Aalto University
School of Electrical
Engineering

Exercise Round 6

The deadline of this exercise round is **Wednesday March 04, 2020**. The solutions will be gone through during the exercise session in room T2 in Konemiehentie 2 (CS) on that day starting at 14:15.

The problems should be *solved before the exercise session*, and during the session those who have completed the exercises may be asked to present their solutions on the board/screen.

Exercise 1. (Optimal Importance Distribution)

Recall the following state space model from Exercise 3 on Round 1:

$$\begin{aligned} \mathbf{x}_k &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{x}_{k-1} + \mathbf{q}_{k-1}, \\ y_k &= \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}_k + r_k, \end{aligned} \tag{1}$$

where $\mathbf{x}_k = (x_k \;\; \dot{x}_k)^\mathsf{T}$ is the state, y_k is the measurement, and $\mathbf{q}_k \sim \mathrm{N}(\mathbf{0}, \mathrm{diag}(1/10^2, 1^2))$ and $r_k \sim \mathrm{N}(0, 10^2)$ are white Gaussian noise processes. Do the following steps:

(a) Write down the Kalman filter equations for this model.

(b) Derive expression for the optimal importance distribution for the model:

$$\pi(\mathbf{x}_k) = p(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \mathbf{y}_{1:k}). \tag{2}$$

(c) Write pseudo code for the corresponding particle filter algorithm (sequential importance resampling algorithm). Also write down the equations for the weight update.

(d) Compare the number of CPU steps (multiplications and additions) needed by the particle filter and Kalman filter. Which implementation would you choose for a real implementation?

Exercise 2. (Kalman Filter Based Importance Distribution)

Implement the bootstrap filter for the model in Exercise 1 on Round 4 and test its performance against the non-linear Kalman filters (if you implemented them before).

Exercise 3. (Bearings Only Tracking with SIR)

Implement a bootstrap filter and SIR with CKF importance distribution to the bearings only target tracking model in Exercises 4.3 and 5.3. Plot the results and compare RMSE values to those of the non-linear Kalman filters.