

Transistor Theory and DC Characteristics

Dr. Bassam Jamil

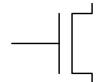
Adopted from slides of the textbook

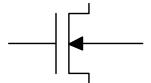
Outline

- □ Transistor Theory
 - Channel Formation and operation Regions
 - I-V Characteristics
 - C-V Characteristics
 - Gate and Diffusion Capacitance
- Nonideal characteristics
- □ DC Response
 - DC Response
 - Logic Levels and Noise Margins

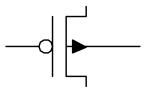
Introduction

- ☐ So far, we have treated transistors as ideal switches
- ☐ An ON transistor passes a finite amount of current
 - Depends on terminal voltages
 - Derive current-voltage (I-V) relationships
- ☐ Transistor gate, source, drain all have capacitance
 - $-I = C (\Delta V/\Delta t) \rightarrow \Delta t = (C/I) \Delta V$
 - Capacitance and current determine speed









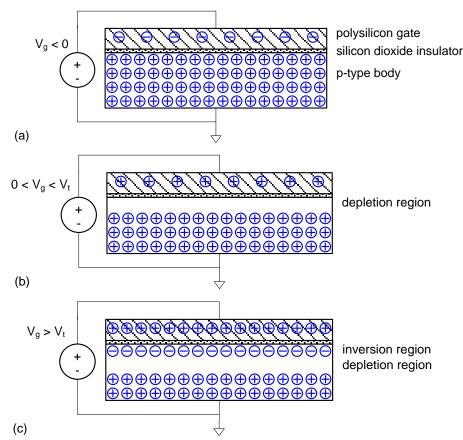
MOS Device Channel Formation

☐ Gate and body form MOS

capacitor

Operating modes

- Accumulation
- Depletion
- Inversion



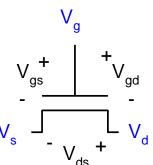
Terminal Voltages

■ Mode of operation depends on V_g, V_d, V_s

$$-V_{gs} = V_g - V_s$$

$$-V_{gd} = V_g - V_d$$

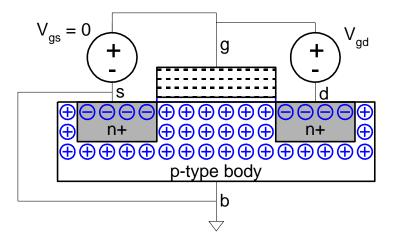
$$-V_{ds} = V_{d} - V_{s} = V_{gs} - V_{gd}$$



- Source and drain are symmetric diffusion terminals
 - By convention, source is terminal at lower voltage
 - Hence $V_{ds} \ge 0$
- nMOS body is grounded. First assume source is 0 too.
- ☐ Three regions of operation
 - Cutoff
 - Linear
 - Saturation

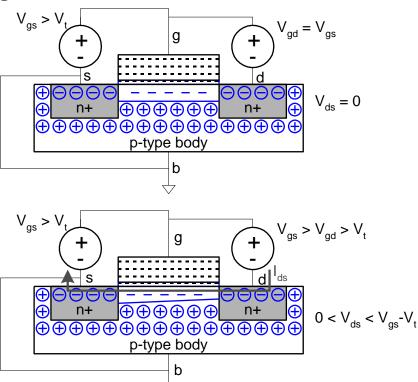
nMOS Cutoff

- No channel
- \Box $I_{ds} \approx 0$



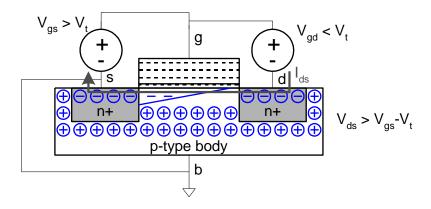
nMOS Linear

- Channel forms
- ☐ Current flows from d to s
 - e⁻ from s to d
- I_{ds} increases with V_{ds}
- □ Similar to linear resistor



nMOS Saturation

- ☐ Channel pinches off
- □ I_{ds} independent of V_{ds}
- ☐ We say current *saturates*
- ☐ Similar to current source



I-V Characteristics

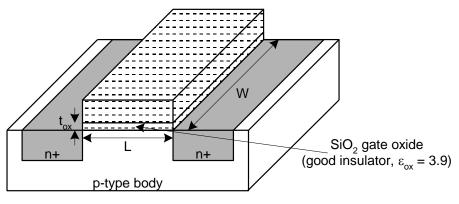
- ☐ In Linear region, I_{ds} depends on
 - How much charge is in the channel?
 - How fast is the charge moving?
- **☐** We will compute the current I_{ds}, where:

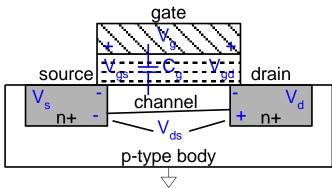
To compute Q: approximate transistor as capacitor

Long Channel I-V

- MOS structure looks like parallel plate capacitor while operating in inversions: Gate – oxide – channel
- $Q_{channel} = CV$
- $C = C_g = WL\epsilon_{ox}/t_{ox} = WLC_{ox}$ $C_{ox} = \epsilon_{ox}/t_{ox}$

- → $V = V_{gc} V_{t} = (V_{gs} V_{ds}/2) V_{t}$





Long Channel I-V

Ids = Qchannel / Time

$$Q_{\text{channel}} = C_g (V_{ge} - V_t)$$

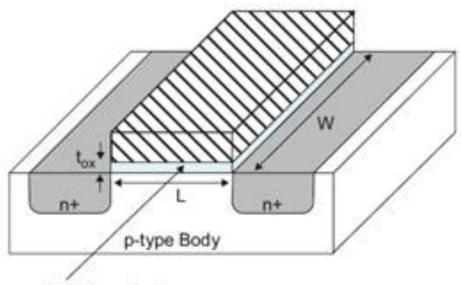
$$C_g = k_{\text{ox}} \varepsilon_0 \frac{WL}{t_{\text{ox}}} = \varepsilon_{\text{ox}} \frac{WL}{t_{\text{ox}}} = C_{\text{ox}} WL$$

Time= L/ν

$$v = \mu E$$
 Where, $E = \frac{V_{di}}{L}$

$$\begin{split} I_{dt} &= \frac{Q_{\text{channel}}}{L/v} \\ &= \mu C_{\text{ox}} \frac{W}{L} \Big(V_{gs} - V_{t} - V_{dt} / 2 \Big) V_{dt} \\ &= \beta \Big(V_{GT} - V_{dt} / 2 \Big) V_{dt} \end{split}$$

$$\beta = \mu C_{\rm ox} \frac{W}{L}; \ V_{GT} = V_{gs} - V_{t}$$



 SiO_2 Gate Oxide (insulator, $\varepsilon_{OX} = 3.9\varepsilon_0$)

nMOS Saturation I-V

- \Box If $V_{qd} < V_t$, channel pinches off near drain
 - When $V_{ds} > V_{dsat} = V_{gs} V_{t}$
- Now drain voltage no longer increases current

$$I_{ds} = \frac{\beta}{2} V_{GT}^2$$

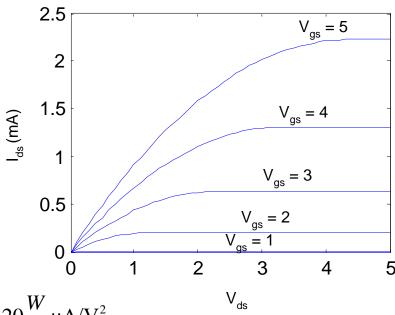
nMOS I-V Summary

☐ Shockley 1st order transistor models

$$I_{ds} = \begin{cases} 0 & V_{gs} < V_{t} & \text{cutoff} \\ \beta \left(V_{gs} - V_{t} - \frac{V_{ds}}{2}\right) V_{ds} & V_{ds} < V_{dsat} & \text{linear} \\ \frac{\beta}{2} \left(V_{gs} - V_{t}\right)^{2} & V_{ds} > V_{dsat} & \text{saturation} \end{cases}$$

