

# **Combinational Circuit Design**

**Dr. Bassam Jamil**

**Adopted from slides of the  
textbook**

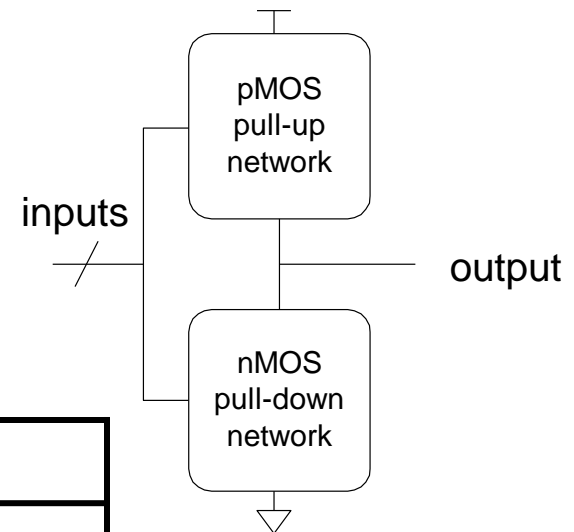
# Topics

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- ❑ CMOS Gate
  - Complementary N and P networks
  - Complex Gates
  - Pass transistor and transmission gates
  - Tristates, multiplexers

# Complementary CMOS

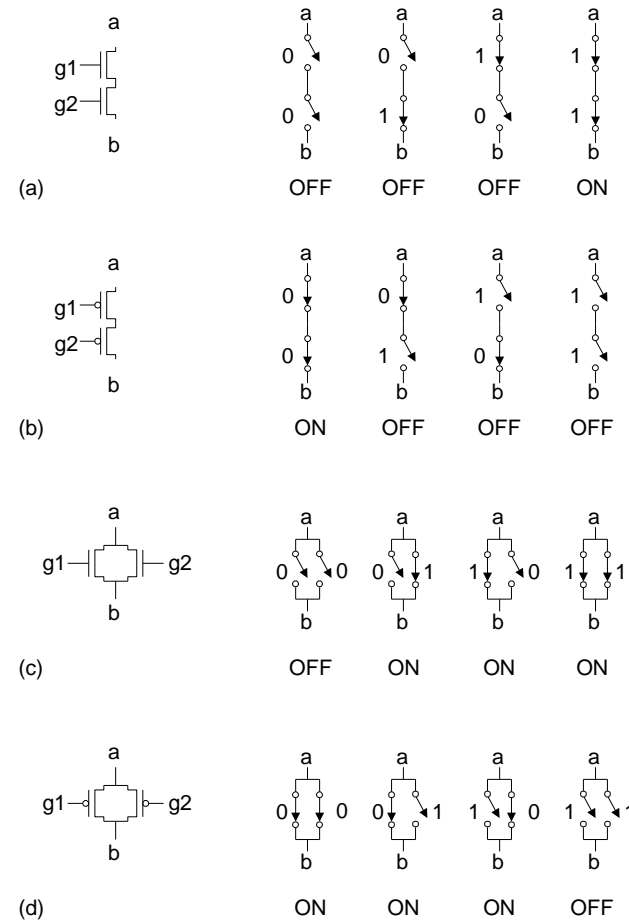
- ❑ Complementary CMOS logic gates
  - nMOS *pull-down network*
  - pMOS *pull-up network*
  - a.k.a. static CMOS



	Pull-up OFF	Pull-up ON
Pull-down OFF	Z (float)	1
Pull-down ON	0	X (crowbar)

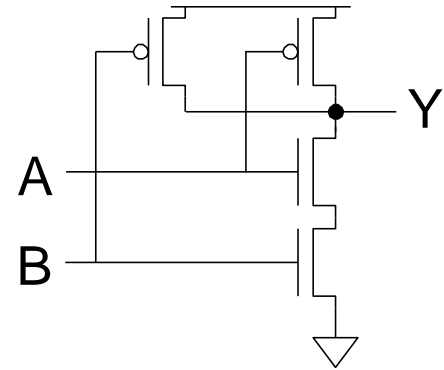
# Series and Parallel

- ❑ nMOS: 1 = ON
- ❑ pMOS: 0 = ON
- ❑ *Series*: both must be ON
- ❑ *Parallel*: either can be ON



# Conduction Complement

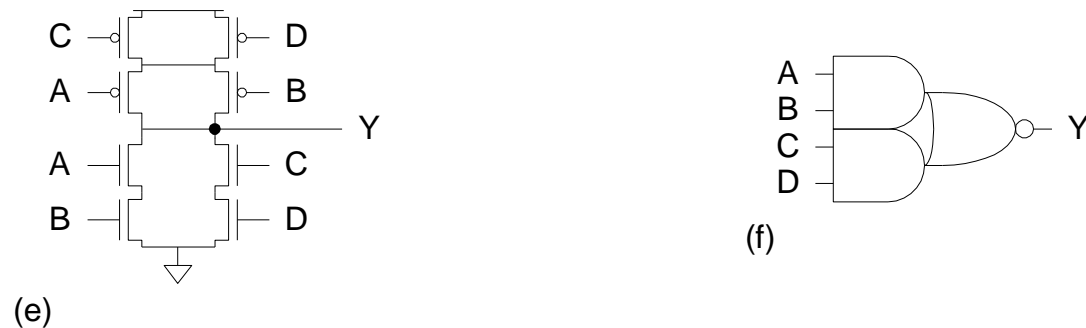
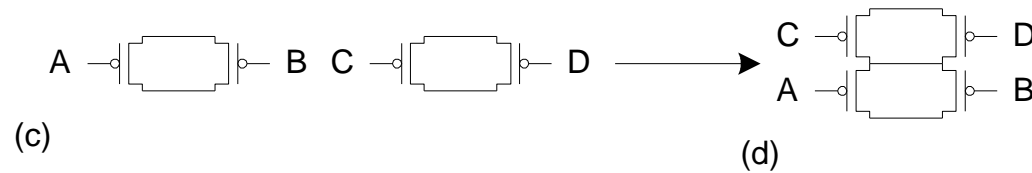
- ❑ Complementary CMOS gates always produce 0 or 1
- ❑ Ex: NAND gate
  - Series nMOS:  $Y=0$  when both inputs are 1
  - Thus  $Y=1$  when either input is 0
  - Requires parallel pMOS
- ❑ Rule of *Conduction Complements*
  - Pull-up network is complement of pull-down
  - Parallel  $\rightarrow$  series, series  $\rightarrow$  parallel



# Compound Gates

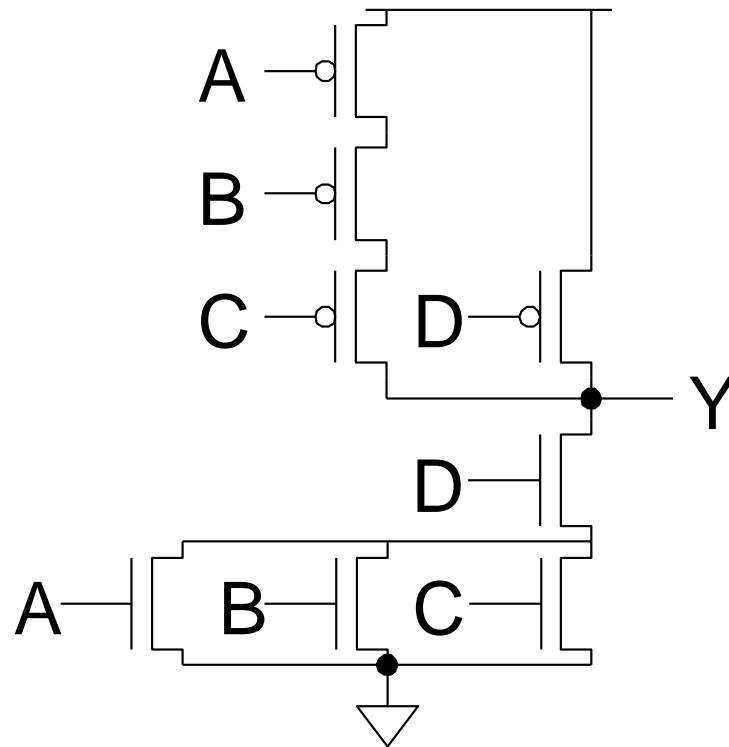
❑ *Compound gates can do any inverting function*

❑ Ex:  $Y = (AB + CD)'$



# Example: O3AI

$$\square Y = ( (A + B + C) D )'$$



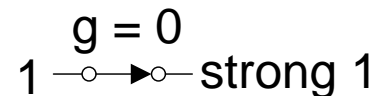
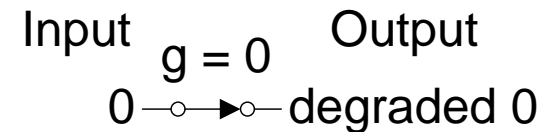
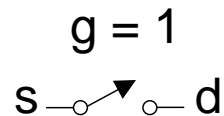
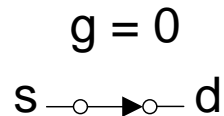
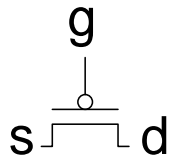
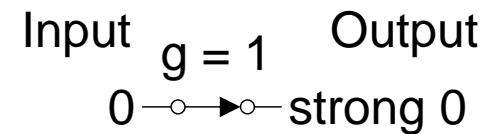
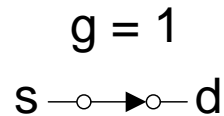
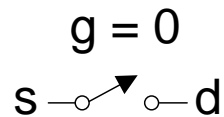
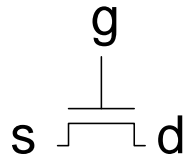
# Signal Strength

- ❑ *Strength* of signal
  - How close it approximates ideal voltage source
- ❑  $V_{DD}$  and GND rails are strongest 1 and 0
- ❑ nMOS pass strong 0
  - But degraded or weak 1
- ❑ pMOS pass strong 1
  - But degraded or weak 0
- ❑ Thus nMOS are best for pull-down network



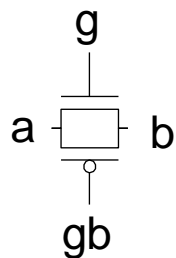
# Pass Transistors

- Transistors can be used as switches

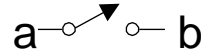


# Transmission Gates

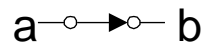
- ❑ Pass transistors produce degraded outputs
- ❑ *Transmission gates* pass both 0 and 1 well



$g = 0, gb = 1$



$g = 1, gb = 0$



Input

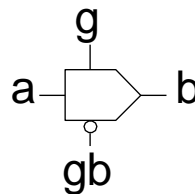
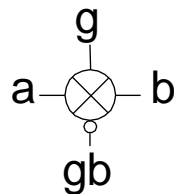
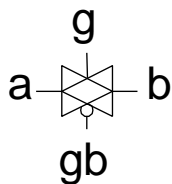
Output

$g = 1, gb = 0$

$0 \rightarrow \text{strong } 0$

$g = 1, gb = 0$

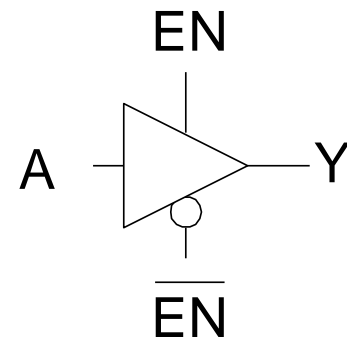
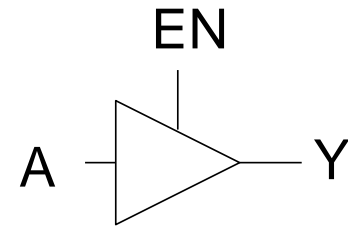
$1 \rightarrow \text{strong } 1$



