

Project of L^AT_EX
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Mohammadamin Raisi
Student Number :970087192
Unit : Tehran Shomal - Shahriar

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Solution :

Present State	Next State	
	0	1
$[q_0]$	$[q_0, q_3]$	$[q_0, q_1]$
$[q_0, q_3]$	$[q_0, q_3, q_4]$	$[q_0, q_1]$
$[q_0, q_3, q_4]$	$[q_0, q_3, q_4]$	$[q_0, q_1, q_4]$
$[q_0, q_1, q_4]$	$[q_0, q_3, q_4]$	$[q_0, q_1, q_4]$
$[q_0, q_1]$	$[q_0, q_3]$	$[q_0, q_1]$

For simplification, let us replace $[q_0]$ by A, $[q_0, q_3,]$ by B, $[q_0, q_3, q_4]$ by C, $[q_0, q_1, q_4]$ by D, and $[q_0, q_1]$ by E. Here, A is the initial state, and C and D are the final states as they contain the state q_0 . The simplified DFA is

Present State	Next State	
	0	1
A	B	E
B	C	E
C	C	D
D	C	D
E	B	E

6. Convert the following NFA to an equivalent DFA.

Solution :

Σ		
States	0	1
q_0	q_0	q_0, q_1
q_1	q_2	q_2
q_0	-	q_1

($[q_0]$ is the initial state and $[q_1]$ is the final state)

Solution: Conversion is done in the following ways:

Σ		
States	0	1
$[q_0]$	$[q_0]$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_2]$	$[q_0, q_1, q_2]$
$[q_0, q_1, q_2]$	$[q_0, q_2]$	$[q_0, q_1, q_2]$
$[q_0, q_2]$	$[q_0]$	$[q_0, q_1, q_2]$

Rename $[q_0]$ as A, $[q_0, q_1]$ as B, $[q_0, q_1, q_2]$ as C, and $[q_0, q_2]$ as D. The beginning state is A, and final states are B and C.

Σ		
States	0	1
A	A	B
B	D	C
C	D	C
D	A	C

7. Convert the following NFA to an equivalent DFA. [UPTU 2005]

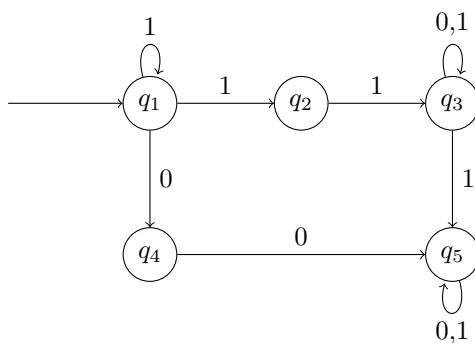
Σ		
States	0	1
p	{q,s}	{q}
q	{e}	{q,r}
r	{s}	{p}
s	\emptyset	{p}

where p is the initial state and q and s are the final states. **Solution:**

Σ		
States	0	1
{p}	{q,s}	{q}
{q}	{r}	{q,r}
{r}	{s}	{p}
{s}	\emptyset	{p}
{q,r}	{r,s}	{p,q,r}
{r,s}	{s}	{p}
{p,q,r}	{q,r,s}	{p,q,r}
{q,r,s}	{r,s}	{p,q,r}
{q,s}	{r}	{p,q,r}
$\{\emptyset\}$	$\{\emptyset\}$	$\{\emptyset\}$

Here {p} is the beginning state and {q}, {s}, {q, r}, {r, s}, {p, q, r}, {q, r, s}, and {q, s} are the final states. \emptyset is the dead state.

8. Construct a DFA equivalent to the following NFA given in the following figure. [UPTU 2004]



Solution: The tabular representation of the NFA is

Present State	Next State	
	0	1
q_0	q_3	$\{q_0, q_1\}$
q_1	\emptyset	q_2
q_2	q_2	$\{q_2, q_4\}$
q_3	q_4	\emptyset
q_4	q_4	q_4