Example

- \Box The following is a 0.6 μ m process:
 - $-t_{0x} = 100 \text{ Å}$
 - $\mu = 350 \text{ cm}^2/\text{V*s}$
 - $V_t = 0.7 V$
- \Box Plot I_{ds} vs. V_{ds}
 - $-V_{qs} = 0, 1, 2, 3, 4, 5$
 - Use W/L = 4/2 λ



$$\beta = \mu C_{ox} \frac{W}{L} = (350) \left(\frac{3.9 \times 8.85 \cdot 10^{-14}}{100 \cdot 10^{-8}} \right) \left(\frac{W}{L} \right) = 120 \frac{W}{L} \, \mu \text{A/V}^2$$

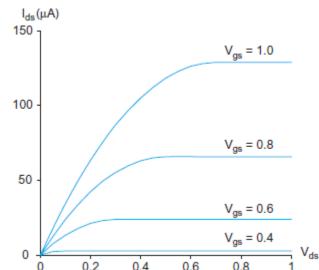
Another Example

Consider an nMOS transistor in a 65 nm process with a minimum drawn channel length of 50 nm ($\lambda = 25$ nm). Let $W/L = 4/2 \lambda$ (i.e., 0.1/0.05 μ m). In this process, the gate oxide thickness is 10.5 Å. Estimate the high-field mobility of electrons to be 80 cm²/V·s at 70 °C. The threshold voltage is 0.3 V. Plot I_{ds} vs. V_{ds} for $V_{gs} = 0$, 0.2, 0.4, 0.6, 0.8, and 1.0 V using the long-channel model.

SOLUTION: We first calculate β .

$$\beta = \mu C_{\text{ox}} \frac{W}{L} = \left(80 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}\right) \left(\frac{3.9 \times 8.85 \times 10^{-14} \frac{\text{F}}{\text{cm}}}{10.5 \times 10^{-8} \text{cm}}\right) \left(\frac{W}{L}\right) = 262 \frac{W}{L} \frac{\text{A}}{\text{V}^2}$$
(2.11)

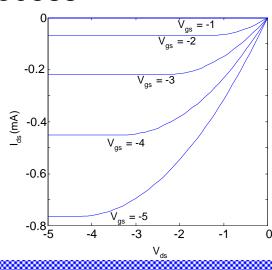
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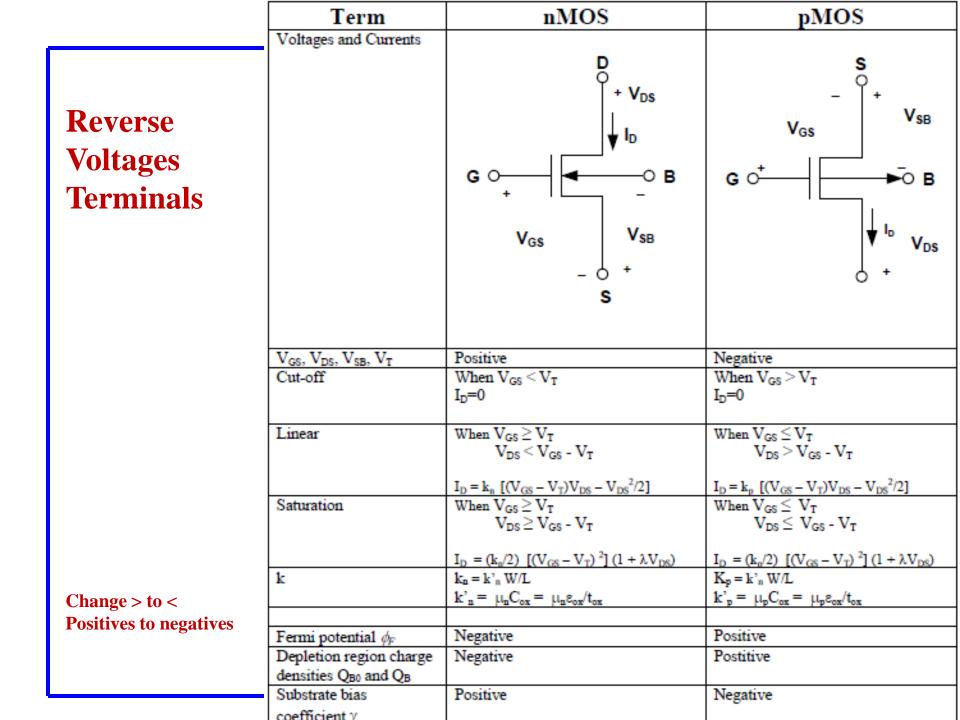


3: CMOS Transistor Th

pMOS I-V

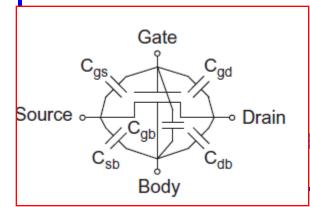
- ☐ All dopings and voltages are inverted for pMOS
 - Source is the more positive terminal
- \Box Mobility μ_{D} is determined by holes
 - Typically 2-3x lower than that of electrons μ_n
 - 120 cm²/V•s in AMI 0.6 μm process
- □ Thus pMOS must be wider to provide same current
 - In this class, assume $\mu_n / \mu_p = 2$



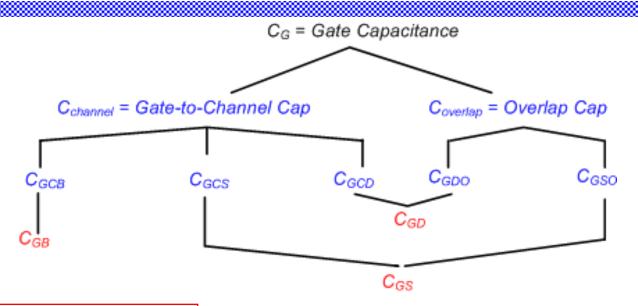


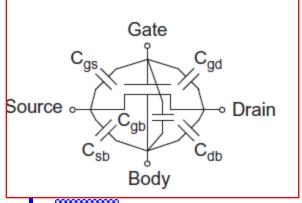
Capacitance

- Any two conductors separated by an insulator have capacitance
- Gate to channel capacitor is very important
 - Creates channel charge necessary for operation
- □ Source and drain have capacitance to body
 - Across reverse-biased diodes
 - Called diffusion capacitance because it is associated with source/drain diffusion