# Tristates

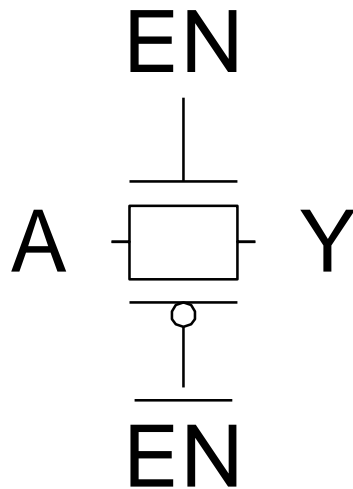
- ❑ *Tristate buffer* produces Z when not enabled

EN	A	Y
0	0	Z
0	1	Z
1	0	0
1	1	1



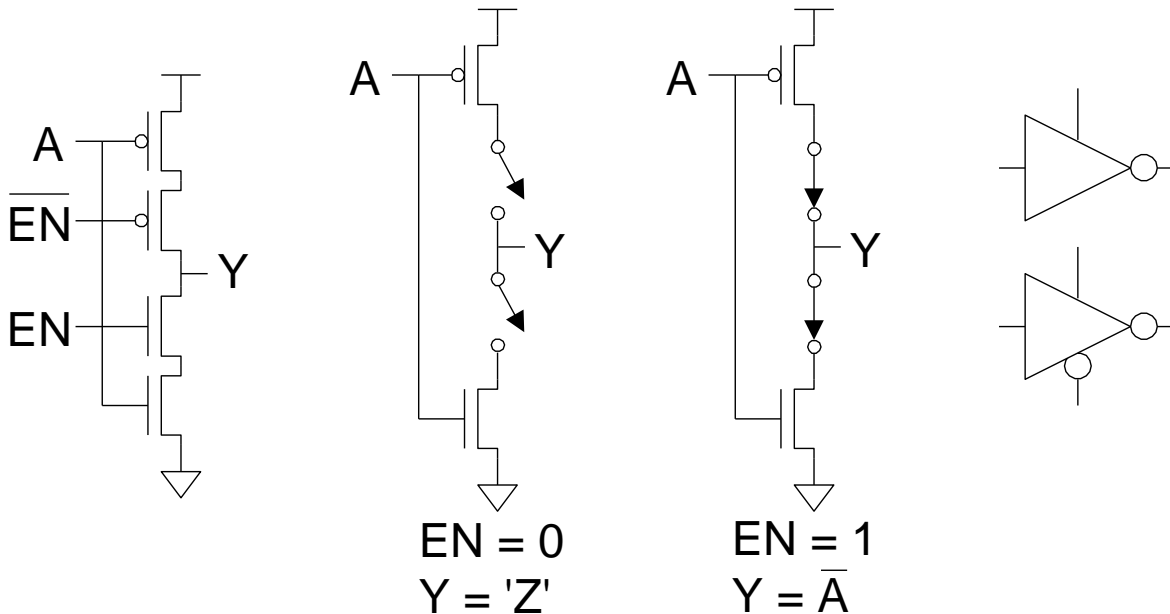
# Nonrestoring Tristate

- ❑ Transmission gate acts as tristate buffer
  - Only two transistors
  - But *nonrestoring*
    - Noise on A is passed on to Y



# Tristate Inverter

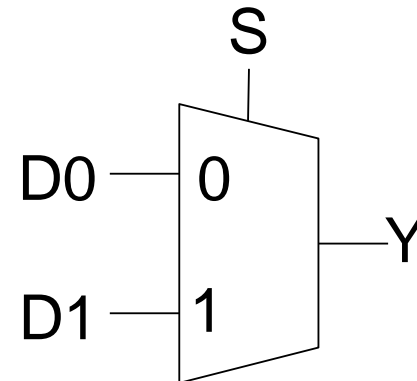
- ❑ Tristate inverter produces restored output
  - Violates conduction complement rule
  - Because we want a Z output



# Multiplexers

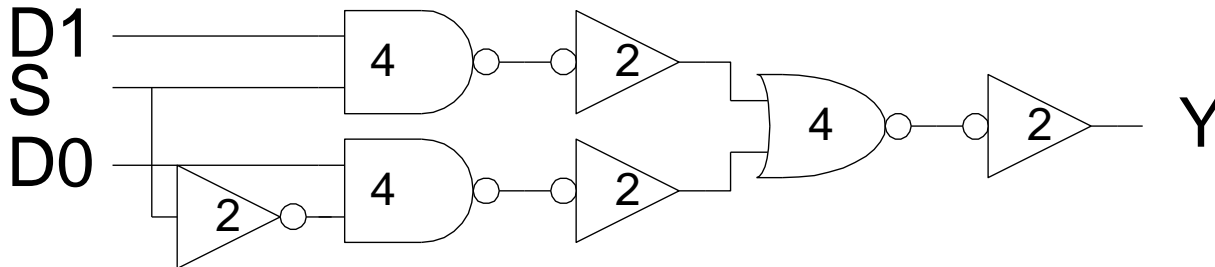
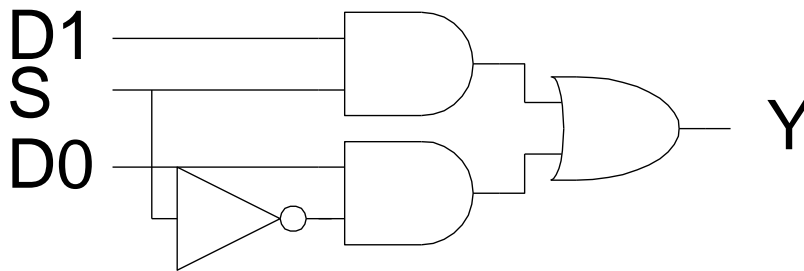
- ❑ 2:1 multiplexer chooses between two inputs

S	D1	D0	Y
0	X	0	0
0	X	1	1
1	0	X	0
1	1	X	1



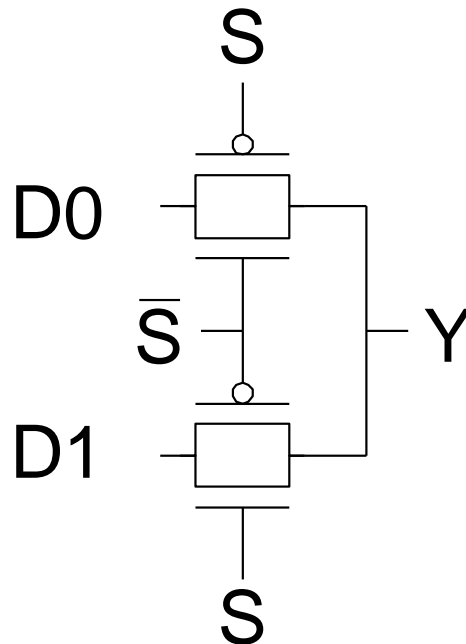
# Gate-Level Mux Design

- ❑  $Y = SD_1 + \bar{S}D_0$  (too many transistors)
- ❑ How many transistors are needed? 20



# Transmission Gate Mux

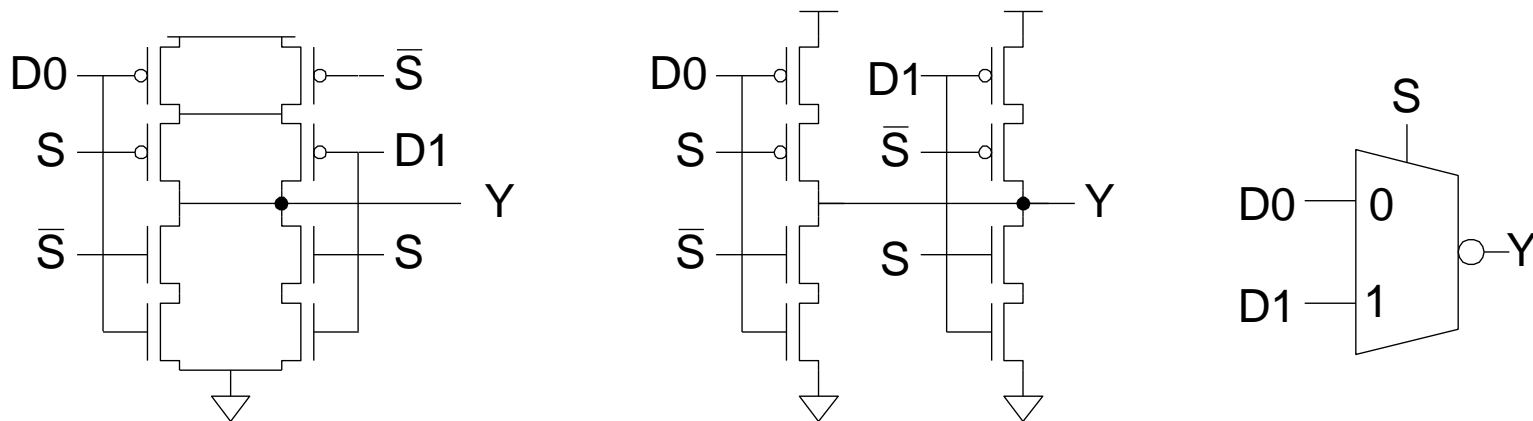
- ❑ Nonrestoring mux uses two transmission gates
  - Only 4 transistors





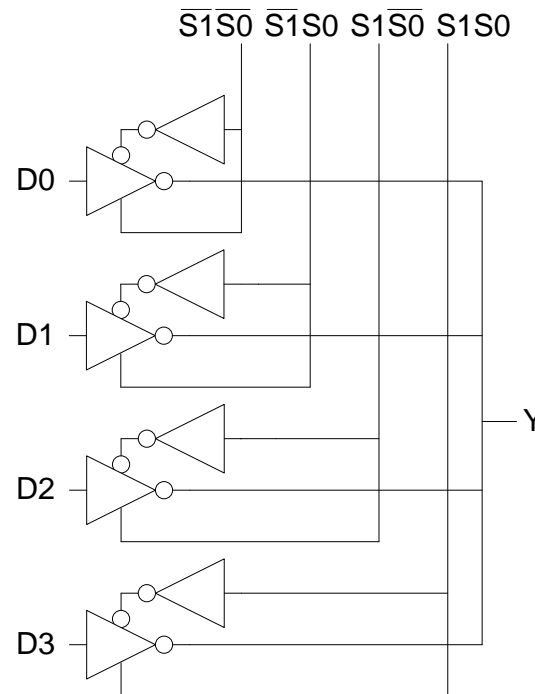
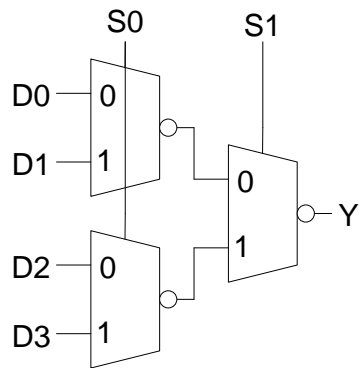
# Inverting Mux

