Startup guide for moment tensor source types

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See webpage with links here:
`http://www.giseis.alaska.edu/input/carl/research/beachball.html`

1 Computing source type coordinates on the fundamental lune

Here we present a guide for converting moment tensors to source types on the fundamental lune, then plotting.

1. Start with a moment tensor represented as a 3×3 symmetric matrix in some orthonormal basis (it does not matter which one). Compute the eigenvalues.

2. Sort the eigenvalues as $\lambda_1 \geq \lambda_2 \geq \lambda_3$. Then compute the (γ, β, ρ) coordinates with *Tape and Tape* (2012, Eq. 21):

$$\begin{aligned}
\tan\gamma &= \frac{-\lambda_1 + 2\lambda_2 - \lambda_3}{\sqrt{3}(\lambda_1 - \lambda_3)} \\
\cos\beta &= \frac{\lambda_1 + \lambda_2 + \lambda_3}{\sqrt{3}\|\mathbf{\Lambda}\|} \\
\rho &= \|\mathbf{\Lambda}\|
\end{aligned}$$

   (Here ρ is not needed, but it represents the seismic moment: $M_0 = \rho/\sqrt{2}$.) For latitude on the lune, use $\delta = \pi/2 - \beta$.

If you are starting with eigenvalues, rather than the moment tensor, then start with step 2.

1.1 Plotting in Mathematica

All 3D plots in the Tape and Tape papers were generated using Mathematica. Some Mathematica plotting scripts are available from the webpage link at the top.

1.2 Plotting in GMT

Generic Mapping Tools (GMT: *Wessel and Smith*, 1991) is open-source software. It provides numerous options for global projections. One example, the equal-area Hammer projection, is shown in Figure 3. The key command in GMT is `-JH0/3i -R-30/30/-90/90`; this specifies the projection and also the region of the fundamental lune (γ-δ coordinates). A script to generate these plots, `lune.pl`, is available from the webpage link above.

The 2D plotting option was intentionally not included in *Tape and Tape* (2012) for two reasons: (1) the projection distracts from the key point that the lune is part of the sphere; (2) the projection, while equal-area, distorts areas that are undistorted on the sphere. Nevertheless, a comparison between Figures 2 and 4 shows that the 2D version might be preferred for representing points that span the full lune.

Figures 5 and 6 show some beachballs on the lune. The moment tensor orientation for each set is shown at the center. Within each plot, all beachballs have the same orientation.

Figures 7–11 is a guide to five different ways to characterize source types of full moment tensors.

References

Aki, K., and P. G. Richards (1980), *Quantitative Seismology, Theory and Methods*, W. H. Freeman, San Francisco, Calif., USA.

Alvizuri, C., and C. Tape (2016), Full moment tensors for small events ($M_{\rm w} < 3$) at Uturuncu volcano, Bolivia, *Geophys. J. Int.*, *206*, 1761–1783, doi:10.1093/gji/ggw247.

Alvizuri, C., V. Silwal, L. Krischer, and C. Tape (2018), Estimation of full moment tensors, including uncertainties, for nuclear explosions, volcanic events, and earthquakes, *J. Geophys. Res. Solid Earth* (in press), doi:10.1029/2017JB015325.

Minson, S. E., D. S. Dreger, R. Bürgmann, H. Kanamori, and K. M. Larson (2007), Seismically and geodetically determined nondouble-couple source mechanisms from the 2000 Miyakejima volcanic earthquake swarm, *J. Geophys. Res.*, *112*, B10308, doi:10.1029/2006JB004847.

Tape, W., and C. Tape (2012), A geometric setting for moment tensors, *Geophys. J. Int.*, *190*, 476–498, doi:10.1111/j.1365-246X.2012.05491.x.

Tape, W., and C. Tape (2013), The classical model for moment tensors, *Geophys. J. Int.*, *195*, 1701–1720, doi:10.1093/gji/ggt302.

Tape, W., and C. Tape (2015), A uniform parameterization of moment tensors, *Geophys. J. Int.*, *202*, 2074–2081, doi:10.1093/gji/ggv262.

Wessel, P., and W. H. F. Smith (1991), Free software helps map and display data, *Eos Trans. Am. Geophys. Un.*, *72*(41), 441 ff.

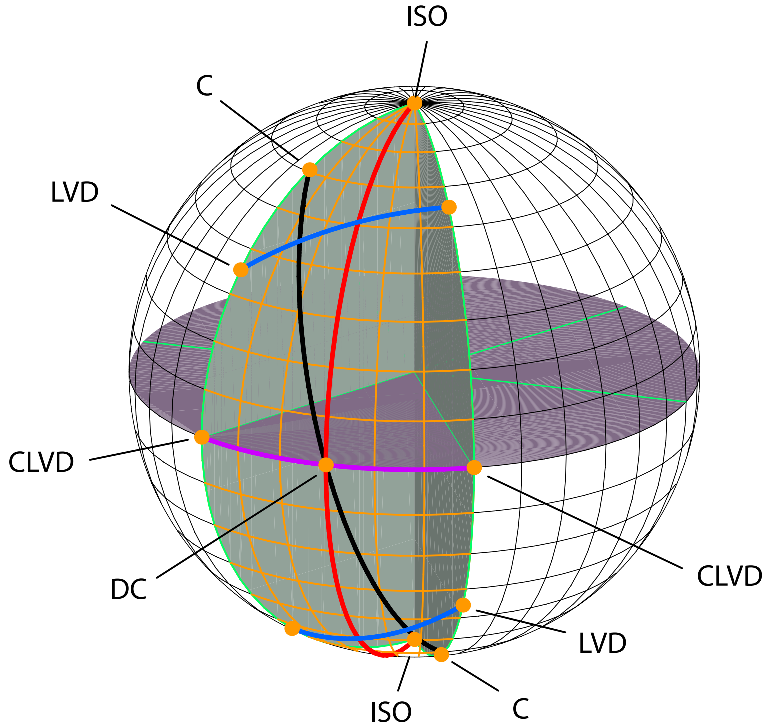

Figure 1: Fundamental lune representation of source types (*Tape and Tape*, 2012, Figure 1). See 2D version in Figure 3.