MOS Device Capacitances

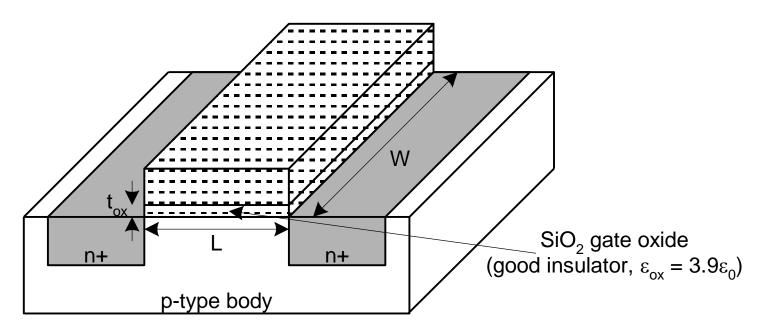




 $C_{ ext{diff}}$: Diffusion Capacitance $C_{ ext{DB}} = CDdiff$ $C_{ ext{SB}} = C_{ ext{Sdiff}}$

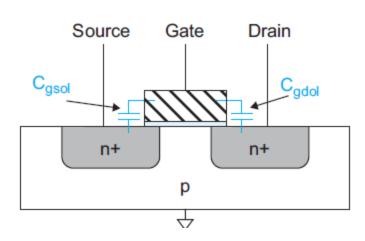
Gate Capacitance

- □ Approximate channel as connected to source
- \Box $C_{gs} = \varepsilon_{ox}WL/t_{ox} = C_{ox}WL = C_{permicron}W$
- C_{permicron} is typically about 2 fF/μm



Gate Overlap Capacitance

The gate overlaps the source and drain in a real device and also has fringing fields terminating on the source and drain. This leads to additional overlap capacitances, as shown in Figure 2.10. These capacitances are proportional to the width of the transistor. Typical values are $C_{gsol} = C_{gdol} = 0.2 - 0.4$ fF/ μ m. They should be added to the intrinsic gate capacitance to find the total.



Approximation of Intrinsic Gate Capacitance

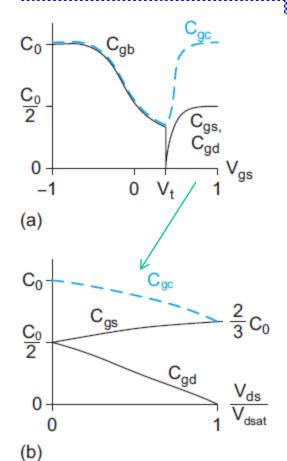


TABLE 2.1 Approximation for intrinsic MOS gate capacitance

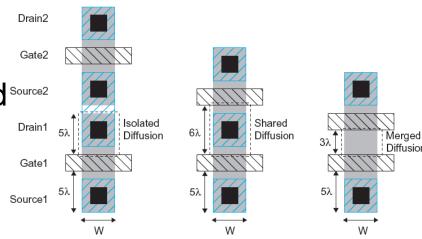
Parameter	Cutoff	Linear	Saturation
C_{gb}	$\leq C_0$	0	0
C_{gs}	0	$C_0/2$	$2/3 C_0$
C_{gd}	0	$C_0/2$	0
$C_g = C_{gs} + C_{gd} + C_{gb}$	C_0	C_0	$2/3 C_0$

FIGURE 2.9 Intrinsic gate capac-

itance $C_{gc} = C_{gs} + C_{gd} + C_{gb}$ as a function of (a) V_{gs} and (b) V_{ds} CMOS VLSI Design ^{4th} Ed.

Diffusion Capacitance

- \Box C_{sb} , C_{db}
- Undesirable, called parasitic capacitance
- ☐ Capacitance depends on area and perimeter
 - Use small diffusion nodes
 - Comparable to C_g
 for contacted diff
 - $\frac{1}{2} C_g$ for uncontacted source 2
 - Varies with process



Diffusion Cap Calculations

☐ Total source diff capacitance

$$C_{sb} = AS \times C_{jbs} + PS \times C_{jbssw}$$

$$C_{jbs} = C_J \left(1 + \frac{V_{sb}}{\psi_0} \right)^{-M_J}$$

$$C_{jbssw} = C_{JSW} \left(1 + \frac{V_{sb}}{\psi_{SW}} \right)^{-M_{JSW}}$$
FIG.

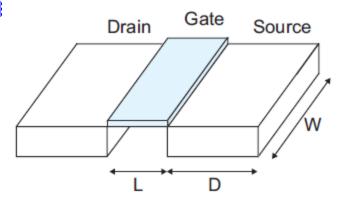


FIGURE 2.12 Diffusion region geometry

In some SPICE models, the capacitance of this sidewall abutting the gate and channel is specified with another set of parameters:

$$C_{jbsswg} = C_{JSWG} \left(1 + \frac{V_{sb}}{\psi_{SWG}} \right)^{-M_{JSWG}}$$

- $\Box \quad C_{sb} = AS \times C_{jbs} + PS \times C_{jbssw} + W \times C_{jbsswg}$
- Source area (AS) = W D
- ☐ Source Perimeter (PS) = W + 2D

Diffusion Cap Example

Example 2.2

Calculate the diffusion parasitic C_{dh} of the drain of a unit-sized contacted nMOS transistor in a 65 nm process when the drain is at 0 V and again at $V_{DD} = 1.0$ V. Assume the substrate is grounded. The diffusion region conforms to the design rules from Figure 2.8 with $\lambda = 25$ nm. The transistor characteristics are CI = 1.2 fF/ μ m², MI = 0.33, $CJSW = 0.1 \text{ fF}/\mu\text{m}$, $CJSWG = 0.36 \text{ fF}/\mu\text{m}$, MJSW = MJSWG = 0.10, and $\psi_0 = 0.7 \text{ V}$ at room temperature.

SOLUTION: From Figure 2.8, we find a unit-size diffusion contact is $4 \times 5 \lambda$, or 0.1×10^{-5} $0.125 \mu m$. The area is $0.0125 \mu m^2$ and perimeter is $0.35 \mu m$ plus $0.1 \mu m$ along the channel. At zero bias, $C_{ibd} = 1.2 \text{ fF/}\mu\text{m}^2$, $C_{ibdsw} = 0.1 \text{ fF/}\mu\text{m}$, and $C_{ibdswg} = 0.36 \text{ fF/}\mu\text{m}$ μm. Hence, the total capacitance is

$$C_{db}(0 \text{ V}) = \left(0.0125 \mu\text{m}^2\right) \left(1.2 \frac{\text{fF}}{\mu\text{m}^2}\right) + \left(0.35 \mu\text{m}\right) \left(0.1 \frac{\text{fF}}{\mu\text{m}}\right) + \left(0.1 \mu\text{m}\right) \left(0.36 \frac{\text{fF}}{\mu\text{m}}\right) = 0.086 \text{ fF}$$