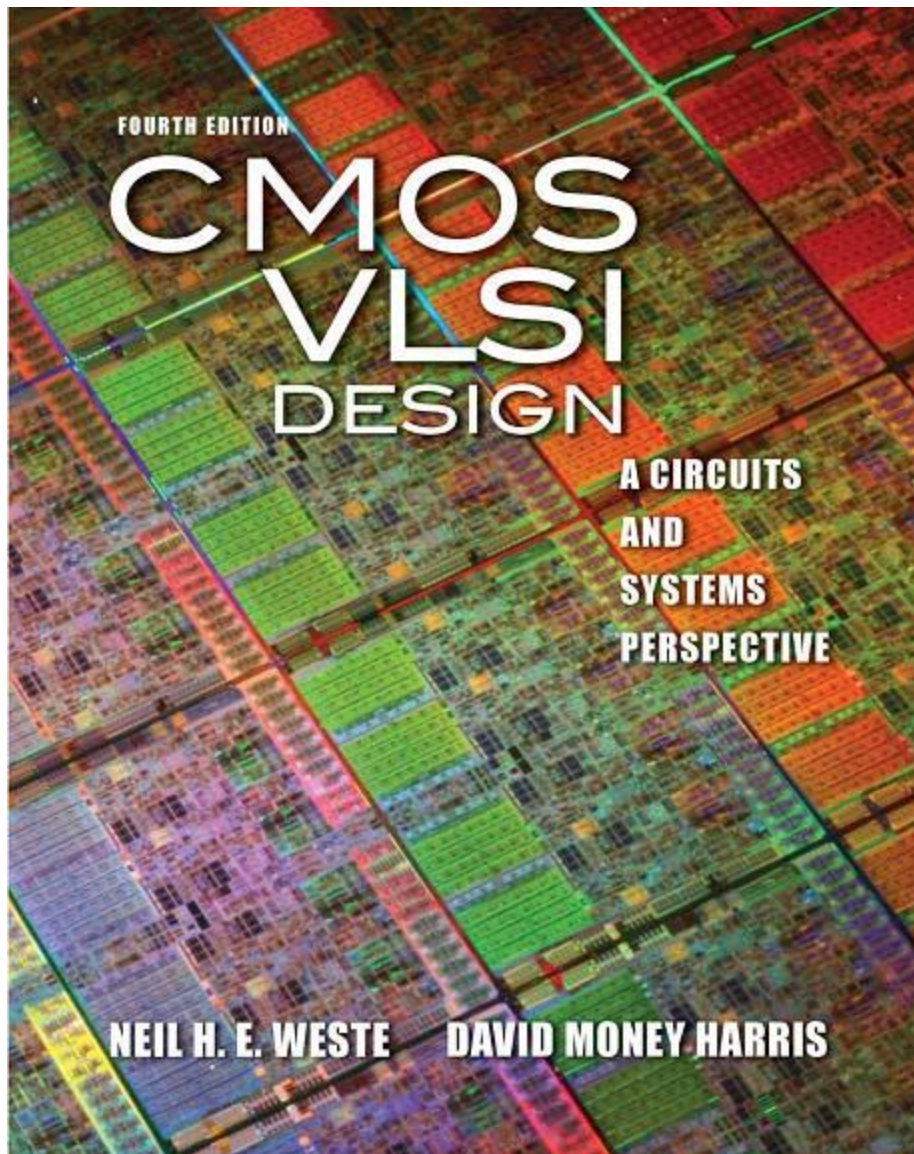
- ❑ Inverting multiplexer
  - Use compound AOI22
  - Or pair of tristate inverters
  - Essentially the same thing
- ❑ Noninverting multiplexer adds an inverter



# 4:1 Multiplexer

- ❑ 4:1 mux chooses one of 4 inputs using two selects
  - Two levels of 2:1 muxes
  - Or four tristates





# Chapter 9: Combinational Circuit Design

# Outline

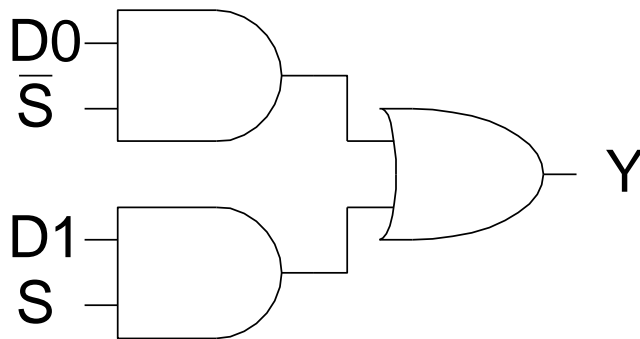
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- ☐ Bubble Pushing
- ☐ Compound Gates
- ☐ Logical Effort Example
- ☐ Input Ordering
- ☐ Asymmetric Gates
- ☐ Skewed Gates
- ☐ Best P/N ratio

# Example 1

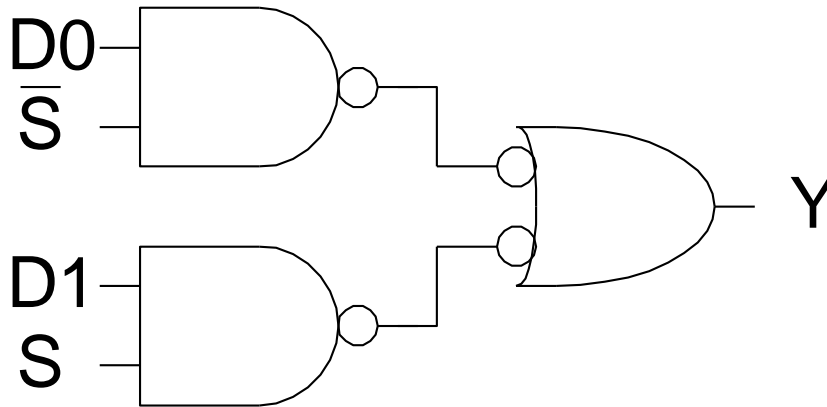
```
module mux(input  s, d0, d1,  
           output y);  
  
    assign y = s ? d1 : d0;  
endmodule
```

1) Sketch a design using AND, OR, and NOT gates.



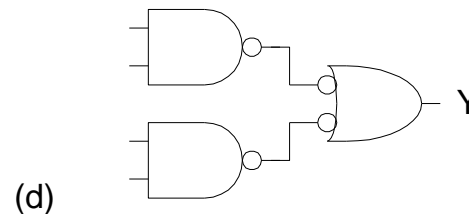
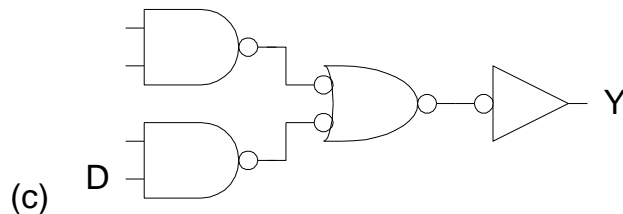
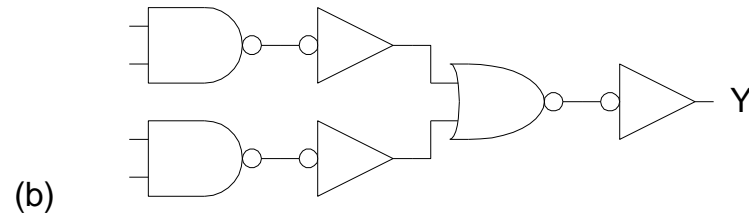
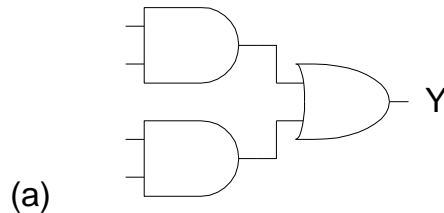
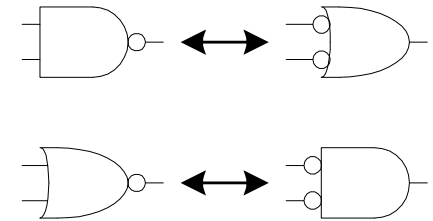
# Example 2

2) Sketch a design using NAND, NOR, and NOT gates.  
Assume  $\sim S$  is available.



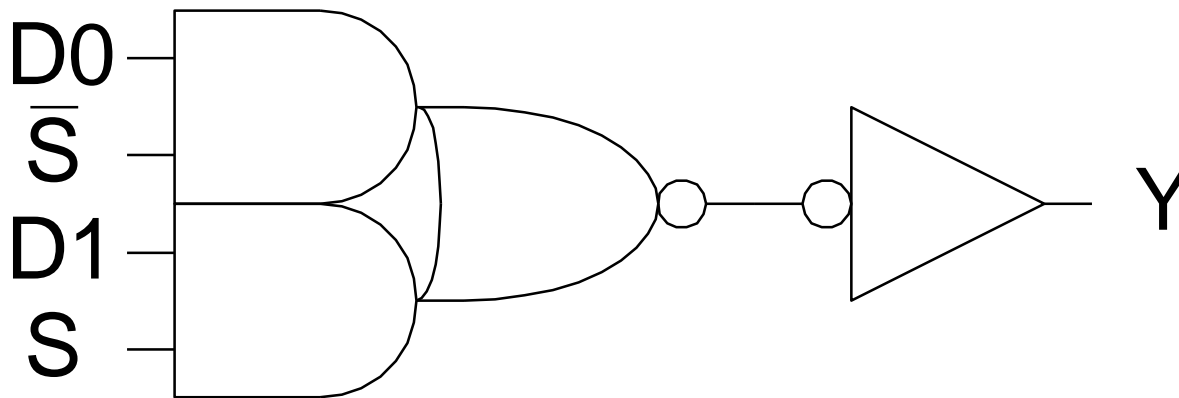
# Bubble Pushing

- ❑ Start with network of AND / OR gates
- ❑ Convert to NAND / NOR + inverters
- ❑ Push bubbles around to simplify logic
  - Remember DeMorgan's Law



# Example 3

3) Sketch a design using one compound gate and one NOT gate. Assume  $\sim S$  is available.



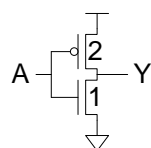
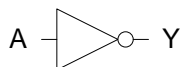


# Compound Gates

## □ Logical Effort of compound gates

unit inverter

$$Y = \overline{A}$$

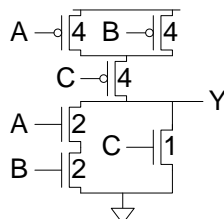


$$g_A = 3/3$$

$$p = 3/3$$

AOI21

$$Y = \overline{A \square B + C}$$



$$g_A = 6/3$$

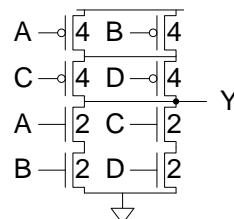
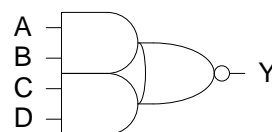
$$g_B = 6/3$$

$$g_C = 5/3$$

$$p = 7/3$$

AOI22

$$Y = \overline{A \square B + C \square D}$$



$$g_A = 6/3$$

$$g_B = 6/3$$

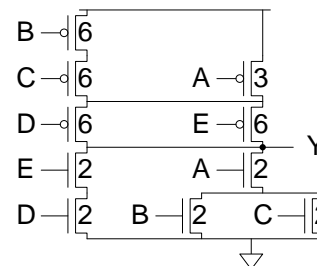
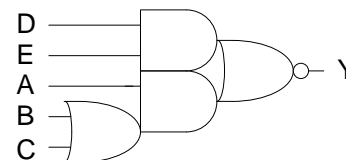
$$g_C = 6/3$$

$$g_D = 6/3$$

$$p = 12/3$$

Complex AOI

$$Y = \overline{A \square (B + C) + D \square E}$$



$$g_A = 5/3$$

$$g_B = 8/3$$

$$g_C = 8/3$$

$$g_D = 8/3$$

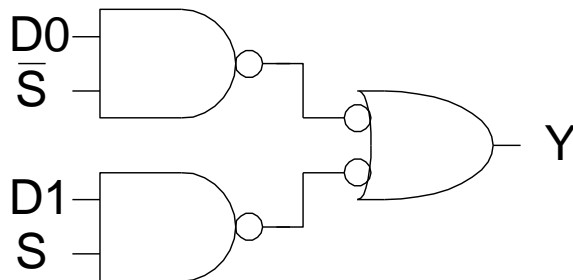
$$g_E = 8/3$$

$$p = 16/3$$

# Example 4

- ❑ The multiplexer has a maximum input capacitance of 16 units on each input. It must drive a load of 160 units. Estimate the delay of the two designs.