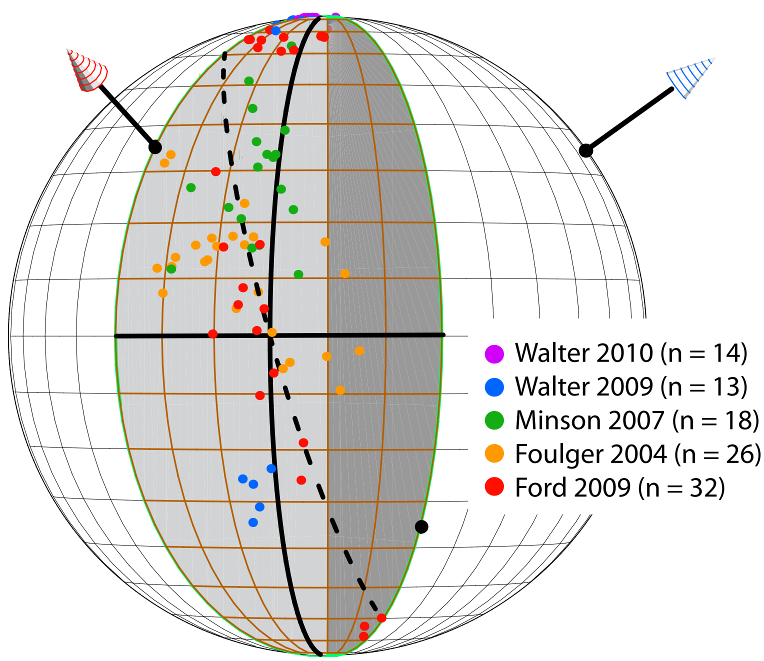

Figure 2: Five full moment tensor data sets plotted on the fundamental lune (*Tape and Tape*, 2012, Figure 25). See 2D version in Figure 4.