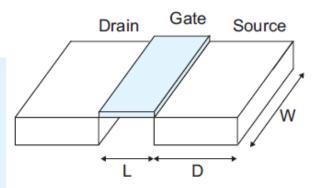
At a drain voltage of V_{DD} , the capacitance reduces to

$$C_{db}(1 \text{ V}) = (0.0125 \mu\text{m}^2) \left(1.2 \frac{\text{fF}}{\mu\text{m}^2}\right) \left(1 + \frac{1.0}{0.7}\right)^{-0.33} + C_{jbssw} = C_{JSW} \left(1 + \frac{1}{\psi} \frac{sb}{W_{SW}}\right) \left(0.35 \mu\text{m}\right) \left(0.1 \frac{\text{fF}}{\mu\text{m}}\right) + \left(0.1 \mu\text{m}\right) \left(0.36 \frac{\text{fF}}{\mu\text{m}}\right) \left(1 + \frac{1.0}{0.7}\right)^{-0.10} = 0.076 \text{ fF}$$

$$C_{jbssw} = C_{JSW} \left(1 + \frac{V_{sb}}{\psi_{SWG}}\right)^{-M_{JSWG}}$$

$$C_{jbssw} = C_{JSW} \left(1 + \frac{V_{sb}}{\psi_{SWG}}\right)^{-M_{JSWG}}$$

For the purpose of manual performance estimation, this nonlinear capacitance is too much effort. An effective capacitance averaged over the switching range is quite satisfactory for digital applications. In this example, the effective drain capacitance would be approximated as the average of the two extremes, 0.081 fF.



$$\begin{array}{ll} \lambda = 25 \text{ nm} & \text{eometry} \\ W = 4 \ \lambda = 0.1 \mu m \\ D = 5 \ \lambda = 0.125 \mu m \end{array}$$

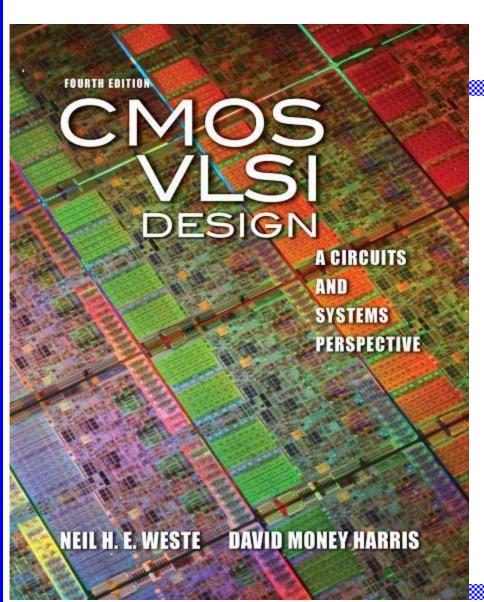
$$C_{sb} = AS \times C_{ibs} + PS \times C_{jbssw} + W \times C_{jbsswg}$$

$$C_{jbs} = C_J \left(1 + \frac{V_{sb}}{\psi_0} \right)^{-M_J}$$

$$C_{jbssw} = C_{JSW} \left(1 + \frac{V_{sb}}{\psi_{SW}} \right)^{-M_{JSW}}$$

$$C_{jbsswg} = C_{JSWG} \left(1 + \frac{V_{sb}}{\psi_{SWG}} \right)^{-M_{JSWG}}$$

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Nonideal Transistor Theory

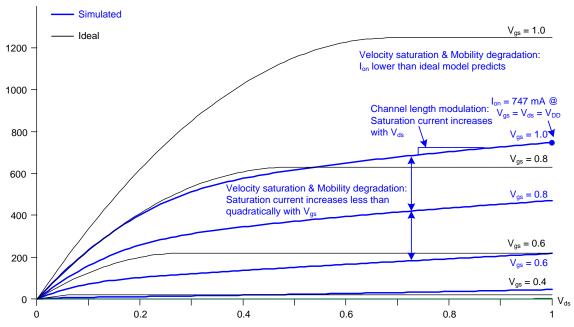
CMOS VLSI Design 4th Ed.

Nonideal Transistor Behavior

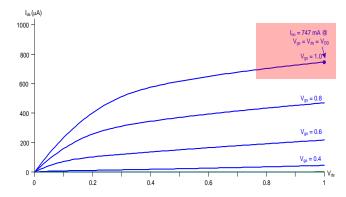
- ☐ High Field Effects
 - Mobility Degradation
 - Velocity Saturation
- □ Channel Length Modulation
- ☐ Threshold Voltage Effects
 - Body Effect
 - Drain-Induced Barrier Lowering
 - Short Channel Effect
- □ Leakage
 - Subthreshold Leakage
 - Gate Leakage
 - Junction Leakage

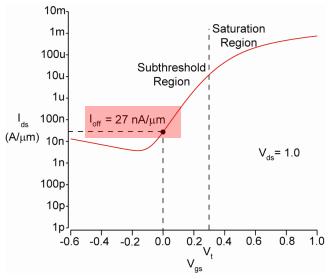
Ideal vs. Simulated nMOS I-V Plot

- \Box 65 nm IBM process, $V_{DD} = 1.0 \text{ V}$
- ☐ This is due to:
 - Velocity Saturation
 - Mobility depredation



ON and OFF Current





Nonideal Transistor Behavior: High Field Effects

- □ The saturation current increases less than quadratically with increasing Vgs because of:
 - velocity saturation
 - mobility degradation.
- At high lateral field strengths (Vds /L), carrier velocity ceases to increase linearly with field strength. This is called **velocity** saturation and results in lower lds than expected at high Vds.
- At high vertical field strengths (Vgs /tox), the carriers scatter off the oxide interface more often, slowing their progress. This mobility degradation effect also leads to less current than expected at high Vgs

Electric Fields Effects

- \Box Vertical electric field: $E_{vert} = V_{gs} / t_{ox}$
 - Attracts carriers into channel
 - Long channel: Q_{channel} ∝ E_{vert}
- \Box Lateral electric field: $E_{lat} = V_{ds} / L$
 - Accelerates carriers from drain to source
 - Long channel: $v = \mu E_{lat}$

Mobility Degradation

- ☐ High E_{vert} effectively reduces mobility
 - Collisions with oxide interface

$$\mu_{\text{eff}-n} = \frac{540 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}}{1 + \left(\frac{V_{gs} + V_t}{0.54 \frac{\text{V}}{\text{nm}} t_{\text{ox}}}\right)^{1.85}}$$

$$\mu_{\text{eff}-p} = \frac{185 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}}{1 + \frac{\left| V_{gs} + 1.5 V_t \right|}{0.338 \frac{\text{V}}{\text{nm}} t_{\text{ox}}}}$$

Example for μ_{eff}

Example 2.3

Compute the effective mobilities for nMOS and pMOS transistors when they are fully ON. Use the physical parameters from Example 2.1.

SOLUTION: Use $V_{gs} = 1.0$ for ON transistors, remembering that we are treating voltages as positive in a pMOS transistor. Substituting $V_t = 0.3$ V and $t_{ox} = 1.05$ nm into EQ (2.23) gives:

$$\mu_{\text{eff-n}}(V_{gs} = 1.0) = 96 \text{ cm}^2/\text{V}, \mu_{\text{eff-p}}(V_{gs} = 1.0) = 36 \text{ cm}^2/\text{V}$$

$$\mu_{\text{eff}-n} = \frac{540 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}}{1 + \left(\frac{V_{gs} + V_t}{0.54 \frac{\text{V}}{\text{nm}} t_{\text{ox}}}\right)^{1.85}} \qquad \mu_{\text{eff}-p} = \frac{185 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}}{1 + \frac{\left|V_{gs} + 1.5V_t\right|}{0.338 \frac{\text{V}}{\text{nm}} t_{\text{ox}}}}$$

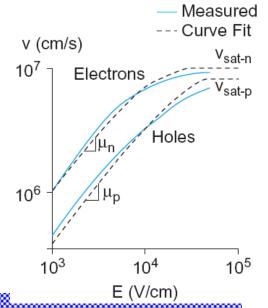
Velocity Saturation

- □ At high E_{lat}, carrier velocity rolls off
 - Carriers scatter off atoms in silicon lattice
 - Velocity reaches v_{sat}
 - Electrons: 10⁷ cm/s
 - Holes: 8 x 10⁶ cm/s
- ☐ The critical voltage Vc is the drain-source voltage at which the critical effective

field is reached:
$$V_c = E_c L$$
.

$$v = \begin{cases} \frac{\mu_{\text{eff}} E}{1 + \frac{E}{E_c}} & E < E_c \\ v_{\text{sat}} & E \ge E_c \end{cases}$$

$$E_c = \frac{2v_{\text{sat}}}{\mu_{\text{eff}}}$$



Example for Critical Voltage

Example 2.4

Find the critical voltage for fully ON nMOS and pMOS transistors using the effective mobilities from Example 2.3.