$$H = 160 / 16 = 10 \quad B = 1 \quad N = 2$$



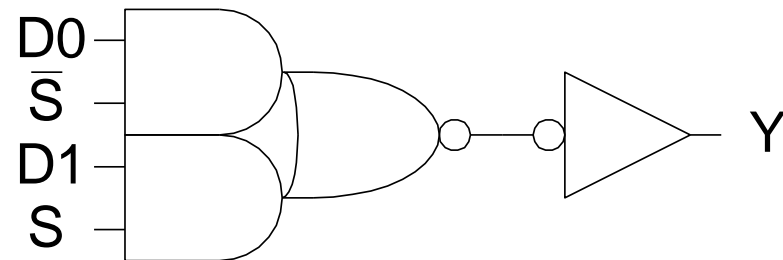
$$P = 2 + 2 = 4$$

$$G = (4/3) \times (4/3) = 16/9$$

$$F = GBH = 160/9$$

$$\hat{f} = \sqrt[3]{F} = 4.2$$

$$D = N\hat{f} + P = 12.4\tau$$



$$P = 4 + 1 = 5$$

$$G = (6/3) \times (1) = 2$$

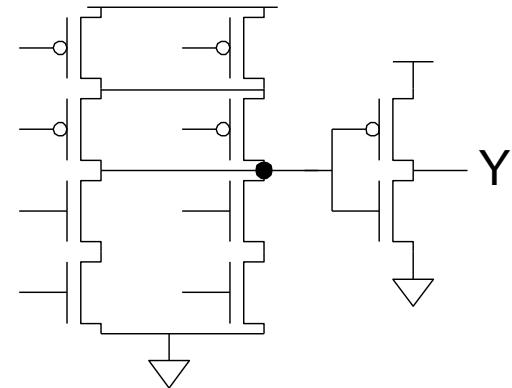
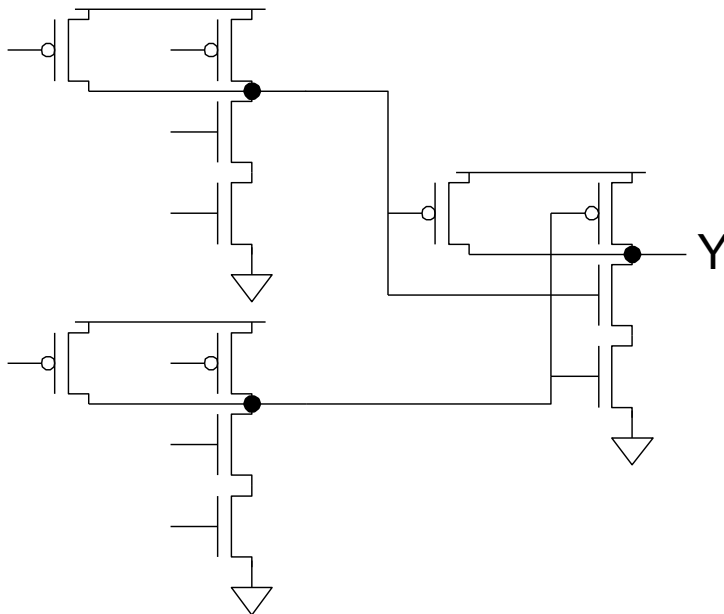
$$F = GBH = 20$$

$$\hat{f} = \sqrt[3]{F} = 4.5$$

$$D = N\hat{f} + P = 14\tau$$

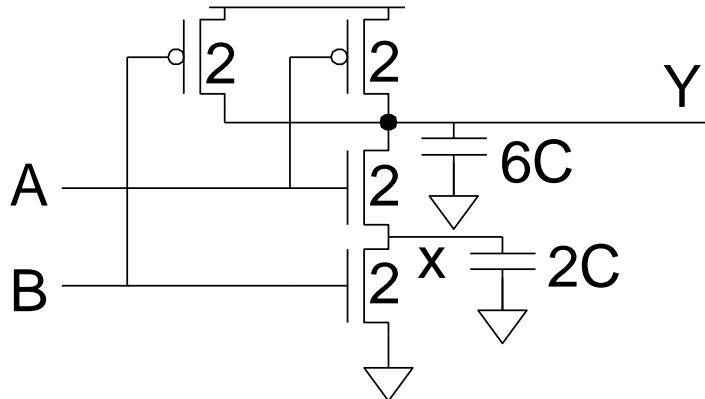
# Example 5

- ❑ Annotate your designs with transistor sizes that achieve this delay.



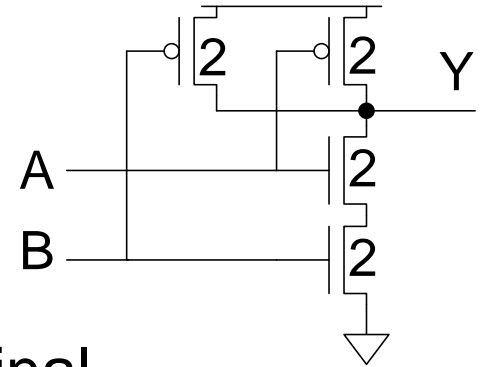
# Input Order

- ❑ Our parasitic delay model was too simple
  - Calculate parasitic delay for Y falling
    - If A arrives latest?
    - If B arrives latest?



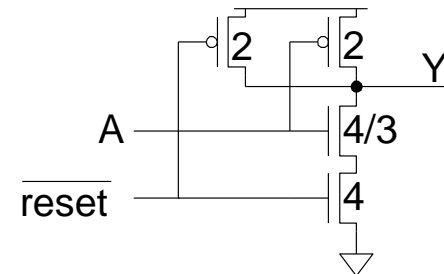
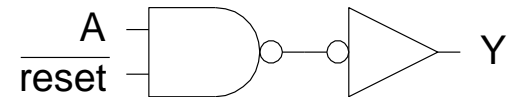
# Inner & Outer Inputs

- ❑ *Inner* input is closest to output (A)
- ❑ *Outer* input is closest to rail (B)
- ❑ If input arrival time is known
  - Connect latest input to inner terminal



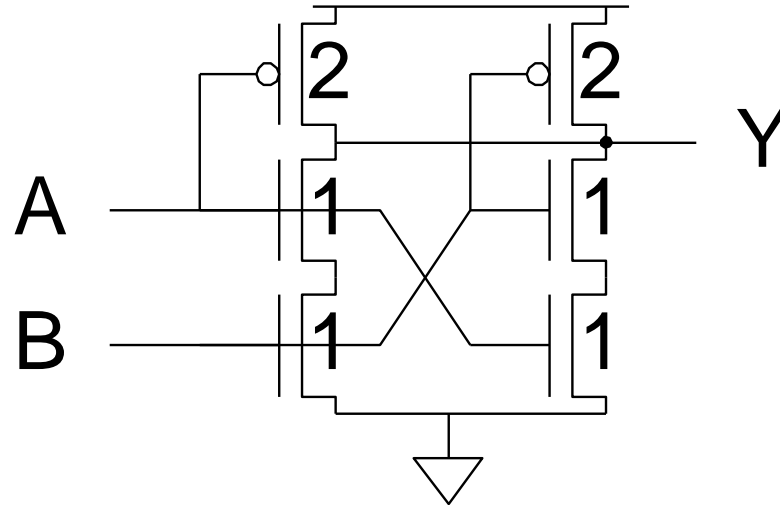
# Asymmetric Gates

- ❑ Asymmetric gates favor one input over another
- ❑ Ex: suppose input A of a NAND gate is most critical
  - Use smaller transistor on A (less capacitance)
  - Boost size of noncritical input
  - So total resistance is same
- ❑  $g_A = 10/9$
- ❑  $g_B = 2$
- ❑  $g_{\text{total}} = g_A + g_B = 28/9$
- ❑ Asymmetric gate approaches  $g = 1$  on critical input
- ❑ But total logical effort goes up



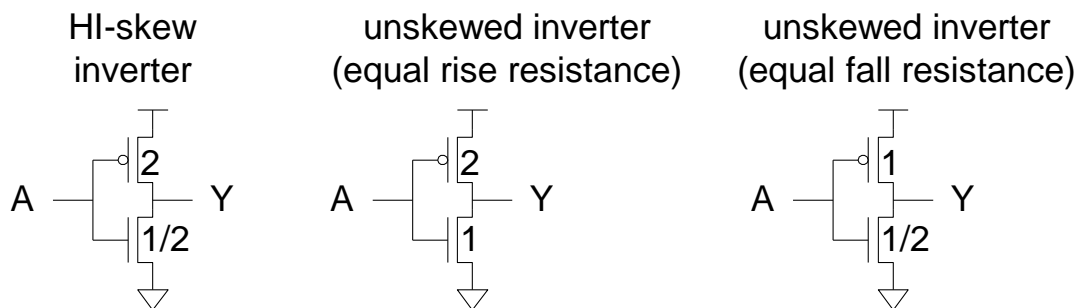
# Symmetric Gates

- Inputs can be made perfectly symmetric



# Skewed Gates

- ❑ Skewed gates favor one edge over another
- ❑ Ex: suppose rising output of inverter is most critical
  - Downsize noncritical nMOS transistor



- ❑ Calculate logical effort by comparing to unskewed inverter with same effective resistance on that edge.
  - $g_u = 2.5 / 3 = 5/6$
  - $g_d = 2.5 / 1.5 = 5/3$



# HI- and LO-Skew

- ❑ Def: Logical effort of a skewed gate for a particular transition is the ratio of the input capacitance of that gate to the input capacitance of an unskewed inverter delivering the same output current for the same transition.
- ❑ Skewed gates reduce size of noncritical transistors
  - HI-skew gates favor rising output (small nMOS)
  - LO-skew gates favor falling output (small pMOS)
- ❑ Logical effort is smaller for favored direction
- ❑ But larger for the other direction

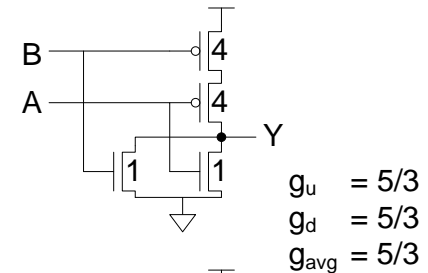
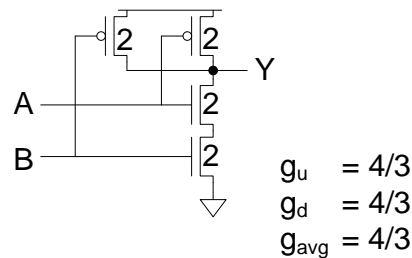
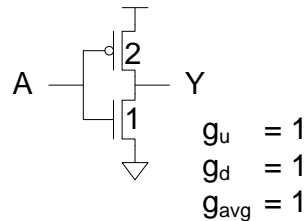
# Catalog of Skewed Gates

