \middle| y_{1:k-1}\right) = P(x_k) \cdot P(y_k \mid x_k)$$

$$= N\left(\begin{bmatrix} x_k \\ y_k \end{bmatrix}, \begin{pmatrix} m_k^- \\ H m_k^- + m_2 \end{pmatrix}, \begin{pmatrix} P_k^- & P_k^- H^T \\ H P_k^- & H P_k^- H^T + R \end{pmatrix} \right)$$

Step 4 :-

$$P(x_k \mid y_k, y_{1:k-1}) = P(x_k \mid y_{1:k})$$

$$= N\left(x_k \mid m_k^- + P_k^- H^T (H P_k^- H^T + R)^{-1} (y_k - H m_k^- - m_2),\ P_k^- - P_k^- H^T (H P_k^- H^T + R)^{-1} H P_k^-\right)$$

$$m_k = m_k^- + P_k^- H^T (\underbrace{H P_k^- H^T + R}_{S_k})^{-1} (y_k - H m_k^- - m_2)$$

$$\boxed{S_k = (H P_k^- H^T + R)}$$

$$m_k = m_k^- + \underbrace{P_k^- H^T S_k^{-1}}_{K_k} (y_k - H m_k^- - m_2)$$

$$\boxed{K_k = P_k^- H^T S_k^{-1}}$$

$$\boxed{m_k = m_k^- + K_k (y_k - H m_k^- - m_2)}$$

$$\boxed{m_k^- = A m_{k-1} + m_q}$$

$$P_k = P_k^- - P_k^- H^T (H P_k^- H^T + R^{-1}) H P_k^-$$

$$= P_k^- - P_k^- H^T S_k^{-1} H P_k^{-*}$$

$$= P_k^- - K_k \cancel{H} P_k^-$$

$$H P_k^{-*} = S_k^{-1} H P_k^{*}$$

$$\boxed{P_k = P_k^- - K_k S_k K_k^T}$$

$$m_k^- =$$

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2) Finite state

<u>Prediction</u>

$$P(x_k = i \mid y_{1:k-1}) = \sum_i P(x_k = i \mid x_{k-1} = j) \, P(x_{k-1} = j \mid y_{1:k-1})$$

<u>Update</u>

$P(x_k = i \mid \cancel{y_{1:k}})$

$P(y_k \mid x_k = j)$

$$P(x_k = i \mid y_{1:k}) = \frac{P(y_k \mid x_k = i) \, P(x_k = i \mid y_{1:k-1})}{\sum_i P(y_k \mid x_k = i) \, P(x_k = i \mid y_{1:k-1})}$$