SOLUTION: Using EQ (2.25)

$$V_{c-n} = \frac{2\left(10^7 \frac{\text{cm}}{\text{s}}\right)}{96 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}} \left(5 \times 10^{-6} \text{cm}\right) = 1.04 \text{ V}$$

$$V_{c-p} = \frac{2\left(8 \times 10^6 \frac{\text{cm}}{\text{s}}\right)}{36 \frac{\text{cm}^2}{\text{cm}^2}} \left(5 \times 10^{-6} \text{cm}\right) = 2.22 \text{ V}$$

$$E_c = \frac{2v_{\text{sat}}}{\mu_{\text{eff}}}$$

$$V_c = E_c L$$

The nMOS transistor is velocity saturated in normal operation because V_{c-n} is comparable to V_{DD} . The pMOS transistor has lower mobility and thus is not as badly velocity saturated.

Vel Sat I-V Effects

☐ Ideal transistor ON current increases with V_{DD}²

$$I_{ds} = \mu C_{ox} \frac{W}{L} \frac{(V_{gs} - V_{t})^{2}}{2} = \frac{\beta}{2} (V_{gs} - V_{t})^{2}$$

□ Velocity-saturated ON current increases with V_{DD}

$$I_{ds} = C_{ox}W(V_{gs} - V_{t})v_{max}$$

- Real transistors are partially velocity saturated
 - Approximate with α -power law model
 - I_{ds} \propto V_{DD} $^{\alpha}$
 - 1 < α < 2 determined empirically (≈ 1.3 for 65 nm)

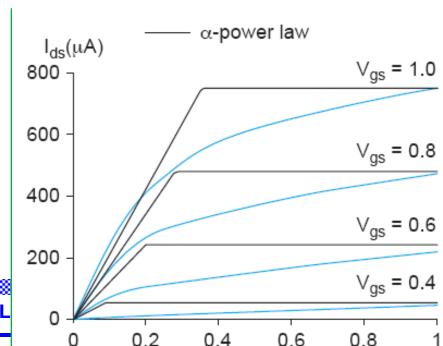
α -Power Model: curve fitting

$$I_{ds} = \begin{cases} 0 & V_{gs} < V_{t} & \text{cutoff} \\ I_{dsat} \frac{V_{ds}}{V_{dsat}} & V_{ds} < V_{dsat} & \text{linear} \\ I_{dsat} & V_{ds} > V_{dsat} & \text{saturation} \end{cases}$$

$$I_{dsat} = P_{c} \frac{\beta}{2} \left(V_{gs} - V_{t} \right)^{\alpha}$$

$$V_{dsat} = P_{v} \left(V_{gs} - V_{t} \right)^{\alpha/2}$$

 α , P_c and P_v are determined empirically from curve fit I-V



Simulated

4: Nonideal Transistor Theory

CMOS VL

Nonideal Transistor Behavior

- ☐ High Field Effects
 - Mobility Degradation
 - Velocity Saturation
- □ Channel Length Modulation
- ☐ Threshold Voltage Effects
 - Body Effect
 - Drain-Induced Barrier Lowering
 - Short Channel Effect
- □ Leakage
 - Subthreshold Leakage
 - Gate Leakage
 - Junction Leakage

Nonideal Transistor Behavior: Other Effects

- The saturation current of the nonideal transistor increases somewhat with Vds. This is caused by **channel length modulation**, in which higher Vds increases the size of the depletion region around the drain and thus effectively shortens the channel.
- ☐ There are other fields in the transistor have some effect on the channel, effectively modifying the threshold voltage.
 - Increasing the potential between the source and body raises the threshold through the **body effect**.
 - Increasing the drain voltage lowers the threshold through drain-induced barrier lowering.
 - Increasing the channel length raises the threshold through the short channel effect.

Body Effect vs. DIBL

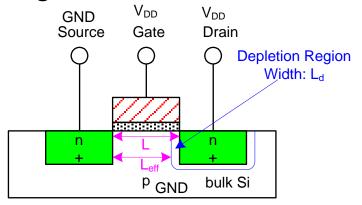
- ☐ Both are created by high drain voltage
- ☐ But:
 - Channel length modulation: Transistor is ON
 - DIBL: Transistor is OFF

Channel Length Modulation

- ☐ Reverse-biased p-n junctions form a *depletion region*
 - Region between n and p with no carriers
 - Width of depletion L_d region grows with reverse bias

$$-L_{eff} = L - L_{d}$$

$$I_{ds} = \frac{\beta}{2} \left(V_{gs} - V_t \right)^2 \left(1 + \lambda V_{ds} \right)$$



 λ = channel length modulation coefficient

- not feature size
- Empirically fit to I-V characteristics

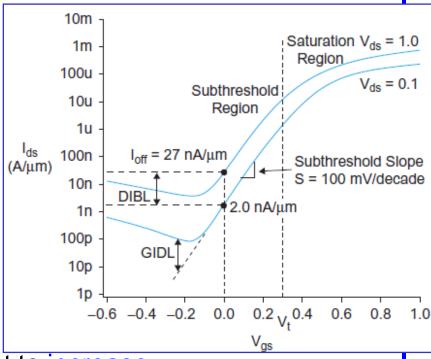
Threshold Voltage Effects

- \Box V_t is V_{qs} for which the channel starts to invert
- ☐ Ideal models assumed V_t is constant
- ☐ Really depends (weakly) on almost everything else:
 - Body voltage: Body Effect
 - Drain voltage: Drain-Induced Barrier Lowering
 - Channel length: Short Channel Effect

DIBL

- ☐ Electric field from drain affects channel
- More pronounced in small transistors where the drain is closer to the channel
 10m →
- Drain-Induced Barrier Lowering
 - Drain voltage also affect V_t

$$V_t' = V_t - \eta V_{ds}$$



☐ High drain voltage causes current to increase.