Inverter

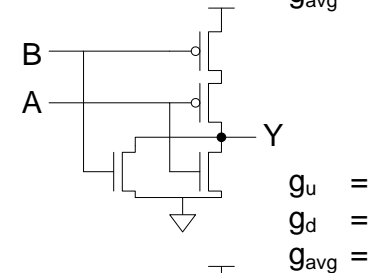
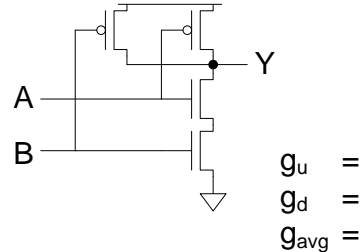
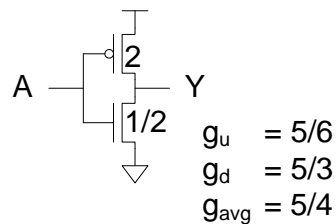
NAND2

NOR2

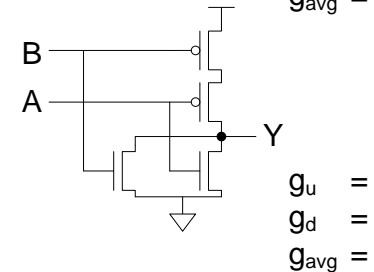
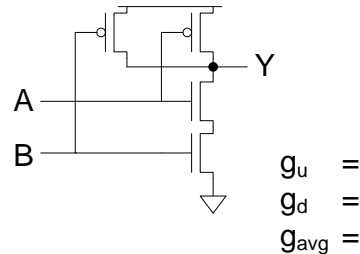
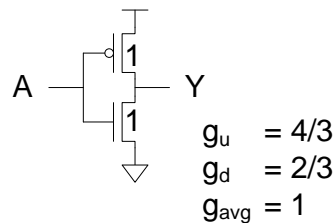
unskewed



HI-skew

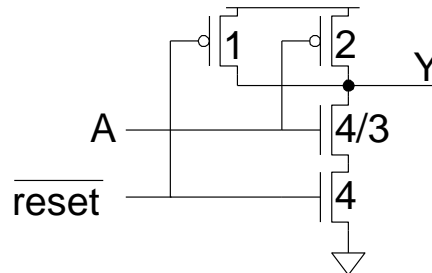
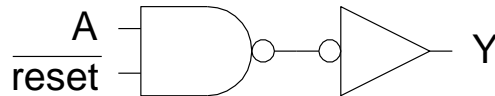


LO-skew



# Asymmetric Skew

- ❑ Combine asymmetric and skewed gates
  - Downsize noncritical transistor on unimportant input
  - Reduces parasitic delay for critical input



# Best P/N Ratio

- ❑ We have selected P/N ratio for unit rise and fall resistance ( $\mu = 2$ -3 for an inverter).
- ❑ Alternative: choose ratio for least average delay
- ❑ Ex: inverter

- Delay driving identical inverter

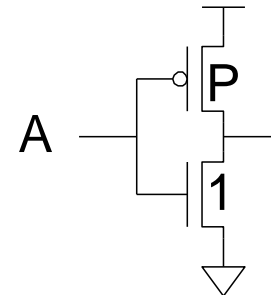
- $t_{pdf} = (P+1)$

- $t_{pdr} = (P+1)(\mu/P)$

- $t_{pd} = (P+1)(1+\mu/P)/2 = (P + 1 + \mu + \mu/P)/2$

- $dt_{pd} / dP = (1 - \mu/P^2)/2 = 0$

- Least delay for  $P = \sqrt{\mu}$

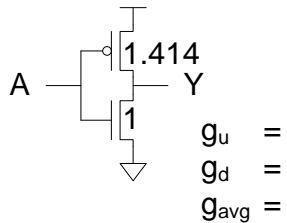


# P/N Ratios

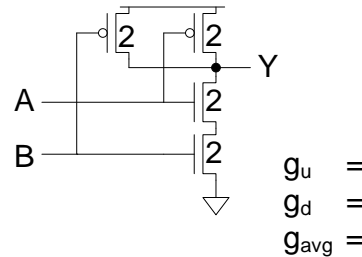
- ❑ In general, best P/N ratio is sqrt of equal delay ratio.
  - Only improves average delay slightly for inverters
  - But significantly decreases area and power

Inverter

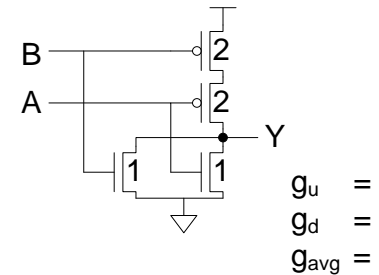
fastest  
P/N ratio



NAND2

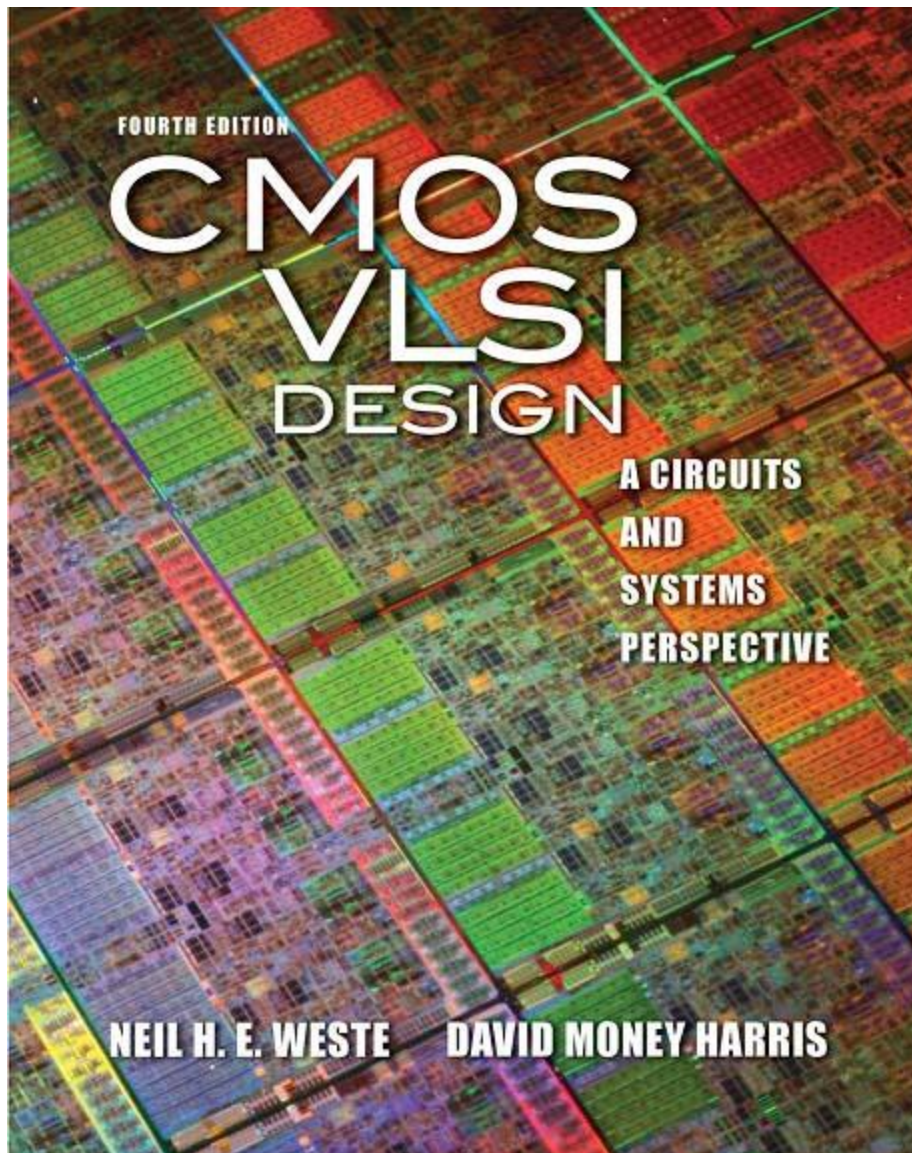


NOR2



# Observations

- ❑ For speed:
  - NAND vs. NOR
  - Many simple stages vs. fewer high fan-in stages
  - Latest-arriving input
- ❑ For area and power:
  - Many simple stages vs. fewer high fan-in stages



# Lecture 10: Circuit Families

# Outline

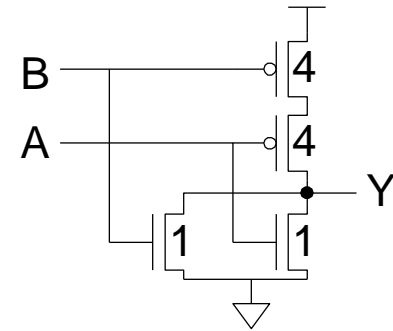
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- ☐ Pseudo-nMOS Logic
- ☐ Dynamic Logic
- ☐ Pass Transistor Logic



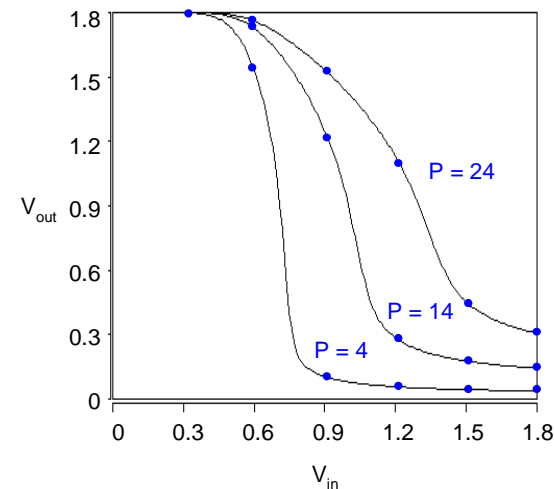
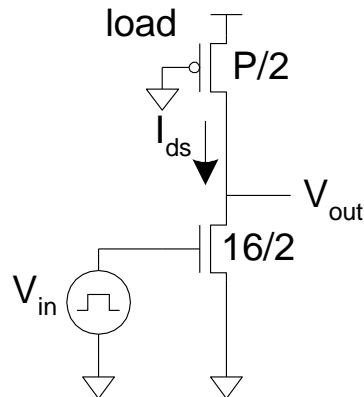
# Introduction

- ❑ What makes a circuit fast?
  - $I = C \, dV/dt \rightarrow t_{pd} \propto (C/I) \, \Delta V$
  - low capacitance
  - high current
  - small swing
- ❑ Logical effort is proportional to  $C/I$
- ❑ pMOS are the enemy!
  - High capacitance for a given current
- ❑ Can we take the pMOS capacitance off the input?
- ❑ Various circuit families try to do this...