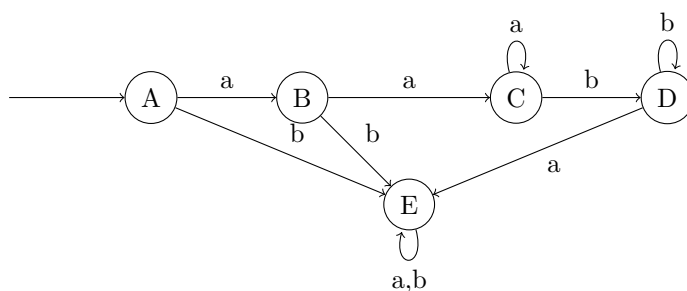
(q_0 is the initial state and q_4 is the final state)

The corresponding DFA is

Σ		
	0	1
$\{q_0\}$	$\{q_3\}$	$\{q_0, q_1\}$
$\{q_3\}$	$\{q_4\}$	$\{\emptyset\}$
$\{q_4\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_1\}$	$\{q_2\}$	$\{q_2, q_4\}$
$\{q_2\}$	$\{q_2\}$	$\{q_2, q_4\}$
$\{q_2, q_4\}$	$\{q_2, q_4\}$	$\{q_2, q_4\}$
$\{\emptyset\}$	$\{\emptyset\}$	$\{\emptyset\}$

Here $\{q_0\}$ is the beginning state, and $\{q_4\}$, and $\{q_0, q_1\}$ are the final states.
(Draw a transitional diagram to complete the answer.)

9. Find the minimal DFAs for the language $L = \{a^n b^m, n \geq 2, m \geq 1\}$ **Solution:** All 'a' will appear before 'b'. There is atleast 2 'a' and 1 'b'. The DFA is the following.



10. Design a DFA for the language $L = \{0^m 1^n, m \geq 0, n \geq 1\}$ [JNTU 2007]

Solution: All '0's will appear before '1'. There is at least one '1', but the number of '0's may be zero. The DFA is shown in Fig. 3.58.

11. Construct a DFA which accepts the set of all binary strings that, interpreted as the binary representation of an unsigned decimal integer, is divisible by 5.

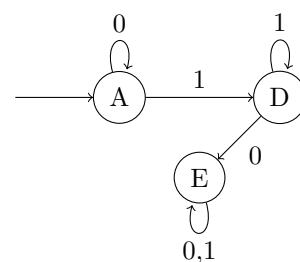


Fig. 3.58

[WBUT 2008]

Solution: For Mod 5, the remainders are 0, 1, 2, 3, and 4. We can assign the states as q_0, q_1, q_2, q_3 , and q_4 . For any binary string, if we add a bit at LSB, then the previous value becomes doubled. (Let the string be 101. The decimal value is 5. If we add another 1 at LSB, the string becomes 1011. The decimal value of the previous 101 becomes 10.) In general, we can write that 'n' becomes $2n + b$, where n is the previous number and b is the added bit.

$$(2n + b) \bmod 5 = 2n \bmod 5 + b \bmod 5$$

As b is either 0 or 1, $b \bmod 5 = b$. $2n \bmod 5$ is any one of 0, 1, 2, 3, or 4, i.e., $2 \times \text{X (state number)} + a$. For this machine, the input alphabets are 0 and 1.

$$\begin{aligned} \delta(q_0, 0) &\rightarrow 2 \times 0 + 0 = 0 \text{ means } q_0 \\ \delta(q_0, 1) &\rightarrow 2 \times 0 + 1 = 1 \text{ means } q_1 \\ \delta(q_1, 0) &\rightarrow 2 \times 1 + 0 = 2 \text{ means } q_2 \\ \delta(q_1, 1) &\rightarrow 2 \times 1 + 1 = 3 \text{ means } q_3 \\ \delta(q_2, 0) &\rightarrow 2 \times 2 + 0 = 4 \text{ means } q_4 \\ \delta(q_2, 1) &\rightarrow 2 \times 2 + 1 = 5\%5 = 0 \text{ means } q_0 \\ \delta(q_3, 0) &\rightarrow 2 \times 3 + 0 = 6\%5 = 1 \text{ means } q_1 \\ \delta(q_3, 1) &\rightarrow 2 \times 3 + 1 = 7\%5 = 2 \text{ means } q_2 \\ \delta(q_4, 0) &\rightarrow 2 \times 4 + 0 = 8\%5 = 3 \text{ means } q_3 \\ \delta(q_4, 1) &\rightarrow 2 \times 4 + 1 = 9\%5 = 4 \text{ means } q_4 \end{aligned}$$