Body Effect

- Body is a fourth transistor terminal
- V_{sb} affects the charge required to invert the channel
 - Increasing V_s or decreasing V_b increases V_t

$$V_{t} = V_{t0} + \gamma \left(\sqrt{\phi_{s} + V_{sb}} - \sqrt{\phi_{s}} \right)$$

 \Box $\phi_s = surface potential$ at threshold

$$\phi_s = 2v_T \ln \frac{N_A}{n_i}$$

- Depends on doping level N_A
- And intrinsic carrier concentration n_i
- \square γ = body effect coefficient

$$\gamma = \frac{t_{\text{ox}}}{\varepsilon_{\text{ox}}} \sqrt{2q\varepsilon_{\text{si}}N_A} = \frac{\sqrt{2q\varepsilon_{\text{si}}N_A}}{C_{\text{ox}}}$$

Body Effect Cont. (read)

☐ For small source-to-body voltage, treat as linear

$$V_t = V_{t0} + k_{\gamma} V_{sb}$$

$$k_{\gamma} = \frac{\gamma}{2\sqrt{\phi_s}} = \frac{\sqrt{\frac{q\varepsilon_{si}N_A}{v_T \ln \frac{N_A}{n_i}}}}{2C_{ox}}$$

Body Effect Example

Example 2.5

Consider the nMOS transistor in a 65 nm process with a nominal threshold voltage of 0.3 V and a doping level of 8×10^{17} cm⁻³. The body is tied to ground with a substrate contact. How much does the threshold change at room temperature if the source is at 0.6 V instead of 0?

SOLUTION: At room temperature, the thermal voltage $v_T = kT/q = 26$ mV and $n_i = 1.45 \times 10^{10}$ cm⁻³. The threshold increases by 0.04 V.

$$\phi_{s} = 2(0.026 \text{ V}) \ln \frac{8 \times 10^{17} \text{ cm}^{-3}}{1.45 \times 10^{10} \text{ cm}^{-3}} = 0.93 \text{ V}$$

$$\phi_{s} = 2v_{T} \ln \frac{N_{A}}{n_{i}}$$

$$\gamma = \frac{10.5 \times 10^{-8} \text{ cm}}{3.9 \times 8.85 \times 10^{-14} \frac{\text{F}}{\text{cm}}} \sqrt{2(1.6 \times 10^{-19} \text{C})(11.7 \times 8.85 \times 10^{-14} \frac{\text{F}}{\text{cm}})(8 \times 10^{17} \text{ cm}^{-3})} = 0.16$$

$$V_{t} = 0.3 + \gamma \left(\sqrt{\phi_{s} + 0.6 \text{ V}} - \sqrt{\phi_{s}}\right) = 0.34 \text{ V}$$

$$V_{t} = V_{t0} + \gamma \left(\sqrt{\phi_{s} + V_{sb}} - \sqrt{\phi_{s}}\right)$$

Short Channel Effect

- ☐ In small transistors, source/drain depletion regions extend into the channel
 - Impacts the amount of charge required to invert the channel
 - And thus makes V_t a function of channel length
- ☐ Short channel effect: V_t increases with L
 - Some processes exhibit a reverse short channel effect in which V_t decreases with L

Nonideal Transistor Behavior

- ☐ High Field Effects
 - Mobility Degradation
 - Velocity Saturation
- □ Channel Length Modulation
- ☐ Threshold Voltage Effects
 - Body Effect
 - Drain-Induced Barrier Lowering
 - Short Channel Effect
- □ Leakage
 - Subthreshold Leakage
 - Gate Leakage
 - Junction Leakage

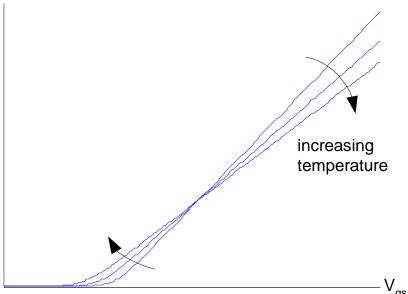
Nonideal Transistor Behavior: Leakage & Temperature

- Sources of leakage result in current flow in nominally OFF transistors:
 - Subthreshold conduction: when Vgs < Vt , the current drops off exponentially rather than abruptly becoming zero.
 - Gate leakage: The current into the gate Ig is ideally 0. However, as the thickness of gate oxides reduces to only a small number of atomic layers, electrons tunnel through the gate, causing some gate leakage current.
 - Diffusion leakage: The source and drain diffusions are typically reverse-biased diodes and also experience junction leakage into the substrate or well.
- Both mobility and threshold voltage decrease with rising temperature. The mobility effect tends to dominate for strongly ON transistors, resulting in lower Ids at high temperature. The threshold effect is most important for OFF transistors, resulting in higher leakage current at high temperature. In summary, MOS characteristics degrade with temperature.

Temperature Sensitivity

- □ Increasing temperature
 - Reduces mobility
 - Reduces V_t

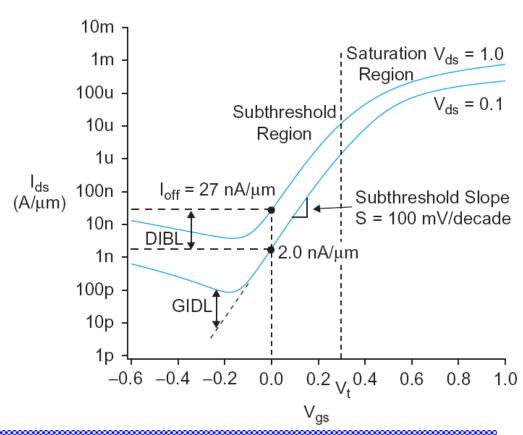




The mobility effect tends to dominate for strongly ON transistors, resulting in lower Ids at high temperature. The threshold effect is most important for OFF transistors, resulting in higher leakage current at high temperature.

Leakage

- What about current in cutoff?
- Simulated results
- What differs?
 - Current doesn't go to 0 in cutoff



Leakage Sources

- Subthreshold conduction
 - Transistors can't abruptly turn ON or OFF
 - Dominant source in contemporary transistors
- □ Gate leakage
 - Tunneling through ultrathin gate dielectric
- Junction leakage
 - Reverse-biased PN junction diode current

Subthreshold Leakage (Read)

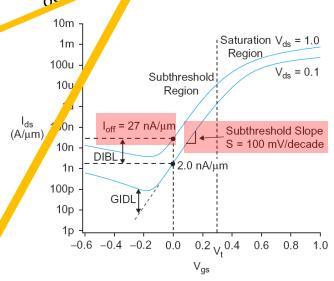
Subthreshold leakage exponential with V_{as}

$$I_{ds} = I_{ds0} e^{\frac{V_{gs} - V_{t0} + \eta V_{ds} - k_{\gamma} V_{sb}}{n v_T}} \left(1 - e^{\frac{-V_{ds}}{v_T}} \right)$$

- n is process dependent
 - typically 1.3-1.7
- Rewrite relative to I_{off} on log scale

$$I_{ds} = I_{\text{off}} 10^{\frac{V_{gs} + \eta \left(V_{ds} - V_{dd}\right) - k\gamma V_{sb}}{S}} \left(1 - e^{\frac{-V_{ds}}{v_t}}\right) \qquad S = \left\lceil \frac{d \left(\log_{10} I_{ds}\right)}{dV_{gs}}\right\rceil^{-1} = nv_T \ln 10$$





$$S = \left[\frac{d \left(\log_{10} I_{ds} \right)}{dV_{gs}} \right]^{-1} = nv_T \ln 10$$

Gate Leakage (read)

- Carriers tunnel thorough very thin gate oxides
- \square Exponentially sensitive to t_{ox} and V_{DD}