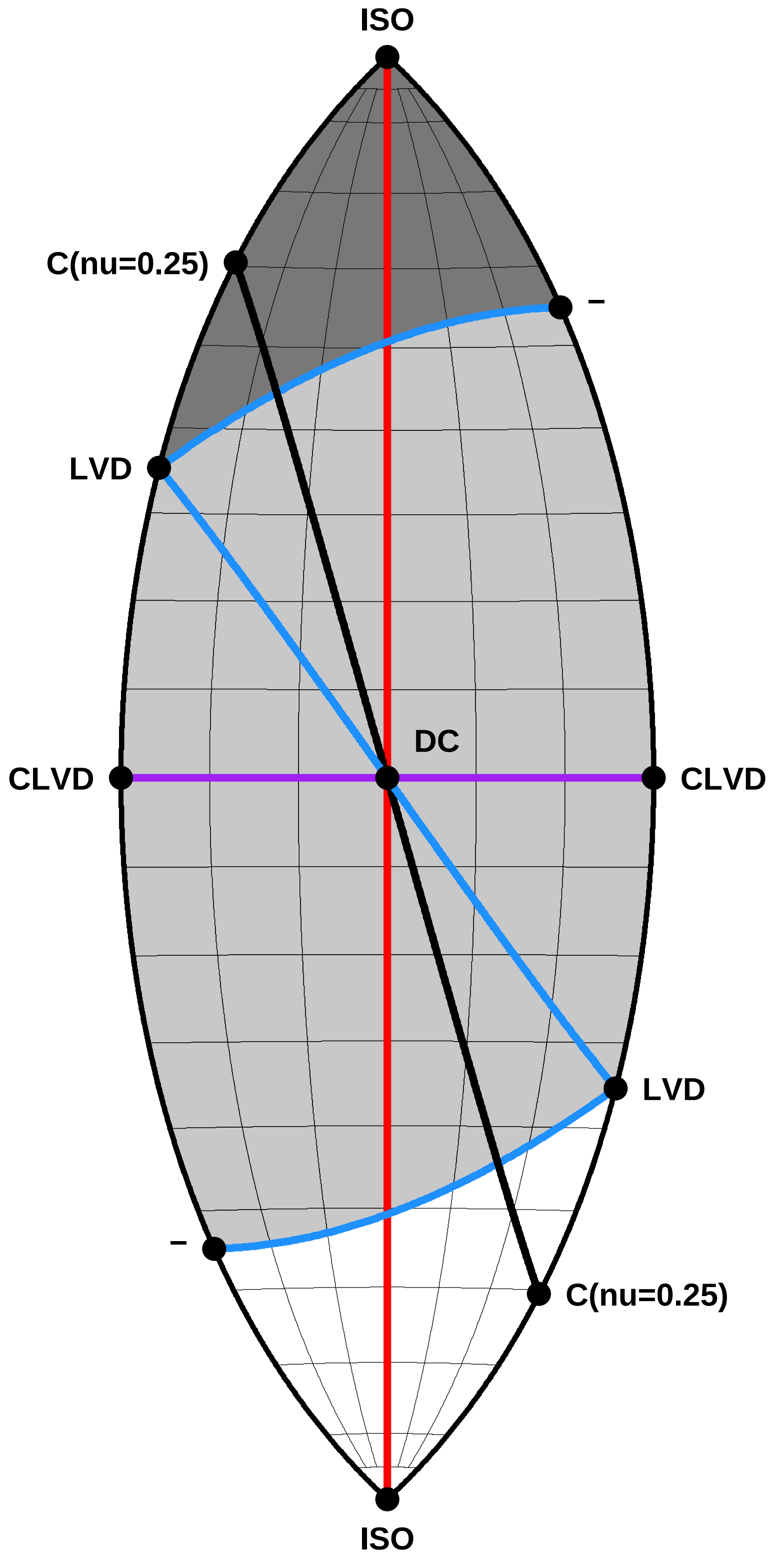

Figure 3: Home for source types, 2D version. See 3D version in Figure 1. The dark gray region at the top represents source types for which all vectors point outward. The white region at the bottom represents source types for which all vectors point inward.

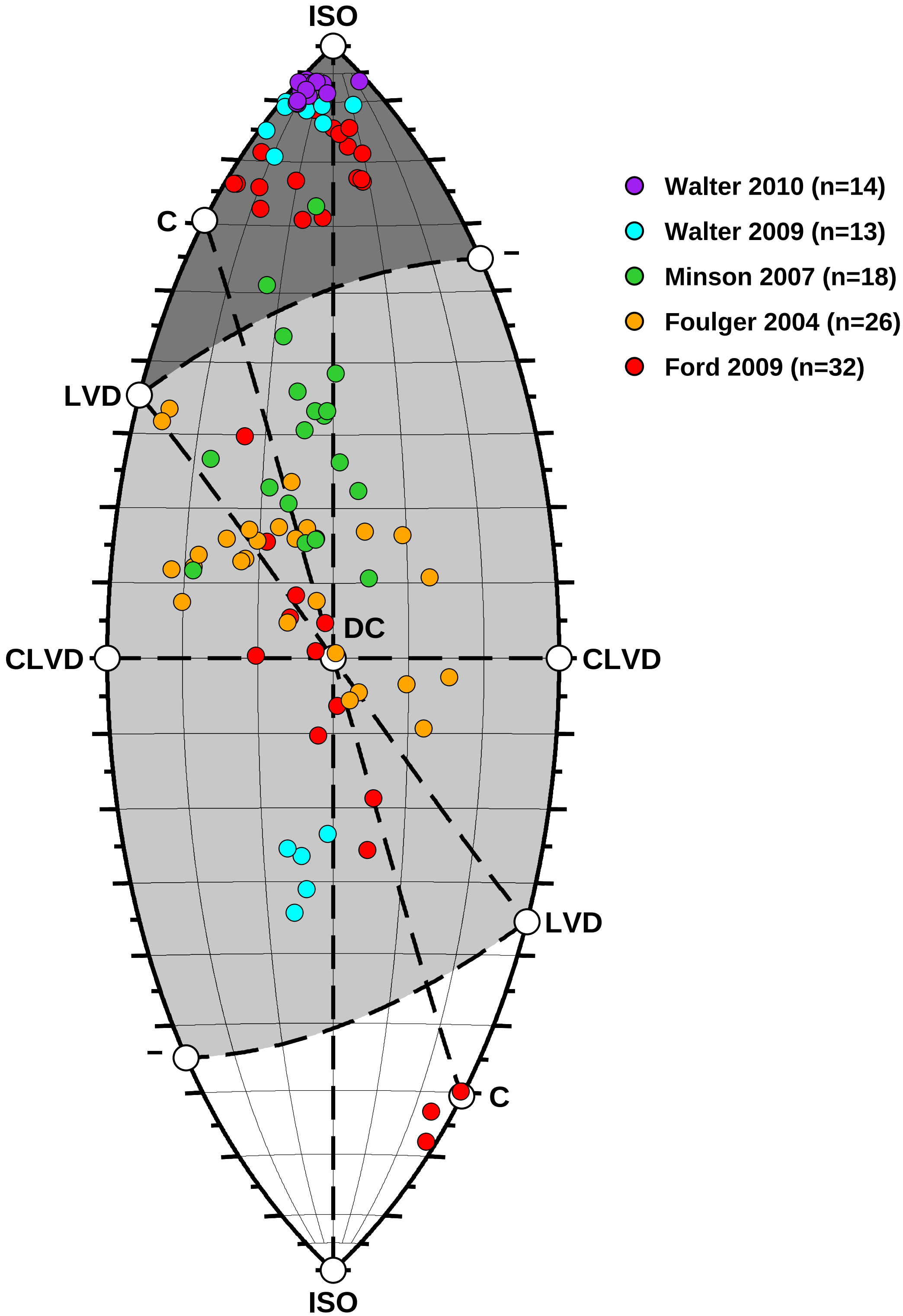

Figure 4: Five full moment tensor data sets plotted on the fundamental lune. See 3D version in Figure 2.

