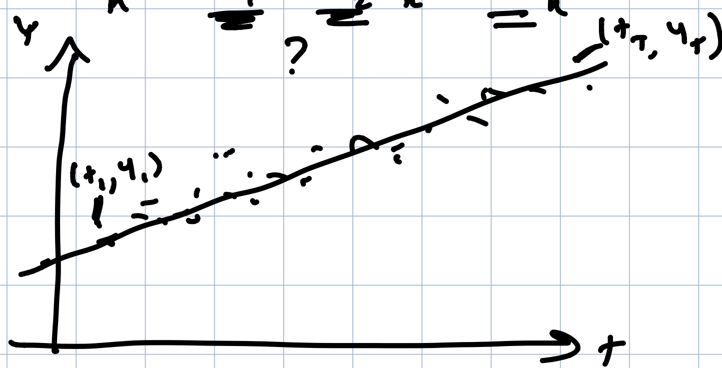
$$y_k = \underline{\theta_1} + \underline{\theta_2}\, t_k + \underline{\varepsilon}_k$$

?



$$p(y_k \mid \theta_1, \theta_2, t_k) = N(y_k \mid \theta_1 + \theta_2 t_k, \sigma^2)$$

$$\theta_1 + \theta_2 t_k = \underbrace{(1 \quad t_k)}_{H_k} \underbrace{\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}}_{\vec{\theta}} = H_k \vec{\theta}$$

$$p(y_k \mid \vec{\theta}, t_k) = N(y_k \mid H_k \vec{\theta}, \sigma^2)$$

ML : $\hat{\theta}_{ML} = \max_{\vec{\theta}} p(y_1, \dots, y_T \mid \vec{\theta})$

$$= \max_{\vec{\theta}} \prod_k p(y_k \mid \vec{\theta})$$

$$p(y_{1:T} \mid \vec{\theta}) = \prod_k p(y_k \mid \vec{\theta})$$

$p(\vec{\theta})$ = given

$$p(\vec{\theta} \mid y_{1:T}) = \frac{p(y_{1:T} \mid \hat{\theta})\, p(\vec{\theta})}{\int p(y_{1:T} \mid \vec{\theta})\, p(\vec{\theta})\, d\vec{\theta}}$$

$$\propto \underline{p(y_{1:T} \mid \vec{\theta})\, p(\vec{\theta})}$$

$$p(y_{1:T} \mid \vec{\theta}) = \prod_k N(y_k \mid H_k \vec{\theta}, \sigma^2)$$

$$p(\vec{\theta}) = N(\vec{\theta} \mid m_0, P_0)$$

$$p(\vec{\theta} \mid y_{1:T}) \propto \left[\prod_k N(y_k \mid H_k \vec{\theta}, \sigma^2)\right] N(\vec{\theta} \mid m_0, P_0)$$

$$= \left(\prod_k |2\pi\sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2\sigma^2}(y_k - H_k\vec{\theta})^2\right)\right)$$

$$|2\pi P_0|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\vec{\theta} - m_0) P_0^{-1} (\vec{\theta} - m_0)\right)$$

$$\propto \exp\left(-\frac{1}{2\sigma^2} \sum_k (y_k - H_k\vec{\theta})^2 - \frac{1}{2}(\vec{\theta} - m_0)^T P_0^{-1} (\vec{\theta} - m_0)\right)$$

$$\vec{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix}, \quad H = \begin{pmatrix} H_1 \\ \vdots \\ H_T \end{pmatrix}$$

$$\sum_k (y_k - H_k\vec{\theta})^2 = (y - H\vec{\theta})^T (y - H\vec{\theta})$$

$$= \exp\Big(\underbrace{-\frac{1}{2\sigma^2}(\vec{y} - H\vec{\theta})^T(\vec{y} - H\vec{\theta}) - \frac{1}{2}(\vec{\theta} - m_0)^T P_0^{-1}(\vec{\theta} - m_0)}_{= -\frac{1}{2}(\vec{\theta} - \vec{m}_T)^T P_T^{-1} (\vec{\theta} - \vec{m}_T)}\Big)$$