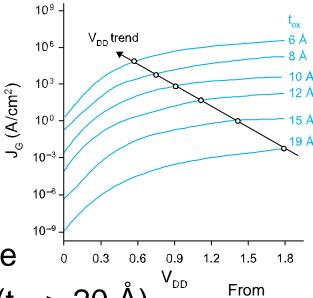
$$I_{\text{gate}} = WA \left(\frac{V_{DD}}{t_{\text{ox}}}\right)^{2} e^{-B\frac{t_{\text{ox}}}{V_{DD}}}$$



- Greater for electrons
 - So nMOS gates leak more



□ Critically important at 65 nm and below (t_{ox} ≈ 10.5 Å)



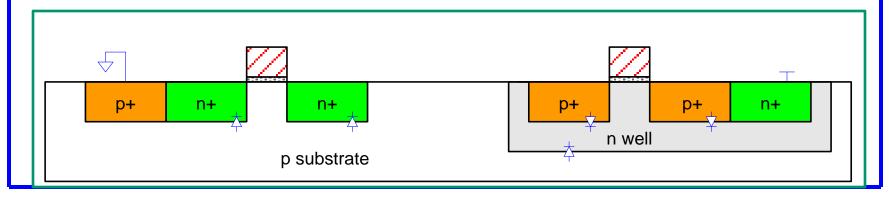
Junction Leakage

- ☐ Reverse-biased p-n junctions have some leakage
 - Band-to-band tunneling (BTBT)

BTBT occurs across the junction between the source or drain and the body when the junction is reverse-biased. It is a function of the reverse bias and the doping levels.

Gate-induced drain leakage (GIDL)

GIDL occurs where the gate partially overlaps the drain. This effect is most pronounced when the drain is at a high voltage and the gate is at a low voltage. GIDL current is proportional to gate-drain overlap area and hence to transistor width. It is a strong function of the electric field and hence increases rapidly with the drain-to-gate voltage.



Diode Leakage

□ Reverse-biased p-n junctions have some leakage

$$I_D = I_S \left(e^{\frac{V_D}{v_T}} - 1 \right)$$

- \Box At any significant negative diode voltage, $I_D = -I_s$
- I_s depends on doping levels
 - And area and perimeter of diffusion regions
 - Typically < 1 fA/μm² (negligible)

Example of DC Anlysis

Example

Determine the required width of the transistor (for $L = 0.25\mu m$) such that X equals 1.5V.

Neglect short channel effects and assume that $\lambda_p = 0$.

PMOS:
$$k'_p = 30 \mu A/V^2$$
, $VT0 = -0.4 V$

$$Vds = -Vx$$

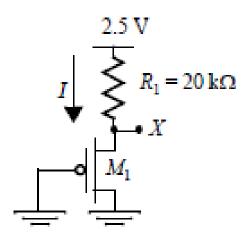
 $Vgs = -Vx = -1.5 \rightarrow transistor is ON.$

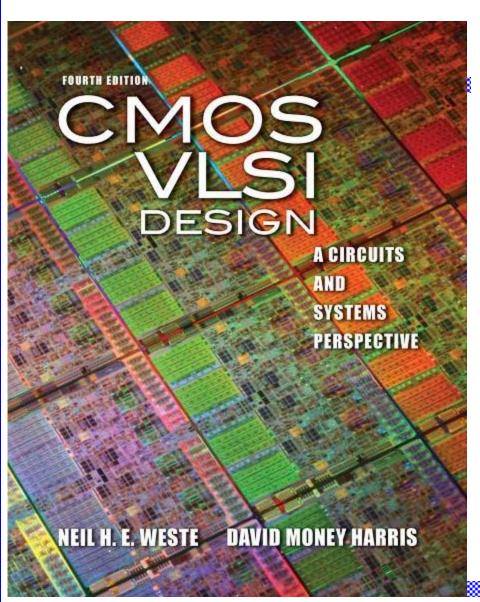
And, $Vds \leq Vgs \Rightarrow$ transistor is in Saturation.

$$I_{ds} = (2.5 - 1.5) / 20K = 50 \mu A$$

 $I_{ds} = k'_{p} (W/L) (Vgs-Vt)^{2}/2$
 $50 \mu = 30\mu (W/0.25) (-1.5 + 0.4)^{2}/2$

$$W = 0.688 \mu$$





DC Characteristics

Dr. Bassam Jamil

Adopted from slides of the textbook

CMOS VLSI Design 4th Ed.

Outline

- Pass Transistors
- □ DC Response
- Logic Levels and Noise Margins
- ☐ Transient Response
- □ RC Delay Models
- Delay Estimation

Pass Transistors

- We have assumed source is grounded
- What if source > 0?
 - e.g. pass transistor passing V_{DD}

$$\Box$$
 $V_g = V_{DD}$

$$-$$
 If $V_s > V_{DD}-V_t$, $V_{gs} < V_t$

- Hence transistor would turn itself off
- □ nMOS pass transistors pull no higher than V_{DD}-V_{tn}
 - Called a degraded "1"
 - Approach degraded value slowly (low l_{ds})
- pMOS pass transistors pull no lower than V_{tp}
- ☐ Transmission gates are needed to pass both 0 and 1

Pass Transistor Ckts

$$V_{DD} \perp V_{S} = V_{DD} - V_{tn}$$

$$V_{DD}$$
 V_{DD} V_{DD}
 V_{DD} V_{DD}
 V_{DD} V_{DD}
 V_{DD} V_{DD}

$$V_s = |V_{tp}|$$
 V_{SS}

$$V_{DD} \perp V_{DD} - V_{tn}$$
 $V_{DD} \perp V_{DD} - 2V_{tr}$

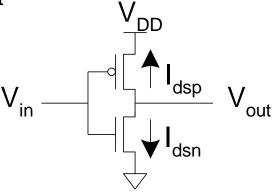
DC Response

- ☐ DC Response: V_{out} vs. V_{in} for a gate
- Ex: Inverter

- When
$$V_{in} = 0$$
 -> $V_{out} = V_{DD}$

- When
$$V_{in} = V_{DD}$$
 -> $V_{out} = 0$

- In between, V_{out} depends on transistor size and current
- By KCL, must settle such that $I_{dsn} = |I_{dsp}|$
- We could solve equations
- But graphical solution gives more insight

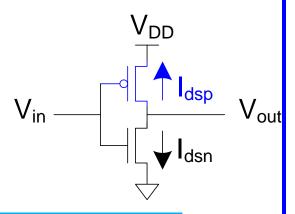


pMOS Operation

For PMOS: $V_{tp} < 0$

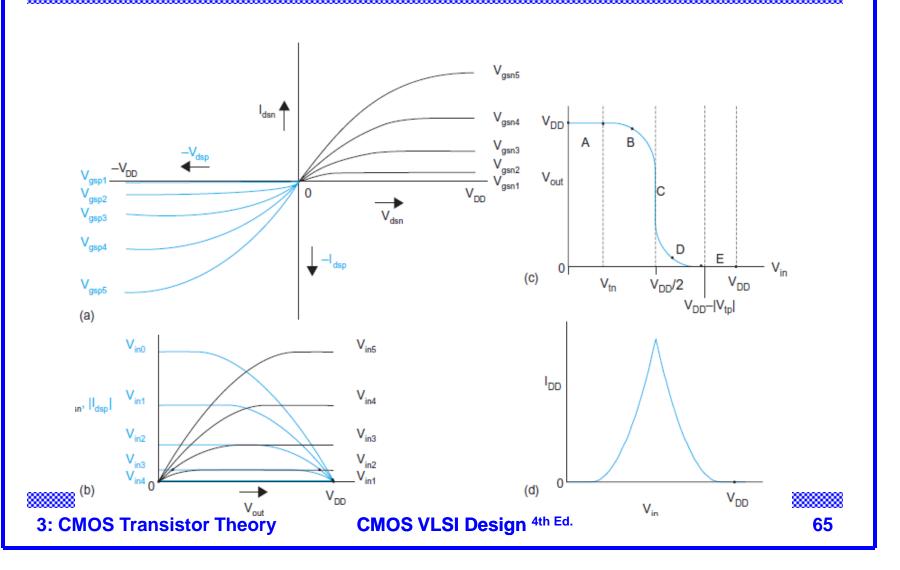
$$V_{\text{gsp}} = V_{\text{in}} - V_{\text{DD}}$$
 , $V_{\text{dsp}} = V_{out} - V_{DD}$