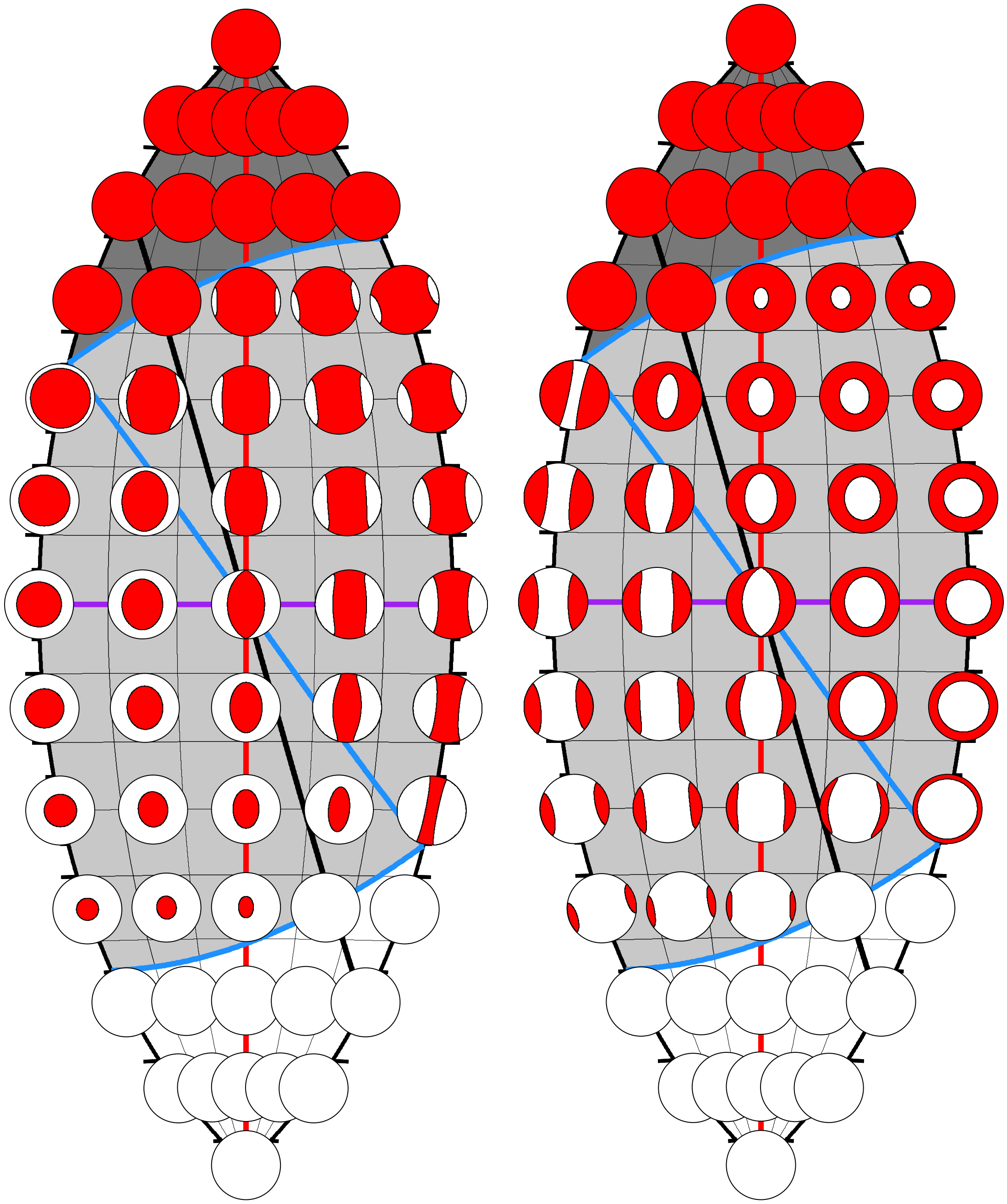

Figure 5: Beachballs on fundamental lune. All beachballs within each lune have the same basis U, which is easiest to identify in the double couple beachball at the center of the lune. Two additional examples are shown in Figure 6.