$$m_T = \min_{\vec{\theta}} \underbrace{\frac{1}{2\sigma^2}(\vec{y} - H\vec{\theta})^T (y - H\vec{\theta}) + \frac{1}{2}(\vec{\theta} - m_0)^T P_0^{-1} (\vec{\theta} - m_0)}_{E(\vec{\theta})}$$

$$\frac{\partial E}{\partial \vec{\theta}} = \frac{1}{\sigma^2} H^T (H\vec{\theta} - \vec{y}) + P_0^{-1}(\vec{\theta} - m_0) = 0$$

$$\Downarrow$$

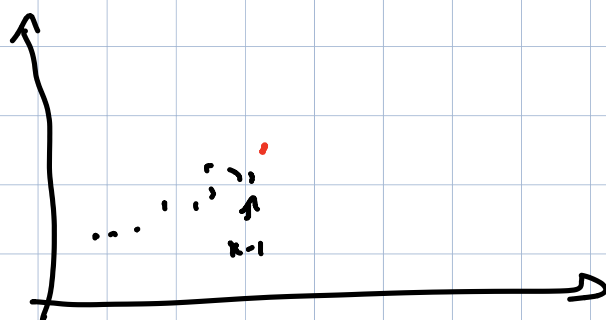
$$\vec{\theta} = \left[P_0^{-1} + \frac{1}{\sigma^2} H^T H\right]^{-1} \left[\frac{1}{\sigma^2} H^T y + P_0^{-1} m_0\right]$$

$$= \vec{m}_T$$

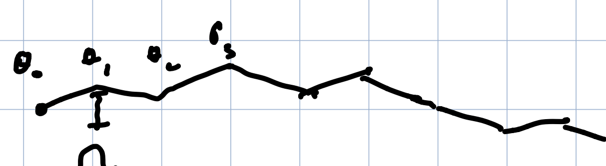
$$\frac{\partial^2 E}{\partial \hat{\theta} \partial \hat{\theta}^T} = \frac{1}{\sigma^2} H^T H + P_0^{-1} = P_T^{-1}$$

$$p(\vec{\theta} \mid y_{1:T}) \propto \exp\left(-\frac{1}{2} (\vec{\theta} - \hat{m}_T)^T P_T^{-1} (\vec{\theta} - \hat{m}_T)\right)$$

$$\Rightarrow \quad p(\vec{\theta} \mid y_{1:T}) = N(\vec{\theta} \mid \vec{m}_T, P_T)$$



$$p(\vec{\theta} \mid y_{1:k-1})$$
$$= N\left(\vec{\theta} \mid \left[P_0^{-1} + \frac{1}{\sigma^2} H_{1:k-1}^T H_{1:k-1}\right]^{-1} \ldots\right.$$



step size $\sim N(0, Q)$

$$\theta_k = \theta_{k-1} + q_{k-1}, \quad q_{k-1} \sim N(0, Q)$$

$$\Rightarrow \quad p(\theta_k \mid \theta_{k-1}) = N(\theta_k \mid \theta_{k-1}, Q)$$

$$p(A, B \mid C) = p(A \mid B, C)\, p(B \mid C)$$

$$p(\theta_k, \theta_{k-1} \mid y_{1:k-1})$$

$$= p(\theta_k \mid \theta_{k-1}, \cancel{y_{1:k-1}})\, p(\theta_{k-1} \mid y_{1:k-1})$$

Markov & cond. ind.

$$= \underbrace{p(\theta_k \mid \theta_{k-1})}_{\text{dyn. model}}\, \underbrace{p(\theta_{k-1} \mid y_{1:k-1})}_{\text{prev. posterior}}$$

$$p(\theta_k \mid y_{1:k-1})$$

$$= \int p(\theta_k, \theta_{k-1} \mid y_{1:k-1})\, d\theta_{k-1}$$

$$= \int p(\theta_k \mid \theta_{k-1})\, p(\theta_{k-1} \mid y_{1:k-1})\, d\theta_{k-1}$$

$$= \int N(\theta_k \mid \theta_{k-1}, Q)\, N(\theta_{k-1} \mid m_{k-1}, P_{k-1})\, d\theta_{k-1}$$

$$= N(\theta_k \mid m_{k-1}, P_{k-1} + Q)$$

$$\Rightarrow \int N(y \mid A x, Q)\, N(x \mid m, P)\, dx$$

$$= N(y \mid A m, A P A^T + Q)$$