For NMOS: $V_{gsp} = V_{in}$, $V_{dsp} = V_{out} - V_{DD}$



	Cutoff	Linear	Saturated
nMOS	$V_{gsn} < V_{tn}$	$V_{gsn} > V_{tn}$	$V_{gsn} > V_{tn}$
	$V_{\rm in} < V_{tn}$	$V_{\rm in} > V_{tn}$	$V_{\rm in} > V_{tn}$
		$V_{dsn} < V_{gsn} - V_{tn}$	$V_{dsn} > V_{gsn} - V_{tn}$
		$V_{\rm out} < V_{\rm in} - V_{tn}$	$V_{\rm out} > V_{\rm in} - V_{tn}$
pMOS	$V_{gsp} > V_{tp}$	$V_{gsp} < V_{tp}$	$V_{gsp} < V_{tp}$
	$V_{\rm in} > V_{tp} + V_{DD}$	$V_{\rm in} < V_{tp} + V_{DD}$	$V_{\rm in} < V_{tp} + V_{DD}$
		$V_{dsp} > V_{gsp} - V_{tp}$	$V_{dsp} < V_{gsp} - V_{tp}$
		$V_{\rm out} > V_{\rm in} - V_{tp}$	$V_{\rm out} < V_{\rm in} - V_{tp}$

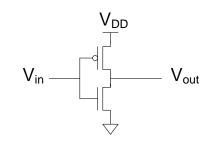
DC Characteristic

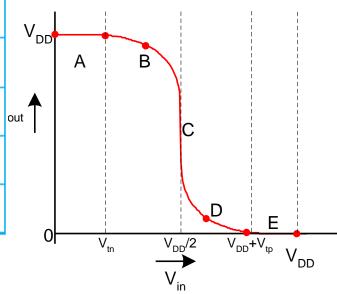


Operating Regions

- □ Revisit transistor operating regions
- □ Define input threshold (Vinv) as when Vinv=Vin=Vout

Region	Condition	p-device	n-device	Output
A	$0 \le V_{\text{in}} < V_{tn}$	linear	cutoff	$V_{\text{out}} = V_{DD}$
В	$V_{tn} \le V_{\rm in} < V_{DD}/2$	linear	saturated	$V_{\rm out} > V_{DD}/2$
С	$V_{\rm in} = V_{DD}/2$	saturated	saturated	$V_{ m out}$ drops sharply
D	$V_{DD}/2 < V_{\text{in}} \le V_{DD} - V_{tp} $	saturated	linear	$V_{\rm out} < V_{DD}/2$
Е	$V_{\rm in} > V_{DD} - V_{tp} $	cutoff	linear	$V_{\text{out}} = 0$





Calculate V_{inv}

$$I_{dn} = \frac{\beta_n}{2} (V_{inv} - V_{fn})^2$$

$$I_{dp} = \frac{\beta_p}{2} (V_{inv} - V_{DD} - V_{fp})^2$$
(2.54)

By setting the currents to be equal and opposite, we can solve for $V_{
m inv}$ as a function of r:

$$V_{\text{inv}} = \frac{V_{DD} + V_{tp} + V_{tn} \sqrt{\frac{1}{r}}}{1 + \sqrt{\frac{1}{r}}}$$

$$r = \beta_{p} / \beta_{n}$$
(2.55)

In the limit that the transistors are fully velocity saturated, EQ (2.29) shows

$$\begin{split} I_{dn} &= W_n C_{\text{ox}} v_{\text{sat-}n} (V_{\text{inv}} - V_{tn}) \\ I_{dp} &= W_p C_{\text{ox}} v_{\text{sat-}p} (V_{\text{inv}} - V_{DD} - V_{tp}) \end{split} \tag{2.56}$$

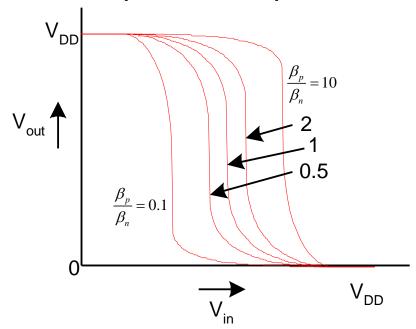
Redefining $r = W_p v_{\text{sat-}p} / W_n v_{\text{sat-}n}$, we can again find the inverter threshold

$$V_{\text{inv}} = \frac{V_{DD} + V_{tp} + V_{tn} \frac{1}{r}}{1 + \frac{1}{r}}$$
 (2.57)

In either case, if $V_{tn} = -V_{tb}$ and r = 1, $V_{inv} = V_{DD}/2$ as expected.

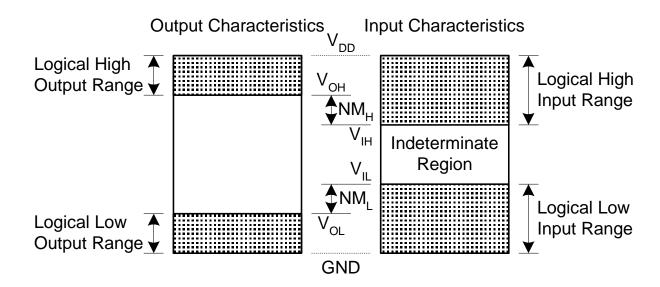
Beta Ratio

- □ If $\beta_p / \beta_n \neq 1$, switching point will move from $V_{DD}/2$
- ☐ Called *skewed* gate
- Other gates: collapse into equivalent inverter



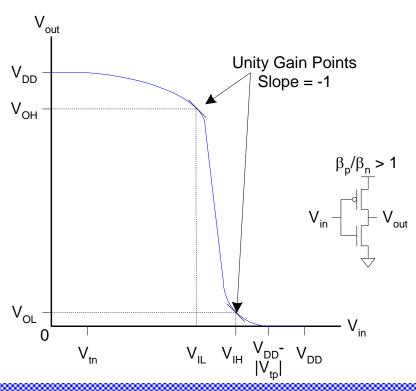
Noise Margins

☐ How much noise can a gate input see before it does not recognize the input?



Logic Levels

- □ To maximize noise margins, select logic levels at
 - unity gain point of DC transfer characteristic



$$NM_H = V_{OH} - V_{IH}$$

 $NM_L = V_{IL} - V_{OL}$