1.) Derivation $\Rightarrow$ RTS for non-zero mean

To derive

$$x_k = A x_{k-1} + m_q + q_{k-1} \qquad q_k \sim N(0, Q)$$

$$y_k = H x_k + m_r + r_k \qquad r_k \sim N(0, R)$$

$$p(x_k, x_{k+1} \mid y_{1:k}) = p(x_{k+1} \mid x_k)\, p(x_k \mid y_{1:k})$$

$p(x_k$

$$x_{k+1} \sim N(m_k, m_q, P_k)$$

$$p(x_{k}) = p(x_k \mid m_k, m_q, P_k)$$

$$p(x_{k+1} \mid x_k) = N(x_{k+1} \mid A x_k, m_q, Q)$$

$$p(x_k, x_{k+1} \mid y_{1:k}) = N\left[\begin{pmatrix} x_k \\ x_{k+1} \end{pmatrix}, m_1, P_1\right]$$

$$m_1 = \begin{pmatrix} m_k \\ A m_k + m_q \end{pmatrix}, \qquad P_1 = \begin{pmatrix} P_k & P_k A^T \\ A P_k & A P_k A^T + Q \end{pmatrix}$$

$$p(x_k \mid x_{k+1}, y_{1:T}) = p(x_k \mid x_{k+1}, y_{1:k})$$

$$= N(x_k \mid m_2, P_2)$$

where

$$G_k = P_k A_k^T (A_k P_k A_k^T + Q_k)^{-1}$$

$$m_2 = m_k + G_k (x_{k+1} - A m_k - m_q)$$

$$P_2 = P_k - G_k (A P A^T + Q) G_k^T$$

$$p(x_{k+1}, x_k \mid y_{1:T}) = \frac{p(x_k \mid x_{k+1}, y_{1:T})}{p(x_{k+1} \mid y_{1:T})}$$

$$= N(x_k \mid m_2, P_2)\, N(x_{k+1} \mid m_{k+1}^s, P_{k+1}^s)$$

$$= N\left(\begin{bmatrix} x_{k+1} \\ x_k \end{bmatrix} \Big| m_3, P_3\right)$$

$$m_3 = \begin{pmatrix} m_{k+1}^s \\ m_k + G_k(m_{k+1}^s - A m_k - m_q) \end{pmatrix}$$

$$P_3 = \begin{pmatrix} P_{k+1}^s & P_{k+1}^s G_k^T \\ G_k P_{k+1}^s & G_k . P_{k+1}^s G_k + P_2 \end{pmatrix}$$

$$m_k^s = m_k + G_k(m_{k+1}^s - A_k m_k - m_q)$$

$$P_k^s = P_k + G_k\,(P_{k+1}^s - A P_k A^T - Q_k)\, G_k^T$$

$$p(x_k \mid y_{1:T}) = N(x_k \mid m_k^s, P_k^s)$$

Backward recursion equations.

$$m_{k+1}^- = A m_k + m_q$$

$$P_{k+1}^- = A P A^T + Q.$$

$$G_k = P_k A^T [P_{k+1}^-]^{-1}$$

$$m_k^s = m_k + G_k[m_{k+1}^s - m_{k+1}^-]$$

$$P_k^s = P_k + G_k[P_{k+1}^s - P_{k+1}^-]\, G_k^T$$