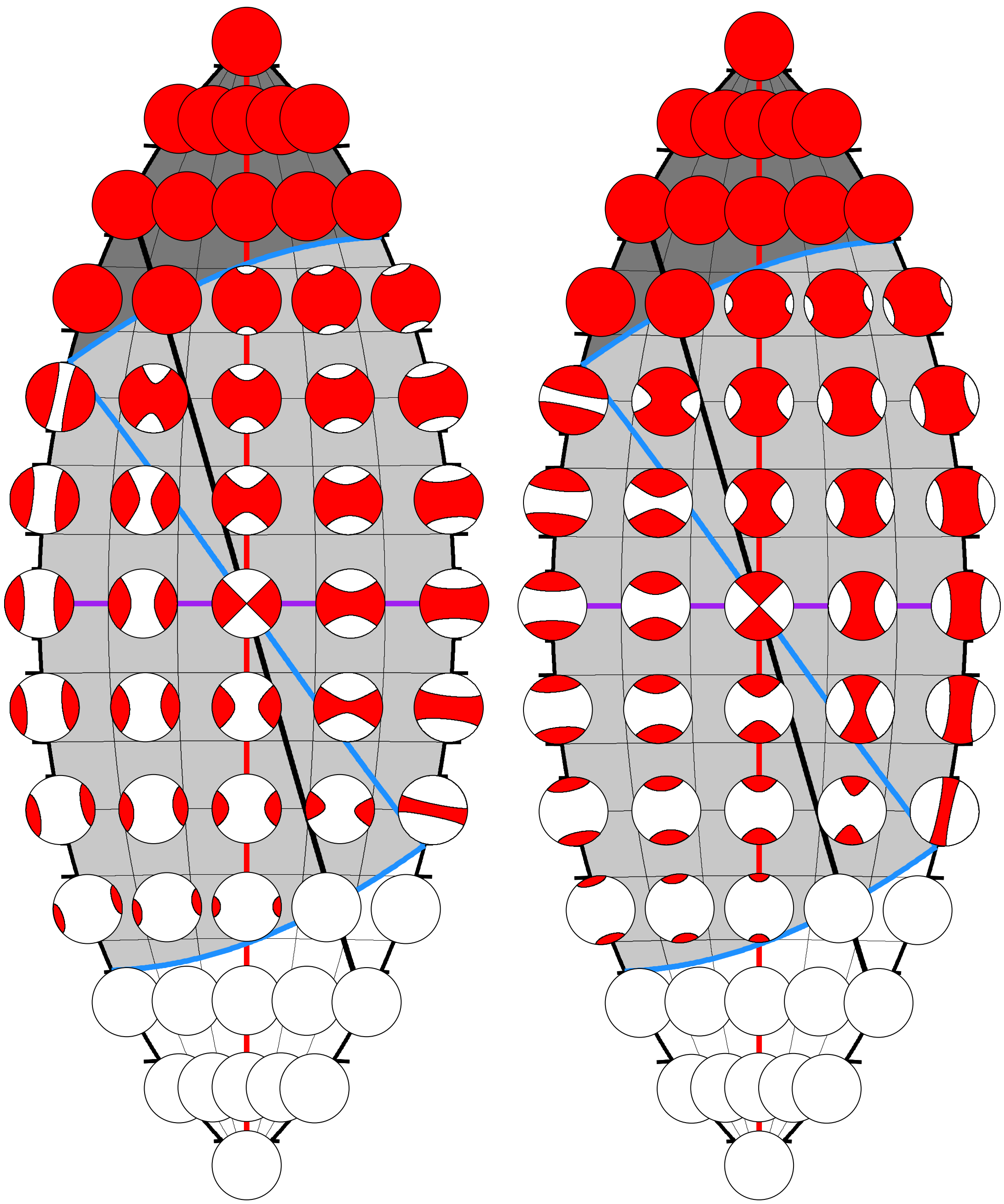

Figure 6: Same as Figure 5, but for two different reference orientations.

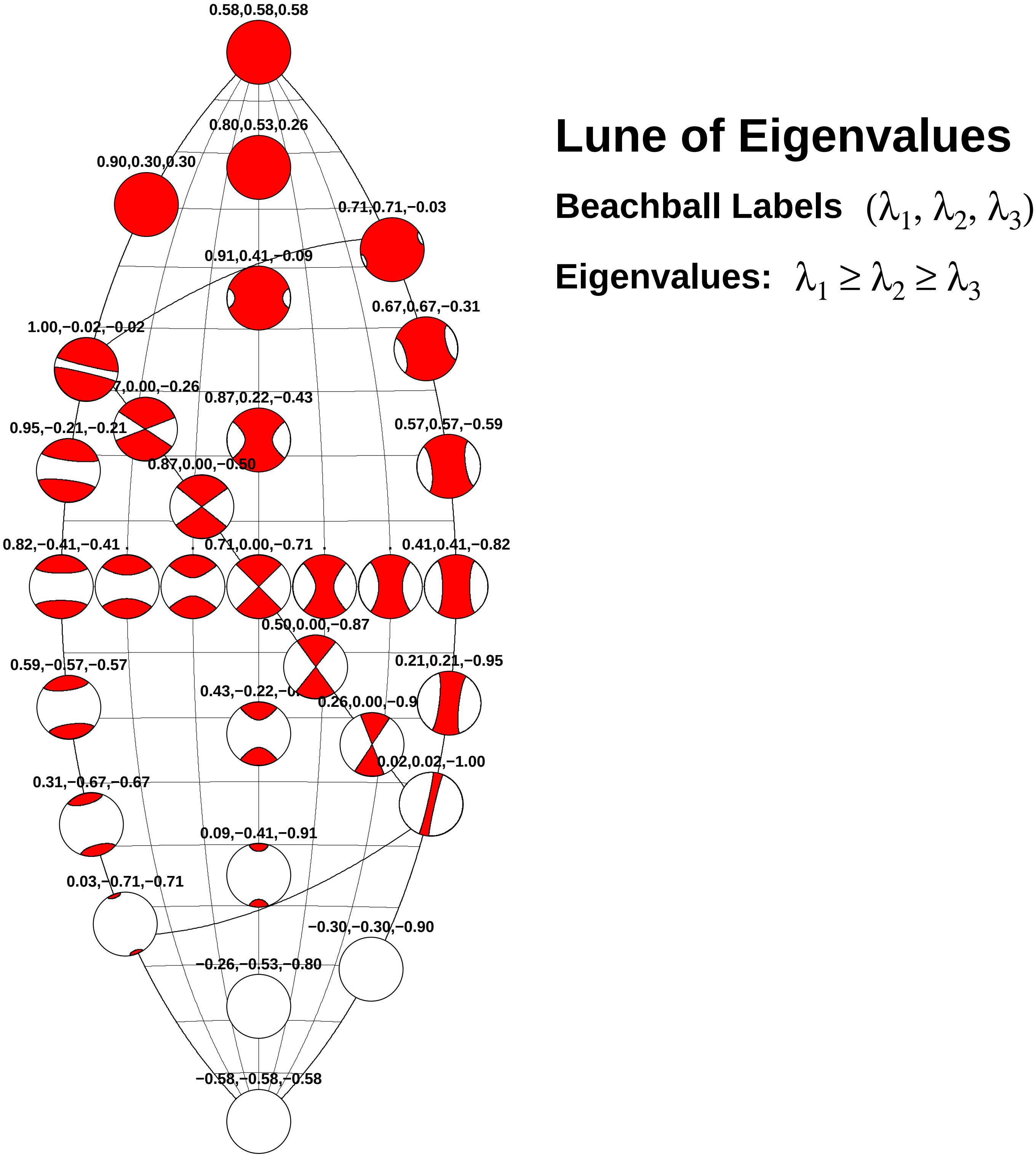

Figure 7: Reference beachballs on the lune, labeled as $(\lambda_1, \lambda_2, \lambda_3)$.