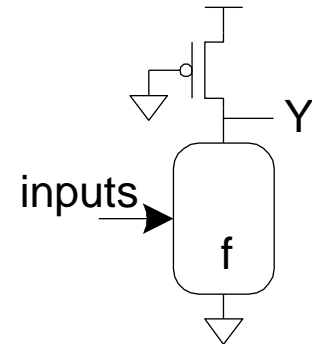
# Pseudo-nMOS

- ❑ In the old days, nMOS processes had no pMOS
  - Instead, use pull-up transistor that is always ON
- ❑ In CMOS, use a pMOS that is always ON
  - *Ratio* issue
  - Make pMOS about  $\frac{1}{4}$  effective strength of pulldown network

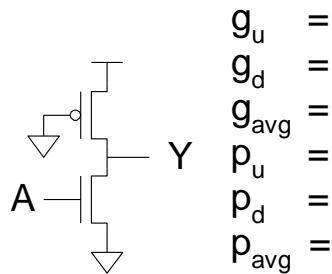


# Pseudo-nMOS Gates

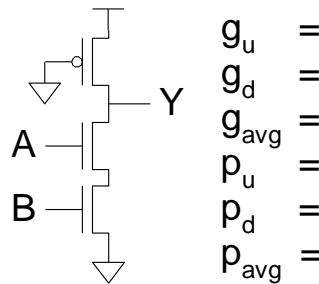
- ❑ Design for unit current on output to compare with unit inverter.
- ❑ pMOS fights nMOS



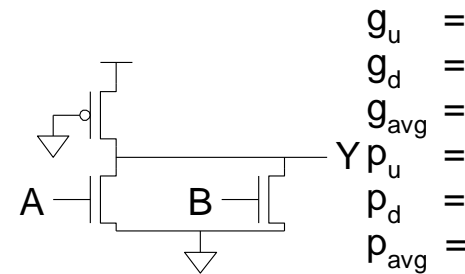
Inverter



NAND2

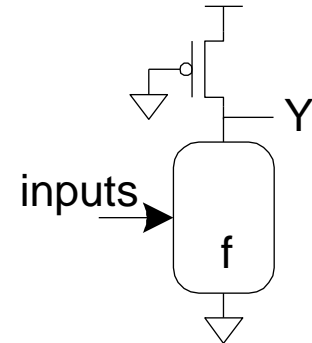


NOR2

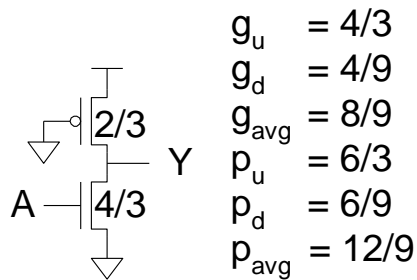


# Pseudo-nMOS Gates

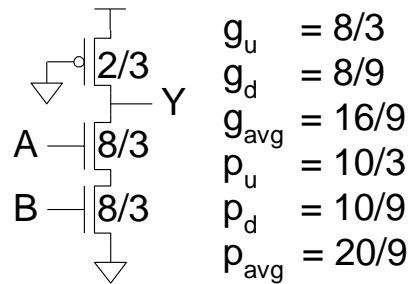
- ❑ Design for unit current on output to compare with unit inverter.
- ❑ pMOS fights nMOS



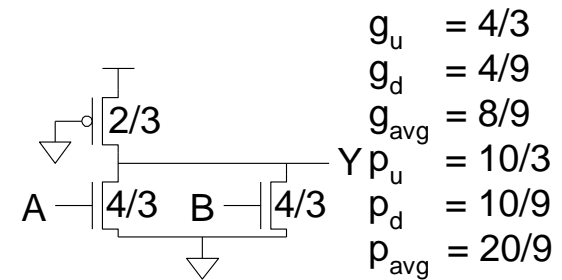
Inverter



NAND2



NOR2



# Pseudo-nMOS Design

- ❑ Ex: Design a k-input AND gate using pseudo-nMOS. Estimate the delay driving a fanout of H

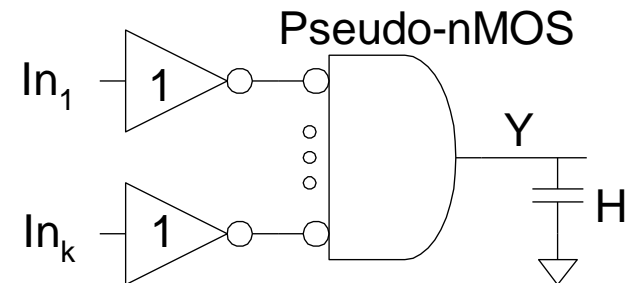
- ❑  $G =$

- ❑  $F =$

- ❑  $P =$

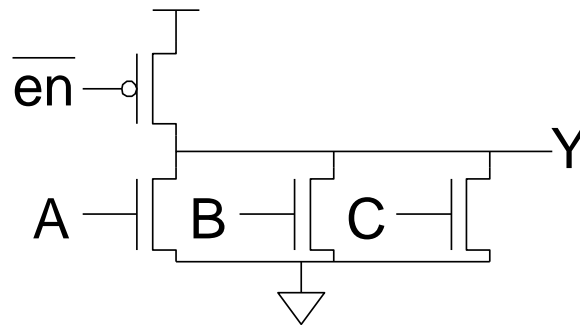
- ❑  $N =$

- ❑  $D =$



# Pseudo-nMOS Power

- ❑ Pseudo-nMOS draws power whenever  $Y = 0$ 
  - Called static power  $P = I_{DD} V_{DD}$
  - A few mA / gate \* 1M gates would be a problem
  - Explains why nMOS went extinct
- ❑ Use pseudo-nMOS sparingly for wide NORs
- ❑ Turn off pMOS when not in use



# Ratio Example

- ❑ The chip contains a 32 word x 48 bit ROM
  - Uses pseudo-nMOS decoder and bitline pullups
  - On average, one wordline and 24 bitlines are high
- ❑ Find static power drawn by the ROM
  - $I_{\text{on-p}} = 36 \mu\text{A}$ ,  $V_{\text{DD}} = 1.0 \text{ V}$

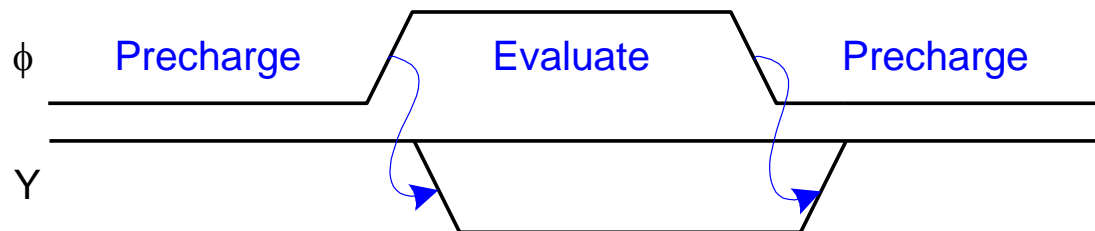
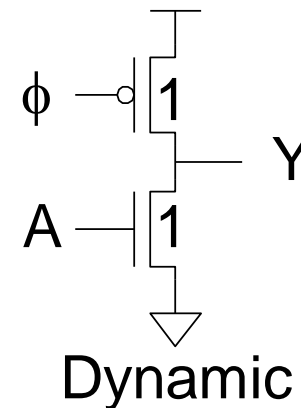
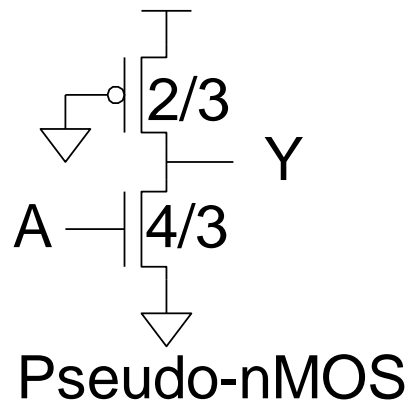
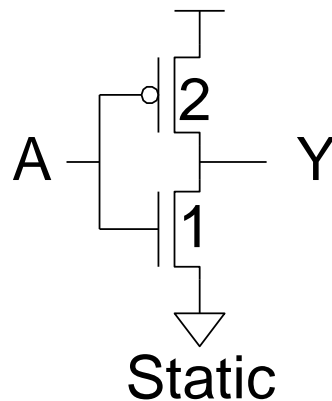
❑ Solution:

$$P_{\text{pull-up}} =$$

$$P_{\text{static}} =$$

# Dynamic Logic

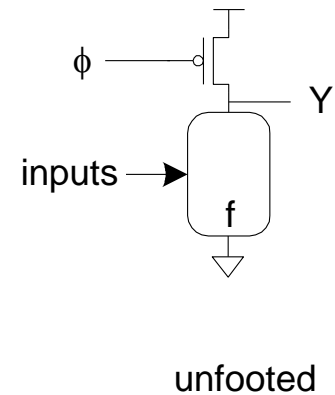
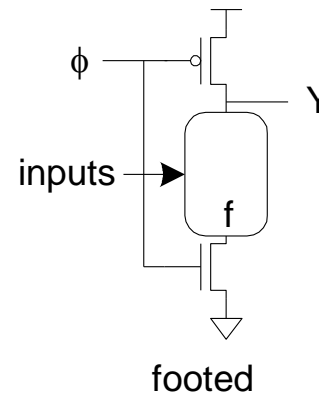
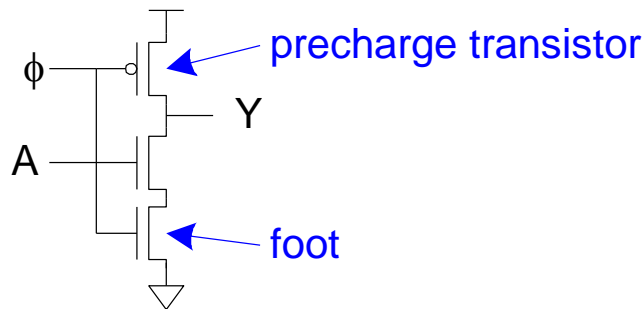
- ❑ *Dynamic* gates uses a clocked pMOS pullup
- ❑ Two modes: *precharge* and *evaluate*





# The Foot

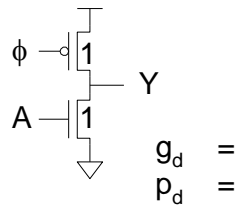
- ❑ What if pulldown network is ON during precharge?
- ❑ Use series evaluation transistor to prevent fight.



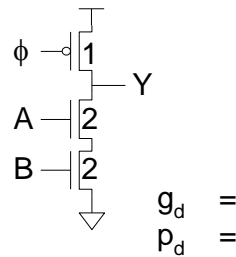
# Logical Effort

unfooted

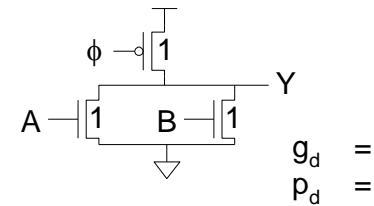
Inverter



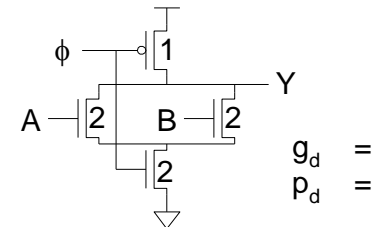
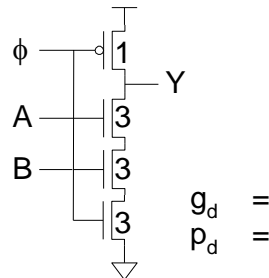
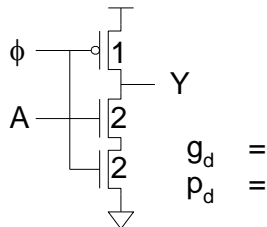
NAND2



