Assignment 3

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Advanced Machine Learning

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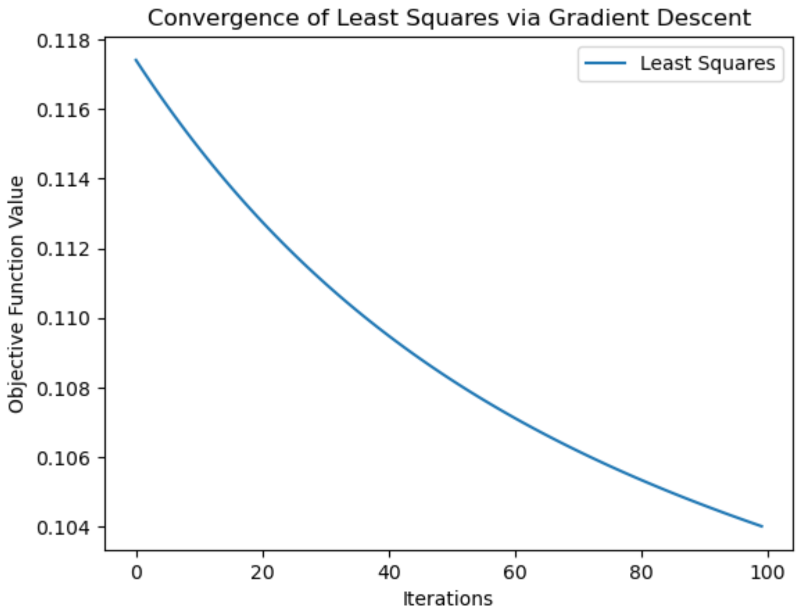
Problem 1:

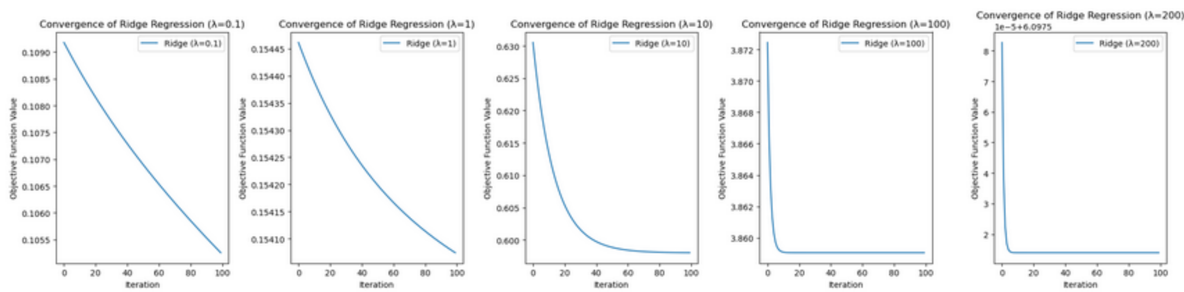
```
(512, 512, 3)
0.018112803
127.87512195121951
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).
0.009907449
51.15004878048781
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).
0.0028579345
12.787512195121952
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).
0.0010130886
5.11500487804878
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).
0.00050557224
3.196878048780488
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).
0.00034230365
2.55750243902439
```

As shown above, the higher components we get the more image quality we observe. But the idea from the SVD is to simplify our calculations with the data reduction! So, it doesn’t make any sense to take high components which is useless at some point. In my opinion, and as the framework shows the image with 50 components has 5.115 compression ratio and 0.00101 MSE which is enough to show the required data and all features. Also, visually you can tell that the image has a good quality and by that we reduced the data and simplified the calculations and got a good image quality.

Problem2: Q2 & 3:

```
The max lamb of AtA is 297.7329864000428
```





As shown above, the first image is to show that the convergence rate is linear. And the second row images to show you the faster convergence rate with higher lambda value. Since the learning rate is smaller.

Problem 3: Q2:

The Computer died here using Jupyter Notebook since my laptop isn't working properly. But I tried google colab and it worked and it gave me no zeros which proves that works correctly for the data completion.

Problem 2: Least Squares problem:

$\min_{B} \frac{1}{2} \|Y - AB\|_2^2$ where $A \in \mathbb{R}^{n \times p}$

1) If $P > n \rightarrow$ vanilla Sol. $(A^TA)^{-1} A^T Y$ is not applicable any more.

Sol. : Our main concern is the inversion. let's see if (A^TA) is inverted $\rightarrow$ if so, the solution stands if not, not applicable.

$(A^T A)$

$[P \times n][n \times P] = [P \times P]$

when: $P > n$; means more columns than rows. But the rank will ~~st~~ be # of rows at most which is n.

$\text{rank}(A) = n$

$\text{rank}(A^TA) = n$ (at max, cannot exceed)

with that being said (A^TA) with $[P \times P]$ has rank of n. means not invertible.

~~th~~ then, there's no solution for $(A^TA)^{-1}$, well, no solution for vanilla.

Problem 3:

Q1:

$SVD(Z) = U \Sigma V^T$

$U \in R^{n \times r}$, $\Sigma \in R^{r \times r}$, $V \in R^{d \times r}$

If:

$A = U \Sigma^{\frac{1}{2}}$, $B = V \Sigma^{\frac{1}{2}}$

According to: Monterio Method

$A B^T \stackrel{?}{=} Z$

$(U \Sigma^{\frac{1}{2}})(V \Sigma^{1/2})^T = U \Sigma V^T$ #

This is the factorization AB^T which reconstr
Z.

Now: Considering the obj func. ②:

$\min_{A,B} \frac{1}{2} \| P_\Omega(X - AB^T) \|_F^2$

$= \min_{A,B} \frac{1}{2} \| P_\Omega(X) - P_\Omega(AB^T) \|_F^2$

$= \min_{Z} \frac{1}{2} \| P_\Omega(X) - P_\Omega(Z) \|_F^2$

where $Z = AB^T$ (Factorization low rank property)

and the second term is also the same which is regularization term. #.