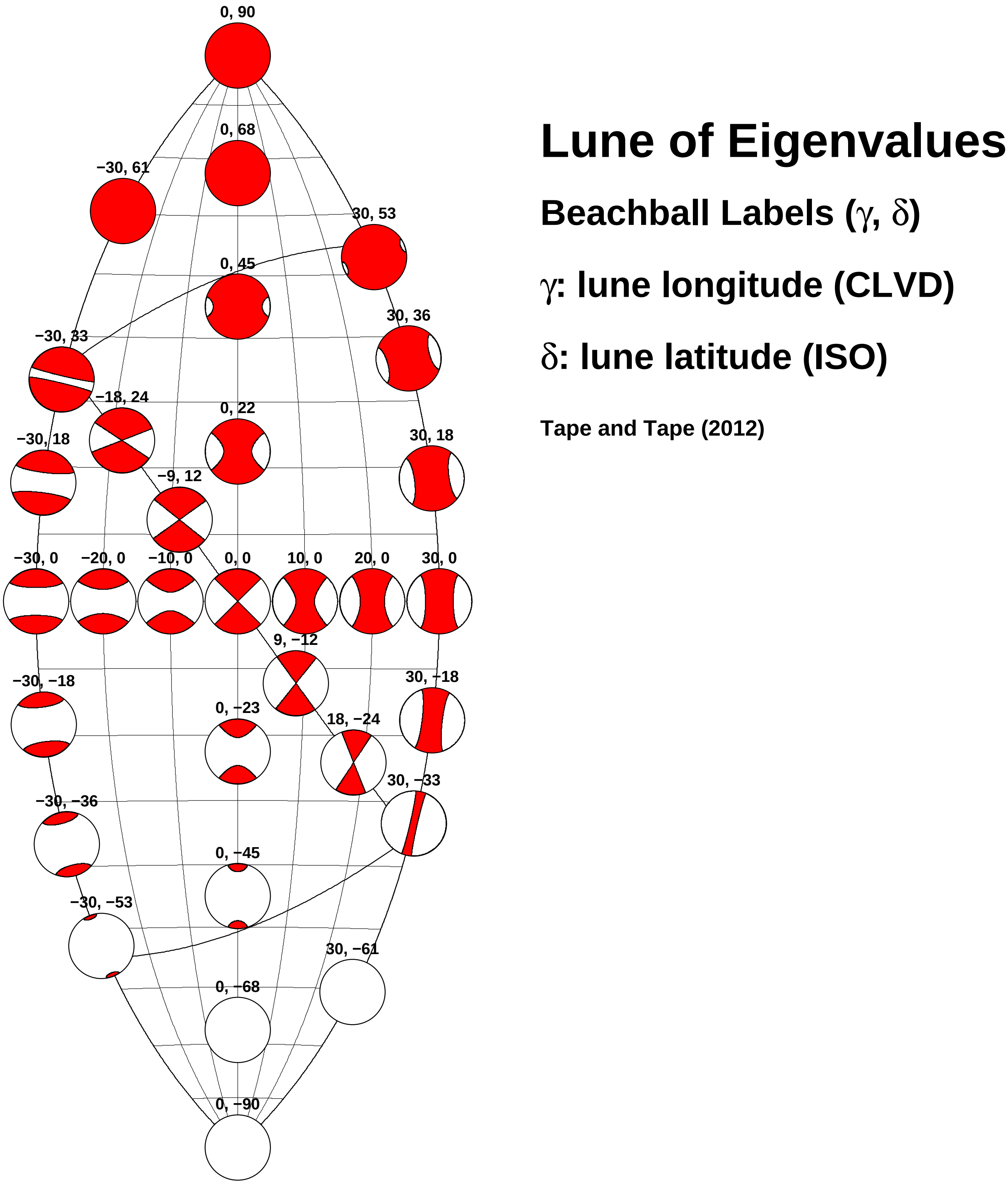

Figure 8: Reference beachballs on the lune, labeled as (γ, δ) (*Tape and Tape*, 2012). See *Alvizuri and Tape* (2016) for an application.