NOR2



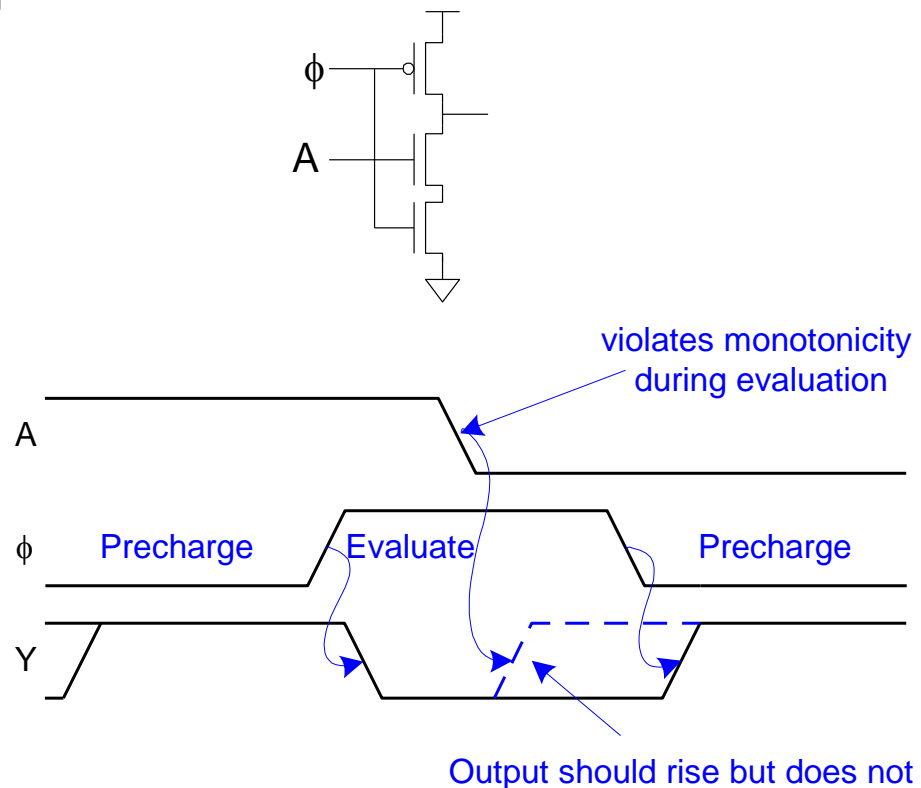
footed



# Monotonicity

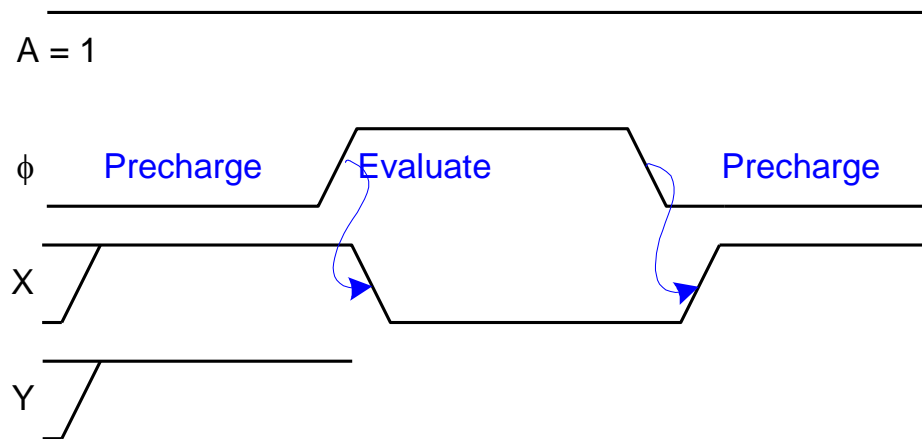
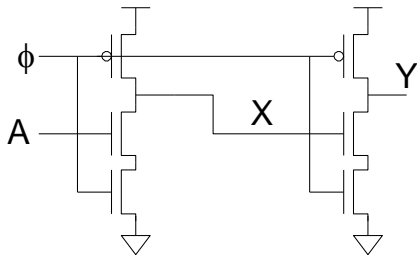
❑ Dynamic gates require *monotonically rising* inputs during evaluation

- 0  $\rightarrow$  0
- 0  $\rightarrow$  1
- 1  $\rightarrow$  1
- But not 1  $\rightarrow$  0



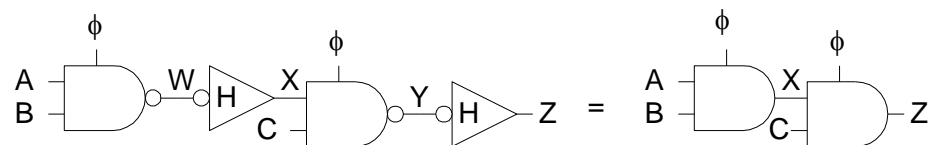
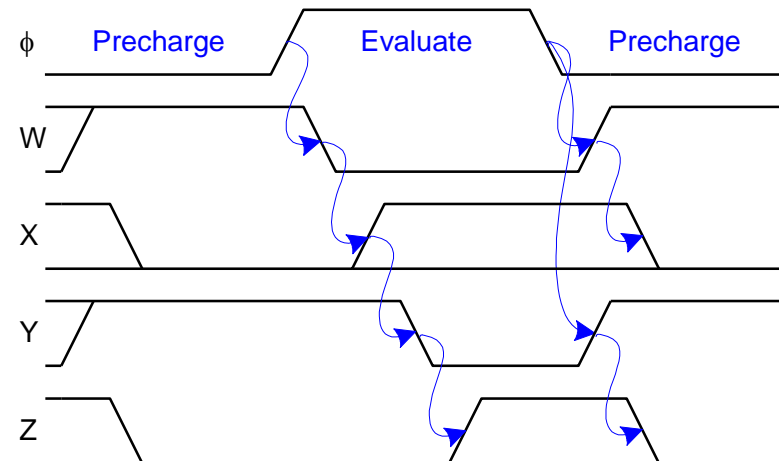
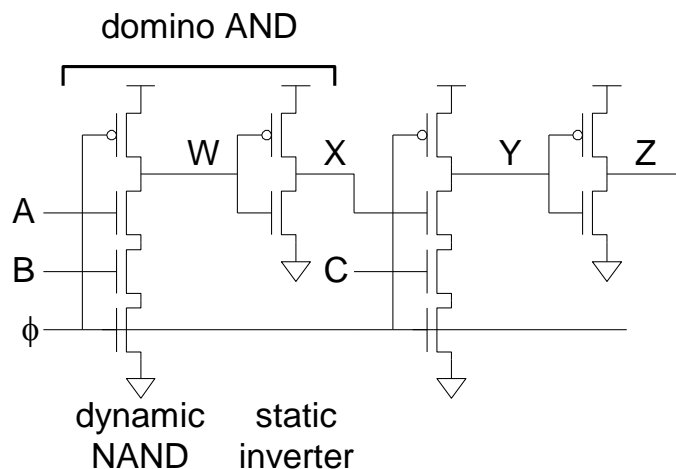
# Monotonicity Woes

- ❑ But dynamic gates produce monotonically falling outputs during evaluation
- ❑ Illegal for one dynamic gate to drive another!



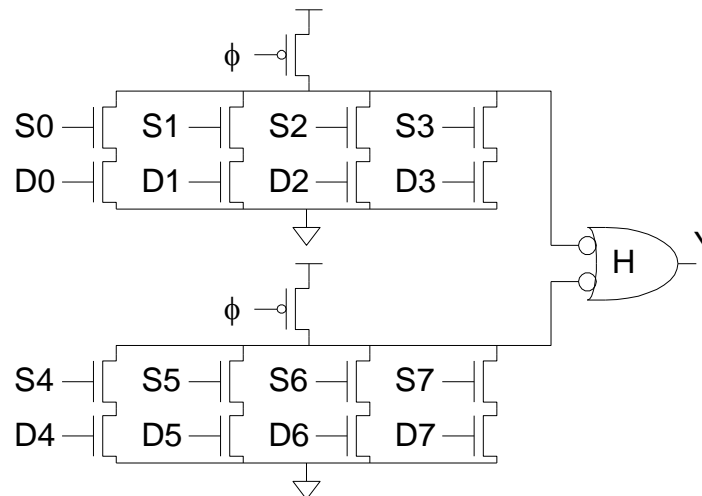
# Domino Gates

- ❑ Follow dynamic stage with inverting static gate
  - Dynamic / static pair is called domino gate
  - Produces monotonic outputs



# Domino Optimizations

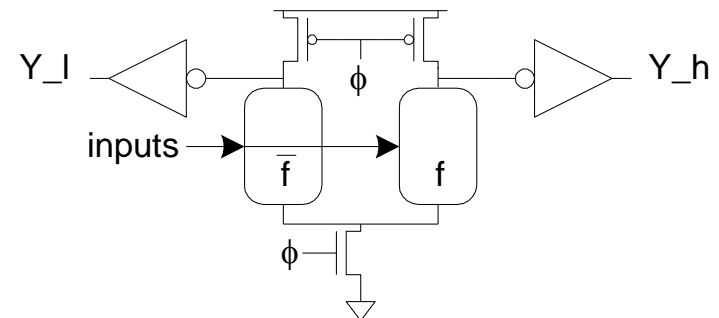
- ❑ Each domino gate triggers next one, like a string of dominos toppling over
- ❑ Gates evaluate sequentially but precharge in parallel
- ❑ Thus evaluation is more critical than precharge
- ❑ HI-skewed static stages can perform logic



# Dual-Rail Domino

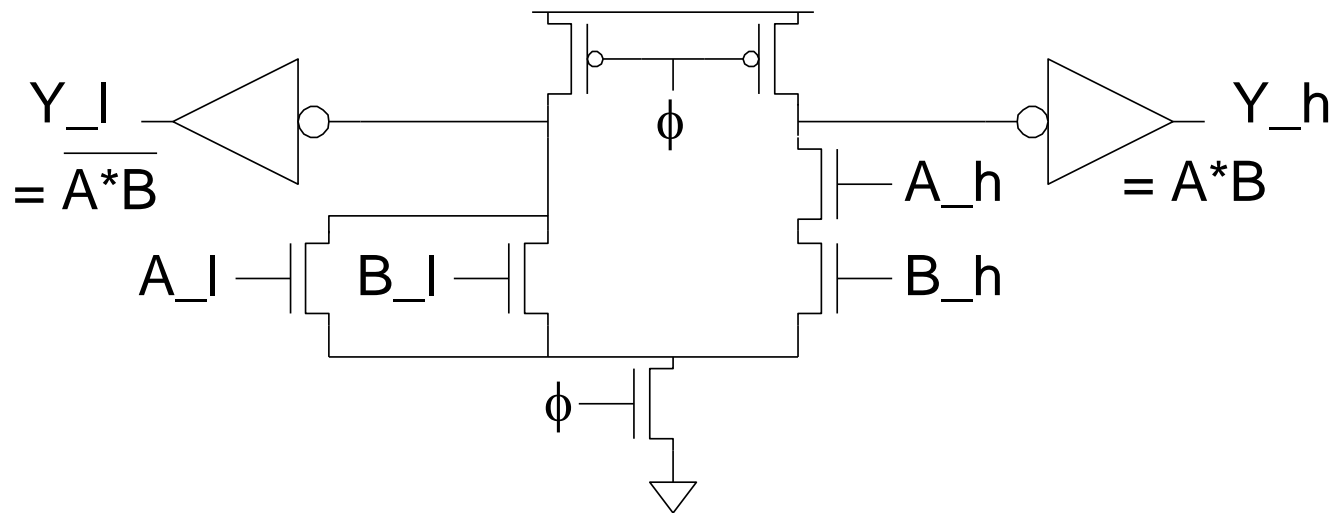
- ❑ Domino only performs noninverting functions:
  - AND, OR but not NAND, NOR, or XOR
- ❑ Dual-rail domino solves this problem
  - Takes true and complementary inputs
  - Produces true and complementary outputs

sig_h	sig_l	Meaning
0	0	Precharged
0	1	'0'
1	0	'1'
1	1	invalid



# Example: AND/NAND

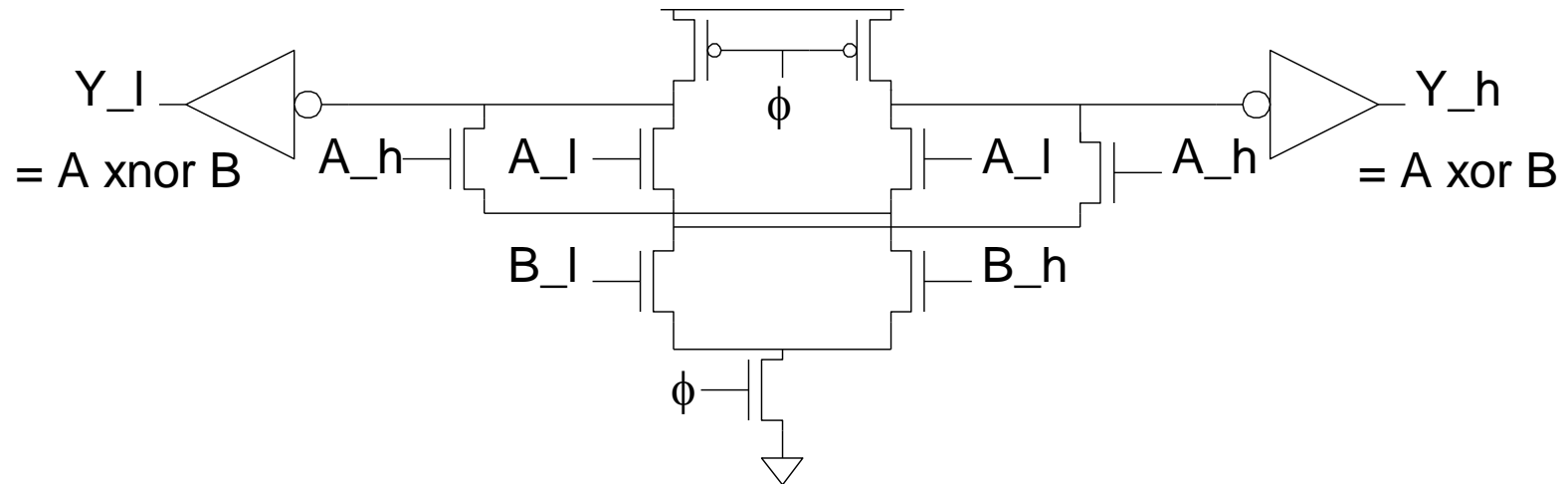
- ❑ Given  $A_h, A_l, B_h, B_l$
- ❑ Compute  $Y_h = AB, Y_l = \overline{AB}$
- ❑ Pulldown networks are conduction complements





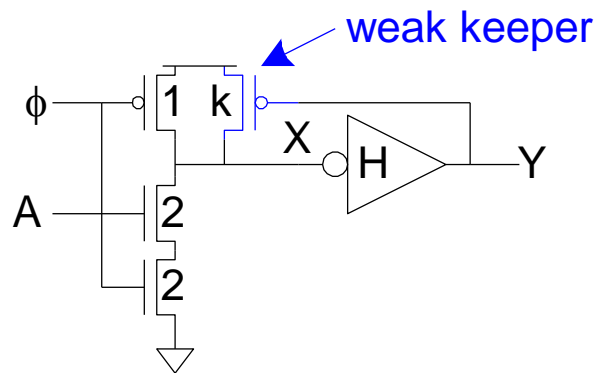
# Example: XOR/XNOR

- Sometimes possible to share transistors



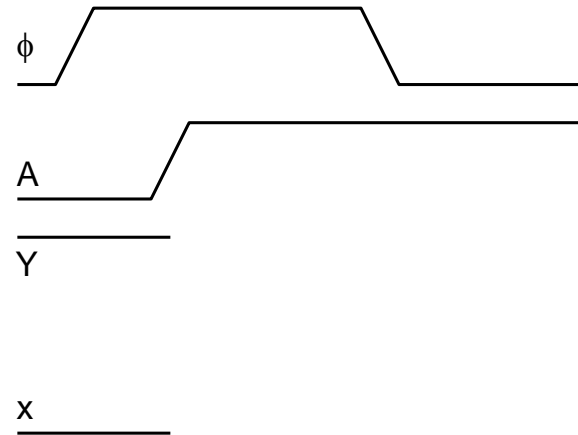
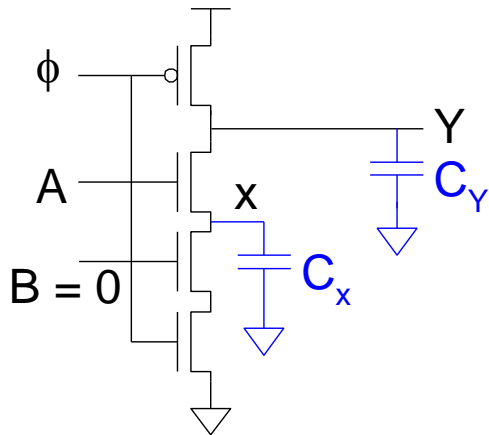
# Leakage

- ❑ Dynamic node floats high during evaluation
  - Transistors are leaky ( $I_{OFF} \neq 0$ )
  - Dynamic value will leak away over time
  - Formerly milliseconds, now nanoseconds
- ❑ Use keeper to hold dynamic node
  - Must be weak enough not to fight evaluation



# Charge Sharing

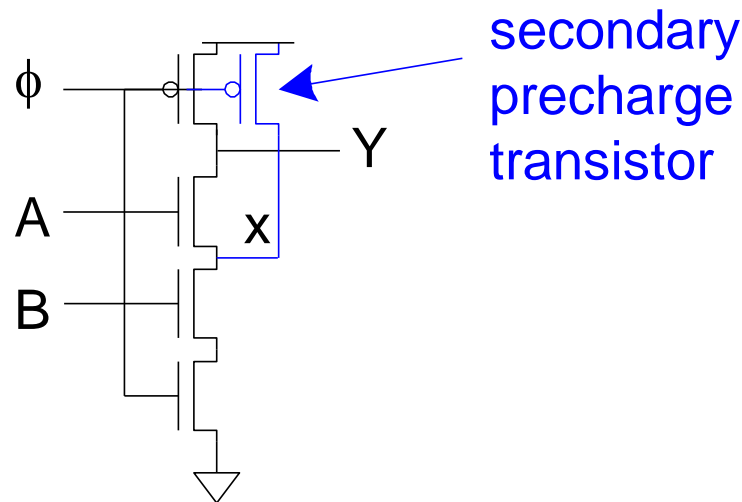
- ❑ Dynamic gates suffer from charge sharing



$$V_x = V_Y = \frac{C_Y}{C_x + C_Y} V_{DD}$$

# Secondary Precharge

- ❑ Solution: add secondary precharge transistors
  - Typically need to precharge every other node
- ❑ Big load capacitance  $C_Y$  helps as well



# Noise Sensitivity

- ❑ Dynamic gates are very sensitive to noise
  - Inputs:  $V_{IH} \approx V_{tn}$
  - Outputs: floating output susceptible noise
- ❑ Noise sources
  - Capacitive crosstalk
  - Charge sharing
  - Power supply noise
  - Feedthrough noise
  - And more!

# Power

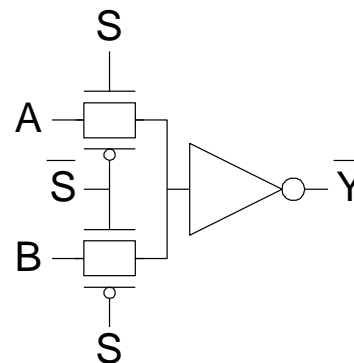
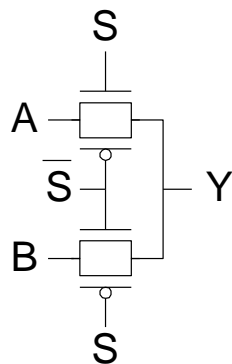
- ❑ Domino gates have high activity factors
  - Output evaluates and precharges
    - If output probability = 0.5,  $\alpha = 0.5$ 
      - Output rises and falls on half the cycles
  - Clocked transistors have  $\alpha = 1$
- ❑ Leads to very high power consumption

# Domino Summary

- ❑ Domino logic is attractive for high-speed circuits
  - 1.3 – 2x faster than static CMOS
  - But many challenges:
    - Monotonicity, leakage, charge sharing, noise
- ❑ Widely used in high-performance microprocessors in 1990s when speed was king
- ❑ Largely displaced by static CMOS now that power is the limiter
- ❑ Still used in memories for area efficiency

# Pass Transistor Circuits

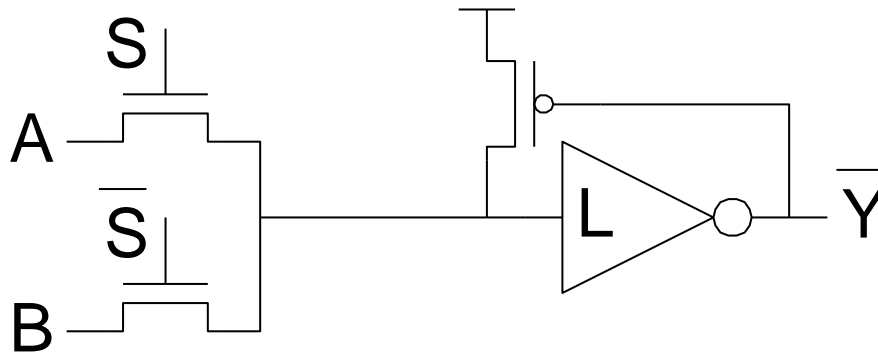
- ❑ Use pass transistors like switches to do logic
- ❑ Inputs drive diffusion terminals as well as gates
- ❑ CMOS + Transmission Gates:
  - 2-input multiplexer
  - Gates should be restoring





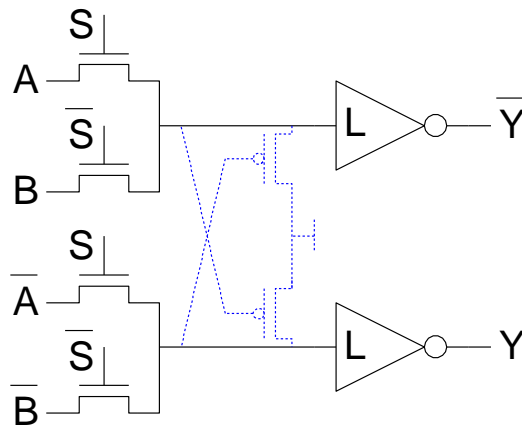
# LEAP

- ❑ **LEA**n integration with **P**ass transistors
- ❑ Get rid of pMOS transistors
  - Use weak pMOS feedback to pull fully high
  - Ratio constraint



# CPL

- ❑ Complementary Pass-transistor Logic
  - Dual-rail form of pass transistor logic
  - Avoids need for ratioed feedback
  - Optional cross-coupling for rail-to-rail swing



# Pass Transistor Summary

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- ❑ Researchers investigated pass transistor logic for general purpose applications in the 1990's
  - Benefits over static CMOS were small or negative
  - No longer generally used
- ❑ However, pass transistors still have a niche in special circuits such as memories where they offer small size and the threshold drops can be managed