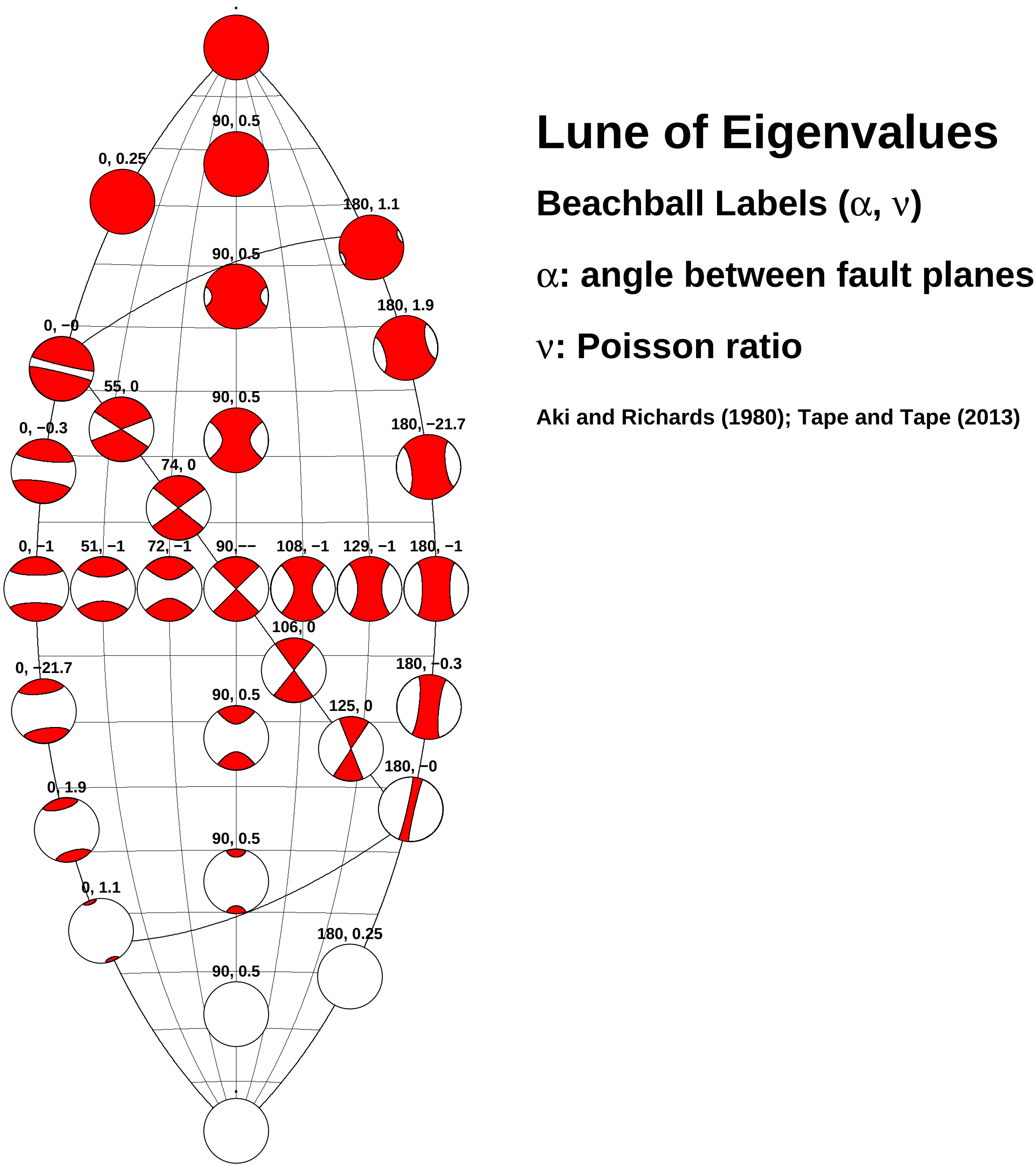

Figure 9: Reference beachballs on the lune, labeled as (α, ν) (*Aki and Richards*, 1980; *Tape and Tape*, 2013).

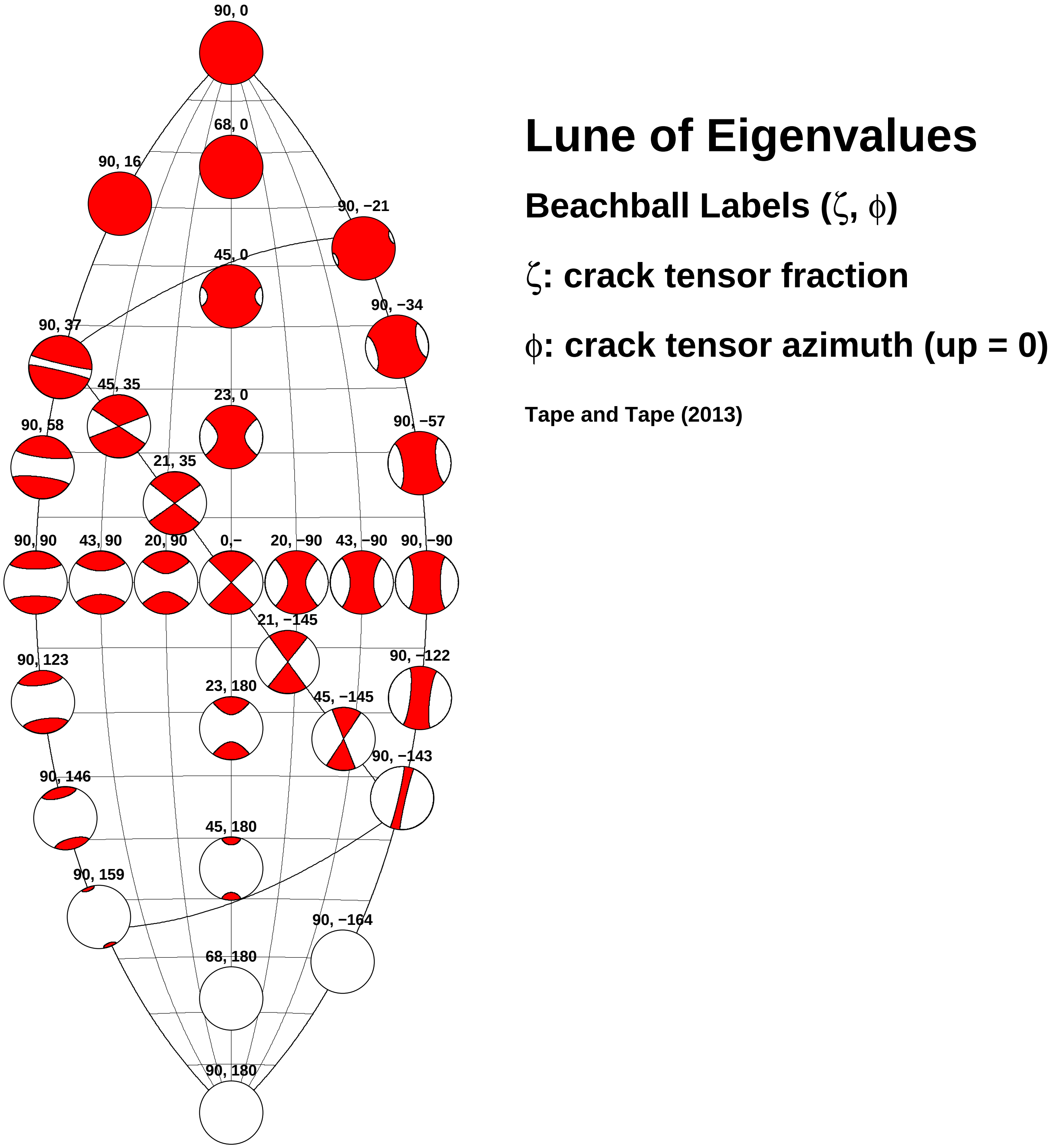

Figure 10: Reference beachballs on the lune, labeled as (ζ, ϕ) (*Tape and Tape*, 2013). See *Minson et al.* (2007) for motivation behind the crack-plus-double-couple representation.

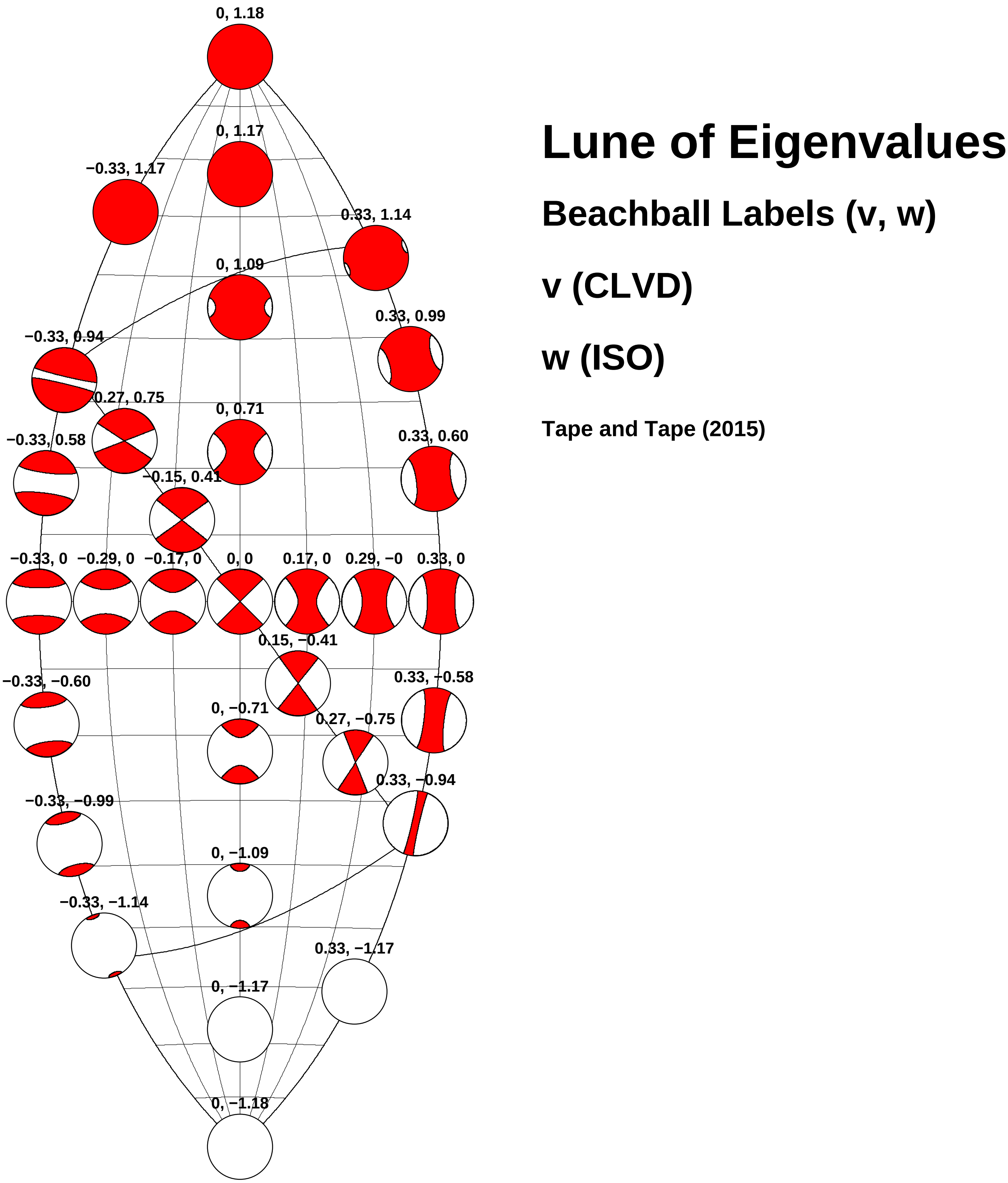

Figure 11: Reference beachballs on the lune, labeled as (v, w) (*Tape and Tape*, 2015), where $w = \frac{3\pi}{8} - u$. See *Alvizuri et al.* (2018) for an application.